Lecture # 13

Applications of system of linear algebraic equations.

EXAMPLE 01: If 20 pounds of rice and 10 pounds of potatoes cost $16.20 and 30 pounds of rice and 12 pounds of potatoes cost $23.04, how much will 10 pounds of rice and 50 pounds of potatoes cost?

Solution: Let the cost of 1 pound of rice be $$x$ and the cost of 1 pound of potatoes be $$y$. Then according to the question,

$$20x + 10y = 16.20 \qquad (i)$$

$$30x + 12y = 23.04 \qquad (ii)$$

Multiply (i) by 6 and (ii) by 4, we have

$$120x + 60y = 97.2 \qquad (iii)$$

$$120x + 48y = 92.16 \qquad (iv)$$

Subtracting (iv) from (iii), we have

$$12y = 5.04 \Rightarrow y = 0.42 \cdot$$

Substituting this value of y into (i), we obtain

$$20x + 10(0.42) = 16.20 \Rightarrow x = 0.6 \cdot$$

Cost of rice = $ 0.6 / Lbs   Cost of potatoes = $ 0.42 / Lbs

Thus, 10 pounds of rice cost $\$10 \times 0.6 = \$6,$ and 50 pounds of potatoes cost $\$50 \times 0.42 = \$21 \cdot$

EXAMPLE 02: An animal feed is to be made from corn, soybeans, and cotton seed. Determine how many units of each ingredient are needed to make a feed that supplies 1800 units of fiber, 2800 units of fat, and 2200 units of protein, given that one unit of each ingredient provides the numbers of units shown in the table below. The table states, for example, that a unit of corn provides 10 units of fiber, 30 units of fat, and 20 units of protein.

	Corn x	Soybean y	Cotton seedz	Total
Units of Fiber	**10**	**20**	**30**	**1800**
Units of Fat	**30**	**20**	**40**	**2800**
Units of Protein	**20**	**40**	**25**	**2200**

Solution:

Let x represent the required number units of corn, y the number of units of soybeans, and z the number of units of cottonseed.

The above data gives the equations as

$$10x+20y+30z=1800\cdot$$
$$30x+20y+40z=2800\cdot$$
$$20x+40y+25z=2200\cdot$$

Then to solve the above system we are using Gauss – Jordan method

Step1. We change the system of linear equations in matrix form $\boldsymbol{Ax}=\boldsymbol{b}\cdot$

$$\boldsymbol{A}=\begin{bmatrix}10&20&30\\30&20&40\\20&40&25\end{bmatrix},\ \boldsymbol{x}=\begin{bmatrix}x\\y\\z\end{bmatrix},\text{ and }\ \boldsymbol{b}=\begin{bmatrix}1800\\2800\\2200\end{bmatrix}\cdot$$

Step2. We form the augmented coefficient matrix A_b by including the constants, an extra column in the matrix.

$$A_b=\begin{bmatrix}10&20&30&1800\\30&20&40&2800\\20&40&25&2200\end{bmatrix}\cdot$$

Step3. We convert this augmented coefficient matrix into reduced echelon form using elementary row operations.

$$A_b=\begin{bmatrix}10&20&30&1800\\30&20&40&2800\\20&40&25&2200\end{bmatrix}\left(\frac{1}{10}\right)R_1,\left(\frac{1}{10}\right)R_2,\left(\frac{1}{5}\right)R_3$$

$$\approx\begin{bmatrix}1&2&3&180\\3&2&4&280\\4&8&5&440\end{bmatrix}R_2+(-3)R_1,\ R_3+(-4)R_1$$

$$\approx\begin{bmatrix}1&2&3&180\\0&-4&-5&-260\\0&0&-7&-280\end{bmatrix}\left(-\frac{1}{4}\right)R_2,\left(-\frac{1}{7}\right)R_3\approx\begin{bmatrix}1&2&3&180\\0&1&1.25&65\\0&0&1&40\end{bmatrix}R_1+(-2)R_2$$

$$\approx\begin{bmatrix}1&0&0.5&50\\0&1&1.25&65\\0&0&1&40\end{bmatrix}R_1+(-0.5)R_3,\ R_2+(-1.25)R_3$$

$$\approx\begin{bmatrix}1&0&0&30\\0&1&0&15\\0&0&1&40\end{bmatrix}\cdot\ \Rightarrow\ \mathbf{x=30,\ \ y=15,\ \ z=40}$$

Hence, the feed should contain 30 units of corn, 15 units of soybeans, and 40 units of cottonseed.

