Liquidity and the Strategic Value of Information

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First draft March 2020. This draft October 2024

Abstract

In Kyle (1985), the ratio of fundamental variance to price impact measures the value of information to a monopolist strategic informed investor. We show this same statistic provides an approximation for the value of information in a more general setting with multiple differentially-informed investors, and estimate it using high-frequency stocks data. We find that the value of information rises during crises. The value of information is higher for large, growth, and momentum stocks. Its most dramatic spikes occur at the start of the Covid-19 pandemic and the financial crisis of 2008, when the Fed announces liquidity facilities.

Keywords: value of information, liquidity, strategic trading, Covid-19

We thank Craig Holden for sharing code that greatly expedited our analysis. We thank Marcus Opp (editor), an anonymous referee, Yakov Amihud, Maryam Farboodi (discussant), Nicolae Garleanu, Alan Moreira, Laura Veldkamp and seminar participants at City UHK, Columbia, Cornerstone, CUNY, and Washington University in St. Louis and participants of the AFA Meetings for helpful comments. We thank Jiaen Li and Songrun He for excellent research assistance. E-mail: ohad.kadan@asu.edu or amanela@wustl.edu.

1 Introduction

What is the value of information to an investor? This is a question of paramount importance in security markets where investors, analysts, and professional money managers allocate their research capacity among a plethora of assets. Quantifying the amount investors would pay for information also has important practical implications such as determining the compensation for security analysts, valuing financial data, and even penalizing insider trading.

In Kyle-type models, the ratio of fundamental variance to price impact measures the value of information to a monopolist strategic informed investor. It reflects that a steeper reduction in uncertainty by an informative signal and a lower price impact are both conducive to a higher value of information. This simple, intuitive statistic is straightforward to estimate at high frequency. Our theoretical contribution is to show that this same statistic provides an approximate value of information in a more general setting, with multiple informed investors who each receives an imperfect signal about fundamentals. Our empirical contribution is to estimate this statistic on a daily basis for US-listed stocks, and to study its time-series and cross-sectional properties. While a large literature estimates variance and a separate large literature estimates price impact, this important ratio has thus far not been estimated.

We derive lower and upper bounds on the value of information by relying on the framework developed in Back, Cao, and Willard (2000, hereafter BCW), which generalizes the continuous-time version of Kyle (1985) by incorporating several realistic features. First, this framework allows for multiple informed strategic traders each trading to maximize profits. Recent empirical work shows that private information is pervasive yet hardly reflected by equity prices (Collin-Dufrense and Fos 2015; Kacperczyk and Pagnotta 2019b; Akey, Gregoire, and Martineau 2020). Second, the model is dynamic – reflecting that informed traders often smooth their trades over time to minimize price impact (Koudijs

2015; Kacperczyk and Pagnotta 2019a). Third, the model allows for informed traders to only possess partial information on the true value of the asset, and this information may be correlated across traders.

Using this framework, we show that the dollar expected profits of traders informed about an asset is given by

$$\Omega = c\left(n,\rho\right)\frac{var\left(\tilde{v}\right)}{\lambda\left(0\right)}, \tag{1}$$

where $var\left(\tilde{v}\right)$ is the variance of the asset's fundamental value, $\lambda\left(0\right)$ is the sensitivity of the asset's price to share order flow (Kyle's lambda) at the beginning of the trading period (time zero), and c is a coefficient of proportionality, which depends on the number of informed traders n, and on the correlation between their signals ρ. This simple expression underscores the fundamental tension governing the value of information. Indeed, stocks that are associated with a high level of uncertainty are often also relatively illiquid. It is the ratio of these two that determines the profitability of information acquisition about an asset. This ratio is proportional to the value of information in a fairly large class of models of strategic trading á-la Kyle (1985), including Back (1992), Caballé and Krishnan (1994), Back, Cao, and Willard (2000), and Collin-Dufresne and Fos (2016).

Our main theoretical result is that the coefficient of proportionality, $c\left(n,\rho\right)$, is at least 0.92, for all possible values of the exogenous parameters, and no greater than 1 when private signals are uncorrelated. Thus, the statistic

$$\tilde{\Omega} = \frac{var\left(\tilde{v}\right)}{\lambda\left(0\right)}, \tag{2}$$

the ratio of fundamental variance to Kyle's lambda, serves as an approximate value of information even with multiple informed investors, as long as their signal correlation is low. While volatility and illiquidity are highly correlated, their ratio has thus far not been studied empirically, despite its centrality to the value of information. It is important to note that Kyle's lambda is endogenous in the model and depends on its exogenous parameters including signal correlation and the number of informed traders.

The key to our estimation approach is that, while the aforementioned exogenous parameters are typically unobservable and hard to estimate, the estimation of lambda is standard (e.g., Hasbrouck 2009; Holden and Jacobsen 2014), as is the estimation of price volatility (e.g., Aït-Sahalia, Mykland, and Zhang 2005). Thus, we replace the hard problem of estimating the exact value of information relying on unobservable parameters, with the problem of estimating a simple statistic, exploiting observable data. This statistic, which we refer to hereafter as the *value of information*, quantifies the value of information through the lens of a monopolist informed trader and provides an approximation more generally.

In our empirical analysis, we estimate the value of information daily using intraday data for US publicly-listed common stocks over the September 2003 to December 2020 period. The idea is to measure the value an investor would be willing to pay at the close of a trading day to learn the closing stock price at the end of the next trading day. We find that the average daily value of information is \$11,500, which annualizes to an average of \$2.9 million for becoming informed on an average stock each day.

Our approach allows us to provide insight into fundamental questions regarding how the value of information varies over time an across firms. Is information more valuable during calm or turbulent times? Is gathering information more profitable for large or small firms? The answers to these questions are not obvious because return volatility and liquidity are tightly linked (Nagel 2012; Drechsler, Moreira, and Savov 2021). For example, during turbulent times, one would expect uncertainty to rise, but also liquidity to fall, which would raise both numerator and denominator of (2). Whether the net effect on the value of information is positive is thus an empirical question.

We begin our analysis by considering time variation in the value of information. We find that the value of information rises dramatically during turbulent times. Evidently, the increase in uncertainty overshadows the rise in illiquidity during these times, leading to information becoming more valuable. The most dramatic increase in the value of information occurs during the Covid-19 pandemic of 2020. We observe nine of the ten highest value of information days in our sample in March 2020, when financial market participants fly to liquidity and the Federal Reserve intervenes at an unprecedented scale (Haddad, Moreira, and Muir 2021).[1] We also observe notable spikes in the value of information during the financial crisis of 2008, several days after Lehman Brothers collapses, when the Fed announced several novel liquidity facilities. During both of these rare crises, it appears that the Fed, perhaps unintentionally, increases the value of information by enhancing market liquidity. Such large rewards to collecting information can be beneficial if they encourage a more efficient capital allocation. But they can be harmful to short-term debt funded financial intermediaries, if they instigate coordinate failures and runs (He and Manela 2016).

We also document interesting seasonality patterns. The value of information increases monotonically during the work week, and drops markedly toward the end of the calendar year. Many of the lowest value of information days in our sample occur between Christmas and New Year.

Cross-sectionally, one may argue that gathering information on small stocks is more profitable, because such stocks are likely neglected by investors and analysts (e.g., Arbel, Carvell, and Strebel 1983; Hou and Moskowitz 2005). But, a countervailing argument is that trading on information discovered on small stocks is less profitable, because of their lack of liquidity. The expression in (2) encapsulates this trade-off. Empirically, we find that, perhaps surprisingly, the value of information is significantly higher for larger stocks. While it is certainly true that the adverse-selection associated with large stocks is smaller, this effect is dwarfed by their superior liquidity. In fact, we document that volatility is

1. In Section 5 we discuss the extent to which our estimates could be picking up uncertainty rather than the value of information.

significantly less sensitive to size than liquidity. We further find that low book-to-market (growth) stocks and stocks that experienced high returns over the past year (momentum), tend to have higher values of information.

Our paper is the first to empirically estimate the value of information to strategic investors who internalize their price impact. The foundations for our analysis are set in the seminal work of Grossman and Stiglitz (1980) and Kyle (1985) who considered the value of information in a rational expectations setting. Ai (2007), Epstein, Farhi, and Strzalecki (2014), Croce, Marchuk, and Schlag (forthcoming) and Kadan and Manela (2019) study the value of macro-level information in a setting employing Epstein and Zin (1989) preferences. This approach allows them to separate between the psychic and instrumental values of information. Ai et al. (2019) augment the Kadan and Manela (2019) option-based approach to generalized risk preferences. Farboodi et al. (2021) compute the value of information implied by a structural noisy rational expectations model and find it is higher for large growth firms.[2] The high frequency nature of our measure allows us to document for the first time that the value of information rises during turbulent times and falls at year and quarter ends.

The above papers consider the value of information to a small risk-averse agent having no effect on prices. The value of information is finite in these models because risk aversion prevents the informed agent from taking arbitrarily large positions in systematically risky assets, and prices therefore do not fully reveal their private information. By contrast, the essence of the value of information considered here is that it applies to a investors whose trades affect prices, and liquidity plays a key role in our analysis. Here, the value of information is finite and prices are partially revealing because our agents internalize their marginal effect on prices. Accordingly, we consider risk-neutral traders

2. A large literature on information choice studies the CARA-normal (mean-variance) noisy rational expectations framework. See Veldkamp (2011) for a survey, and see Malamud and Rostek (2017) and Dávila and Parlatore (2021) for recent examples. Recent work suggests that moving beyond CARA utility can be important (Savov 2014; Breon-Drish 2015; Malamud 2015).

and focus our attention on asset-specific as opposed to macro-level information. This approach complements the earlier work and may be more suitable in cases where systematic risk can be neutralized, for example by taking long-short positions that efficiently trade on asset-specific information, or when investment decisions are delegated to risk neutral money managers (Gârleanu and Pedersen 2018). The value of information statistic that we derive is also simple and has easy to interpret units.[3]

Our paper also relates to recent work that studies the informational efficiency of financial markets (Fama 1970). Bai, Philippon, and Savov (2016) document that price informativeness has increased since 1960, and that it is concentrated among growth stocks. Farboodi et al. (2021) show that this increase is driven by large, growth stocks, while the informational efficiency of smaller assets' prices or prices of assets with less growth potential have been flat or declining. They argue that more data has been processed for large growth firms because the value of information for these firms has been higher. Dávila and Parlatore (2018) estimate price informativeness by regressing prices on fundamentals and find it is higher for stocks with greater size and trade volume. Kacperczyk, Sundaresan, and Wang (2021) find that greater foreign ownership increases stock price informativeness. Dávila and Parlatore (forthcoming) study the relation between volatility and informativeness. We provide easy to estimate bounds on the value of information, which is distinct from price informativeness. The value of information measures the willingness to pay for information while price informativeness measures the amount of information inferred from prices. For example, proper analyst compensation, the value of data, or penalties for insider trading, all depend on the value of information, while price informativeness does not directly provide a quantitative answer to these questions. These concepts are nonetheless related because a private value to information incentivizes market participants to seek private information and through their trades, improve capital alloca-

3. See Kyle and Obizhaeva (2016), Kyle and Obizhaeva (2018), Kyle, Obizhaeva, and Wang (2018), and Kyle and Obizhaeva (2019) for careful work on microstructure invariants and units of measurement.

tion efficiency and managerial decisions (Bond, Edmans, and Goldstein 2012; Brogaard, Ringgenberg, and Sovich 2019; Goldstein 2023).[4]

The paper proceeds as follows. Section 2 develops bounds on the value of information. Section 3 discusses estimation. The empirical results are in Section 4. Section 5 discusses alternative interpretations. Section 6 concludes. Proofs and robustness tests are in the Appendix.

2 Theory

Our aim in this section is to develop estimable lower and upper bounds for the value of information in a strategic trading setting. For this purpose, we rely on the model presented in Back, Cao, and Willard (2000, hereafter BCW), which is a generalization of the continuous-time version of Kyle (1985). The model in BCW incorporates several realistic features. First, it allows for asymmetric information with potentially multiple informed strategic traders. Second, the BCW model is dynamic – reflecting that informed traders often break their trades into pieces to minimize price impact. Third, the model reflects that informed traders may only possess partial information on the true value of the asset, and this information may be correlated across traders. Thus, the value of information we derive based on the BCW model reflects the number of informed traders in the market, the accuracy of their information and any correlation structure in their signals, as well as the optimal smoothing of their trades over time.[5]

4. A related theoretical literature studies information and strategic trading starting with Grinblatt and Ross (1985) and Kyle (1989), with more recent contributions by Rostek and Weretka (2012), Vives (2014), and Kacperczyk, Nosal, and Sundaresan (forthcoming).

