



RUTGERS
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Recitation 5

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One's Complement

- Represent negative numbers by complementing positive numbers
- It has two zero representation

000	001	010	011	100	101	110	111
0	1	2	3	-3	-2	-1	-0

Two's Complement

- Advantages – only 1 zero & convenient for arithmetic computation
- Flip the bits and add 1 (One's complement + 1)
- Ex)
 - $40 = 0010\ 1000$
 - > Flip $1101\ 0111$
 - > Add1 $1101\ 1000$ (-40 in two's complement form)

 - $-40 = 1101\ 1000$
 - > Flip $0010\ 0111$
 - > Add1 $0010\ 1000$ (two's complement of -40)

Two's complement

- What is the range that can represent with n bits?

$$[-2^{n-1}, 2^{n-1} - 1]$$

- More negative numbers than positive numbers
 - Since we only have 1 zero

000	001	010	011	100	101	110	111
0	1	2	3	-4	-3	-2	-1

Examples

- Convert these negative decimal into negative binary using 2's complement
- -192
- -16
- -1
- -0

Examples

- Convert these negative decimal into negative binary using 2's complement
- -192 \Rightarrow 0100 0000
- -16 \Rightarrow 1111 0000
- -1 \Rightarrow 1111 1111
- -0 \Rightarrow 0000 0000

Arithmetic of two's complement

- Arithmetic addition

$$\begin{array}{r} + 6 \quad 0000 \ 0110 \\ +13 \quad \underline{0000 \ 1101} \\ +19 \quad 0001 \ 0011 \end{array}$$

$$\begin{array}{r} - 6 \quad 1111 \ 1010 \\ +13 \quad \underline{0000 \ 1101} \\ + 7 \quad 0000 \ 0111 \end{array}$$

Arithmetic of two's complement

- Arithmetic subtraction

$$\begin{array}{r} -5 \quad 1111 \ 1011 \\ - \ 6 \quad 0000 \ 0110 \\ \hline -11 \quad 1111 \ 0101 \end{array}$$

$$\begin{array}{r} -5 \quad 1111 \ 1011 \\ - -6 \quad 1111 \ 1010 \\ \hline + \ 1 \quad 0000 \ 0001 \end{array}$$

Two's complement overflow

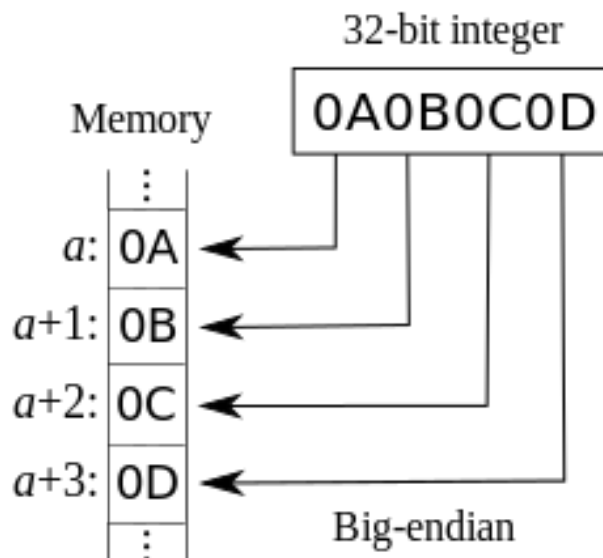
- It needs one extra bit but the sign bit will be wrong
- How to detect an overflow?
 - Adding 2 positive numbers -> But negative result
 - Adding 2 negative numbers -> But positive result

$$\begin{array}{rcl} 6 & 0110 & \\ + 5 & \underline{0101} & \\ -5 & 1011 & \end{array}$$

$$\begin{array}{rcl} -6 & 1010 & \\ + -6 & \underline{1010} & \\ 4 & 0100 & \end{array}$$

