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0077-BSCS-20
(B)

GC University Lahore
Midterm Exam Fall 2021

Semester: III | **Time: 1 hour**
Subject: Differential Equation | **Total Marks: 20**

(SECTION-I)

Note: Question **ONE** is compulsory. (10)

$\frac{09}{10}$

Q1) Multiple Choice Questions.

✓ 1)The order of a differential equation is defined as:
(a) The highest degree of variable (b) The order of highest derivatives
(c) The power of variable in solution (d) None of these

✓ 2) The general solution of differential equation $\frac{dy}{dx} = \frac{y}{x}$ is:
(a) $\log y = kx$ (b) $y = kx$ (c) $y = \frac{k}{x}$ (d) $y = k \log x$

✓ 3) Integrating factor of differential equation $\cos x \frac{dy}{dx} + y \sin x = 1$ is:
(a) sin x (b) cos x (c) sec x (d) tan x

✓ 4) If p & q are the order and degree of differential equation $y\frac{dy}{dx} + x^2 \left(\frac{d^2y}{dx^2}\right)^3 + xy = \cos x$ then 2, 3
(a) p<q (b) p=q (c) p>q (d) None of these

✓ 5) The differential equation $Mdx + Ndy = 0$ is defined as an exact differential equation if:
(a) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$ (b) $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ (c) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ (d) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

✓ 6) The order of differential equation $\frac{\partial^2 y}{\partial x^2} + y^2 = x + e^x$ is
(a) 2 (b) 3 (c) 0 (d) 1

✓ 7)If $\frac{dy}{dx} = \frac{f(x,y)}{\emptyset(x,y)}$ is a homogeneous differential equation, then it can be made in the form " separable variables" by putting:
(a) $y^2 = vx$ (b) $x^2 = vy$ (c) $y = vx$ (d) $x = vy$

✓ 8)The differential equation $\frac{dy}{dx} = \frac{ax+by+c}{a'x+b'y+c'}$ is:
(a) Homogeneous (b) Non Homogeneous (c) Non-Linear (d) Exact Equation

✓ 9) The differential equation $[1 + (y')^2]^{\frac{1}{2}} = y''$ has the order and degree respectively.
(a) 2, 1 (b) 2, 2 (c) 1, 2 (d) 2, $\frac{1}{2}$

✗ 10)A differential equation is linear if:
(a) the derivative of y are all of the first degree (b) y appears in the first degree only
n'a (c) y and its derivatives are not multiplied together (d) All of these

(SECTION- II)

Note: Attempt any **TWO** Questions. Each question carries equal Marks.

Q#2: Solve $(x^2 + xy + y^2)dx - x^2dy = 0.$ (5)

Q#3: Solve (by finding an I.F) $dx + \left(\frac{x}{y} - \sin y\right) dy = 0$ (5)

Q#4: Find an equation of orthogonal trajectories of the family of the curve. (5)

$$r^n = a^n \cos n\theta$$

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Answer Script for
Mid Term / Final Examination

Script No. 96527

Name Waseem Akram Roll No. 0077-BSCS-20 Class BSCS (Section: B)

Session 2020-24 Academic Year 2nd Semester 3rd Course Code

Title Differential Equations Signature of the Candidate Waseem Signature of Teacher

Q. No.	1	2	3	4	5	6	7	8	9	10	Total in Figures	Total in Words
Marks	09		05	05							19	

START WRITING FROM HERE

Signature of Examiner

Q.No.1 Q.No 2

$$(x^2+xy+y^2)\,dx - x^2\,dy = 0$$

Solution:

Here, $M(x,y) = x^2+xy+y^2$, $N(x,y) = -x^2$

$\frac{\partial M}{\partial y} = 0 + x + 2y$, $\frac{\partial N}{\partial x} = -2x$

$\frac{\partial M}{\partial y} = x + 2y$,

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ So, given equation is not exact.