5. Foster and Viswanathan (1996) study a dynamic model with multiple informed traders holding imperfect signals in discrete time. We rely on BCW because their continuous-time approach simplifies some of the analysis.

2.1 Model and Equilibrium

Consider an asset with normally distributed value $\tilde{v}$ to be revealed at time 1.[6] Trading of this asset and a risk free asset takes place continuously during the time interval $[0, 1]$. There are three types of traders: (i) n risk-neutral informed traders who learn a signal correlated with v at time 0; (ii) noise traders submitting exogenous orders; and (iii) risk neutral market makers who set prices to clear the market.

As in BCW, assume that the value at the end of the trading day is

$$\tilde{v} = \sum_{i=1}^{n} \tilde{s}^i,$$

where $\tilde{s}^i$ is the signal obtained by informed trader i, and $(\tilde{s}^1, ..., \tilde{s}^n)$ are symmetrically joint-normally distributed, with correlation coefficient $0 \leq \rho < 1$ between $\tilde{s}^i$ and $\tilde{s}^j$ for $\bar{\imath} \neq j$.[7] The cumulative noise trading process is given by

$$dZ(t) = \sigma_z dW(t),$$

where $W(t)$ is a Wiener process and σ_z is a volatility parameter.[8]

BCW show the existence of equilibria in this game that are linear in price $P(t)$ and signal s^i. That is, there are functions $\alpha(t)$, $\beta(t)$, and $\lambda(t)$ such that the profit-maximizing rate of trade of each informed trader i at time t is given by

$$\alpha(t) P(t) + \beta(t) \tilde{s}^i,$$

6. One can think about $\tilde{v}$ as the price at which the asset will be traded at time 1, rather than the actual value of the asset at time 1.

7. BCW show that in the case $\rho = 1$ an equilibrium fails to exist. We thus do not consider this case.

8. Note that BCW assume $\sigma_z = 1$ saying "We could easily make this an arbitrary constant σ_z. However, this parameter will affect market depth, the profits of informed traders, and so on." In the context of our paper, we cannot assume that $\sigma_z = 1$ because the value of information is affected by this parameter.

and the break-even price set by the market makers follows the process

$$dP(t) = \lambda(t)\left\{dZ(t) + \sum_{i=1}^{n}\left[\alpha(t)P(t) + \beta(t)\tilde{s}^i\right]dt\right\}.$$

Similar to BCW, denote

$$\phi(n,\rho) = \frac{var(\tilde{v})}{var(n\tilde{s}^i)} = \frac{1}{n} + \frac{n-1}{n}\rho, \tag{3}$$

and

$$\zeta(n,\rho) = \frac{1-\phi(n,\rho)}{n\phi(n,\rho)}. \tag{4}$$

For brevity, we will often omit the arguments of $\phi(n,\rho)$ and $\zeta(n,\rho)$ and will refer to them as ϕ and ζ. Denote also

$$\kappa = \int_1^{\infty} x^{\frac{2(n-2)}{n}} e^{-2x\zeta} dx.$$

Theorem 1 in BCW (generalized to allow for arbitrary σ_z) establishes the existence of a unique linear equilibrium satisfying:

$$\beta(t) = \sigma_z\left(\frac{\kappa}{\Sigma(0)}\right)^{\frac{1}{2}}\left(\frac{\Sigma(t)}{\Sigma(0)}\right)^{\frac{n-2}{n}}\exp\left[\zeta\frac{\Sigma(0)}{\Sigma(t)}\right],$$

$$\alpha(t) = -\beta(t)/n,$$

and

$$\lambda(t) = \frac{\beta(t)\Sigma(t)}{\sigma_z^2},$$

where $\Sigma(t)$ is the conditional variance of $\tilde{v}$ given the market maker's time t information.

2.2 A Lower Bound on the Value of Information

Denote the total value of information to all informed traders by Ω. It is equal to the expected loss of noise traders (Treynor 1995), namely

$$\Omega = \int_0^1 dP(t)\, dZ(t).$$

Corollary 3 in BCW (but allowing for an arbitrary σ_z) shows that this total value of information is given by

$$\Omega = \sigma_z \left(\frac{var(\tilde{v})}{\kappa}\right)^{\frac{1}{2}} \int_1^\infty x^{-\frac{2}{n}} \exp[-\zeta x]\, dx.$$

It follows that,

$$\begin{aligned}
\Omega\lambda(0) &= \sigma_z \left(\frac{var(\tilde{v})}{\kappa}\right)^{\frac{1}{2}} \int_1^\infty x^{-\frac{2}{n}} \exp[-\zeta x]\, dx \cdot \frac{\beta(0)\,\Sigma(0)}{\sigma_z^2} \\
&= \frac{1}{\sigma_z} \left(\frac{var(\tilde{v})}{\kappa}\right)^{\frac{1}{2}} \int_1^\infty x^{-\frac{2}{n}} \exp[-\zeta x]\, dx \cdot (var(\tilde{v}))^{\frac{1}{2}} \cdot \sigma_z \kappa^{\frac{1}{2}} e^{\zeta} \\
&= var(\tilde{v})\, e^{\zeta} \int_1^\infty x^{-\frac{2}{n}} \exp[-\zeta x]\, dx.
\end{aligned}$$

We thus have the following result.

Proposition 1 *The total value of information to all informed traders can be written as*

$$\Omega(n,\rho) = c(n,\rho)\, \frac{var(\tilde{v})}{\lambda(0)}, \tag{5}$$

where the coefficient of proportionality $c(n,\rho) > 0$ *does not depend on* σ_z *and is given by*

$$c(n,\rho) = e^{\zeta} \int_1^\infty x^{-\frac{2}{n}} e^{-x\zeta} dx. \tag{6}$$

In words, the value of information equals the ratio of fundamental variance to Kyle's lambda, both measured at time 0, multiplied by a constant that depends only on the number of informed traders n and their signal correlation ρ . To help interpret this result, we consider first the case $n = 1$, which serves as a useful benchmark. We then discuss the general case in detail.

2.2.1 Benchmark: Monopolistic and Perfectly Informed Trader ($n = 1$)

When $n = 1$, the model in BCW coincides with the continuous-time version of Kyle (1985). In this case, λ is time-invariant, and it is easy to verify that $c\,(n, \rho) = 1$. Thus, Proposition 1 simplifies to

$$\Omega\,(1, \rho) = \sigma_v \sigma_z = \frac{\sigma_v^2}{\lambda}. \tag{7}$$

The literature starting with Kyle (1985) emphasizes the first equality in (7), which says that the informed trader expects higher profits when fundamental volatility is higher or when noise trading is larger. It is empirically useful, however, to focus instead on the last expression which says that the value of information equals the ratio of price volatility to price impact. This is intuitive, as the value of information increases with fundamental uncertainty but decreases in λ, which impedes trading on information.

Note that both σ_v and λ are estimable from data, which is the key to our estimation approach. Thus, this expression provides a useful benchmark for our analysis. If we knew that there is only one informed trader possessing a perfect signal on the fundamental value – we could have stopped here. In practice, however, there may be multiple informed traders each possessing an imperfect (yet correlated) signal on the fundamental value.

2.2.2 The general case: Multiple Imperfectly Informed Traders ($n \geq 1$)

Proposition 1 establishes that when $n \geq 1$, the total value of information is proportional (but typically not equal) to the variance of the fundamental value divided by Kyle's lambda measured at the beginning of the trading period. A key to estimating the value of information then becomes the magnitude of the coefficient of proportionality $c(n, \rho)$, which depends on the number of informed traders n and the correlation between signals ρ. However, both n and ρ are not observable in the data, posing a challenge to estimating the value of information. In the next proposition, we present our first theoretical result, which addresses this challenge. We establish a general lower bound on the total value of information that does not depend on n and ρ. The key to this result is that $c(n, \rho)$ is proportional to a well known mathematical form known as the generalized exponential integral, which has well established properties (Chiccoli, Lorenzutta, and Maino 1990). We use these properties to show that the case $n = 2$ and $\rho = 0$ provides a lower bound on $c(n, \rho)$ for any $n \geq 1$ and $\rho \geq 0$. One can readily calculate that $c(2, 0) \approx 0.9229$. We thus have,

Proposition 2 *For any $n \geq 1$, $\rho \geq 0$, and for any $\sigma_z > 0$, the coefficient $c(n, \rho) \geq 0.92$, and the total value of information satisfies*

$$\Omega(n, \rho) \geq 0.92 \frac{var(\tilde{v})}{\lambda(0)}.$$

Proof: See Appendix.

Proposition 2 says that, regardless of the degree of competition among informed traders and the extent to which their imperfect signals are correlated, the total value of information cannot be lower than 92% of the ratio of fundamental variance to initial price impact.

2.3 An Upper Bound on the Value of Information

We can further derive an upper bound on the value of information when the private signals obtained by informed agents are uncorrelated, that is when $\rho = 0$. We derive this bound by showing that when $\rho = 0$, the proportionality coefficient $c(n, 0)$ is no greater than 1. The proof of this result relies on the properties of the generalized exponential integral to show that $c(n, 0)$ can be written as a limit of a generalized continuous fraction. A detailed analysis of the recursive structure of these fractions allows us to establish that they are reflecting sequences whose limits are weakly lower than 1, which implies the requisite upper bound.

Proposition 3 *For any $n \geq 1$, and for any $\sigma_z > 0$, when $\rho = 0$, the coefficient $c(n, \rho) \leq 1$, and the total value of information satisfies*

$$\Omega(n, \rho) \leq \frac{var(\tilde{v})}{\lambda(0)}. \tag{8}$$

Proof: See Appendix.

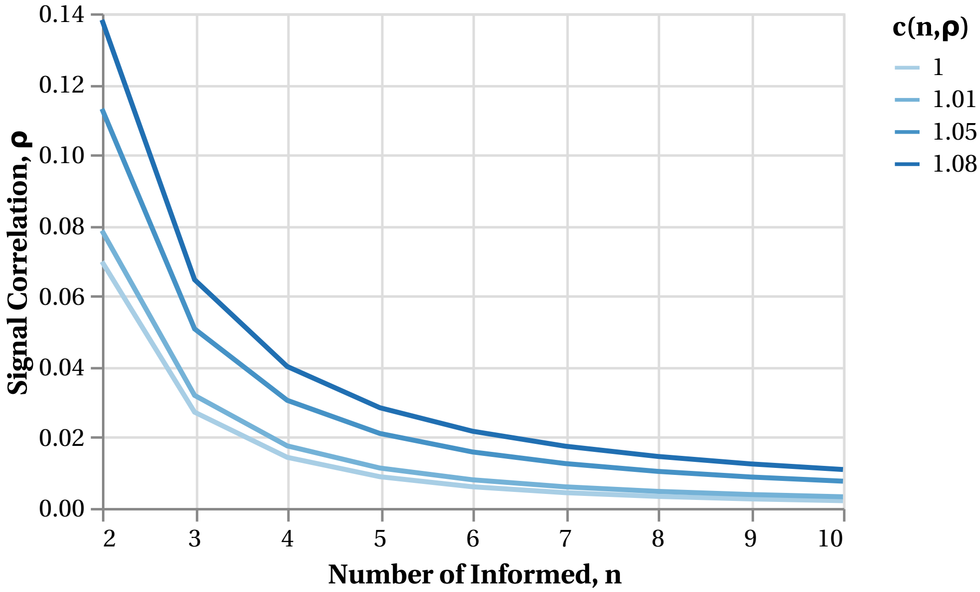
Proposition 3 as stated applies only for the case $\rho = 0$. However, by continuity, as long as the signal correlation is modest, the right hand side of (8) provides a good approximation for the upper bound.

Figure 1 quantifies the accuracy of the approximate upper bound for different values of the unknown parameters in our model, n and ρ. It shows, for example, that if the signal correlation is actually $\rho = 0.02$, then as long as the number of informed is no greater than $n = 5$, the approximate upper bound of Proposition 3 is within 5% of its true value.

