Questions

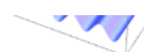
What is Interference of Light? Explain the constructive interference and destructive interference

Interference: The phenomenon of redistribution of energy due to super position of light waves from two coherent sources is called interference.

Constructive Interference: In constructive interference the amplitude of the resultant wave is greater than that of either individual wave.

Destructive Interference: In destructive interference the amplitude of the resultant wave is less that that of either individual wave.

State the Conditions for steady Interference Pattern



Necessary conditions for Interference: „

- The sources must be monochromatic.
- Two coherent sources are required.
- There must be a phase difference or path difference between the two waves.
- The amplitude of the waves must be same.
- The distance between the two coherent sources of light must be small as possible.
- The two waves must travel with the same velocity.

What are the coherent sources?

Coherent sources: Two sources of light are said to coherent if they emit light waves of the same frequency, nearly the same amplitude and always have a constant phase difference between them. Therefore, two sources must emit radiations of the same wavelength/colour.

What is the Importance of Young's Experiment?

Young's Experiment

In 1801, Young devised and performed an experiment to measure the wavelength of light. Thomas Young recognized that if light behaved like a wave, it would be possible to create patterns of constructive and destructive interference using light. He devised an experiment that would force two beams of light to travel different distances before interfering with each other when they reached a screen.

Write down the Conditions for bright band and dark band in terms of path difference

Special cases:

(i) When the phase difference

$\delta = 0, 2\pi, 2(2\pi), \ldots\ldots\ldots\ldots n(2\pi)$ *or* the path difference $x = 0, \lambda, 2\lambda \ldots\ldots\ldots n\lambda$

$$I = 4a^2$$

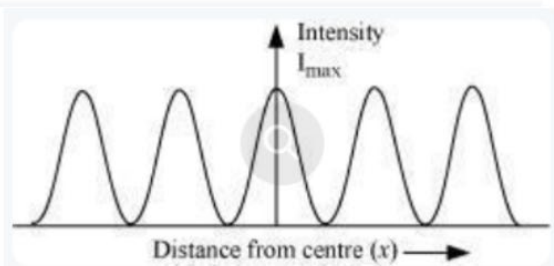
Intensity is maximum when the phase difference is a whole number multiple of 2π or the path difference is a whole number multiple of wavelength.

(ii) When the phase difference

$\delta = \pi, 3\pi, \ldots\ldots\ldots\ldots (2n+1)\pi$ *or* the path difference $x = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots\ldots\ldots (2n+1)\frac{\lambda}{2}$

$$I = 0$$

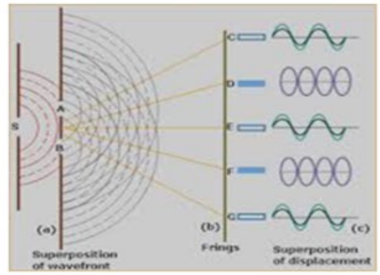
Intensity is minimum when the phase difference is a odd number multiple of half wavelength.



Derive the equation of resultant intensity due to interference of light waves.

Young's Double Slit Experiment: Intensity Distribution
(Conditions for Constructive and Destructive Interference)

Consider a monochromatic source of light S emitting waves of wavelength λ and two narrow pin holes A and B. A and B are equidistant from S and act as two virtual sources. Let $\boldsymbol{a}$ be the amplitude of the waves. The phase difference between the two waves reaching the point P, at any instant is δ.



If y_1 and y_2 are the displacements

$$y_1 = a\sin\omega t$$

$$y_2 = a\sin(\omega t + \delta)$$

$$y = y_1 + y_2 = a\sin\omega t + a\sin(\omega t + \delta)$$

$$= a\sin\omega t + a\sin\omega t\cos\delta + a\cos\omega t\sin\delta$$

$$= a\sin\omega t(1+\cos\delta) + a\cos\omega t\sin\delta$$

Taking $\quad a(1+\cos\delta) = R\cos\theta \quad$ ..... (1)

and $\quad a\sin\delta = R\sin\theta \quad$ ..... (2)

$$y = R\sin\omega t\cos\theta + R\cos\omega t\sin\theta$$

We get $\quad y = R\sin(\omega t + \theta) \quad$ ..... (3)

