نظم القياس الإلكترونية
Electronic Measurement Systems (EMS)

كلية الهندسة الكهربائية والالكترونية – جامعة حلب
د. أسعد كعدان

المحاضرتين 5-6 – أخطاء القياس والضجيج

مصادر المحاضرة

- Electronic Instrumentation, Prof. Dr. Kofi Makinwa
  https://ocw.tudelft.nl/courses/electronic-instrumentation/
  - Lecture 3 – Errors and Noise

Deterministic Errors

Two kinds of errors (uncertainty):

a. Deterministic errors
   e.g. source loading, offset, gain error
b. Random (stochastic) errors
   e.g. thermal noise

Assuming that the output of a system *z = f(a,b,c, ...)*

Then the total error due to deteministic parameter uncertainty is:

$$\Delta z = \left|\frac{\partial f(a,b,c,\ldots)}{\partial a}\right|\Delta a + \left|\frac{\partial f(a,b,c,\ldots)}{\partial b}\right|\Delta b + \left|\frac{\partial f(a,b,c,\ldots)}{\delta c}\right|\Delta c + \ldots$$

where Δz denotes the uncertainty in z due to parameter a, which is specified with uncertainty Δa etc..

The derivatives above are called sensitivities.

Additive Errors

Example: Combining additive errors:

For $z = a + b$ or $z = a - b$ with Δa the uncertainty in a and Δb the uncertainty in b, the overall uncertainty follows as:

$$\Delta z = \left|\frac{\partial(a \pm b)}{\partial a}\right| \Delta a + \left|\frac{\partial(a \pm b)}{\partial b}\right| \Delta b = \Delta a + \Delta b$$

By extension, the effect of deterministic errors (e.g. offset, gain error etc.) in a linear time-invariant (LTI) system can be determined by superposition.

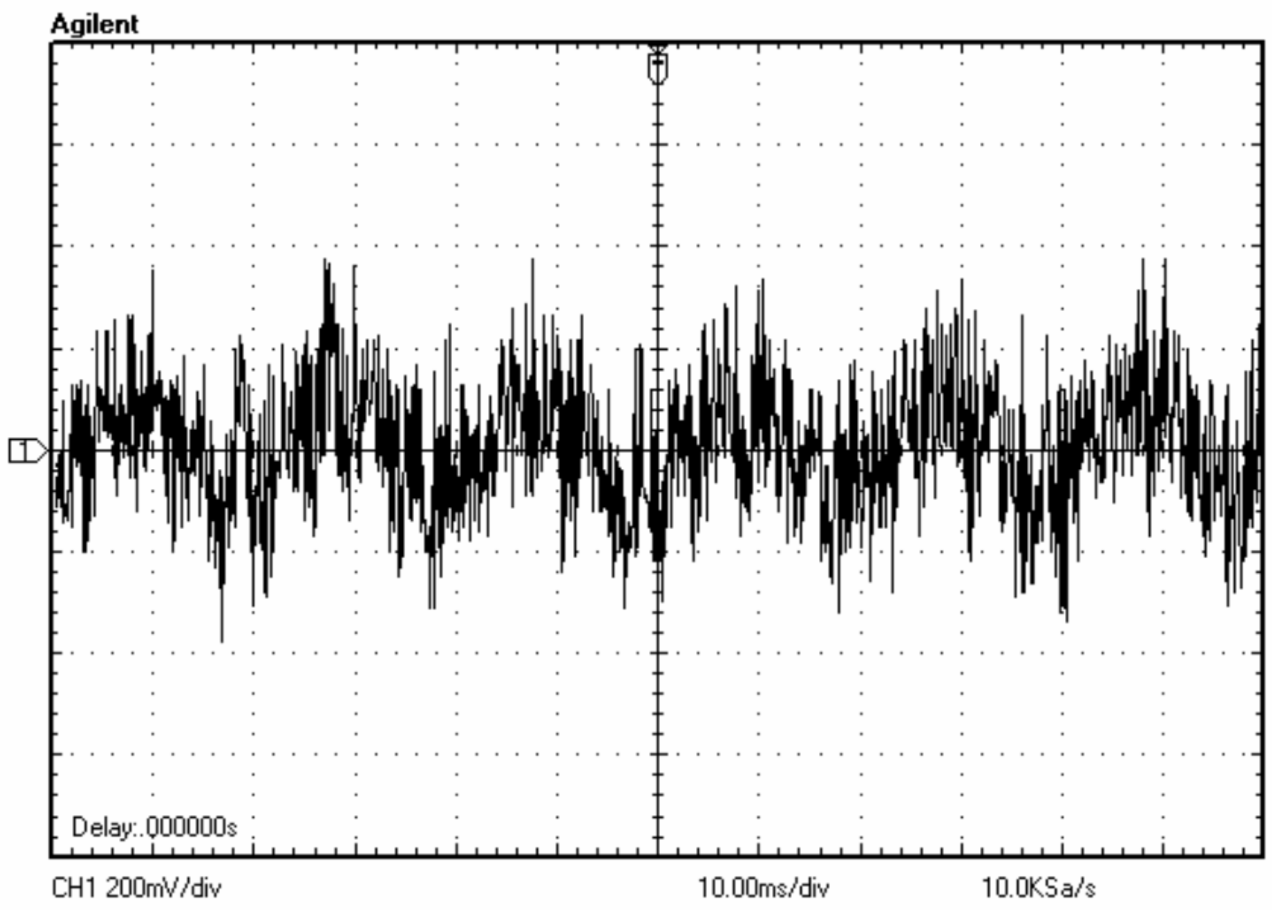
Noise, interference & distortion

- *Interference* is caused by *external* signals e.g. 50 Hz hum
- *Noise* is caused by stochastic processes in the measurement system itself e.g. thermal noise.

- Noise and interference are usually additive error signals
  $\Rightarrow$ they can both be modeled by extra error sources to the system

- Note that interference and noise should not be confused with signal *distortion*, which is caused non-linearities in either the amplitude or the frequency domain.

Noise + 50 Hz interference



Thermal noise

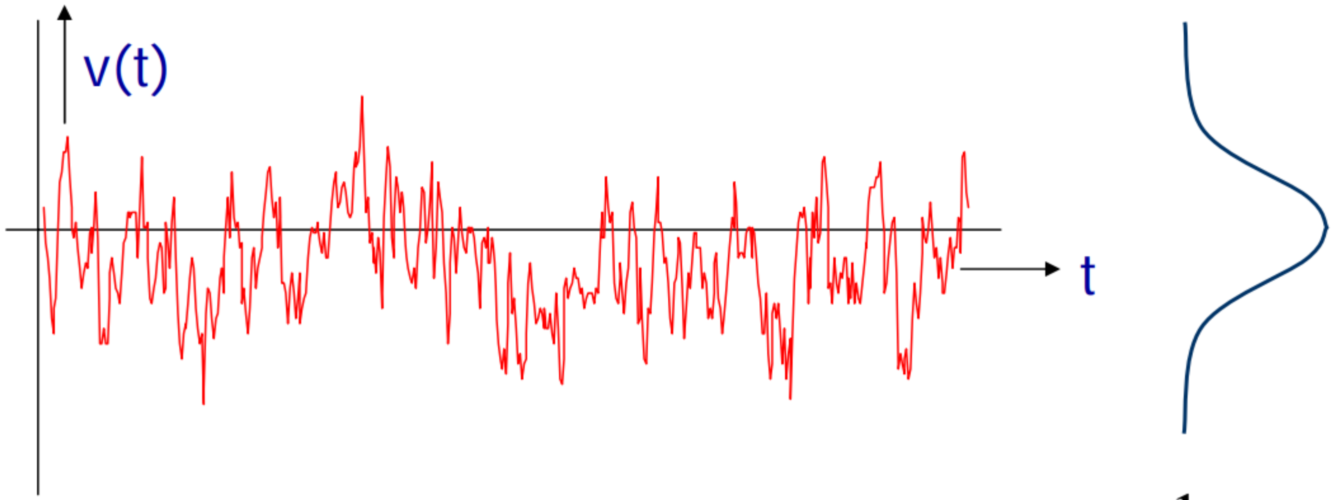
- ALL resistors exhibit thermal noise
- Thermal noise has a uniform (white) frequency spectrum
- noise power density: $P_n = 4kT$ (W/Hz)
  - where k is Boltzmann's constant and T is absolute temperature

- At room temperature (293 K): $P_n = 4kT = 1.62 \times 10^{-20}$ (W/Hz)

- For a resistor: $P = V_{rms}^2/R = I_{rms}^2 R$
- => noise voltage with spectral density: $v_n = \sqrt{(4kTR)}$ (V/Hz$^{1/2}$)
- => noise current with spectral density: $i_n = \sqrt{(4kT/R)}$ (A/Hz$^{1/2}$)

Thermal noise (1)

- (Thermal) noise is stochastic, i.e. its amplitude at a given time t cannot be predicted
- Thermal noise has a Gaussian amplitude distribution



$$p(V)\mathrm{d}V = \frac{1}{V_\mathrm{n}\sqrt{2\pi}} e^{-\left(\frac{V^2}{2V_\mathrm{n}^2}\right)}\mathrm{d}V$$

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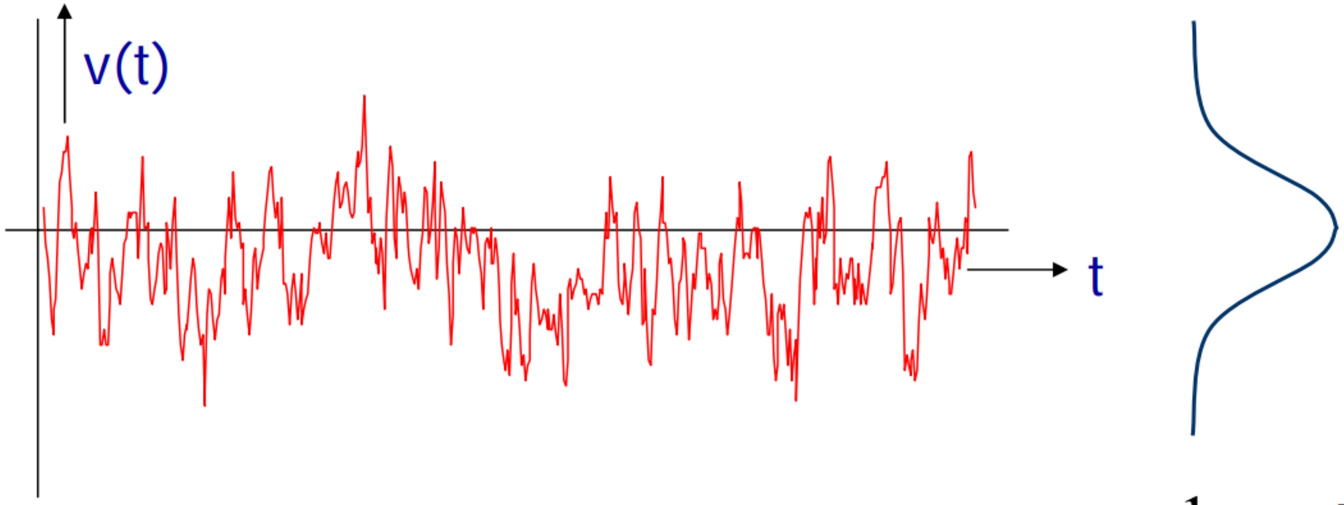
Thermal noise (2)

- In the presence of thermal noise, the detection limit is usually specified as the standard deviation (or sigma) Vn
- A good estimate of its peak value (e.g. as seen on a wide-band scope) is 5-sigma



$$p(V)\mathrm{d}V = \frac{1}{V_n\sqrt{2\pi}} e^{-\left(\frac{V^2}{2V_n^2}\right)} \mathrm{d}V$$

Shot noise

- Shot noise is the result of the fact that an electrical current is the result of the motion of discrete charge carriers.
- This can be compared with hail on a roof: even if the average current (mass flow) is constant, the amount of charge carrriers (ice lumps), measured in different intervals with the same length Δt, will, in general, be different. The relative fluctuation becomes greater as Δt becomes smaller and smaller.
- Like thermal noise, shot noise has a uniform frequency spectrum and so is also a form of white noise.
- If a current I consists of charge carriers with charge q that move independently of each other, then the spectral noise current is given by $i_n = \sqrt{(2qI)}$ $(A/Hz^{1/2})$.
- This equation applies to p-n junctions (diodes, transistors, etc.) but not to metallic conductors where there is more long-range correlation between the movement of the charge carriers. In such conductors, the shot noise will thus be smaller!