2.4 The Value of Information Statistic

Combining the lower bound result in Proposition 2 with the upper bound result in Proposition 3, we obtain a tight band for the value of information, when the signal correlation

Figure 1: Isoquants of the constant of proportionality $c(n, \rho)$



Notes: Each curve represents an isoquant where the constant of proportionality $c(n, \rho)$ takes a particular value as the number of informed traders n and their signal correlation ρ vary.

is small:

$$0.92\frac{var\left(\tilde{v}\right)}{\lambda\left(0\right)} \leq \Omega\left(n,\rho\right) \leq \frac{var\left(\tilde{v}\right)}{\lambda\left(0\right)}. \tag{9}$$

Therefore, the statistic

$$\tilde{\Omega} = \frac{var\left(\tilde{v}\right)}{\lambda\left(0\right)}, \tag{10}$$

which quantifies the value of information estimated through the lens of the benchmark monopolist trader case, provides an approximate value of information more generally when signal correlation is small, satisfying

$$0.92 \leq \frac{\Omega\left(n,\rho\right)}{\tilde{\Omega}\left(n,\rho\right)} \leq 1. \tag{11}$$

For brevity, below we refer to $\tilde{\Omega}$ as the value of information.[9]

It is important to note that both $var\left(\tilde{v}\right)$ and $\lambda\left(0\right)$ are estimable from data. Interestingly, $\lambda\left(0\right)$ is an endogenous object that depends on the exogenous parameters n, ρ, and σ_z. But, while the latter three exogenous parameters are not easily observed in the data, estimating Kyle's lambda is well established in the literature, facilitating our empirical approach.

2.5 Relation to price informativeness

Our focus is on the estimation of the value of information, which is distinct from price informativeness. The value of information captures the dollar amount a potentially informed agent would be willing to pay for information. Price informativeness, by contrast, measures the amount of information that can be inferred from prices.

The literature has offered several related definitions for price informativeness. Kyle (1985) defines price informativeness as the reduction in the variance of the fundamental

9. Our focus thus far has been on the total value of information to all informed traders. However, the approximations established in Propositions 2 and 3 translate easily to the individual value of information by simply dividing them by n.

value conditional on the price. Bai, Philippon, and Savov (2016) define price informativeness as the covariance of the price with the fundamental, scaled by price variance, which they measure using regressions of future earnings (the fundamental) on market prices. As Kacperczyk, Nosal, and Sundaresan (forthcoming) explain, this measure can be mapped to the reduction in the variance of posterior beliefs of a Bayesian agent who learns from the price.

To illustrate the distinction between the value of information and price informativeness, in Kyle's setup, the value of information to the informed trader depends on the volatility of noise trading, as does the price impact coefficient λ. However, price informativeness is independent of the amount of noise trading (using either definition). The BCW model employed here inherits this property that the variance of the fundamental conditional on the price is independent of noise trading volatility.

Intuitively, one might expect that when noise trading increases prices becomes less informative about the fundamental value. But in these models, the informed would simultaneously trade more aggressively on their information, which exactly offsets the effect on the conditional variance, leaving price informativeness constant. Nonetheless, the value of information increases reflecting that the informed trade more aggressively when faced with a more liquid market.

Price informativeness and the value of information are therefore related yet different economic concepts.

3 Estimation

Our goal is to estimate the value of information statistic (10) on a daily basis for stocks traded on US stock exchanges. When going to data on stocks, the fact that in the model σ_v is the volatility of prices rather than returns may be problematic to estimate due to lack of stationarity. For this reason, the empirical literature focusing on the estimation of price impact from intraday data considers versions of Kyle's lambda in which order flow affects returns rather than prices (e.g., Hasbrouck 2009; Holden and Jacobsen 2014).

The value of information statistic expressed in returns rather than prices becomes

$$\tilde{\Omega} = \frac{\sigma_v^2}{\lambda(0)} = \frac{\sigma_r^2}{\lambda_r(0)} P(0), \tag{12}$$

where σ_r is the standard deviation of returns, $\lambda_r(0)$ is the initial percent price impact per share order flow, and $P(0)$ is the initial price. The unit of measurement of this statistic is dollars (or whatever denominates prices). Indeed, both σ_r^2 and $\lambda_r(0)$ are unitless, implying that the statistic inherits its units of measurement from $P(0)$.[10]

The expression for the statistic is intriguing empirically, because uncertainty and illiquidity are known to be highly correlated.[11] In fact, the market microstructure literature traditionally considers asset volatility as a fundamental determinant of asset illiquidity (Stoll 1978; Copeland and Galai 1983; Admati and Pfleiderer 1988). And yet, the two are not perfectly correlated and, as we document below, their ratio fluctuates dramatically both cross-sectionally and over time. It is this ratio that determines the value of acquiring information on the asset.

To establish daily values of this statistic we rely on intraday data obtained from the NYSE TAQ database, which covers all US publicly traded stocks. Our sample includes 7,400 common stocks over the September 11, 2003 to December 31, 2020 sample period, which we could match by trading symbol and date to CRSP/Compustat. Following Amihud (2002) we drop all stocks with a previous-day closing stock price smaller than \$5 to

10. While the theoretical model assumes the fundamental is normally distributed, which allows negative prices, in practice, stock prices cannot be negative because of limited liability. This can be resolved by setting the mean of the fundamental sufficiently high relative to its standard deviation, making the probability of a negative price negligible. Note that this mean plays no role in the value of information statistic.

11. In our sample, the correlation between return variance and price impact per dollar is 0.3. A univariate regression of log price impact per dollar on log return variance estimates their elasticity at 0.9, or if we control for date and stock fixed effect, at 0.4.

avoid market microstructure effects.[12] We then inflation-adjust all prices using the CPI to December 2020 values.

Empirically, we interpret the daily value of information statistic for a particular stock as the dollar amount informed investors would be willing to pay at the close of the trading day to learn the closing price at the end of the next trading day. As Back (1992) explains, because of risk neutrality, v in the model can be thought of as an unbiased signal of the end-of-period price. This price in turn, is an unbiased expectation of the next period's value. Thus, for each stock j on date t we estimate a daily statistic,

$$\hat{\Omega}_{jt} = \frac{\hat{\sigma}^2_{jt}}{\hat{\lambda}_{jt}} P_{jt-1}, \tag{13}$$

which is the ratio of realized log return variance $\hat{\sigma}^2_{jt}$ to price impact $\hat{\lambda}_{jt}$, both estimated from 1-minute log returns and order flow, times the previous day's closing price P_{jt-1}.

Specifically, let $p_{jt\tau} = \log P_{jt\tau}$ denote the log price of asset j on day t, at time $\tau \in [0, T]$, where T denotes the time in years that passes on day t. We observe equidistant observations over time intervals $\Delta = 1$ minute, so we estimate variance at time T based on $N + 1$ discrete observations recorded at times τ_0, τ_1, ..., $\tau_N = N\Delta = T$. We annualize volatility (and the value of information), by setting $T = 1/252$. Let $r_{jti} = p_{jt\tau_i} - p_{jt\tau_{i-1}}$ be the log return of asset j on day t over interval $i = 1, \ldots, N$.

Following common practice (e.g., Aït-Sahalia, Mykland, and Zhang 2005), we estimate the intraday log return variance of stock j on day t as

$$\hat{\sigma}^2_{jt,\text{intraday}} = \frac{1}{T} \sum_{i=1}^{N} r^2_{jti}. \tag{14}$$

Recent work documents that a substantial amount idiosyncratic risk (Bogousslavsky 2019), as well as systematic risk (Hendershott, Livdan, and Rösch, forthcoming; Muravyev

12. In the Appendix we show how our estimates change if we keep penny stocks.

and Ni 2020) is resolved when stock markets are closed. We therefore augment this estimator with the squared overnight log return $r_{jt,\text{overnight}} = p_{jt\tau_0} - p_{jt-1\tau_N}$, properly scaled, to attain our estimator of annualized log return variance,

$$\hat{\sigma}^2_{jt} = \frac{1}{T} r^2_{jt,\text{overnight}} + \hat{\sigma}^2_{jt,\text{intraday}}. \tag{15}$$

Below we study the total (overnight+intraday) value of information as well as the intraday-only value.

Let $Y_{jt\tau}$ denote cumulative signed order flow and let $y_{jti} = Y_{jt\tau_i} - Y_{jt\tau_{i-1}}$ be share order flow over intraday interval i. Several approaches to signing TAQ trades as buys $(+1)$ or sells (-1) have been proposed by prior work. Our baseline estimates are based on the Chakrabarty et al. (2007) approach, using the algorithms developed by Holden and Jacobsen (2014). In the Appendix we show that our qualitative and quantitative conclusions are not sensitive to this choice.

We estimate price impact using a regression of 1-minute log returns on contemporaneous share order flow,

$$r_{jti} = \hat{\lambda}_{jt} y_{ijt} + \varepsilon_{ijt}. \tag{16}$$

We follow Hasbrouck (2009) and Holden and Jacobsen (2014) who fix the intercept at zero because the expected log return is theoretically zero when order flow is zero. By imposing this null we remove a degree of freedom from the regressions to get more accurate estimates of λ_r.

Theoretically, $\hat{\lambda}_{jt}$ is aiming to estimate the time zero price impact coefficient $\lambda_r(0)$. There is a tradeoff here between estimating it based on the first or first few minutes of the day and alternatively measuring it with an average over the entire day. While the first is theoretically more accurate in population, averaging more observations tends to yield more accurate estimates. We opt for the latter approach for its simplicity, and note that the direction of any potential bias is unclear because price impact changes non-monotonically over the trading day (see BCW, Figure 3B).

Finally, we use the closing price of the preceding trading day, $P_{jt-1\tau_N}$ as the initial asset price P_0 in (13). We note that while in theory λ_r is positive, it could be negative in any finite sample. An alternative approach to estimating λ_r could be to take its theoretical definition exactly, without error, and measure it as the ratio of mean returns to mean order flow. But because the latter is often close to zero, this alternative approach can produce many arbitrarily large price impact coefficients. Moreover, $\hat{\lambda}_{jt}$ could suffer from selection bias if traders employ price-dependent strategies and cancel expensive orders (Obizhaeva 2011). The are also other measures of price impact suggested by the large literature on its measurement (Holden and Jacobsen 2014). We prefer ours because it corresponds closely to the model we use to interpret the data, which means we can easily interpret the units of the value of information it yields.

The statistic (13) is a ratio of two random variables measured with error. Small or negative λ point estimates in particular can be problematic because they would result in arbitrarily large (or small) values of information. Therefore, sample means must be taken with care. Using the delta method it can be shown that a first-order approximation for the mean over a sample of observations s is just the ratio of the mean variance to mean price impact per dollar over that subsample.

Formally, let $\mu_{rs} = \frac{1}{|s|}\sum_{jt\in s}\hat{\sigma}^2_{jt}$ be an estimate of mean variance over subsample s, and let $\mu_{\lambda s} = \frac{1}{|s|}\sum_{jt\in s}\hat{\lambda}_{jt}/P_{jt-1}$ be an estimate of mean price impact per dollar.

Then by the delta method, a first-order estimator for the mean is

$$E\hat{\Omega}_s = \frac{\mu_{rs}}{\mu_{\lambda s}}. \tag{17}$$

To estimate the uncertainty about information value estimates, let

$$\Sigma_s = \begin{bmatrix} \Sigma_{rs} & \Sigma_{r\lambda s} \\ \Sigma_{r\lambda s} & \Sigma_{\lambda s} \end{bmatrix} = Cov([\hat{\sigma}^2_{jt}, \hat{\lambda}_{jt}/P_{jt-1}])$$

be an estimate of the 2×2 variance-covariance matrix of variance and price impact over the same subsample. Using a second-order expansion yields the following estimator for the variance of the value of information:

$$Var\hat{\Omega}_s = \frac{1}{\mu^2_{\lambda s}} \left(\Sigma_{\lambda s} \frac{\mu^2_{rs}}{\mu^2_{\lambda s}} + \Sigma_{rs} - 2\Sigma_{r\lambda s} \frac{\mu_{rs}}{\mu_{\lambda s}} \right). \tag{18}$$

Naturally, the ratio is more variable when the mean price impact $\mu_{\lambda s}$ is small, but also when the covariance between return variance and price impact $\Sigma_{r\lambda s}$ is low. Below we rely on Σ_s estimates that allow for clustering over time (day) to estimate the variance of the value of information.