Which represents the equation of simple harmonic vibration of amplitude R.
Squaring and adding equation (1) and (2)

$$R^2 = a^2\sin^2\delta + a^2(1+\cos\delta)^2$$

$$= 2a^2 + 2a^2\cos\delta$$

$$= 2a^2(1+\cos\delta)$$

$$= 4a^2\cos^2\frac{\delta}{2}$$

The intensity at a point is given by the square of the amplitude

$\therefore \quad I = R^2$

or $\quad I = 4a^2\cos^2\dfrac{\delta}{2}$

Since $\boldsymbol{I_0} \propto \boldsymbol{a^2}$, $\quad I = 4I_0 Cos^2\left(\dfrac{\delta}{2}\right)$

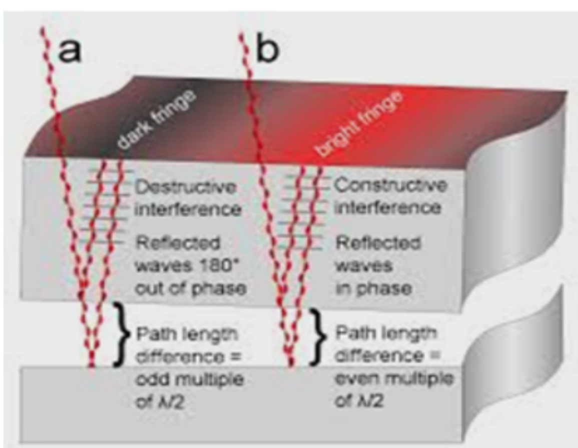
What is Newton’s ring? Find out the radius of curvature of the lens and also derive the equation for the radius of dark and bright rings.

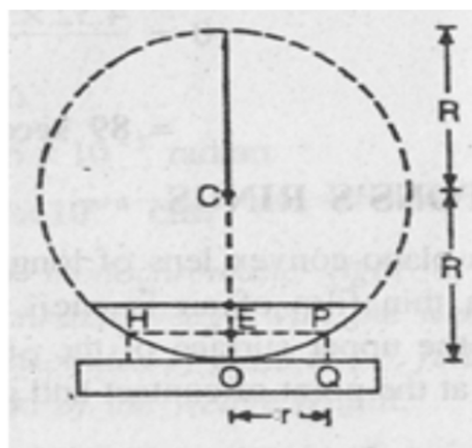
Newton's rings

Newton’s ring is a noteworthy illustration of the interference of light waves reflected from the opposite surfaces of the thin film of variable thickness.

Formation of Newton’s ring by reflected light

Suppose the radius of curvature of the lens is R and the air film is of thickness t at a distance of OQ = r, from the point of contact O. Here interference is due to reflected light. Therefore for bright rings, the optical path difference (*optical path is the product of geometrical distance and refractive index of the medium and in a given time, light travels the same optical path in different media*)





$$2\mu t\cos\theta = (2n-1)\frac{\lambda}{2} \qquad \text{……….. (1)}$$

Where n = 1, 2, 3, ………..etc. *[When light is reflected from the surface of an optically denser medium, a phase change π equivalent to a path difference $\frac{\lambda}{2}$ occurs.]*

Here, θ is small, therefore $\cos\theta = 1$ and for air, $\mu = 1$

$$\therefore\ 2t = (2n-1)\frac{\lambda}{2} \qquad \text{……….. (ii)}$$

For dark rings, $2\mu t\cos\theta = n\lambda$

Or $2t = n\lambda$, where n = 0, 1, 2, 3, ……… etc. ……….. (iii)

From the figure, $HE \times EP = OE \times (2R - OE)$

But $HE = EP = r,\ \ OE = PQ = t$

$\therefore\ 2R - t = 2R$ (approximately)

$r^2 = 2R.t$

or $t = \frac{r^2}{2R}$

Substituting the value of ***t*** in equation (ii) and (iii),

For n^{th} dark rings, $r = \sqrt{n\lambda R}$ ……….... (iv)

or $D_n^2 = 4n\lambda R$ ……….... (v)

and for $(n+m)^{th}$ dark rings, $D_{n+m}^2 = 4(n+m)\lambda R$ ……….... (vi)

Therefore $D_{n+m}^2 - D_n^2 = 4(n+m)\lambda R - 4n\lambda R$ ……….... (vii)

