Introduction to Casual Inference - 097400

Winter 2021 - HW 3

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Question 1:

Consider the next DAG:

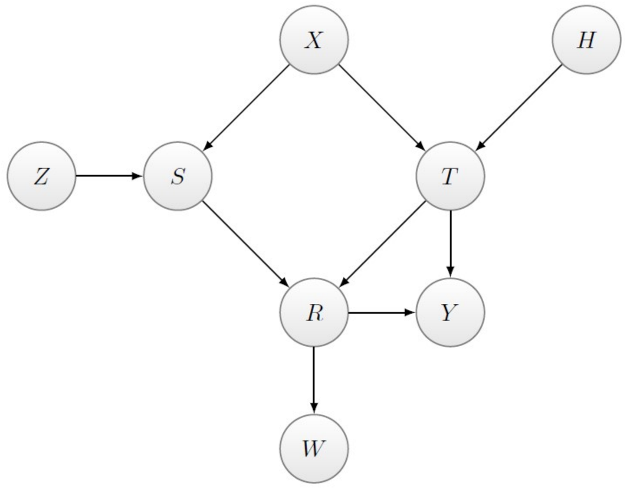

Figure 1:

1)

$Z \to Y$

a)

all possible paths:

$$i : Z \to S \to R \to Y$$

$$ii : Z \to S \to R \leftarrow T \to Y$$

$$iii : Z \to S \leftarrow X \to T \to Y$$

$$iv : Z \to S \leftarrow X \to T \to R \to Y$$

b)

to block i we need to know S or R

to block ii we need to know S or T or to not know R, W and Y

to block iii we need to know X or T or to not know S, R, W and Y

to block iv we need to know X or T or R or to not know S, R, W and Y

possible set of nodes to block all paths (we show only the minimal sets):

$$\{S, T\}, \{S, X\}, \{R, T\}$$

2)

$X \to W$

a)

all possible paths:

$$i : X \to S \to R \to W$$

$$ii : X \to T \to R \to W$$

$$iii : X \to T \to Y \leftarrow R \to W$$

b)

to block i we need to know S or R

to block ii we need to know T or R

to block iii we need to know T or R or to not know Y

possible set of nodes to block all paths (we show only the minimal sets):

$$\{R\}$$

3)

$H \to S$

a)

all possible paths:

$$i : H \to T \leftarrow X \to S$$

$$ii : H \to T \to R \leftarrow S$$

$$iii : H \to T \to Y \leftarrow R \leftarrow S$$

b)

to block i we need to know X or to not know T, Y, R and W

to block ii we need to know T or to not know Y, R and W

to block iii we need to know T or R or to not know Y

possible set of nodes to block all paths (we show only the minimal sets):

$$\emptyset$$

Problem 2:

Consider the next casual graph:

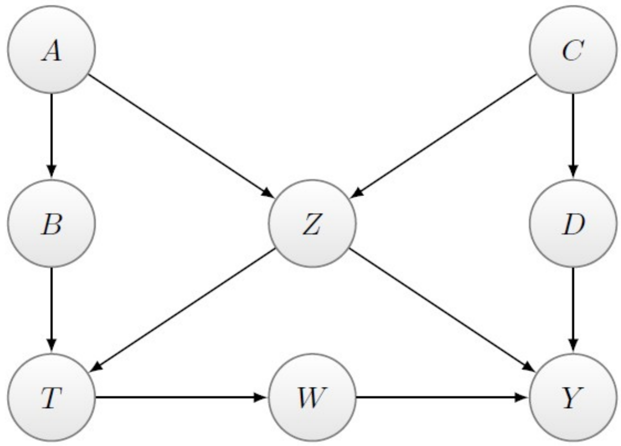

Figure 2:

i)

all possible paths from T to Y that contains an arrow into T are:

$$i : Y \leftarrow Z \rightarrow T$$

$$ii : Y \leftarrow Z \leftarrow A \rightarrow B \rightarrow T$$

$$iii : Y \leftarrow D \leftarrow C \rightarrow Z \rightarrow T$$

$$iv : Y \leftarrow D \leftarrow C \rightarrow Z \leftarrow A \rightarrow B \rightarrow T$$

to block i we need to know Z

to block ii we need to know Z or A or B

to block iii we need to know D or C or Z

to block iv we need to know D or C or A or B

in addition the sets can't contain W which is a descendant of T

thus the sets of variables that satisfy the backdoor criteria are:

$$\{Z, A\}, \{Z, B\}, \{Z, C\}, \{Z, D\}$$

$$\{Z, A, B\}, \{Z, A, C\}, \{Z, A, D\}, \{Z, B, C\}, \{Z, B, D\}, \{Z, C, D\}$$

$$\{Z, A, B, C\}, \{Z, A, B, D\}, \{Z, A, C, D\}, \{Z, B, C, D\}$$

$$\{Z, A, B, C, D\}$$

ii)

minimal sets of variables that satisfy the backdoor criteria:

$$\{Z, D\}, \{Z, C\}, \{Z, A\}, \{Z, B\}$$

iii)

all possible paths from D to Y that contains an arrow into D are:

$$i : Y \leftarrow Z \leftarrow C \rightarrow D$$

$$ii : Y \leftarrow W \leftarrow T \leftarrow Z \leftarrow C \rightarrow D$$

$$iii : Y \leftarrow W \leftarrow T \leftarrow B \leftarrow A \rightarrow Z \leftarrow C \rightarrow D$$

to block i we need to know Z or C

to block ii we need to know W or T or Z or C

to block iii we need to know W or T or B or A or C or to not know Z, T and W

D has no descendants besides Y

thus the minimal set of variables that needs to be measured is:

$$\{C\}$$

Problem 3:

1.

The casual graph that describes the given experiment.

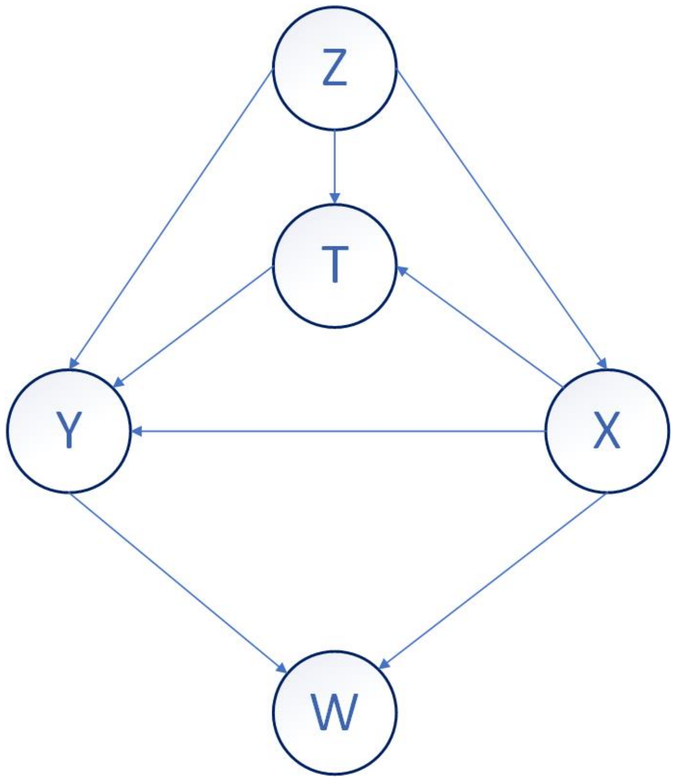

Figure 3:

2.

now let's estimate the conditional distributions corresponding to the graph using the data table:

$P_Z(0)$	$P_Z(1)$
$\frac{9}{22}$	$\frac{13}{22}$

	$P_{X\|Z}(0)$	$P_{X\|Z}(1)$
$Z=0$	$\frac{6}{9}=\frac{2}{3}$	$\frac{3}{9}=\frac{1}{3}$
$Z=1$	$\frac{7}{13}$	$\frac{6}{13}$

	$P_{T\|Z,X}(0)$	$P_{T\|Z,X}(1)$
$Z=0, X=0$	$\frac{2}{6}=\frac{1}{3}$	$\frac{4}{6}=\frac{2}{3}$
$Z=0, X=1$	$\frac{1}{3}$	$\frac{2}{3}$
$Z=1, X=0$	$\frac{3}{7}$	$\frac{4}{7}$
$Z=1, X=1$	$\frac{4}{6}=\frac{2}{3}$	$\frac{2}{6}=\frac{1}{3}$