In some cases it is useful to consider the logarithmic version of (13), which allows for an additive interpretation. Denoting the value of information in logs as $\omega = \log \Omega$ and correspondingly the logarithm of the value of information static $\tilde{\omega} = \log \tilde{\Omega}$ we have

$$\tilde{\omega} = \log \sigma^2_r - \log \frac{\lambda_r}{P_0} = \omega - \log c(n, \rho), \tag{19}$$

where the interpretation is that the log value of information equals the log of return variance less the log of percent price impact per dollar order flow. Of course, when we use this decomposition, we are restricted to observations where price impact is estimated to be positive. But an advantage of the logarithmic version is that its expectations are better behaved.

The second equality in Equation (19) implies that as long as the variation in $\log c$ is small relative to the variation ω, or is absorbed by control variables, estimates of the variation of $\tilde{\omega}$ closely correspond to variation in ω. For example, if the coefficient c is fairly constant over time, then time series variation in ω is well approximated by time series variation in $\tilde{\omega}$. This assumption is reasonable if the institutional response by potential information acquirers is slow relative to the frequency of the empirical analysis. The coefficient can still vary across firms. For such cases, we recommend and implement in our analysis below, that firm fixed effects be used to absorb such unobserved heterogeneity in the value of information.

4 Empirical Analysis of the Value of Information

4.1 Estimated Information Values and Other Summary Statistics

Table 1 reports sample moments of the value of information as well as summary statistics for the sampled firms. The mean and standard deviation of the value of information are estimated using the delta method as described in Section 3 above.

While our estimates pertain to the value from acquiring information to be realized over one day, because our measure of return volatility is annualized, so is the value of information we report throughout. Thus, reported values of information correspond to the minimum willingness to pay for daily information on a stock over a year. The average annualized value of information in our sample is \$2.9 million (\$11,500 per day). The value of information is highly variable, with a standard deviation of about \$87 million.

The median annualized intraday return volatility is 37%, and the median volatility accounting for overnight returns is 39%. As a result, about 6 percent of the median volatility accrues overnight. The median price impact (λ_r divided by stock price) is 0.01, reflecting that \$1 million of order flow changes prices by 1 percent.[13] About 4 percent of the observations have a negative price impact (and thus information value) estimate. These obser-

13. Our estimate of median price impact is similar to that of Hasbrouck (2009, Table II), where a \$1 million order would move the log price by 0.7%.

Table 1: Information values and other summary statistics

variable	Mean	Std	Min	q25	Median	q75	Max	Obs
Information value, $M	2.92	86.77	-323.23	3.22	12.05	46.66	2,188.49	11,857,178
Information value (intraday), $M	2.50	70.16	-272.14	2.95	10.99	41.22	1,563.89	11,857,178
Volatility	0.50	0.36	0.09	0.26	0.39	0.60	2.12	11,857,178
Volatility (intraday)	0.46	0.33	0.06	0.25	0.37	0.57	1.91	11,857,178
Price impact per $M	0.13	0.43	-0.09	0.00	0.01	0.06	3.34	11,857,178
Stock price	42.56	114.01	5.00	14.93	27.97	49.69	252,438.54	11,857,178
Size (lnME)	13.97	1.78	10.29	12.68	13.86	15.12	18.60	11,857,178
Book-to-market ratio	0.61	0.45	0.03	0.29	0.51	0.80	2.57	11,857,178
Momentum	0.17	0.47	-0.64	-0.10	0.10	0.33	2.31	11,857,178

Notes: Daily common stocks panel from NYSE TAQ, CRSP, and Compustat, September 2003 to December 2020. The value of information is annualized variance divided by price impact and reported in millions of dollars. The mean and standard deviation of the value of information are estimated with the delta method to alleviate measurement error of this ratio. Dollar figures are CPI adjusted to December 2020 values. Volatility is the square root of the annualized sum of squared one-minute log returns. Price impact is estimated by regressing one-minute log returns on contemporaneous share order flow and divided by the previous trading day's closing stock price. The intraday value of information is based on intraday volatility alone (excluding overnight returns). Size is log market equity over the previous month. Momentum is the return over the prior 2–12 months. Book-to-market is book equity as of last June divided by market equity as of last December (Fama-French conventions). All variables are 1% winsorized.

vations are included when we report information value in levels, but omitted whenever we report results in logs below.[14] The median market capitalization of firms in the sample is $1 billion, the median book-market ratio is 51 percent. Median momentum shows that the median stock appreciated by 10 percent over the preceding 11 months.

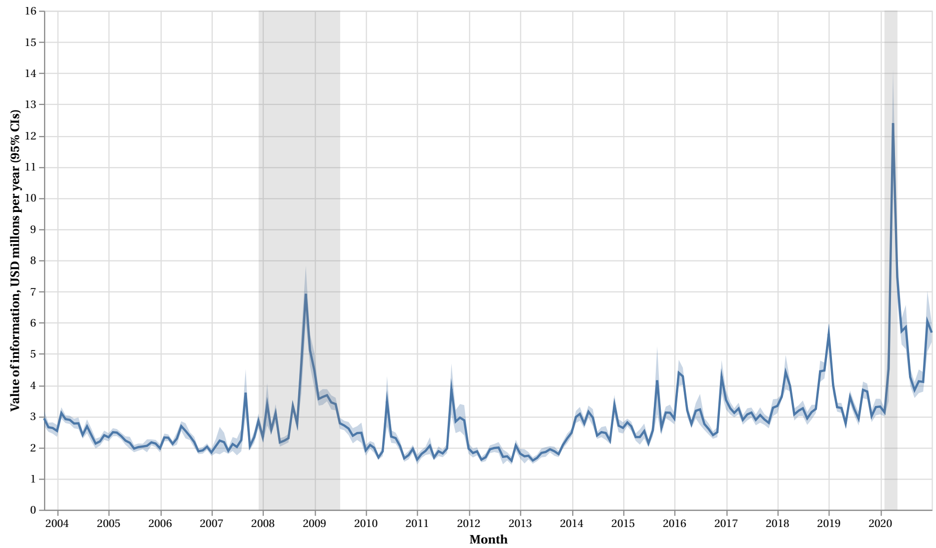
4.2 Time Trends in the Value of Information

Figure 2 describes how the value of information changes over time. Panel (a) reports the value of information $\tilde{\Omega}$ in levels, averaged over stocks and days each month. Panel (b) reports a decomposition into log volatility and log price impact, which sum up to the value of information in logs. Recession periods are shaded in the graph. The figure shows that during turbulent times, when volatility goes up, and in particular during recessions, stocks become less liquid as reflected in an increased price impact. Thus, as expected, a high level of uncertainty goes hand-in-hand with low market liquidity. However, the figure demonstrates that the increase in volatility during turbulent times is far steeper than the increase in price impact, leading to an increase in the value of information. This pattern is particularly stark during the financial crisis of 2007–2009 and the Covid-19 pandemic of 2020. In both cases, both volatility and illiquidity increase dramatically, but the increase in volatility is much steeper, leading to a spike in the value of information.

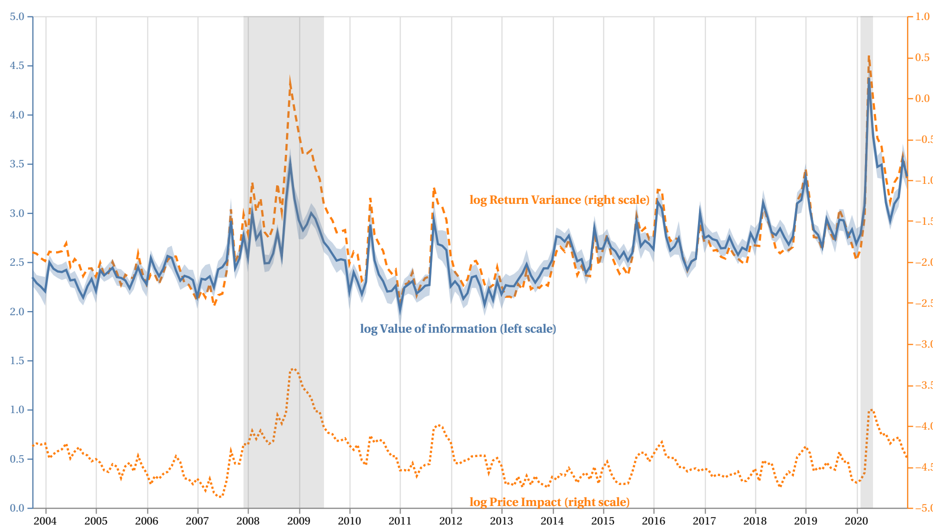
Figure 3a zooms in on the value of information during the recent Covid-19 pandemic period. The two highest value of information days in our sample are Monday, March 16, 2020, when the annualized value of information reaches $19.3 million ($77k per day), and Friday, March 20, 2020 with an information of value of $19.7 million ($78k per day). During that week it became clear that the virus has been spreading in the US for quite some time, and New York City schools, restaurants and bars shut down. The Federal Reserve slashed interest rates to near-zero and unveiled a remarkable set of programs to backstop

14. For robustness, in Figure A.2 we report values of information that winsorize at greater levels to the point where negative price impact coefficients are removed.

Figure 2: Value of information over time



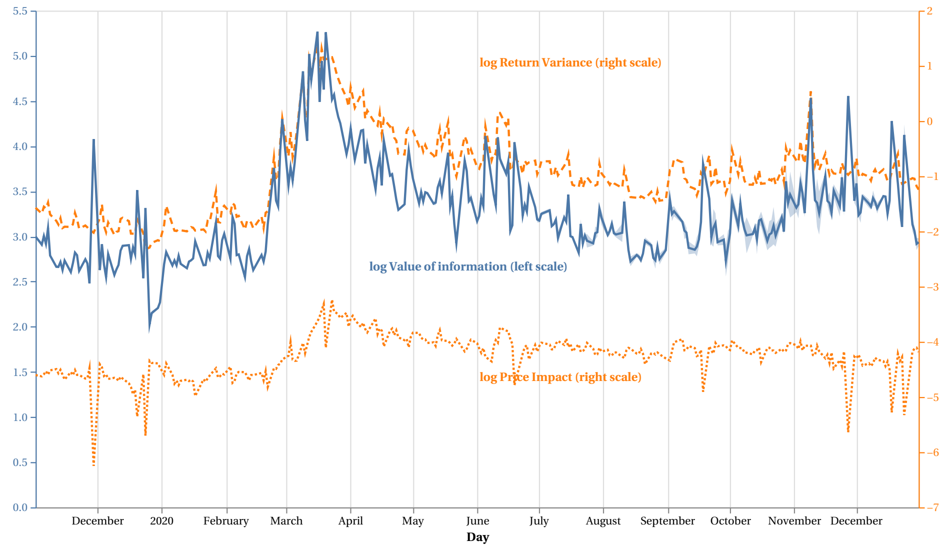
(a) Value of information in levels



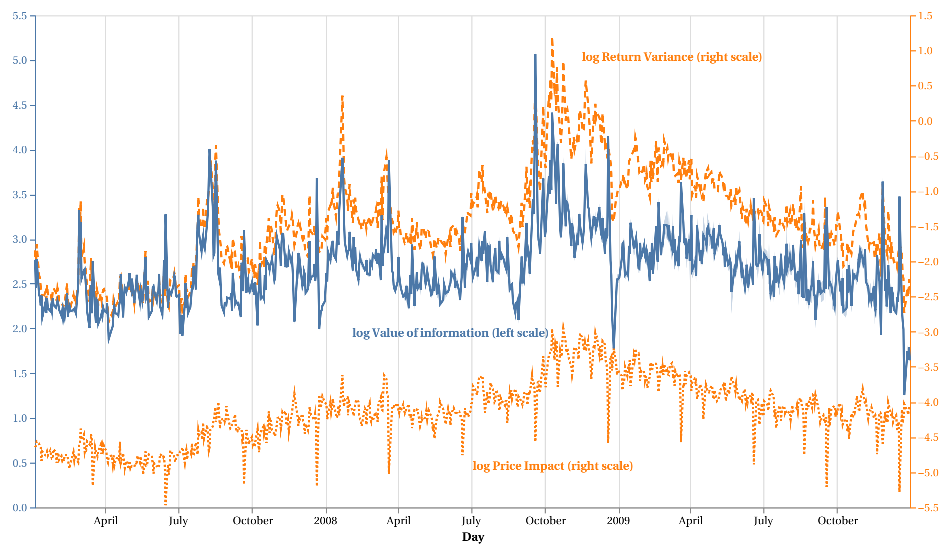
(b) Value of information in logs and its components

Notes: The solid line is the monthly information value averaged over stocks and days surrounded by its 95 percent confidence interval, based on time-clustered variance estimates. The value of information is annualized variance divided by price impact and reported in millions of dollars. The mean and standard deviation of the value of information are estimated with the delta method to alleviate measurement error of this ratio. The log information value equals log annualized return variance less log price impact, so we also report the mean of log annualized return variance (dashed line) and the mean of log price impact (dotted line). Shades indicate recessions.

