# An Empirical Evaluation of the Black-Litterman Approach to Portfolio Choice

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- The Black-Litterman approach is applied to allocate billions of dollars.
  - Applications (GS, Software, etc.)
  - Books
  - Papers
  - 2008 Financial Engineer of the Year Award
- Uses equilibrium weights to learn about "market views"
  - Account for information available to other investors
  - Addresses the problem of extreme portfolio weights
- Useful for active portfolio managers
  - Allows to incorporate private signals
  - Views on portfolios of assets
  - Allows to combine private views with market views
- Benefits of the empirical evaluation
  - Practitioners
  - Academia

- Since Black and Litterman introduced the approach, one of the building blocks, the CAPM, has been rejected empirically
- Extend their approach to linear multi-factor models.
- Extend their approach to account for asset pricing model uncertainty.
- Evaluate empirically.
    - Industry portfolios and portfolios formed on book-to-market and size.
    - Long time-series of data.
    - A novel approach to simulate private signals.

- Input to portfolio problem: distribution of returns

- Use historical covariance matrix

- Prior about expected returns

- Define the set of models and provide probability for each model

- For each model:
    - Learn from prices about expected excess returns
    - Posterior about expected excess returns

- Bayesian Model Averaging

- Portfolio decision

- In equilibrium:

$$\mu_{\omega,t} = E_t(R_{t+1}^e) = \underbrace{\Sigma[\omega_{m,t} \; \omega_{s,t}]\Sigma_f^{-1}}_{\beta_t} \left[ \begin{array}{c} E_t(R_{t+1}^{em}) \\ E_t(R_{t+1}^{es}) \end{array} \right] \tag{1}$$

The likelihood $\ell(\omega|\theta) = \ell([\omega_m \; \omega_s]|\mu, \Sigma)$ for expected returns based on the ICAPM is then:

$$\mu|\omega \sim \mathcal{N}(\mu_\omega, V_\omega) \tag{2}$$

where $V_\omega$ is an unbiased estimator of the sample mean's covariance $\left(\frac{1}{T-1}\right)\Sigma$ and $T$ is the length of the time-series used to estimate the covariance matrix $\Sigma$.

- The Black-Litterman approach assumes there is only market factor.
- Approximation: ICAPM and CAPM hold only for homogeneous information.
- Admati (1985) models heterogeneously informed agents and CAPM does not hold in her setting.

An investor provides a $P = K \times N$ matrix of portfolio weights and $Q = K \times 1$ vector of expected returns such that his views are described by:

$$P\mu = Q + \varepsilon, \ \varepsilon \sim \mathcal{N}(0, \Omega) \tag{3}$$

The use of a conjugate normal prior parameterized by $P$, $Q$ and $\Omega$ with a normal likelihood parameterized by $\mu_\omega$ and $V_\omega$ leads to the following normal posterior:

$$\mu | P, Q, \Omega, \omega \sim \mathcal{N}(\mu^*, V^*) \tag{4}$$

where

$$\mu^* = [P'\Omega^{-1}P + V_\omega^{-1}]^{-1}[P'\Omega^{-1}Q + V_\omega^{-1}\mu_\omega]$$
$$V^* = [P'\Omega^{-1}P + V_\omega^{-1}]^{-1}$$

Finally, we must update the returns covariance matrix:

$$\Sigma^* = \Sigma + V^* \tag{5}$$

So that the posterior distribution of returns, which updates the investor's views with those of the average investor are distributed:

$$R | P, Q, \Omega, \omega \sim \mathcal{N}(\mu^*, \Sigma^*) \tag{6}$$

## Bayesian Model Averaging

The unconditional expected return distribution is the expected return distribution conditional on a model times the probability of each model:

$$p(\mu|\psi,\omega) = \sum_{j=1}^{J} p(\mu|\psi,\omega,M_j)f(M_j|\psi,\omega) \tag{7}$$