Sothat the radius of curvature of the lower surface of the plano-convex lens $R = \frac{D_{n+m}^2 - D_n^2}{4m\lambda}$

For n^{th} bright rings, $r = \sqrt{\frac{(2n-1)\lambda R}{2}}$ ……….... (viii)

And similarly, the radius of curvature of the lower surface of the plano-convex lens can be proved as $R = \frac{D_{n+m}^2 - D_n^2}{4m\lambda}$

Problems on interference of tight

(1) In a Young's Slits experiment, the separation between the first and fifth bright fringe is 2.5mm when the wavelength used is 6.2x10^{-7}m. The distance from the slits to the screen is 0.80m. Calculate the separation of the two slits

Ans: We have $x = \frac{\lambda D}{d}$, where d is the slit separation

$4x = 2.5 \times 10^{-3}$ m

$$d = \frac{\lambda D}{x} = \frac{6.2\times10^{-7}\times0.8}{25\times10^{-3}/4} = 8\times10^{-4}m$$

(2). In an experiment with Newton's rings, using reflected light, the diameters of two consecutive rings are 2cm and 2.02cm; what is the radius of curvature of the lens surface in contact with plane glass? (λ for light used 5897 Å)

Ans: we know = $\lambda = \frac{d^2 n+m - d^2 n}{4Rm}$

$$R = \frac{(2.02)^2 - 2^2}{4\times5897\times10^{-8}\times1} = 340.4cm$$

- In a Newton's rings experiment the diameter of the 15th ring was found to be 0.590cm and that of the 5th ring was 0.336cm. If the radius of the piano-convex lens is 100cm, Calculate the wavelength of light used.

  Ans: D_5 = 0.36 cm, D_{15} = 0.590 cm, R = 100 cm, m =10

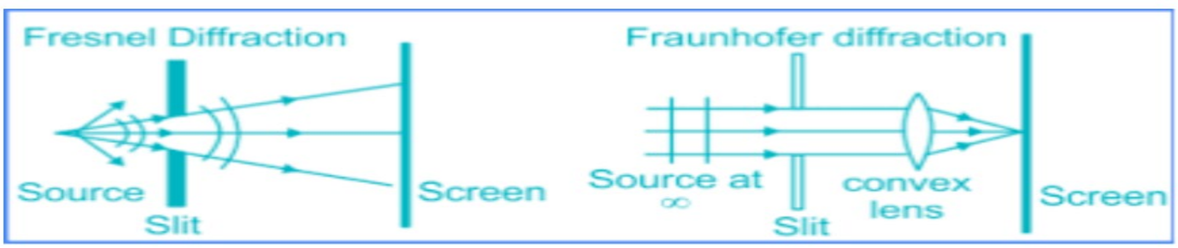
$$\lambda = \frac{D^2 n+m - D^2 n}{4Rm} = \frac{(0.590)^2 - (0.336)^2}{4\times100\times10} = 5880\times10^{-8}cm$$

What is diffraction of light?

DIFFRACTION of Light

Diffraction refers to a phenomenon that occurs when a wave encounters an obstacle or opening. It is defined as the bending of waves around the corners of an obstacle or through an aperture. The diffracting object or aperture effectively becomes a secondary source of the propagating wave.

What are the differences in between the Fresnel and Fraunhofar types of diffraction?



The basic difference between **Fresnel and Fraunhofer diffraction**

Fresnel Diffraction	Fraunhofer diffraction
1: If the source of light and screen is at a finite distance from the obstacle, then the diffraction called Fresnel diffraction.	1:If the source of light and screen is at infinite distance from the obstacle then the diffraction is called Fraunhofer diffraction
2:The corresponding rays are not parallel.	2:The corresponding rays are parallel.
3:The wave fronts falling on the obstacle are not plane.	3:The wave fronts falling on the obstacle are planes.
4.Incident wave fronts are spherical. 5. No lens is required	4.Incident wave fronts on the diffracting obstacle are plane. 5. Lens is required

What is the difference in between Interference and diffraction?

