

**SVKM'S NMIMS**  
**MUKESH PATEL SCHOOL OF TECHNOLOGY MANAGEMENT & ENGINEERING/**  
**SCHOOL OF TECHNOLOGY MANAGEMENT**

Academic Year: 2023-2024

Program/s: BTech/ MBA Tech/ BTech-Integrated

Year: II/IV Semester: IV/VIII

Stream/s: \_\_\_\_CS/Computer

Subject: Design and Analysis of Algorithms

Time: 3 hrs (10 am to 1 pm)

Date: 24/Apr/2024

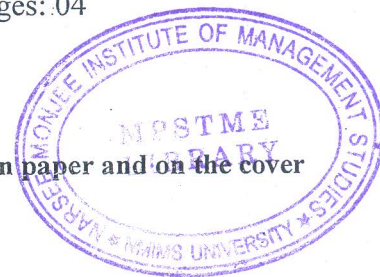
No. of Pages: 04

Marks: 100

**Final Examination/Re-Examination (2022-23)**

**Instructions: Candidates should read carefully the instructions printed on the question paper and on the cover of the Answer Book, which is provided for their use.**

- 1) Question No. 1 is compulsory.
- 2) Out of remaining questions, attempt any 4 questions.
- 3) In all 5 questions to be attempted.
- 4) All questions carry equal marks.
- 5) Answer to each new question to be started on a fresh page.
- 6) Figures in brackets on the right hand side indicate full marks.
- 7) Assume Suitable data if necessary.



<b>Q1</b>		Answer briefly:	[20]
CO- 1; SO-2; BL-1	a.	Define the term Algorithms? "The choice of algorithmic design paradigm is an important aspect of algorithm synthesis". Justify the statement by giving suitable examples.	[5]
CO-2; SO-2; BL-3	b:	Illustrate the control abstraction for Divide and Conquer algorithm. Also specify the generic complexity representation for divide and conquer based recursions.	[5]
CO- 2; SO-2; BL-2	c.	State the difference between optimal solution and feasible solution. Explain giving a specific example when Greedy paradigm does not produce an optimal solution, rather gives an approximate (near optimal) solution in reasonable time.	[5]
CO- 3; SO-1; BL-3	d.	Interpret the control abstraction of the Dynamic Programming approach.	[5]
<b>Q2</b>	a.	Differentiate between Divide and Conquer Technique and Greedy Algorithmic Paradigm on the following parameters (i) Basic Approach (ii) Optimality (iv) Problem Decomposition (iii) Typical Problems (v) Memory usage.	[5]
	b.	<p>Compute time complexity and space complexity for following function.</p> <p><b>Multiply (A,B,n)</b></p> <pre> {     for(i=0; i&lt;n; i++)     {         for(j=0; j&lt;n; j++)         {             C[i,j] = 0;              for(k=0; k&lt;n; k++)             {                 C[i,j] = C[i,j] + A[i,k] * B[k,j];             }         }     } } </pre>	[5]

