

# BEST-FIRST SEARCH (BEST-FS)

- The search space is evaluated according to a heuristic function.
- Nodes yet to be evaluated are kept on an OPEN list(represented as a priority queue)
- Nodes that have already been evaluated are stored on a CLOSED list.
- The OPEN list is then built in **order of  $f(n)$** .

This makes best-first search fundamentally greedy because it always chooses the best local opportunity in the search frontier.

# Frontier

- Best-first search
- **Idea**: use an evaluation function  $f(n)$  for each node
- $f(n)$  provides an estimate for the total cost.
- Expand the node  $n$  with smallest  $f(n)$ .
- Implementation: Order the nodes in fringe increasing order of cost.
- Special cases:
  - greedy best-first search
  - $A^*$  search

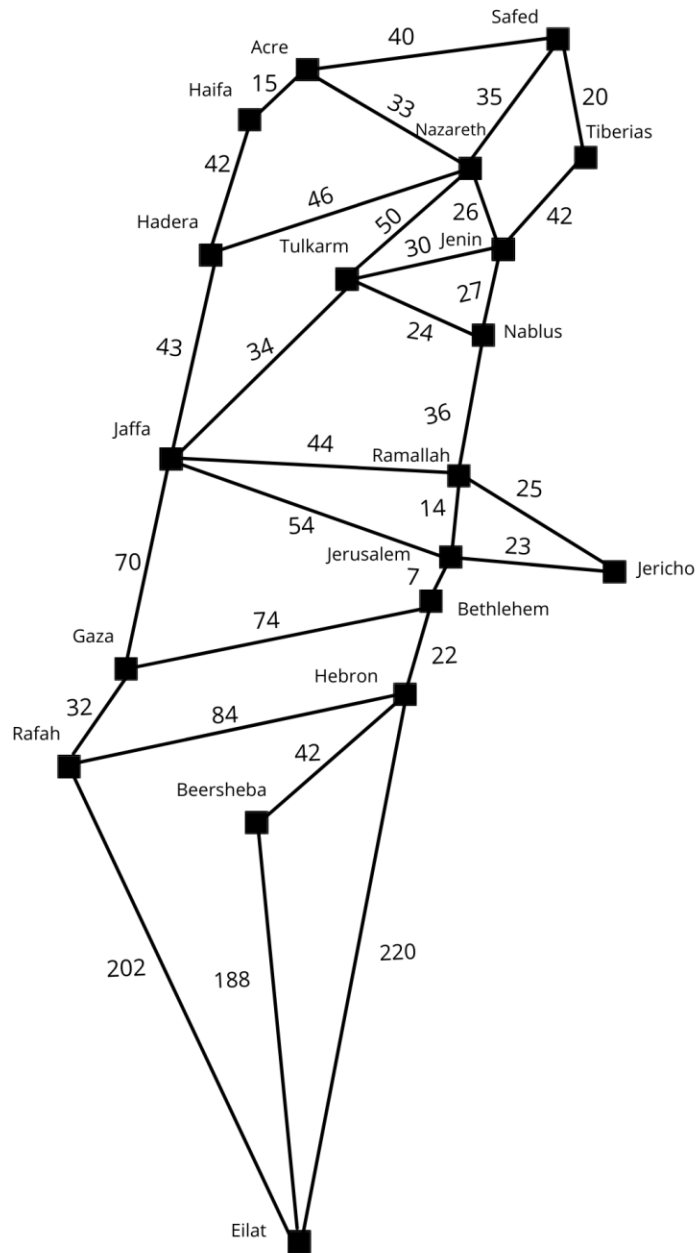
# Frontier

- The search frontier is defined as the set of node opportunities that can be searched next.
- In Best-First search, the frontier is a priority queue sorted in  $f(n)$  order.
- Given the strict order of  $f(n)$ , the selection of the node to evaluate from the priority queue is greedy

# Properties of best-first search

- **Complete?** No – can get stuck in loops.
- **Time?**  $O(b^m)$ , but a good heuristic can give dramatic improvement
- **Space?**  $O(b^m)$  -- keeps all nodes in memory
- **Optimal?** No  
a solution can be found in a longer path (higher  $h(n)$  with a lower  $g(n)$  value.

# Palestine with step costs in km



## Straight-line Distance to Eilat

Safed	382
Eilat	0
Acre	375
Nazareth	351
Tiberias	364
Tulkarm	307
Haifa	360
Hadera	320
Nablus	298
Ramallah	262
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Rafah	202
Gaza	222
Beersheba	189

# Best-first search

- Evaluation function  $f(n) = h(n)$  (heuristic)  
= estimate of cost from  $n$  to goal
- e.g.,  $h_{\text{SLD}}(n)$  = straight-line distance from  $n$  to Bucharest
- Greedy best-first search expands the node that appears to be closest to goal

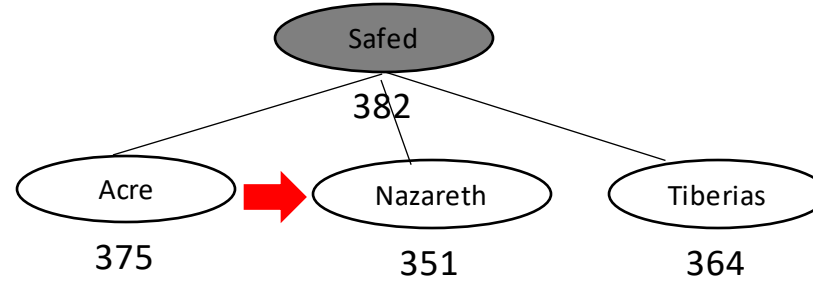
# Greedy Best-first search example



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# Greedy Best-first search example