Distinguish between interference and diffraction

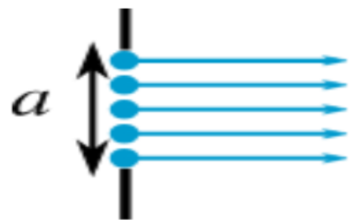
Interference	Diffraction
Interference pattern is result of superposition of light coming from two different wave fronts of two coherent sources.	Diffraction pattern is result of superposition of light coming from different parts of the same wave front.
Interference fringes are of equal width.	Diffraction fringes have unequal width.
In interference pattern all bright bands have same intensity.	In diffraction pattern all bright bands are not of same intensity. Central maximum has highest intensity.
In Interference pattern dark band is perfectly dark.	In diffraction pattern dark band is not perfectly dark.

Discuss about the Fraunhofer diffraction due to a single slit.

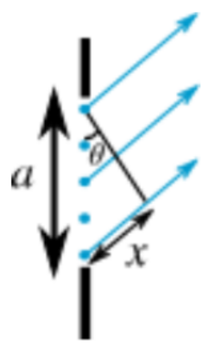
Diffraction is the spreading out of waves as they pass through an aperture or around objects. It occurs when the size of the aperture or obstacle is of the same order of magnitude as the wavelength of the incident wave.

In an aperture with width smaller than the wavelength, the wave transmitted through the aperture spreads all the way round and behaves like a point source of waves

The diffraction pattern made by waves passing through a slit of width a (larger than λ) can be understood by imagining a series of point sources all in phase along the width of the slit. The waves moving directly forward are all in phase (they have zero path difference), so they form a large central maximum.



If the wave travels at an angle θ from the normal to the slit, then there is a path difference x between the waves produced at the two ends of the slit.



If the path difference between the top and middle waves is $\lambda/2$, then they are exactly out of phase and cancel each other out. at this angle, so there is no resultant wave at this angle. Thus, a minimum in the diffraction pattern is obtained. **Diffraction gratings** are formed by large numbers of equally spaced slits or lines that diffract the light falling on them.

Solved Examples

1. Fraunhofer diffraction at a single slit is performed using a 700 nm light. If the first dark fringe appears at an angle 30°, find the slit width.

Solution: Using the diffraction formula for a single slit of width a, the nth dark fringe occurs for,

$a \sin\theta = n\lambda$

At angle $\theta = 30°$, the first dark fringe is located. Using $n = 1$ and $\lambda = 700$ nm $= 700 \times 10^{-9}$ m,

$a \sin 30° = 1 \times 700 \times 10^{-9}$ m

$a = 14 \times 10^{-7}$m

$a = 1400$ nm

The slit width is 1400 nm.

- A screen is placed at a distance 40cm. from a narrow slit illuminated by a parallel beam of light of wave length 5896 Å. Calculate the width of the slit if the first dark line lies 2 mm. on either side of the central maximum.

We have, $X_n = \dfrac{nf.\lambda}{a}$

Hence, n = 1, X_1 = 0.2 cm; $\lambda = 5896 \times 10^{-8}$ cm and f = 40 cm

$$\therefore \quad 0.2 = \frac{1 \times 40 \times 5896 \times 10^{-8}}{a}$$

$$\text{or} \quad a = \frac{40 \times 5896 \times 10^{-8}}{0.2} = 0.012 \text{ cm}$$

- In Fraunhofer diffraction due to a narrow slit, a screen is placed 2 m away from the lens to obtain the pattern. If the slit width is 0.2 mm and the first minima lie 5 mm on either side of the central maximum, find the wavelength of light.

We have, $X_n = \dfrac{nf.\lambda}{a}$

Hence, n = l, X_1 = 5mm = 0.5 cm; f = 2 m = 2 × 100 = 200 cm, and a = 0.2 mm = 0.02 cm

$$\therefore \; 0.5 = \frac{2 \times 100 \times \lambda}{0.02}$$

$$\therefore \lambda = \frac{0.02 \times 0.5}{100 \times 2} \; 5 \text{ x } 10^{-5} \text{ cm} = 5000 \text{ Å}$$

