# Fundamentals of Perpetual Futures

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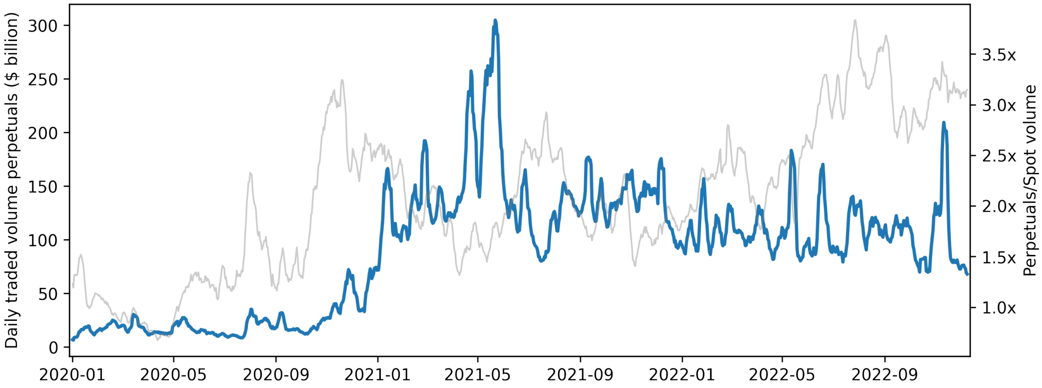
# What are perpetual futures?

- Perpetual futures are swaps that never expire
- Efficient way to hedge and speculate
  - Allow high leverage
  - No need to take delivery of crypto
  - No rollover
  - Concentrates liquidity
- First introduced in Shiller (1993)

Image by Asaf Manela × Midjourney

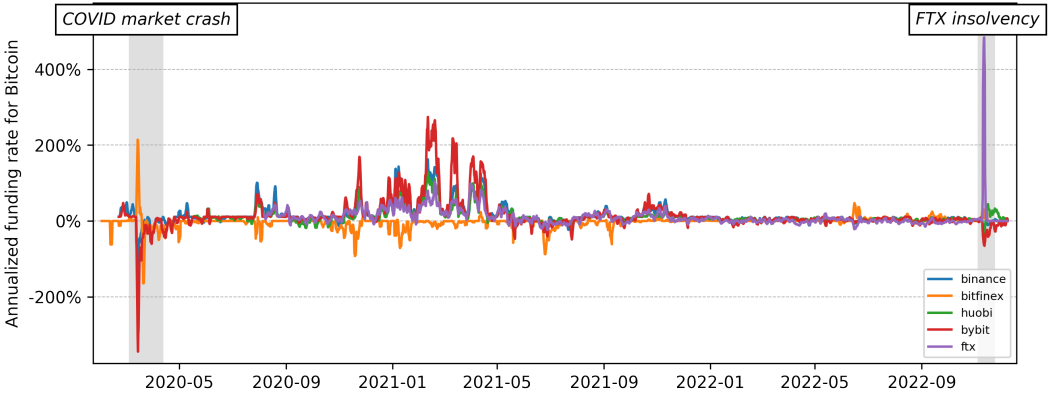
# Most popular derivative traded in crypto markets

## $100 billion daily volume



# Among the most actively-traded products on FTX before it imploded

## Alameda Research was counterparty to many such leveraged trades



# Questions

1. What are the fundamental values of perpetual futures in theory?
2. How large are deviations from these fundamentals in practice?

# Our paper

Theoretic contribution

- Derive no-arbitrage prices for perpetual futures in frictionless markets
- Derive no-arbitrage bounds for markets with trading costs

Empirical findings

- Deviations of crypto perpetual futures from no-arbitrage prices are large
- Deviations comove across cryptocurrencies
- Diminish over time as crypto markets develop and become more efficient
- Simple trading strategy generates large Sharpe ratios

# Related work

- Descriptive evidence on perpetual futures: Alexander, Choi, Park, and Sohn (2020), Franz and Schmeling (2021), De Blasis and Webb (2022), and Ferko, Moin, Onur, and Penick (2022)
- Theory on perpetuals: Angeris, Chitra, Evans, and Lorig (2022) derive no-arb prices for perpetuals assuming payoff is a fixed function of the spot price
- Recent work on fixed-maturity crypto futures by Schmeling, Schrimpf, and Karamfil (2022) find large profits to carry strategies (Du, Tepper, and Verdelhan, 2018); Koijen, Moskowitz, Pedersen, and Vrugt, 2018)

# Fixed-maturity futures refresher

## Definition

- Two counterparties agree at time $t = 0$ to exchange underlying asset at future expiration time $\tau$
- Underlying asset price $S_t$ fluctuates over time
- Future price $F_{0,\tau}$ is fixed at initiation
- At expiration, short counterparty delivers to long counterparty the underlying in exchange for the futures price[1]

[1] This technically describes a forward contract. A futures contract is guaranteed by the exchange and is marked-to-market periodically against a margin account.