Figure 3: Value of information rises sharply at the onset of crises



(a) Covid-19 pandemic: November 2019 to December 2020



(b) Financial crisis: November 2007 to December 2009

Notes: Solid lines are daily information values averaged over stocks surrounded by their 95 percent confidence interval, based on time-clustered variance estimates. The value of information is annualized variance divided by price impact and reported in millions of dollars. The log information value equals log annualized return variance less log price impact, so we also report the mean of log annualized return variance (dashed line) and the mean of log price impact (dotted line).

the US economy (Smialek and Irwin 2020). The heightened uncertainty coupled with the Fed's interventions to increase market liquidity made information more valuable than on any other time in our sample. In fact, nine of the ten highest values of information during our sample all occur during March 2020 (see Table A.1), as financial market participants fled to liquidity and the Fed continued to intervene at an unprecedented scale (Haddad, Moreira, and Muir 2021). The only other day to make the top 10 is Monday, November 9, 2020—the first trading day after the 2020 US election results became clear and when news arrived that Pfizer's Covid-19 vaccine is highly effective.

Figure 3b focuses on the financial crisis of 2007–2009. It shows that the value of information peaked on September 19, 2008, just a few days after the collapse of Lehman Brothers. Volatility increases that day by 0.50 log points at the same time that market liquidity improves, with price impact declining by 0.76 log points relative to the previous day's values. On this day the Fed announced new asset-backed commercial paper and money market mutual fund liquidity facilities, and that it will purchase mortgage-backed securities from primary dealers.[15] The combined spike in volatility and improved market liquidity lead to a dramatic increase in the value of information.

These two turbulent episodes are rare, so learning from them is subject to caveats about small sample inference. That said, it appears that the Fed, perhaps unintentionally, tends to increase the value of information during crises, by enhancing market liquidity. Such large rewards to collecting information can be beneficial if investors learn about solvency by facilitating a more efficient capital allocation. But they can be harmful if investors reap these rewards by researching funding liquidity, which can instigate coordination failures and debt runs (He and Manela 2016).

The only other event to make the top 20 days list is the stock market crash of August 24, 2015. This global market price drop has been attributed to concerns about China's econ-

15. Federal Reserve Bank of St. Louis' Financial Crisis Timeline. https://fraser.stlouisfed.org/timeline/financial-crisis, accessed on June 10, 2020.

omy (Driebusch 2015) and to the Fed beginning to raise short-term interest rates (Feldstein 2015) after years near the zero lower bound following the 2008 crisis.

Figure 4 documents seasonality patterns of the value of information over the year, over the quarter, and over the trading week. Two main patterns emerge. First, the value of information drops sharply toward the end of the year, and second, the value of information increases gradually during the work week. A potential explanation for the end-of-year drop is that when traders work less during the holiday period volatility drops because less information is being generated, and at the same time market makers are supplying less liquidity. This can be seen from Panel 4a, where volatility drops and price impact rises during the last weeks of the average year.

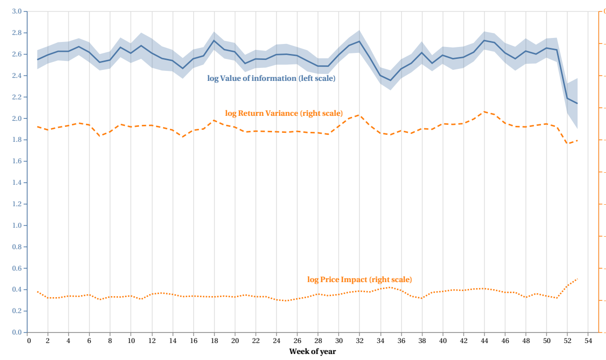
4.3 The Value of Information in the Cross-section of Stocks

We next turn to studying how the value of information varies cross-sectionally. We begin by considering firm size. It is often argued that the value of acquiring information is higher for small stocks. Such stocks are more likely to be neglected (Arbel, Carvell, and Strebel 1983) and they suffer from frictions preventing information from being fully incorporated into their price (Hou and Moskowitz 2005). These arguments, however, do not account for the fact that small stocks are typically illiquid, and thus, trading on the information acquired on such stocks is associated with significant price impact. Thus, a priori, it is not obvious whether the value of information is higher for large or small stocks.

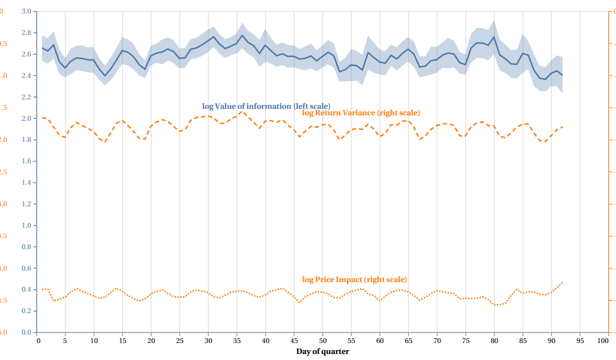
Figure 5 shows that for some stocks, the value of information is considerably larger than the $2.9 million for the average firm reported in Table 1. For example, the value for information on Google (GOOG) is about a $1 billion a year. From the scatter plot we can see that size explains some but not all of this variation. Evidently, the value of information for many smaller stocks is minimal.

Figure 6 shows that, perhaps surprisingly, the value of information is consistently higher

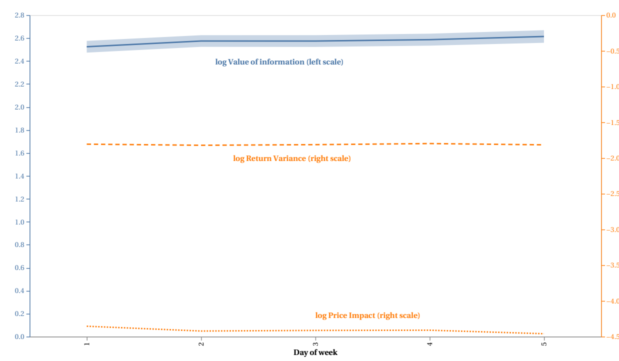
Figure 4: Seasonality in the Value of information



(a) Year-end decline



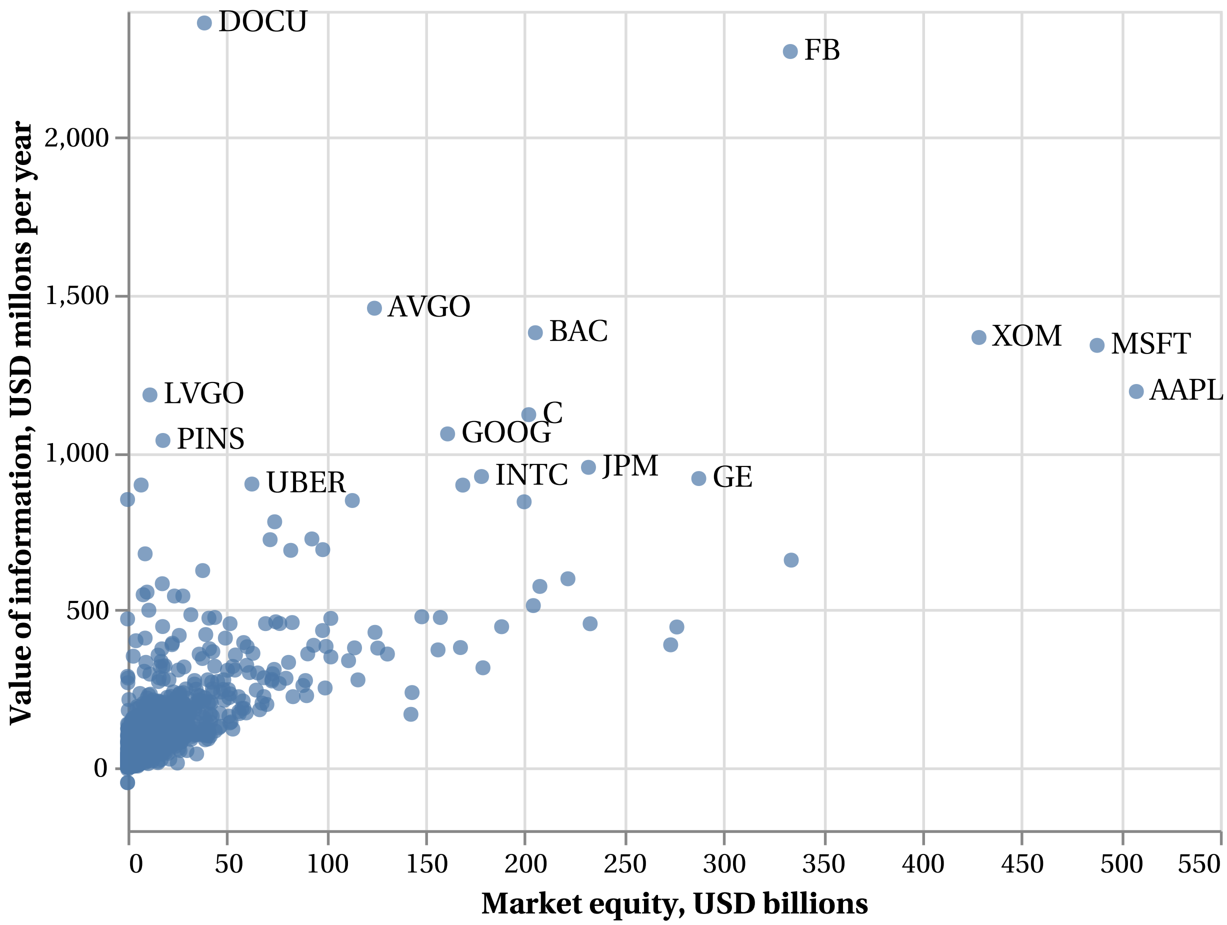
(b) Quarter-end decline



(c) Gradual rise over the trading week

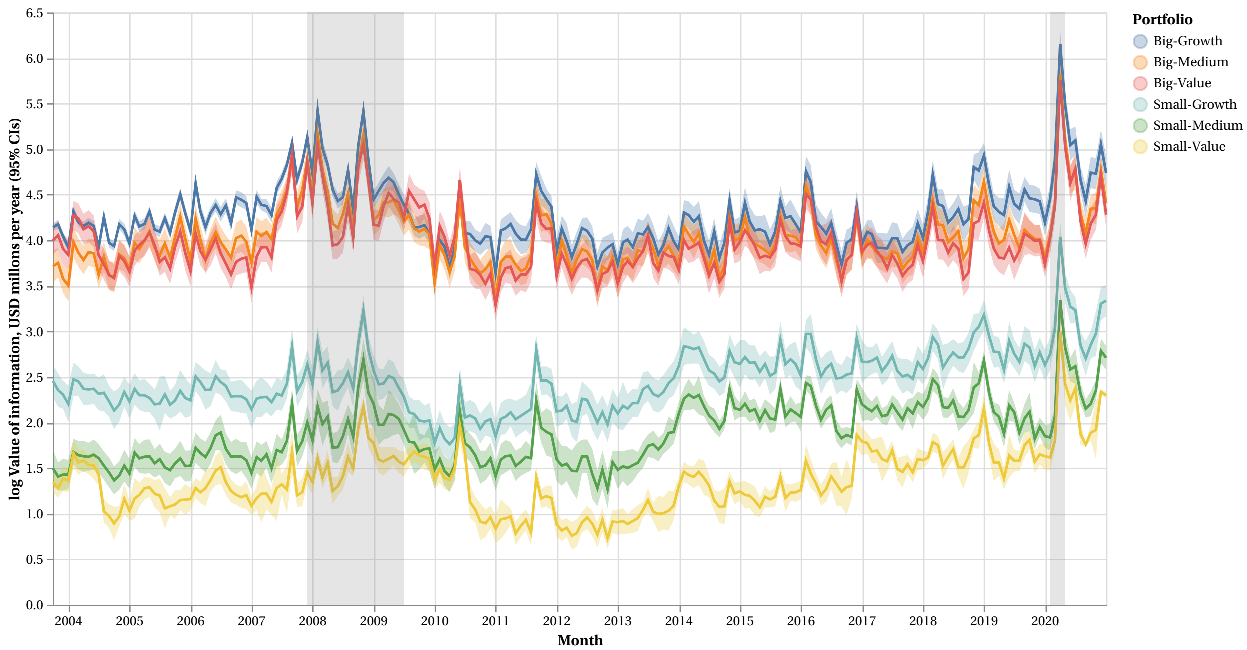
Notes: Solid lines are daily information values averaged over stocks surrounded by their 95 percent confidence interval, based on time-clustered variance estimates. The value of information is annualized variance divided by price impact and reported in millions of dollars. The log information value equals log annualized return variance less log price impact, so we also report the mean of log annualized return variance (dashed line) and the mean of log price impact (dotted line).

