Fundamentals of Perpetual Futures

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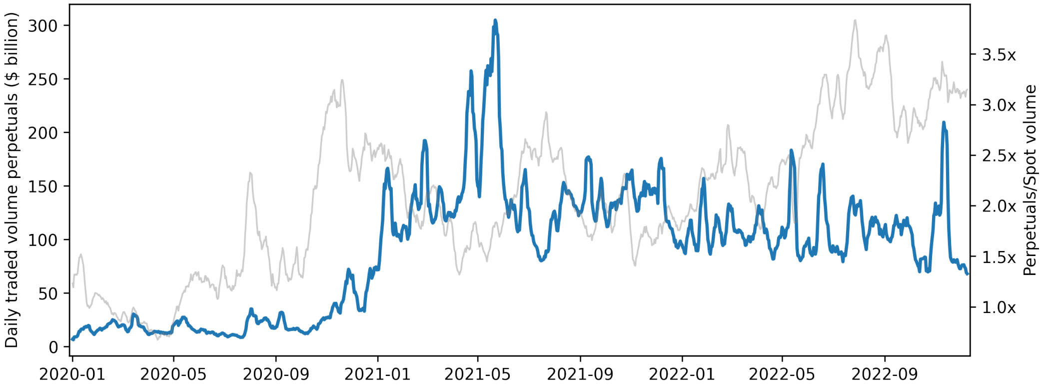
What are perpetual futures?

- Perpetual futures are swaps that never expire
- Efficient way to hedge and speculate
  - Allow high leverage
  - No need to take delivery of crypto
  - No rollover
  - Concentrates liquidity
- First introduced in Shiller (1993)

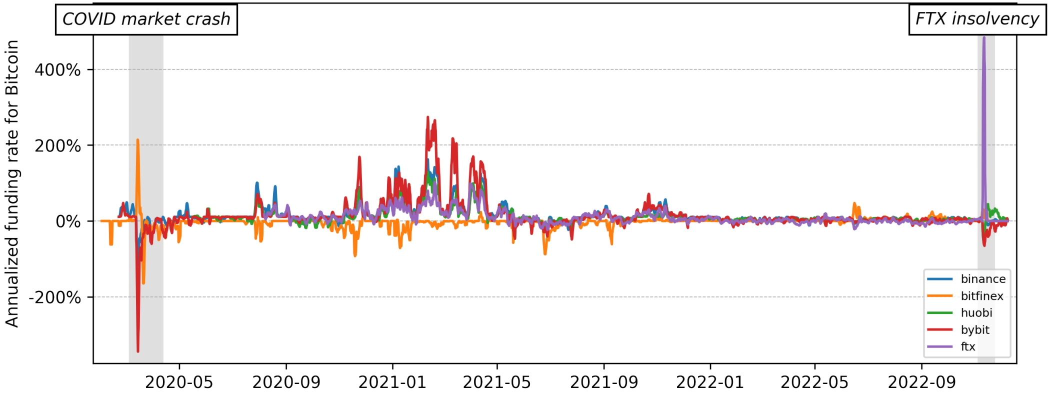
Most popular derivative traded in crypto markets

$100 billion daily volume



Among the most actively-traded products on FTX before it imploded

Alameda Research was counterparty to many such leveraged trades



Questions

1. What are the fundamental values of perpetual futures in theory?
2. How large are deviations from these fundamentals in practice?

Our paper

Theoretic contribution

- Derive no-arbitrage prices for perpetual futures in frictionless markets
- Derive no-arbitrage bounds for markets with trading costs

Empirical findings

- Deviations of crypto perpetual futures from no-arbitrage prices are large
- Deviations comove across cryptocurrencies
- Diminish over time as crypto markets develop and become more efficient
- Simple trading strategy generates large Sharpe ratios

Related work

- Descriptive evidence of a "carry trade" in perpetual futures: Alexander, Choi, Park, and Sohn (2020), Franz and Schmeling (2021), De Blasis and Webb (2022), Ferko, Moin, Onur, and Penick (2022), Christin, Routledge, Soska, and Zetlin-Jones (2023)
- Theory on perpetuals: Angeris, Chitra, Evans, and Lorig (2022) derive no-arb prices for perpetuals assuming payoff is a fixed function of the spot price
- Recent work on fixed-maturity crypto futures by Schmeling, Schrimpf, and Todorov (2022) find large profits to carry strategies (Du, Tepper, and Verdelhan, 2018); Koijen, Moskowitz, Pedersen, and Vrugt, 2018), Cong, He, and Tang (2022)