With normal conditional distributions, the unconditional distribution is also normal and as shown by Pastor and Stambaugh (2000) is given by:

$$\mu|\psi \sim \mathcal{N}(\mu_M^*, V_M^*) \tag{8}$$

$$\mu_M^* = \sum_{j=1}^{J} \mu_j^* f(M_j|\psi)$$

$$V_M^* = \sum_{j=1}^{J} V_j^* f(M_j|\psi) + \sum_{j=1}^{J} (\mu_j^* - \mu_M^*)(\mu_j^* - \mu_M^*)' f(M_j|\psi)$$

Finally, we must update the returns covariance matrix for the posterior mean:

$$\Sigma_M^* = \Sigma + V_M^* \tag{9}$$

The resulting posterior distribution of excess returns is:

$$R \sim \mathcal{N}(\mu_M^*, \Sigma_M^*) \tag{10}$$

This distribution incorporates three sources of information:

- Investor's beliefs about different equilibrium asset pricing models
- Investor's private views
- Views of the average investor calculated from prices

This distribution of returns enters the optimization problem of the investor.

- Private information is by definition unobservable.
- We let each hypothetical investor observe a noisy signal of next-period returns:

$$Q_t = R_{t+1}^e + \sigma \tilde{\varepsilon}_t \tag{11}$$

where $\varepsilon_t$ is randomly drawn from a zero mean normal distribution with covariance matrix of excess returns $\Sigma$.

- As $\sigma \rightarrow \infty$ the signals become uninformative and the investor only uses market implied expected returns to form his posterior.
- Our specification for the noise preserves the correlation structure between the assets and allows us to control the precision of the signal with a single parameter.
- We report performance estimates for six levels of signal noise factor ($\sigma$) ranging from 0.1 to 10.

## Empirical Evaluation

- Equilibrium Models
    - No model (Dogmatic informed investor)
    - CAPM (Black-Litterman)
    - FF3F model (ICAPM)

- Investment Opportunity Sets
    - 5,10 and 49 industry portfolios and 100 book-to-market and size portfolios from Ken French's website
    - Commonly used, bias-free, long time-series
    - B/M and Size decile portfolios allow for exact solution for the weights in HML and SMB.

- Track performance of the Maximum Sharpe ratio portfolio
    - Sensitive to the inputs of expected returns
    - Simple approach to evaluate informativeness of equilibrium prices
    - Not effected by leverage
    - Normalization eliminates variation in the dollar size of the bet

$$\omega_t^* = \frac{\widehat{\Sigma}_t^{-1} \mu_t}{\mathbf{1}|\widehat{\Sigma}_t^{-1} \mu_t|} \tag{12}$$

**Table 1**

**Assets: 100 Portfolios Formed on Size and Book-to-Market**

Period: (7/1937-9/2009)
Strategy Frequency: Monthly
Minimum History Length: 240
Number of Simulations: 100