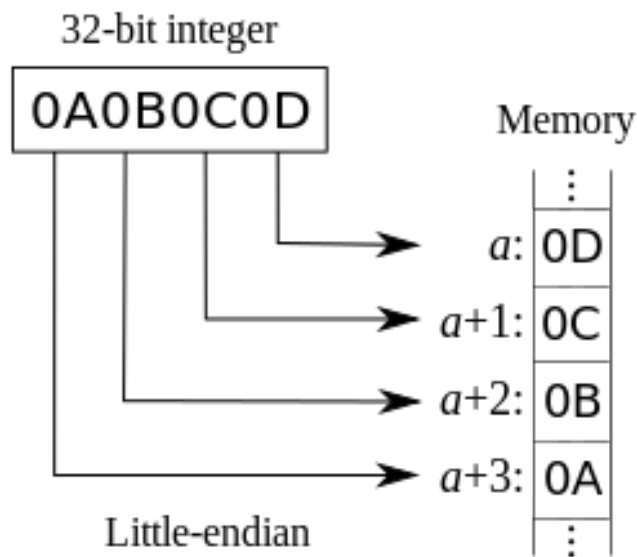
Endianness

- The order of the bytes stored in memory
- Big endian
 - MSB is stored at a particular address and the subsequent bytes are stored in the following higher memory addresses
 - LSB is stored at the highest memory address



Endianness

- Little endian
 - LSB is stored at the lower memory address and the subsequent bytes are stored in the following higher memory addresses
 - MSB is stored at the highest memory address



Endianness (Byte ordering example)

- Consider the following word (32 bit) of memory

Little Endian LSB
Big Endian MSB

Little Endian MSB
Big Endian LSB

	AB	CD	00	00
Memory address	0	1	2	3

- Big Endian interprets as
 - AB CD 00 00 (2882338816)
- Little Endian interprets as
 - 00 00 CD AB (52651)

Boolean Algebra

- Developed by George Boole in 19th Century
 - Algebraic representation of logic
 - Encode 1 as “True” and 0 as “False”

And

- $A \& B = 1$ when both $A=1$ and $B=1$

$\&$	0	1
0	0	0
1	0	1

Or

- $A | B = 1$ when either $A=1$ or $B=1$

$ $	0	1
0	0	1
1	1	1

Not

- $\sim A = 1$ when $A=0$

\sim	
0	1
1	0

Exclusive-Or (Xor)

- $A \wedge B = 1$ when either $A=1$ or $B=1$, but not both

\wedge	0	1
0	0	1
1	1	0

General Boolean Algebras

- Operate on Bit Vectors
 - Operations applied bitwise

01101001	01101001	01101001	01101001
& 01010101	01010101	^ 01010101	~ 01010101
<hr/>	<hr/>	<hr/>	<hr/>
01000001	01111101	00111100	10101010

- All of the properties of Boolean Algebra Apply

Example: Representing & Manipulating Sets

- Representation

- Width w bit vector represents subsets of $\{0, \dots, w-1\}$
- $a_j = 1$ if $j \in A$

- **01101001** $\{0, 3, 5, 6\}$

- 7**65**4**32**10

- **01010101** $\{0, 2, 4, 6\}$

- 7**65**4**32**10

- Operations

- & Intersection 01000001 $\{0, 6\}$
- | Union 01111101 $\{0, 2, 3, 4, 5, 6\}$
- ^ Symmetric difference 00111100 $\{2, 3, 4, 5\}$
- ~ Complement 10101010 $\{1, 3, 5, 7\}$

Bit-Level Operations in C

- Operations %, |, ~, ^ are available in C
 - Apply to any “integral” data type
 - View arguments as bit vectors
 - Arguments applied bit-wise

C expression	Binary expression	Binary result	Hexadecimal result
~0x41	~[0100 0001]	[1011 1110]	0xBE
~0x00	~[0000 0000]	[1111 1111]	0xFF
0x69 & 0x55	[0110 1001] & [0101 0101]	[0100 0001]	0x41
0x69 0x55	[0110 1001] [0101 0101]	[0111 1101]	0x7D

Contrast: Logic Operations in C

- Contrast to Logical Operators
 - `&&`, `||`, `!`
 - View 0 as “False”
 - Anything nonzero as “True”
 - Always return 0 or 1

C expression	Result
<code>!0x41</code>	<code>0x00</code> = “False”
<code>!0x00</code>	<code>0x01</code> = “True”
<code>0x69 && 0x55</code>	<code>0x01</code> = “True”
<code>0x69 0x55</code>	<code>0x01</code> = “True”