	$P_{W\|X,Y}(0)$	$P_{W\|X,Y}(1)$
$X=0, Y=0$	$\frac{6}{8}=\frac{3}{4}$	$\frac{2}{8}=\frac{1}{4}$
$X=0, Y=1$	$\frac{2}{5}$	$\frac{3}{5}$
$X=1, Y=0$	$\frac{1}{2}$	$\frac{1}{2}$
$X=1, Y=1$	$\frac{1}{5}$	$\frac{4}{5}$

	$P_{Y\|Z,X,T}(0)$	$P_{Y\|Z,X,T}(1)$
$Z=0, X=0, T=0$	1	0
$Z=0, X=0, T=1$	$\frac{1}{2}$	$\frac{1}{2}$
$Z=0, X=1, T=0$	1	0
$Z=0, X=1, T=1$	$\frac{1}{2}$	$\frac{1}{2}$
$Z=1, X=0, T=0$	$\frac{2}{3}$	$\frac{1}{3}$
$Z=1, X=0, T=1$	$\frac{1}{2}$	$\frac{1}{2}$
$Z=1, X=1, T=0$	$\frac{1}{2}$	$\frac{1}{2}$
$Z=1, X=1, T=1$	0	1

3.

we are trying to estiamte the causal effect of T on Y

all possible paths from T to Y that contains an arrow into T are:

$$i : T \leftarrow X \rightarrow Y$$

$$ii : T \leftarrow X \rightarrow W \leftarrow Y$$

$$iii : T \leftarrow X \leftarrow Z \rightarrow Y$$

to block i we need to know X

to block ii we need to know X or to not know W

to block iii we need to know X or Z

in addition the sets can't contain W which is a descendant of T

thus the minimal set of variables that need to be measured in order to identify the effect of T on Y is:

$$\{X\}$$

In order to estiamte the ATE we will need the distribution of Y depending on X and T:

	$P_{Y\|X,T}(0)$	$P_{Y\|X,T}(1)$
$X=0, T=0$	$\frac{4}{5}$	$\frac{1}{5}$
$X=0, T=1$	$\frac{1}{2}$	$\frac{1}{2}$
$X=1, T=0$	$\frac{3}{5}$	$\frac{2}{5}$
$X=1, T=1$	$\frac{1}{4}$	$\frac{3}{4}$

and the distribution of X :

$P_X(0)$	$P_X(1)$
$\frac{13}{22}$	$\frac{9}{22}$

now using the backdor adjustment:

$$\begin{aligned} ATE &= \mathbb{E}\left[Y|do\left(T=1\right)\right] - \mathbb{E}\left[Y|do\left(T=0\right)\right] = \\ &= \mathbb{E}_X\left[\mathbb{E}_Y\left[Y|T=1,X\right]\right] - \mathbb{E}_X\left[\mathbb{E}_Y\left[Y|T=0,X\right]\right] \end{aligned}$$

$$\begin{aligned} \mathbb{E}_X\left[\mathbb{E}_Y\left[Y|T=1,X\right]\right] &= \mathbb{E}\left[Y|T=1,X=0\right]P\left(X=0\right) + \mathbb{E}\left[Y|T=1,X=1\right]P\left(X=1\right) = \\ &= \frac{1}{2}\cdot\frac{13}{22} + \frac{3}{4}\cdot\frac{9}{22} \end{aligned}$$

$$\begin{aligned} \mathbb{E}_X\left[\mathbb{E}_Y\left[Y|T=0,X\right]\right] &= \mathbb{E}\left[Y|T=0,X=0\right]P\left(X=0\right) + \mathbb{E}\left[Y|T=0,X=1\right]P\left(X=1\right) = \\ &= \frac{1}{5}\cdot\frac{13}{22} + \frac{2}{5}\cdot\frac{9}{22} \end{aligned}$$

$$\begin{aligned} ATE &= \frac{1}{2}\cdot\frac{13}{22} + \frac{3}{4}\cdot\frac{9}{22} - \frac{1}{5}\cdot\frac{13}{22} - \frac{2}{5}\cdot\frac{9}{22} = \\ &= \frac{13}{22}\left(\frac{1}{2} - \frac{1}{5}\right) + \frac{9}{22}\left(\frac{3}{4} - \frac{2}{5}\right) = \\ &= \frac{13}{22}\cdot\frac{3}{10} + \frac{9}{22}\cdot\frac{7}{20} = \frac{141}{440} = 0.32 \end{aligned}$$

$$\boxed{ATE = 0.32}$$