Figure 5: Value of information by stock



Notes: Mean value of information plotted against mean market equity for each specific stock (permno). The value of information is annualized variance divided by price impact and reported in millions of dollars. The mean and standard deviation of the value of information are estimated with the delta method to alleviate measurement error of this ratio. We label the 15 most information-valuable stocks with their latest ticker symbols.

Figure 6: Information on large growth stocks is more valuable



Notes: Monthly log information value averaged over stocks and days surrounded by their 95 percent confidence interval for stocks sorted to size and book-to-market portfolios. Following Fama-French conventions for constructing size-based portfolios, big stocks have market equity greater than the median NYSE stock over the previous month, and the rest are defined as small. Value stocks have a book-to-market equity ratio greater than the 70th percentile NYSE stock, growth stocks below the 30th percentile, and the rest are classified as medium. The value of information is annualized variance divided by price impact and reported in millions of dollars. Shades indicate recessions.

for large stocks than it is for small stocks. While the uncertainty associated with large stocks is lower, their illiquidity as measured by price impact is much lower to an extent that makes investment in acquiring information on such stocks more valuable. Note that the higher value of information for large stocks is consistent throughout our sample period. The spread between the value of information for large/small stocks widens during turbulent times, as stock volatility of small stocks is much more sensitive to macroeconomic shocks that the volatility of large stocks.

To formally test the hypothesis that the value of information is higher for large stocks, we regress the value of information on stock size (measured as log of market equity) as well as stock and day fixed effects. The results reported in Table 2a show a strong positive association between firm size and the value of information for all specifications. The coefficients can be interpreted as elasticities with respect to size. Specification (1) implies that the value of information rises by 0.81% when market equity rises by 1%. Before adding any fixed effects, the large R-squared in this univariate regression means size explains a substantial share of the variation in information values. Specifications (2) through (4) show that these conclusions are quite robust to controlling for firm-specific and time-specific heterogeneity. Specifications (5) through (8) show that the intraday value of information, which is somewhat smaller on average (Table 1), has a higher size elasticity.

Table 2b shows that the main reason for the higher value of information for large stocks is that price impact is more sensitive than volatility to size. To see this, we regress both log variance and log price impact on log market capitalization. Comparing, for example, specifications (1) and (5) we find that a 1% increase in size reduces variance by about 0.32% but it reduces price impact by about 1.12%.

Table 3 studies the extent to which the value of information can be explained by commonly studied characteristics: size, book-to-market, and momentum. Panel 3a shows that growth and momentum stocks have higher values of information. Unlike book-to-market, which is subsumed by the inclusion of both stock and day fixed effects, stocks that appreci-

Table 2: Information on large stocks is more valuable

	log Information value				log Information value (intraday)			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Size	0.81***	0.81***	0.69***	0.71***	0.82***	0.82***	0.72***	0.72***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Date (day) FE		Yes		Yes		Yes		Yes
Stock FE			Yes	Yes			Yes	Yes
N	11,431,389	11,431,389	11,431,344	11,431,344	11,431,389	11,431,389	11,431,344	11,431,344
R^2	0.46	0.50	0.57	0.60	0.46	0.49	0.58	0.61

(a) Size explains a substantial share of information value variation

	log Return variance				log Price impact			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Size	-0.32***	-0.33***	-0.34***	-0.34***	-1.12***	-1.13***	-1.04***	-1.05***
	(0.00)	(0.00)	(0.01)	(0.01)	(0.00)	(0.00)	(0.01)	(0.01)
Date (day) FE		Yes		Yes		Yes		Yes
Stock FE			Yes	Yes			Yes	Yes
N	11,431,389	11,431,389	11,431,344	11,431,344	11,431,389	11,431,389	11,431,344	11,431,344
R^2	0.20	0.40	0.36	0.54	0.68	0.70	0.72	0.74

(b) Larger stocks have lower volatility, but even lower price impact

Notes: Panel (a) shows regressions of the log value of information and its components on size. The log information value equals log annualized return variance less log price impact, so in Panel (B) we regress log return variance and log price impact on the same variables. Volatility is the square root of the annualized sum of squared one-minute log returns. Price impact is estimated by regressing one-minute log returns on contemporaneous share order flow and divided by the previous trading day's closing stock price. Size is log market equity over the previous month. The intraday value of information is based on intraday volatility alone (excluding overnight returns). Observations with negative price impact are omitted. Standard errors clustered by date and stock are in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table 3: Value of information and stock characteristics

	log Information value				log Information value (intraday)			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Size	0.79***	0.79***	0.68***	0.68***	0.80***	0.80***	0.70***	0.68***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Book-to-market	-0.27***	-0.24***	-0.05***	-0.01	-0.30***	-0.28***	-0.05***	-0.01
	(0.02)	(0.02)	(0.02)	(0.01)	(0.02)	(0.02)	(0.02)	(0.01)
Momentum	0.20***	0.32***	0.05***	0.17***	0.21***	0.34***	0.05***	0.18***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Date (day) FE		Yes		Yes		Yes		Yes
Stock FE			Yes	Yes			Yes	Yes
N	11,431,389	11,431,389	11,431,344	11,431,344	11,431,389	11,431,389	11,431,344	11,431,344
R^2	0.47	0.50	0.57	0.60	0.47	0.50	0.58	0.61

(a) Information values are higher for growth and momentum stocks

	log Return variance				log Price impact			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Size	-0.33***	-0.34***	-0.31***	-0.35***	-1.11***	-1.12***	-1.00***	-1.03***
	(0.00)	(0.00)	(0.01)	(0.01)	(0.00)	(0.00)	(0.01)	(0.01)
Book-to-market	-0.18***	-0.16***	0.05***	0.04***	0.07***	0.06***	0.09***	0.04***
	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Momentum	0.01	0.24***	-0.11***	0.12***	-0.18***	-0.08***	-0.16***	-0.05***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Date (day) FE		Yes		Yes		Yes		Yes
Stock FE			Yes	Yes			Yes	Yes
N	11,431,389	11,431,389	11,431,344	11,431,344	11,431,389	11,431,389	11,431,344	11,431,344
R^2	0.20	0.41	0.36	0.54	0.68	0.70	0.72	0.74

(b) Momentum stocks are more volatile and more liquid

Notes: Panel (a) shows regressions of the log value of information and its components on stock characteristics. The log information value equals log annualized return variance less log price impact, so in Panel (B) we regress log return variance and log price impact on the same variables. Size is log market equity over the previous month. Momentum is the return over the prior 2–12 months. Book-to-market is book equity as of last June divided by market equity as of last December (Fama-French conventions). The intraday value of information is based on intraday volatility alone (excluding overnight returns). Observations with negative price impact are omitted. Standard errors clustered by date and stock are in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

ated over the preceding year (high momentum stocks) have higher values of information regardless of the regression specification.

Comparing the R-squares in Table 2 to those in Table 3 we can see that size explains most of the variation in information values. By contrast, the increase in explanatory power from the addition of book-to-market ratios and momentum is quite modest.

Overall, our results show that, while small stocks may sound appealing for research, the value of information is actually higher for large stocks. The higher liquidity of large stocks more than makes up for the reduced uncertainty associated with their cash flows.

5 Discussion

It should be noted that an underlying assumption in our estimation approach is that price impact of order flow reflects information asymmetry. Indeed, market makers in Kyle-type models are assumed risk neutral, and price reactions to order flow are used to compensate them against expected losses from adverse selection. In practice, it is possible that the price impact of order flow also reflects the uncertainty of the asset's fundamental value due to the riskiness associated with holding inventory. Stoll (1978), Ho and Stoll (1980), and Ho and Stoll (1981) study settings with no adverse selection in which uncertainty alone results in a price impact (reflected in a bid-ask spread) due to dealers' risk aversion. Thus, a caveat to our results is that our estimates may, to some extent, be picking up uncertainty rather than the value of information. This may be more of an issue during periods or within asset classes for which uncertainty is high while adverse selection is less likely to be present.

For a specific example, consider our results showing that the value of information rose dramatically during the initial days of the Covid pandemic (Figure 3a). These results are driven by a spike in fundamental uncertainty associated with a relatively modest increase in price impact. Given that the value of information statistic is the ratio of the two – we document a sharp increase in the value of information. The question is whether the increase in price impact we document is indeed reflective of adverse selection or risk aversion. As the latter is outside the scope of our model, it may be that our estimates of the value of information during this volatile time period are inflated.

A related caveat is that the risk-neutral Kyle model is best suited for modeling information to a trader who receives idiosyncratic or stock-specific information and who can neutralize systematic risk from their portfolio. A reasonable objection in that case is that the estimate of variance reduction should be net of systematic risk. In the Appendix we redo our analysis with estimates of idiosyncratic variance. That analysis shows that the value of information in that case is slightly smaller and that its relation to the characteristics we study remains about the same. We also verify that this idiosyncratic value of information generates similar peaks around turbulent times as reported below.

6 Conclusion

In this paper we offer intuitive and empirically useful bounds on the value of gathering firm-specific information to strategic informed investors. A simple statistic—the ratio of return variance to price impact measured using a version of Kyle's lambda—quantifies the value of information to a monopolist informed trader and provides an approximation more generally. While volatility and illiquidity are known to be correlated, we find that the ratio of the two varies markedly both over time and in the cross section, giving rise to interesting patterns in the value of information.

In the time series, we find that the value of information rises in turbulent times and during recessions. In particular, the value of information spikes dramatically during both the great recession of 2007–2009 and the Covid-19 crisis of 2020. During these times, stock volatility increases more steeply than illiquidity, and information becomes much more valuable. In the cross section, we find that information on large stocks is consistently more valuable than information on small stocks, despite informational frictions that are known to be associated with the latter. This follows from the lack of liquidity in small stocks, which makes it harder to exploit information associated with them.

Our measure of the value of information has a variety of potential applications–both in academic work and in practice. Future empirical work could use our approach to rank information sources such as analysts, newsletters, and alternative data. It can also be used to rank and evaluate the degree to which different types of corporate announcements add value to investors. Moreover, our measure can be employed by the courts to assess penalties in cases of insider trading.

Data Availability

The data used in this project is proprietery and cannot be shared publicly. The code replication package provides example random data that can be used to understand the structure of the original data.

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A Appendix

A.1 Proofs

Proof of Proposition 2: When $n = 1$, $c(1, \rho) = 1$ and we are done. Thus, assume $n \geq 2$, From (6), we have that

$$c(n, \rho) = e^{\zeta} E_{\frac{2}{n}}(\zeta), \tag{A.1}$$

where $E_p(z)$ is the generalized exponential integral defined as

$$E_p(z) = \int_1^{\infty} x^{-p} e^{-xz} dx.$$

To show that $c(n, \rho) \geq c(2, 0) \approx 0.92$, we establish three claims.

Claim 1: For any $n \geq 2$ and $\rho \geq 0$, $\frac{\partial \zeta(n,\rho)}{\partial \rho} < 0$. Thus, for any $n \geq 2$, $\zeta(n, \rho)$ is maximized at $\rho = 0$, where the maximal value is $\zeta(n, 0) = \frac{n-1}{n}$.

The proof of this claim follows from direct differentiation of (4).