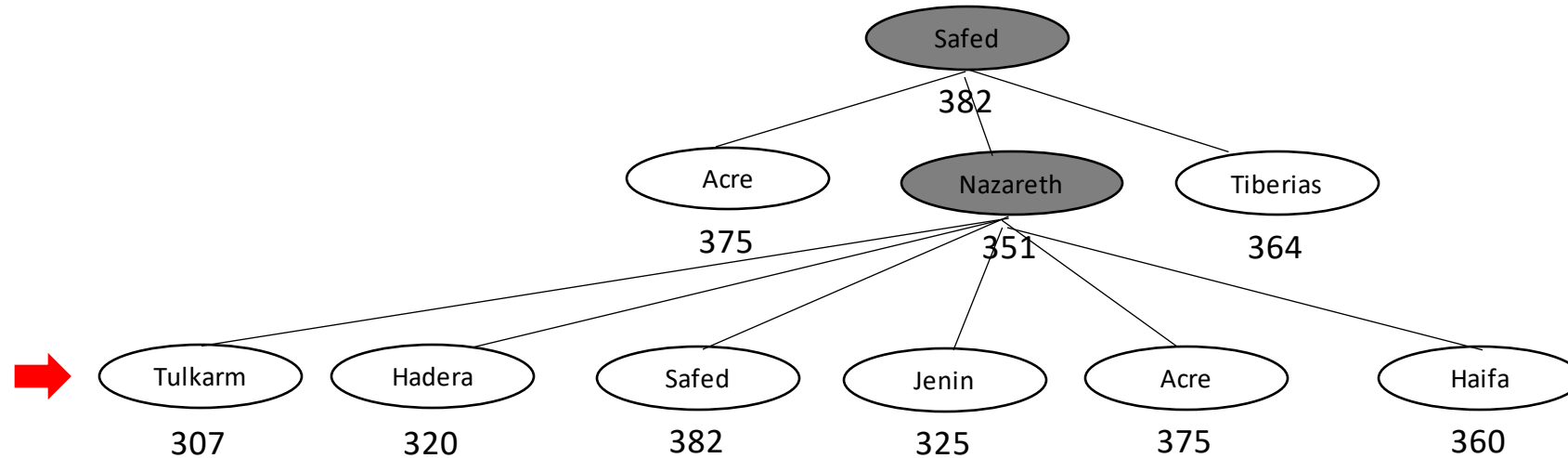


Straight-line Distance to Eilat

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<b>Acre</b>	<b>375</b>
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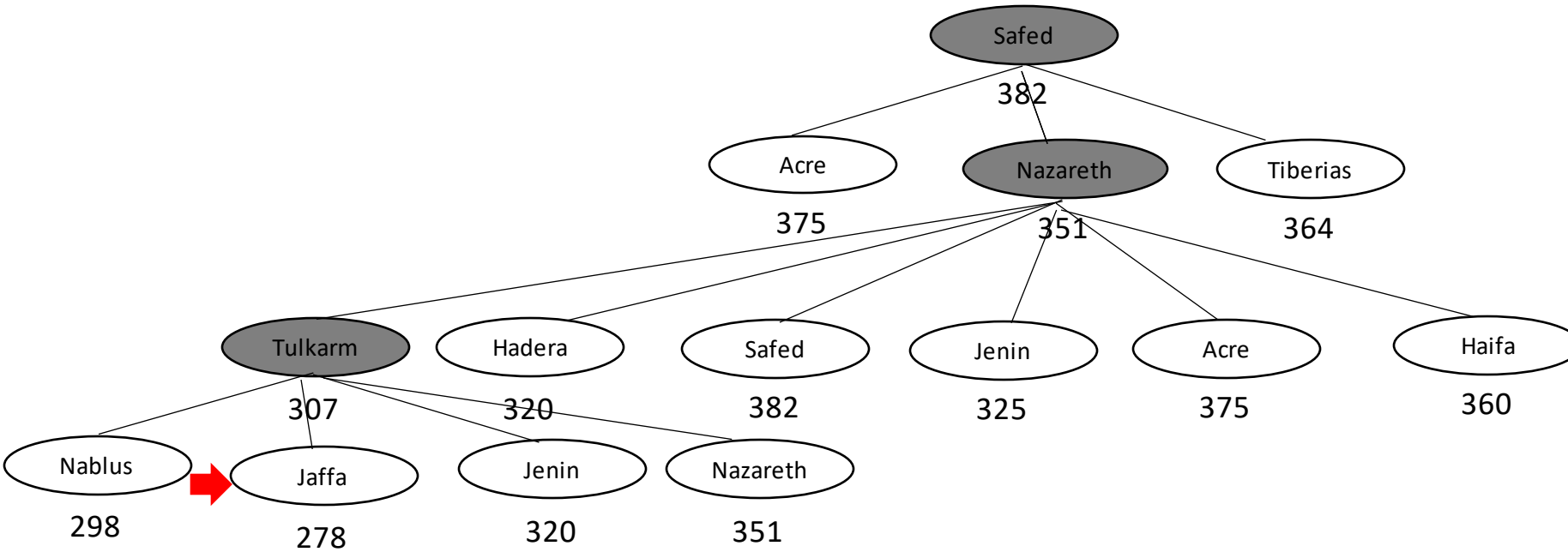
# Greedy Best-first search example



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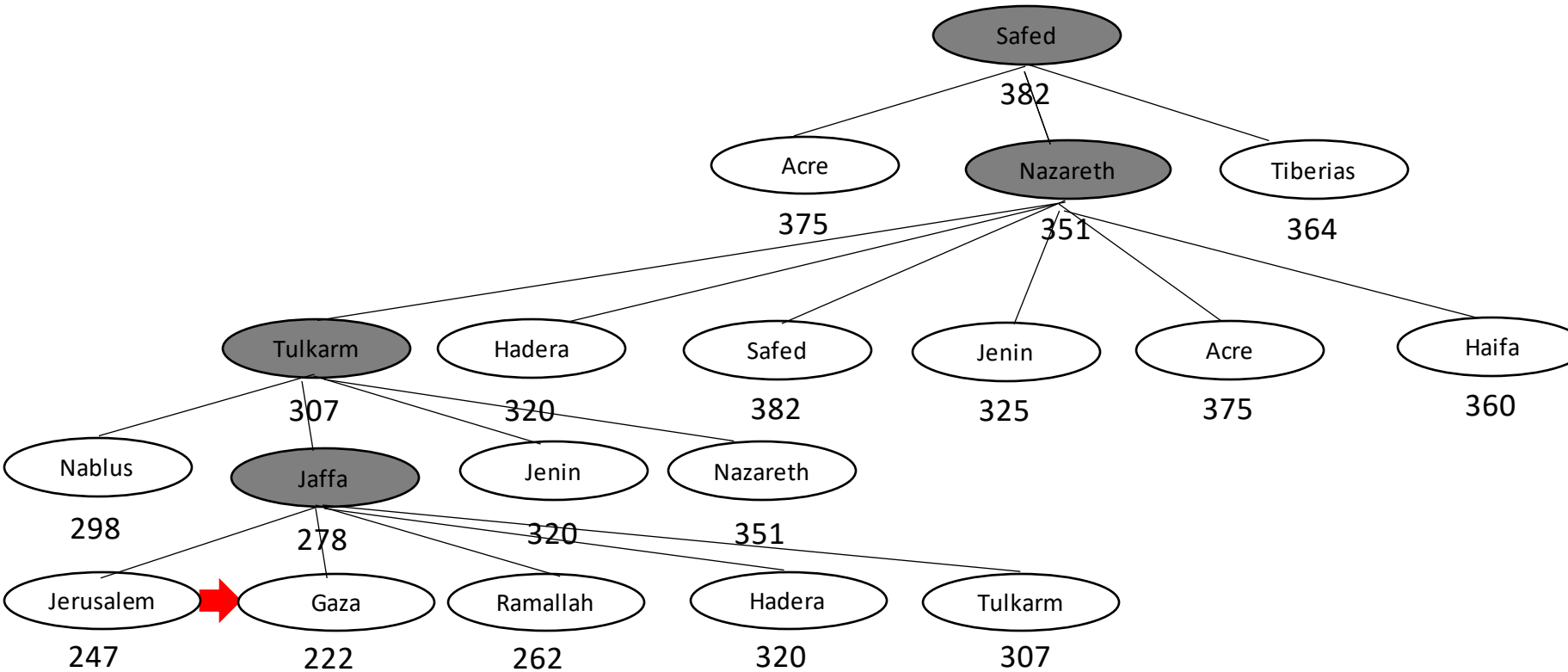
# Greedy Best-first search example



