Subject Title: **LINEAR ALGEBRA AND ANALYTICAL GEOMETRY**

The title of the subject suggests that it consists three main topics:

1. Linear Algebra 2. Analytical Geometry 3. Multiple Integrals

Linear Algebra

The portion of linear algebra consists the study of a matrix, its types, operations and applications.

James Joseph Sylvester first introduced the term "Matrix" in 1848. Cayley, Hamilton, Hermann Grossmann, Frobenius and Von Neumann are among the famous mathematicians who had worked on matrix theory.

Matrices (plural) have a long history of applications in solving linear equations. Leibniz, one of the two founders of Calculus, developed the theory of determinants in 1693. Cramer developed the theory further, presenting Cramer's Rule in 1750. Carl Friedrich Gauss and Wilhelm Jordan developed Gauss-Jordan elimination method in 1800s.

1.1.1 Applications of Matrices

Tons of applications, including

- Solving system of linear equations
- Computer graphics, Image processing
- Models within many areas of Computational Science and Engineering
- Quantum Mechanics, Quantum Computing
- Many, many more …

Definition: A *matrix* is a rectangular array (or table) of numbers enclosed in square brackets (or parentheses). Generally the matrices are denoted by capital letters of English alphabet while the small letters or numerals denote the elements or entries of a matrix. For instance,

$$A=\begin{bmatrix} 2 & 0.4 & 7 \\ 0.5 & -3 & 0 \end{bmatrix},\ B=\begin{bmatrix} 0 \\ 4 \end{bmatrix},\ C=\begin{bmatrix} a & b \\ c & d \end{bmatrix},\ D=\begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix}$$

are matrices. It may be noted that the horizontal arrangement of the elements of a matrix is known as a "ROW" and the vertical one is called a "COLUMN".

Definition: If a matrix has *m* rows and *n* columns, then the ***order of the matrix*** is $m \times n$ read as ***m*** by ***n***. It may be, noted that $m \times n$ is not a multiplication of *m* and *n*. Thus, the matrix *A* as shown above is of order 2×3 , *B* is a matrix of order 2×1 , *C* is a matrix of order 2×2 , and the matrix *D* is of order 1×3. In general, a matrix of order $m \times n$ is written as:

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & a_{m3} \cdots & a_{mn} \end{bmatrix}$$

It may be noted that each element has two subscripts. The first subscript indicates the row and the second one indicates the column in which the element is located. For example, a_{12} is the element of matrix *A* positioned in the first row and the second column etc. Generally, a_{ij} is the element positioned in the i^{th} row and j^{th} column of the matrix *A*. This element is known as the $(i, j)th$ element. The matrix *A* above can be abbreviated / written as $A = (a_{ij})$.

A matrix with real elements or entities is called a ***real matrix***. For example,

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 2 \\ 0 & 8 \end{bmatrix} \quad \text{is a 3 x 2 real matrix.}$$

A matrix with complex elements is called a ***complex matrix***. For example,

$$A = \begin{bmatrix} 6 & 1-i \\ 1+i & 5 \end{bmatrix} \text{ is a 2×2 complex matrix.}$$

Matrices allow us to store given data in a concise manner. For example, at a boarding house there are 35 boys and 3 teachers who take breakfast, 65 boys and 7 teachers who take lunch and 48 boys and 5 teachers who take the dinner. It is easy to put this information in matrix form as follows:

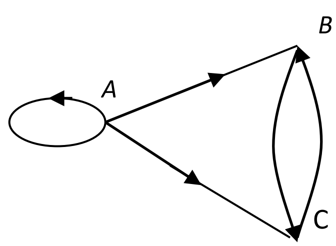
$$\begin{array}{l c} & \begin{array}{cc} \text{Boys} & \text{Teachers} \end{array} \\ \begin{array}{l} \text{Breakfast} \\ \text{Lunch} \\ \text{Dinner} \end{array} & \begin{bmatrix} 35 & 3 \\ 65 & 7 \\ 48 & 5 \end{bmatrix} \end{array}$$

Consider an example of computer communication links as shown below:

The matrix so formed from this net-work is known as "*Route Matrix*"

A, B and *C* are the cities and the arrows show the route or path of communication net-work . If there is a path between cities we indicate it by "1" and if not we indicate it by "0". The loop shows that there is

a network within the city itself. The above graph is known as "Directed graph". It may be noted that there is path from city A to C but not from *C* to *A*, hence zero is shown at the intersection of *C* and *A*.



$$\begin{array}{l c} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{l} \text{A} \\ \text{B} \\ \text{C} \end{array} & \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$