Lecture # 2

Definition: If a matrix has *m* rows and *n* columns, then the ***order of the matrix*** is $m \times n$ read as ***m*** by ***n***. It may be, noted that $m \times n$ is not a multiplication of *m* and *n*. In general, a matrix of order $m \times n$ is written as:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & a_{m3} \cdots & a_{mn} \end{bmatrix}$$

The entry a11 represents the element of first row and first column while the entry a35 represents the element of third row and fifth column.

1. Row and Column Matrices

A matrix having a single row is called a ***row matrix*** and a matrix having a single column is called a ***column matrix***.

For example, $\begin{bmatrix} 6 & 7 & 1 \end{bmatrix}$ is a row matrix and

$\begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix}$ is a column matrix.

Row and column matrices are sometimes called row vectors and column vectors.

2. Rectangular Matrix

A matrix where the number of rows and columns are not equal is, called a ***rectangular matrix***.
For instance,

$$\begin{bmatrix} 2 & -1 \\ 0 & 8 \\ 4 & 4 \\ -2 & 2 \end{bmatrix} \text{ and } \begin{bmatrix} 9 & -3 \\ 5 & 8 \\ 0 & 5 \end{bmatrix}$$

are examples of rectangular matrix.

3. Null or Zero Matrix

A matrix in which each element is zero is called a ***null matrix***. For example,

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \text{ and } \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

are examples of null matrices. A null matrix or zero matrix is usually denoted by *O*. If the null matrix is a square matrix of say order 2, we may denote it by O_2. If it is a rectangle matrix say of order 2×3, we may denote it by $O_{2\times3}$. However, where ever the order is immaterial, we may simply denote the matrix by *O*. It may further be noted that although above two matrices are null matrices, nevertheless, they are not equal as their orders are different.

4. Horizontal Matrix

An $m \times n$ rectangular matrix in which the number of rows is less than the number of its columns $(m < n)$ is called a ***horizontal matrix***. For example,

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

is a 2×3 horizontal matrix.

5. Vertical Matrix

An $m \times n$ rectangular matrix in which the number of rows is greater than the number of its columns $(m > n)$ is called a ***vertical matrix***. For example,

$$C = \begin{bmatrix} 1 & 1 \\ -3 & 5 \\ 6 & 0 \end{bmatrix}$$

is a 3×2 vertical matrix.

6. Square Matrix

A matrix in which the number of rows is equal to the number of columns is called a ***square matrix***. For example,

$$\begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix} \text{ and } \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

are square matrices of order 2 and 3,respectively. It may be noted that for the square matrix

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & a_{n3} \cdots & a_{nn} \end{bmatrix}$$

elements a_{11}, a_{22}, ..., a_{nn} form a ***leading diagonal or main diagonal or principal diagonal***.

NOTE: The elements that lie on the main diagonal are known as *leading or diagonal elements*.

7. Diagonal Matrix

A square matrix in which all elements other than the diagonal elements are zero is called a ***diagonal matrix***.It may further be noted that if at least one diagonal element of a diagonal matrix is non-zero it is still called diagonal matrix. For example,

$$\begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix} \quad \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix} \quad \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 5 \end{bmatrix} \quad \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

are diagonal matrices.

8. Scalar Matrix

A diagonal matrix in which all the diagonal elements are equal is called a ***scalar matrix***. Thus,

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \text{ is a scalar matrix.}$$

9. Identity Matrix

A scalar matrix in which each diagonal element is unity is called ***an identity matrix***. For example

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

is an identity matrix of order 3 and is denoted by I_3 · An identity matrix of order *n* is denoted by I_n · When the order is immaterial the identity matrix is usually denoted by *I*.

NOTE: ***If we use double subscript notation, the diagonal, scalar and unit matrices may be defined as follows:***

Let $A=(a_{ij})$ be a square matrix. Then *A* is called

i. a diagonal matrix if not all $a_{ij}=0$ when $i \neq j \cdot$

ii. a scalar matrix if $a_{ij}=\begin{cases} k & when\, i=j \\ 0 & when\; i \neq j, \end{cases}$ *k* being a scalar.

iii. an identity matrix if $a_{ij}=\begin{cases} 1 & when\, i=j \\ 0 & when\; i \neq j \end{cases}.$

10. Unit Matrix

A matrix with all entries equal to 1 is called a unit matrix. A unit matrix may be square or rectanguler. For example, $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and $B=\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ are unit matrices.

11. Upper and Lower Triangular Matrices

A square matrix in which every element below the principal diagonal is zero is, said to be ***upper triangularmatrix***and a square matrix in which every element above the `principal diagonal is zero is, called ***lower triangularmatrix***.

For example, $\begin{bmatrix} 7 & 4 & 3 \\ 0 & 8 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ is an **upper triangular matrix** and

$\begin{bmatrix} 4 & 0 & 0 & 0 \\ 2 & 3 & 0 & 0 \\ 1 & 5 & 8 & 0 \\ 6 & 3 & 7 & 8 \end{bmatrix}$ is a **lower triangular matrix.**

NOTE: ***If we use double subscript notation, the upper and lower triangular matrices may be defined as follows:***

Let $A=(a_{ij})$ be a square matrix. Then *A* is called

i. upper triangular matrix if : $a_{ij}=0$ for all $i>j$

ii. lower triangular matrix if: $a_{ij}=0$ for all $i<j$

12. Triangular Matrix

A matrix is said to be a ***triangular matrix***if it is either upper triangular or lower triangular matrix.

13. Trace of a Matrix

The sum of the diagonal elements of a matrix is called the *trace of the matrix.* For example, if

$$A=\begin{bmatrix} 1 & 0 & 4 \\ 0 & 3 & 5 \\ 0 & 0 & -2 \end{bmatrix},$$

then the trace of $A=1+3+(-2)=2 \cdot$

14. Equality of Matrices

Two matrices *A* and *B* are said to be ***equal*** if and only if they have the same order and each element of one is equal to the corresponding element of the other.Thus if

$$A=\begin{bmatrix} 6 & 2 \\ 5 & 7 \end{bmatrix} \text{ and } B=\begin{bmatrix} \sqrt{36} & 1+1 \\ 3+2 & 9-2 \end{bmatrix}$$

then*A* = *B* because the order of matrices A and B is same and $a_{ij}=b_{ij}$ for every $i, j \cdot$

EXAMPLE 01: Find the values of *x*, *y*, *z* and *a* which satisfy the matrix equation

$$\begin{bmatrix} x+3 & 2y+x \\ z-1 & 4a-6 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 3 & 2a \end{bmatrix}.$$

Solution: By the definition of equality of matrices

$x + 3 = 0$	$2y + x = -7$	$z - 1 = 3$	$4a - 6 = 2a$
$x = -3$	$2y - 3 = -7$	$z = 3 + 1$	$4a - 2a = 6$
$x = -3$	$2y = -4$	$z = 4$	$2a = 6$

Solving these equations, we obtain: $x = -3$, $y = -2$, $z = 4$, $a = 3$.