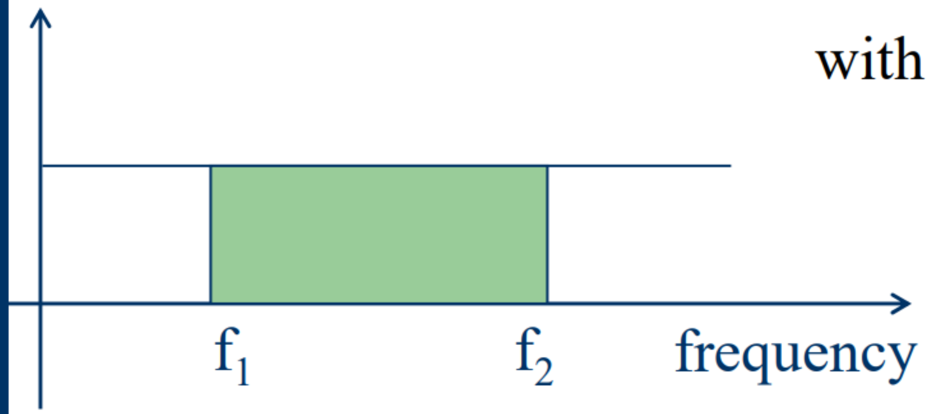
Calculating shot noise

i_n is proportional to the square-root of current!

$$i_n = \sqrt{2qI} \qquad (\mathrm{A/Hz^{1/2}})$$

$$I_n(\mathrm{rms}) = i_n\sqrt{B} = \sqrt{2qI_{DC}B} \qquad (\mathrm{A})$$

with bandwidth $B = f_2 - f_1$



Example (for B = 10 Hz):

- I = 1 A ⇒ noise current I_n(rms) = 57 nA (0.000006%)
- I = 1 pA ⇒ noise current I_n(rms) = 56 fA (5.6%)
- Shot noise is important at low current levels

1/*f* (flicker) noise

- Both thermal and shot noise are the result of fundamental physical processes and so form a fundamental detection limit.
- Apart from these types of noise, "real" electronic components also suffer from several sources of "excess noise."
- Resistors exhibit current-dependent 1/*f* noise: $V_n(\text{rms}) = c \cdot I$ where the constant *c* is determined by resistor quality (price).
- Transistors (especially MOSFETs) exhibit area-dependent 1/*f* noise: where $V_n(\text{rms}) \propto 1/\text{sqrt}(\text{Area})$
- 1/*f* noise has a 1/*f* POWER spectrum, i.e. the spectral noise power per decade (or per octave) is constant.
- 1/*f* noise is sometimes referred to as "pink" or "flicker" noise

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1/*f* noise example

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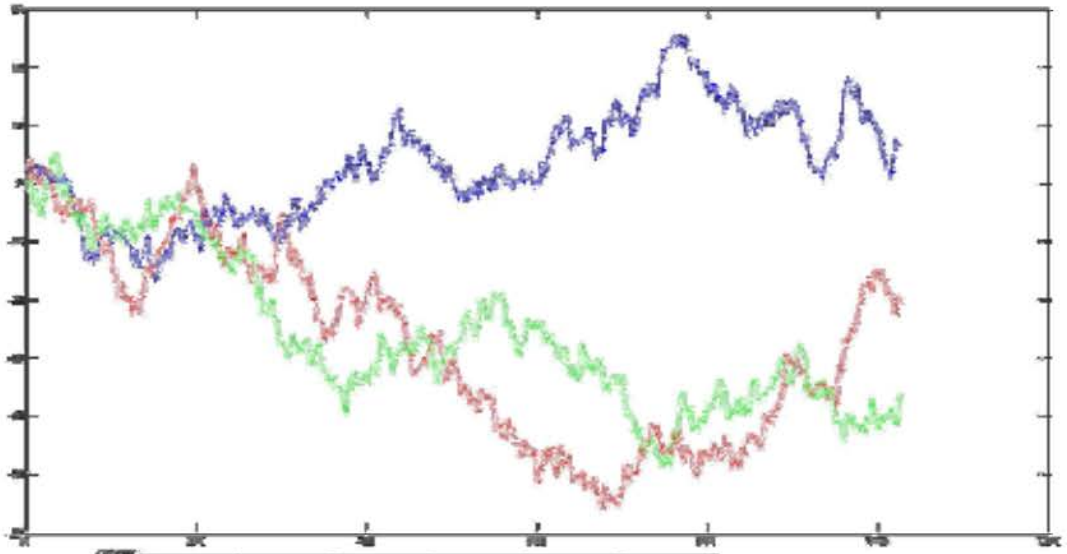
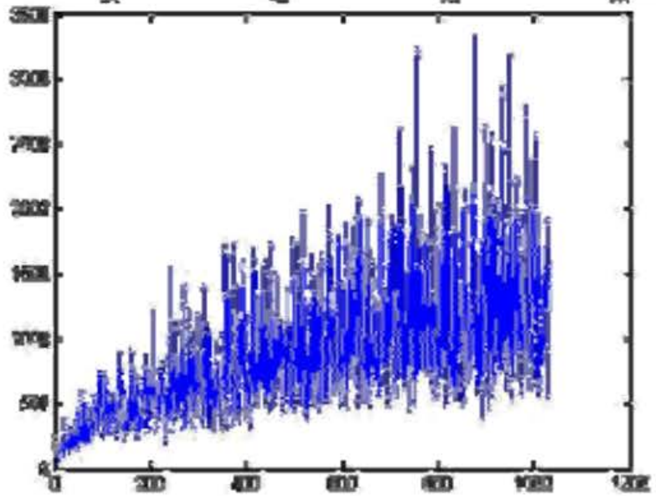


3 instances
of 1/*f* ruis



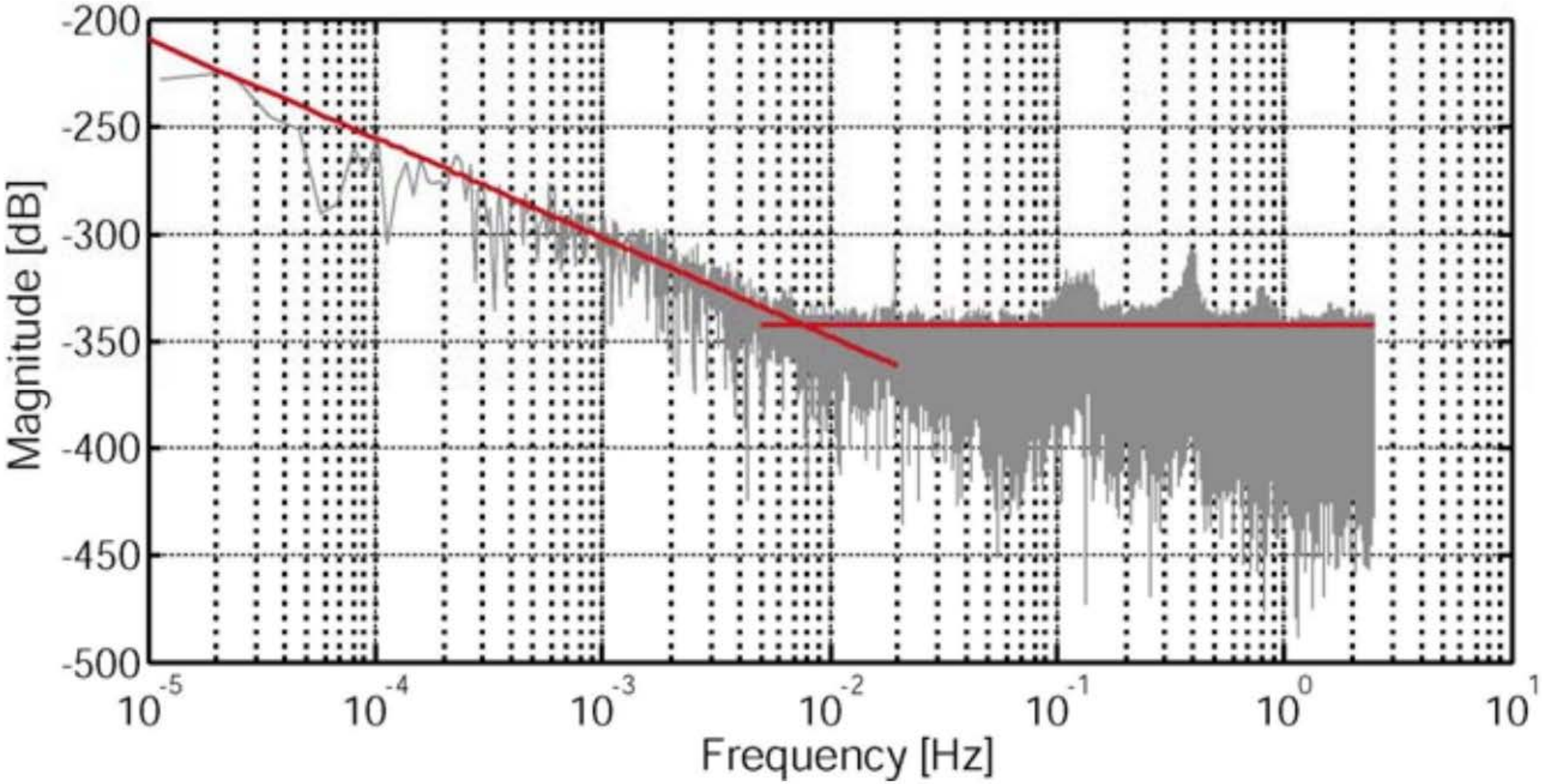
Peak amplitude increases for as measurement time increases?!

x ~ t

More and more important for long (low-frequency) measurements!

Real noise spectrum

Spectrum of a critically damped micro-mechanical structure



Two spectral regimes: a. 1/f noise and
b. frequency independent white noise.

On-line noise demo

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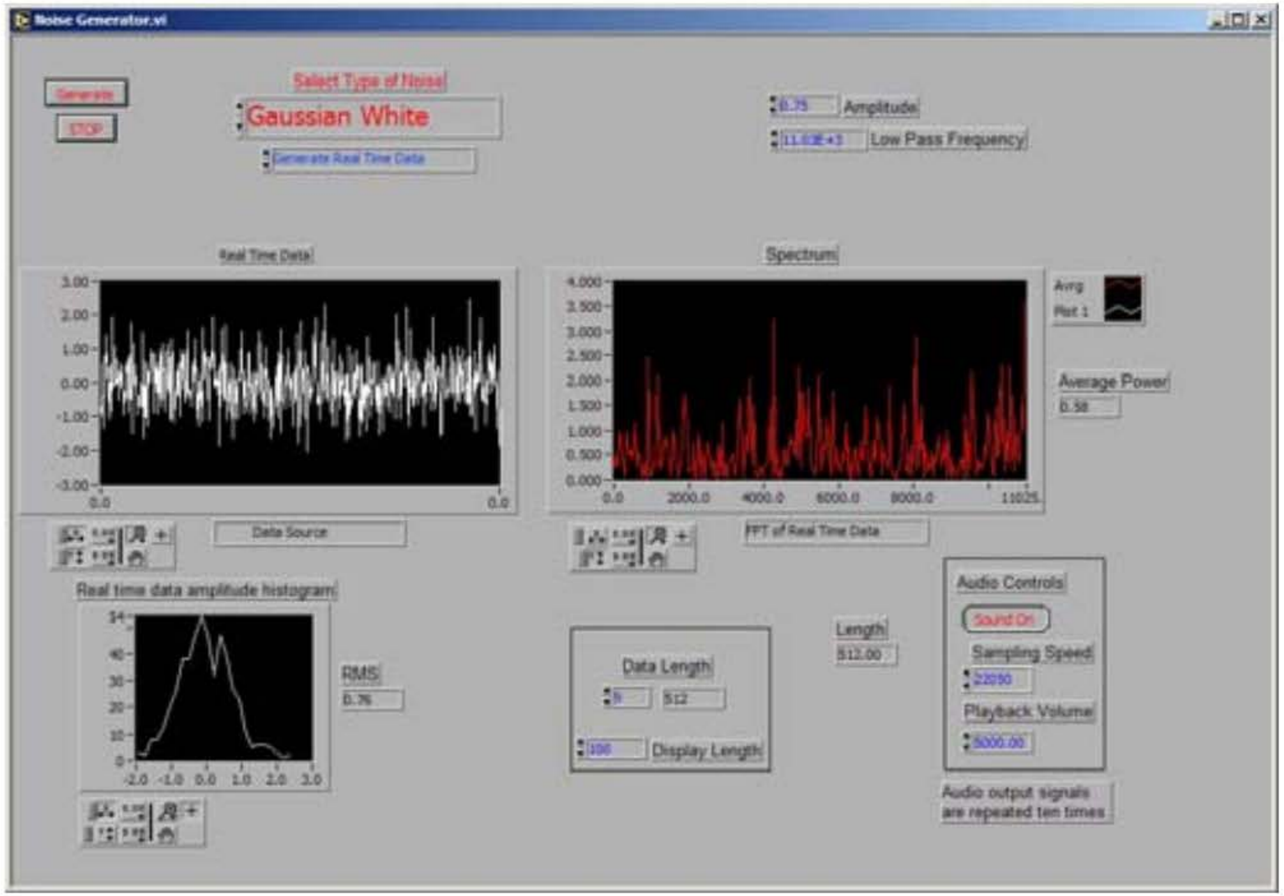