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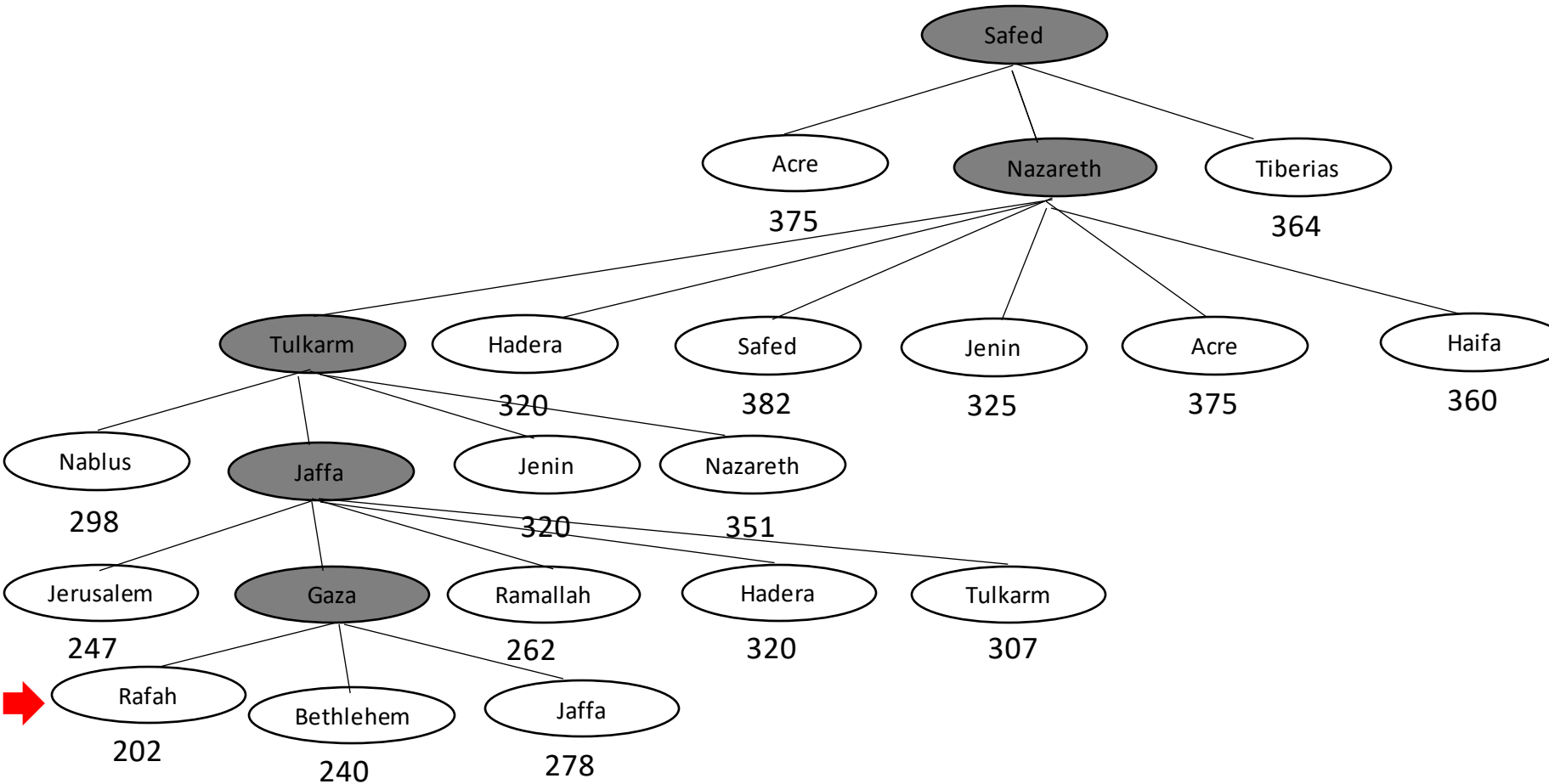
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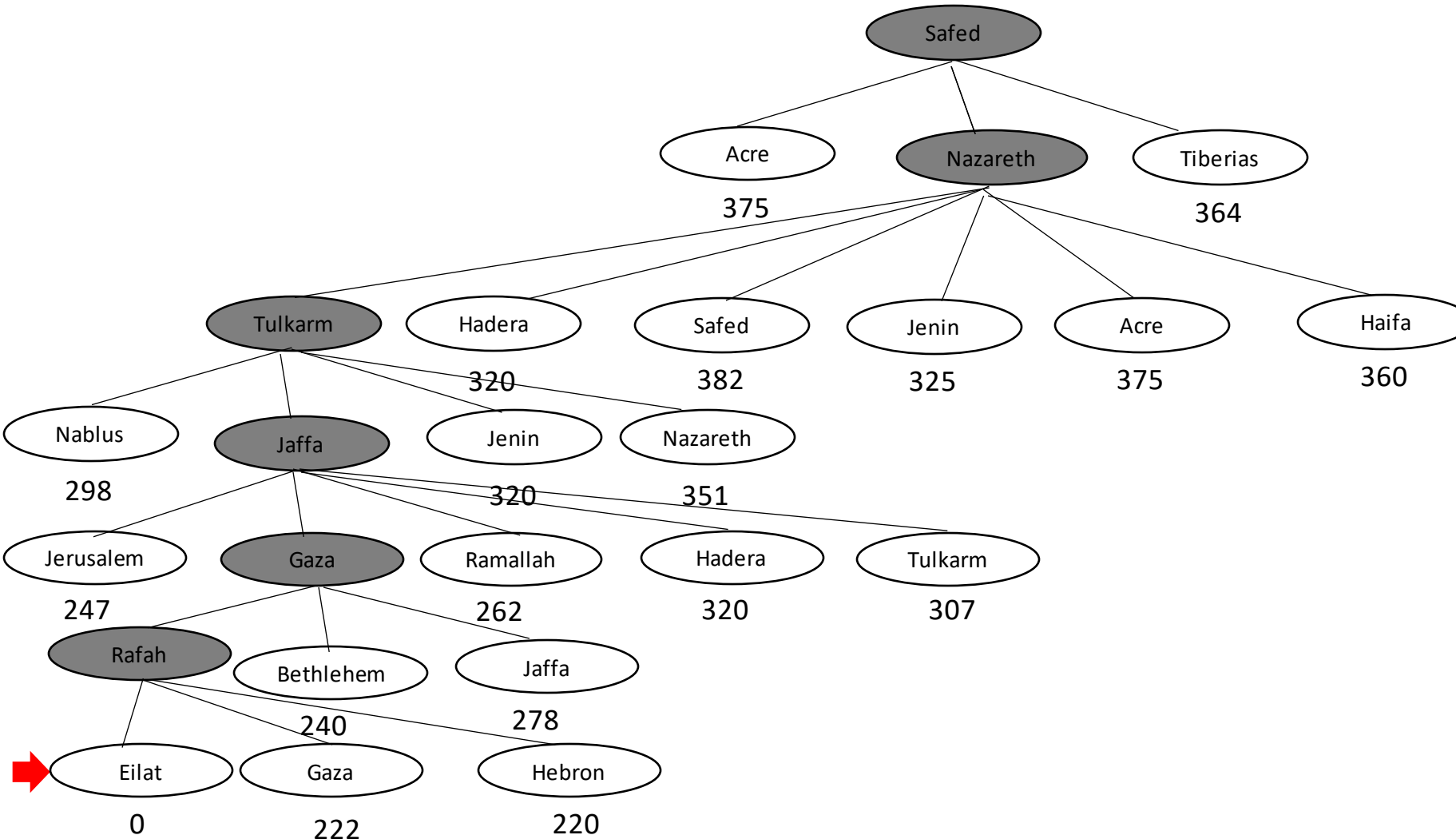
# Greedy Best-first search example



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# Greedy Best-first search example



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# A \* search

- Idea: avoid expanding paths that are already expensive
- Evaluation function  $f(n) = g(n) + h(n)$
- $g(n)$  = cost so far to reach  $n$
- $h(n)$  = estimated cost from  $n$  to goal
- $f(n)$  = estimated total cost of path through  $n$  to goal

# A\*SEARCH



Priority Queue



Heuristics

Best

Average

Worst

$O(b*d)$

$O(b^d)$

$O(b^d)$



Set

**search** (initial, goal)

1. initial.depth = 0
2. open = new PriorityQueue
3. closed = new Set
4. insert (open, copy(initial))
5. **while** (open is not empty) **do**
6.     n = minimum (open)
7.     insert (closed, n)
8.     **if** (n = goal) **then return** "Solution"
9.     **foreach** valid move m at n **do**
10.         next = state when playing m at n
11.         next.depth = n.depth + 1
12.         **if** (closed contains next) **then**
13.             prior = state in closed matching next
14.             **if** (next.score < prior.score) **then**
15.                 remove (closed, prior)
16.                 insert (open, next)
17.         **else**
18.             insert (open, next)
19.     **return** "No Solution"
- end**

initial 00 target 21 open ← 00

(1) after first time through loop

open ← 10 01

closed:

00

(2) after second time through loop

open ← 20 11 01

closed:

00 10

(3) after third time through loop

open ← 21 11 01 30

closed:

00 10  
20

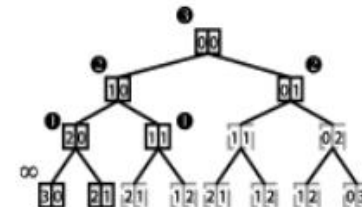
(4) in fourth time through loop, goal found

open ← 11 01 30

closed:

00 10  
20 21

- closed
- explored
- open
- unexplored





# Admissible heuristic

- a **heuristic function** is said to be **admissible** if it never overestimates the cost of reaching the goal, i.e. the cost it estimates to reach the goal is not higher than the lowest possible cost from the current point in the path.