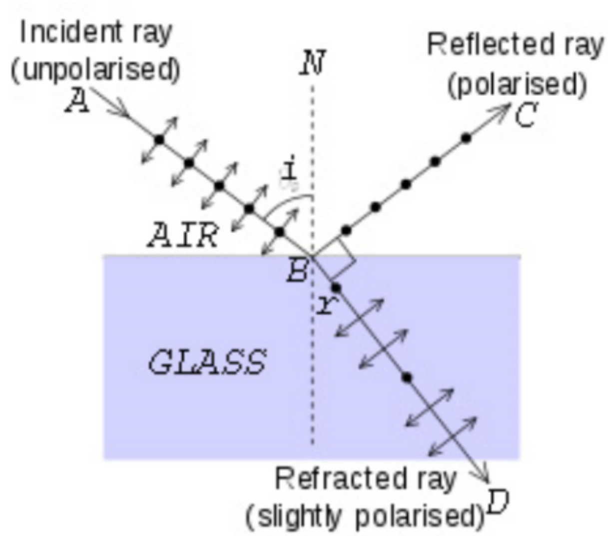
Using Brewster's law, show that at the angle of polarization, the angle between the reflected and refracted ray is 90°.

Brewster's Law

In 1811, Brewster found that ordinary light is completely polarized in the plane of incidence when it gets reflected from a transparent medium at a particular angle known as the angle of polarization. He was also able to prove that the tangent of the angle of polarization is numerically equal to the refractive index of the medium. Moreover, the reflected and refracted rays are perpendicular to each other

Brewster's Law

Suppose that unpolarized light is incident at an angle equal to the polarizing angle on the glass surface. It is reflected along BC and refracted along BD.



From Snell's law, $\mu = \dfrac{\sin i}{\sin r}$ .... (1)

From Brewster's law, $\mu = \tan i = \dfrac{\sin i}{\cos i}$ .... (2)

Comparing (1) and (2) $\cos i = \sin r = \cos\left(\dfrac{\pi}{2} - r\right)$

$$i = \frac{\pi}{2} - r \quad or \quad i + r = \frac{\pi}{2}$$

As $i + r = \dfrac{\pi}{2}$, $\angle CBD$ is also equal to $\dfrac{\pi}{2}$. Therefore, the reflected and refracted rays are at right angles to each other.

It is clear that the light vibrating in the plane of incidence is not reflected along *BC*. The refracted ray will have both the vibrations *(i)* in the plane of incidence and *(ii)* at right angles to the plane of incidence.

Discuss about quarter and half waveplate.

Half wave plate: When the thickness of crystal is such that path difference between E-ray and O-ray is $\lambda/2$ or phase difference is π then wave plate is known as half wave plate.

If the thickness of the plate is t and the refractive indices for the ordinary and extraordinary rays are μ_O and μ_E respectively, then

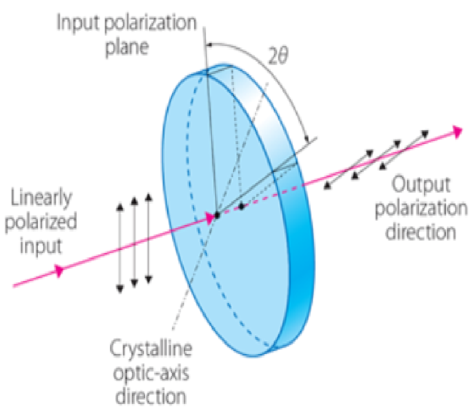
For negative crystals, path difference $=(\mu_O-\mu_E)t$

For positive crystals, path difference $=(\mu_E-\mu_O)t$

Therefore in calcite, $(\mu_O-\mu_E)t=\dfrac{\lambda}{2}$

Or $$t=\frac{\lambda}{2(\mu_O-\mu_E)}$$

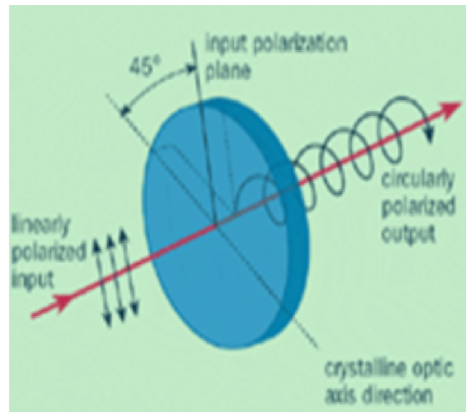
And in quartz, $$t=\frac{\lambda}{2(\mu_E-\mu_O)}$$



Performance of half (λ/2) waveplate

Quarter wave plate: When the thickness of crystal is such that path difference between E-ray and O-ray is λ/4 or phase difference is π/2 then wave plate is known as quarter wave plate. A quarter-waveplate can also be used to create plane-polarization from circular polarization

