COP 3502 – Computer Science 1

Lecture 07

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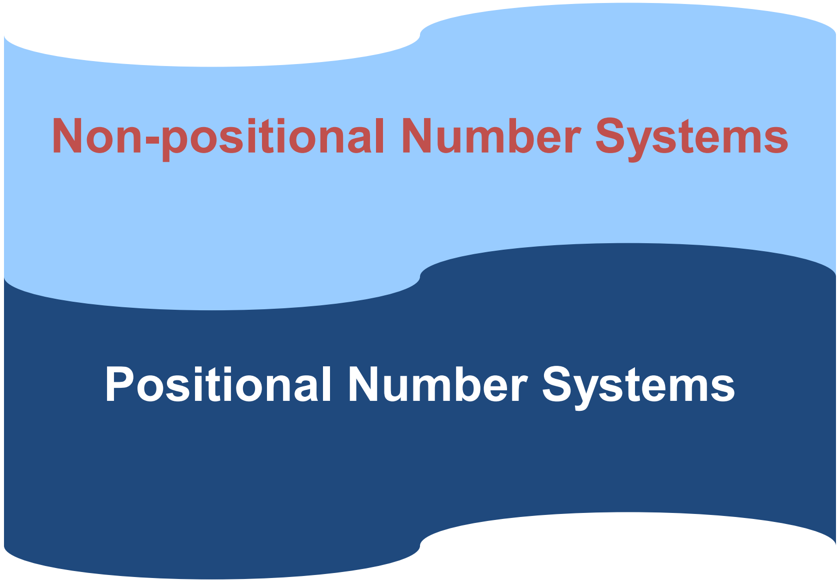
Number system and base conversion

Number System



Non-positional Number Systems

- In early days, human being counted on fingers,. When ten fingers are not adequate, stones, pebbles, or sticks were used to indicate values. This method of counting uses an Non-positional Number Systems.

- In this system symbols such as I for 1. II for 2, III for 3, IIII for 4 etc.

Positional Number Systems

- In a positional number system, there are only a few symbols called digits, and these symbols represent different values depending on the position they occupy in the number. The value of each digit in such a number is determined by three considerations:

  - the digit itself
  - the position of the digit in the number and
  - the **base of the number system**

- **What is base of a number system?**
  - **The number of different symbols available in the number system is known as base of the number system**

Positional Number Systems

Decimal Number System

Binary Number System

Octal Number System

Hexadecimal Number System

Decimal Number System

- The number system that we use in our day to day life is called the Decimal Number System.

- We have ten symbols or digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) that can be used in this number system.
- See the bellow example, the position of each digit can significantly change the value of the number

Example: 1234_{10}

Binary Number System

- The binary number system is exactly like the decimal design except that the base is 2 instead of 10.

- We have only two symbols or digits (0 and 1) that can be used in this number system

Example: 1010_2

Here is First 16 binary numbers

Decimal Number	4-bit Binary Number	Hexadecimal Number
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Why Binary ?

- **The reasons as follows :**

  - The first and the foremost reason is that electronic and electrical components, by their very nature, operate in a binary mode.

  - Information is handled in the computer by electrical/ electronic components such as transistors, semiconductors, wires, etc. all of which can only indicate two states or conditions- on (1) or off (0).

  - Transistors are either conducting (1) or non-conducting (0); magnetic materials are either magnetized (1) non- magnetized (0) in one direction; a pulse or voltage is present (1) or not present (0) in wire.

Why Binary

- All information is represented within the computer by the presence or absence of these various signals.

- The binary number system, which has only two digit (o and 1), is most suitable and is conveniently used to express the two possible states.

- The second reason is that computer circuits only have to handle two binary digits rather than ten decimal digits. The result is that the internal circuit design of computers is simplified to a great extent. This ultimately results in less expensive and more reliable circuits for computers.

Octal Number System

- In the octal number system the base is 8.
- So in this system there are eight symbols or digits : 0, 1, 2, 3, 4, 5, 6 and 7

Example : 2057_8

Decimal Number	3-bit Binary Number	Octal Number
0	000	0
1	001	1
2	010	2
3	011	3
4	100	4
5	101	5
6	110	6
7	111	7

Hexadecimal Number System

- The Hexadecimal Number System is one with a base of 16. The base of 16 suggests choices of 16 single character digits or symbols. Then first 10 digits are the digits of a decimal system 0, 1, 2, 3, 4, 5, 6, 7, 8, ,9. The remaining six digits are denoted by A, B,C, D, E, F representing the decimal values 10, 11, 12, 13, 14, 15 respectively.

Example : $69BBA_{16}$

Decimal Number	4-bit Binary Number	Hexadecimal Number
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Converting One Number System to Another

- Converting to decimal from another base

Formula to converting from any base to decimal
(d_i: is the ith digit of the number starting from i = 0)

$$d_{n-1}d_{n-2}\ldots d_2d_1d_{0\ (\text{in base b})} = d_{n-1}xb^{n-1} + d_{n-2}xb^{n-2} + \ldots + d_2xb^2 + d_1xb + d_0$$

- **Example:**

$314_5 = 3 \times 5^2 + 1 \times 5^1 + 4 \times 5^0 = 84_{10}$

Try:

1. $1110101_2 = ?_{10}$
2. $4706_8 = ?_{10}$
3. $1AC1_{16} = ?_{10}$
4. $4052_6 = ?_{10}$
5. $11001_4 = ?_{10}$
6. $1AC1_{13} = ?_{10}$

Answers to the exercises

- $1110101_2 = 117_{10}$
- $4706_8 = 2502_{10}$
- $1AC1_{16} = 6849_{10}$
- $4052_6 = 896_{10}$
- $11001_4 = 321_{10}$
- $1AC1_{13} = 4044_{10}$

