SVKM'S NMIMS

MUKESH PATEL SCHOOL OF TECHNOLOGY MANAGEMENT& ENGINEERING

Academic Year: 2022-2023

Program: B.Tech/MBATech Stream: Computer Engg./ Computer Science Year: II Semester: IV

Subject: Complex Variables and Transforms Time: 3 hrs (10:00am to 1:00pm)

Date: 19 /April 2023. No. of Pages: 2

Marks: 100

Final Examination

Instructions: Candidates should read carefully the instructions printed on the question paper and on the cover of the Answer Book, which is provided for their use.

1) Question No. _1___ is compulsory.
2) Out of remaining questions, attempt any __4___ questions.
3) In all ____5_ questions to be attempted.
4) All questions carry equal marks.
5) Answer to each new question to be started on a fresh page.
6) Figures in brackets on the right hand side indicate full marks.
7) Assume Suitable data if necessary.

Q1 CO-1; SO-1; BL-1	a.	Write down the value of $L\{t\,H(t-4)+t^2\delta(t-4)\}$.	4
CO-1; SO-1; BL-2	b.	Find line integral $\int_{(1-i)}^{(2+i)} (2x+iy+1)\,dz$ along the curve $x=t+1, y=2t^2-1$.	4
CO-1; SO- 1; BL-1	c.	Find the fixed points of the bilinear transformation $w=(3z-5)/(z+1)$.	4
CO-1; SO- 1; BL-3	d.	Express the Fourier series to represent $f(x)=x$ in $(-\pi,\pi)$.	4
CO-1; SO- 1; BL-2	e.	Calculate $f(x)$ if its finite Fourier sine transform is $F_s(p)=\dfrac{1-\cos p\pi}{p^2\pi^2}$ for $p=1,2,3,\ldots$, $0<x\le\pi$.	4
Q2 CO-2; SO-1; BL-2	a.	Determine whether the function $f(z)=ze^z$ is analytic and if so find derivative.	6
CO-2; SO-1; BL-5	b.	Evaluate $\int_0^\infty e^{-t}t^3\sin t\,dt$ using Laplace transform.	6
CO-3; SO-2; BL-5	c.	Apply Cauchy Residue theorem to evaluate $\int_C \frac{z^2}{(z^4-1)}dz$, where C is the circle: i) $\lvert z\rvert=\frac{3}{4}$, ii) $\lvert z-1\rvert=1$, iii) $\lvert z+i\rvert=\frac{1}{2}$.	8
Q3 CO-2; SO-1; BL-2	a.	Determine the analytic function $w=u+iv$ whose real part is $e^{2x}(x\cos 2y-y\sin 2y)$.	6
CO-2; SO-1; BL-6	b.	Solve half range cosine series for $f(x)=\sin x, 0<x\le\pi$.	6

CO-2; SO-2; BL-4	c.	Compute Laurentz's series expansion of $f(z) = \frac{z-1}{z^2-2z-3}$ about $z = 0$ valid for $(i)\ \lvert z\rvert < 1,\ (ii)\ 1 < \lvert z\rvert < 3$.	8
Q4 CO-2; SO-1; BL-3	a.	Build Fourier series for $f(x) = 2x^3 + 4x^2 - x + 5$ in terms of Legendre's polynomial $P_n(x)$ on interval $-1 < x \leq 1$ with $w(x) = 1$.	6
CO-2; SO-1; BL-5	b.	Find the image of $\lvert z - 2i\rvert = 2$ under the transformation $w = 1/z$.	6
CO-3; SO-2; BL-5	c.	Apply Laplace transform to solve differential equation $(D^2 + 3D + 2)y = 2(t^2 + t + 1)$ with $y(0) = 2$ and $y'(0) = 0$.	8
Q5 CO-2; SO-2; BL-3	a.	Use method of partial fraction to find inverse Laplace transform of $\frac{5s^2 + 8s - 1}{(s+3)(s^2+1)}$.	6
CO-2; SO-1; BL-4	b.	Analyze Fourier series expansion of $f(x) = \frac{(\pi - x)^2}{4},\ 0 < x \leq 2\pi$.	6
CO-2; SO-1; BL-2	c.	Find the Fourier sine transform of $f(x) = e^{-x},\ x \geq 0$ and hence deduce that $\int_0^{\infty} \frac{x \sin mx}{1+x^2} dx = \frac{\pi}{2} e^{-m} (m > 0)$.	8
Q6 CO-2; SO-2; BL-5	a.	Evaluate inverse Laplace transform of $\left[\frac{4s+12}{s^2+8s+12}\right]$.	6
CO-3; SO-2; BL-3	b.	Use Cauchy's Integral formula to evaluate $\int_C \frac{z+2}{(z-3)(z-4)} dz$ where C is the circle $\lvert z\rvert = 3.5$.	6
CO-3; SO-2; BL-6	c.	Using Fourier series expansion, solve the heat conduction equation in one dimension $\frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2}$ with the Dirichlet boundary conditions: $T = T_1$ if $z = 0$ and $T = T_2$ if $x = L$. The initial temperature distribution is given by $T(x,0) = f(x)$.	8
Q7 CO-2; SO-1; BL-2	a.	Illustrate inverse Laplace transform of $\left[\frac{1}{(s-2)^4 (s+3)}\right]$ by convolution method.	6
CO-2; SO-1; BL-4	b.	Estimate Fourier cosine transform of $f(x) = \begin{cases} x, & 0 < x \leq 1 \\ 2-x, & 1 < x \leq 2 \\ 0, & x > 2 \end{cases}$.	6
CO-3; SO-2; BL-5 .	c.	Evaluate Fourier series of the function piecewise function $f(x) = \begin{cases} 0; & -\pi < x \leq 0 \\ \sin x; & 0 < x \leq \pi \end{cases}$ and deduce that $\frac{1}{2} = \frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \ldots$	8