Shift Operations

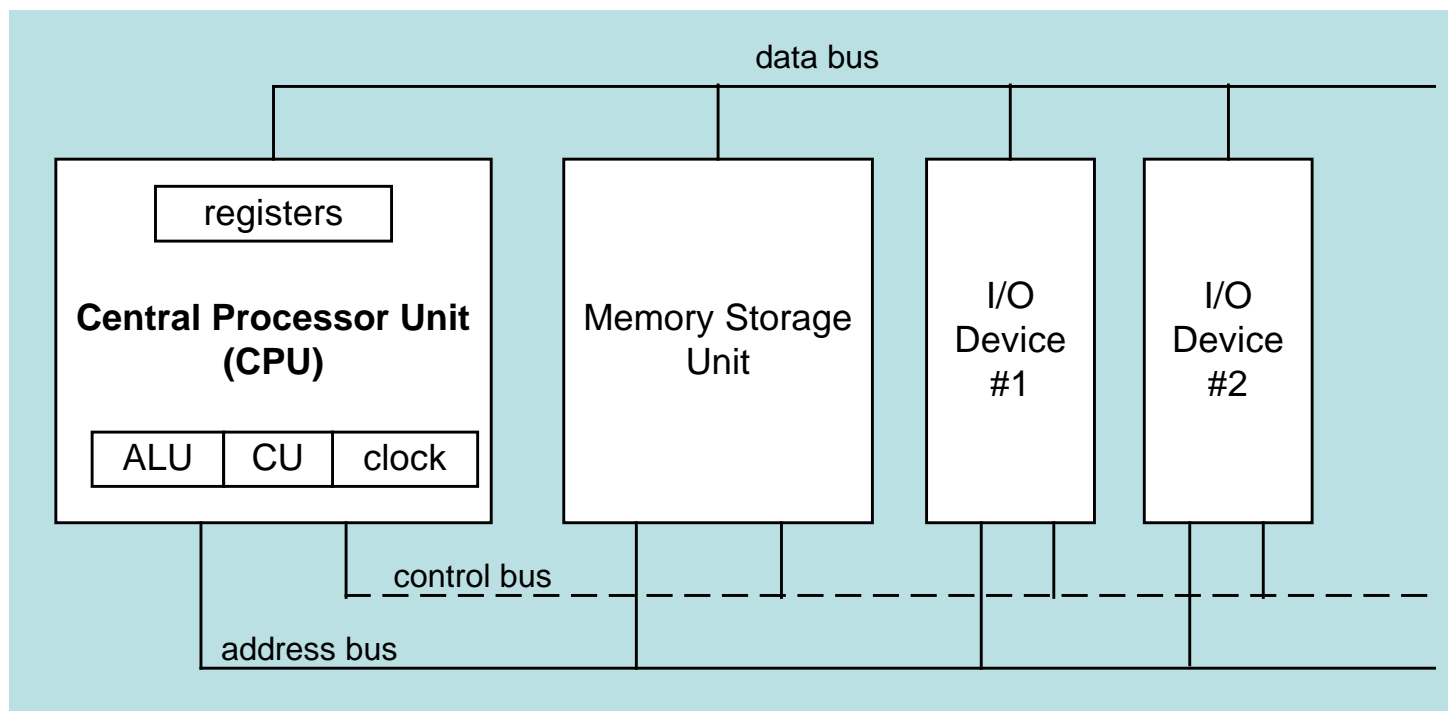
- Left Shift: $x \ll y$
 - Shift bit-vector x left y positions
 - Throw away extra bits on left
 - Fill with 0's on right
- Right Shift: $x \gg y$
 - Shift bit-vector x right y positions
 - Throw away extra bits on right
 - Logical shift
 - Fill with 0's on left
 - Arithmetic shift
 - Replicate most significant bit on left
- Undefined Behavior
 - Shift amount < 0 or \geq word size

Argument x	01100010
$\ll 3$	00010000
Log. $\gg 2$	00011000
Arith. $\gg 2$	00011000

Argument x	10100010
$\ll 3$	00010000
Log. $\gg 2$	00101000
Arith. $\gg 2$	11101000