	<p>c.</p> <p>CO-1; SO-1; BL-1,3</p>	<p>With reference to Asymptotic Notations</p> <p>i) Define Big-Oh, Theta and Omega Asymptotic notations.</p> <p>ii) Sketch the Worst-case time complexity relation between Linear, Logarithmic, Exponential, Log Linear, Constant, Polynomial and Factorial function.</p> <p>iii) Prove that <math>10n^2 + 4n + 2 = O(n^2)</math></p>	[10]																			
Q-3	<p>a.</p> <p>CO-1; SO-1; BL-3</p>	<p>State the Master's Theorem for Decreasing functions. Also Solve the following recurrence relation using Master's theorem-</p> $T(n) = 2T(n/4) + n^{0.51}$	[5]																			
	<p>b.</p> <p>CO-3; SO-1; BL-6</p>	<p>Consider the instance of a sum of subset problem where <math>X = \{3, 5, 6, 7\}</math> and <math>Y = \{15\}</math>. Generate state space tree to find the possible subset of X that sum to Y using Backtracking.</p>	[5]																			
	<p>c</p> <p>CO-2; SO-1; BL-1,3</p>	<p>With reference to Divide and Conquer algorithmic paradigm</p> <p>i) Use algorithmic explanation to show how Divide and Conquer technique can be used to sort an array of numbers using Merge Sort.</p> <p>ii) Derive the recurrence relation for Merge Sort using the algorithm</p> <p>Solve the recurrence relation using Master's Theorem to derive the worst time complexity of Merge Sort.</p>	[10]																			
Q-4	<p>a.</p> <p>CO-3; SO-1; BL-1,3</p>	<p>i) Solve for a minimum cost path from 's' to 't' in the multistage graph of the Figure given below. Adopt a forward approach.</p> <p>ii) State the Recurrence relation for solving the Multistage Graph problem by Forward Approach.</p>	[10]																			
	<p>b.</p> <p>CO-2; SO-2; BL-3,5</p>	<p>Assume that the numbers given below represent counts of letters in hundreds from a file. For example, in the file there will be exactly 20 * 100 occurrences of the letter 'a', 11*100 occurrence of the letter 'c', etc. accordingly the frequencies list is as follows:</p> <table><tr><td>Letter</td><td>a</td><td>c</td><td>d</td><td>e</td><td>o</td><td>m</td><td>s</td><td>t</td><td>u</td></tr><tr><td>frequency</td><td>20</td><td>11</td><td>2</td><td>10</td><td>15</td><td>8</td><td>10</td><td>22</td><td>2</td></tr></table>	Letter	a	c	d	e	o	m	s	t	u	frequency	20	11	2	10	15	8	10	22	2
Letter	a	c	d	e	o	m	s	t	u													
frequency	20	11	2	10	15	8	10	22	2													



Based on the given data perform the below mentioned task.

a) Solve for optimal Huffman code based on the following set of frequencies.

- 1) Draw the tree.
- 2) Fill in the table with the Huffman encoding for each letter.
- 3) Encode the file using the Huffman codes produced above
- 4) Calculate the total length of encoded bits using Huffman coding.

Letter	Encoding	
	Huffman	Fixed length
a		
c		
d		
e		
o		
m		
s		
t		
u		

b) Solve for Fixed Length Encoding.

- 1) Fill in the table the fixed-length encoding for each letter.
- 2 Calculate the total length of encoded bits using fixed-length coding.

Justify which of the encoding scheme is best and why?

Q5

a.  
CO-1,4;  
SO-1;  
BL-1,3

- i) State the conditions which make a decision problem belong to
- a. Class P
  - b. Class NP
  - c. Class NP -Hard
  - d. NP Complete.
- Depict the relationship between these classes using a Venn diagram.

- ii) What do you mean by Time Complexity and Space Complexity?
- Solve for the time complexity of the following code:
- ```

var a = 0, b = 0, i, j, k, N;
for (i = 0; i < N; i++) {
    for (j = 0; j < N; j++) {
        a = a + j;
    }
}
for (k = 0; k < N; k++) {
    b = b + k;
}

```

[10]

b.  
CO-3;  
SO-2;  
BL-1,3

- i) State the problem statement of Matrix chaining multiplication Problem. Also write the recurrence relation to solve Matrix chaining multiplication Problem using Dynamic programming. Consider the following 4 matrices
- A : 4 x 4  
B : 4 x 5