$n$  is a node

$h$  is a heuristic

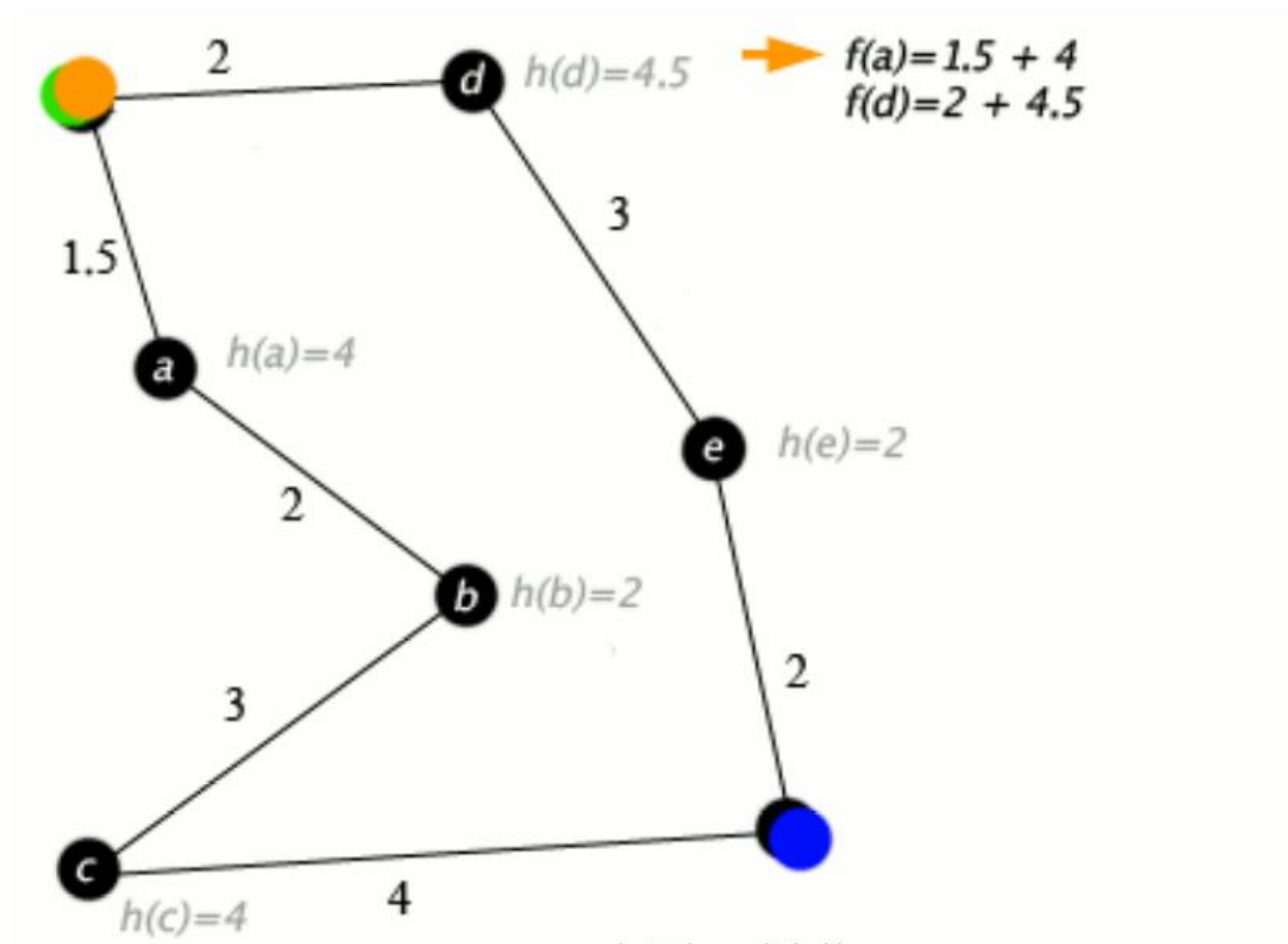
$h(n)$  is cost indicated by  $h$  to reach a goal from  $n$

