SVKM'S NMIMS

MUKESH PATEL SCHOOL OF TECHNOLOGY MANAGEMENT& ENGINEERING

Academic Year: 2022-2023

Program: B.Tech/MBATech Stream: Computer Engg./ Computer Science Year: II Semester: IV

Subject: Complex Variables and Transforms Time: 3 hrs (10:00 am to 1:00 pm)

Date: 26 / 06 / 2023. No. of Pages: 2

Marks: 100

Re-Examination (2022-23)

Instructions: Candidates should read carefully the instructions printed on the question paper and on the cover of the Answer Book, which is provided for their use.

1) Question No. _1___ is compulsory.
2) Out of remaining questions, attempt any __4___ questions.
3) In all ___5_ questions to be attempted.
4) All questions carry equal marks.
5) Answer to each new question to be started on a fresh page.
6) Figures in brackets on the right hand side indicate full marks.
7) Assume Suitable data if necessary.

Q1 CO-1; SO-1; BL-1	a.	Write down the value of $L\{t^4 H(t-2)+t^2\delta(t-2)\}$.	4
CO-1; SO-1; BL-2	b.	Find line integral $\int_{(1+i)}^{(2+4i)} (x^2+ixy)\,dz$ along the curve $x=t, y=t^2$.	4
CO-1; SO- 1; BL-1	c.	Find the fixed points of the bilinear transformation $w=(2z-5)/(z+4)$.	4
CO-1; SO- 1; BL-3	d.	Express the Fourier series to represent $f(x)=x^2$ in $(-\pi,\pi)$.	4
CO-1; SO- 1; BL-2	e.	Calculate $f(x)$ if its finite Fourier sine transform is $F_s(p)=\dfrac{16(-1)^{p-1}}{p^3}$ for $p=1,2,3,\ldots$, $0<x\le 8$.	4
Q2 CO-2; SO-1; BL-2	a.	Determine the constant a,b,c,d if $f(z)=(x^2+axy+by^2)+i(cx^2+dxy+y^2)$ is analytic.	6
CO-2; SO-1; BL-5	b.	Evaluate $\int_0^\infty e^{-t}t^3\cos t\,dt$ using Laplace transform.	6
CO-3; SO-2; BL-5	c.	Apply Cauchy Residue theorem to evaluate $\int_C \dfrac{(z+6)}{(z^2-4)}dz$ where C is the circle (i) $\lvert z\rvert=1$, (ii) $\lvert z-2\rvert=1$, (iii) $\lvert z+2\rvert=1$.	8
Q3 CO-2; SO-1; BL-2	a.	Determine the analytic function $w=u+iv$ if $v=\log(x^2+y^2)+x-2y$.	6

CO-2; SO-1; BL-6	b.	Obtain half range sine series for $f(x)=lx-x^2, 0<x\le l$.	6
CO-2; SO-2; BL-4	c.	Compute Laurentz's series expansion of $f(z)=\frac{1}{z^2+4z+3}$ about $z=0$ valid for $(i)\ 1<\|z\|<3,\ (ii)\ \|z\|>3$.	8
Q4 CO-2; SO-1; BL-3	a.	Build Fourier series for $f(x)=\begin{Bmatrix}0, & -1<x\le 0\\ x, & 0<x\le 1\end{Bmatrix}$ in terms of Legendre's polynomial $P_n(x)$ on interval $-1<x\le 1$ with $w(x)=1$.	6
CO-2; SO-1; BL-5	b.	Find the image of the infinite strip $\frac{1}{4}\le y\le\frac{1}{2}$ under the transformation $w=1/z$.	6
CO-3; SO-2; BL-5	c.	Apply Laplace transform to solve differential equation $(D^2+2D+5)y=e^{-t}\sin t$ with $y(0)=0$ and $y'(0)=1$.	8
Q5 CO-2; SO-2; BL-3	a.	Use method of partial fraction to find inverse Laplace transform of $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$.	6
CO-2; SO-1; BL-4	b.	Analyze Fourier series expansion of $f(x)=\sqrt{1-\cos x},\ 0<x\le 2\pi$.	6
CO-2; SO-1; BL-2	c.	Find the Fourier transform of $f(x)=\begin{cases}(1-x^2); & \|x\|\le 1\\ 0\ ; & \|x\|>1\end{cases}$ and hence evaluate $\int_0^\infty\left(\frac{x\cos x-\sin x}{x^3}\right)\cos\frac{x}{2}dx$.	8
Q6 CO-2; SO-2; BL-5	a.	Evaluate inverse Laplace transform of $\left[\frac{2s+3}{s^2+2s+2}\right]$.	6
CO-3; SO-2; BL-3	b.	Use Cauchy's Integral formula to evaluate $\int_C\frac{(z+3)}{(z^2+2z+5)}dz$ where C is the circle $\|z+1-i\|=2$.	6
CO-3; SO-2; BL-6	c.	Using Fourier series expansion, a boundary value problem involving the heat equation: $\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2}$; $u(x,0)=f(x),\ 0<x\le l,\ u(0,t)=u(l,t)=0$.	8
Q7 CO-2; SO-1; BL-2	a.	Illustrate inverse Laplace transform of $\left[\frac{1}{(s-2)(s+2)^2}\right]$ by convolution method.	6
CO-2; SO-1; BL-4	b.	Estimate Fourier cosine transform of $f(x)=\begin{cases}x, & 0<x\le 1/2\\ 1-x, & 1/2<x\le 1\\ 0, & x>1\end{cases}$.	6
CO-3; SO-2; BL-5	c.	Evaluate Fourier series for $f(x)=\begin{cases}x-\pi; & -\pi<x\le 0\\ \pi-x; & 0<x\le\pi\end{cases}$ and deduce that $\frac{\pi^2}{8}=\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\dots$.	8