3.5 $x_2(t) = x_1(1-t) + x_1(t-1)$

由于尺度变换，翻转不影响频率 T 不变

$\therefore \omega_2 = \omega_1$ 由表 3.1 结论

$\because x_1(t) \overset{FS}{\longleftrightarrow} a_k.$

$x_1(t-1) \overset{FS}{\longleftrightarrow} a_k \cdot e^{-jk\omega_0 t_0}$

$x_1(-t) \overset{FS}{\longleftrightarrow} a_{-k}$

$x_1(1-t) = x_1(-(t-1)) \overset{FS}{\longleftrightarrow} a_{-k} e^{-jk\omega_0 t_0}$

$\therefore x_2(t) \longleftrightarrow a_{-k} e^{-jk\omega_0 t} + a_k e^{-jk\omega_0 t_0}$

$\therefore b_k = a_{-k} e^{-jk\omega_0 t} + a_k e^{-jk\omega_0 t_0}$

1. ⇒ $a_k$ 是虚奇的（书后答案）

3.8 ① $x(t)$ 为实信号，则 $x(t) = x^*(t)$

奇信号性质，又有 $x(t) = -x(-t)$

② 又由 $T = \frac{2\pi}{\omega_0} = 2$. $\therefore \omega_0 = \pi$

③ $|k|>1$ $a_k = 0$ 故其傅里叶级数只有

$a_{-1}$. $a_0$ $a_1$ 三项 且 $\begin{cases} x(t) \longleftrightarrow a_k \\ x(-t) \longleftrightarrow a_{-k} \end{cases}$

则 $a_1 + a_{-1} = 0$ $a_0 = 0$

④ 由帕塞瓦尔定理 $T=2$ $\boxed{\frac{1}{T}\int_{-\infty}^{+\infty} |x(t)|^2 dt = \sum a_k}$

则 $\frac{1}{2}\int_0^2 |x(t)|^2 dt^* = |a_1|^2 + |a_2|^2$ 因为 $a_k = \frac{1}{T}X(jk\omega_0)$

$= \frac{1}{T}X(jk\omega_0)$

$\therefore 2|a_1|^2 = 1 \quad \therefore |a_1| = \frac{1}{\sqrt{2}}$ $\frac{1}{2\pi}\int 1 \, , d\omega .$

由 $a_1 + a_{-1} = 0$. 可发现 $a_1, a_{-1}$ 为纯虚数

$\therefore a_1 = \frac{1}{\sqrt{2}}j$ 或 $-\frac{1}{2}j$ $a_{-1}$ 正好相反

$\therefore$ 两个信号为 $x_1(t) = a_1 e^{j\omega_0 kt} + a_1 e^{j\omega_0 kt}$

$= -\frac{\sqrt{2}}{2}j e^{j\omega_0(-1)t} + \frac{\sqrt{2}}{2}j e^{j\omega_0 t}$

或 $x_2(t) = \frac{\sqrt{2}}{2}j e^{j\omega_0 t} - \frac{\sqrt{2}}{2}j e^{-j\omega_0 t}$ .

共轭.

3.22 (a)

(1) 锯齿波信号. 基波周期 $T_0 = 2 \quad \omega = \pi$

在一个周期内. $a_k = \frac{1}{T}\int_T x(t) e^{-jk\omega t} dt$

选择在 $(-1,1)$ 内积分, $k \neq 0$ 时.

$$a_k = \frac{1}{2}\int_{-1}^{1} t e^{-jk\omega t} dt$$

$$= \frac{1}{-2jk\pi}\left(e^{-j\pi k} + e^{-j\pi k} + \frac{1}{jk\pi}\left(e^{-j\pi k} - e^{j\pi k}\right)\right)$$

$$= -\frac{1}{2jk\pi}\left[2\cos k\pi + \frac{1}{jk\pi}2j\sin(-k\pi)\right]$$

$$= \frac{-(-1)^k}{jk\pi} = \frac{(-1)^k}{k\pi} \quad (k \neq 0)$$

当$k=0$ $a_0=\frac{1}{2}\int_{-1}^{1}x(t)dt=\frac{1}{2}\int_{-1}^{1}tdt$

综上所述 $a_k=\begin{cases}\frac{(-1)^k}{k\pi} & k\neq 0\\ 0 & k=0\end{cases}$ $=\frac{1}{4}t^2\Big|_{-1}^{1}=0$