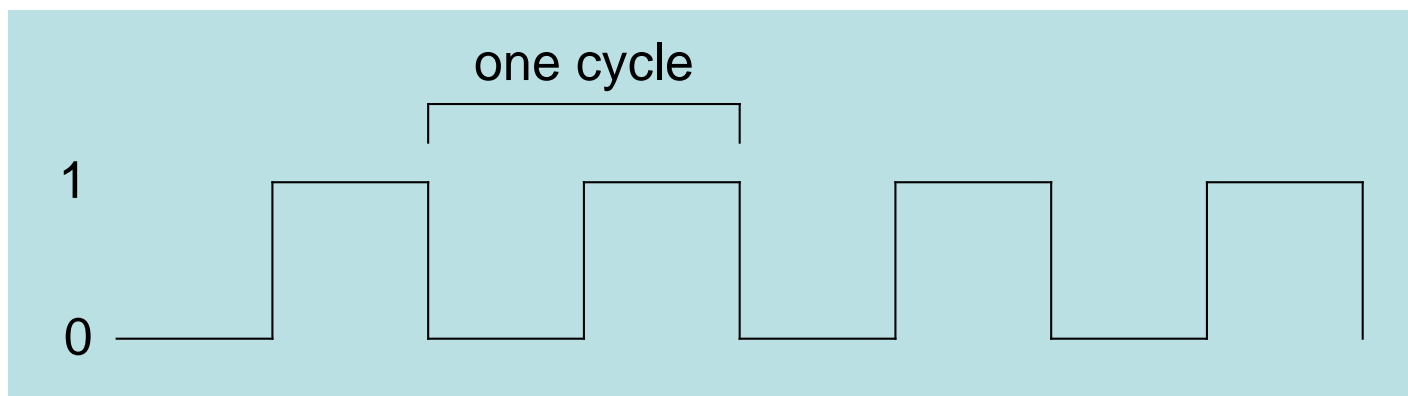
Basic Hardware Organization

- Clock synchronizes CPU operations
- Control Unit coordinates sequence of execution steps
- ALU performs arithmetic and bitwise processing



Clock

- Clock synchronizes all CPU and BUS operations
- Clock cycle measures time of a single operation
- Clock is used to trigger events



Instruction Execution Cycle

- Basic operation cycle of a computer
 - **Fetch:** The next instruction is fetched from the memory that is currently stored in the program counter
 - **Decode:** The encoded instruction present in the IR is interpreted
 - **Execute:** The control unit passes the instruction to the ALU to perform mathematical or logic functions and writes the result to the register.

Instruction Execution Cycle

Loop

fetch next instruction

advance the program counter (PC)

decode the instruction

if memory operand needed read from memory

execute the instruction

if result is memory operand, write to memory

Continue loop

CISC and RISC

- CISC – Complex instruction set computer
 - Large instruction set
 - High-level operations
 - Requires microcode interpreter
- RISC – Reduced instruction set computer
 - Simple, atomic instructions
 - Small instruction set
 - Directly executed by hardware