$h^*(n)$  is the actual cost to reach a goal from  $n$

$h(n)$  is admissible if,  $\forall n$

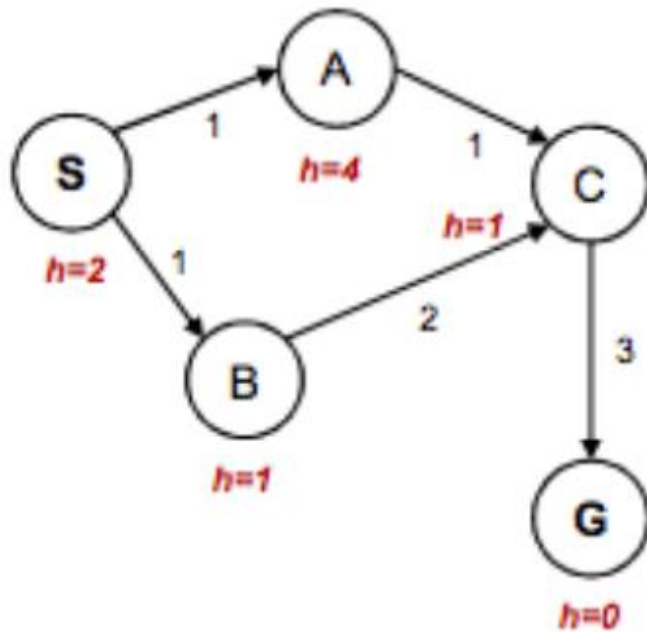
$$h(n) \leq h^*(n)$$

# A\* search

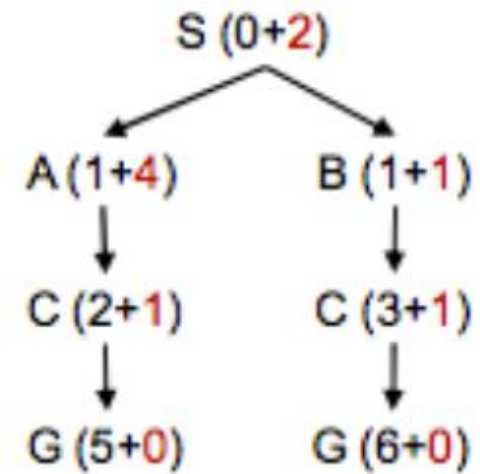


# A\* search

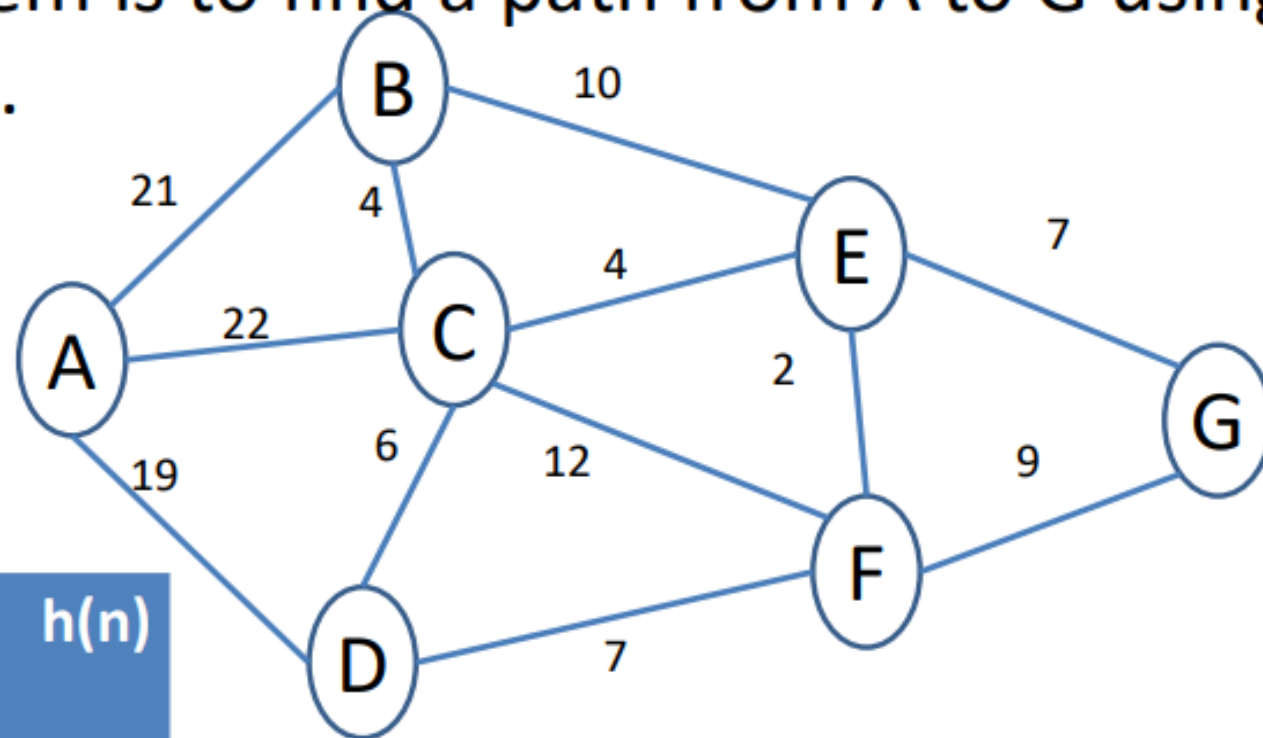
State space graph



Search tree

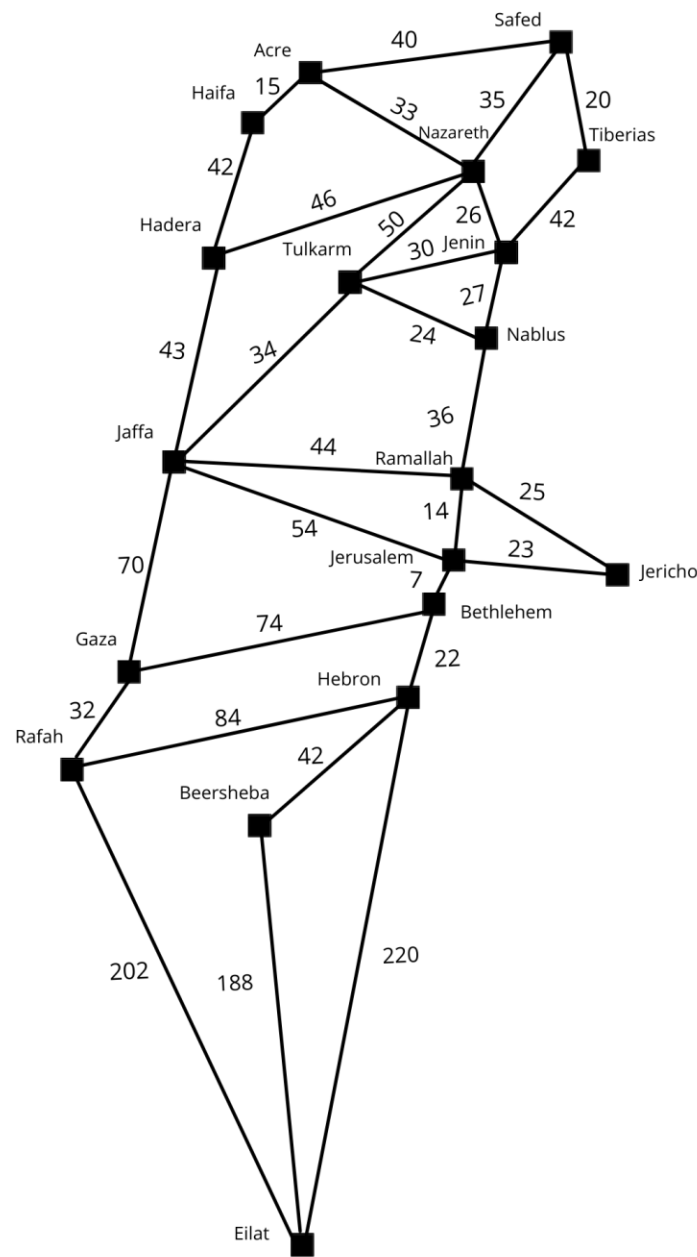


the problem is to find a path from A to G using A\* algorithm.



nodes	$h(n)$
A	34
B	19
C	14
D	18
E	7
F	9
G	0

# Palestine with step costs in km



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Open List: Safed

# A\* search Example



Straight-line Distance to Eilat

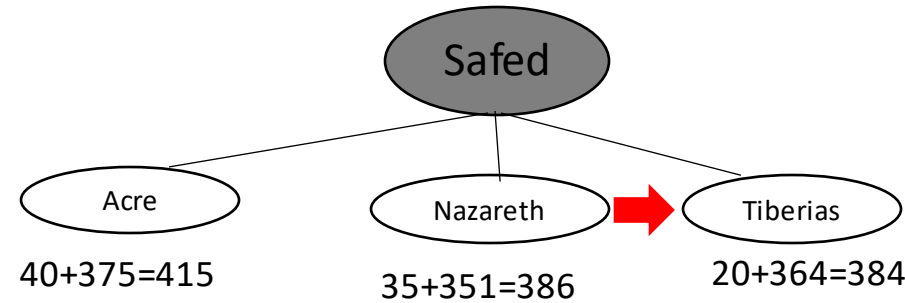
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We start with our initial state Safed.

We make a node and add it to the open list. Since it's the only thing on the open list, we expand the node. Think of the open list as a priority queue (or heap) that sorts the nodes inside of it according to their  $g()+h()$  score

Open List:  
Tiberias  
Nazareth  
Acre

# A\* search Example

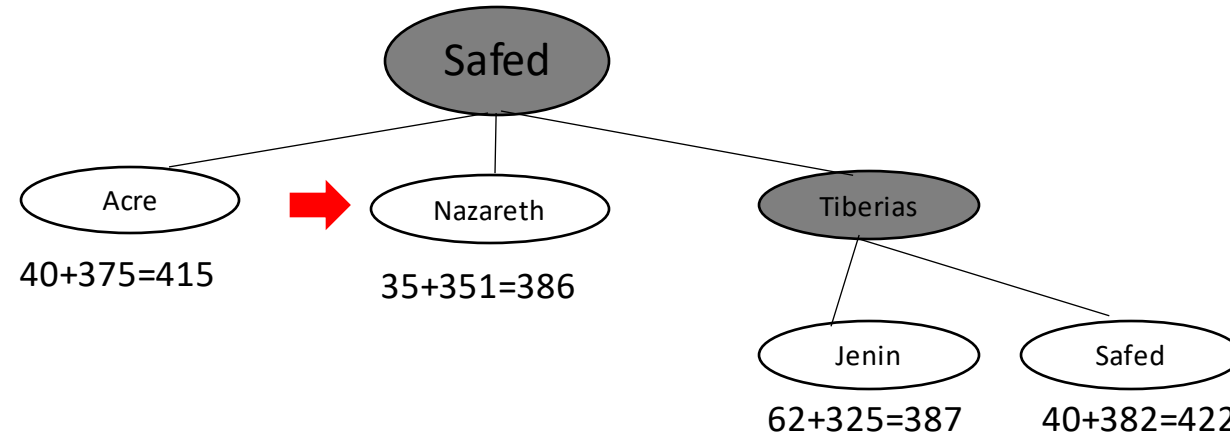


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We add the three nodes we found to the open list. We sort them according to the  $g()+h()$  calculation.

Open List:  
Nazareth  
Jenin  
Acre

# A \* search Example



When we expand Tiberias, we run into Safed again. But we've already expanded this node once; so, we don't add it to the open list again.

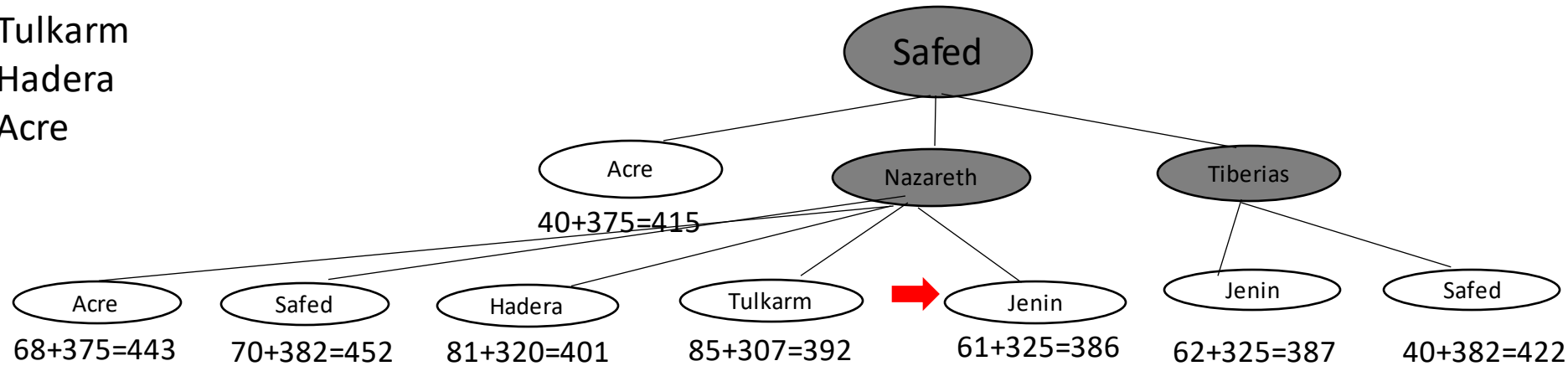
We see that Nazareth is at the top of the open list; so, it's the next node we will expand.



# A\* search Example

Open List:  
Jenin  
Tulkarm  
Hadera  
Acre

Straight-line Distance to Eilat



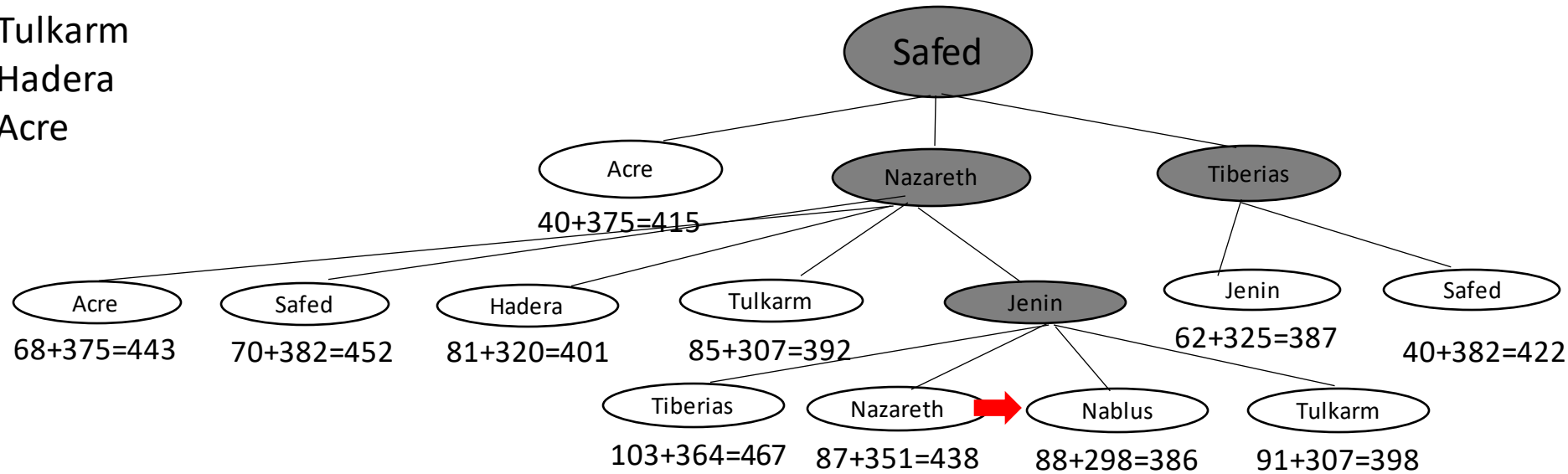
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When we expand Nazareth, we run into Acre and Safed again. But we've already expanded those nodes once; so, we don't add it to the open list again.