http://socrates.berkeley.edu/~phylabs/bsc/Supplementary/NoiseGenerator.html

Non-electrical noise

- Noise is also present in non-electrical domains
- Thermal noise $\Rightarrow$ some kind of energy dissipation

- In the mechanical domain this is due to damping
- In very small structures (MEMS), Brownian noise is also important
- In the thermal domain this is due to thermal resistance

- Shot noise $\Rightarrow$ small amount of energy-carrying particles
- In the optical domain, photon shot noise is significant

Error propagation

Errors (uncertainties) due to:

a. Noise,

b. Electro-Magnetic Interference (EMI)

are referred to as Stochastic (random) sources of error (uncertainty).

The effect of stochastic error(s) on overall system specifications can be evaluated using Gauss law of error propagation:

$$\underline{z} = f(\underline{a}, \underline{b}, \underline{c}, \ldots)$$

$$\sigma_z^2 = \left(\frac{\partial f(a,b,c,\ldots)}{\partial a} \right)^2_{\underline{a},\underline{b},\underline{c},\ldots} \cdot \sigma_a^2 + \left(\frac{\partial f(a,b,c,\ldots)}{\partial b} \right)^2_{\underline{a},\underline{b},\underline{c},\ldots} \cdot \sigma_b^2 + \left(\frac{\partial f(a,b,c,\ldots)}{\partial c} \right)^2_{\underline{a},\underline{b},\underline{c},\ldots} \cdot \sigma_c^2 + \ldots$$

where σ_z^2 denotes the variance (uncertainty) in parameter z with average value $\underline{z}$ due to parameter a, which is specified with uncertainty σ_a etc..

Error propagation and equivalent input error

Combining additive errors:

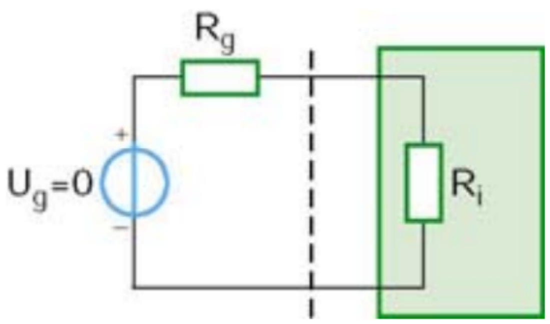
For z= a+ b or z= a-b the overall uncertainty follows as:

$$\underline{z} = \underline{a} \pm \underline{b}$$

$$\sigma_z^2 = \left(\frac{\partial(a \pm b)}{\partial a}\right)^2_{\underline{a},\underline{b},\underline{c},\ldots} \cdot \sigma_a^2 + \left(\frac{\partial(a \pm b)}{\partial b}\right)^2_{\underline{a},\underline{b},\underline{c},\ldots} \cdot \sigma_b^2 \qquad = \sigma_a^2 + \sigma_b^2$$

Hence, the variances of stochastic errors in a linear system can be linearly added.

In case of distributed noise voltages or currents:



Error propagation and equivalent input error

Combining additive errors:

For z= a+ b or z= a-b, the overall uncertainty follows as:

$$\underline{z} = \underline{a} \pm \underline{b}$$

$$\sigma_z^2 = \left(\frac{\partial(a \pm b)}{\partial a}\right)^2_{\underline{a},\underline{b},\underline{c},\ldots} \cdot \sigma_a^2 + \left(\frac{\partial(a \pm b)}{\partial b}\right)^2_{\underline{a},\underline{b},\underline{c},\ldots} \cdot \sigma_b^2 \qquad = \sigma_a^2 + \sigma_b^2$$

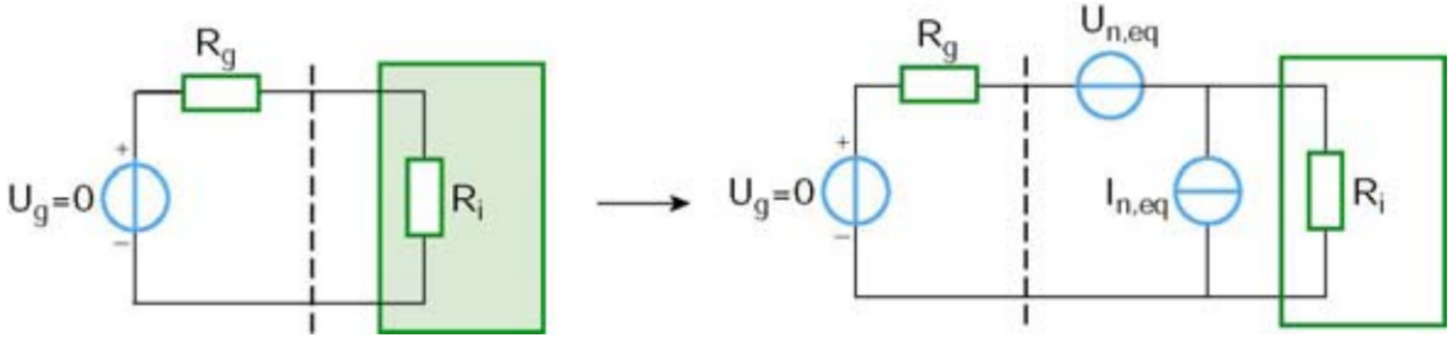
Hence, the variances of stochastic errors in a linear system can be linearly added.

In case of distributed noise voltages or currents:

$$u_{n,eq} = \sqrt{u_{n1}^2 + u_{n2}^2}$$

$$i_{n,eq} = \sqrt{i_{n1}^2 + i_{n2}^2}$$

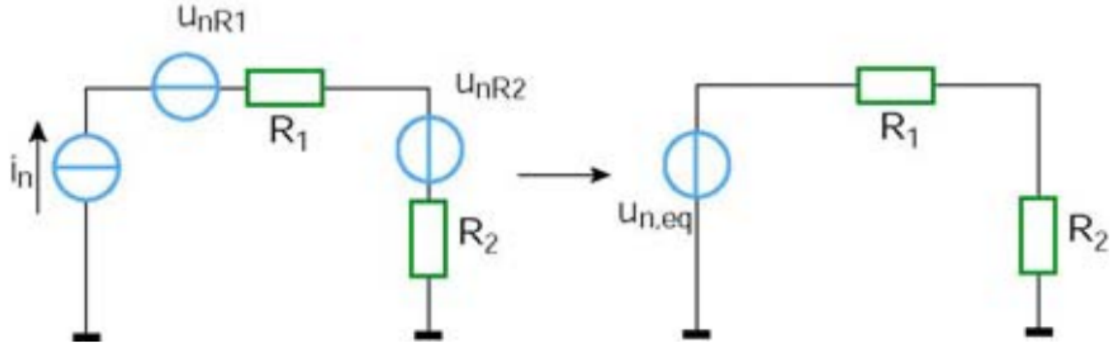
Provided these are independent !



Error propagation and equivalent input error

Noise paradox:

What is the equivalent noise voltage in a circuit with two resistors connected to a noise current source ?



Adding the thermal noise powers of R_1 and R_2 to the noise power due to i_n flowing through R_1 and I_n flowing through R_2:

X

$$(a)\overline{u^2_{n1,eq}} = \overline{u^2_{n,R1}} + \overline{u^2_{n,R2}} + \overline{i^2_n}R_1^2 + \overline{i^2_n}R_2^2 = \overline{u^2_{n,R1}} + \overline{u^2_{n,R2}} + \overline{i^2_n}\left(R_1^2 + R_2^2\right)$$

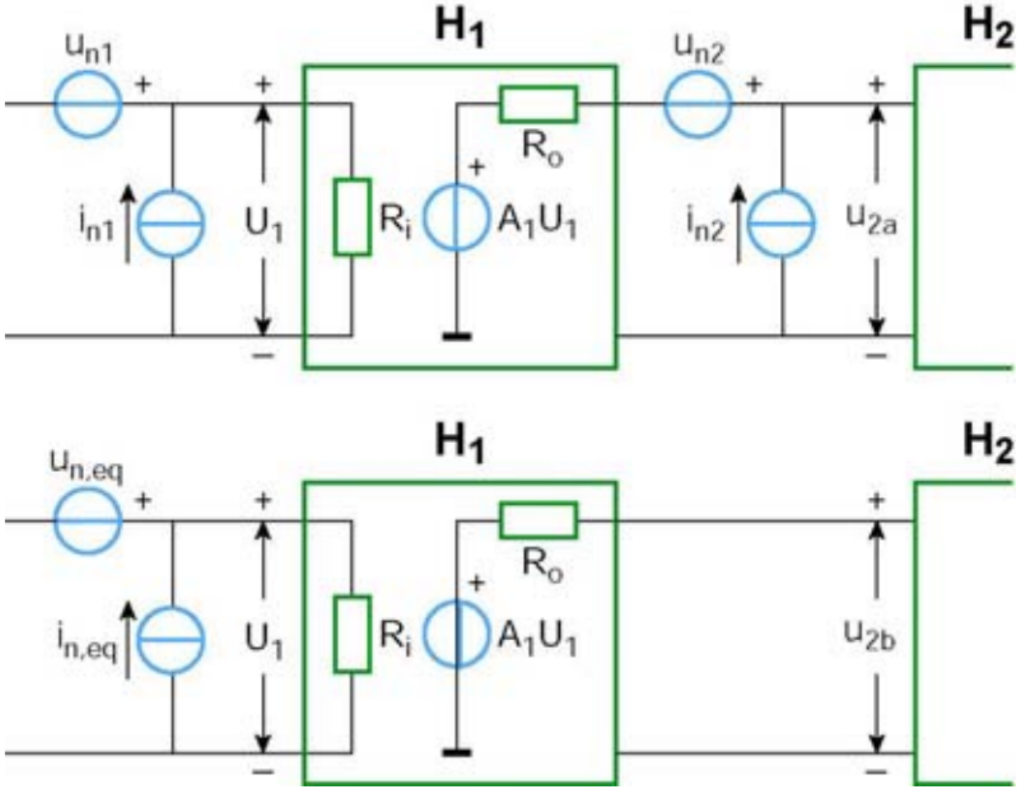
Adding the thermal noise powers of R_1 and R_2 to the noise power due to i_n flowing through R_1 plus R_2:

V

$$(b)\overline{u^2_{n2,eq}} = \overline{u^2_{n,R1}} + \overline{u^2_{n,R2}} + \overline{i^2_n}\left(R_1 + R_2\right)^2$$

Equivalent input sources

General approach for finding equivalent input noise sources:



Note: equivalent noise is independent of source impedance!