Converting One Number System to Another

- Converting from base 10 to a new base
  (division- remainder technique)
  $117_{10} = ?_2$

Step	**Operation**	**Result (n/base)**	**Remainder (n%base)**
Step 1	117 / 2	58	1
Step 2	58 / 2	29	0
Step 3	29/ 2	14	1
Step 4	14 / 2	7	0
Step 5	7 / 2	3	1
Step 6	3/2	1	1
Step 7	1/2	0	1

Result: Read the reminders in reverse order 1110101_2

Converting One Number System to Another

- Converting from base 10 to a new base

  (division- remainder technique)

Try:. 1. $381_{10} = ?_{16}$

2. $175_{10} = ?_{3}$

3. $1792_{10} = ?_{16}$

4. $48879_{10} = ?_{16}$

Answers to the exercises

e.g. 1. $381_{10} = 17D_{16}$

2. $175_{10} = 20111_3$

3. $1792_{10} = 700_{16}$

4. $48879_{10} = BEEF_{16}$

Converting One Number System to Another

- Converting from a base other than 10 to base other than 10
- Example: 1. $545_6 = ?_4$

  2. $10110_2 = ?_8$

- ***Step 1*** *– Convert the original number to a decimal number (base 10).*
- ***Step 2*** *– Convert the decimal number so obtained to the new base number.*
- You already know from the previous examples how to perform step 1 and then step2!

Answers to the Exercises

- $545_6 = 3101_4$
- $10110_2 = 26_8$

Shortcut method for Binary to Octal conversion (a must approach to learn!)

$101110_2 = ?_8$

101	110
5	6

1. Each octal digit will fit into 3 bits binary. So, segment the binary numbers by 3 bits from right to left.
2. Convert each 3 bits segment into Octal.
3. Take help from the following table during the conversation (use it as side note)

2^2	2^1	2^0
4	2	1
1 (add 4)	1(add 2)	0
So: 110 = 4+2 = 6		
1 (add 4)	0	1(add 1)
So: 101 = 4+1=5		

This is just a reference table to compare your conversion

Binary	Decimal
000	0
001	1
010	2
011	3
100	4
101	5
110	6
111	7

Ans: $101110_2 = 56_8$

Solve this using shortcut method

- $\underbrace{010}_{2}\underbrace{110}_{6}{}_2 = ?_8$

Ans: 26_8

side note

4 2 1

1 1 0 = 6

0 1 0 = 2

Shortcut method for octal to binary conversion

Example: $562_8 = ?_2$

1. Convert each of the octal digit into 3 bit binary number from right to left
2. Take help of the following table during conversion.

2^2	2^1	2^0
4	2	1

Shortcut method for octal to binary conversion

$$562_8 = ?_2$$

$\underbrace{101}\ \underbrace{110}\ \underbrace{010}$

Ans: $101\ 110\ 010_2$

Side note

4	2	1	
0	1	0	= 2
1	1	0	= 6
1	0	1	= 5

Shortcut method for binary to hexadecimal conversion

Example: $11010011_2=?_{16}$

Each hexa decimal digit is converted into 4 bits binary as 4 bits are enough to represent any digit of hexa decimal number.
The conversion steps are similar to conversion from binary to Octal, but now it is 4 bit (instead of 3 bit)

1101	0011
D	3

2^3	2^2	2^1	2^0
8	4	2	1

3 = 0 0 1 1

13 = 1 1 0 1

8 + 4 + 1

$11010011_2=D3_{16}$

Shortcut method for hexadecimal to binary conversion

e.g. 1. $3AD_{16}=?_2$

2. $ABC_{16}=?_2$

2^3	2^2	2^1	2^0
8	4	2	1

Shortcut method for hexadecimal to binary conversion

$3AD_{16} = ?_2$

0011 1010 1101

8 4 2 1

13 ⇒ 1 1 0 1

10 ⇒ 1 0 1 0

3 ⇒ 0 0 1 1

Ans: 0011 1010 1101

Converting between bases that are Perfect powers of 2

- **If you are converting between two bases that are perfect powers of 2, the following procedure works more quickly:**

1) Convert from base B_1 to base 2.

2) Convert from base 2 to base B_2.

$$\mathbf{A3D_{16} = \underline{\qquad\quad}_{8}}$$

What is a ‘bit’ in computer terminology

- “*Binary digit*” is often to referred to by the common abbreviation bit. Hence, a bit in computer terminology means either a 0 or a 1.
- Bit Single Binary Digit (1 or 0)
- Byte = 8 bits
- Kilobyte (KB) = 1,024 Bytes
- Megabyte (MB) = 1,024 Kilobytes
- Gigabyte (GB) = 1,024 Megabytes
- Terabyte (TB) = 1,024 Gigabytes
- Petabyte (PB) = 1,024 Terabytes
- Exabyte (EB) = 1,024 Petabytes
- A binary number consisting of ‘n’ bits is called n-bit number.

‘bit’ in computer terminology

- How many different patterns of bits are possible with n-bits?

Examples: 1. 6 bits

2. 7 bits

3. 8 bits

Writing code for base conversion (You don’t have to submit it)

- Write code to convert a binary number to decimal number.
  - Take the binary number as input as string
  - Read char by char in a loop and based on the position use the formula we have discussed
  - While reading the character convert ‘1’ to integer 1 by ‘1’-’0’.

- Helpful links:
  - For conversion: https://www.tutorialspoint.com/computer_logical_organization/digital_number_system.htm
  - https://www.tutorialspoint.com/computer_logical_organization/number_system_conversion.htm
  - Exercise: http://www.free-test-online.com/binary/binary2hex.htm