Problem 1 William Chen

1. $Y = AC + \bar{A}\bar{B}C = C(A + \bar{A}\bar{B}) = C(A + A\bar{B} + \bar{A}\bar{B})$

$A = A + A\bar{B}$

$= C(A + \bar{B}(A + \bar{A})) = C(A + \bar{B})$

1

Truth Table

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

Karnaugh Map

AB \ C	0	1
00	0	1
01	0	0
11	0	1
10	0	1

2. $Y = \bar{A}\bar{B} + \bar{A}B\bar{C} + (\overline{A + \bar{C}})$

Demorgan's

$= \bar{A}\bar{B} + \bar{A}B\bar{C} + \bar{A}C = \bar{A}(\bar{B} + B\bar{C} + C)$

$C = C + CB$

$= \bar{A}(\bar{B} + B\bar{C} + C + CB) = \bar{A}(C + B(C + \bar{C}) + \bar{B})$

$= \bar{A}(C + B + \bar{B}) = \bar{A}(C + 1) = \bar{A}$

Truth Table

A	B	C	Y
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

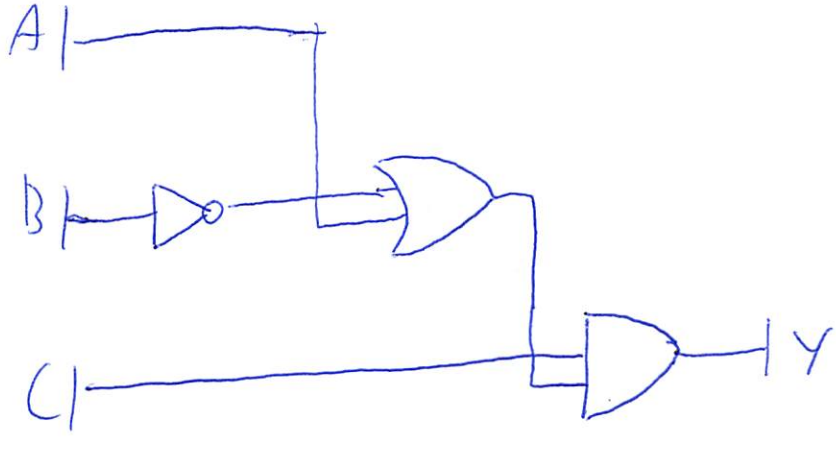
Karnaugh Map

AB \ C	0	1
00	1	1
01	1	1
11	0	0
10	0	0

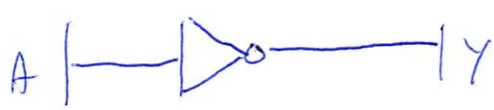
Problem 1

William Chen

1.



2.



Problem 2

William Chen

A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

SOP:

$Y = \bar{A}\bar{B}\bar{C} + ABC$

A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

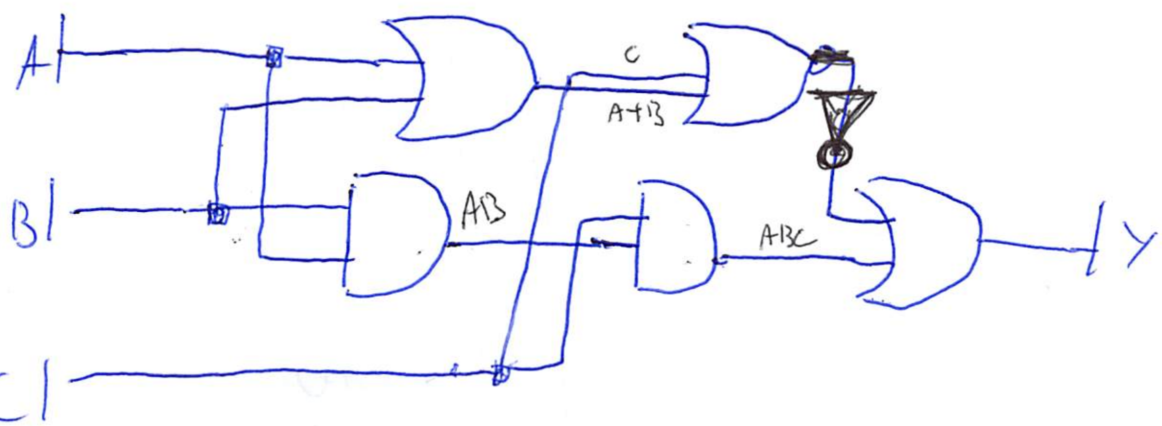
SOP:

$Y = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + A\bar{B}\bar{C} + AB\bar{C}$

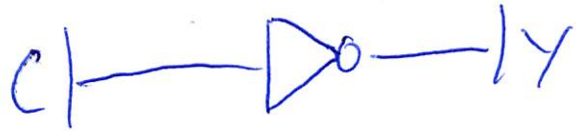
Problem 3

1. $Y = \bar{A}\bar{B}\bar{C} + ABC$

$\bar{A}\bar{B}\bar{C} = \overline{(A \cdot B \cdot C)} = \overline{(A + \overline{BC})}$
$= \overline{(A+B+C)}$



2. $Y = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + A\bar{B}\bar{C} + AB\bar{C}$
$= \bar{B}\bar{C}(\bar{A}+A) + B\bar{C}(\bar{A}+A)$
$= \bar{B}\bar{C} + B\bar{C} = \bar{C}(\bar{B}+B) = \bar{C}$



Problem 4

William Chen

	x_1	x_2	x_3	$f(x_1,x_2,x_3)$	$\bar{f}(x_1,x_2,x_3)$
0	0	0	0	1	0
1	0	0	1	1	0
2	0	1	0	1	0
3	0	1	1	1	0
4	1	0	0	1	0
5	1	0	1	0	1
6	1	1	0	1	0
7	1	1	1	1	0

$$\bar{f}(x_1, x_2, x_3) = x_1 \cdot \bar{x_2} \cdot x_3 \quad \cancel{= \overline{f(\bar{x_1} + x_2 + \bar{x_3})}}$$

$$f = \bar{\bar{f}}(x_1, x_2, x_3) = \overline{x_1 \bar{x_2} x_3} = \bar{x_1} + x_2 + \bar{x_3}$$

Karnaugh Map

AB \ C	0	1
00	1	1
01	1	1
11	1	1
10	1	0

Alternate

$$f(x_1,x_2,x_3) = \bar{x}_1\bar{x}_2\bar{x}_3 + \bar{x}_1\bar{x}_2x_3 + \bar{x}_1x_2\bar{x}_3 + \bar{x}_1x_2x_3 + x_1\bar{x}_2\bar{x}_3$$
$$+ x_1x_2\bar{x}_3 + x_1x_2x_3$$

Problem 5 William Chen

SOP:

$$f(x_1, x_2, x_3) = \bar{x}_1 \bar{x}_2 x_3 + x_1 \bar{x}_2 \bar{x}_3 + x_1 \bar{x}_2 x_3 + x_1 x_2 \bar{x}_3$$

POS:

$$f(x_1, x_2, x_3) = (x_1 + x_2 + x_3)(x_1 + \bar{x}_2 + x_3)(x_1 + \bar{x}_2 + \bar{x}_3)(\bar{x}_1 + \bar{x}_2 + \bar{x}_3)$$

Minimal cost:

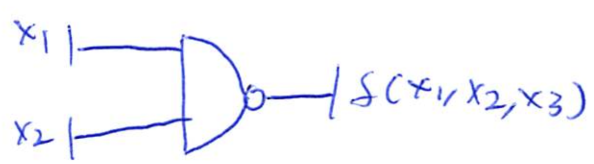
$$\begin{aligned} f(x_1, x_2, x_3) &= \bar{x}_1 \bar{x}_2 x_3 + x_1 \bar{x}_2 \bar{x}_3 + x_1 \bar{x}_2 x_3 + x_1 x_2 \bar{x}_3 \\ &= x_3(\bar{x}_1 \bar{x}_2 + x_1 \bar{x}_2) + \bar{x}_3(x_1 \bar{x}_2 + x_1 x_2) \\ &= x_3(\bar{x}_2(x_1 + \bar{x}_1)) + \bar{x}_3(x_1(\bar{x}_2 + x_2)) \\ &= x_3 \bar{x}_2 + \bar{x}_3 x_1 = x_1 \bar{x}_3 + \bar{x}_2 x_3 \end{aligned}$$