Extreme case 1:
open input.

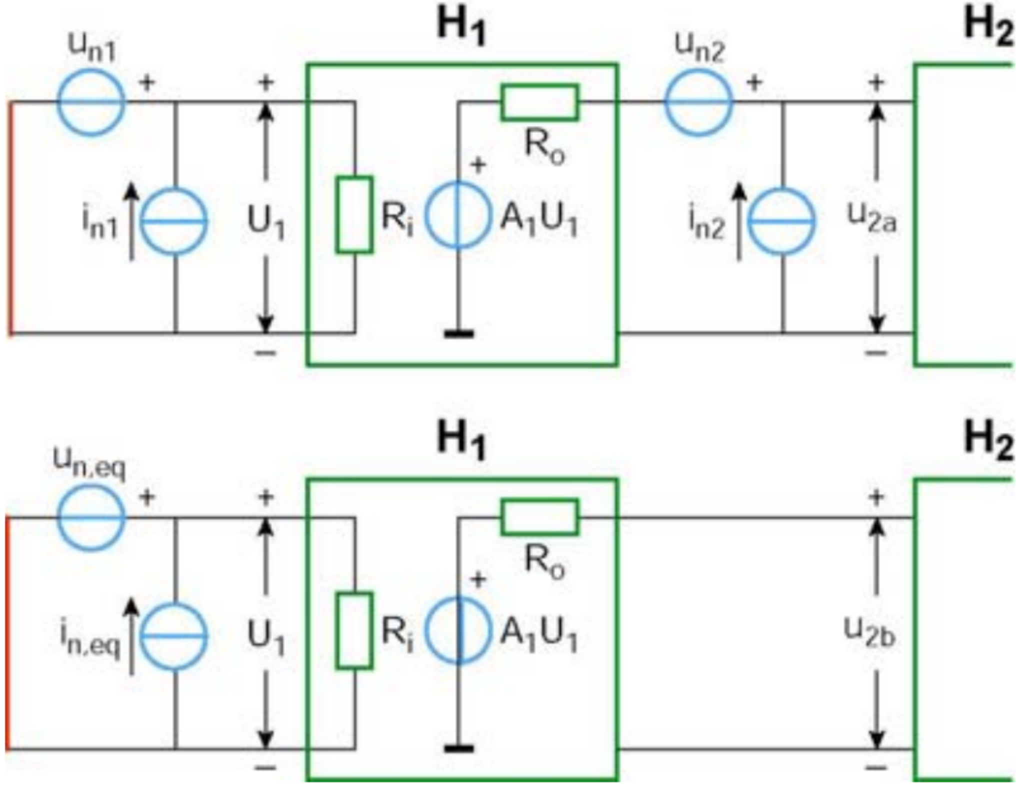
$$\left.\begin{aligned} u_{2a}^2 &= (i_{n1}R_iA_1)^2 + (i_{n2}R_o)^2 + u_{n2}^2 \\ u_{2b}^2 &= (i_{n,eq}R_iA_1)^2 \\ u_{2a}^2 &= u_{2b}^2 \end{aligned}\right\}$$

Hence:

$$i_{n,eq}^2 = i_{n1}^2 + \frac{u_{n2}^2 + i_{n2}^2 R_o^2}{A_1^2 R_i^2}$$

Equivalent input sources

General approach for finding equivalent input noise sources:



Note: equivalent noise is independent of source impedance!

Extreme case 2:
short-circuited input.

$$\left.\begin{aligned} u_{2a}^2 &= (u_{n1}A_1)^2 + (i_{n2}R_o)^2 + u_{n2}^2 \\ u_{2b}^2 &= (u_{n,eq}A_1)^2 \\ u_{2a}^2 &= u_{2b}^2 \end{aligned}\right\}$$

Hence:

$$u_{n,eq}^2 = u_{n1}^2 + \frac{u_{n2}^2 + i_{n2}^2 R_o^2}{A_1^2}$$

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Equivalent input sources

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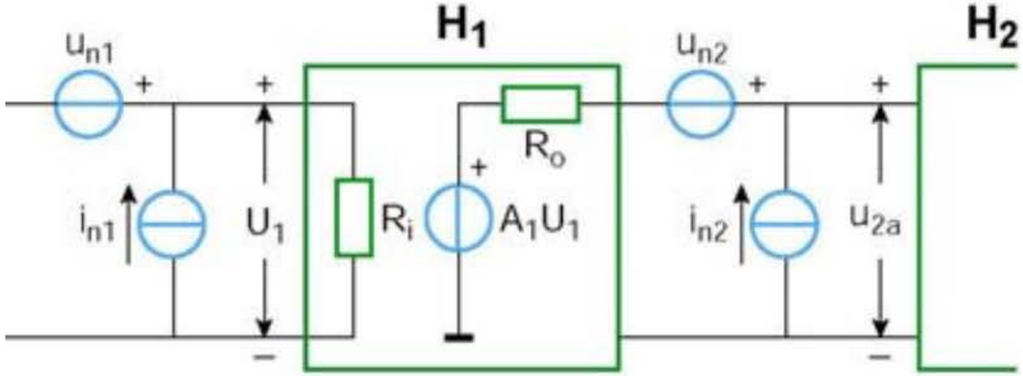
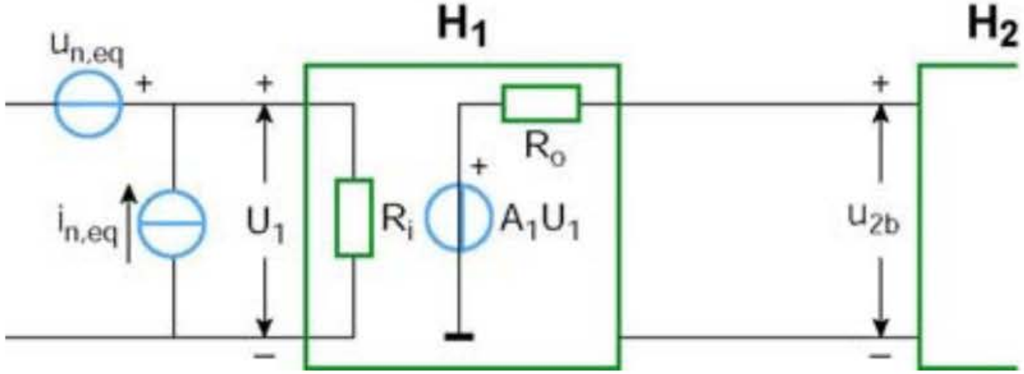
General approach for finding equivalent input noise sources:





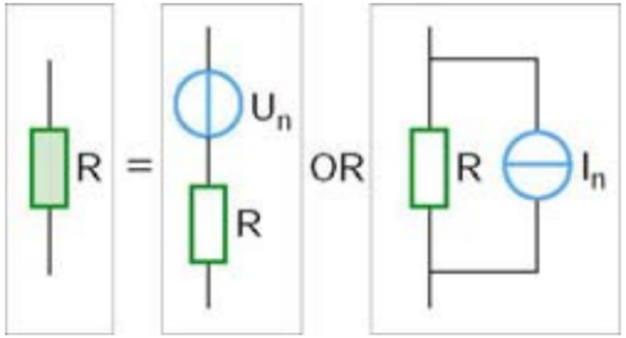
$$u_{n,eq}^2 = u_{n1}^2 + \frac{u_{n2}^2 + i_{n2}^2 R_o^2}{A_1^2}$$

$$i_{n,eq}^2 = i_{n1}^2 + \frac{u_{n2}^2 + i_{n2}^2 R_o^2}{A_1^2 R_i^2}$$

Recommendations for low-noise performance:
Focus design effort on first stage: low noise and high gain, A_1.

Noise bandwidth

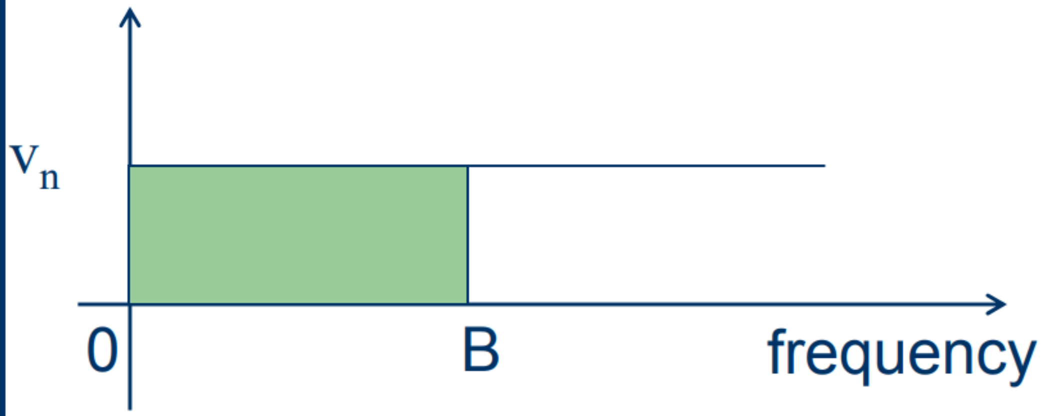
Noise in components is usually specified in terms of the noise spectral power, s_n. In case of a resistor:



$$s_{nu,Z} = 4k_B T\,|Z| = 4k_B TR\,[\mathrm{V^2/Hz}]$$

$$s_{ni,Z} = \frac{4k_B T}{|Z|} = \frac{4k_B T}{R} \qquad [\mathrm{A^2/Hz}]$$

Noise bandwidth is the bandwidth of a theoretical low-pass filter with an abrupt cut-off frequency at B. A practical low-pass filter will have a more gradual transition from pass-band to stop-band, H(ω).

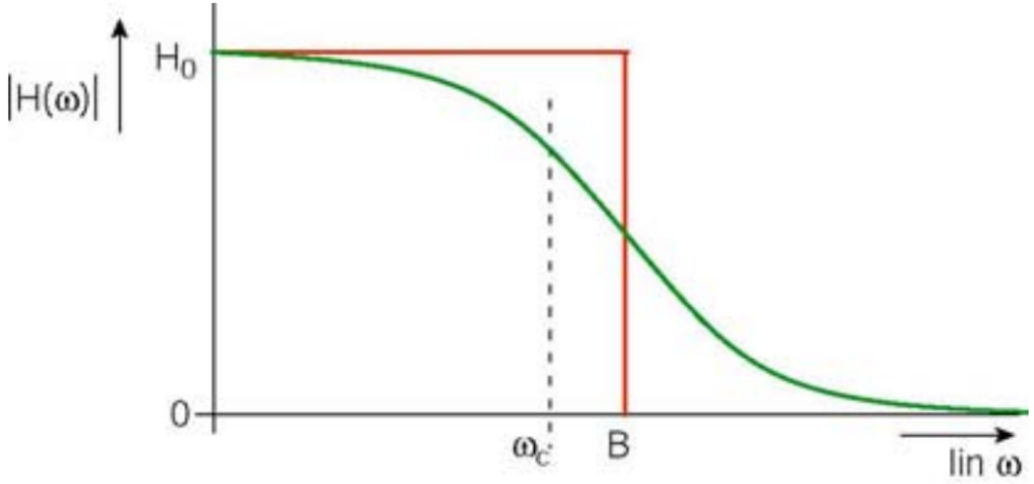


At 20°C and into a 10kHz bandwidth, a 10kΩ resistor generates 1.3 μVrms.

Noise bandwidth

is defined as the bandwidth of a “brick wall” filter with an abrupt cut-off frequency at B. A practical system is specified by a low-pass filter with a more gradual transition from pass-band to stop-band, H(ω).