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Q. No. (3)

$$dx + \left(\frac{x}{y} - \sin y\right) dy = 0 \quad \text{—— (i)}$$

Solution

$M(x,y) = 1$, $N(x,y) = \frac{x}{y} - \sin y$

$\frac{\partial M}{\partial y} = 0$

, $\frac{\partial N}{\partial x} = \frac{1}{y} - 0$

, $\frac{\partial N}{\partial x} = \frac{1}{y}$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ So, given equation is Not exact.

Now,

$$\frac{N_x - M_y}{M} = \frac{\frac{1}{y} - 0}{1}$$

$$\frac{N_x - M_y}{M} = \frac{1}{y}$$

So,

$$I.F = e^{\int \frac{1}{y} dy}$$

$$I.F = e^{\ln y}$$

$$I.F = y$$

Multiplying equation (i) by I.F

$$y\,dx + (x - y\sin y)\,dy = 0$$

$M(x,y) = y$, $N(x,y) = x - y\sin y$

$\frac{\partial M}{\partial y} = 1$, $\frac{\partial N}{\partial x} = 1 - 0$

, $\frac{\partial N}{\partial x} = 1$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ So, it is an Exact equation.

The solution is,

$$\int M(x,y)\,dx + \int N(x,y)\begin{pmatrix}\text{Terms Independent} \\ \text{of } x \text{ only}\end{pmatrix} dy = C$$

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$$\int y \, dx + \int (-y \sin y) \, dy = c$$

$$y \int 1 \, dx + (-) \int y \sin y \, dy = c$$

$$y \int 1 \, dx - \left[y(-\cos y) - \int 1 (-\cos y) \, dy \right] = c$$

$$y(x) + y \cos y - \int \cos y \, dy = c$$

$$xy + y \cos y - \sin y = c$$

Here 'c' is the constant of Integration

5/5

Q. No [redacted] (4)

$$r^n = a^n \cos n\theta$$

Solution:

First, we find the differential equation of given family of curves.

Taking 'ln' on both sides.

$$\ln r^n = \ln(a^n \cos n\theta)$$

$$n \ln r = \ln a^n + \ln \cos n\theta$$

$$n \ln r = n \ln a + \ln \cos n\theta$$

Differentiating with respect to 'θ'.

$$n \frac{1}{r} \frac{dr}{d\theta} = 0 + \frac{1}{\cos n\theta}(-\sin n\theta)(n)$$

$$\cancel{n} \frac{1}{r} \frac{dr}{d\theta} = \cancel{n} \frac{-\sin n\theta}{\cos n\theta}$$

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$$\frac{1}{r}\frac{dr}{d\theta} = -\frac{\sin n\theta}{\cos n\theta}$$

$$r\frac{d\theta}{dr} = -\frac{\cos n\theta}{\sin n\theta}$$

Now, Differential equation of orthogonal trajectories

$$r\frac{d\theta}{dr} = +\frac{\sin n\theta}{\cos n\theta}$$

Now, we will solve it

$$r\,d\theta = \frac{\sin n\theta}{\cos n\theta}\,dr$$

$$\frac{\cos n\theta}{\sin n\theta}\,d\theta = \frac{1}{r}\,dr$$

$$\frac{1}{r}\,dr = \frac{\cos n\theta}{\sin n\theta}\,d\theta$$

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Integrating both sides.

$$\int \frac{1}{r}\,dr = \int \frac{\cos n\theta}{\sin n\theta}\,d\theta$$

$$\int \frac{1}{r}\,dr = \frac{1}{n}\int \frac{n\cos n\theta}{\sin n\theta}\,d\theta$$

$$n\ln r = \ln \sin n\theta + \ln b$$

$$\ln r^n = \ln b \sin n\theta$$

$$r^n = b \sin n\theta$$

Here, b is the constant of integration.
Which is the required equation of orthogonal trajectories of given family of curves.

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ So, given equation is not exact.