Karnaugh map

$x_1x_2 \backslash x_3$	0	1
00	1	X
01	1	1
11	0	0
10	1	X

SOP:

$$f(x_1, x_2, x_3) = \bar{x}_1 + \bar{x}_2 = \overline{x_1 x_2}$$



Problem 6

William Chen

Karnaugh Map

AB \ CD	00	01	11	10
00	X	X	0	0
01	0	X	X	0
11	1	1	1	X
10	1	0	1	X

$Y = AB + A\bar{C}\bar{D} + AC$

$= A(B + \bar{C}\bar{D} + C)$ $\quad \bar{C}\bar{D} + C \to C + \bar{D}$

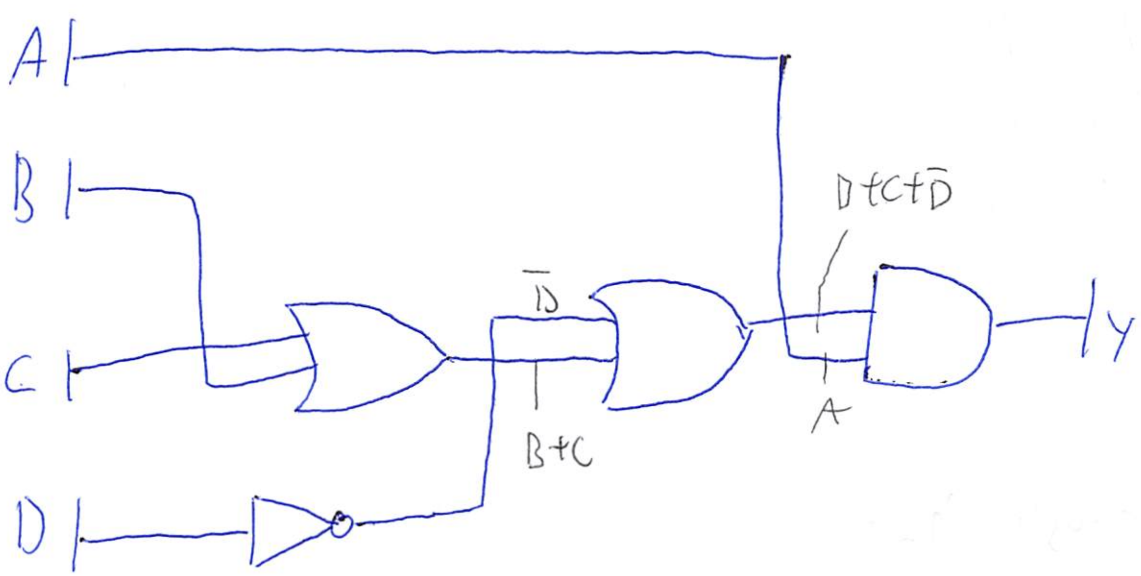
$= A(B + C + \bar{D})$

$= AC + AB + A\bar{D}$

$Y = AC + AB + A\bar{D}$

$= A(C + B + \bar{D})$

Circuit



Problem 7

$$P = \overline{A}\,B\,\overline{C} + \overline{B\,\overline{C}} = \overline{A}\,B\,\overline{C} + \overline{B} + C = \overline{A} + \overline{B} + C$$

Truth Table

A	B	C	Y
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

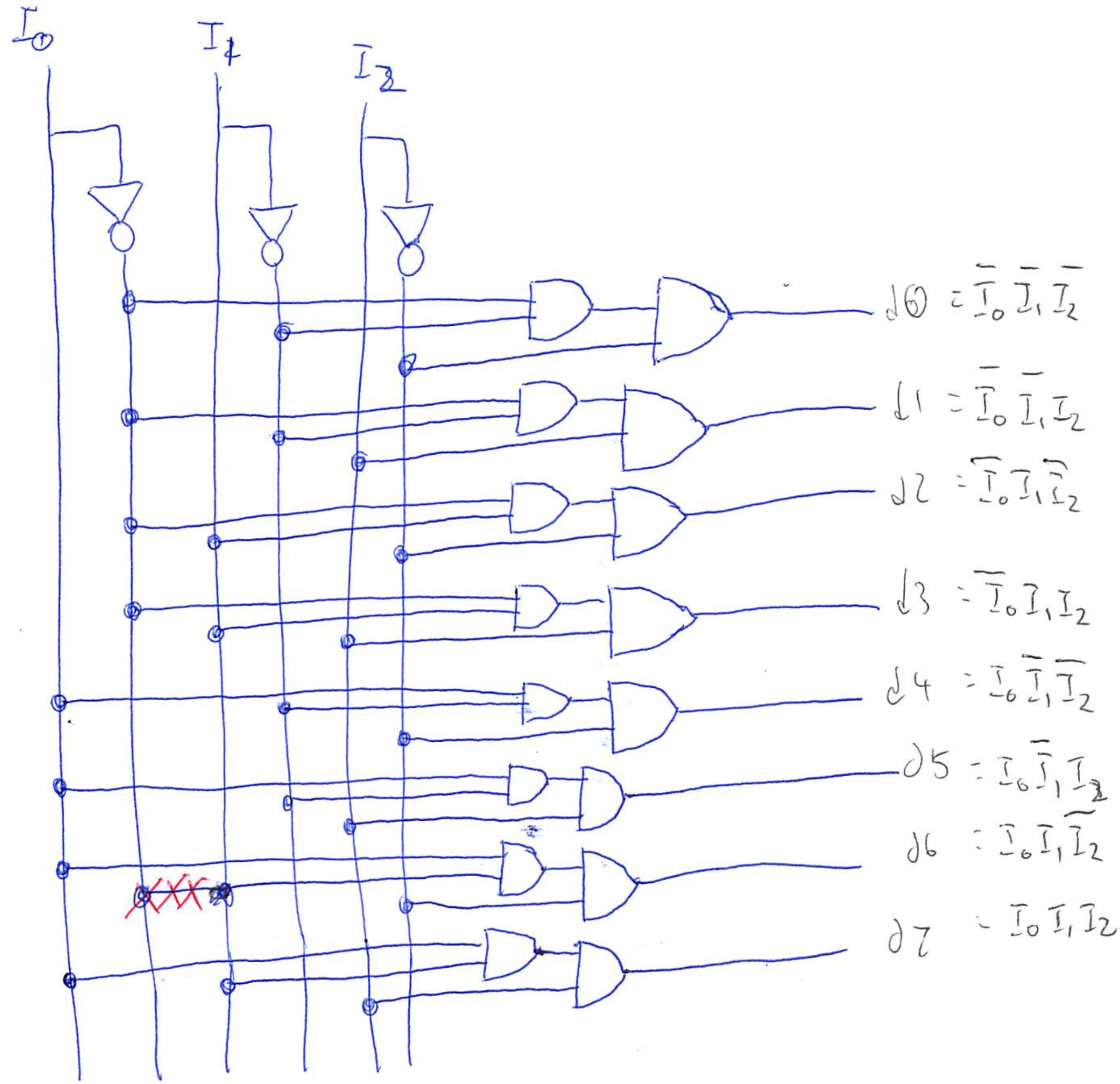
Karnaugh Map

AB \ C	0	1
00	1	1
01	1	1
11	0	1
10	1	1

$\cancel{P=\overline{A}B}$

$P = \overline{A} + \overline{B} + C$

Problem 8 William Chen



19 Gates Total:

16 AND Gates

3 NOT Gates

For an n-2^n decoder, $\boxed{n}$ NOT Gates and $\boxed{2^n \cdot (n-1)}$ AND Gates