The Noise bandwidth B of a practical low-pass filter can be obtained from its transfer function, H(ω) as follows:



$$P_n = |H_o|^2 B = \int_0^\infty |H(\omega)|^2 d\omega \rightarrow B = \frac{1}{|H_o|^2}\int_0^\infty |H(\omega)|^2 d\omega$$

Noise bandwidth of a first-order system

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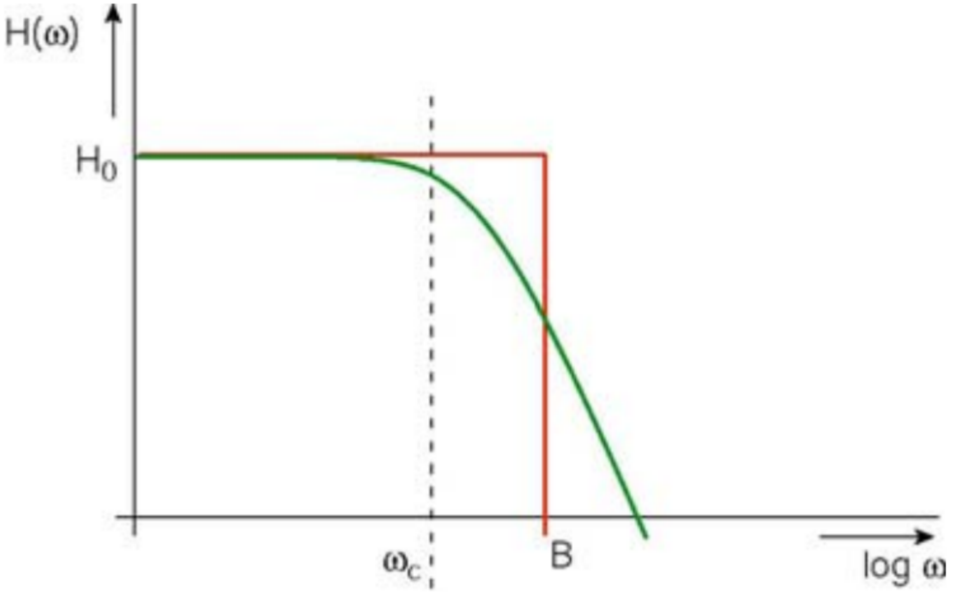
$$P_n = |H_o|^2 B = \int_0^\infty |H(\omega)|^2 d\omega \rightarrow B = \frac{1}{|H_o|^2}\int_0^\infty |H(\omega)|^2 d\omega$$

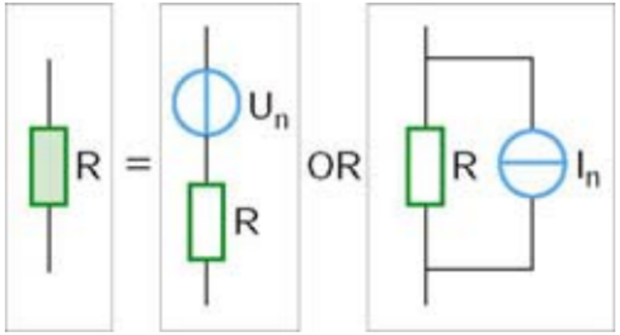


For $H_o=1$: $B = \int_0^\infty \frac{d\omega}{1+(\omega\tau)^2} = \frac{1}{\tau} arctg(\omega\tau)\Big|_0^\infty = \frac{\pi}{2\tau} = \frac{\pi}{2}\omega_c\,[rad/s]$,

For noise calculations, a "correction factor" $\pi/2$ is required

Noise in passive components

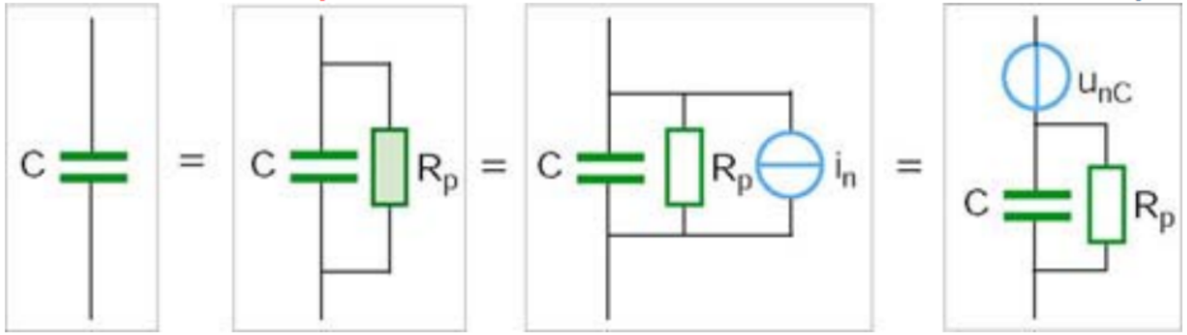
Noise in impedance, Z, is primarily due to the dissipating (resistive) part, R:



$$s_{nu,Z} = 4k_BT\,|Z| = 4k_BTR\,[\mathrm{V^2/Hz}]$$

$$s_{ni,Z} = \frac{4k_BT}{|Z|} = \frac{4k_BT}{R} \qquad [\mathrm{A^2/Hz}]$$

Reactive components are not free of noise due to parasitics:



$$\overline{u_{nC}^2} = \int_0^\infty i_n^2 |Z(\omega)|^2 d\omega = i_n^2 \int_0^\infty R_p^2 \frac{d\omega}{1+(\omega R_p C)^2} = i_n^2 R_p^2 \frac{2\pi}{4R_pC} = \frac{4k_BT}{2\pi R_p} R_p^2 \frac{\pi}{2R_pC} = \frac{k_BT}{C}$$

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Noise in passive components

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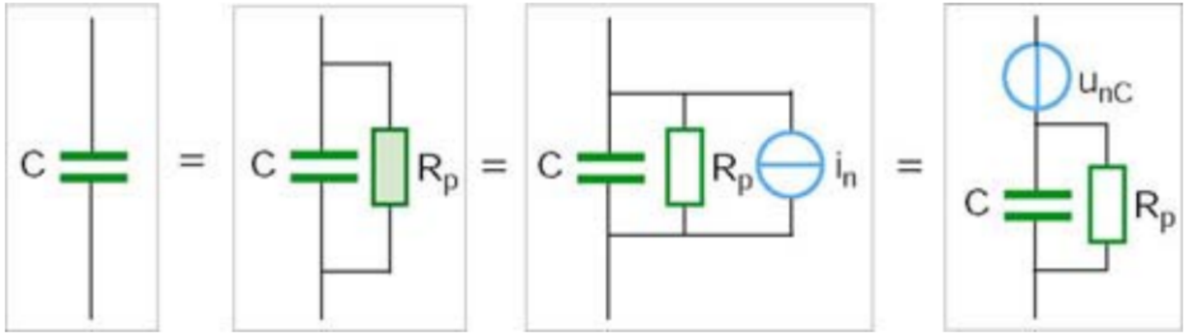
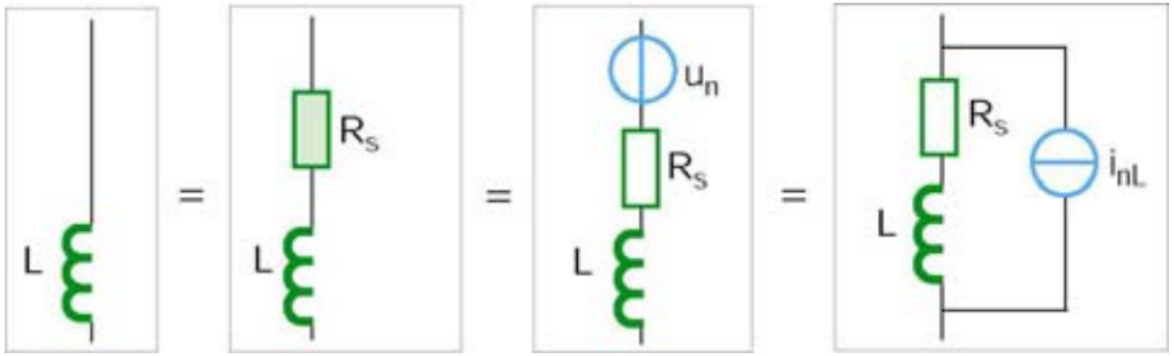
Reactive components are not free of noise due to parasitics:

In a capacitor:



$$\overline{u_{nC}^2} = \int_0^\infty i_n^2 |Z(\omega)|^2 d\omega = i_n^2 \int_0^\infty R_p^2 \frac{d\omega}{1+(\omega R_p C)^2} = i_n^2 R_p^2 \frac{2\pi}{4R_pC} = \frac{4k_BT}{2\pi R_p} R_p^2 \frac{\pi}{2R_pC} = \frac{k_BT}{C}$$

Similarly,
in an inductor:



$$\overline{i_{nL}^2} = \int_0^\infty \frac{u_n^2}{|Z(\omega)|^2} d\omega = u_n^2 \int_0^\infty \frac{d\omega}{R_s^2+(\omega L)^2} = \frac{4k_BTR_s}{2\pi}\times\frac{1}{R_s^2}\int_0^\infty \frac{d\omega}{1+\left(\frac{\omega L}{R_s}\right)^2} = \frac{k_BT}{L}$$

Signal-to-noise ratio (SNR)

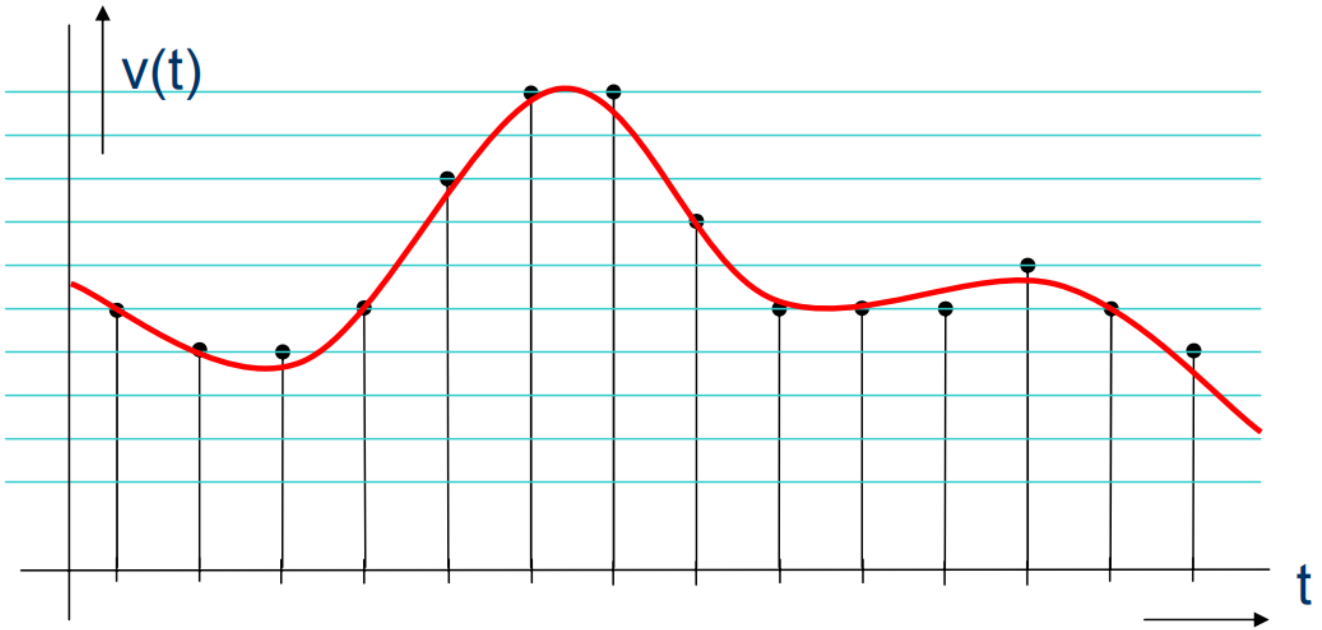
SNR is defined as the ratio between signal power and noise power expressed in dB:

$$\frac{\mathrm{S}}{\mathrm{N}} = 10\log\left(\frac{P_{\mathrm{S}}}{P_{\mathrm{N}}}\right) = 10\log\left(\frac{V_{\mathrm{S}}(\mathrm{rms})}{V_{\mathrm{N}}(\mathrm{rms})}\right)^2 = 20\log\left(\frac{V_{\mathrm{S}}(\mathrm{rms})}{V_{\mathrm{N}}(\mathrm{rms})}\right)$$

Notes:

- Since the form of the signal and that of the noise may differ significantly, SNR should not be calculated from signal amplitudes but from rms values.
- A given SNR is always associated with a certain bandwidth.
- For maximum SNR, the bandwidth of a measurement system should be about the same as the expected signal bandwidth

Analog to Digital Conversion



Signal is quantized in both amplitude and in time

Quantization Noise

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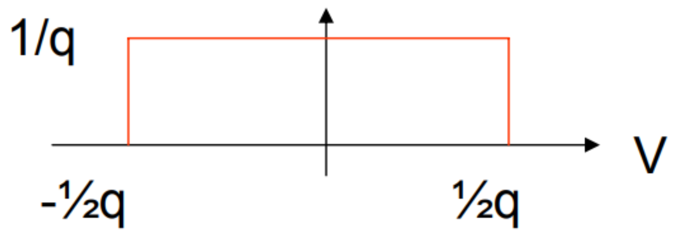
- Range: 0 - V_{ref}
- Number of bits: n
- Number of levels: 2^n
- "step-size": $q = V_{ref}.2^{-n}$
- error: $-\frac{1}{2}q < V < \frac{1}{2}q$
- For a busy signal (one that jumps between many quantization levels), this error can be modeled as random noise with a uniform pdf
- The associated standard deviation *e* is then given by:

Probability distr. function p(V)



$$e = \sqrt{\frac{1}{q}\int_{-q/2}^{q/2} V^2 dV} = \frac{1}{\sqrt{12}}q$$

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Sources of interference

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Interference source → Coupling mechanism → Interference sensitive system

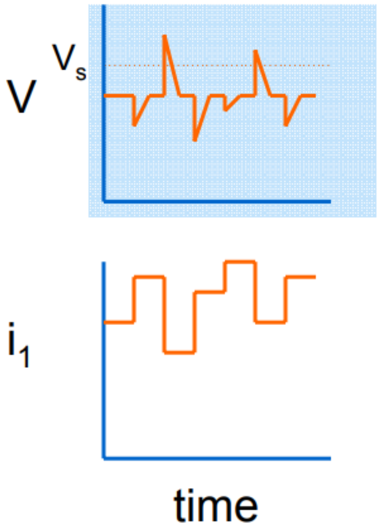
Interference source	Coupling mechanism	Interference sensitive system
electrostatic discharge	**conduction**	integrated circuit
transformer (50 Hz)	**magnetic field**	audio system
Switched-mode PSU	**electric field**	high-impedance sensor
HF digital circuit	**electromagnetic field**	FM radio

Interference via conduction

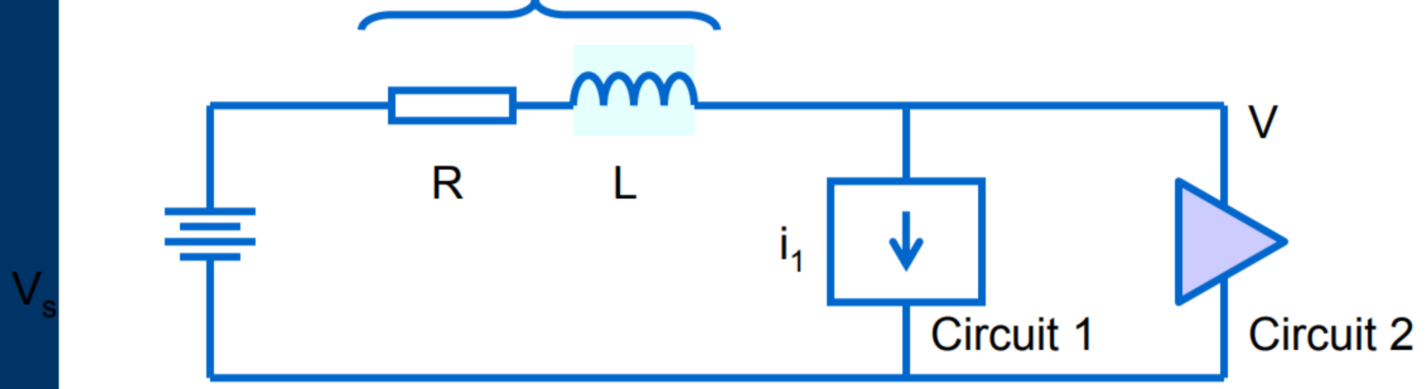
- Coupling via a common impedance
- E.g. via a common supply line or a ground connection

Circuit 1 : HF digital circuit

$\Rightarrow$ i_1 is a rapidly varying current (high di/dt)

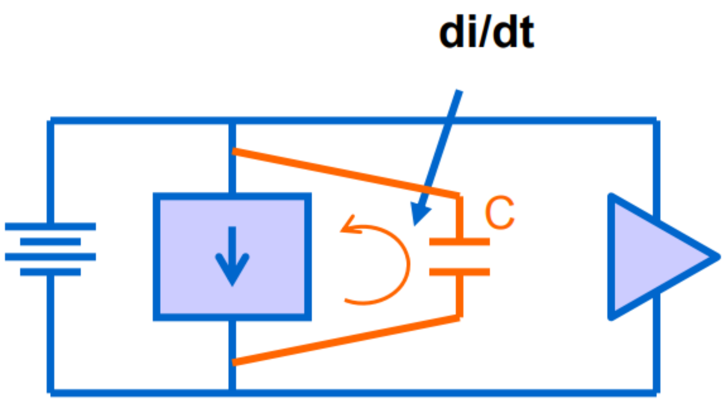


$$\underbrace{\Delta V = R\, i_1 + L\, di_1/dt}$$

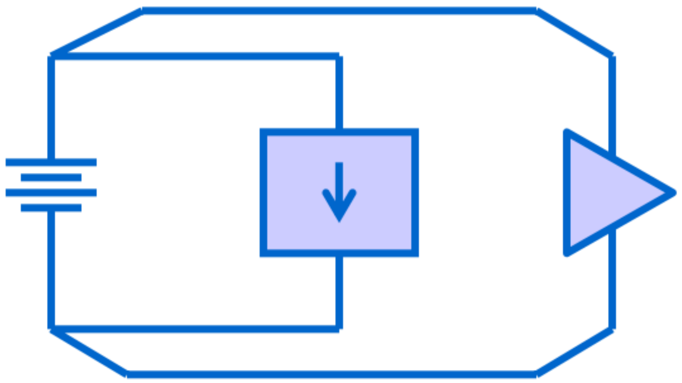


Common impedances: solutions

- Quick fix: use a bypass capacitor near a HF circuit to locally deliver its rapidly changing supply currents (this can be seen as a form of filtering)
- Good design: avoid shared impedances



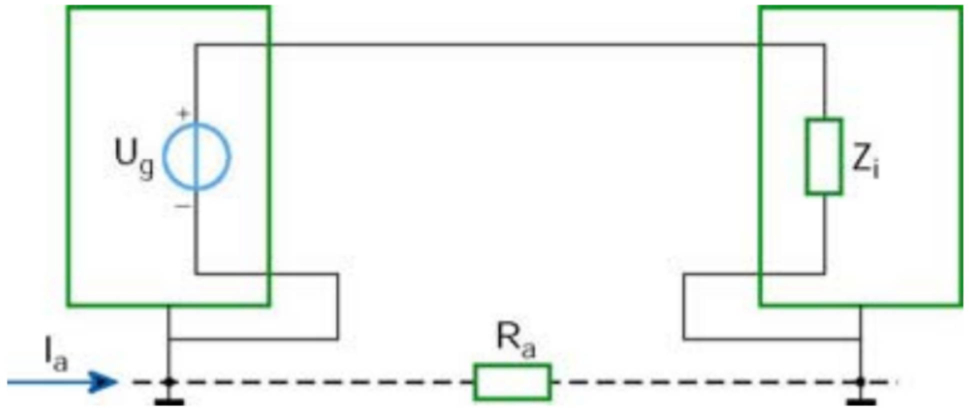
Local charge storage



Seperate supply lines

External sources of error

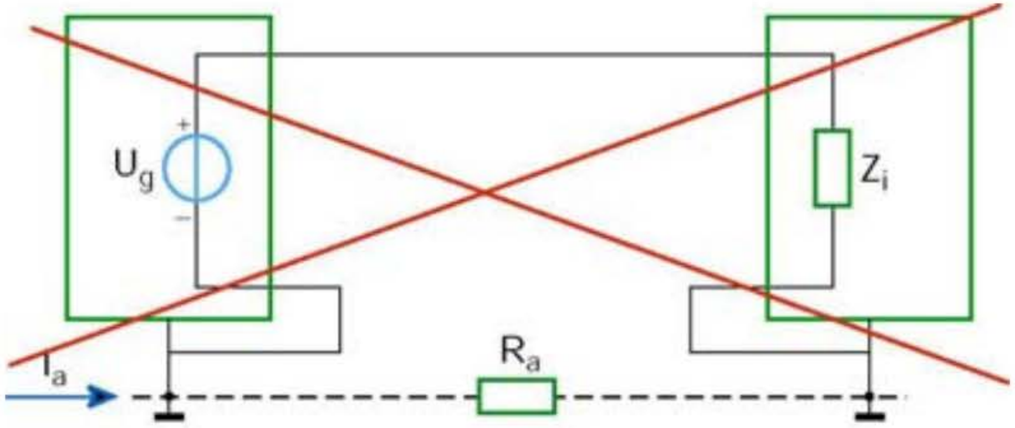
Ground loops



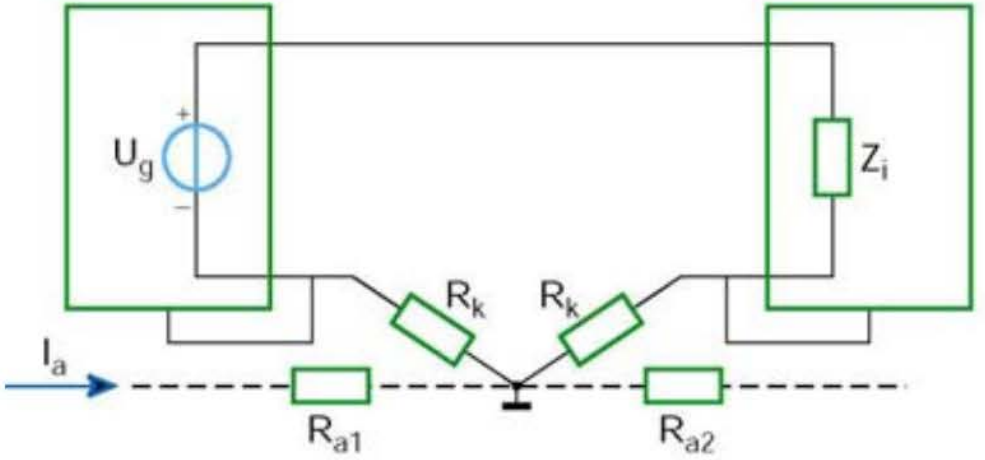
Additive error due to $I_a R_a$ in series with u_g.