Fixed-maturity futures refresher

Definition

- Two counterparties agree at time $t = 0$ to exchange underlying asset at future expiration time τ
- Underlying asset price S_t fluctuates over time
- Future price $F_{0,\tau}$ is fixed at initiation
- At expiration, short counterparty delivers to long counterparty the underlying in exchange for the futures price[1]

[1] This technically describes a forward contract. A futures contract is guaranteed by the exchange and is marked-to-market periodically against a margin account.

Fixed-maturity futures refresher

No arbitrage pricing

- In the absence of arbitrage, the futures price $F_{0,\tau}$ only depends on the initial underlying price S_0 and the interest rate r

$$F_{0,\tau} = S_0 e^{r\tau} \tag{1}$$

- Just the future value of the spot price
- Easily extends to dividend-paying assets and storage costs

Fixed-maturity futures refresher

Arbitrage strategy

- Suppose the futures price is actually higher

$$F_{0,\tau} > S_0 e^{r\tau}$$

- Arbitrageur would today
  1. Open a short futures position (0 cashflow today)
  2. Borrow dollars in cash markets ($+S_0$)
  3. Buy the underlying at spot price ($-S_0$)
- At expiration time τ
  1. Collect futures price ($+F_{0,\tau}$)
  2. Deliver underlying asset
  3. Repay dollar loan ($-S_0 e^{r\tau}$)
- Net cashflow at expiration $F_{0,\tau} - S_0 e^{r\tau} > 0$

Perpetual futures (swap)

Definition

- No expiration date!
- No initial cash
- Can be closed at any time
- At termination, short pays long $F_t - F_0$
- Long counterparty pays short one at each interval ds a funding rate:

$$\text{Funding Rate}_s = \kappa(F_s - S_s)ds$$

- In most exchanges, funding rate is paid every 8 hours and is roughly the futures-spot difference
- In this case, $\kappa = 3 \times 365 = 1095$
- Arbitrage is risky: Nothing guarantees that futures price converge to the spot!

Random-maturity arbitrage

Certain profits at an uncertain future time

Definition (Traditional)

A riskless arbitrage opportunity is defined with respect to payoff x at a certain future time T and its price $p(x)$. If (1) $x \geq 0$ almost surely, (2) $x > 0$ with some positive probability, (3) its price satisfies $p(x) \leq 0$, then this payoff is an arbitrage opportunity.

Definition (Our extension)

A random-maturity arbitrage opportunity is defined with respect to a bounded random payoff x at a future random time τ, $\tau \in (0, \infty)$, and its price $p(x)$. If (1) $x \geq 0$ almost surely, (2) $x > 0$ with some positive probability, (3) its price satisfies $p(x) \leq 0$, then this payoff is a random-maturity arbitrage opportunity.

Random-maturity arbitrage

Objection

- "But random-maturity arbitrage opportunities are not riskless!"
- Yes, but no arbitrage prices are always just a useful fiction
- Real markets have transactions costs, margin requirements, and risk of liquidation
- Random-maturity no arb prices are similarly a useful benchmark

Perpetual futures - No arbitrage pricing

Assumptions

A1 *The perpetual futures-spot gap can temporarily explode but will eventually shrink*

$$\liminf_{t \to \infty} |F_t - S_t| < \infty$$

A2 *The risk-free rate r for arbitrageurs is constant*

Proposition

The absence of random-maturity arbitrage opportunities implies

$$F_t = S_t \left(1 + \frac{r}{\kappa}\right) \tag{2}$$

The perpetual futures to spot gap is small when interest rates r are low relative to the funding rate coefficient κ

