第十六讲
离散时间信号的采样

杜清河
西安交通大学
信息与通信工程学院
2025

本讲覆盖章节

- 7.5

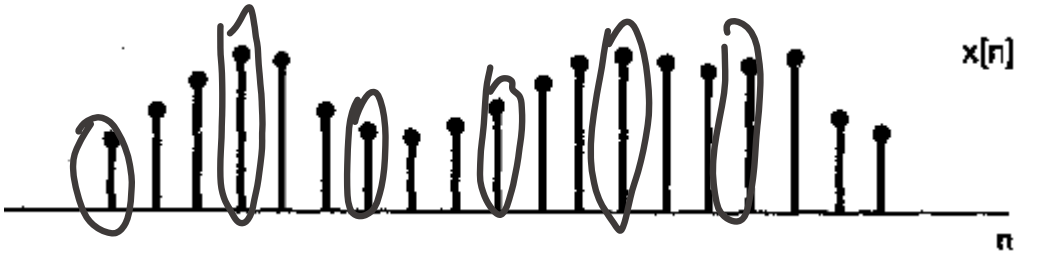
内容提要

- 脉冲串采样
- 离散时间抽取与内插

内容提要

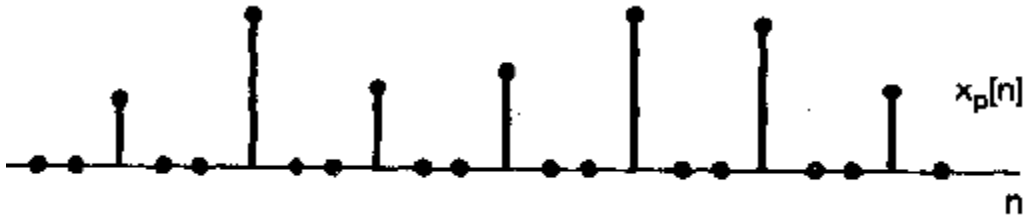
- 脉冲串采样
- 离散时间抽取与内插

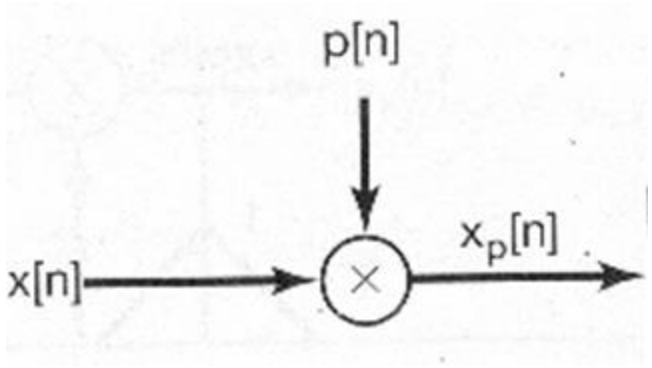
脉冲串采样



抽取
or
采样

$$p[n]=\sum_{k=-\infty}^{+\infty}\delta[n-kN]$$





脉冲串采样的频域分析

$$x[n] \leftrightarrow X\left(e^{j\omega}\right)$$

$\frac{1}{N} = a_k$

$$p[n] = \sum_{k=-\infty}^{+\infty} \delta[n - kN] \leftrightarrow P\left(e^{j\omega}\right) = \frac{2\pi}{N} \sum_{k=-\infty}^{+\infty} \delta\left(\omega - k\omega_s\right)$$

