

$$z = wx + b$$

$$y = \frac{1}{1 + e^{-z}}$$

$$\mathcal{L}(y, t) = -t \log(y) + (t - 1) \log(1 - y)$$

$$\frac{\partial \mathcal{L}}{\partial y} = \frac{-t}{y} + \frac{t - 1}{1 - y}$$

$$\frac{dy}{dz} = \frac{e^{-z}}{(1 + e^{-z})^2}$$

$$\frac{\partial z}{\partial w} = x$$

$$\Rightarrow \frac{\partial \mathcal{L}}{\partial w} = \left( \frac{-t}{y} + \frac{t - 1}{1 - y} \right) \left( \frac{e^{-z}}{(1 + e^{-z})^2} \right) x$$

$$\Rightarrow \frac{\partial \mathcal{L}}{\partial b} = \left( \frac{-t}{y} + \frac{t - 1}{1 - y} \right) \left( \frac{e^{-z}}{(1 + e^{-z})^2} \right)$$