# Fixed-maturity futures refresher

## No arbitrage pricing

- In the absence of arbitrage, the futures price $F_{0,\tau}$ only depends on the initial underlying price $S_0$ and the interest rate $r$

$$F_{0,\tau} = S_0 e^{r\tau} \tag{1}$$

- Just the future value of the spot price

# Fixed-maturity futures refresher

## Arbitrage strategy

- Suppose the futures price is actually higher

$$F_{0,\tau} > S_0 e^{r\tau}$$

- Arbitrageur would today
  1. Open a short futures position (0 cashflow today)
  2. Borrow dollars in cash markets ($+S_0$)
  3. Buy the underlying at spot price ($-S_0$)
- At expiration time $\tau$
  1. Collect futures price ($+F_{0,\tau}$)
  2. Deliver underlying asset
  3. Repay dollar loan ($-S_0 e^{r\tau}$)
- Net cashflow at expiration $F_{0,\tau} - S_0 e^{r\tau} > 0$

# Perpetual futures (swap)

## Definition

- No expiration date!
- No initial cash
- Can be closed at any time
- At termination, short pays long $F_t - F_0$
- Long counterparty pays short one at each interval $ds$ a funding rate:

$$\text{Funding Rate}_s = \kappa(F_s - S_s)ds$$

- In most exchanges, funding rate is paid every 8 hours and is roughly the futures-spot difference
- In this case, $\kappa = 3 \times 365 = 1095$
- Arbitrage is risky: Nothing guarantees that futures price converge to the spot!

# Random-maturity arbitrage

## Certain profits at an uncertain future time

### Definition (Traditional)

A riskless arbitrage opportunity is defined with respect to payoff $x$ at a certain future time $T$ and its price $p(x)$. If (1) $x \geq 0$ almost surely, (2) $x > 0$ with some positive probability, (3) its price satisfies $p(x) \leq 0$, then this payoff is an arbitrage opportunity.

### Definition (Our extension)

A random-maturity arbitrage opportunity is defined with respect to a bounded random payoff $x$ at a future random time $\tau$, $\tau \in (0, \infty)$, and its price $p(x)$. If (1) $x \geq 0$ almost surely, (2) $x > 0$ with some positive probability, (3) its price satisfies $p(x) \leq 0$, then this payoff is a random-maturity arbitrage opportunity.

# Random-maturity arbitrage

## Objection

- "But random-maturity arbitrage opportunities are not riskless!"
- Yes, but no arbitrage prices are always just a useful fiction
- Real markets have transactions costs, margin requirements, and risk of liquidation
- Random-maturity no arb prices are similarly a useful benchmark

# Perpetual futures

## No arbitrage pricing

### Assumptions

A1 *The perpetual futures-spot gap is bounded*

A2 *The risk-free rate $r$ for arbitrageurs is constant*

### Proposition

*Arbitrageurs will trade the perpetual futures toward*

$$F_t = S_t\left(1 + \frac{r}{\kappa}\right) \tag{2}$$

The perpetual futures to spot gap is small when interest rates $r$ are low relative to the funding rate coefficient $\kappa$

# Perpetual futures

## Arbitrage strategy

- Suppose the futures price is actually higher

$$F_0 > S_0 \left(1 + \frac{r}{\kappa}\right)$$

- Arbitrageur would today
  1. Open a short futures position (0 cashflow today)
  2. Borrow dollars in cash markets ($+S_0$)
  3. Buy the underlying at spot price ($-S_0$)
- At (random) unwinding time $t$
  1. Close futures position ($+F_0 - F_t$)
  2. Sell the underlying at spot price ($+S_t$)
  3. Repay dollar loan ($-S_0 e^{rt}$)
  4. Funding payments accrued ($\kappa \int_0^t (F_s - S_s) e^{r(t-s)} ds$)
- Net discounted payoff $= e^{-rt} F_0 - S_0 - e^{-rt}(F_t - S_t) + \kappa \int_0^t (F_s - S_s) e^{-rs} ds$

# Perpetual futures

## Arbitrage strategy (continued)

- Suppose this is not an arbitrage opportunity
- Then for all $t$, payoff is nonpositive

$$\underbrace{e^{-rt}F_0 - S_0}_{\text{traditional spread}} + \underbrace{\kappa \int_0^t (F_s - S_s)e^{-rs}ds}_{\text{funding payments}} \leq \underbrace{e^{-rt}(F_t - S_t)}_{\text{spread at unwinding}} \tag{3}$$

- Turns out we can bound this process from below and show that

$$e^{-rt}(F_t - S_t) \geq \frac{F_0 re^{-rt}}{\kappa + r} + \left(\frac{F_0}{1+\frac{r}{\kappa}} - S_0\right) e^{\kappa t} \tag{4}$$

- But $F_0 > S_0\left(1 + \frac{r}{\kappa}\right)$ implies RHS $\rightarrow \infty$ violating bounded spread assumption

$\Rightarrow$ Arbitrage! Some $t$ exists when arb can unwind at a positive discounted payoff