EXAMPLE 03: The housing department of the Punjab Government plans to undertake four housing projects and lists material requirements for each house in each of the project as follows:

	x1	**x2**	**x3**	**x4**
	Project1	**Project2**	**Project3**	**Project4**
Paint (in 100 gallons)	**1**	**2**	**1**	**1.5**
Wood (in 10,000 cu.ft.)	**3**	**4**	**2.5**	**2.5**
Bricks (in millions)	**1**	**2**	**1.5**	**1**
Labour (in 1000 hours)	**10**	**10**	**9**	**8**

If the supplier delivers 6800 gallons of paint, 1,420,000 cubic feet of wood, 67 million bricks and 4,48,000 hours of labor, find the number of houses built for each project.

Solution: Let x_1, x_2, x_3 and x_4 be the numbers of houses built under Project-1, Project-2, Project-3 and Project-4 respectively. Then according to the conditions of the problem:

$$1\times x_1 + 2\times x_2 + 1\times x_3 + 1.5\times x_4 = 68$$
$$3\times x_1 + 4\times x_2 + 2.5\times x_3 + 2.5\times x_4 = 142$$
$$1\times x_1 + 2\times x_2 + 1.5\times x_3 + 1\times x_4 = 64$$
$$10\times x_1 + 10\times x_2 + 9\times x_3 + 8\times x_4 = 448$$

The augmented matrix is:

$$A_b = \begin{bmatrix} 1 & 2 & 1 & 1.5 & 68 \\ 3 & 4 & 2.5 & 2.5 & 142 \\ 1 & 2 & 1.5 & 1 & 64 \\ 10 & 10 & 9 & 8 & 448 \end{bmatrix}.$$

We convert this augmented coefficient matrix into reduced echelon form using elementary row operations:

$$A_b = \begin{bmatrix} 1 & 2 & 1 & 1.5 & 68 \\ 3 & 4 & 2.5 & 2.5 & 142 \\ 1 & 2 & 1.5 & 1 & 64 \\ 10 & 10 & 9 & 8 & 448 \end{bmatrix} R_2 + (-3)R_1,\ R_3 + (-1)R_1,\ R_4 + (-10)R_1$$

$$\approx \begin{bmatrix} 1 & 2 & 1 & 1.5 & 68 \\ 0 & -2 & -0.5 & -2 & -62 \\ 0 & 0 & 0.5 & -0.5 & -4 \\ 0 & -10 & -1 & -7 & -232 \end{bmatrix} R_1 + R_2,\ \left(-\frac{1}{2}\right)R_2,\ 2R_3,\ (-1)R_4$$

$$\approx \begin{bmatrix} 1 & 0 & 0.5 & -0.5 & 6 \\ 0 & 1 & 0.25 & 1 & 31 \\ 0 & 0 & 1 & -1 & -8 \\ 0 & 10 & 1 & 7 & 232 \end{bmatrix} R_4 + (-10)R_2$$

$$\approx \begin{bmatrix} 1 & 0 & 0.5 & -0.5 & 6 \\ 0 & 1 & 0.25 & 1 & 31 \\ 0 & 0 & 1 & -1 & -8 \\ 0 & 0 & -1.5 & -3 & -78 \end{bmatrix} R_1 + (-0.5)R_3,\ \ R_2 + (-0.25)R_3,\ \ R_4 + 1.5R_3$$

$$\approx \begin{bmatrix} 1 & 0 & 0 & 0 & 10 \\ 0 & 1 & 0 & 1.25 & 33 \\ 0 & 0 & 1 & -1 & -8 \\ 0 & 0 & 0 & -4.5 & -90 \end{bmatrix} \left(-\frac{1}{4.5}\right) R_4$$

$$\approx \begin{bmatrix} 1 & 0 & 0 & 0 & 10 \\ 0 & 1 & 0 & 1.25 & 33 \\ 0 & 0 & 1 & -1 & -8 \\ 0 & 0 & 0 & 1 & 20 \end{bmatrix} R_2 + (-1.25)R_4,\ \ R_3 + R_4 \approx \begin{bmatrix} 1 & 0 & 0 & 0 & 10 \\ 0 & 1 & 0 & 0 & 8 \\ 0 & 0 & 1 & 0 & 12 \\ 0 & 0 & 0 & 1 & 20 \end{bmatrix}$$

Solving, we obtain**:** $x_1 = 10, x_2 = 8, x_3 = 12,\ x_4 = 20 \cdot$

This means that with the available material, ***10*** houses can be built for Project-1, ***8*** houses for Project-2, ***12*** houses for Project-3 and ***20*** houses for Project-4.

