SVKM'S NMIMS CE

MUKESH PATEL SCHOOL OF TECHNOLOGY MANAGEMENT& ENGINEERING

Academic Year: 2023-2024

Program: B.Tech (Computer Engineering/ Computer Science), MBATech(Computer)

Year: II Semester: IV

Subject: Complex Variables and Transforms

Time: 3 hrs (10:00am to 1:00pm)

Date: 6 / 7 / 24.

No. of Pages:02

Marks: 100

Re- Examination (2022-23)

Instructions: Candidates should read carefully the instructions printed on the question paper and on the cover of the Answer Book, which is provided for their use.

1) Question No. _1___ is compulsory.
2) Out of remaining questions, attempt any __4___ questions.
3) **In all ___5_ questions to be attempted.**
4) All questions carry equal marks.
5) **Answer to each new question to be started on a fresh page.**
6) **Figures in brackets on the right hand side indicate full marks.**
7) **Assume Suitable data if necessary.**

Q1			
CO-1; SO-1; BL-1	a.	Find the constants a,b,c,d if $f(z)=(x^2+2axy+by^2)+i(cx^2+2dxy+y^2)$ is analytic.	4
CO-1; SO-1; BL-2	b.	Evaluate the integral $\int_{1-i}^{2+i}(2x-iy+1)\,dz$ along the straight line joining $1-i$ and $2+i$.	4
CO-1; SO- 1; BL-1	c.	Write Laplace transform of $L\left[(1+2t-3t^2+4t^3)H(t-2)\right]$.	4
CO-1; SO- 1; BL-3	d.	Express $f(x)=x\|x\|$ as Fourier series in $(-1,1)$.	4
CO-1; SO- 1; BL-2	e.	Find the Fourier transform of $f(x)=\begin{cases} k & for\ \|x\|\le a \\ 0 & for\ \|x\|>a \end{cases}$.	4
Q2 CO-2; SO-1; BL-3	a.	Check whether the function $f(z)=ze^{z}$ is analytic or not. If yes find its derivative.	6
CO-2; SO-1; BL-2	b.	Find $L\left\{\dfrac{e^{-2t}\sin^2 3t}{t}\right\}$.	6
CO-3; SO-2; BL-3	c.	Using Cauchy residue theorem evaluate $\int_c \dfrac{z^2+2z+3}{z(z-1)(z+3)}dz$, where C is $\|z-1+i\|=4$.	8
Q3 CO-2; SO-1; BL-3	a.	Find the analytic function whose real part is $u=x^3-3xy^2+3x^2-3y^2+1$, also find its imaginary part.	6

CO-2; SO-1; BL-3	b.	Construct the complex form of Fourier series for $f(x)=\begin{cases}1, & 0<x<1\\ 0, & 1<x<2\end{cases}$.	6
CO-2; SO-2; BL-4	c.	Obtain Laurent's expansion of $f(z)=\dfrac{4z+3}{z(z-3)(z+2)}$ about $z=0$ for (i) $2<\lvert z\rvert<3$, (ii) $\lvert z\rvert>3$.	8
Q4 CO-2; SO-1; BL-2	a.	Find the bilinear transformation, which maps the points $z=1,i,-1$ into the points $w=i,0,-i$ respectively.	6
CO-2; SO-1; BL-4	b.	Prove that the functions $f_1(x)=1$ and $f_2(x)=x$ are orthogonal over $(-1,1)$. Determine the constants a and b such that the function $f_3(x)=-1+ax+bx^2$ is orthogonal to both $f_1(x)$ and $f_2(x)$ on that interval.	6
CO-3; SO-2; BL-3	c.	Using Laplace transform, solve the following differential equation $(D^2-3D+2)y=4e^{2t}$, given that $y(0)=-3$ and $y'(0)=5$.	8
Q5 CO-2; SO-1; BL-3	a.	Apply method of partial fraction to find $L^{-1}\left\{\dfrac{s}{(s-3)(s^2+9)}\right\}$.	6
CO-2; SO-1; BL-3	b.	Analyze Fourier series of $f(x)=(2\pi-x)$ in $[0,2\pi]$.	6
CO-2; SO-1; BL-2	c.	Find the finite Fourier cosine and sine transform of $f(x)=e^{-x}, 0<x<\pi$.	8
Q6 CO-2; SO-2; BL-5	a.	Evaluate the infinite integral using Laplace transform $\int_0^\infty e^{-2t}\left(\int_0^t u^2\sin 3u\,du\right)dt$.	6
CO-3; SO-2; BL-3	b.	Use Cauchy's Integral formula to evaluate $\int_c \dfrac{e^{2z}}{(z-1)(z-2)}dz$, where c is the circle $\lvert z\rvert=3$.	6
CO-3; SO-2; BL-3	c.	Using Fourier Series, solve the heat flow equation $\dfrac{\partial u}{\partial t}=c^2\dfrac{\partial^2 u}{\partial x^2}$ with respect to the boundary conditions: $u(0,t)=0$, $u(\pi,t)=0$ and $u(x,0)=(\pi x-x^2)$.	8
Q7 CO-2; SO-1; BL-3	a.	Evaluate the Laplace transform of the piecewise function $f(t)=\begin{cases}(t-1)^2 & 1<t<2\\ 0 & \text{otherwise}\end{cases}$.	6
CO-2; SO-1; BL-3	b.	Estimate the Fourier transform of the function $f(x)=\begin{cases}x^2, & \lvert x\rvert<a\\ 0, & \lvert x\rvert\ge a\end{cases}$.	6
CO-3; SO-2; BL-5	c.	Expand the function $f(x)=x\sin x$ as Fourier series in the interval $-\pi\le x\le\pi$. Hence, deduce that $\dfrac{\pi-2}{4}=\dfrac{1}{1\cdot3}-\dfrac{1}{3\cdot5}+\dfrac{1}{5\cdot7}-\dfrac{1}{7\cdot9}+\cdots$.	8