External sources of error

Ground loops



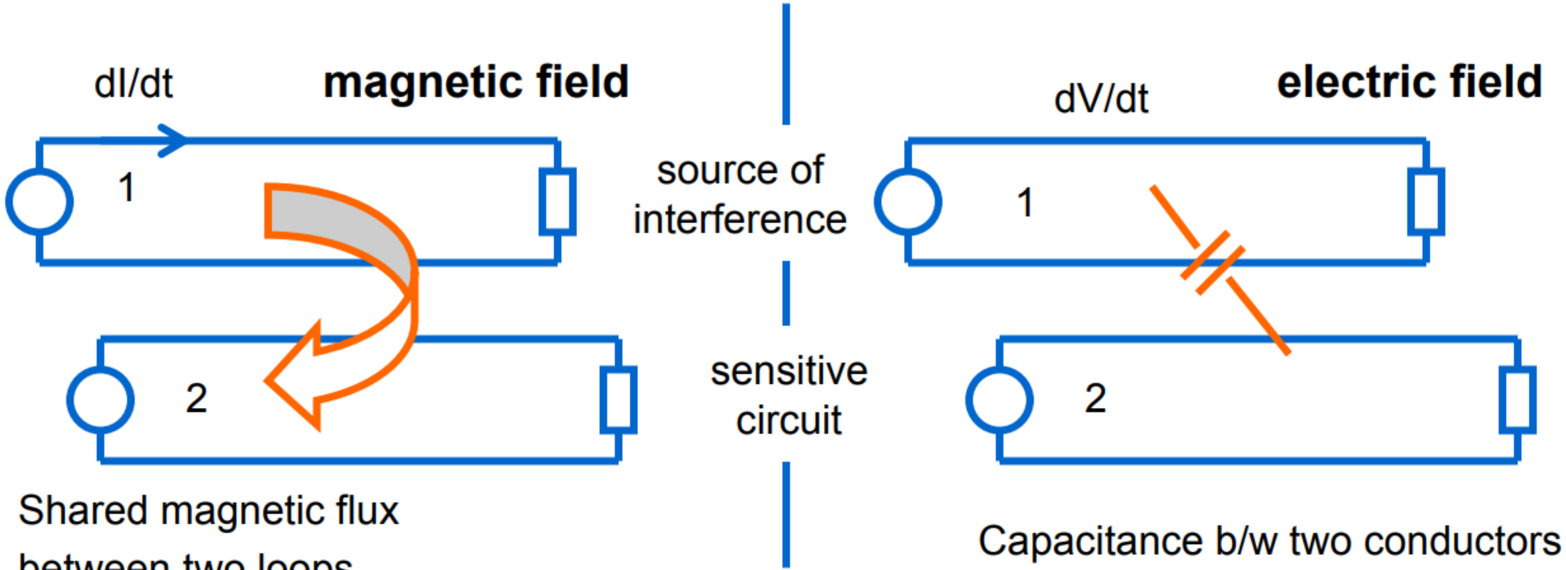
“star” connection to limit error due to $I_a R_a$.



Electric & magnetic coupling

ET8.017 El. Instr.

Electric (capacitive) coupling is due to changing voltages, i.e. dV/dt
Magnetic (inductive) coupling is due to changing currents, i.e. dI/dt



Shared magnetic flux between two loops

Capacitance b/w two conductors

- **In the same circuit there can be multiple coupling mechanisms**
  - Depends on source and load-impedances, size and geometry

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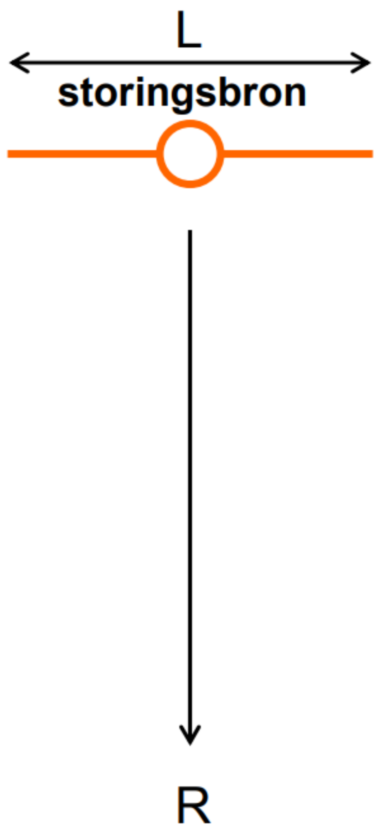
Electromagnetic interference

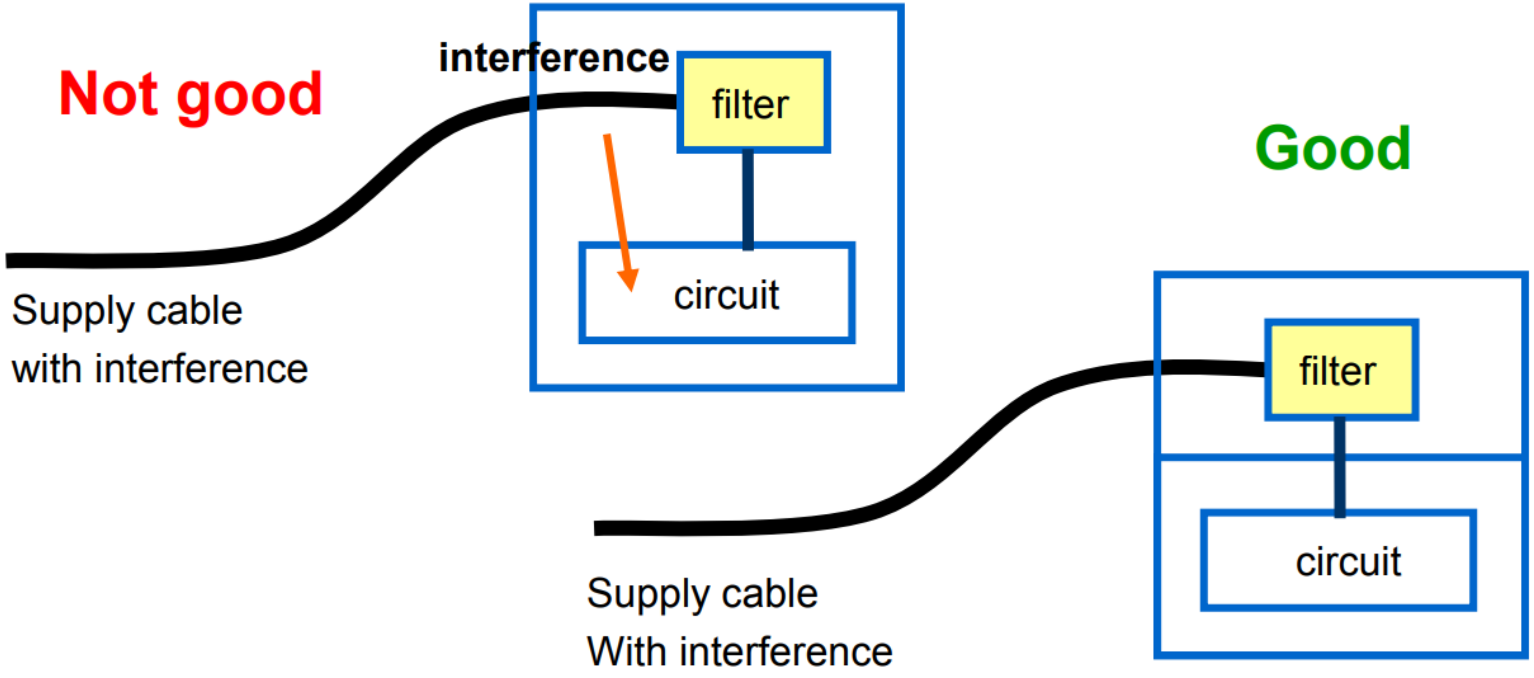
ET8.017
El. Instr.

- When an “antenna” is driven by a source, it creates
  - an electromagnetic field
  - an electric field
  - a magnetic field
- In a given situation, the dominant field depends on the shape and size of the antenna, the frequency and the distance to the antenna
- Field strengths vary with:
  - $1/R^2$ and $1/R^3$ voor electric and magnetic fields, resp. $1/R$ for EM-waves (R = distance to antenna)
  - $R < \lambda$: E and M fields can dominate (near field)
  - $R > \lambda$: EM waves can dominate (far field)
- The following applies to EM waves:
  - $L = \lambda/2$: good antenna (lots of EM interference)
  - $L < \lambda/20$: bad antenna (less problems)

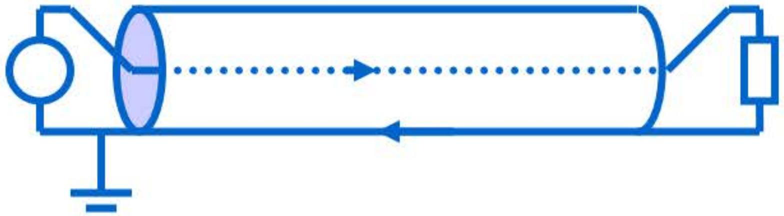


Interference reduction: Filtering

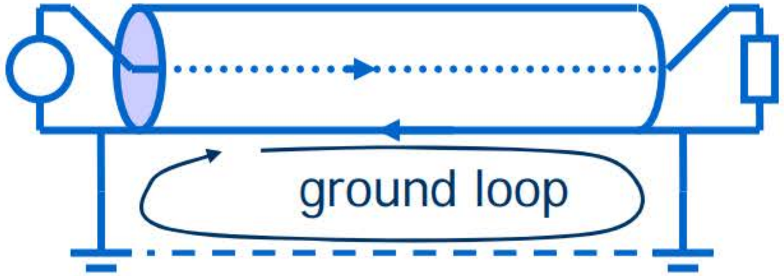
- Filtering can help suppress interference that is coupled into a circuit via its supply lines
- Filtering can also be used on signal lines if the interference does not occur at the same frequencies as the wanted signal



Shielding: Coax cable



If the outer conductor of a coax cable (on 1 side) is grounded, electric field cannot reach the inner conductor. The outer conductor also forms the return path *and* the signal's reference.



If the outer is grounded at both sides (often the case) a ground loop will be created. This can lead to interference because of the voltage differences between the two grounds and/or magnetic coupling via the loop.

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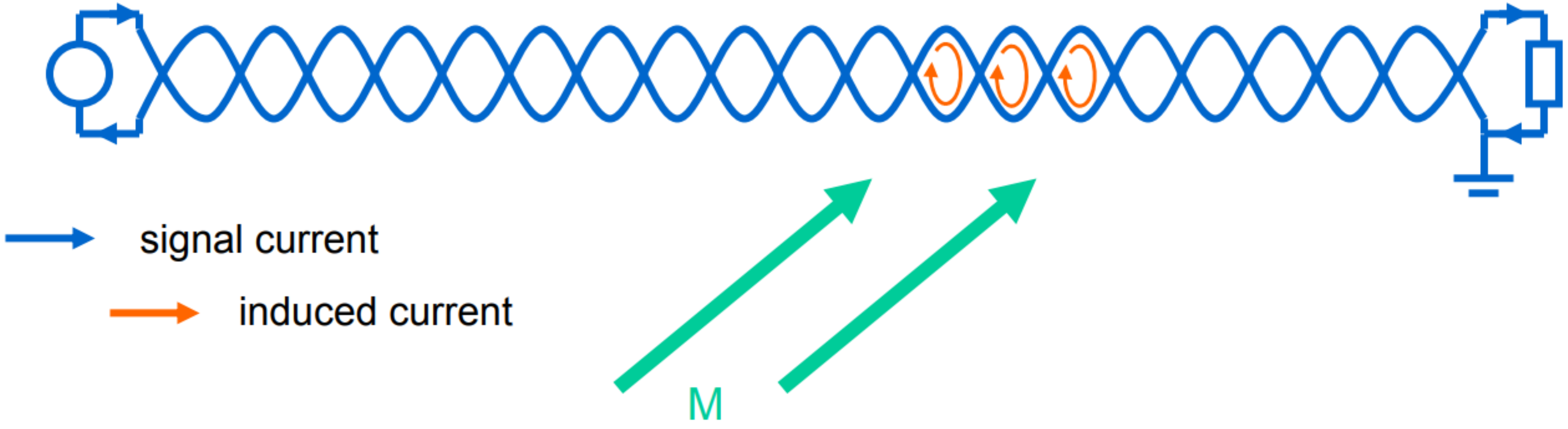
Twisted pair

ET8.017
El. Instr.