What is Assembly Language

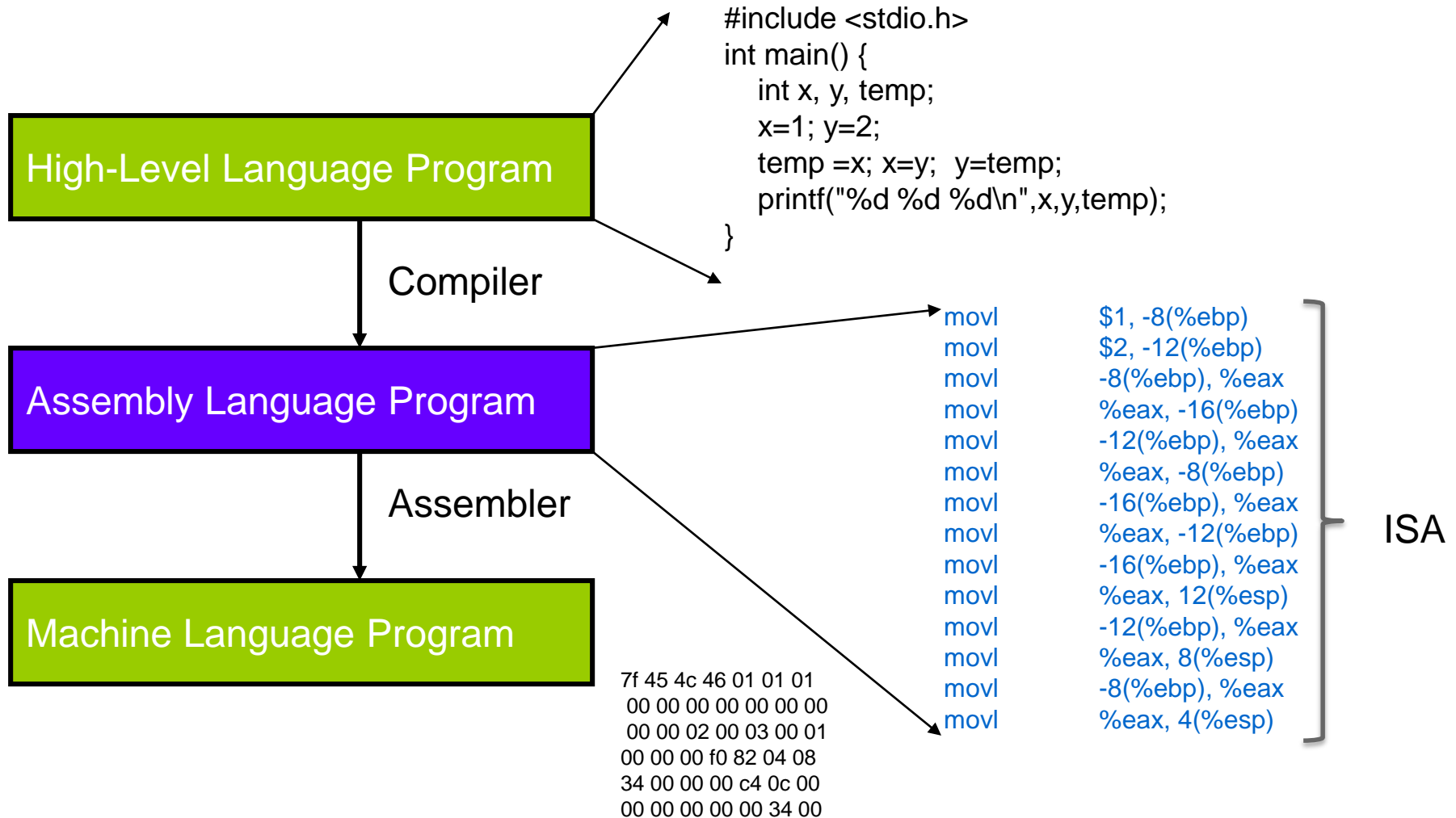
- It is used to write programs in terms of the basic operations of a processor
- A processor understands only machine language instructions
- Machine language is too obscure and complex
- So low-level assembly language is designed for the processors

Advantages of Assembly Language

- Requires less memory and execution time
- Allows hardware-specific complex jobs in an easier way
- Suitable for time-critical jobs



Brief structure



Assembly Instructions

- Assembled into machine code by assembler
- Executed at runtime by the CPU
- Parts
 - Label (optional)
 - Opcode (also called as mnemonic)
 - Operand
 - Format
 - [label:]
opcode operands
 - Example)
movl %eax, %ebx

Labels

- Act as place markers
 - Marks the address of code and data
- Code label
 - Target of jump or loop instructions
 - Ex) L1: (followed by colon)

Opcode

- Instruction opcode
 - MOV
 - ADD
 - SUB
 - MUL
 - JMP
 - CALL

Operands

- Constant (immediate value)
 - Ex) 96
- Constant expression
 - Ex) $2 + 4$
- Register
 - Ex) EAX

Registers

- Registers are CPU components that hold data and address
- Much faster to access than memory
- It is used to speed up CPU operations
- Categories
 - General registers
 - Data registers
 - Pointer registers
 - Index registers
 - Control registers
 - Segment registers

Q & A

- Any questions?