# Perpetual futures

## No arbitrage bounds

### Proposition

*If there are constant round-trip trading costs $C > 0$, arbitrageurs would trade the perpetual futures until it lies within a bound of the spot:*

$$S_t \left(1 + \frac{r}{\kappa}\right) - C \leq F_t \leq S_t \left(1 + \frac{r}{\kappa}\right) + C$$

# Data

Focus on the 5 largest cryptocurrencies with a total market cap of $529B
64.15% of crypto spot market in Nov 2022

| Crypto | start date | end date | N |
|---|---|---|---|
| BTC | 2019-09-10 | 2022-11-13 | 27,895 |
| ETH | 2019-11-27 | 2022-11-13 | 25,985 |
| BNB | 2020-02-10 | 2022-11-13 | 24,184 |
| DOGE | 2020-07-10 | 2022-11-13 | 20,559 |
| ADA | 2020-01-31 | 2022-11-13 | 24,424 |

- Perpetual futures, spot, and funding rate at 1-hour frequency from Binance
- Funding rate is paid every 8 hours
- Market is open 24/7
- Risk-free interest rates from Aave

# Deviations from no-arbitrage benchmarks

Define $\rho$ as the annualized interest rate deviation that rationalizes an observed future-spot spread

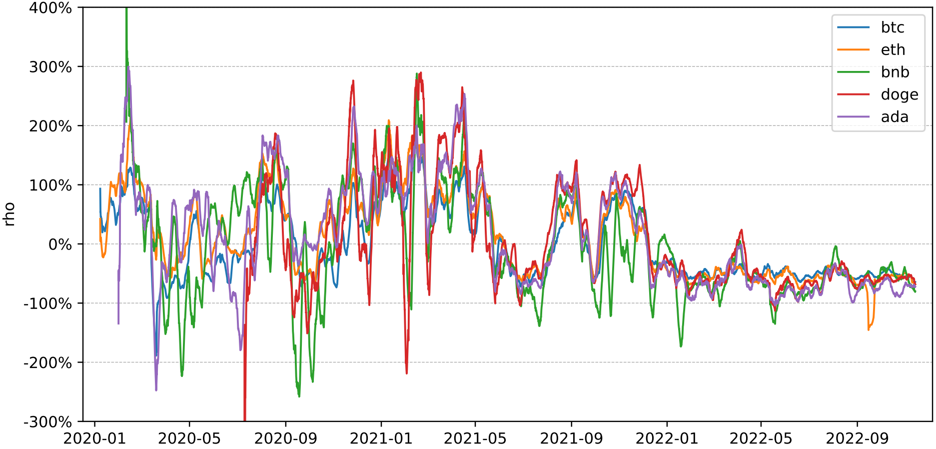
$$F = S\left(1 + \frac{r + \rho}{\kappa}\right)$$

or approximately

$$\rho \approx \kappa(\log F - \log S) - r$$

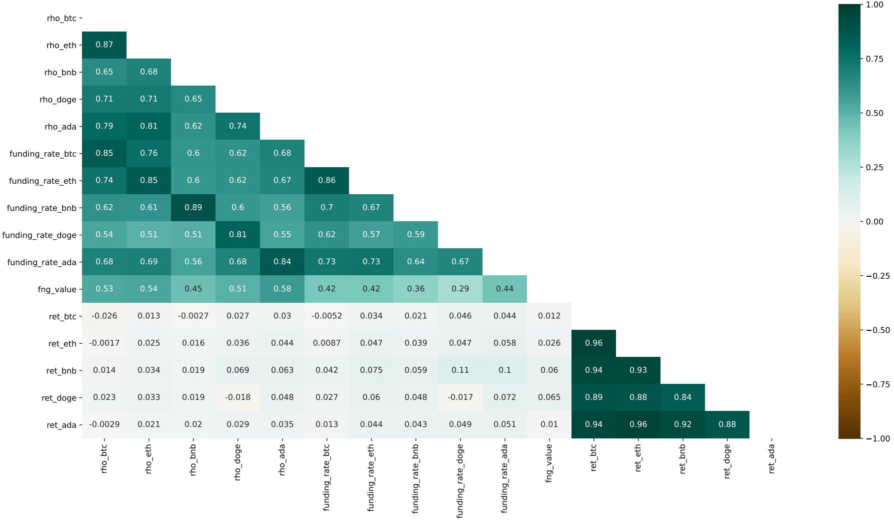
# Deviations of perpetual futures from no-arbitrage benchmarks

Mean absolute deviation is about 60–100% per year
Considerably larger than deviations Du-Tepper-Verdelhan (2018) find in traditional FX markets
Deviations shrink in early 2022