$\delta[n-kN]$ 视为 1

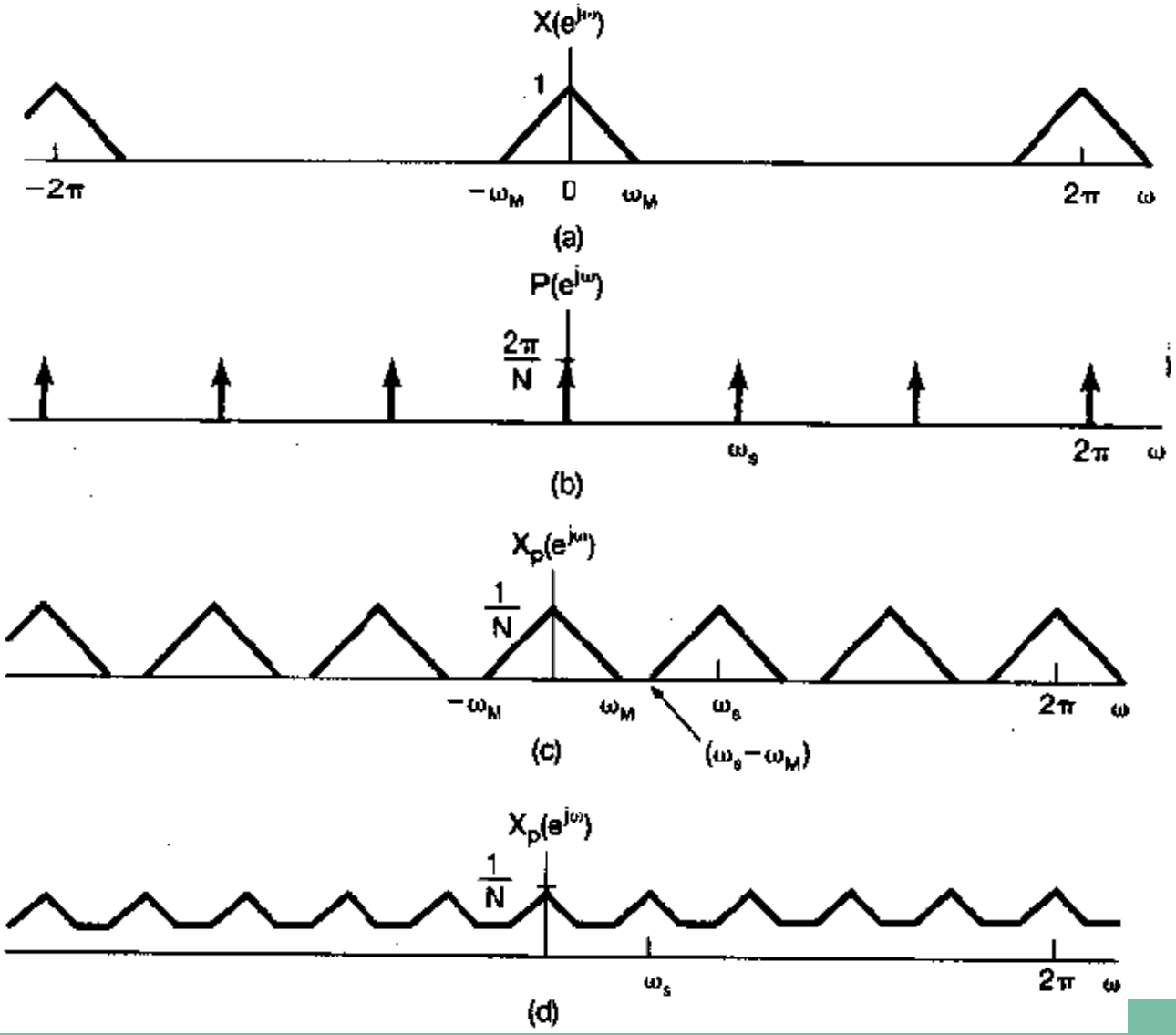
$\delta[n-k\cdot\frac{2\pi}{\omega_s}]$

所以：

$$X_p\left(e^{j\omega}\right) = \frac{1}{2\pi}\int_{2\pi} X(e^{j\theta})P(e^{j(\omega-\theta)})d\theta$$