Problem 9

William Chen

$$f = \bar{C}\bar{D}\underset{\overset{\downarrow}{1}}{I_1} + \bar{C}D\underset{\overset{\downarrow}{0}}{I_2} + C\bar{D}\underset{\overset{\downarrow}{0}}{I_3} + CD\underset{\overset{\downarrow}{0}}{I_4}$$

$$f = \bar{C}\bar{D}$$

$$Y = \bar{A}\bar{B}\underset{\overset{\downarrow}{1}}{I_1} + \bar{A}B\underset{\overset{\downarrow}{\bar{C}\bar{D}}}{I_2} + A\bar{B}\underset{\overset{\downarrow}{\bar{C}\bar{D}}}{I_3} + AB\underset{\overset{\downarrow}{0}}{I_4}$$

$$Y = \bar{A}\bar{B} + \bar{C}\bar{D}(\bar{A}B + A\bar{B})$$

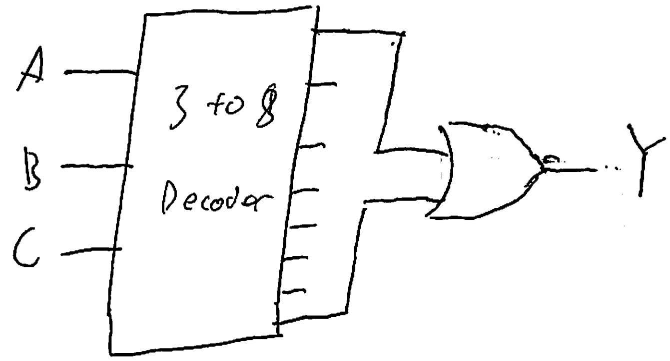
~~Problem 10~~

William Chen

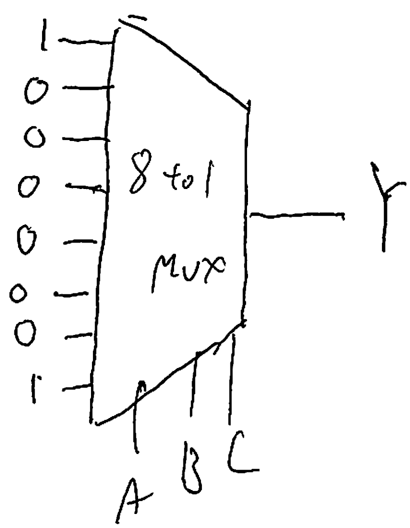
$Y = \bar{A}\bar{B}\bar{C} + ABC$

Problem 10

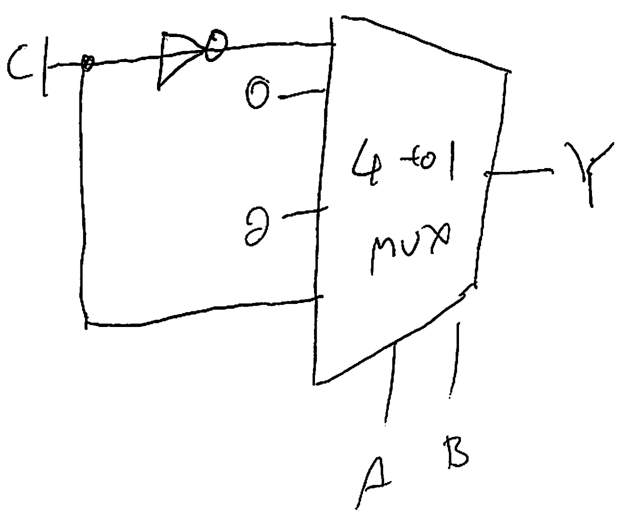
1.



2.

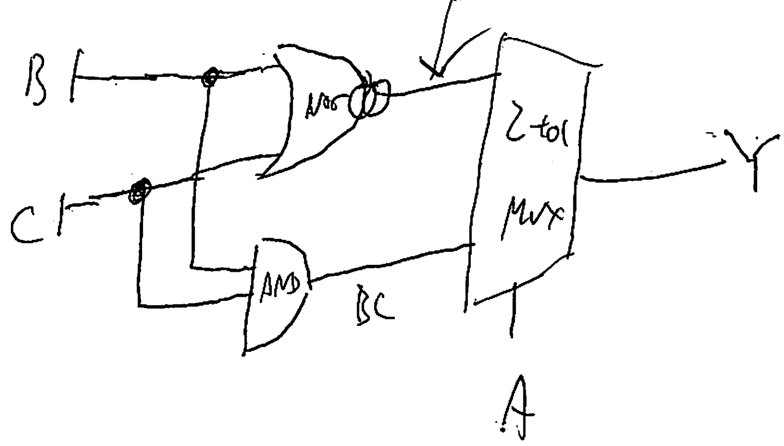


3.



4.

$\overline{(B+C)} = \bar{B}\bar{C}$



Problem 11

William Chen

$f = \bar{C}\bar{D}I_1 + \bar{C}DI_2 + C\bar{D}I_3 + CDI_4$

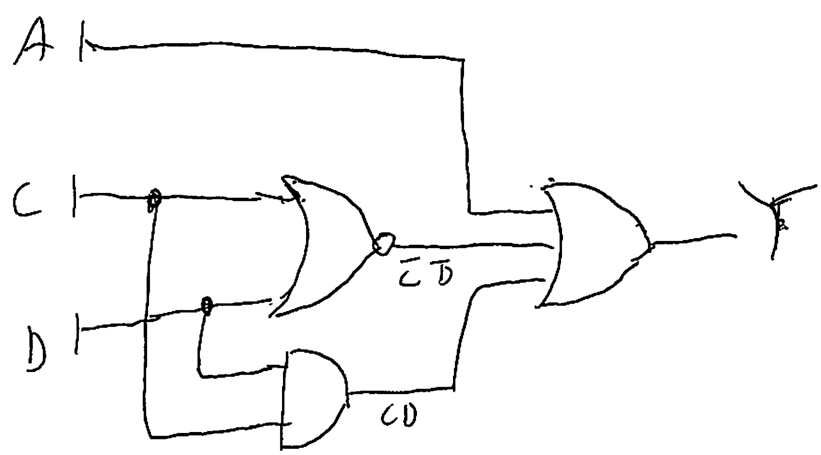
$I_1 = 1,\quad I_2 \to 0,\quad I_3 \to 0,\quad I_4 = 1$