Claim 2: For any $n \geq 2$ and $\rho \geq 0$, $c(n, \rho) \geq c(n, 0)$.

To prove this, we fix $n \geq 2$ and differentiate $c(n, \rho)$ by ρ applying (A.1):

$$\begin{aligned} \frac{\partial}{\partial \rho} c(n, \rho) &= e^{\zeta} \frac{\partial \zeta}{\partial \rho} E_{\frac{2}{n}}(\zeta) + e^{\zeta} \left(\frac{\partial}{\partial \zeta} E_{\frac{2}{n}}(\zeta) \frac{\partial \zeta}{\partial \rho} \right) \\ &= e^{\zeta} \frac{\partial \zeta}{\partial \rho} \left(E_{\frac{2}{n}}(\zeta) + \frac{\partial}{\partial \zeta} E_{\frac{2}{n}}(\zeta) \right). \end{aligned}$$

From Eq. (3) in Chiccoli, Lorenzutta, and Maino (1990),

$$\frac{\partial}{\partial \zeta} E_{\frac{2}{n}}(\zeta) = -E_{\frac{2}{n}-1}(\zeta).$$

It follows that

$$\frac{\partial}{\partial \rho} c(n, \rho) = e^{\zeta} \frac{\partial \zeta}{\partial \rho} \left(E_{\frac{2}{n}} (\zeta) - E_{\frac{2}{n}-1} (\zeta) \right).$$

Now, $\frac{\partial \zeta}{\partial \rho} < 0$ (from Claim 1), and $E_{\frac{2}{n}} (\zeta) - E_{\frac{2}{n}-1} (\zeta) < 0$ (from Eq. (8) in Chiccoli, Lorenzutta, and Maino (1990)). It follows that $\frac{\partial}{\partial \rho} c(n, \rho) > 0$, and thus $c(n, \rho) \geq c(n, 0)$.

Claim 3: We now show that $c(n, \rho) \geq 0.92$ for all $n \geq 2$ and $\rho \geq 0$.

By Claim 2, it is sufficient to show that $c(n, 0) \geq c(2, 0) \approx 0.92$ for all $n \geq 2$. For $n = 2, ..., 11$, we calculate that $c(n, 0) = e^{\zeta(n,0)} E_{\frac{2}{n}} (\zeta(n, 0)) > 0.92$. For $n \geq 12$, we apply Eq. (13) in Chiccoli, Lorenzutta, and Maino (1990), which (using Claim 1) implies that

$$c(n, 0) = e^{\zeta(n,0)} E_{\frac{2}{n}} (\zeta(n, 0)) > \frac{1}{\frac{2}{n} + \frac{n-1}{n}} = \frac{n}{n+1} \geq \frac{12}{13} > 0.92.$$

This concludes the proof. ■

Proof of Proposition 3: The case $n = 1$ corresponds to Kyle (1985), in which $c(1, 0) = 1$. Using direct calculation, we have checked that for $n = 2, 3, 4$, $c(n, 0) < 1$. We will now show that $c(n, 0) \leq 1$ for all $n \geq 5$.

We know from the proof of Proposition 2 that

$$c(n, 0) = e^{\frac{n-1}{n}} E_{\frac{2}{n}} \left(\frac{n-1}{n} \right).$$

By Chiccoli et al (1990, Section 7), this expression can be written as a generalized continued fraction

$$e^{\frac{n-1}{n}} E_{\frac{2}{n}} \left(\frac{n-1}{n} \right) = \frac{a_{1,n}}{b_{1,n}+} \frac{a_{2,n}}{b_{2,n}+} \frac{a_{3,n}}{b_{3,n}+} \frac{a_{4,n}}{b_{4,n}+} \frac{a_{5,n}}{b_{5,n}+} \cdots$$

where

$$
\begin{aligned}
a_{1,n} &= 1, b_{1,n} = \frac{n-1}{n}; a_{2,n} = \frac{2}{n}; b_{2,n} = 1; a_{3,n} = 1; b_{3,n} = \frac{n-1}{n}; \\
a_{4,n} &= \frac{2}{n} + 1; b_{4,n} = 1; a_{5,n} = 2; b_{5,n} = \frac{n-1}{n}, \\
a_{6,n} &= \frac{2}{n} + 2; b_{6,n} = 1; a_{7,n} = 3; b_{7,n} = \frac{n-1}{n}, \ldots
\end{aligned}
$$

By Lorentzen and Waadeland (2008, Lemma 1.1), for any n, the kth convergent of this continued fraction, which we denote by $\alpha_{k,n}$, is given by

$$
\alpha_{k,n} = \frac{A_{k,n}}{B_{k,n}}, \tag{A.2}
$$

where $A_{k,n}$ and $B_{k,n}$ are known as continuants and are given recursively as follows:

$$
A_{-1,n} = 1,\ A_{0,n} = 0,\ B_{-1,n} = 0,\ B_{0,n} = 1,
$$

and for $k \geq 1$,

$$
\begin{aligned}
A_{k,n} &= b_{k,n} A_{k-1,n} + a_{k,n} A_{k-2,n} \qquad \text{(A.3)} \\
B_{k,n} &= b_{k,n} B_{k-1,n} + a_{k,n} B_{k-2,n}.
\end{aligned}
$$

As $k \to \infty$, $\alpha_{k,n}$ converge to $c(n, 0)$. Thus, to show that $c(n, 0) \leq 1$ for all $n \geq 5$, it is sufficient to show that for all $n \geq 5$ and k sufficiently large, $\alpha_{k,n} < 1$. We will now establish this result.

Fix an arbitrary $\hat{n} \geq 5$. We will show using induction on k that for all $k \geq 4$, $\alpha_{k,\hat{n}} < 1$.

Lemmas 1 and 2 (below) establish the base case of the induction: $\alpha_{4,\hat{n}} < 1$ and $\alpha_{5,\hat{n}} < 1$. To established the inductive step, assume that for $k \geq 4$, both $\alpha_{k-2,\hat{n}} < 1$ and $\alpha_{k-1,\hat{n}} < 1$. We will show that $\alpha_{k,\hat{n}} < 1$.

By (A.2), $A_{k-2,\hat{n}} < B_{k-2,\hat{n}}$ and $A_{k-1,\hat{n}} < B_{k-1,\hat{n}}$. Then,

$$\begin{aligned} A_{k,\hat{n}} &= b_{k,\hat{n}} A_{k-1,\hat{n}} + a_{k,\hat{n}} A_{k-2,\hat{n}} \\ &< b_{k,\hat{n}} B_{k-1,\hat{n}} + a_{k,\hat{n}} B_{k-2,\hat{n}} \\ &= B_{k,\hat{n}}, \end{aligned}$$

where the two equalities follow from (A.3), and the inequality follows from the induction hypothesis and the fact that $a_{k,n}$ and $b_{k,n}$ are non-negative for all k and n. We have thus shown that $A_{k,\hat{n}} < B_{k,\hat{n}}$, which by (A.2) means that $\alpha_{k,\hat{n}} < 1$ as needed. This completes the proof. ■

Lemma 1: For all $n \geq 5$, $\alpha_{4,n} < 1$.

Proof of Lemma 1: For any $n \geq 5$, the recursion (A.3) implies

$$\begin{aligned} A_{1,n} &= 1,\ B_{1,n} = \frac{n-1}{n},\ \alpha_{1,n} = \frac{n}{n-1} \\ A_{2,n} &= 1,\ B_{2,n} = \frac{n+1}{n},\ \alpha_{2,n} = \frac{n}{n+1} \\ A_{3,n} &= \frac{2n-1}{n},\ B_{3,n} = \frac{2n^2-n-1}{n^2},\ \alpha_{3,n} = \frac{n\,(2n-1)}{(n-1)\,(2n+1)} \\ A_{4,n} &= \frac{3n+1}{n},\ B_{4,n} = \frac{3n^2+2n+1}{n^2},\ \alpha_{4,n} = \frac{3n^2+n}{3n^2+2n+1}. \end{aligned}$$

Inspection of $\alpha_{4,n}$ shows that the numerator is smaller than the denominator for any positive n. and therefore $\alpha_{4,n} < 1$ as claimed. ■

Lemma 2: For all $n \geq 5$, $\alpha_{5,n} < 1$.

Proof of Lemma 2: Using Recursion (A.3) once more yields:

$$A_{5,n} = \frac{7n^2-4n-1}{n^2},\ B_{5,n} = \frac{(n-1)\,(7n^2+4n+1)}{n^3},\ \alpha_{5,n} = \frac{n\,(7n^2-4n-1)}{(n-1)\,(7n^2+4n+1)}. \quad \text{(A.4)}$$

We will use the expression for $\alpha_{5,n}$ to prove the following three claims:

Claim 1: $\alpha_{5,5} < 1$.

Claim 2: $\lim_{n\to\infty} \alpha_{5,n} = 1$.

Claim: 3: $\alpha_{5,n}$ is strictly increasing in n for all $n \geq 5$.

These three claims jointly imply the lemma. Indeed, given that $\alpha_{5,5} < 1$ (Claim 1) and $\lim_{n\to\infty} \alpha_{5,n} = 1$ (Claim 2), the only way in which there is some $n > 5$ for which $\alpha_{5,n} \geq 1$ is if $\alpha_{5,n}$ were to be weakly decreasing over some range of n values. But, this contradicts Claim 3.

Claim 1 follows by plugging $n = 5$ in ($A.4$) which yields

$$\alpha_{5,5} = \frac{55}{56} < 1.$$

Claim 2 follows by taking the limit as $n \to \infty$ in ($A.4$).

To establish Claim 3, differentiate (A.4)

$$\frac{d}{dn}\alpha_{5,n} = \frac{7n^4 - 28n^3 - 12n^2 + 8n + 1}{\left(7n^3 - 3n^2 - 3n - 1\right)^2}.$$

The denominator of this expression is positive, while the numerator is positive for $n = 5$ and increasing for $n \geq 5$, (as can be established by direct differentiation of the numerator). This shows Claim 3 and completes the proof of the lemma. ■

A.2 Robustness

Here we include some additional robustness tests.

To study the sensitivity of our estimates to how we estimate price impact, in Figure A.1 we report the mean value of information across days and stocks for three different algorithms for signing trades as buys or sells: Chakrabarty et al. (2007, CLNV), Lee and Ready (1991, LR), and Ellis, Michaely, and O'Hara (2000, EMO). None of these have a major effect on the mean value of information.

Our baseline analysis omits penny stocks with more than $5 in price. The figure further shows that including such stocks would reduce the mean value of information. This is intuitive as penny stocks likely have large price impact coefficients.

The last specification in Figure A.1 shows the effect of subtracting market variance from stock-specific variance to measure the values of idiosyncratic information. This has only a minor effect of the mean value of information because most of the variance for the average stock is idiosyncratic. For this same reason, Figure A.3, shows that focusing on idiosyncratic variance only yields about the same time series for the value of information.

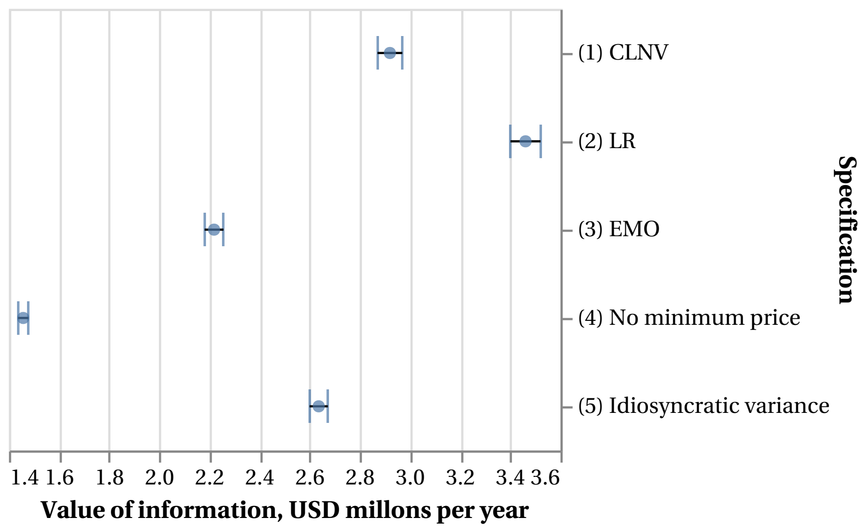
Figure A.4 reports similar robustness tests, but this time for the regression analysis reported in the baseline regression tables in the paper (e.g. Table 3). Again, we find that our conclusions are robust to these alternative specifications.