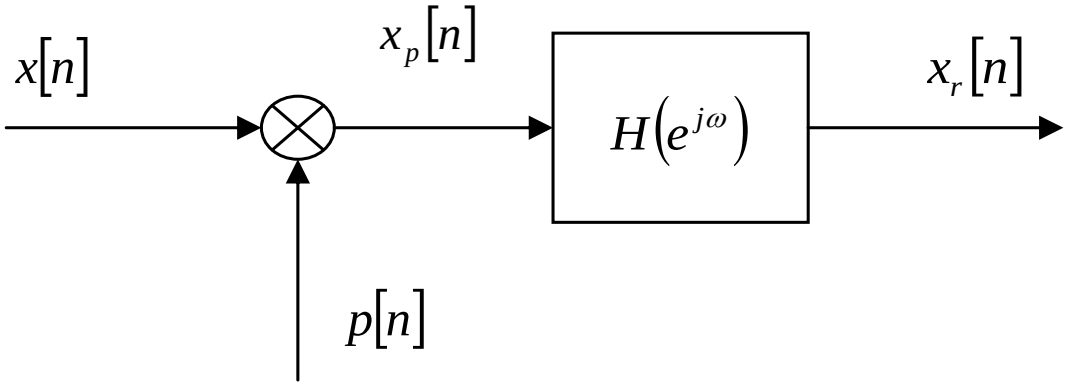
$$\Rightarrow \quad X_p\left(e^{j\omega}\right) = \frac{1}{N}\sum_{k=-\infty}^{+\infty} X\left(e^{j(\omega-k\omega_s)}\right)$$

脉冲串采样的频域分析


X(e^jω)
1
−2π
−ω_M
0
ω_M
2π
ω
(a)
P(e^jω)
2π/N
ω_s
2π
ω
(b)
X_p(e^jω)
1/N
−ω_M
ω_M
ω_s
2π
ω
(ω_s−ω_M)
(c)
X_p(e^jω)
1/N
ω_s
2π
ω
(d)

离散时间序列的恢复

才开始讲离散信号和连续一样



$$h[n]=\frac{N\omega_c}{\pi}\frac{\sin\omega_c n}{\omega_c n}\overset{\omega_c=\omega_s/2}{=}\frac{\sin\omega_c n}{\omega_c n}$$

