TLS continued

- What if $\lambda_{\min}(\mathbf{A}^T\mathbf{A}) \le \lambda_{\min}(\mathbf{B})$?
- This means $\lambda_{\min}(\mathbf{A}^T\mathbf{A}) = \lambda_{\min}(\mathbf{B})$
- Let $\mathbf{v}$ be the eigenvector corresponding to the min eigenvalue of $\mathbf{A}^T\mathbf{A}$

$$f(\alpha) = \frac{\|\alpha\mathbf{A}\mathbf{v} - \mathbf{b}\|^2}{\|\alpha\mathbf{v}\|^2 + 1} = \frac{\|\mathbf{A}\mathbf{v} - \frac{\mathbf{b}}{\alpha}\|^2}{\|\mathbf{v}\|^2 + \frac{1}{\alpha^2}} \ge \lambda_{\min}(\mathbf{B}),\ \forall\alpha$$

- Moreover, taking $\alpha \to \infty$ we have that

$$\lim_{\alpha\to\infty} f(\alpha) = \frac{\|\mathbf{A}\mathbf{v}\|^2}{\|\mathbf{v}\|^2} = \lambda_{\min}(\mathbf{A}^T\mathbf{A}) = \lambda_{\min}(\mathbf{B}).$$

- Thus, the optimal value is the same as in the case where $\lambda_{\min}(\mathbf{A}^T\mathbf{A}) > \lambda_{\min}(\mathbf{B})$, but it is not attained.