If the thickness of the plate is ***t*** and the refractive indices for the ordinary and extraordinary rays are μ_O and μ_E respectively, then



For negative crystals, path difference $=\left(\mu_O-\mu_E\right)t$

For positive crystals, path difference $=\left(\mu_E-\mu_O\right)t$

Therefore in calcite, $\left(\mu_O-\mu_E\right)t=\dfrac{\lambda}{4}$

Or $$t=\frac{\lambda}{4\left(\mu_O-\mu_E\right)}$$

And in quartz, $$t=\frac{\lambda}{4\left(\mu_E-\mu_O\right)}$$

Problem:

(1) The refractive index for plastic is 1.25. Calculate the angle of refraction for a ray of light inclined as polarizing angle.

Solution. From Brewster's law we have

Here, $\mu = \tan i = 1.25$

$\therefore \quad i = \tan^{-1}(1.25) = 51.4°$

Now, $i + r = 90° \quad \therefore \quad r = 90° - 51.4° = 38.6°$

(2) The critical angle for certain wavelength of light in the case of a piece of glass is 40°. Find the polarizing angle for glass.

Solution. We have critical angle, $C = 40°$

Refractive index for glass, $\mu = 1/\sin C = 1/\sin 40° = 1/0.6427 = 1.55$

From Brewster's law, $\mu = \tan i$, 0r $i = \tan^{-1} \mu = \tan^{-1}(1.55) = 57.3°$

Example 12 Calculate the thickness of the mica sheet required to make a quarter-wave plate and a half-wave plate for λ=5460 Å. The indices of refraction for the ordinary and extraordinary waves in mica are 1.586 and 1.592 respectively.

Solution: The data given are μ_o=1.586, μ_e=1.592, λ=5460× 10^{-8}cm.
Thickness for the quarter- wave plate in the positive crystals is given by

$$t_4 = \frac{1}{\mu_e - \mu_0}\left(\frac{\lambda}{4}\right)$$

or

$$t_4 = \frac{1}{1.592 - 1.586}\left(\frac{5460\times10^{-8}}{4}\right) = 2.275\times10^{-3}\ \text{cm}.$$

Thickness for the half-wave plate in positive crystals is

$$t_2 = \frac{1}{\mu_e - \mu_0}\left(\frac{\lambda}{2}\right)$$

$$t_2 = \frac{1}{1.592 - 1.586}\left(\frac{5460\times10^{-8}}{2}\right) = 4.55\times10^{-3}\ \text{cm}.$$

Example 12.1: Plane polarized light is incident on a piece of quartz cut parallel to the axis. Calculate the least thickness for which the ordinary ray and the extraordinary ray combine to form the pane polarized light. Given $\mu_o = 1.5442$, $\mu_e = 1.5533$, and $\lambda = 5\times10^{-5}$cm.

Solution: The ordinary ray and the extraordinary ray combine to form the plane polarized light on emergence if the plate introduce a phase difference of π or a path difference of $\lambda/2$ between the ordinary ray and the extraordinary ray. The plate, which introduces a phase difference of π or path difference of $\lambda/2$ between the ordinary ray and extraordinary ray, is the half wave plate.

The data given are μ_o=1.5442, μ_e=1.5533, λ=5×10^{-5}cm

Here,
$$t = \frac{1}{(\mu_e - \mu_o)}\left(\frac{\lambda}{2}\right)$$

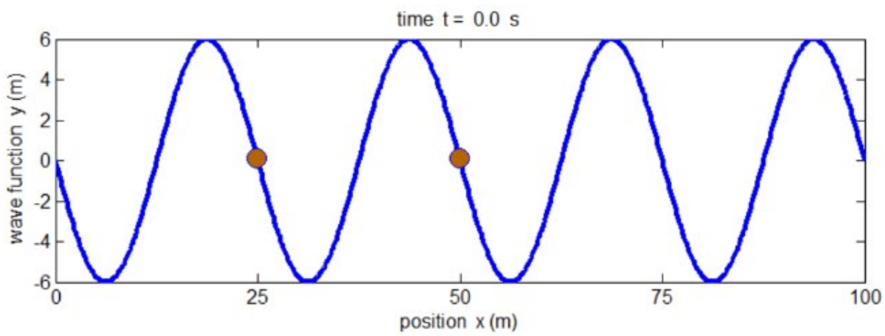
is the least thickness of a half wave plate in positive crystals