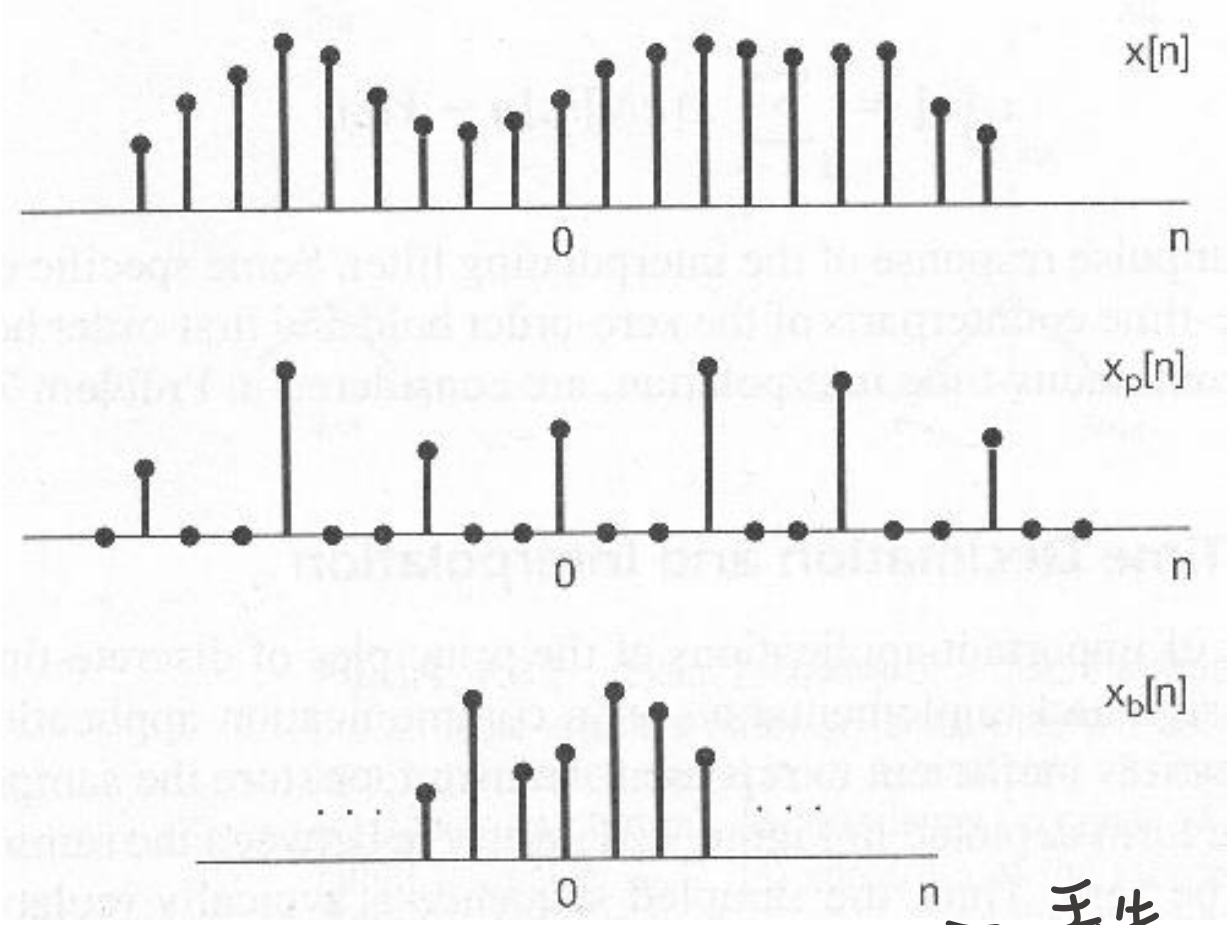
从而有：

$$x_r[n]=\sum_{k=-\infty}^{+\infty}x[kN]\frac{\sin\omega_c(n-kN)}{\omega_c(n-kN)}$$

内容提要

- 脉冲串采样
- 离散时间抽取与内插

离散时间抽取



丢失

$$x_b[n] = x[nN]$$

离散时间抽取过程的分析

$$x[n] \Rightarrow x_p[n]=\begin{cases} x[n], & n=kN \\ 0, & n\neq kN \end{cases} \Rightarrow x_b[n]=x_p[nN]$$

之前已经得到：

$$X_p(e^{j\omega})=\frac{1}{N}\sum_{k=-\infty}^{\infty}X\left(e^{j(\omega-k\frac{2\pi}{N})}\right)$$