$f = \bar{C}\bar{D} + CD$

$Y = \bar{A}f + AI_2$

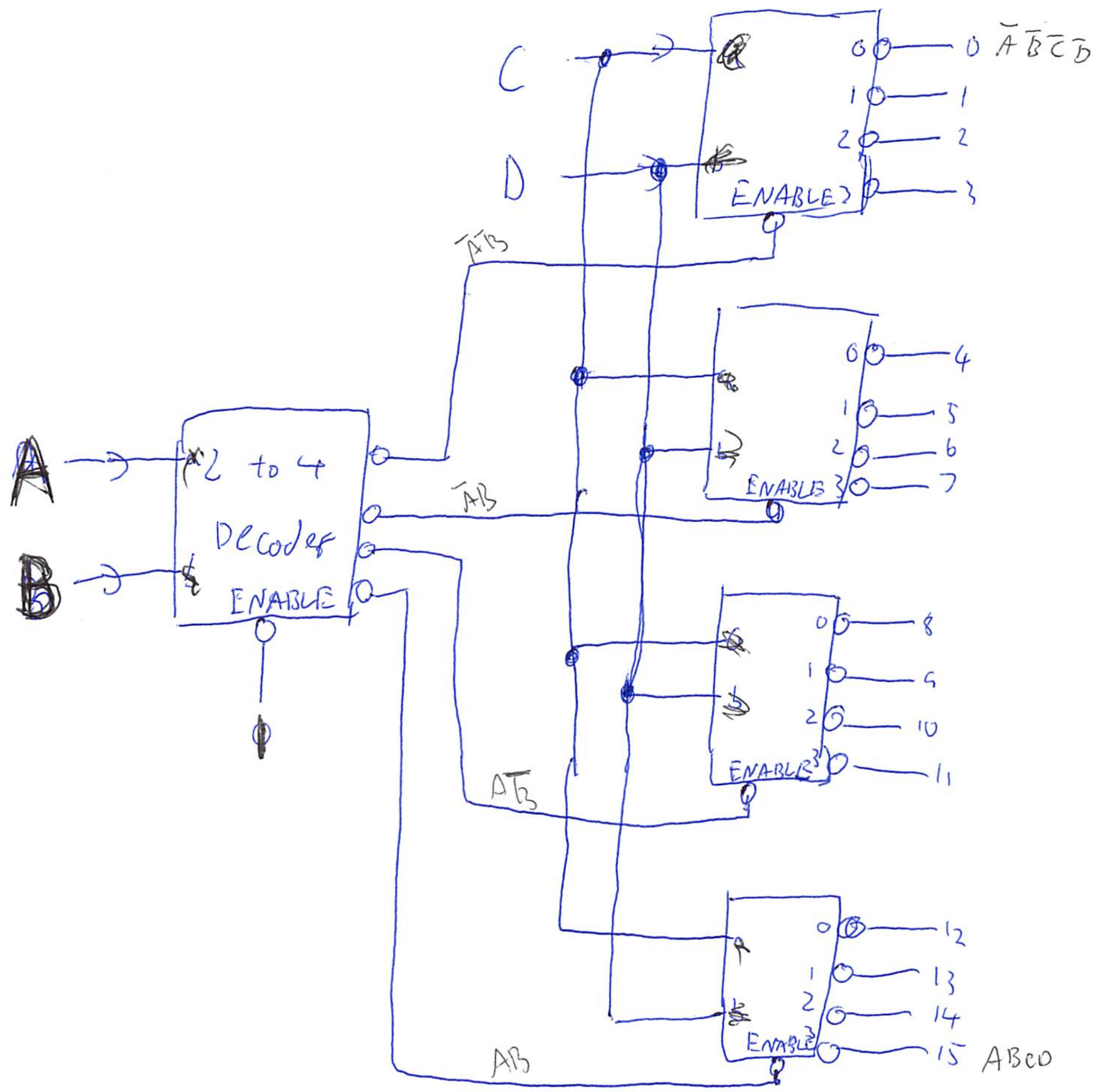
$I_2 \to 1$

$Y = \bar{A}(\bar{C}\bar{D} + CD) + A = A + CD + \bar{C}\bar{D}$



Problem 12

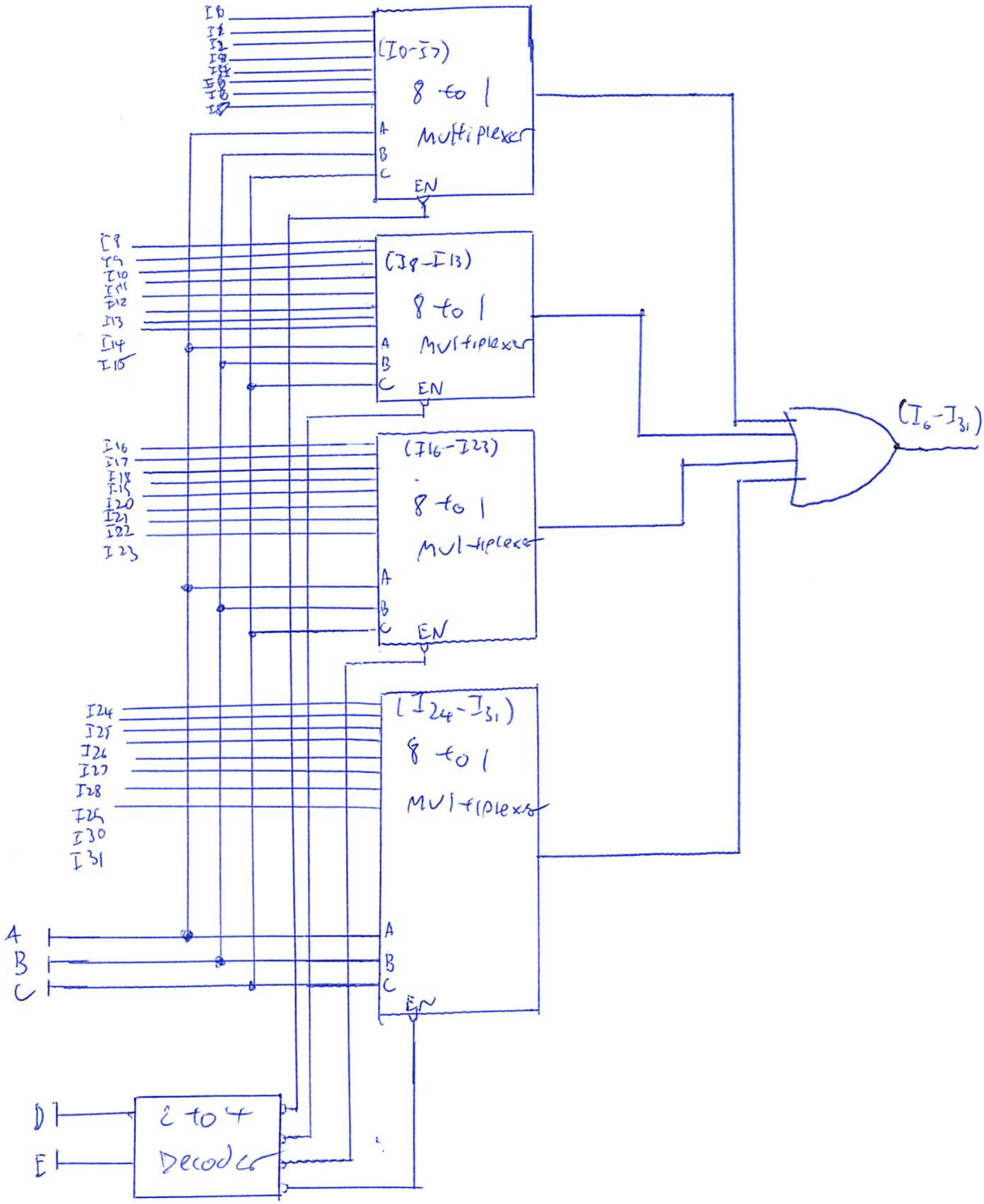
William Chen



Problem 13

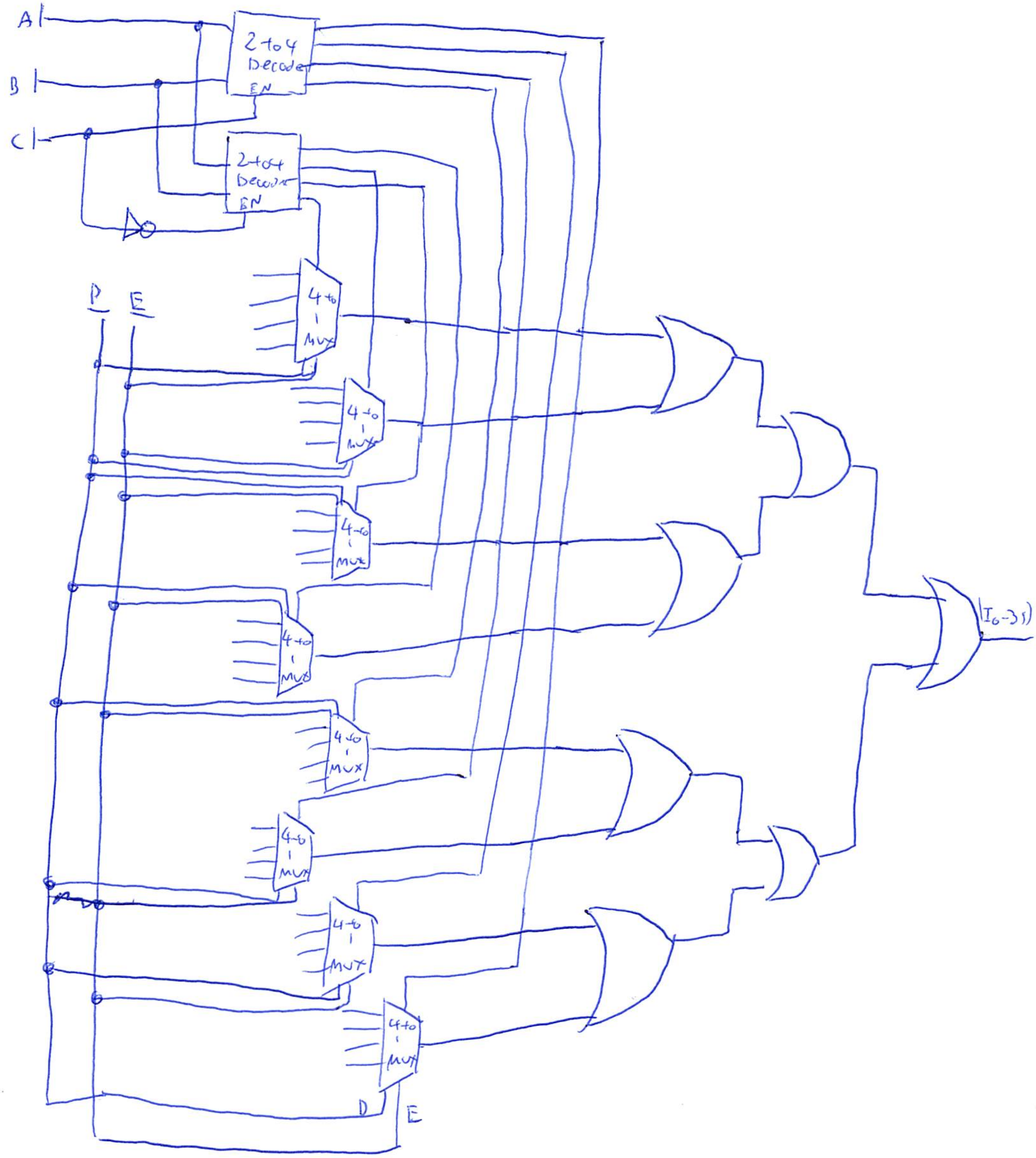
William Chen

1.



Problem 13 William Chen

2.



Problem 14-1

William Chen