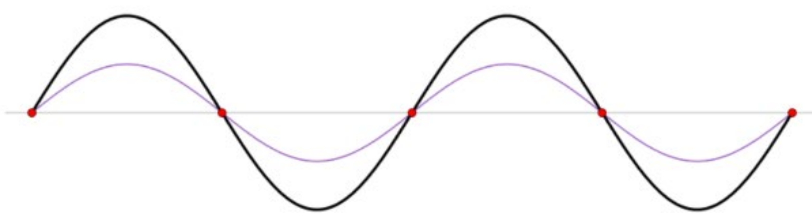
Therefore,
$$t = \frac{1}{(1.5533 - 1.5442)} \times \left(\frac{5\times10^{-5}}{2}\right) cm = 2.75\times10^{-3}\, cm$$

Discuss on the difference between stationary wave and progressive wave.

Progressive waves and Stationary waves

Progressive waves transfer energy from one place to another, without transferring the matter. Stationary waves do not transfer energy from one place to another. Besides, Progressive waves bring a net amount of energy through the path of the wave. A stationary wave does not carry net energy through the path.





Derive an equation of the total energy of a particle executing simple harmonic motion.

Simple harmonic oscillator (Finding out its differential equation and total energy)

The simple harmonic oscillator has no driving force, and no friction (damping), so the net force is just:

$$F = -kx$$

Using Newton's Second Law of motion,

$$F = ma = -kx$$

The acceleration, ***a*** is equal to the second derivative of ***x***.

$$m\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -kx$$

If we define $\omega_0{}^2 = k/m$, then the equation can be written as follows,

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + \omega_0{}^2 x = 0$$, ***this is the differential equation of simple harmonic oscillator***.

Now let us define,

$$\dot{x} = \frac{\mathrm{d}x}{\mathrm{d}t}$$

We observe that:

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = \ddot{x} = \frac{\mathrm{d}\dot{x}}{\mathrm{d}t}\frac{\mathrm{d}x}{\mathrm{d}x} = \frac{\mathrm{d}\dot{x}}{\mathrm{d}x}\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}\dot{x}}{\mathrm{d}x}\dot{x}$$

and substituting

$$\frac{\mathrm{d}\dot{x}}{\mathrm{d}x}\dot{x} + \omega_0{}^2 x = 0$$

$$\mathrm{d}\dot{x} \cdot \dot{x} + \omega_0{}^2 x \cdot \mathrm{d}x = 0$$

Integrating, $\dot{x}^2 + \omega_0{}^2 x^2 = K$ where, K is the integration constant, set $K = (A\ \omega_0)^2$

$$\dot{x}^2 = A^2{\omega_0}^2 - {\omega_0}^2 x^2$$

$$\dot{x} = \pm\omega_0\sqrt{A^2 - x^2}$$

$$\frac{\mathrm{d}x}{\pm\sqrt{A^2 - x^2}} = \omega_0 \mathrm{d}t$$

integrating, the results (including integration constant φ) are

$$\begin{cases} \arcsin\frac{x}{A} = \omega_0 t + \phi \\ \arccos\frac{x}{A} = \omega_0 t + \phi \end{cases}$$

and has the general solution

$$x = A\cos(\omega_0 t + \phi)$$

where the amplitude A and the phase ϕ are determined by the initial conditions.

Alternatively, the general solution can be written as

$$x = A\sin(\omega_0 t + \phi)$$

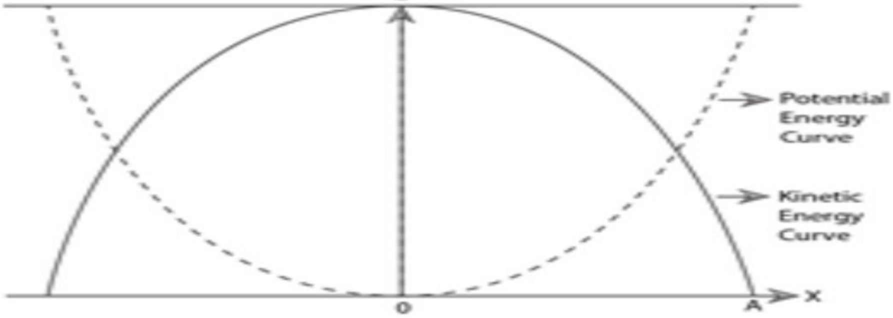
where the value of ϕ is shifted by $\pi/2$ relative to the previous form;