# Correlations

## Deviations are correlated across currencies, but uncorrelated with spot returns



# Random-maturity arbitrage strategy

## No arbitrage bounds prescribe a threshold strategy to exploit divergence from fundamentals

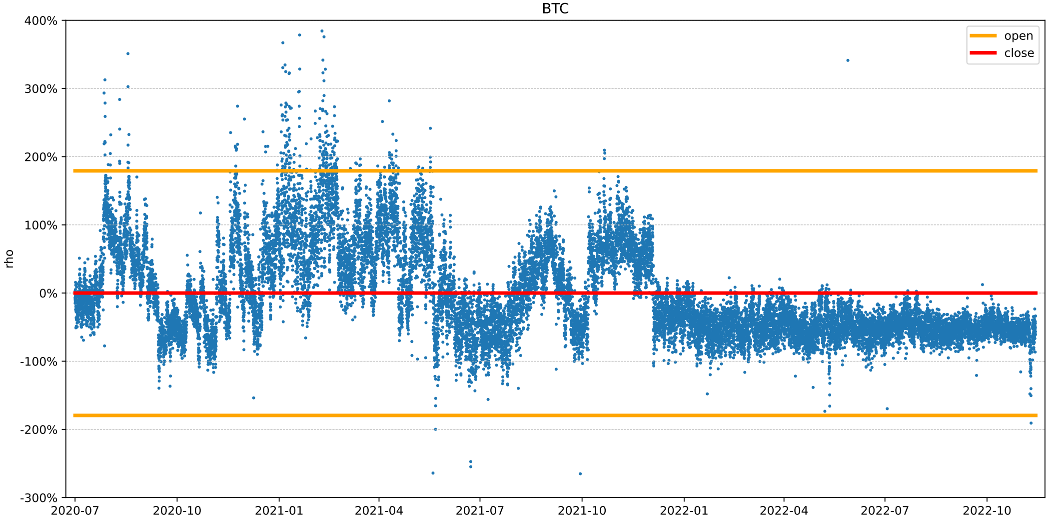
| Fee levels | Spot | Futures | $\rho_l$ | $\rho_u$ |
|---|---|---|---|---|
| No | 0% | 0% | 0.0% | 0.0% |
| Low | 0.0225% | 0.0018% | -53.2% | 53.2% |
| Medium | 0.045% | 0.0072% | -114.4% | 114.3% |
| High | 0.0675% | 0.0144% | -179.5% | 179.2% |

Given $C =$ round-trip percentage trading costs

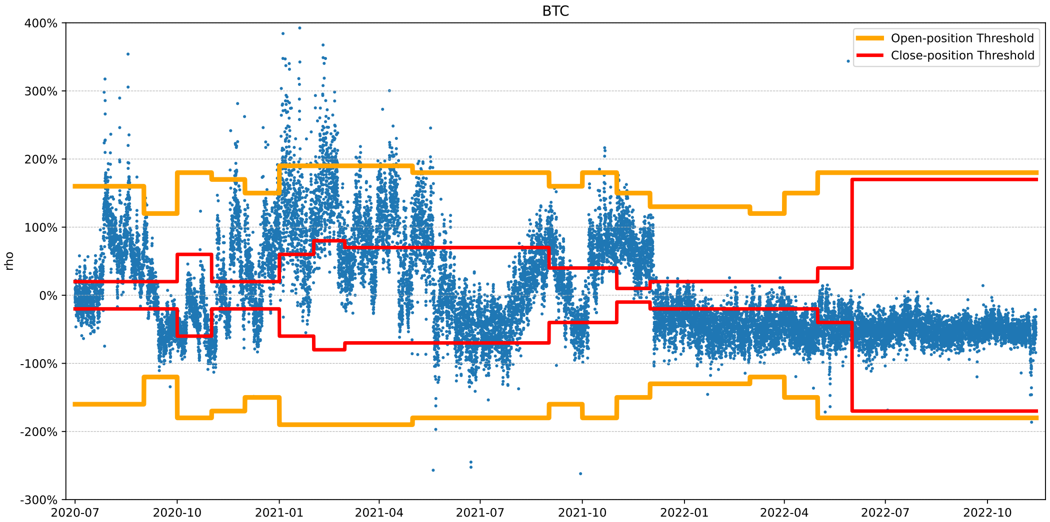
- if $\rho > \rho_u = \kappa log(1 + C)$
- then borrow cash, buy the spot and short the future
- collect funding rate and pay interest
- close the positive when $\rho$ is back to zero