而$x_b[n]$的傅里叶变换可以表示为：

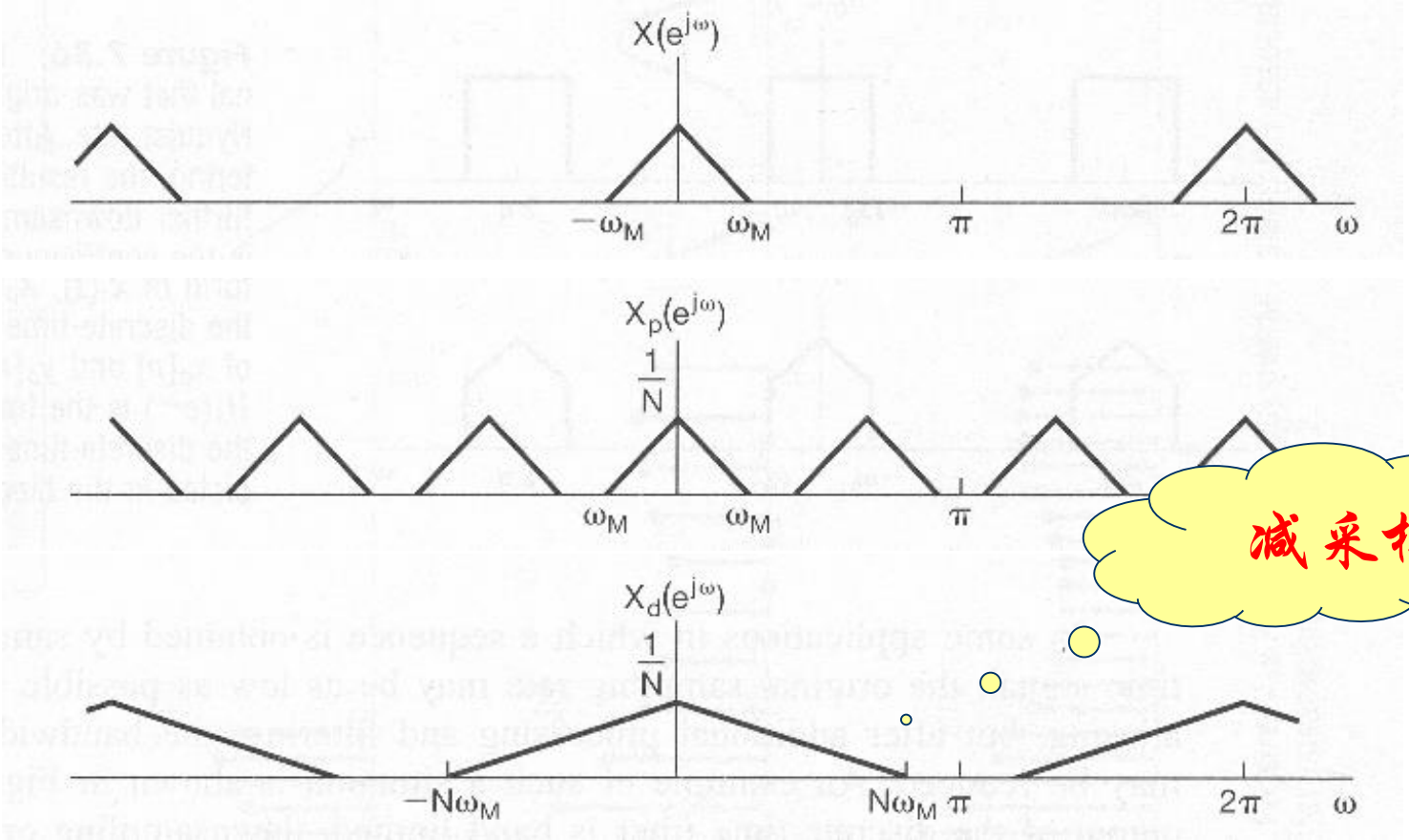
$$X_b(e^{j\omega})=\sum_{k=-\infty}^{\infty}x_b[k]e^{-jk\omega}=\sum_{k=-\infty}^{\infty}x_p[kN]e^{-jk\omega}$$

令$n=kN$，可得：

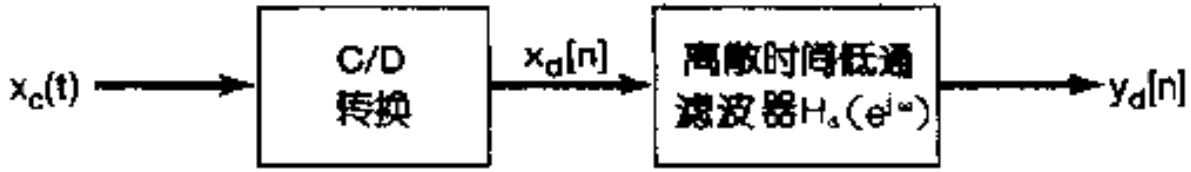
$$X_b(e^{j\omega})=\sum_{\substack{k=-\infty \\ n=kN}}^{\infty}x_p[n]e^{-j\omega n/N}=\sum_{n=-\infty}^{\infty}x_p[n]e^{-j\omega n/N}=X_p(e^{j\omega/N})$$

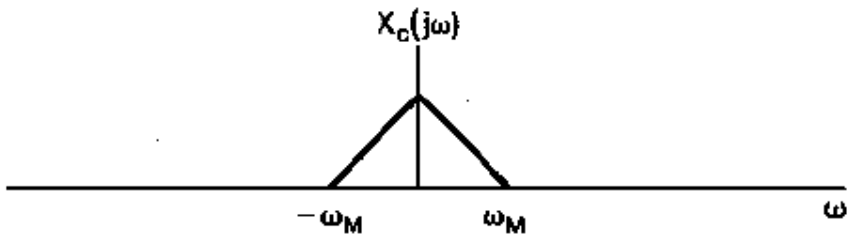
$$\Rightarrow X_b(e^{j\omega})=\frac{1}{N}\sum_{k=-\infty}^{\infty}X\left(e^{j(\frac{\omega}{N}-k\frac{2\pi}{N})}\right)$$