# A\* search Example

Straight-line Distance to Eilat

Open List:  
Nablus  
Tulkarm  
Hadera  
Acre



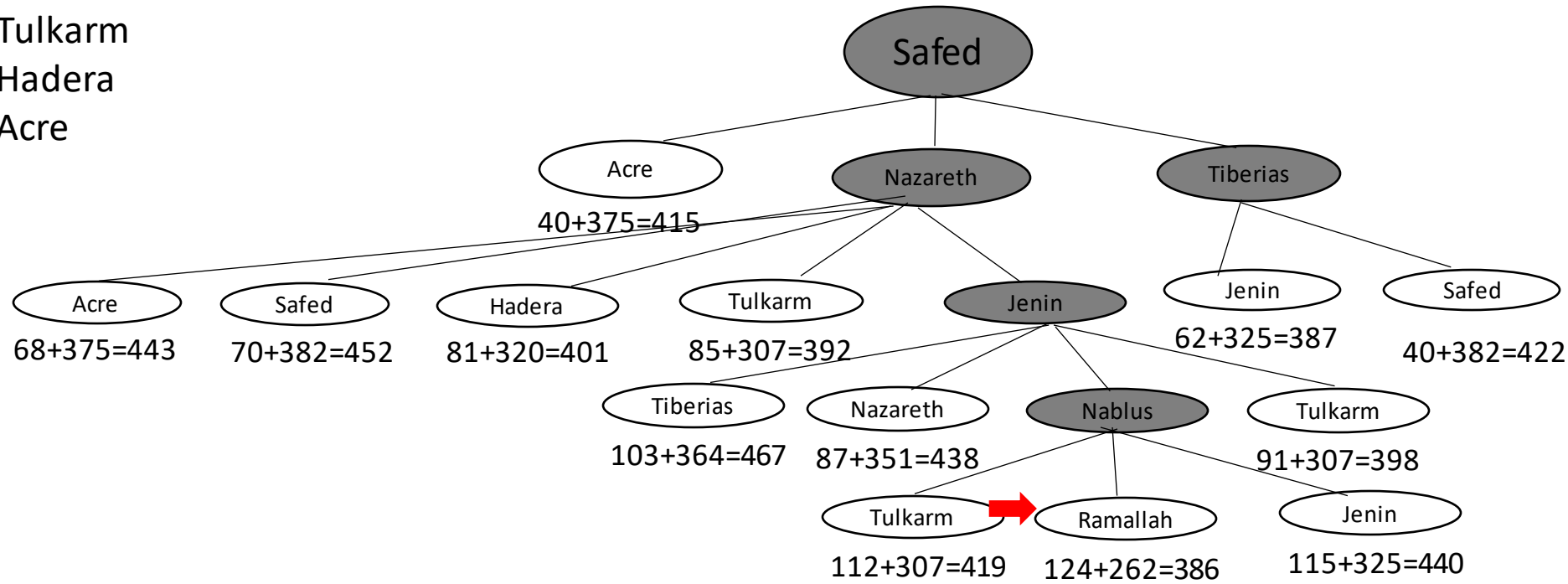
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When we expand Jenin, we run into Nazareth and Tiberias again.  
But we've already expanded those nodes once; so, we don't add them to the open list again.  
Nablus will be the next node we should expand – it's at the top of the sorted open list.

# A\* search Example

Open List:  
Ramallah  
Tulkarm  
Hadera  
Acre

Straight-line Distance to Eilat



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When we expand Nablus, we run into Tulkarm and Jenin again.

But we've already expanded Jenin node once; so, we don't add it to the open list again

Tulkarm Remains in it's old location in the open list

Ramallah will be the next node we should expand – it's at the top of the sorted open list.