Opposite strategy if $\rho < \rho_l = \kappa log(1 - C)$

# Random-maturity arbitrage strategy: BTC, high trading costs



# Data-driven strategy: BTC, high trading costs



# Random-maturity arbitrage strategy: High trading costs

| | | Unrestricted | | | | Long-spot Only | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 2020 | 2021 | 2022 | all | 2020 | 2021 | 2022 | all |
| BTC | N | 4,416 | 8,760 | 7,536 | 20,712 | 4,416 | 8,760 | 7,536 | 20,712 |
| | Active % | 25.34 | 34.36 | 1.01 | 20.30 | 25.34 | 32.07 | 0.03 | 18.97 |
| | Return | 6.67 | 14.80 | 0.26 | 7.78 | 6.34 | 13.96 | 0.10 | 7.29 |
| | Volatility | 3.90 | 6.18 | 0.21 | 4.41 | 3.89 | 6.12 | 0.18 | 4.37 |
| | SR | 1.71 | 2.39 | 1.23 | 1.77 | 1.63 | 2.28 | 0.55 | 1.67 |
| ETH | N | 4,416 | 8,760 | 7,536 | 20,712 | 4,416 | 8,760 | 7,536 | 20,712 |
| | Active % | 40.44 | 34.84 | 9.06 | 26.66 | 38.18 | 34.54 | 0.09 | 22.78 |
| | Return | 16.66 | 18.08 | 1.36 | 11.69 | 15.53 | 17.14 | 0.62 | 10.79 |
| | Volatility | 6.13 | 5.13 | 0.98 | 4.41 | 6.09 | 5.09 | 0.71 | 4.36 |
| | SR | 2.72 | 3.52 | 1.39 | 2.65 | 2.55 | 3.37 | 0.87 | 2.47 |
| BNB | N | 3,672 | 8,760 | 7,536 | 19,968 | 3,672 | 8,760 | 7,536 | 19,968 |
| | Active % | 61.93 | 47.96 | 14.80 | 38.01 | 39.19 | 31.27 | 0.03 | 20.93 |
| | Return | 32.75 | 33.13 | 4.04 | 22.08 | 15.51 | 21.94 | 0.16 | 12.54 |
| | Volatility | 5.44 | 5.02 | 1.55 | 4.17 | 3.47 | 4.39 | 0.24 | 3.27 |
| | SR | 6.02 | 6.60 | 2.61 | 5.29 | 4.47 | 5.00 | 0.68 | 3.84 |
| DOGE | N | | 8,760 | 7,536 | 16,296 | | 8,760 | 7,536 | 16,296 |
| | Active % | | 49.18 | 9.70 | 30.92 | | 41.91 | 0.05 | 22.55 |
| | Return | | 53.51 | 1.94 | 29.66 | | 36.34 | 0.29 | 19.67 |
| | Volatility | | 9.02 | 1.00 | 6.65 | | 7.19 | 0.26 | 5.28 |
| | SR | | 5.93 | 1.93 | 4.46 | | 5.05 | 1.11 | 3.73 |
| ADA | N | 4,416 | 8,760 | 7,536 | 20,712 | 4,416 | 8,760 | 7,536 | 20,712 |
| | Active % | 49.55 | 42.16 | 32.40 | 40.18 | 43.30 | 39.53 | 0.00 | 25.95 |
| | Return | 18.56 | 24.18 | 3.15 | 15.33 | 16.10 | 20.36 | 0.00 | 12.04 |
| | Volatility | 4.71 | 7.09 | 1.39 | 5.17 | 4.37 | 6.92 | 0.00 | 4.93 |
| | SR | 3.94 | 3.41 | 2.27 | 2.97 | 3.68 | 2.94 | 0.00 | 2.44 |

# Random-maturity arbitrage strategy: High trading costs

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| | Return | 6.67 | 14.80 | 0.26 | 7.78 | 6.34 | 13.96 | 0.10 | 7.29 |
| | Volatility | 3.90 | 6.18 | 0.21 | 4.41 | 3.89 | 6.12 | 0.18 | 4.37 |
| | SR | 1.71 | 2.39 | 1.23 | 1.77 | 1.63 | 2.28 | 0.55 | 1.67 |
| ETH | N | 4,416 | 8,760 | 7,536 | 20,712 | 4,416 | 8,760 | 7,536 | 20,712 |
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| | Volatility | 5.44 | 5.02 | 1.55 | 4.17 | 3.47 | 4.39 | 0.24 | 3.27 |
| | SR | 6.02 | 6.60 | 2.61 | 5.29 | 4.47 | 5.00 | 0.68 | 3.84 |
| DOGE | N | | 8,760 | 7,536 | 16,296 | | 8,760 | 7,536 | 16,296 |
| | Active % | | 49.18 | 9.70 | 30.92 | | 41.91 | 0.05 | 22.55 |
| | Return | | 53.51 | 1.94 | 29.66 | | 36.34 | 0.29 | 19.67 |
| | Volatility | | 9.02 | 1.00 | 6.65 | | 7.19 | 0.26 | 5.28 |
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| ADA | N | 4,416 | 8,760 | 7,536 | 20,712 | 4,416 | 8,760 | 7,536 | 20,712 |
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| | Volatility | 4.71 | 7.09 | 1.39 | 5.17 | 4.37 | 6.92 | 0.00 | 4.93 |
| | SR | 3.94 | 3.41 | 2.27 | 2.97 | 3.68 | 2.94 | 0.00 | 2.44 |