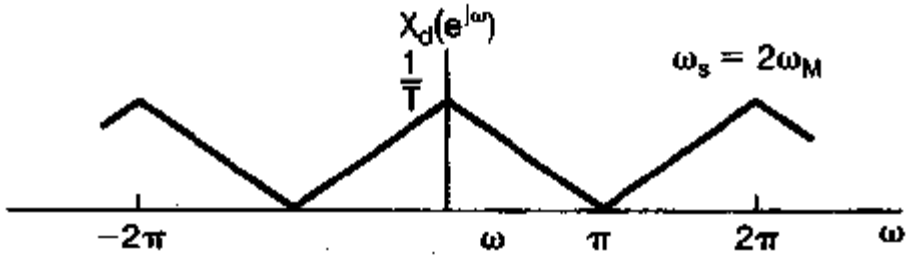
离散时间抽取过程的分析

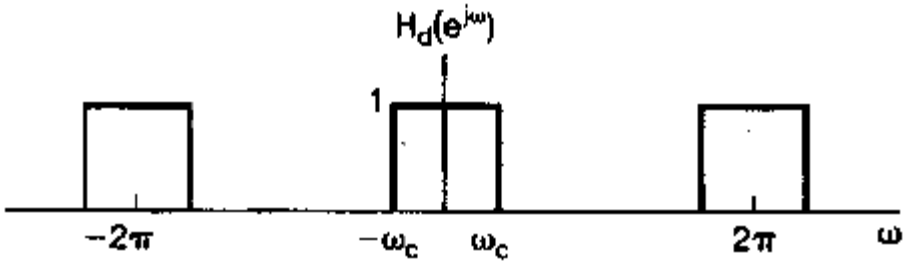


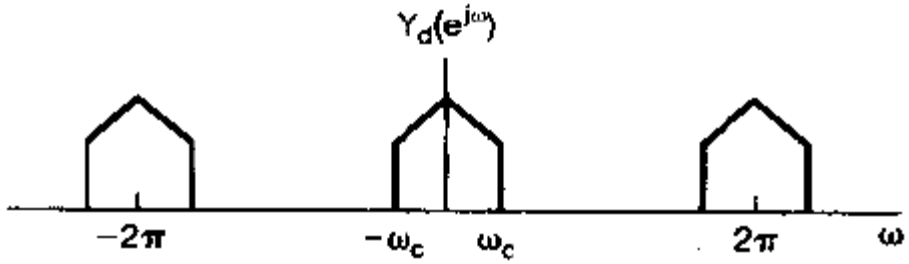
离散时间抽取过程的分析



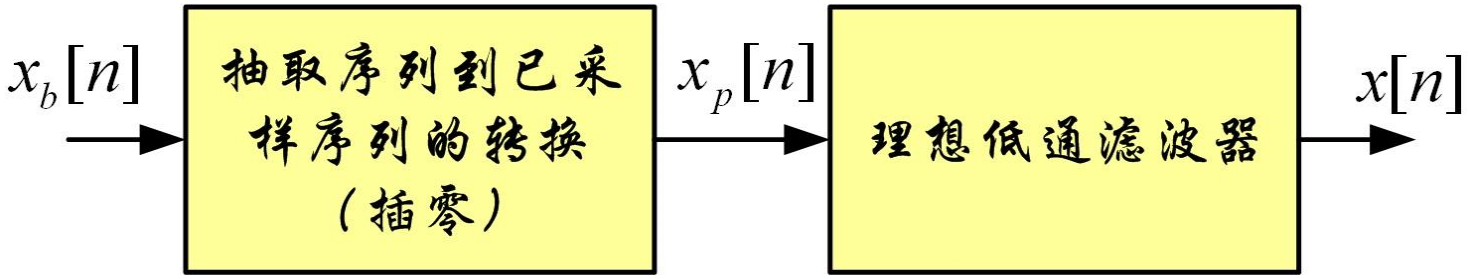