Truth Table for $f(x_1, x_2, x_3) = \sum M(0, 2, 4, 6, 7)$

x_1	x_2	x_3	$f(x_1, x_2, x_3)$	
0	0	0	1	(0)
0	0	1	0	(1)
0	1	0	1	(2)
0	1	1	0	(3)
1	0	0	1	(4)
1	0	1	0	(5)
1	1	0	1	(6)
1	1	1	1	(7)

Logic Expression

$$f(x_1, x_2, x_3) = \bar{x}_1\bar{x}_2\bar{x}_3 + \bar{x}_1 x_2 \bar{x}_3 + x_1\bar{x}_2\bar{x}_3 + x_1 x_2 \bar{x}_3 + x_1 x_2 x_3$$

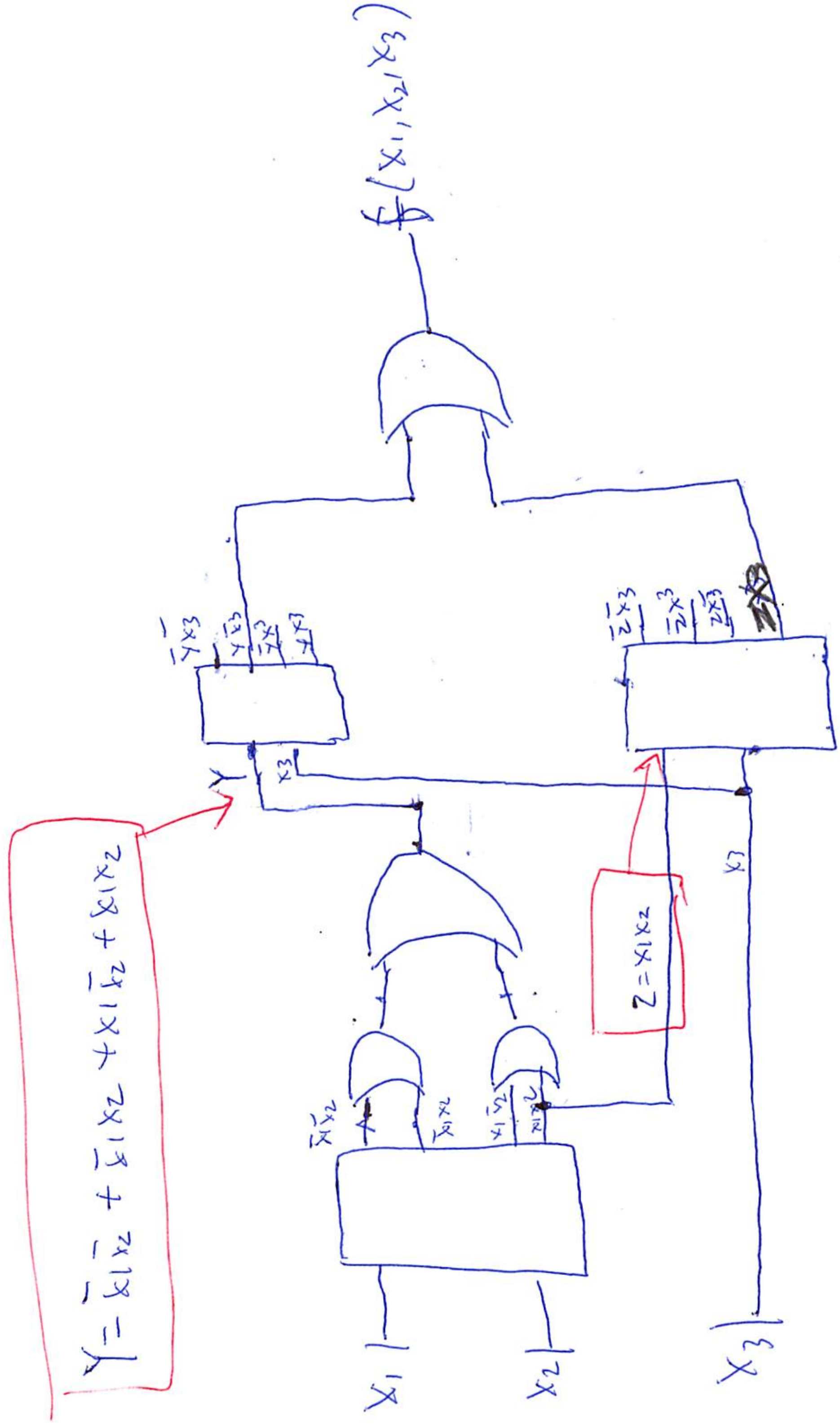
Question 1:

- simplist expression into:

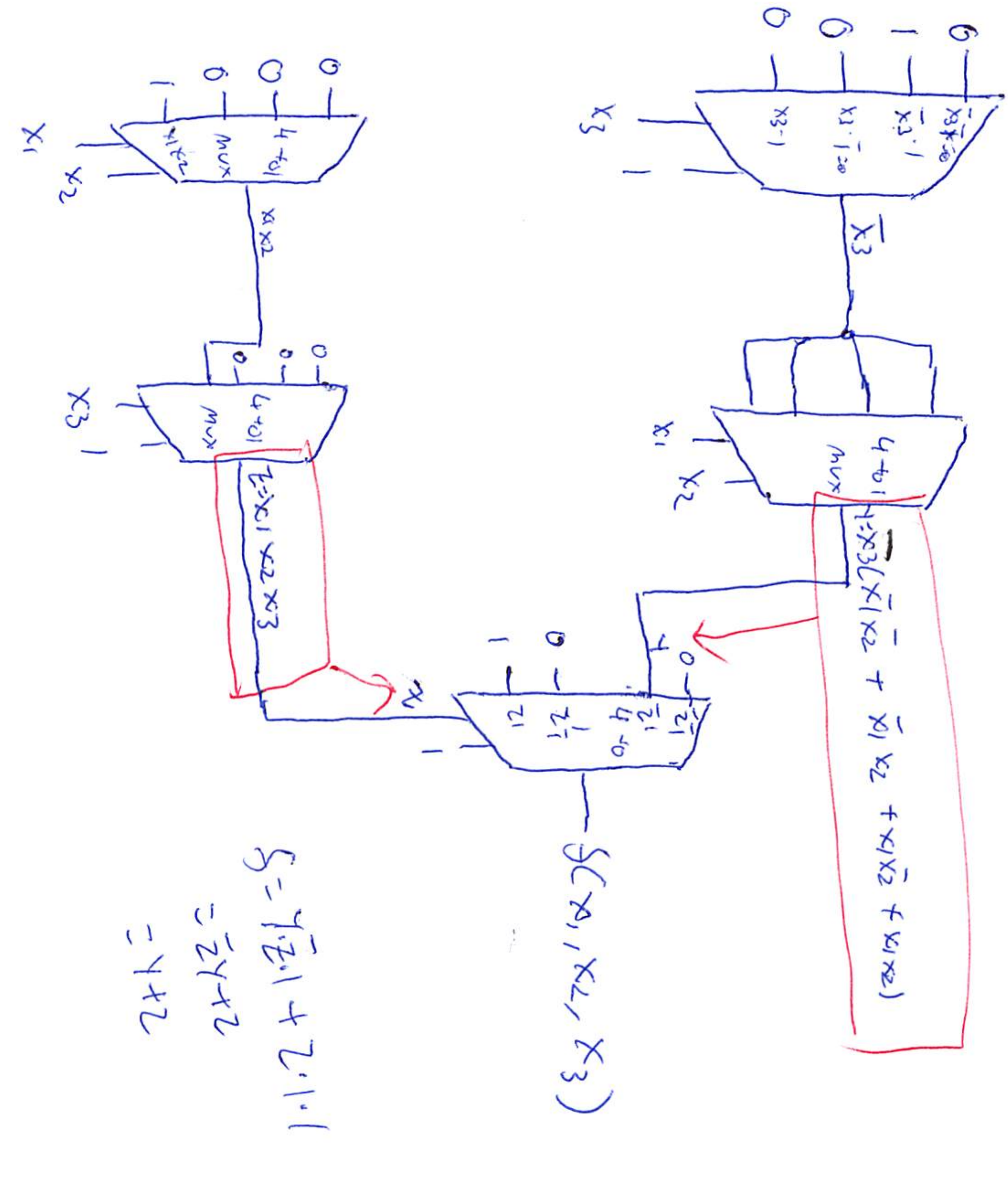
$$f = (\bar{x}_1\bar{x}_2 + \bar{x}_1 x_2 + x_1\bar{x}_2 + x_1 x_2)\bar{x}_3 + x_1 x_2 x_3$$

Problem 14-2

William Chen



Problem 14-3



Problem 15

William Chen

$$f(x_1,x_2,x_3,x_4,x_5) = \bar{x}_1\bar{x}_2\bar{x}_4\bar{x}_5 + x_1x_2 + x_1x_3 + x_1x_4 + x_3x_4x_5$$

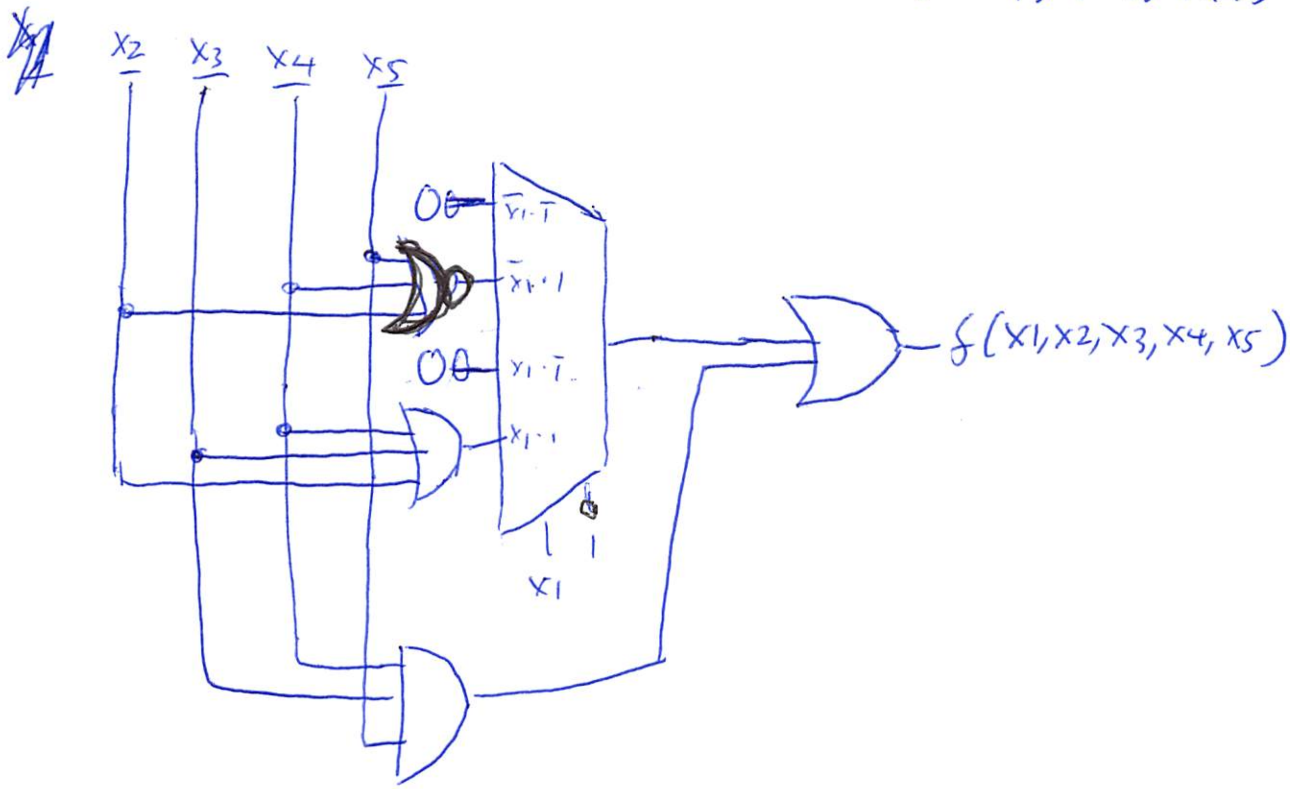
$$= \bar{x}_1(\bar{x}_2\bar{x}_4\bar{x}_5) + x_1(x_2+x_3+x_4) + x_3x_4x_5$$

$$= \bar{x}_1\left(\overline{\overline{\bar{x}_2(\bar{x}_4\bar{x}_5)}}\right) + x_1(x_2+x_3+x_4) + x_3x_4x_5$$

$$= \bar{x}_1\left(\overline{(\bar{\bar{x}}_2 + (\overline{\bar{x}_4\bar{x}_5})}\right) + x_1(x_2+x_3+x_4) + x_3x_4x_5$$

$$= \bar{x}_1\left(\overline{x_2 + \bar{\bar{x}}_4 + \bar{\bar{x}}_5}\right) + x_1(x_2+x_3+x_4) + x_3x_4x_5$$

$$= \bar{x}_1\left(\overline{x_2 + x_4 + x_5}\right) + x_1(x_2+x_3+x_4) + x_3x_4x_5$$