| **Max Sharpe Ratio portfolio** | Mean Excess Return | | Sharpe Ratio | | Alpha | | Alpha (FF3F) | |
|---|---|---|---|---|---|---|---|---|
| *Signal noise factor:* $\sigma = 0.10$ | | | | | | | | |
| Dogmatic informed investor | 2.36 | ( 0.00) | 3.04 | ( 0.00) | 2.36 | ( 0.00) | 2.37 | ( 0.00) |
| CAPM and signals | 2.36 | ( 0.00) | 3.05 | ( 0.00) | 2.36 | ( 0.00) | 2.37 | ( 0.00) |
| ICAPM (FF3F) and signals | 2.36 | ( 0.00) | 3.06 | ( 0.00) | 2.37 | ( 0.00) | 2.37 | ( 0.00) |
| *Signal noise factor:* $\sigma = 0.25$ | | | | | | | | |
| Dogmatic informed investor | 2.33 | ( 0.00) | 3.00 | ( 0.00) | 2.33 | ( 0.00) | 2.34 | ( 0.00) |
| CAPM and signals | 2.38 | ( 0.00) | 3.27 | ( 0.00) | 2.37 | ( 0.00) | 2.38 | ( 0.00) |
| ICAPM (FF3F) and signals | 2.38 | ( 0.00) | 3.27 | ( 0.00) | 2.37 | ( 0.00) | 2.35 | ( 0.00) |
| *Signal noise factor:* $\sigma = 0.50$ | | | | | | | | |
| Dogmatic informed investor | 2.19 | ( 0.00) | 2.85 | ( 0.00) | 2.19 | ( 0.00) | 2.20 | ( 0.00) |
| CAPM and signals | 2.18 | ( 0.00) | 2.98 | ( 0.00) | 2.15 | ( 0.00) | 2.15 | ( 0.00) |
| ICAPM (FF3F) and signals | 2.04 | ( 0.00) | 2.67 | ( 0.00) | 2.01 | ( 0.00) | 1.95 | ( 0.00) |
| *Signal noise factor:* $\sigma = 1.00$ | | | | | | | | |
| Dogmatic informed investor | 1.76 | ( 0.01) | 2.54 | ( 0.01) | 1.76 | ( 0.01) | 1.77 | ( 0.01) |
| CAPM and signals | 1.69 | ( 0.01) | 1.74 | ( 0.00) | 1.59 | ( 0.01) | 1.59 | ( 0.01) |
| ICAPM (FF3F) and signals | 1.20 | ( 0.00) | 1.19 | ( 0.00) | 1.15 | ( 0.00) | 1.00 | ( 0.00) |
| *Signal noise factor:* $\sigma = 2.00$ | | | | | | | | |
| Dogmatic informed investor | 1.09 | ( 0.01) | 2.19 | ( 0.01) | 1.09 | ( 0.01) | 1.10 | ( 0.01) |
| CAPM and signals | 1.07 | ( 0.01) | 0.61 | ( 0.00) | 0.85 | ( 0.00) | 0.84 | ( 0.00) |
| ICAPM (FF3F) and signals | 0.58 | ( 0.00) | 0.47 | ( 0.00) | 0.52 | ( 0.00) | 0.32 | ( 0.00) |
| *Signal noise factor:* $\sigma = 10.00$ | | | | | | | | |
| Dogmatic informed investor | 0.23 | ( 0.00) | 0.88 | ( 0.01) | 0.24 | ( 0.00) | 0.24 | ( 0.00) |
| CAPM and signals | 0.58 | ( 0.00) | 0.15 | ( 0.00) | 0.09 | ( 0.00) | 0.08 | ( 0.00) |
| ICAPM (FF3F) and signals | 0.29 | ( 0.00) | 0.21 | ( 0.00) | 0.22 | ( 0.00) | -0.00 | ( 0.00) |
| **Reference Portfolios:** | | | | | | | | |
| Market | 0.59 | | 0.13 | | | | | |
| SMB | 0.14 | | 0.05 | | | | | |
| HML | 0.35 | | 0.13 | | | | | |
| Max Sharpe Ratio portfolio (historical mean) | 0.06 | | 0.19 | | | | | |

In parenthesis are standard errors for the mean values across the simulations.
Only months in which all assets have valid returns are included.

**Table 2**

**Assets: 5 Industry Portfolios**
Period: (7/1926-12/2007)
Strategy Frequency: Monthly
Minimum History Length: 240
Number of Simulations: 100