# Random-maturity arbitrage strategy: High trading costs

| | | Unrestricted | | | | Long-spot Only | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 2020 | 2021 | 2022 | all | 2020 | 2021 | 2022 | all |
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| | Active % | 25.34 | 34.36 | 1.01 | 20.30 | 25.34 | 32.07 | 0.03 | 18.97 |
| | Return | 6.67 | 14.80 | 0.26 | 7.78 | 6.34 | 13.96 | 0.10 | 7.29 |
| | Volatility | 3.90 | 6.18 | 0.21 | 4.41 | 3.89 | 6.12 | 0.18 | 4.37 |
| | SR | 1.71 | 2.39 | 1.23 | 1.77 | 1.63 | 2.28 | 0.55 | 1.67 |
| ETH | N | 4,416 | 8,760 | 7,536 | 20,712 | 4,416 | 8,760 | 7,536 | 20,712 |
| | Active % | 40.44 | 34.84 | 9.06 | 26.66 | 38.18 | 34.54 | 0.09 | 22.78 |
| | Return | 16.66 | 18.08 | 1.36 | 11.69 | 15.53 | 17.14 | 0.62 | 10.79 |
| | Volatility | 6.13 | 5.13 | 0.98 | 4.41 | 6.09 | 5.09 | 0.71 | 4.36 |
| | SR | 2.72 | 3.52 | 1.39 | 2.65 | 2.55 | 3.37 | 0.87 | 2.47 |
| BNB | N | 3,672 | 8,760 | 7,536 | 19,968 | 3,672 | 8,760 | 7,536 | 19,968 |
| | Active % | 61.93 | 47.96 | 14.80 | 38.01 | 39.19 | 31.27 | 0.03 | 20.93 |
| | Return | 32.75 | 33.13 | 4.04 | 22.08 | 15.51 | 21.94 | 0.16 | 12.54 |
| | Volatility | 5.44 | 5.02 | 1.55 | 4.17 | 3.47 | 4.39 | 0.24 | 3.27 |
| | SR | 6.02 | 6.60 | 2.61 | 5.29 | 4.47 | 5.00 | 0.68 | 3.84 |
| DOGE | N | | 8,760 | 7,536 | 16,296 | | 8,760 | 7,536 | 16,296 |
| | Active % | | 49.18 | 9.70 | 30.92 | | 41.91 | 0.05 | 22.55 |
| | Return | | 53.51 | 1.94 | 29.66 | | 36.34 | 0.29 | 19.67 |
| | Volatility | | 9.02 | 1.00 | 6.65 | | 7.19 | 0.26 | 5.28 |
| | SR | | 5.93 | 1.93 | 4.46 | | 5.05 | 1.11 | 3.73 |
| ADA | N | 4,416 | 8,760 | 7,536 | 20,712 | 4,416 | 8,760 | 7,536 | 20,712 |
| | Active % | 49.55 | 42.16 | 32.40 | 40.18 | 43.30 | 39.53 | 0.00 | 25.95 |
| | Return | 18.56 | 24.18 | 3.15 | 15.33 | 16.10 | 20.36 | 0.00 | 12.04 |
| | Volatility | 4.71 | 7.09 | 1.39 | 5.17 | 4.37 | 6.92 | 0.00 | 4.93 |
| | SR | 3.94 | 3.41 | 2.27 | 2.97 | 3.68 | 2.94 | 0.00 | 2.44 |

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| | Return | 6.67 | 14.80 | 0.26 | 7.78 | 6.34 | 13.96 | 0.10 | 7.29 |
| | Volatility | 3.90 | 6.18 | 0.21 | 4.41 | 3.89 | 6.12 | 0.18 | 4.37 |
| | SR | 1.71 | 2.39 | 1.23 | 1.77 | 1.63 | 2.28 | 0.55 | 1.67 |
| ETH | N | 4,416 | 8,760 | 7,536 | 20,712 | 4,416 | 8,760 | 7,536 | 20,712 |
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| | Return | 16.66 | 18.08 | 1.36 | 11.69 | 15.53 | 17.14 | 0.62 | 10.79 |
| | Volatility | 6.13 | 5.13 | 0.98 | 4.41 | 6.09 | 5.09 | 0.71 | 4.36 |
| | SR | 2.72 | 3.52 | 1.39 | 2.65 | 2.55 | 3.37 | 0.87 | 2.47 |
| BNB | N | 3,672 | 8,760 | 7,536 | 19,968 | 3,672 | 8,760 | 7,536 | 19,968 |
| | Active % | 61.93 | 47.96 | 14.80 | 38.01 | 39.19 | 31.27 | 0.03 | 20.93 |
| | Return | 32.75 | 33.13 | 4.04 | 22.08 | 15.51 | 21.94 | 0.16 | 12.54 |
| | Volatility | 5.44 | 5.02 | 1.55 | 4.17 | 3.47 | 4.39 | 0.24 | 3.27 |
| | SR | 6.02 | 6.60 | 2.61 | 5.29 | 4.47 | 5.00 | 0.68 | 3.84 |
| DOGE | N | | 8,760 | 7,536 | 16,296 | | 8,760 | 7,536 | 16,296 |
| | Active % | | 49.18 | 9.70 | 30.92 | | 41.91 | 0.05 | 22.55 |
| | Return | | 53.51 | 1.94 | 29.66 | | 36.34 | 0.29 | 19.67 |
| | Volatility | | 9.02 | 1.00 | 6.65 | | 7.19 | 0.26 | 5.28 |
| | SR | | 5.93 | 1.93 | 4.46 | | 5.05 | 1.11 | 3.73 |
| ADA | N | 4,416 | 8,760 | 7,536 | 20,712 | 4,416 | 8,760 | 7,536 | 20,712 |
| | Active % | 49.55 | 42.16 | 32.40 | 40.18 | 43.30 | 39.53 | 0.00 | 25.95 |
| | Return | 18.56 | 24.18 | 3.15 | 15.33 | 16.10 | 20.36 | 0.00 | 12.04 |
| | Volatility | 4.71 | 7.09 | 1.39 | 5.17 | 4.37 | 6.92 | 0.00 | 4.93 |
| | SR | 3.94 | 3.41 | 2.27 | 2.97 | 3.68 | 2.94 | 0.00 | 2.44 |

