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362-1-6221 – Optimal Control

Optimal Missile Guidance

Statement of the problem

We consider an LQG problem in the realm of missile guidance in the simplified planar approximation case. The classical “guidance triangle” represents the relative geometrical configuration of the pursuer/target system. Let y denote the lateral position of the pursuer with respect to the initial line-of-sight (LOS), let v denote the lateral velocity of the pursuer with respect to the initial LOS, let V denote the constant closing velocity, i.e., the relative velocity of the pursuer along the LOS. The range from the pursuer to the target is thus $V(t_f - t)$, where t is the instantaneous time and t_f is the final time. Let θ denote the angle between the initial LOS and the instantaneous LOS, θ is small when the pursuer is far from the target and quickly grows towards the final time t_f, when the pursuer is close to the target. For small enough θ,

$$\theta \simeq \frac{y}{V(t_f - t)}$$

Let a_P and a_T denote the lateral acceleration of the pursuer and of the target, respectively, with respect to the LOS. The goal is to bring the final value of y (the “miss-distance”) to a small value while avoiding too high efforts on the pursuer acceleration.

Continuous-time description

The continuous-time dynamics of the problem are:

$$\begin{aligned} \dot{y} &= v \\ \dot{v} &= a_P - a_T \end{aligned}$$

where a_P is the control input and a_T is a random process applied by the evader in order to avoid capture. A very common way of modeling the evasion strategy is to assume that a_T is a first-order Markov process satisfying the following equation:

$$da_T = -\frac{1}{\tau} a_T \, dt + d\beta$$

where τ is a known correlation time of the evader maneuvers and β is a Brownian Motion with intensity W.

The initial lateral position, y_0, is zero by definition, and the initial lateral velocity, v_0, is random and assumed to be the result of launching errors. The statistical model is:

$$\begin{aligned} &E\{y_0\} = 0 \quad E\{v_0\} = 0 \\ &E\{y_0^2\} = 0 \quad E\{y_0 v_0\} = 0 \quad E\{v_0^2\} = P_{22}(0) \\ &E\{a_T(0)\} = 0 \quad E\{a_T(0) v_0\} = E\{a_T(0) y_0\} = 0 \quad E\{a_T(0)^2\} = P_{33}(0) \end{aligned}$$

The measurement on-board the pursuer consists of the LOS angle θ which is corrupted by an additive fading and scintillation noise:

$$\begin{aligned} dz &= \theta \, dt + dm \\ &= \begin{bmatrix} \frac{1}{V(t_f - t)} & 0 & 0 \end{bmatrix} \begin{bmatrix} y \\ v \\ a_T \end{bmatrix} dt + dm \end{aligned}$$

where m is a Brownian motion with intensity M expressed as:

$$M = R_1 + \frac{R_2}{(t_f - t)^2}$$

Note that the second term is growing towards the final time. This effect models the increasing scintillation noise when the pursuer gets close to the target. It also reflects the fact that the linear measurement model equation is not valid any more close to the target (θ is not small any more). The measurement equation can be re-written in the classical form:

$$dz = H\mathbf{x} \, dt + dm$$

Numerical Data (for Falcon or Sparrow guided missile)

$$V = 3000 \frac{\text{ft}}{\text{sec}} \quad t_f = 10 \text{ sec} \quad \tau = 2 \text{ sec} \quad R_1 = 15 \times 10^{-6} \frac{\text{rad}^2}{\text{sec}} \quad R_2 = 1.67 \times 10^{-3} \frac{\text{rad}^2}{\text{sec}^3}$$

$$W = 100 \frac{\text{m}^2}{\text{sec}^5}$$

Objective

Given the above assumptions you want to find the optimal acceleration function $a_P(t)$ that minimizes the following performance index:

$$J = E\{ \frac{1}{2} y(t_f)^2 + \frac{b}{2} \int_0^{t_f} a_P^2 \, dt \}$$

where the admissible set for the acceleration $a_P(t)$ is the set of functions of the history of the measurements $\{z_s, 0 \le s \le t\}$.

$$b = 1.52 \times 10^{-2} \quad P_{22}(0) = 16 \frac{\text{m}^2}{\text{sec}^2} \quad P_{33}(0) = 400 \frac{\text{m}^2}{\text{sec}^5}$$

where $P_{22}(0)$, $P_{33}(0)$ denote the initial estimation covariances for the velocity and the target acceleration, respectively.

The evaluation of your solution will be based on three criteria: 1/ its effectiveness, 2/ the clarity in the solution methodology, 3/ the clarity in the presentation of the simulation results.

Good Success !!