| **Max Sharpe Ratio portfolio** | Mean Excess Return | | Sharpe Ratio | | Alpha | | Alpha (FF3F) | |
|---|---|---|---|---|---|---|---|---|
| *Signal noise factor:* $\sigma = 0.10$ | | | | | | | | |
| Dogmatic informed investor | 2.38 | ( 0.00) | 1.65 | ( 0.00) | 2.38 | ( 0.00) | 2.36 | ( 0.00) |
| CAPM and signals | 2.43 | ( 0.00) | 1.63 | ( 0.00) | 2.38 | ( 0.00) | 2.36 | ( 0.00) |
| ICAPM (FF3F) and signals | 2.40 | ( 0.00) | 1.61 | ( 0.00) | 2.35 | ( 0.00) | 2.33 | ( 0.00) |
| *Signal noise factor:* $\sigma = 0.25$ | | | | | | | | |
| Dogmatic informed investor | 2.31 | ( 0.01) | 1.57 | ( 0.01) | 2.31 | ( 0.01) | 2.30 | ( 0.01) |
| CAPM and signals | 2.41 | ( 0.01) | 0.96 | ( 0.00) | 2.15 | ( 0.01) | 2.13 | ( 0.01) |
| ICAPM (FF3F) and signals | 1.99 | ( 0.01) | 0.97 | ( 0.00) | 1.79 | ( 0.01) | 1.73 | ( 0.01) |
| *Signal noise factor:* $\sigma = 0.50$ | | | | | | | | |
| Dogmatic informed investor | 2.13 | ( 0.02) | 1.39 | ( 0.01) | 2.12 | ( 0.02) | 2.11 | ( 0.02) |
| CAPM and signals | 1.51 | ( 0.00) | 0.39 | ( 0.00) | 0.99 | ( 0.00) | 0.99 | ( 0.00) |
| ICAPM (FF3F) and signals | 1.06 | ( 0.01) | 0.40 | ( 0.00) | 0.76 | ( 0.00) | 0.62 | ( 0.00) |
| *Signal noise factor:* $\sigma = 1.00$ | | | | | | | | |
| Dogmatic informed investor | 1.71 | ( 0.03) | 1.08 | ( 0.02) | 1.70 | ( 0.03) | 1.69 | ( 0.03) |
| CAPM and signals | 0.88 | ( 0.00) | 0.21 | ( 0.00) | 0.30 | ( 0.00) | 0.29 | ( 0.00) |
| ICAPM (FF3F) and signals | 0.57 | ( 0.00) | 0.20 | ( 0.00) | 0.24 | ( 0.00) | 0.08 | ( 0.00) |
| *Signal noise factor:* $\sigma = 2.00$ | | | | | | | | |
| Dogmatic informed investor | 1.12 | ( 0.03) | 0.73 | ( 0.02) | 1.12 | ( 0.03) | 1.11 | ( 0.03) |
| CAPM and signals | 0.68 | ( 0.00) | 0.16 | ( 0.00) | 0.09 | ( 0.00) | 0.08 | ( 0.00) |
| ICAPM (FF3F) and signals | 0.42 | ( 0.00) | 0.15 | ( 0.00) | 0.09 | ( 0.00) | -0.07 | ( 0.00) |
| *Signal noise factor:* $\sigma = 10.00$ | | | | | | | | |
| Dogmatic informed investor | 0.25 | ( 0.01) | 0.19 | ( 0.01) | 0.26 | ( 0.01) | 0.25 | ( 0.02) |
| CAPM and signals | 0.61 | ( 0.00) | 0.15 | ( 0.00) | 0.02 | ( 0.00) | 0.01 | ( 0.00) |
| ICAPM (FF3F) and signals | 0.37 | ( 0.00) | 0.13 | ( 0.00) | 0.04 | ( 0.00) | -0.12 | ( 0.00) |
| **Reference Portfolios:** | | | | | | | | |
| Market | 0.59 | | 0.14 | | | | | |
| SMB | 0.12 | | 0.04 | | | | | |
| HML | 0.40 | | 0.15 | | | | | |
| Max Sharpe Ratio portfolio (historical mean) | 0.23 | | 0.14 | | | | | |

In parenthesis are standard errors for the mean values across the simulations.

**Table 3**

**Assets: 10 Industry Portfolios**
Period: (7/1926-12/2007)
Strategy Frequency: Monthly
Minimum History Length: 240
Number of Simulations: 100

