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(18) remember K-set covering problem defined in ex: 15.

Formulation:

Let $X_S \begin{cases} 1 \text{ if set } S \text{ belongs to the cover} \\ 0 \text{ otherwise} \end{cases}$

$$\text{minimize} \quad \sum_{\forall S} X_S \cdot c(S)$$

restrictions : for all $e \in \mathcal{U}$: $\sum_{S: e \in S} X_S \geq 1$

in IP $\rightarrow X_S \in \{0,1\}$
in LP $\rightarrow X_S \in [0,1]$

Notation:

X'_β : LP optimum solution in iteration β

$X'_{\beta i}$: Value of the variable indicating if set i is in the set cover or not, in X'_β.
(LP) ~~iteration β~~.

OUT : output of the algorithm

IPOPT : optimum solution in IP

algorithm

while $U \neq \emptyset$ do
  compute optimum basic LP solution x'
  Choose i with $x'_i \geq \frac{1}{K}$
  Buy set S_i, delete elements in S_i from instance
end while
output bought sets

* In each of the iterations:

$$c(X'_\beta) - c(X'_{\beta+1}) \geq c(S_{\beta i}) \cdot \frac{1}{K},$$

because a solution with the same values can always be constructed, since we have removed all elements in $S_{\beta i}$. The new solution $X'_{\beta+1}$ will have at most cost $c(X'_\beta) - \frac{c(S_{\beta i})}{K}$.

* At the end of the algorithm with m iterations:

$$\underbrace{\sum_{\beta=0}^{m}\left(c(X'_\beta) - c(X'_{\beta+1})\right)}_{\text{telescoping sum}} \geq \frac{\overbrace{\sum c(S_{\beta i})}^{c(OUT)}}{K} \Rightarrow c(X'_1) - c(X'_m) \geq \frac{c(OUT)}{K} \Rightarrow$$

$$\Rightarrow c(IPOPT) \geq c(X'_1) \geq c(X'_1) - c(X'_m) \geq \frac{c(OUT)}{K} \Rightarrow \boxed{c(IPOPT) \geq \frac{c(OUT)}{K}}$$

Therefore, this algorithm gives a K-approximation. :)