[10]

|    |                                |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |      |    |   |   |   |   |   |   |   |   |   |  |   |    |    |   |  |  |   |    |   |  |  |  |   |  |
|----|--------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|----|---|---|---|---|---|---|---|---|---|--|---|----|----|---|--|--|---|----|---|--|--|--|---|--|
|    |                                | <p>C : 5 x 2<br/>D : 2 x 4</p> <p>Using Dynamic Programming approach to solve Matrix chaining Multiplication problem</p> <p>i) compute the values for x, y and z in the table below. Show all the computations involved.</p> <p>ii) Derive the correct order of Matrix Multiplication to reduce the computational complexity.</p> <table><tr><td></td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>1</td><td>0</td><td>x</td><td>y</td><td>z</td></tr><tr><td>2</td><td></td><td>0</td><td>40</td><td>72</td></tr><tr><td>3</td><td></td><td></td><td>0</td><td>40</td></tr><tr><td>4</td><td></td><td></td><td></td><td>0</td></tr></table> |      | 1  | 2 | 3 | 4 | 1 | 0 | x | y | z | 2 |  | 0 | 40 | 72 | 3 |  |  | 0 | 40 | 4 |  |  |  | 0 |  |
|    | 1                              | 2                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | 3    | 4  |   |   |   |   |   |   |   |   |   |  |   |    |    |   |  |  |   |    |   |  |  |  |   |  |
| 1  | 0                              | x                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | y    | z  |   |   |   |   |   |   |   |   |   |  |   |    |    |   |  |  |   |    |   |  |  |  |   |  |
| 2  |                                | 0                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | 40   | 72 |   |   |   |   |   |   |   |   |   |  |   |    |    |   |  |  |   |    |   |  |  |  |   |  |
| 3  |                                |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      | 0    | 40 |   |   |   |   |   |   |   |   |   |  |   |    |    |   |  |  |   |    |   |  |  |  |   |  |
| 4  |                                |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |      | 0  |   |   |   |   |   |   |   |   |   |  |   |    |    |   |  |  |   |    |   |  |  |  |   |  |
| Q6 | a.<br>CO-2;<br>SO-2;<br>BL-3,5 | <p>i) Discuss the algorithm of Fractional Knapsack problem using Greedy Approach.</p> <p>ii) Find and justify the worst-case time complexity of the greedy fractional knapsack.</p> <p>iii) Apply the algorithm to solve the following scenario:<br/>Ramesh wants to buy vegetables from the market, and he can carry a maximum weight of 28 kg in his bag. What vegetables should he take if he can even take fractions of any item?<br/>Given Profit P = {9, 5, 2, 7, 6, 16, 3} and Weight W = {2, 5, 6, 11, 1, 9, 1}</p>                                                                                                                          | [10] |    |   |   |   |   |   |   |   |   |   |  |   |    |    |   |  |  |   |    |   |  |  |  |   |  |
|    | b.<br>CO-3;<br>SO-2;<br>BL-1,6 | <p>With reference to n-Queen problem, answer the following questions</p> <p>i) State the problem statement of n-queen's problem.</p> <p>ii) Explain the Backtracking paradigm of problem solving, in general.</p> <p>iii) Generate a state-space tree for the 4 queen's problem using backtracking approach.</p>                                                                                                                                                                                                                                                                                                                                     | [10] |    |   |   |   |   |   |   |   |   |   |  |   |    |    |   |  |  |   |    |   |  |  |  |   |  |
| Q7 | a.<br>CO-3;<br>SO-1;<br>BL-3   | <p>i) State the Recursive Formulation/Recurrence Relation for the Length of Longest Common Subsequence (LCS) using Dynamic Programming.</p> <p>ii) Find out the similarity between the two strings by computing the length of the longest common subsequence (LCS) between them using Dynamic Programming further, print the longest common subsequence.</p> <p>String1 = "google"</p> <p>String2 = "profile"</p>                                                                                                                                                                                                                                    | [10] |    |   |   |   |   |   |   |   |   |   |  |   |    |    |   |  |  |   |    |   |  |  |  |   |  |
|    | b.<br>CO-1;<br>SO-1;<br>BL-3   | <p>Solve the recurrence relation using both recursion tree and substitution method-</p> <p><math>T(n) = T(n-1) + \log n</math>, when <math>n &gt; 0</math></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | [10] |    |   |   |   |   |   |   |   |   |   |  |   |    |    |   |  |  |   |    |   |  |  |  |   |  |