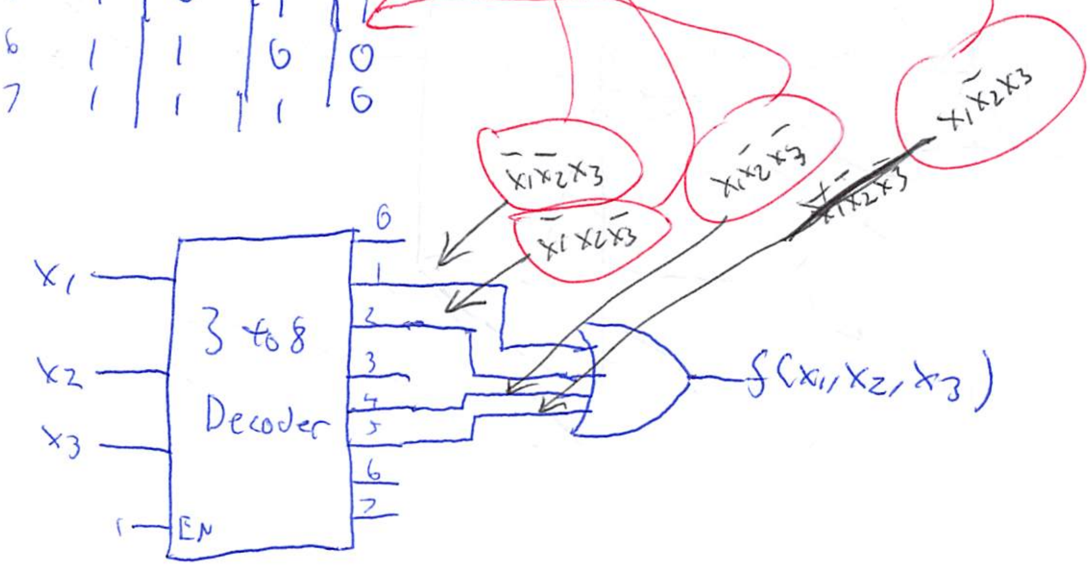
Problem 16

$$f(x_1, x_2, x_3) = x_1\bar{x}_2 + \bar{x}_2 x_3 + \bar{x}_1 x_2 \bar{x}_3$$

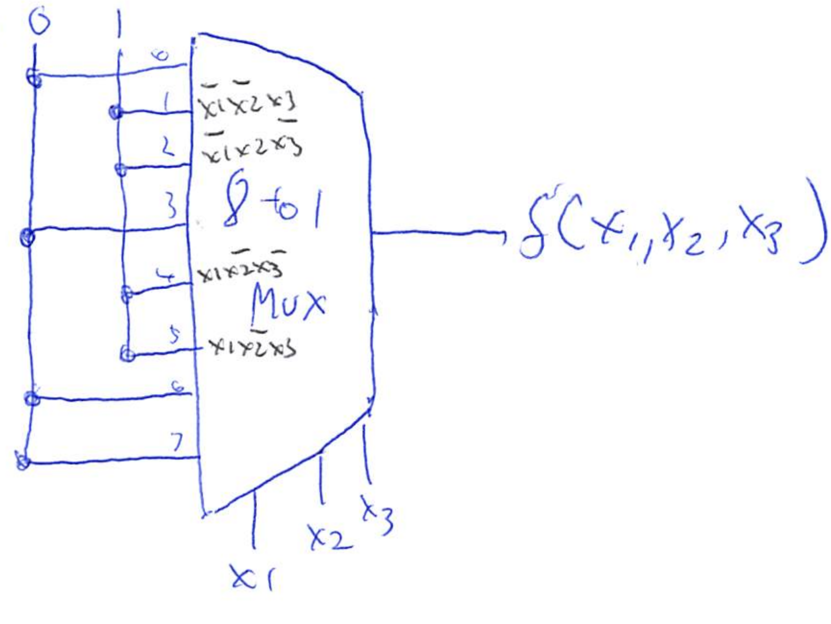
Truth Table

	x_1	x_2	x_3	$f(x_1,x_2,x_3)$
0	0	0	0	0
1	0	0	1	1
2	0	1	0	1
3	0	1	1	1
4	1	0	0	0
5	1	0	1	1
6	1	1	0	0
7	1	1	1	0

1.



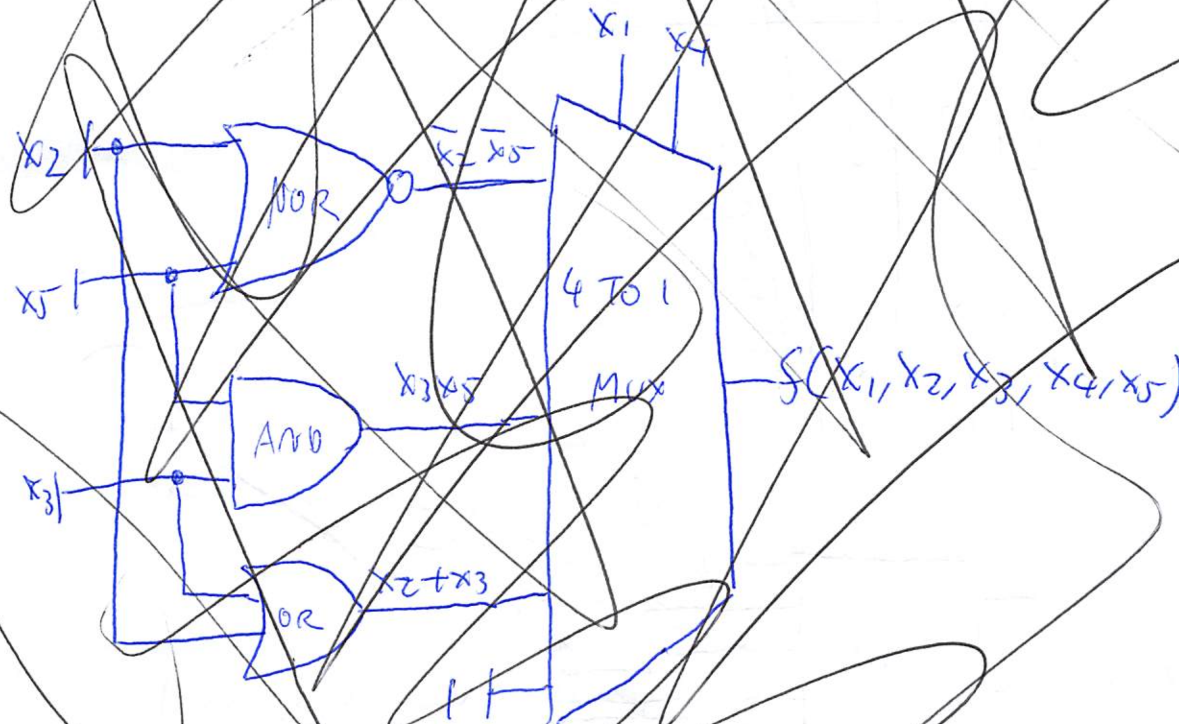
2.