| **Max Sharpe Ratio portfolio** | Mean Excess Return | | Sharpe Ratio | | Alpha | | Alpha (FF3F) | |
|---|---|---|---|---|---|---|---|---|
| *Signal noise factor:* $\sigma = 0.10$ | | | | | | | | |
| Dogmatic informed investor | 2.80 | ( 0.00) | 2.05 | ( 0.00) | 2.79 | ( 0.00) | 2.78 | ( 0.00) |
| CAPM and signals | 2.83 | ( 0.00) | 2.03 | ( 0.00) | 2.79 | ( 0.00) | 2.78 | ( 0.00) |
| ICAPM (FF3F) and signals | 2.81 | ( 0.00) | 2.02 | ( 0.00) | 2.78 | ( 0.00) | 2.76 | ( 0.00) |
| *Signal noise factor:* $\sigma = 0.25$ | | | | | | | | |
| Dogmatic informed investor | 2.72 | ( 0.01) | 1.96 | ( 0.01) | 2.70 | ( 0.01) | 2.69 | ( 0.01) |
| CAPM and signals | 2.79 | ( 0.01) | 1.52 | ( 0.00) | 2.63 | ( 0.01) | 2.62 | ( 0.01) |
| ICAPM (FF3F) and signals | 2.45 | ( 0.01) | 1.52 | ( 0.00) | 2.32 | ( 0.01) | 2.28 | ( 0.01) |
| *Signal noise factor:* $\sigma = 0.50$ | | | | | | | | |
| Dogmatic informed investor | 2.48 | ( 0.02) | 1.74 | ( 0.01) | 2.46 | ( 0.02) | 2.45 | ( 0.02) |
| CAPM and signals | 2.14 | ( 0.01) | 0.67 | ( 0.00) | 1.72 | ( 0.01) | 1.73 | ( 0.01) |
| ICAPM (FF3F) and signals | 1.48 | ( 0.01) | 0.67 | ( 0.00) | 1.23 | ( 0.01) | 1.13 | ( 0.01) |
| *Signal noise factor:* $\sigma = 1.00$ | | | | | | | | |
| Dogmatic informed investor | 1.95 | ( 0.03) | 1.39 | ( 0.02) | 1.94 | ( 0.03) | 1.92 | ( 0.03) |
| CAPM and signals | 1.17 | ( 0.00) | 0.29 | ( 0.00) | 0.60 | ( 0.00) | 0.61 | ( 0.00) |
| ICAPM (FF3F) and signals | 0.69 | ( 0.00) | 0.28 | ( 0.00) | 0.40 | ( 0.00) | 0.27 | ( 0.00) |
| *Signal noise factor:* $\sigma = 2.00$ | | | | | | | | |
| Dogmatic informed investor | 1.26 | ( 0.03) | 1.00 | ( 0.02) | 1.24 | ( 0.03) | 1.23 | ( 0.03) |
| CAPM and signals | 0.76 | ( 0.00) | 0.18 | ( 0.00) | 0.17 | ( 0.00) | 0.17 | ( 0.00) |
| ICAPM (FF3F) and signals | 0.41 | ( 0.00) | 0.16 | ( 0.00) | 0.11 | ( 0.00) | -0.03 | ( 0.00) |
| *Signal noise factor:* $\sigma = 10.00$ | | | | | | | | |
| Dogmatic informed investor | 0.29 | ( 0.01) | 0.29 | ( 0.01) | 0.28 | ( 0.01) | 0.28 | ( 0.01) |
| CAPM and signals | 0.61 | ( 0.00) | 0.15 | ( 0.00) | 0.02 | ( 0.00) | 0.01 | ( 0.00) |
| ICAPM (FF3F) and signals | 0.31 | ( 0.00) | 0.12 | ( 0.00) | 0.01 | ( 0.00) | -0.13 | ( 0.00) |
| **Reference Portfolios:** | | | | | | | | |
| Market | 0.59 | | 0.14 | | | | | |
| SMB | 0.12 | | 0.04 | | | | | |
| HML | 0.40 | | 0.15 | | | | | |
| Max Sharpe Ratio portfolio (historical mean) | 0.17 | | 0.15 | | | | | |

In parenthesis are standard errors for the mean values across the simulations.

**Table 4**

**Assets: 49 Industry Portfolios**
Period: (7/1969-12/2007)
Strategy Frequency: Monthly
Minimum History Length: 240
Number of Simulations: 100