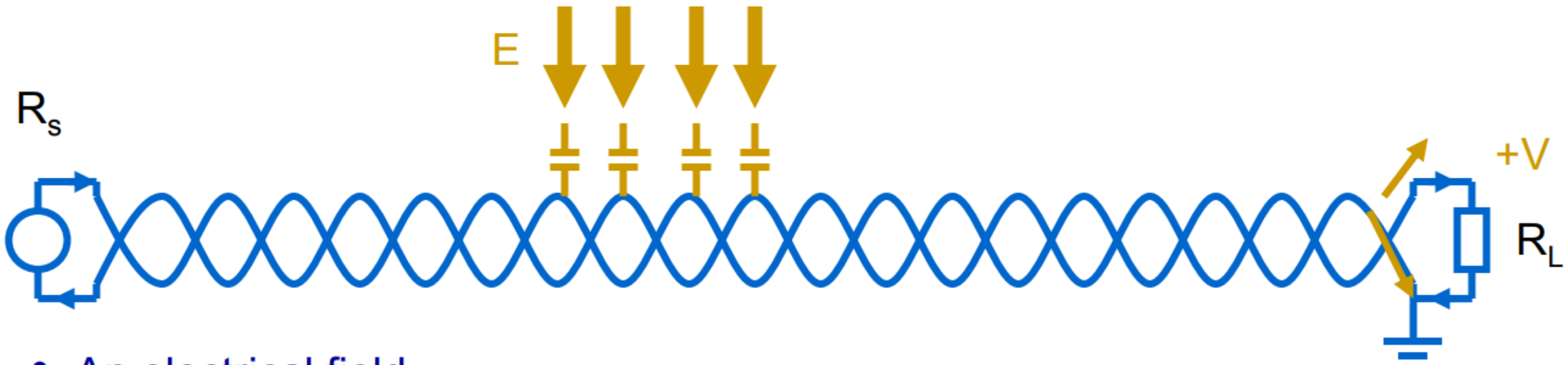


- Magnetic field induces current in coils
  - coils are “twisted”
  - currents in neighboring loops have opposite signs
  - currents cancel each other
  - *net induced current is zero*

	Attn [dB]
untwisted pair (ref)	0
twisted pair	43
untwisted pair + alu foil shield	3
untwisted pair + steel shield	32

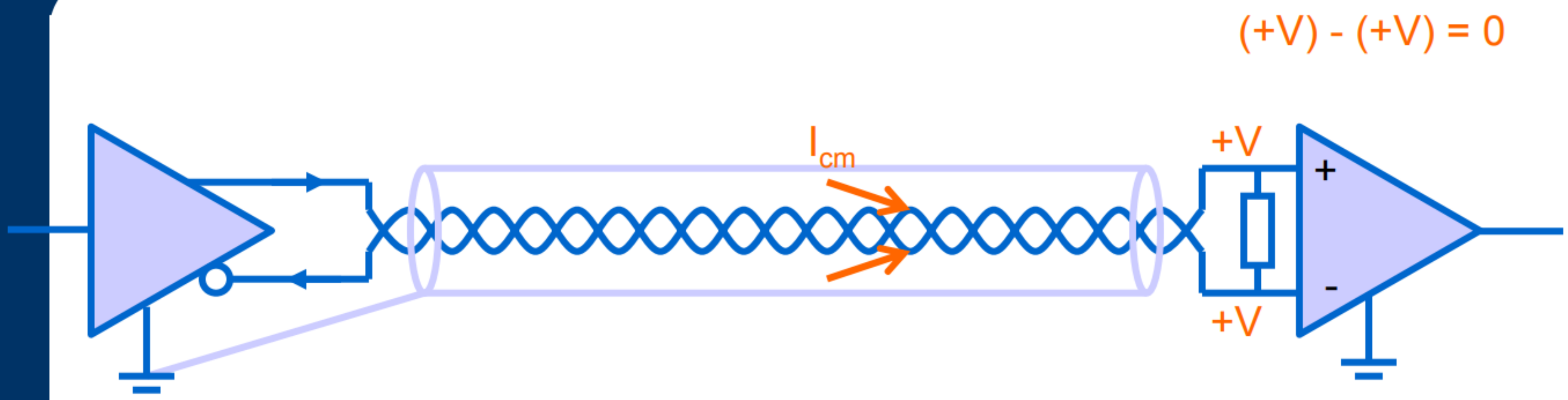
50 Hz **Magnetic** shielding

Twisted pair (2)



- An electrical field
  - Couples capacitively to both conductors
  - Generates equal currents in both signal and return lines
  - Signal line: current flows through $R_s//R_L$ generates +V
  - Return line: current flows to ground (low R)
- Result: electric fields are not shielded!
- Solutions:
  - Add capacitive shielding (e.g. a conducting outer layer)
  - Balance the circuit

Twisted pairs: Balancing



Balancing:

- Use differential inputs and outputs
- Ensure that capacitively coupled currents in both lines are the same (i.e. that they are common-mode signals) and that they "see" the same impedance
- The current-induced voltages will then cancel each other out
- Adding an outer shield will then result in excellent suppression of both magnetic and electric interference

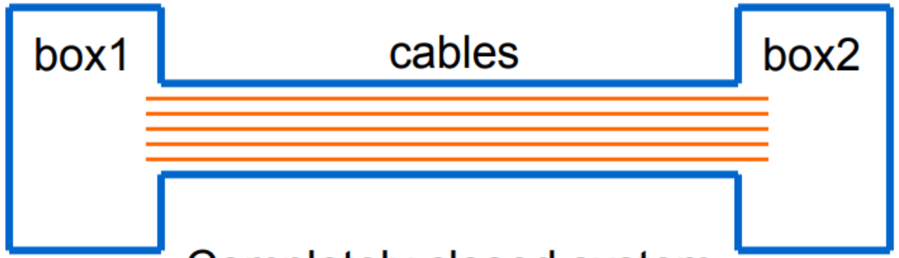
Shielding: EM waves

- EM waves will not penetrate a closed conducting enclosure if it is at least 10 “skin depths” thick
- The enclosure does not need to be grounded
- EM waves can leak into the enclosure via:
    - Cables that enter the enclosure
        - Use filters and/or shield the cables
    - Openings in the enclosure such as:
        - plastic knobs
        - displays
        - Ventilation holes
        - Poorly designed connectors

$$\delta = \frac{1}{\sqrt{\pi.\sigma.\mu.f}}$$

δ = skin depth
σ = conductivity
μ = permeability
f = frequency

1δ = 8.7 dB attn.
(= 2.7 x)



Completely closed system

Types of cable

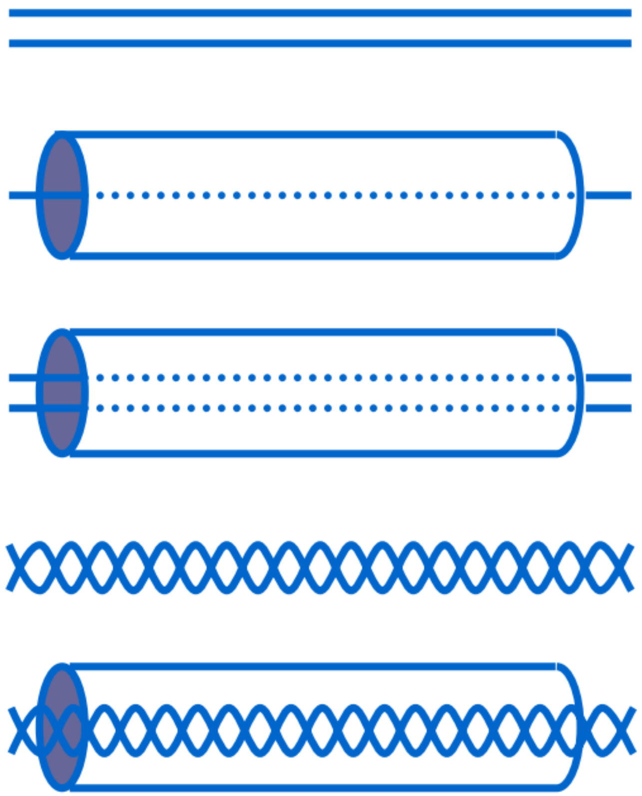
untwisted pair

coaxial

twin-axial

twisted-pair

twisted-pair shielded



Most types of cable can be obtained with an extra outer shield for even better suppression of EM interference

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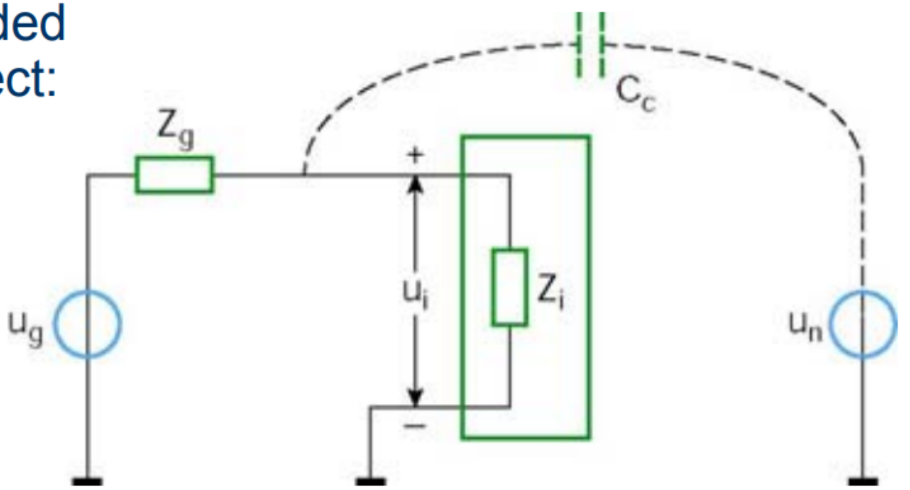
Shielding

Shielding to reduce capacitive coupling

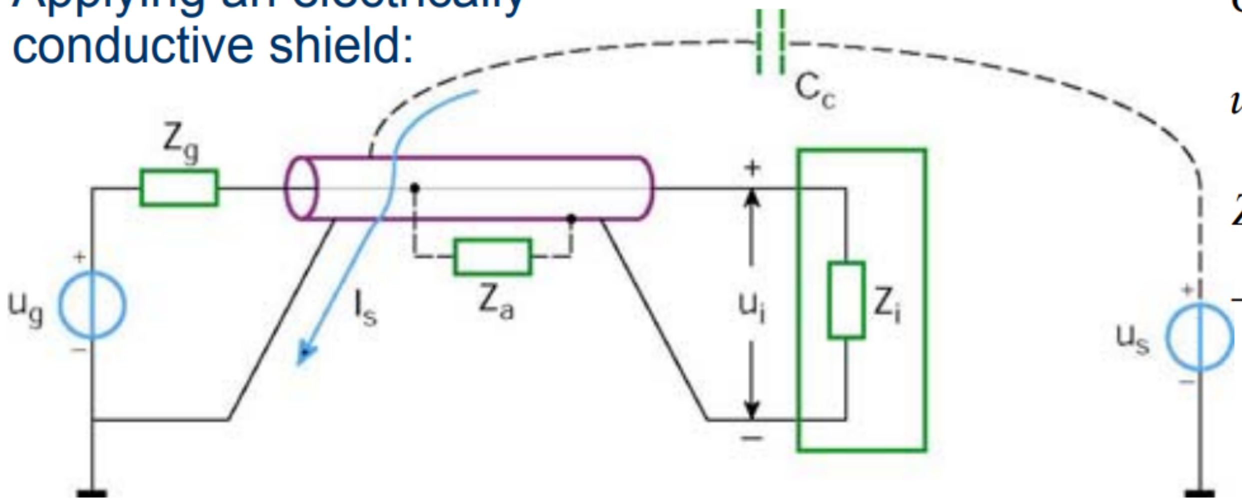
Non-shielded interconnect:



$$u_{i,g} = \frac{Z_i}{Z_g + Z_i} u_g$$

$$u_{i,n} = \frac{Z_g // Z_i}{Z_g // Z_i + \frac{1}{j\omega C_c}} u_n$$

Applying an electrically conductive shield:



$$C_c \downarrow \rightarrow u_{i,n} / u_n \downarrow$$

$$u_{i,g} = \frac{Z_i // Z_a}{Z_g + Z_i // Z_a} u_g$$

$$Z_i // Z_a \ll Z_i$$

$$\rightarrow BW\left(u_{i,g} / u_g\right) \downarrow$$

Suppressing electric fields

Knowing how electric (capacitive) coupling arises, it is “easy” to avoid this type of interference:

- Reduce the signal impedances
  - The coupling cap and the signal impedance form a voltage divider
- Drop the frequency and amplitude of the interfering signal
  - Usually not possible
- Decrease the coupling capacitance
  - Increase the distance between interference and circuit
  - Ground floating conductors between interference and circuit
- Break any return path for interference
  - These usually form ground loops
- Capacitive shielding:
  - Shielded cables (e.g. coax)
  - Conducting box around sensitive circuits
  - Ground the shielding!