Table A.1: Extreme value of information days

Date	Information value, $M	Return variance	Price impact per $M
2020-03-20	19.68	3.34	0.17
2020-03-16	19.29	4.08	0.21
2020-03-12	18.41	3.30	0.18
2020-03-18	17.80	3.87	0.22
2020-03-09	16.88	2.68	0.16
2020-11-09	15.24	2.22	0.15
2020-03-13	14.64	2.76	0.19
2020-03-19	14.53	3.36	0.23
2020-03-24	14.08	3.17	0.23
2020-03-25	13.27	2.83	0.21
2020-03-23	13.11	3.44	0.26
2020-03-17	13.06	2.77	0.21
2015-08-24	12.94	1.94	0.15
2020-03-26	12.79	2.53	0.20
2008-09-19	12.32	2.46	0.20
2008-10-10	11.99	3.66	0.30
2020-02-28	11.34	1.38	0.12
2020-06-05	10.90	1.37	0.13
2020-04-01	10.63	2.22	0.21
2020-04-07	10.37	2.17	0.21
⋮			
2005-11-25	1.24	0.16	0.13
2013-04-12	1.24	0.16	0.13
2012-04-12	1.24	0.16	0.13
2013-02-12	1.22	0.14	0.11
2012-10-08	1.21	0.16	0.13
2012-05-25	1.21	0.18	0.15
2012-10-12	1.20	0.15	0.13
2010-10-11	1.20	0.17	0.14
2010-11-26	1.20	0.15	0.13
2012-08-28	1.20	0.16	0.13
2010-12-23	1.19	0.13	0.11
2012-08-24	1.18	0.16	0.14
2010-12-27	1.17	0.13	0.11
2010-12-29	1.17	0.12	0.10
2010-12-28	1.15	0.12	0.11
2010-12-30	1.09	0.12	0.11
2013-01-16	1.09	0.13	0.12
2012-08-30	1.05	0.14	0.13
2010-12-31	1.05	0.13	0.13
2009-12-24	0.97	0.16	0.16

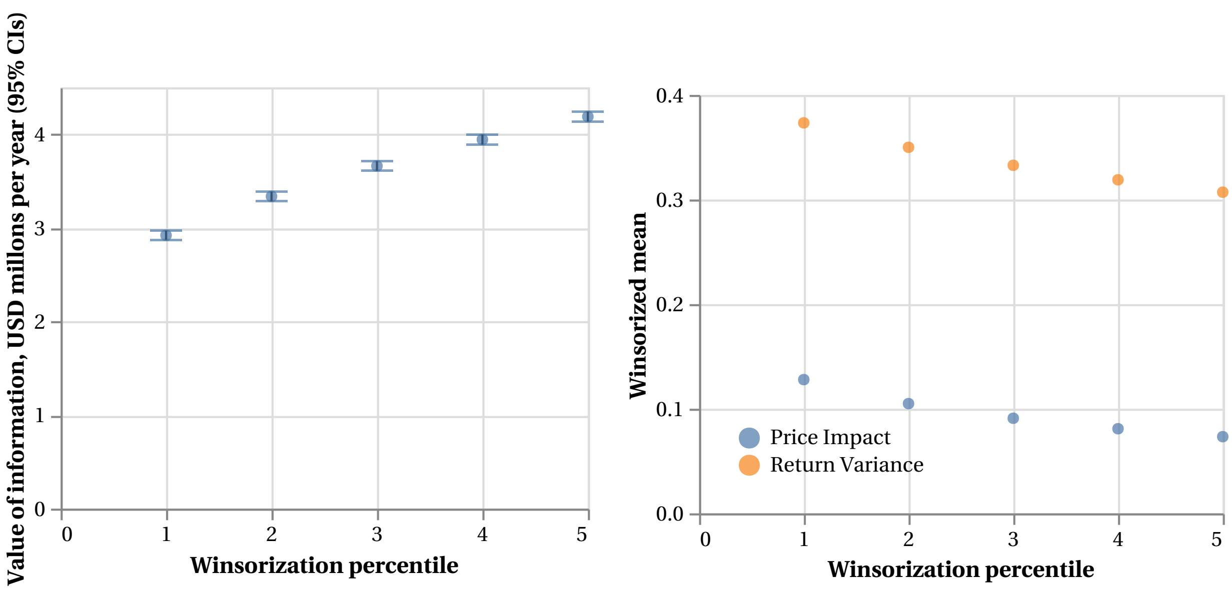
Notes: The 20 highest and lowest information value days, September 2003 to December 2020.

Figure A.1: Robustness: Mean value of information



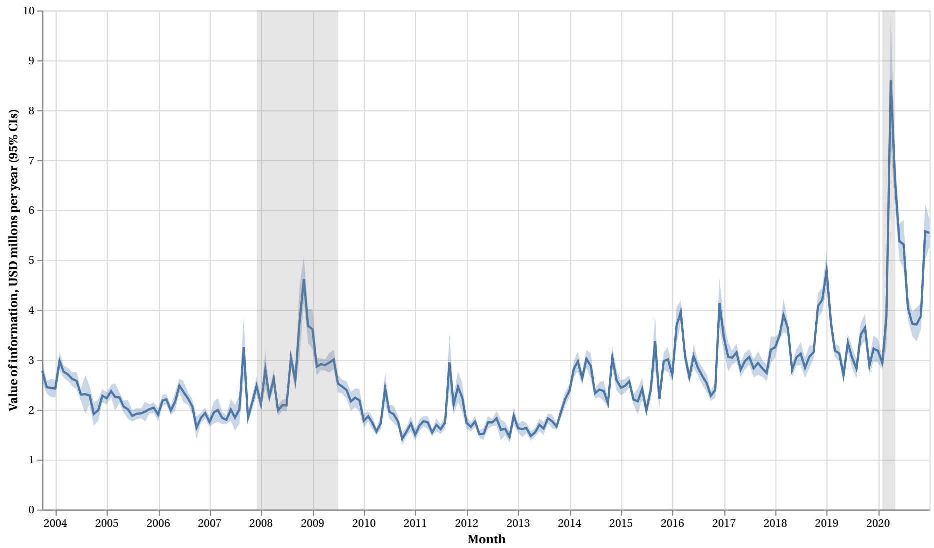
Notes: Reported is the mean value of information across days and stocks for three different algorithms for signing trades as buys or sells: Chakrabarty et al. (2007, CLNV), Lee and Ready (1991, LR), and Ellis, Michaely, and O'Hara (2000, EMO). No minimum price shows the effect of including stocks with nominal price below \$5. Idiosyncratic variance shows the effect of subtracting market variance from stock-specific variance to measure the values of idiosyncratic information. The value of information is annualized variance divided by price impact and reported in millions of dollars. All variables are 1% winsorized.

Figure A.2: Sensitivity to the winsorization level

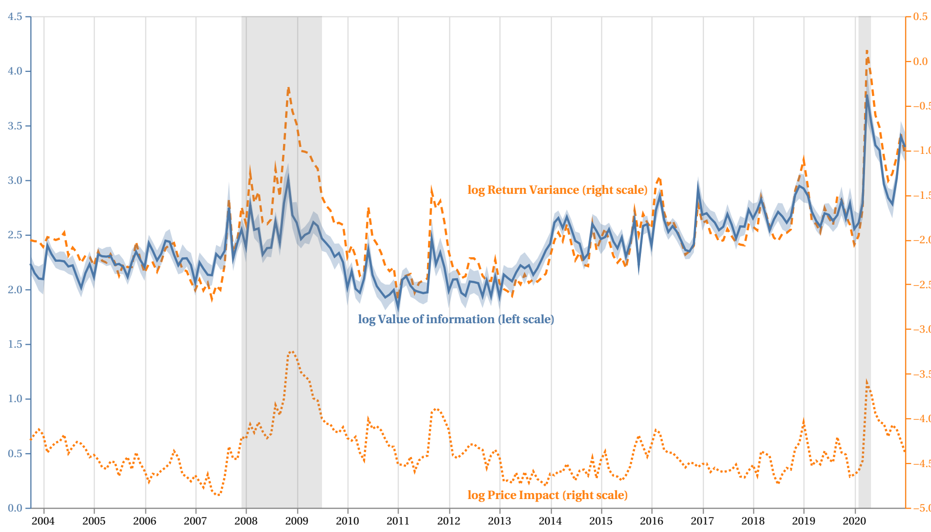


Notes: The left panel shows the mean value of information across days and stocks as function of the winsorization percentile that is applied to return variance and to price impact, which are then used to estimate the value of information. The value of information is annualized variance divided by price impact and reported in millions of dollars. The mean and standard deviation of the value of information are estimated with the delta method to alleviate measurement error of this ratio. The right panel shows the mean return variance and price impact as function of the same winsorization percentiles.

Figure A.3: Value of idiosyncratic information over time



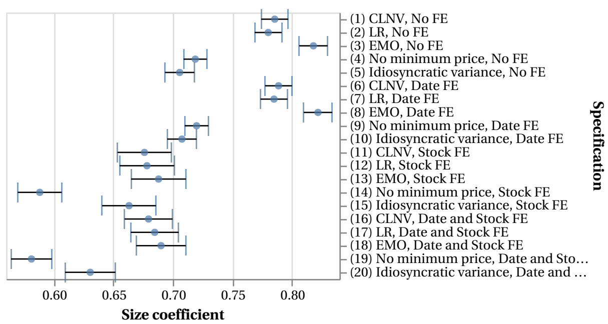
(a) Value of information in levels



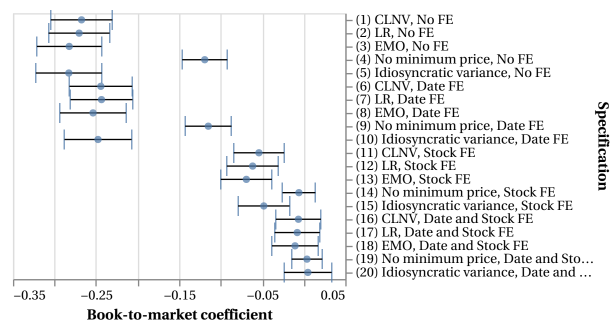
(b) Value of information in logs and its components

Notes: The solid line is the monthly information value averaged over stocks and days surrounded by its 95 percent confidence interval, based on time-clustered variance estimates. The idiosyncratic value of information is annualized idiosyncratic variance divided by price impact and reported in millions of dollars, where idiosyncratic variance is stock-specific variance less market (SPY) variance. The log information value equals log annualized return variance less log price impact, so we also report the mean of log annualized return variance (dashed line) and the mean of log price impact (dotted line). Shades indicate recessions.

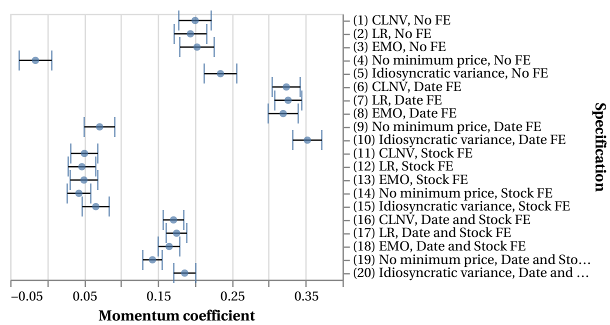
Figure A.4: Robustness: Value of information and stock characteristics



(a) Size



(b) Book-to-market



(c) Momentum

Notes: Reported are coefficients from regressions of the log value of information on stock characteristics. We evaluate several fixed effect specifications and three different algorithms for signing trades as buys or sells: Chakrabarty et al. (2007, CLNV), Lee and Ready (1991, LR), and Ellis, Michaely, and O'Hara (2000, EMO). No minimum price shows the effect of including stocks with nominal price below \$5. Idiosyncratic variance shows the effect of subtracting market variance from stock-specific variance to measure the values of idiosyncratic information. The log information value equals log annualized return variance less log price impact, so in Panel (B) we regress log return variance and log price impact on the same variables. Size is log market equity over the previous month. Momentum is the return over the prior 2–12 months. Book-to-market is book equity as of last June divided by market equity as of last December (Fama-French conventions). Observations with negative price impact are omitted. Standard errors clustered by date and stock are used to construct the 95% errors bars.