| **Max Sharpe Ratio portfolio** | Mean Excess Return | | Sharpe Ratio | | Alpha | | Alpha (FF3F) | |
|---|---|---|---|---|---|---|---|---|
| *Signal noise factor:* $\sigma = 0.10$ | | | | | | | | |
| Dogmatic informed investor | 3.86 | ( 0.00) | 2.67 | ( 0.00) | 3.86 | ( 0.00) | 3.87 | ( 0.00) |
| CAPM and signals | 3.86 | ( 0.00) | 2.68 | ( 0.00) | 3.86 | ( 0.00) | 3.87 | ( 0.00) |
| ICAPM (FF3F) and signals | 3.86 | ( 0.00) | 2.68 | ( 0.00) | 3.86 | ( 0.00) | 3.86 | ( 0.00) |
| *Signal noise factor:* $\sigma = 0.25$ | | | | | | | | |
| Dogmatic informed investor | 3.80 | ( 0.00) | 2.61 | ( 0.00) | 3.80 | ( 0.00) | 3.81 | ( 0.00) |
| CAPM and signals | 3.85 | ( 0.00) | 2.72 | ( 0.00) | 3.83 | ( 0.00) | 3.84 | ( 0.00) |
| ICAPM (FF3F) and signals | 3.66 | ( 0.00) | 2.57 | ( 0.00) | 3.64 | ( 0.00) | 3.62 | ( 0.00) |
| *Signal noise factor:* $\sigma = 0.50$ | | | | | | | | |
| Dogmatic informed investor | 3.59 | ( 0.01) | 2.43 | ( 0.01) | 3.59 | ( 0.01) | 3.60 | ( 0.01) |
| CAPM and signals | 3.56 | ( 0.01) | 2.43 | ( 0.01) | 3.49 | ( 0.01) | 3.50 | ( 0.01) |
| ICAPM (FF3F) and signals | 2.31 | ( 0.01) | 1.76 | ( 0.00) | 2.27 | ( 0.01) | 2.19 | ( 0.01) |
| *Signal noise factor:* $\sigma = 1.00$ | | | | | | | | |
| Dogmatic informed investor | 2.94 | ( 0.02) | 2.07 | ( 0.01) | 2.94 | ( 0.02) | 2.94 | ( 0.02) |
| CAPM and signals | 2.74 | ( 0.01) | 1.50 | ( 0.00) | 2.53 | ( 0.01) | 2.54 | ( 0.01) |
| ICAPM (FF3F) and signals | 0.81 | ( 0.00) | 0.79 | ( 0.00) | 0.76 | ( 0.00) | 0.64 | ( 0.00) |
| *Signal noise factor:* $\sigma = 2.00$ | | | | | | | | |
| Dogmatic informed investor | 1.90 | ( 0.02) | 1.69 | ( 0.01) | 1.90 | ( 0.02) | 1.91 | ( 0.02) |
| CAPM and signals | 1.59 | ( 0.01) | 0.59 | ( 0.00) | 1.20 | ( 0.01) | 1.21 | ( 0.01) |
| ICAPM (FF3F) and signals | 0.28 | ( 0.00) | 0.29 | ( 0.00) | 0.23 | ( 0.00) | 0.11 | ( 0.00) |
| *Signal noise factor:* $\sigma = 10.00$ | | | | | | | | |
| Dogmatic informed investor | 0.43 | ( 0.01) | 0.69 | ( 0.01) | 0.43 | ( 0.01) | 0.43 | ( 0.01) |
| CAPM and signals | 0.67 | ( 0.00) | 0.17 | ( 0.00) | 0.09 | ( 0.00) | 0.08 | ( 0.00) |
| ICAPM (FF3F) and signals | 0.11 | ( 0.00) | 0.11 | ( 0.00) | 0.06 | ( 0.00) | -0.06 | ( 0.00) |
| **Reference Portfolios:** | | | | | | | | |
| Market | 0.61 | | 0.15 | | | | | |
| SMB | 0.09 | | 0.03 | | | | | |
| HML | 0.29 | | 0.09 | | | | | |
| Max Sharpe Ratio portfolio (historical mean) | 0.08 | | 0.11 | | | | | |

In parenthesis are standard errors for the mean values across the simulations.

- Informativeness of prices relative to private information
- CAPM vs. ICAPM
- Sharpe ratios and number of assets

- Black-Litterman approach: Information (Overall No), Wacky-weights (Maybe), Partial Views (Yes)
- If decide to use: consider model uncertainty
- Consistency between equilibrium model and benchmarking (CAPM, FF3F)
- Caveats for the results