# Random-maturity arbitrage strategy: Different trading costs

| | | Trading costs | | | |
|---|---|---|---|---|---|
| | | None | Low | Medium | High |
| BTC | Active % | 100.00 | 83.89 | 41.30 | 20.30 |
| | Return | 17.07 | 10.28 | 9.51 | 7.78 |
| | Volatility | 4.69 | 4.62 | 4.54 | 4.41 |
| | SR | 3.64 | 2.22 | 2.09 | 1.77 |
| ETH | Active % | 100.00 | 86.30 | 51.59 | 26.66 |
| | Return | 23.98 | 15.48 | 13.74 | 11.69 |
| | Volatility | 5.09 | 4.99 | 4.77 | 4.41 |
| | SR | 4.72 | 3.10 | 2.88 | 2.65 |
| BNB | Active % | 100.00 | 89.15 | 66.30 | 38.01 |
| | Return | 56.27 | 34.62 | 27.16 | 22.08 |
| | Volatility | 4.87 | 4.65 | 4.43 | 4.17 |
| | SR | 11.55 | 7.44 | 6.14 | 5.29 |
| DOGE | Active % | 100.00 | 88.64 | 64.97 | 30.92 |
| | Return | 84.14 | 49.50 | 37.52 | 29.66 |
| | Volatility | 7.45 | 7.05 | 6.93 | 6.65 |
| | SR | 11.29 | 7.03 | 5.41 | 4.46 |
| ADA | Active % | 100.00 | 90.42 | 69.22 | 40.18 |
| | Return | 60.78 | 32.90 | 20.71 | 15.33 |
| | Volatility | 5.96 | 5.74 | 5.59 | 5.17 |
| | SR | 10.20 | 5.74 | 3.70 | 2.97 |

# Data-driven strategy: High trading costs

| | | Unrestricted | | | | Long-spot Only | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 2020 | 2021 | 2022 | all | 2020 | 2021 | 2022 | all |
| BTC | N | 4,416 | 8,760 | 7,536 | 20,712 | 7,344 | 8,760 | 7,609 | 23,713 |
| | Active % | 26.11 | 18.06 | 0.29 | 13.31 | 30.27 | 52.45 | 0.03 | 28.76 |
| | Return | 5.78 | 8.51 | 0.13 | 4.88 | 6.24 | 15.21 | 0.10 | 7.58 |
| | Volatility | 3.74 | 3.26 | 0.19 | 2.74 | 3.47 | 6.50 | 0.18 | 4.40 |
| | SR | 1.55 | 2.61 | 0.70 | 1.78 | 1.80 | 2.34 | 0.54 | 1.72 |
| ETH | N | 4,416 | 8,760 | 7,536 | 20,712 | 5,880 | 8,760 | 7,609 | 22,249 |
| | Active % | 22.98 | 23.90 | 12.08 | 19.40 | 44.12 | 54.35 | 0.09 | 33.09 |
| | Return | 9.23 | 13.25 | 0.60 | 7.79 | 11.56 | 18.07 | 0.61 | 10.38 |
| | Volatility | 2.55 | 3.73 | 0.93 | 2.76 | 4.29 | 6.13 | 0.71 | 4.45 |
| | SR | 3.62 | 3.55 | 0.64 | 2.83 | 2.70 | 2.95 | 0.87 | 2.33 |
| BNB | N | 3,672 | 8,760 | 7,536 | 19,968 | 3,672 | 8,760 | 7,609 | 20,041 |
| | Active % | 41.04 | 32.32 | 10.51 | 25.69 | 57.30 | 48.11 | 1.10 | 31.94 |
| | Return | 24.61 | 25.33 | 3.33 | 16.89 | 12.23 | 20.07 | 0.10 | 11.05 |
| | Volatility | 4.24 | 4.23 | 1.32 | 3.44 | 3.77 | 4.80 | 0.30 | 3.57 |
| | SR | 5.80 | 5.99 | 2.52 | 4.91 | 3.24 | 4.18 | 0.33 | 3.10 |
| DOGE | N | | 8,760 | 7,536 | 16,296 | | 8,760 | 7,609 | 16,369 |
| | Active % | | 41.66 | 6.62 | 25.45 | | 54.26 | 1.22 | 29.60 |
| | Return | | 47.57 | 0.98 | 26.02 | | 32.74 | 0.08 | 17.56 |
| | Volatility | | 7.93 | 0.87 | 5.85 | | 7.39 | 0.40 | 5.42 |
| | SR | | 6.00 | 1.14 | 4.45 | | 4.43 | 0.21 | 3.24 |
| ADA | N | 4,416 | 8,760 | 7,536 | 20,712 | 4,416 | 8,760 | 7,609 | 20,785 |
| | Active % | 34.24 | 26.58 | 7.43 | 21.24 | 62.09 | 55.89 | 0.00 | 37.10 |
| | Return | 14.15 | 19.45 | 0.61 | 11.46 | 15.15 | 18.77 | 0.00 | 11.08 |
| | Volatility | 3.62 | 5.13 | 0.89 | 3.77 | 5.07 | 7.20 | 0.00 | 5.23 |
| | SR | 3.91 | 3.79 | 0.68 | 3.04 | 2.99 | 2.61 | 0.00 | 2.12 |