EXAMPLE 04: Find the three numbers whose sum is 34 where the sum of the first and second is 7 and the sum of the second and third is 22.

Solution: Let *x*, *y* and *z* be the required three numbers. Then according to the question, we have

$$\begin{aligned} x + y + z &= 34 \\ x + y \quad\ \ &= 7 \\ y + z &= 22 \end{aligned}$$

To solve the above system of equations, we use the Gauss-Jordan method and have

$$A_b = \begin{bmatrix} 1 & 1 & 1 & 34 \\ 1 & 1 & 0 & 7 \\ 0 & 1 & 1 & 22 \end{bmatrix} R_2 + (-1)R_1 \approx \begin{bmatrix} 1 & 1 & 1 & 34 \\ 0 & 0 & -1 & -27 \\ 0 & 1 & 1 & 22 \end{bmatrix} (-1)R_2$$

$$\approx \begin{bmatrix} 1 & 1 & 1 & 34 \\ 0 & 0 & 1 & 27 \\ 0 & 1 & 1 & 22 \end{bmatrix} R_{23} \approx \begin{bmatrix} 1 & 0 & 0 & 12 \\ 0 & 1 & 1 & 22 \\ 0 & 0 & 1 & 27 \end{bmatrix} R_2 + (-1)R_3$$

$$\approx \begin{bmatrix} 1 & 0 & 0 & 12 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 27 \end{bmatrix} \cdot$$

Thus, $x = 12,\ y = -5,\ z = 27 \cdot$

EXAMPLE 05: Three brands of fertilizer are available that provide nitrogen, phosphoric acid, and soluble potash to the soil. One bag of each brand provides the following units of each nutrient.

Brand	Nitrogen	Phosphoric Acid	Potash
A = x	**1**	**3**	**2**
B = y	**2**	**1**	**0**
C= z	**3**	**2**	**1**
	18	23	18

For ideal growth, the soil in a certain country needs 18 units of nitrogen, 23 units of phosphoric acid, and 13 units of potash per acre. How many bags of each brand of fertilizer should be used per acre for ideal growth?

Solution: Let *x*, *y* and *z* be the number of bags of each brand of fertilizer s should be used per acre for ideal growth respectively. According to the question,

$$\begin{aligned} x+2y+3z &= 18 \\ 3x+y+2z &= 23 \\ 2x \qquad +z &= 13 \end{aligned}$$

To solve the above system of equations, we use the Gauss-Jordan method and have

$$A_b = \begin{bmatrix} 1 & 2 & 3 & 18 \\ 3 & 1 & 2 & 23 \\ 2 & 0 & 1 & 13 \end{bmatrix} R_2 + (-3)R_1,\ R_3 + (-2)R_1$$

$$\approx \begin{bmatrix} 1 & 2 & 3 & 18 \\ 0 & -5 & -7 & -31 \\ 0 & -4 & -5 & -23 \end{bmatrix} (-1)R_2, (-1)R_3 \qquad \approx \begin{bmatrix} 1 & 2 & 3 & 18 \\ 0 & 5 & 7 & 31 \\ 0 & 4 & 5 & 23 \end{bmatrix} R_2 + (-1)R_3$$

$$\approx \begin{bmatrix} 1 & 2 & 3 & 18 \\ 0 & 1 & 2 & 8 \\ 0 & 4 & 5 & 23 \end{bmatrix} R_1 + (-2)R_2,\ R_3 + (-4)R_2 \approx \begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & -3 & -9 \end{bmatrix} \left(-\frac{1}{9}\right)R_3$$

$$\approx \begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 1 & 3 \end{bmatrix} R_1 + R_3,\ R_2 + (-2)R_3 \qquad \approx \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$$\Rightarrow x = 5, y = 2,\ z = 3 \cdot$$

This shows that 5 bags of brand *A*, 2 bags of brand *B*, and 3 bags of brand *C*should be used per acre for ideal growth.