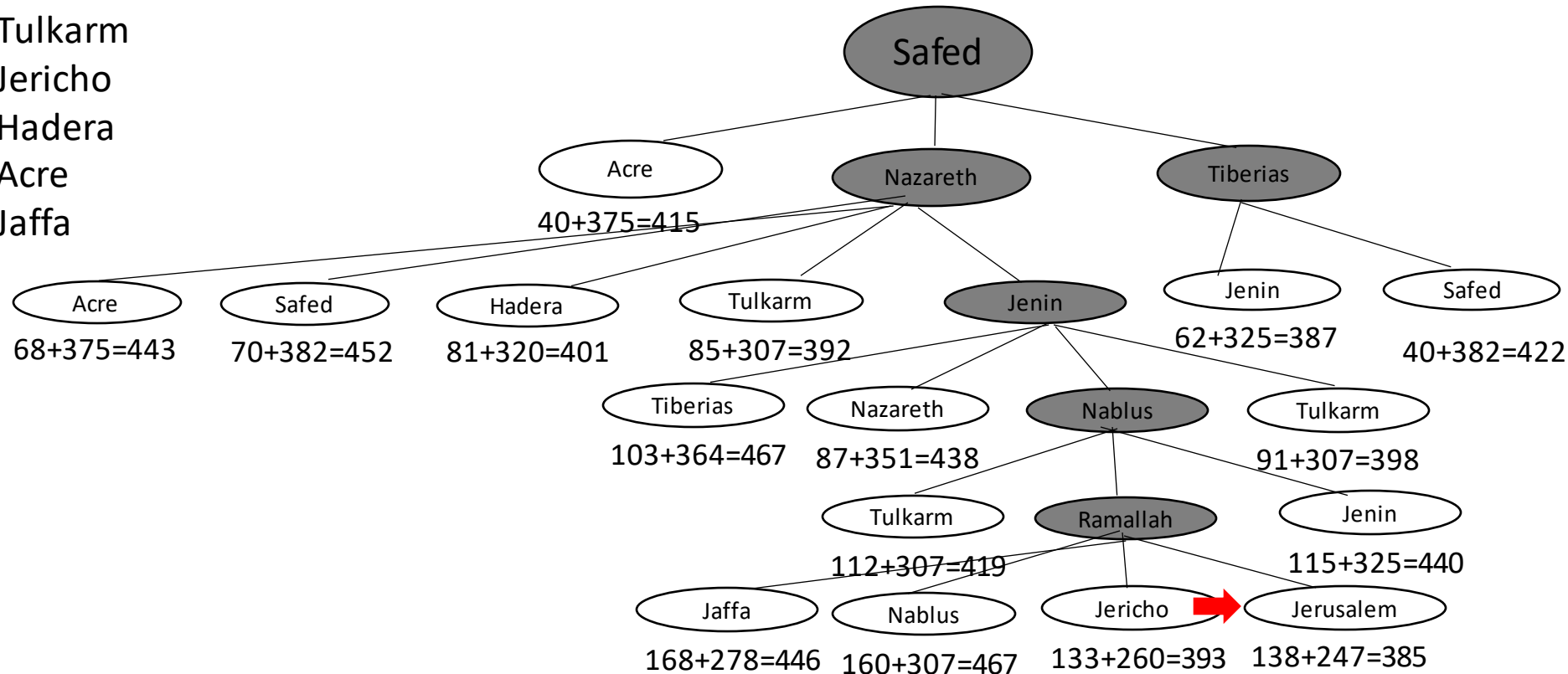
dr. Mohamed Khalil

Palestine Technical University

# A \* search Example

Open List:  
Jerusalem  
Tulkarm  
Jericho  
Hadera  
Acre  
Jaffa

Straight-line Distance to Eilat



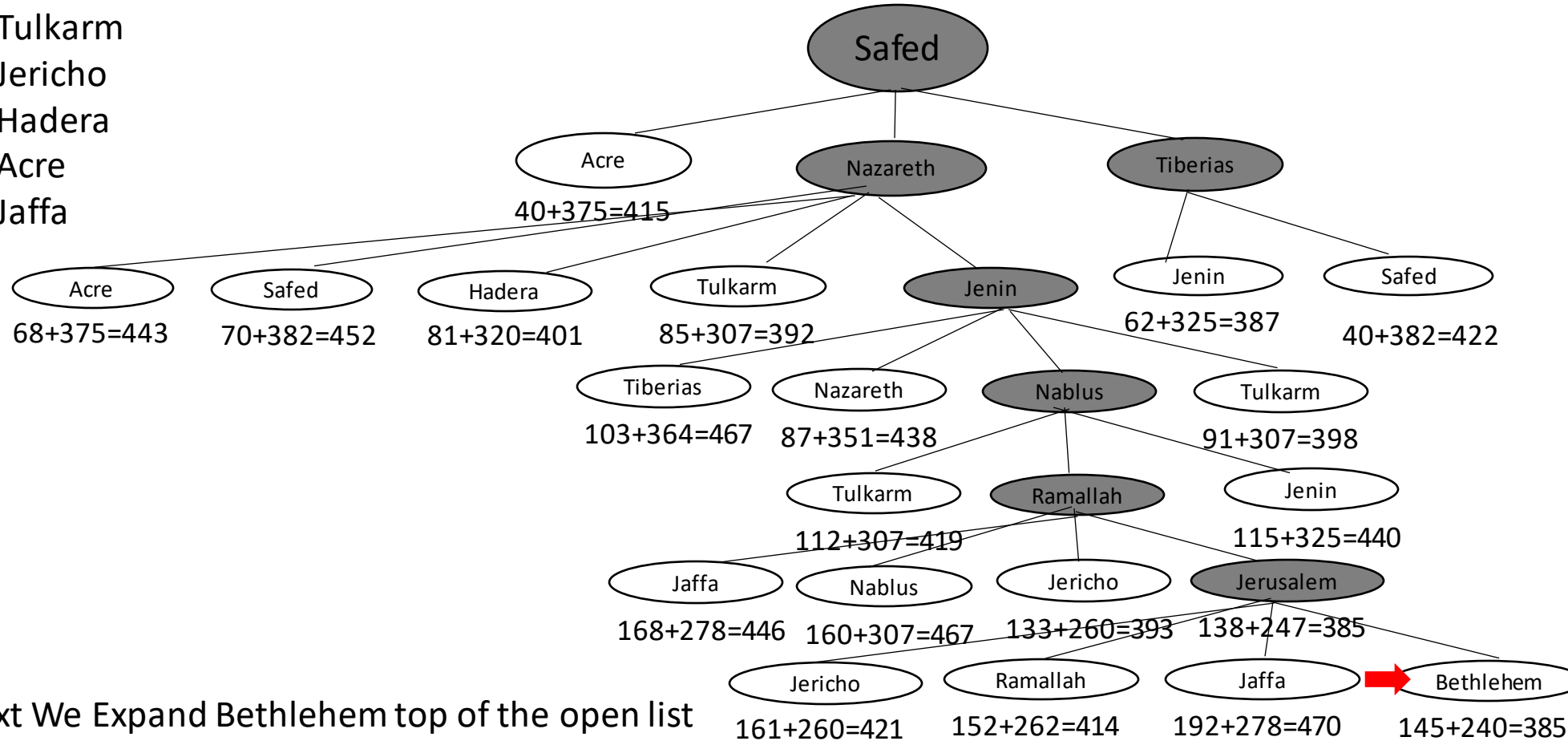
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Next We Expand Jerusalem It has the lowest f(n) (top of the open list)

# A \* search Example

Straight-line Distance to Eilat

Open List:  
Bethlehem  
Tulkarm  
Jericho  
Hadera  
Acre  
Jaffa



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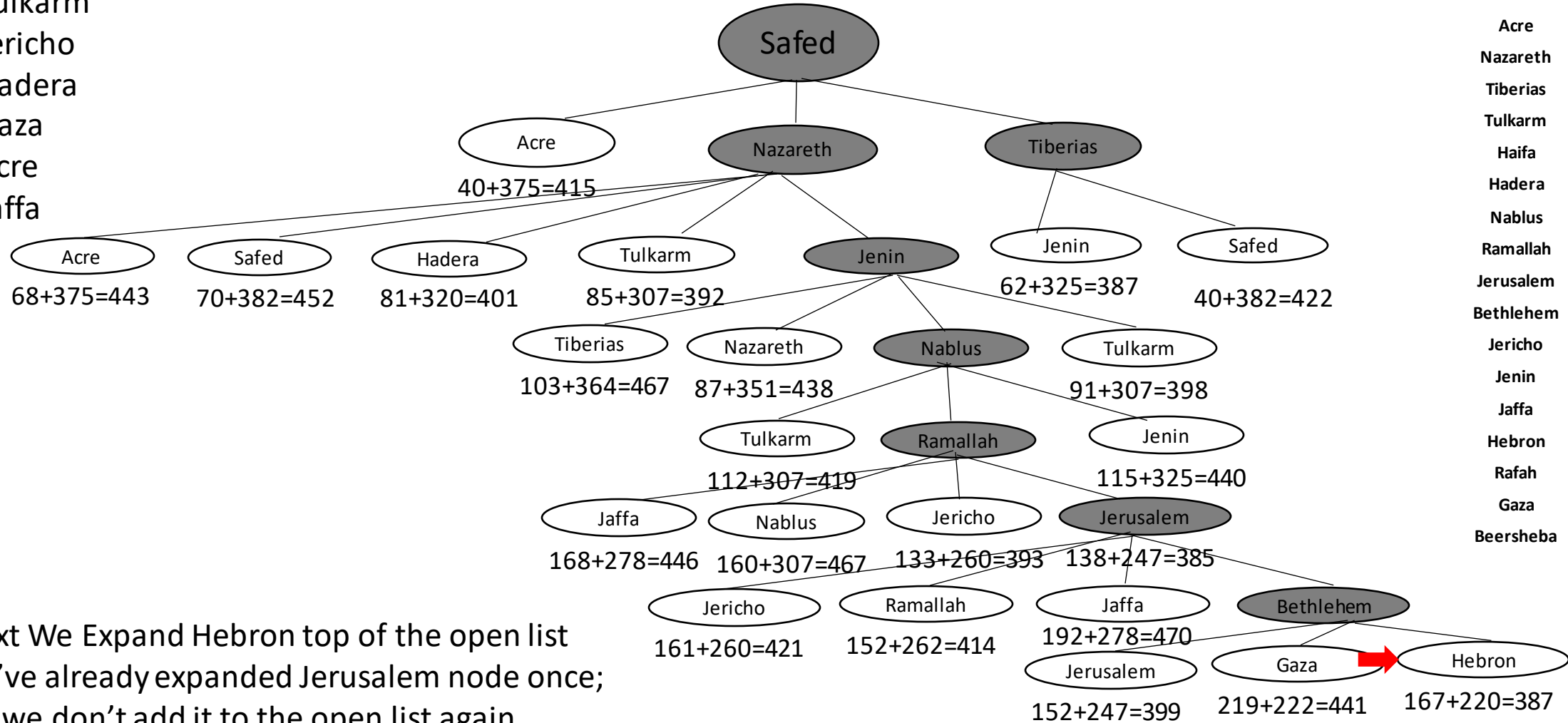
Next We Expand Bethlehem top of the open list  
We run into Ramallah ,Jericho , Jaffa again  
We don't add them again to the list

Open List:  
 Hebron  
 Tulkarm  
 Jericho  
 Hadera  
 Gaza  
 Acre  
 Jaffa

# A \* search Example

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Next We Expand Hebron top of the open list  
 we've already expanded Jerusalem node once;  
 so, we don't add it to the open list again