Perpetual futures

Arbitrage strategy

- Suppose the futures price is actually higher

$$F_0 > S_0 \left(1 + \frac{r}{\kappa}\right)$$

- Arbitrageur would today
  1. Open a short futures position (0 cashflow today)
  2. Borrow dollars in cash markets ($+S_0$)
  3. Buy the underlying at spot price ($-S_0$)
- At (random) unwinding time t
  1. Close futures position ($+F_0 - F_t$)
  2. Sell the underlying at spot price ($+S_t$)
  3. Repay dollar loan ($-S_0 e^{rt}$)
  4. Funding payments accrued ($\kappa \int_0^t (F_s - S_s) e^{r(t-s)} ds$)
- Net discounted payoff $= e^{-rt} F_0 - S_0 - e^{-rt}(F_t - S_t) + \kappa \int_0^t (F_s - S_s) e^{-rs} ds$

Perpetual futures

Arbitrage strategy (continued)

- Suppose this is not an arbitrage opportunity
- Then for all t, the payoff is nonpositive

$$\underbrace{e^{-rt}F_0 - S_0}_{\text{traditional spread}} + \underbrace{\kappa \int_0^t (F_s - S_s)e^{-rs}ds}_{\text{funding payments}} \leq \underbrace{e^{-rt}(F_t - S_t)}_{\text{spread at unwinding}} \tag{3}$$

- Turns out we can bound this process from below and show that

$$\frac{F_0 re^{-rt}}{\kappa + r} + \left(\frac{F_0}{1 + \frac{r}{\kappa}} - S_0\right) e^{\kappa t} \leq e^{-rt}(F_t - S_t) \tag{4}$$

- But $F_0 > S_0 \left(1 + \frac{r}{\kappa}\right)$ implies LHS $\rightarrow \infty$ violating bounded spread assumption

$\Rightarrow$ Arbitrage! Some t exists when arb can unwind at a positive discounted payoff

Perpetual futures - No arbitrage bounds

Proposition

With constant round-trip trading costs $C > 0$, the absence of random-maturity arbitrage opportunities implies that the perpetual futures price must lie within the following bound relative to the spot:

$$\left| F_t - S_t \left(1 + \frac{r}{\kappa} \right) \right| \leq C \tag{5}$$

Data

Focus on the 5 largest cryptocurrencies with a total market cap of $529B
64.15% of crypto spot market in Nov 2022

Crypto	start date	end date	N
BTC	2019-09-10	2022-11-13	27,895
ETH	2019-11-27	2022-11-13	25,985
BNB	2020-02-10	2022-11-13	24,184
DOGE	2020-07-10	2022-11-13	20,559
ADA	2020-01-31	2022-11-13	24,424

- Perpetual futures, spot, and funding rate at 1-hour frequency from Binance
- Funding rate is paid every 8 hours
- Market is open 24/7
- Risk-free interest rates from Aave

Deviations from no-arbitrage benchmarks

Define ρ as the annualized interest rate deviation that rationalizes an observed future-spot spread

$$F = S\left(1 + \frac{r+\rho}{\kappa}\right)$$

or approximately

$$\rho \approx \kappa(\log F - \log S) - r$$

Deviations of perpetual futures from no-arbitrage benchmarks

Mean deviation is no different than zero

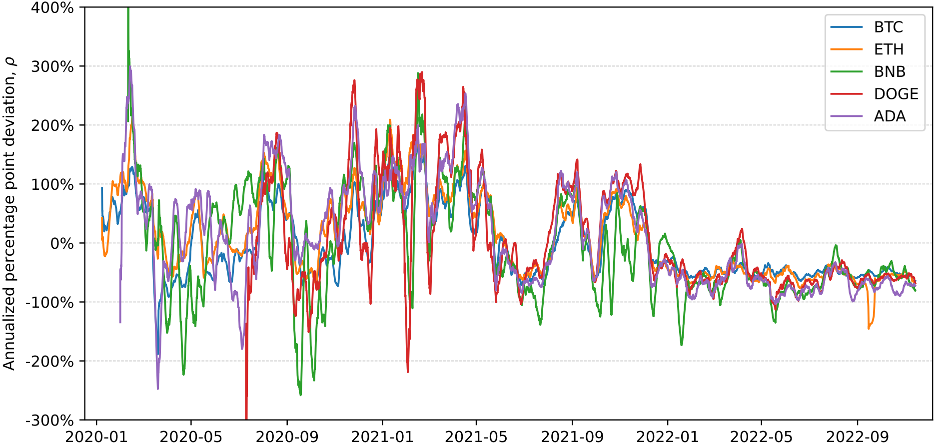
Mean **absolute** deviation is about 60–100% per year