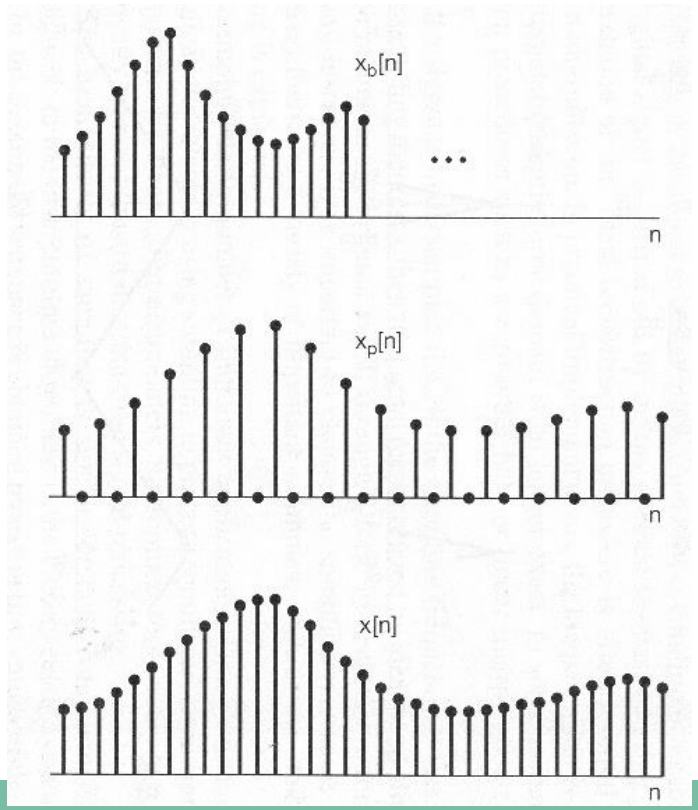


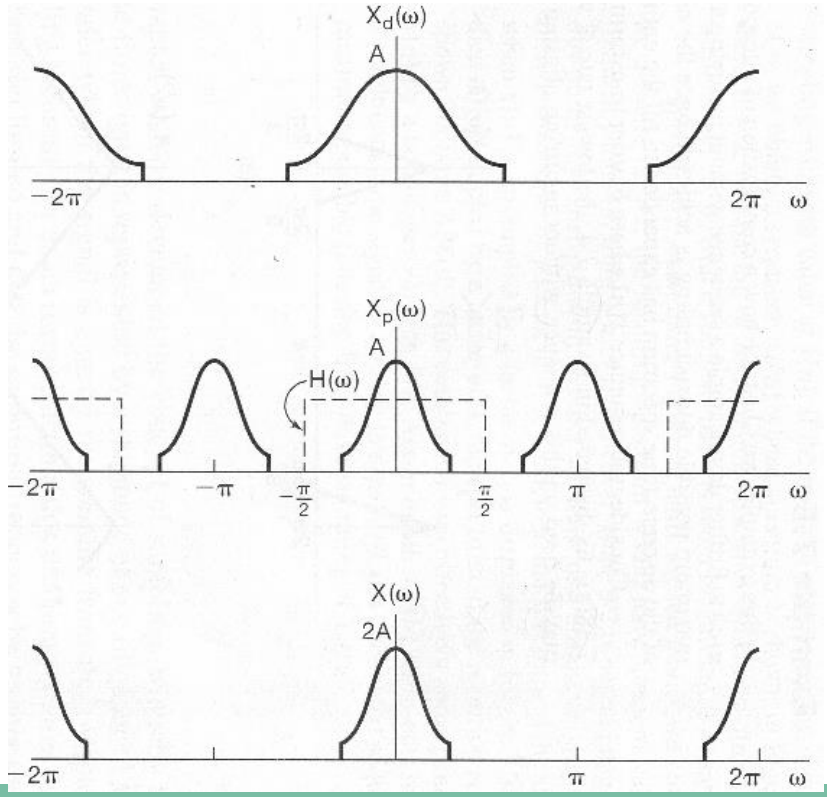




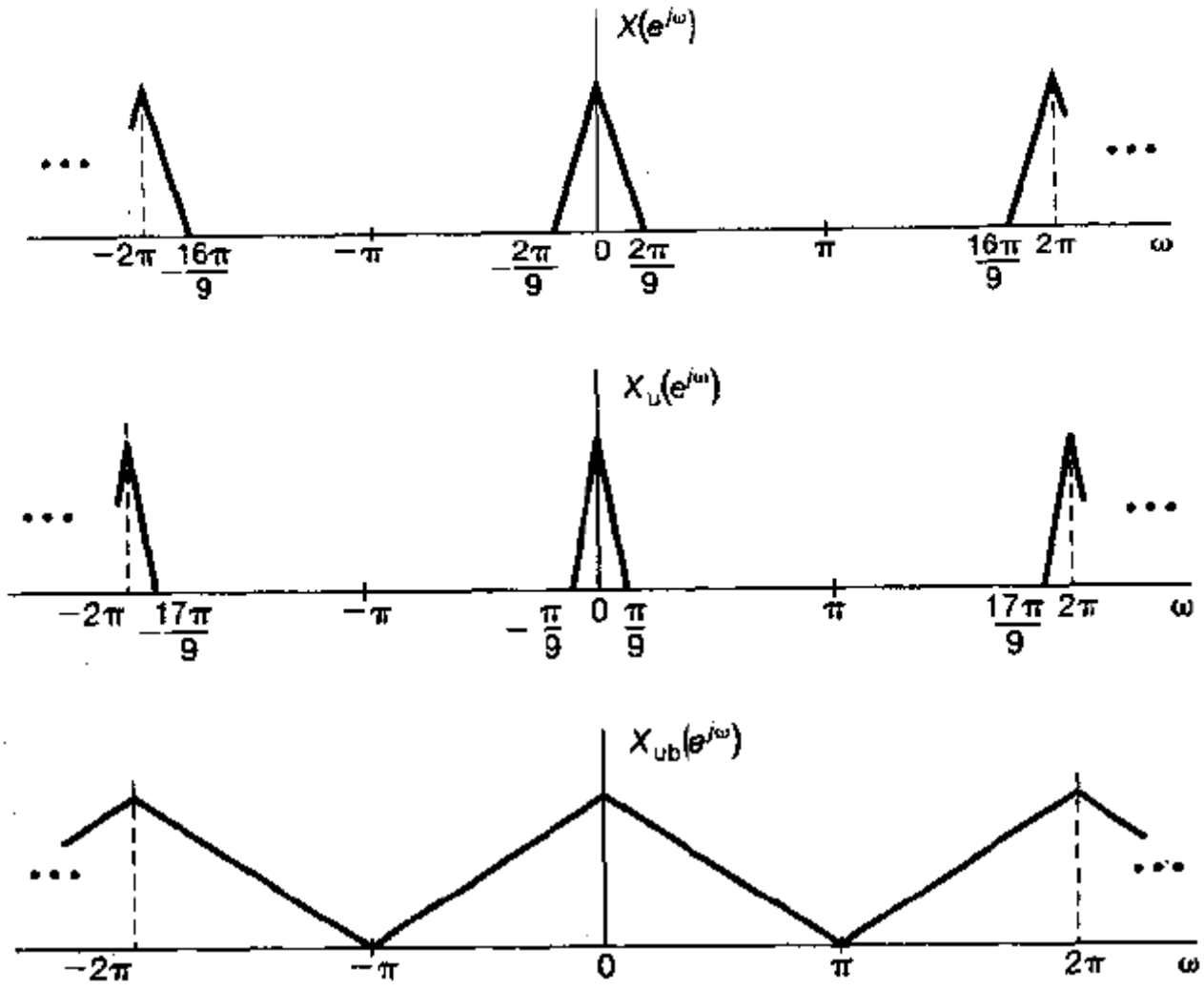
离散时间内插(增采样)


$x_b[n]$
抽取序列到已采样序列的转换（插零）
$x_p[n]$
理想低通滤波器
$x[n]$


$x_b[n]$
n
$x_p[n]$
n
x[n]
n


$X_d(\omega)$
A
-2π
2π
ω
$X_p(\omega)$
A
$H(\omega)$
-2π
$-\pi$
$-\frac{\pi}{2}$
$\frac{\pi}{2}$
π
2π
ω
$X(\omega)$
2A
-2π
π
2π
ω

离散时间抽取与内插举例



谢谢大家！