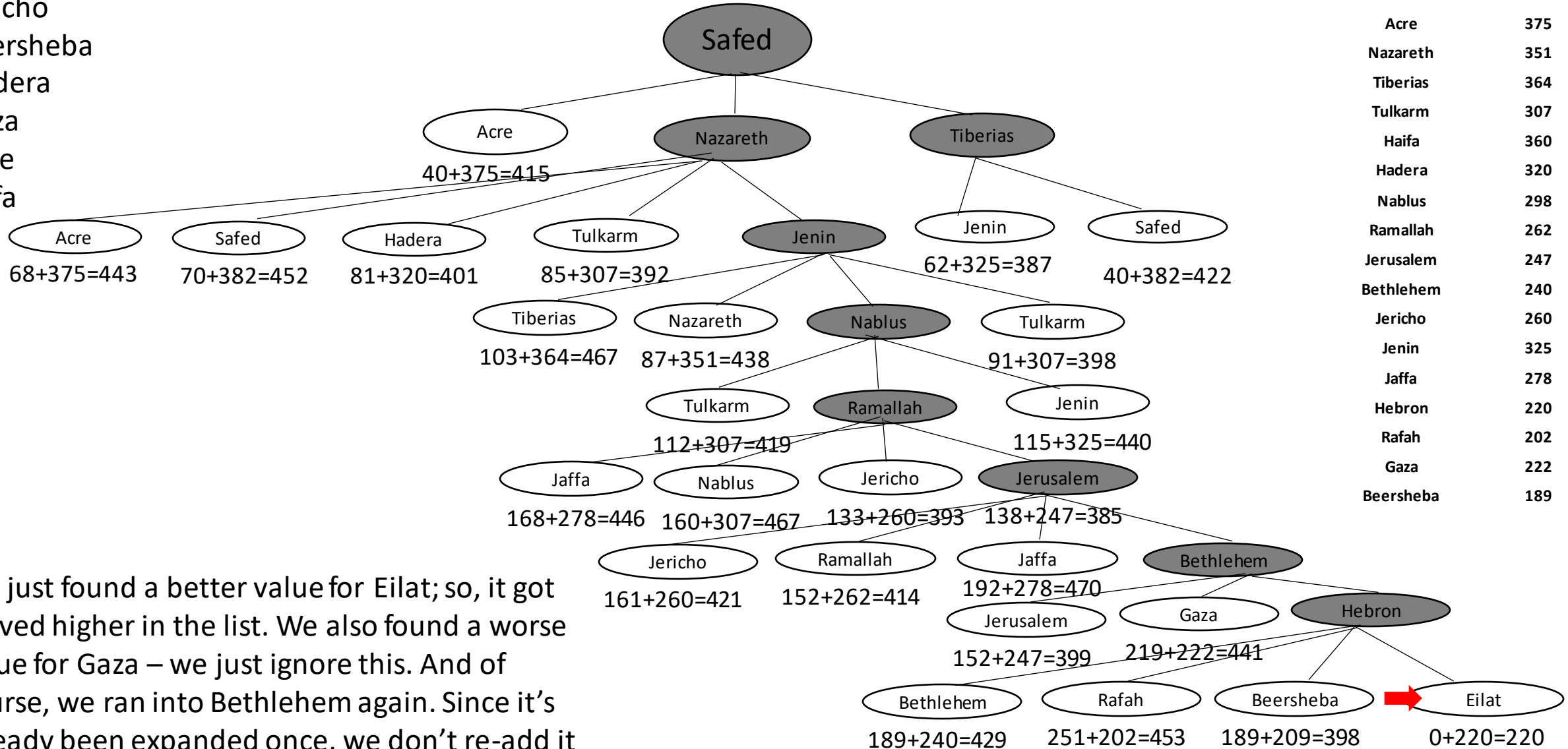
Open List:

Eilat  
Tulkarm  
Jericho  
Beersheba  
Hadera  
Gaza  
Acre  
Jaffa

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We just found a better value for Eilat; so, it got moved higher in the list. We also found a worse value for Gaza – we just ignore this. And of course, we ran into Bethlehem again. Since it's already been expanded once, we don't re-add it to the Open List.

Open List:

Eilat

Tulkarm

Jericho

Beersheba

Hadera

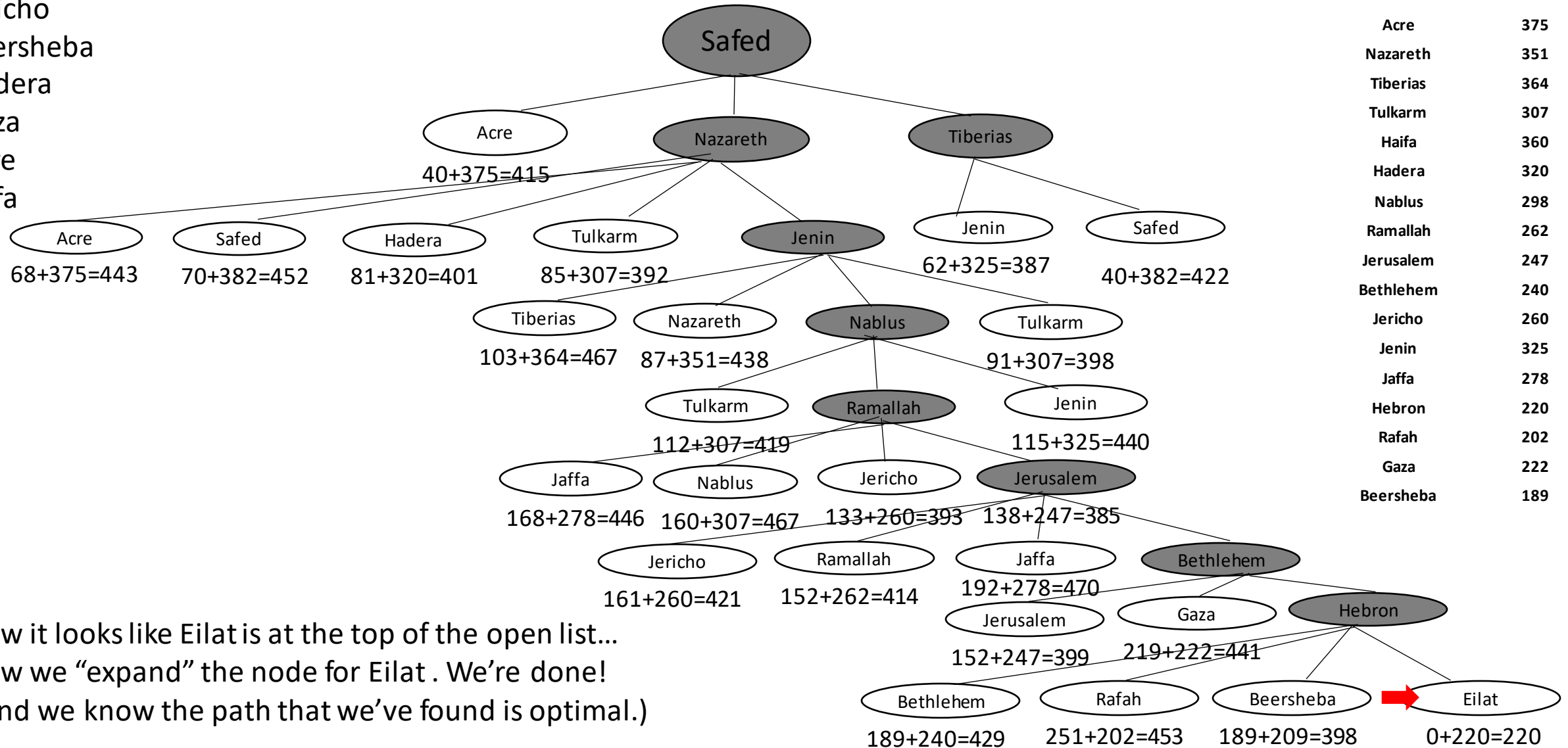
Gaza

Acre

Jaffa

# A \* search Example

Straight-line Distance to Eilat





# Analyzing the heuristic function

- $f(n) = h(n)$  (*Greedy best first search*)
- If  $h(n) = \text{constant}(\text{or zero})$   $g=g+1$  then  $A^*$  as BFS
- If  $h(n) = 0$  for all  $n$  then  $f(n) = g(n)$

This heuristic is admissible,  $A^*$  performs exactly as UCS

- If  $h(n) = h^*(n)$  for all  $n$ 
  - Only nodes on optimal solution path are expanded
  - No unnecessary work is performed %
- The closer  $h$  is to  $h^*$ , The fewer extra nodes that will be expanded.

# Properties of Best First Search( $A^*$ )

- **Complete?** Yes – as long as the memory supports the depth and branching factor of the tree.
- **Time?**  $O(b^d)$ , but a good heuristic can give dramatic improvement.
- **Space?**  $O(b^d)$  -- keeps all nodes in memory.
- **Optimal?** Yes

If  $h$  is admissible,  $A^*$  will always find a least cost path to the goal

# Resources

- [Distance to](#)
- [Wikipedia](#)
- [Utc.city](#)
- [Google Maps](#)

## Palestinian Cities in Arabic

عكا - Acre  
الرشراش ام - Eilat  
نابلس - Nablus  
جنين - Jenin  
طولكرم - Tulakrm  
القدس - Jerusalem

صفد - Safed  
حيفا - Haifa  
طبريا - Tiberias  
الخصيرة - Hadera  
الناصرة - Nazareth  
يافا - Jaffa  
الخليل - Hebron

رام الله - Ramallah  
أريحا - Jericho  
بيت لحم - Bethlehem  
غزة - Gaza  
بئر السبع - Beersheba  
رفح - Rafah