Problem 15

$$f(x_1,x_2,x_3,x_4,x_5) = \bar{x}_1\bar{x}_2\bar{x}_4\bar{x}_5 + x_1x_2 + x_1x_3 + x_1x_4 + x_3x_4x_5$$

$$= \bar{x}_1\bar{x}_4(\bar{x}_2\bar{x}_5) + \bar{x}_1x_4(x_3x_5) + x_1\bar{x}_4(x_2+x_3) + x_1x_2$$



William Chen

Problem 17-1

Truth Table:

Current State				Next State		
X_1	X_2	X_3	Input	Y_1	Y_2	Y_3
0	0	0		0	0	1
0	0	1		0	1	1
0	1	1		0	1	0
0	1	0		1	1	0
1	1	0		1	1	1
1	1	1		1	0	1
1	0	1		1	0	0
1	0	0		0	0	0

$Y_1(X_1, X_2, X_3) = X_2 \overline{X_3} + X_1 X_3$

$X_1X_2 \backslash X_3$	0	1
00	0	0
01	1	0
1ϕ	1	
10	0	1

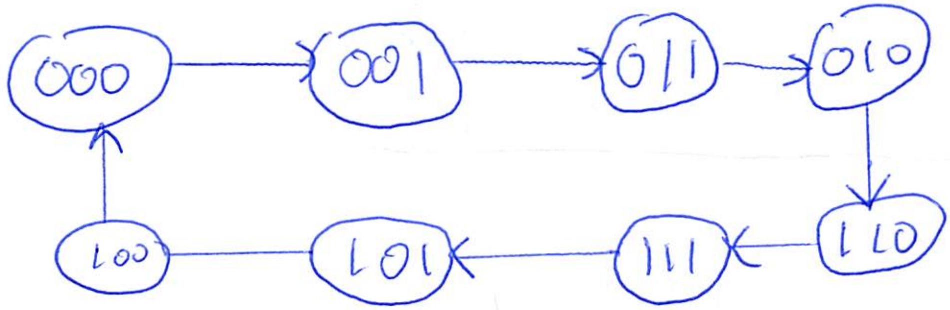
$Y_2(X_1, X_2, X_3) = X_2 \overline{X_3} + \overline{X_1} X_3$

$X_1X_2 \backslash X_3$	0	1
00	0	ϕ
01	1	1
11	1	0
10	0	0

$Y_3(X_1, X_2, X_3) = \overline{X_1}\,\overline{X_2} + X_1 X_2$

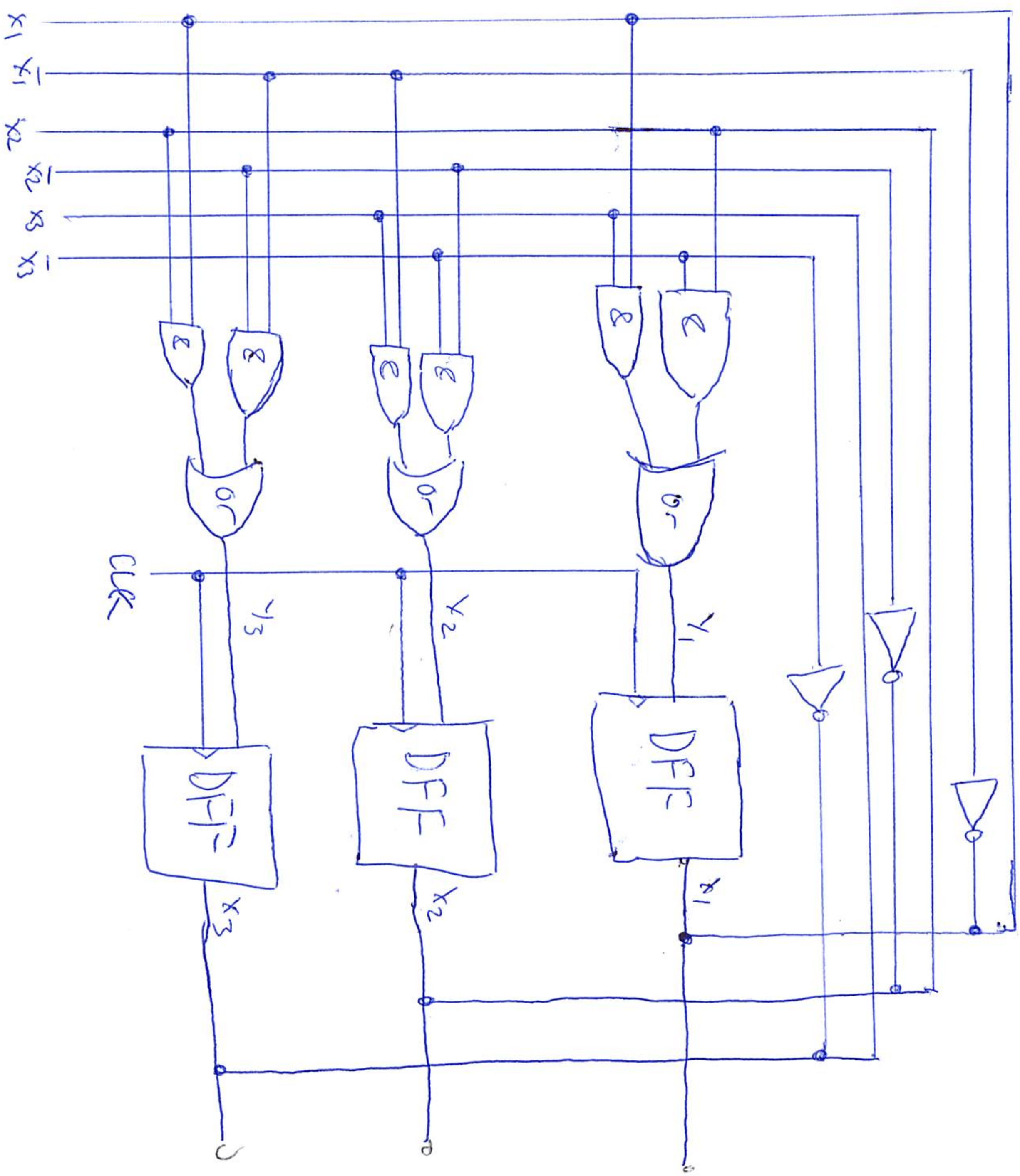
$X_1X_2 \backslash X_3$	0	1
00	1	1
01	0	0
11	1	1
10	0	0

Finite State Machine



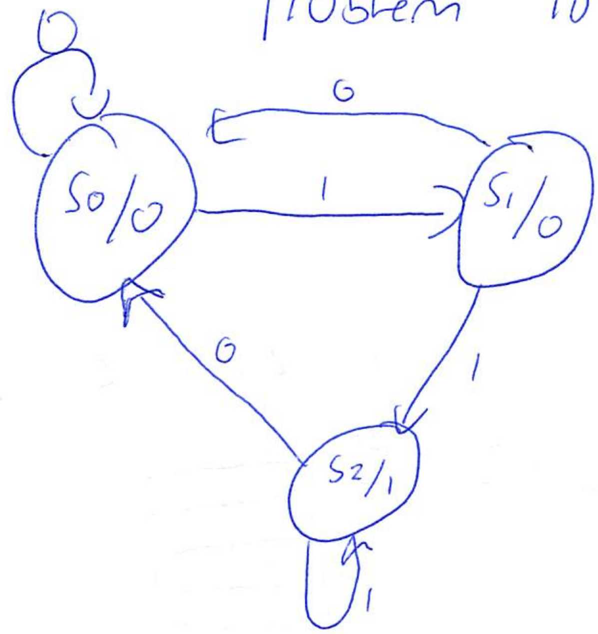
Problem 17-2

William Chen



Problem 18

William Chen



States:

S_0 - Last Input is 0

S_1 - There is a 1 after last zero

S_2 - There are 2 or more 1's after last zero

Input

I_0 : Input 0 on FSM

I_1 : Input 1 on FSM

State Transition Table

Current State	Input	Next State
S_0	I_0	S_0
S_1	I_0	S_0
S_2	I_0	S_0
S_0	I_1	S_1
S_1	I_1	S_2
S_2	I_1	S_2

State and Input

key:

$S_0 \to 00$ $I_0 \to 0$

$S_1 \to 01$ $I_1 \to 1$

$S_2 \to 10$

Current State A	B	Input I	Next State X	Y
0	0	0	0	0
0	1	0	0	0
1	1	0	x	x
1	0	0	0	0
0	0	1	0	1
0	1	1	1	0
1	1	1	x	x
1	0	1	1	0

$X = f(A, B, I) = AI + BI$

AB \ I	0	1
00	0	0
01	0	1
11	x	x
10	0	1

$Y = f(A, B, I) = I$

AB \ I	0	1
00	0	1
01	0	0
11	x	x
10	0	0