Considerably larger than deviations Du-Tepper-Verdelhan (2018) find in traditional FX markets

	ρ				$\lvert\rho\rvert$			
Asset	Mean	Median	Std	p-value	Mean	Median	Std	p-value
BTC	-0.02	-0.25	0.79	0.60	0.59	0.53	0.52	0.00
ETH	0.12	-0.09	0.89	0.13	0.67	0.56	0.61	0.00
BNB	-0.07	-0.18	1.32	0.70	0.94	0.73	0.93	0.00
DOGE	0.08	-0.18	1.62	0.28	1.01	0.72	1.27	0.00
ADA	0.12	-0.06	1.25	0.18	0.91	0.74	0.87	0.00

Deviations of perpetual futures from no-arbitrage benchmarks

Strong comovement
Deviations shrink in mid 2021



Random-maturity arbitrage strategy

No arbitrage bounds prescribe a threshold strategy to exploit divergence from fundamentals

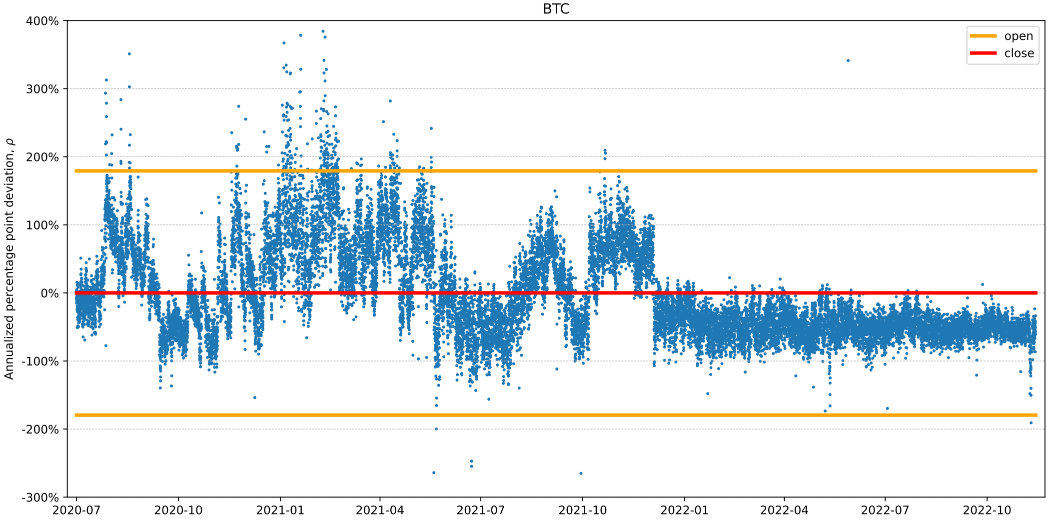
Fee levels	Spot	Futures	ρ_l	ρ_u
No	0%	0%	0.0%	0.0%
Low	0.0225%	0.0018%	-53.2%	53.2%
Medium	0.045%	0.0072%	-114.4%	114.3%
High	0.0675%	0.0144%	-179.5%	179.2%

Given C = round-trip percentage trading costs

- if $\rho > \rho_u = \kappa log(1 + C)$
- then borrow cash, buy the spot and short the future
- collect funding rate and pay interest
- close the positive when ρ is back to zero