# Data-driven strategy: Different trading costs

| | | Trading costs | | | |
|---|---|---|---|---|---|
| | | None | Low | Medium | High |
| BTC | Active % | 23.20 | 20.79 | 16.29 | 13.31 |
| | Return | 15.98 | 7.42 | 6.02 | 4.88 |
| | Volatility | 2.38 | 2.63 | 2.61 | 2.74 |
| | SR | 6.72 | 2.82 | 2.31 | 1.78 |
| ETH | Active % | 32.34 | 25.85 | 20.84 | 19.40 |
| | Return | 23.48 | 11.80 | 9.41 | 7.79 |
| | Volatility | 2.27 | 2.27 | 2.69 | 2.76 |
| | SR | 10.32 | 5.20 | 3.50 | 2.83 |
| BNB | Active % | 37.76 | 30.90 | 25.52 | 25.69 |
| | Return | 54.50 | 30.40 | 22.07 | 16.89 |
| | Volatility | 3.26 | 3.02 | 3.12 | 3.44 |
| | SR | 16.72 | 10.07 | 7.07 | 4.91 |
| DOGE | Active % | 36.62 | 39.81 | 32.46 | 25.45 |
| | Return | 72.95 | 46.87 | 34.45 | 26.02 |
| | Volatility | 5.65 | 5.60 | 5.81 | 5.85 |
| | SR | 12.90 | 8.37 | 5.92 | 4.45 |
| ADA | Active % | 40.00 | 43.22 | 24.38 | 21.24 |
| | Return | 53.06 | 29.63 | 16.53 | 11.46 |
| | Volatility | 3.22 | 3.33 | 3.26 | 3.77 |
| | SR | 16.47 | 8.90 | 5.07 | 3.04 |

# Data-driven strategy: Price convergence vs Funding rate

| | | 2020 | 2021 | 2022 | all |
|---|---|---|---|---|---|
| BTC | Return | 17.70 | 25.69 | 3.69 | 15.98 |
| | Price | 11.77 | 16.10 | 3.58 | 10.62 |
| | Funding | 5.93 | 9.59 | 0.11 | 5.36 |
| ETH | Return | 22.51 | 30.99 | 15.31 | 23.48 |
| | Price | 14.22 | 21.86 | 14.36 | 17.50 |
| | Funding | 8.29 | 9.13 | 0.95 | 5.98 |
| BNB | Return | 74.35 | 64.93 | 32.71 | 54.50 |
| | Price | 61.16 | 56.63 | 28.70 | 46.92 |
| | Funding | 13.19 | 8.29 | 4.01 | 7.58 |
| DOGE | Return | | 96.17 | 45.95 | 72.95 |
| | Price | | 82.50 | 45.58 | 65.43 |
| | Funding | | 13.68 | 0.36 | 7.52 |
| ADA | Return | 44.24 | 56.21 | 54.57 | 53.06 |
| | Price | 31.81 | 45.69 | 53.12 | 45.44 |
| | Funding | 12.43 | 10.52 | 1.44 | 7.62 |

# Explaining deviations

## Regression of the futures-spot gap against explanatory variables

| | BTC | | | ETH | | |
|---|---|---|---|---|---|---|
| Ret | 0.29*** | | 0.25*** | 0.24*** | | 0.20*** |
| | (12.37) | | (9.15) | (6.09) | | (4.93) |
| FnG | | 1.45*** | 0.47*** | | 1.65*** | 0.76*** |
| | | (7.74) | (2.80) | | (6.95) | (5.16) |
| const | -0.14** | -0.62*** | -0.33*** | -0.14 | -0.56*** | -0.43*** |
| | (-2.49) | (-6.61) | (-5.24) | (-1.46) | (-3.59) | (-6.01) |
| R2 | 0.58 | 0.32 | 0.61 | 0.55 | 0.33 | 0.60 |
| N | 928 | 928 | 928 | 928 | 928 | 928 |

# Takeaways

- We derive no-arbitrage prices and bounds for perpetual futures
- Provide valuable benchmarks + strategy to exploit deviations
- Find large deviations of crypto perpetual futures from no-arbitrage prices
- Deviations comove across cryptocurrencies and diminish over time
- Simple trading strategy generates large Sharpe ratios

# AAVE Interest Rate