The kinetic energy is $$T = \frac{1}{2}m\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 = \frac{1}{2}kA^2\sin^2(\omega_0 t + \phi)$$.

and the potential energy is, $$U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\cos^2(\omega_0 t + \phi)$$

so the total energy of the system has the constant value,

$$E = \frac{1}{2}kA^2.$$



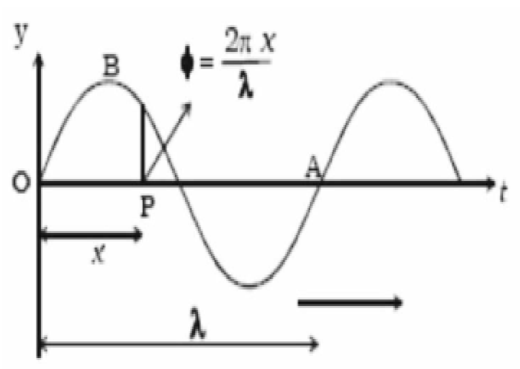
Deduce that the differential equation of the wave motion.

Write down the equation of the plane progressive wave.

Let us consider a wave moves along the positive direction of x with a velocity, 'v'. Let the displacement at any instant of time t at $x = 0$ is

$$y = A \sin \omega t \qquad (1)$$

Where 'A' is the amplitude, 'ω' is the angular frequency of the wave. Here, v be the wave velocity. We have for λ displacement, phase change is 2 π.



Let us consider a particle P at a distance x from the particle O on its right. Let the wave travel with a velocity 'v' from left to right.

Since it takes some time for the disturbance to reach P, its displacement can be written as $y = A \sin(\omega t - \phi) \qquad (2)$

Where φ is the phase difference between the particles O and P.

$$\text{Phase difference, } \varphi = \frac{2\pi x}{\lambda}$$

$$y = A\sin\left(\omega t - \frac{2\pi x}{\lambda}\right) \qquad (3)$$

$$\boxed{\text{But } \omega = \frac{2\pi}{T}}$$

$$\therefore\ y = A\sin\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right) \qquad (4)$$

$$\text{or } y = A\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) \qquad (5)$$

$$\boxed{\text{But } v = \frac{\lambda}{T} \text{ or } T = \frac{\lambda}{v}}$$

When a progressive wave travels through a medium, the displacement of a particle of the medium at any instant of time

$$\therefore\ y = A\sin 2\pi\left(\frac{vt}{\lambda} - \frac{x}{\lambda}\right)$$

$$\text{or } y = A\sin \frac{2\pi}{\lambda}(vt - x) \qquad (6)$$

$$\frac{dy}{dx} = -A\frac{2\pi}{\lambda}\cos\frac{2\pi}{\lambda}(vt - x) \qquad (7)$$

$\frac{dy}{dx}$ represents the strain or the compression. When $\frac{dy}{dx}$ is positive, a rarefaction takes place and when $\frac{dy}{dx}$ is negative, a compression takes place.

If the wave moves towards the negative direction of the x-axis, the displacement

$$\boxed{y = A\sin\frac{2\pi}{\lambda}(vt + x)}$$

We have velocity, the rate of change of displacement with respect to time. From eqn. 6

$$\therefore\ \frac{dy}{dt} = A\frac{2\pi v}{\lambda}\cos\frac{2\pi}{\lambda}(vt - x) \qquad (8)$$

Comparing equations (7) and (8) we get,

$$\frac{dy}{dt} = -v\frac{dy}{dx} \qquad (9)$$

Differentiating equation (7),

$$\frac{d^2y}{dx^2} = -A\frac{4\pi^2}{\lambda^2}\sin\frac{2\pi}{\lambda}(vt-x) \qquad (10)$$

Differentiating equation (8)

$$\frac{d^2y}{dt^2} = -A\frac{4\pi^2}{\lambda^2}v^2\sin\frac{2\pi}{\lambda}(vt-x) \qquad (11)$$

Comparing equation (10) and (11) we get,

$$\boxed{\frac{d^2y}{dt^2} = v^2\frac{d^2y}{dx^2}}$$

This is the differential equation of wave motion.