Opposite strategy if $\rho < \rho_l = \kappa log(1 - C)$

Random-maturity arbitrage strategy: BTC, high trading costs



Performance of Random-maturity arbitrage strategy

		Fee tiers			
		No	Low	Medium	High
BTC	SR	3.94	2.50	2.31	1.92
	Return	17.89	11.21	10.13	8.15
	Volatility	4.54	4.48	4.39	4.25
	MaxDD	-4.24	-4.27	-4.34	-4.43
	α	22.64	10.86	8.24	5.47
	t_α	5.39	2.71	2.10	1.38
	Active %	100.00	84.93	43.87	22.32
	OtC time	15.80	63.90	95.97	113.38
ETH	SR	5.43	3.46	3.13	2.82
	Return	28.03	17.49	15.17	12.68
	Volatility	5.16	5.06	4.85	4.49
	MaxDD	-4.13	-4.21	-3.90	-3.94
	α	45.55	23.09	18.33	14.42
	t_α	7.56	4.25	3.69	3.15
	Active %	99.94	84.63	51.21	28.11
	OtC time	12.10	37.57	70.75	75.77
BNB	SR	12.49	8.07	6.41	5.44
		⋮			
DOGE	SR	12.61	8.34	5.96	4.42
		⋮			
ADA	SR	11.34	6.46	4.07	3.12
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		⋮			

Performance over time: High trading costs tier

		Unrestricted				Long-spot Only			
		2020	2021	2022	All	2020	2021	2022	All
BTC	SR	2.26	2.39	1.23	1.92	1.98	2.28	0.55	1.75
	Return	8.28	14.80	0.26	8.15	6.43	13.96	0.10	7.16
	Volatility	3.67	6.18	0.21	4.25	3.25	6.12	0.18	4.10
	Active %	28.66	34.43	1.01	22.32	22.18	32.07	0.03	18.95
	N	8,616	8,760	7,536	24,912	8,616	8,760	7,536	24,912
ETH	SR	3.08	3.52	1.39	2.82	2.57	3.37	0.87	2.50
	Return	17.11	18.08	1.36	12.68	13.99	17.14	0.62	11.06
	Volatility	5.55	5.13	0.98	4.49	5.44	5.09	0.71	4.42
	Active %	37.80	34.91	9.12	28.11	34.40	34.54	0.09	24.07
	N	8,616	8,760	7,536	24,912	8,616	8,760	7,536	24,912
BNB	SR	6.04	6.60	2.61	5.44	4.45	5.00	0.68	3.94
					⋮				
DOGE	SR	4.90	5.93	1.93	4.42	2.64	5.05	1.11	3.10
					⋮				
ADA	SR	3.87	3.41	2.27	3.12	3.16	2.94	0.00	2.49
					⋮				

Returns mostly come from price convergence rather than funding payments

		2020	2021	2022	All
BTC	Return	21.68	29.60	-0.05	17.89
	Price	14.00	14.29	2.11	10.51
	Funding	7.68	15.31	-2.16	7.39
ETH	Return	40.93	36.66	3.24	28.03
	Price	27.58	19.07	3.75	17.38
	Funding	13.35	17.59	-0.51	10.65
BNB	Return	82.48	76.29	25.36	62.38
	Price	66.70	60.37	20.00	49.80
	Funding	15.78	15.92	5.36	12.57
DOGE	Return	220.81	113.79	49.67	112.09
	Price	214.72	93.24	50.52	102.37
	Funding	6.09	20.55	-0.85	9.72
ADA	Return	90.81	66.73	50.19	69.58
	Price	77.17	49.51	49.05	58.52
	Funding	13.64	17.22	1.13	11.06

Explaining deviations

Regression of the futures-spot gap against explanatory variables

Dependent Variable:	ρ					
	BTC			ETH		
Ret	0.28***		0.28***	0.20***		0.23***
	(7.79)		(7.74)	(3.87)		(4.83)
Vol		-0.02	-0.01		-0.01**	0.01
		(-1.60)	(-0.64)		(-1.99)	(1.58)
Const	-0.12**	0.30	-0.03	-0.06	0.36	-0.28*
	(-2.16)	(1.22)	(-0.16)	(-0.58)	(1.40)	(-1.67)
R2	0.55	0.03	0.56	0.47	0.03	0.49
N	1011	1011	1011	1011	1011	1011

Takeaways

- We derive no-arbitrage prices and bounds for perpetual futures
- Provide valuable benchmarks + strategy to exploit deviations
- Find large deviations of crypto perpetual futures from no-arbitrage prices
- Deviations comove across cryptocurrencies and diminish over time
- Simple trading strategy generates large Sharpe ratios

AAVE Interest Rate