(2). $T=6$ $\omega_0=\frac{2\pi}{T}=\frac{1}{3}\pi$

设 $x(t)=\sum_{k=-\infty}^{+\infty}a_k e^{+jk\frac{1}{3}\pi t}$

$k=0$时 $a_0=\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}x(t)\,dt.$

直接计算梯形面积.

$a_0=\frac{1}{6}\frac{1}{2}(2+4)=\frac{1}{2}$

$k\neq 0$时

$a_k=\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}x(t)e^{-jk\frac{\pi}{3}t}dt$

$=\frac{1}{6}\int_{-2}^{-1}(t+2)e^{-jk\frac{\pi}{3}t}+\frac{1}{6}\int_{-1}^{1}e^{-jk\frac{\pi}{3}t}dt+\frac{1}{6}\int_{1}^{2}(2-t)e^{-jk\frac{\pi}{3}t}dt$

$=\frac{1}{6}\left[\frac{2e^{-jk\frac{\pi}{3}t}}{-jk\frac{\pi}{3}}\Big|_{-2}^{-1}+\frac{1}{-jk\frac{\pi}{3}}(te^{-jk\frac{\pi}{3}t}\Big|_{-2}^{-1}-\frac{e^{-jk\frac{\pi}{3}t}}{-jk\frac{\pi}{3}}\Big|_{-2}^{-1})\right]$

$+\frac{1}{6}\frac{1}{-jk\frac{\pi}{3}}e^{-jk\frac{\pi}{3}t}\Big|_{-1}^{1}+$

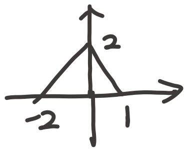
$\frac{1}{6}\left[\frac{2e^{-jk\frac{\pi}{3}t}}{-jk\frac{\pi}{3}}\Big|_{1}^{2}-\frac{1}{-jk\frac{\pi}{3}}te^{-jk\frac{\pi}{3}t}\Big|_{1}^{2}+\frac{1}{(-jk\frac{\pi}{3})^2}e^{-jk\frac{\pi}{3}t}\Big|_{1}^{2}\right]$

$$= \frac{\sin(\frac{2}{3}k\pi) - 2j\sin(\frac{2}{3}k\pi)}{3(jk\frac{\pi}{3})^2}$$

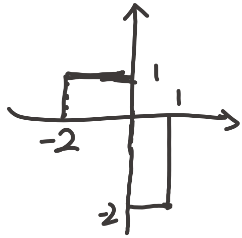
$$\therefore a_k = \begin{cases} \dfrac{\sin(\frac{2}{3}k\pi) - 2j\sin(\frac{2}{3}k\pi)}{3(jk\frac{\pi}{3})^2} & (k \neq 0) \\ \dfrac{1}{2} & (k = 0) \end{cases}$$

(3) $T=3$ $\omega = \frac{2}{3}\pi$ 对 $k \neq 0$.

周期 $\tilde{x}(t)$ 为




$\tilde{x}'(t) =$




对于门信号 $E_g(t) \xleftrightarrow{\mathcal{F}} \frac{2\sin\omega T}{\omega}$

$$\therefore j\omega \tilde{X}(j\omega) = \mathcal{F}\left[E_g(t) * \delta(t+1) - 2E_g(2t) * \delta(t-\tfrac{1}{2})\right]$$

$$= \frac{2\sin\omega}{\omega} \cdot e^{j\omega} - 2 \cdot \frac{2\sin\frac{\omega}{2}}{\omega} \cdot e^{-j\omega\frac{1}{2}}$$

$$\therefore \tilde{X}(j\omega) = \frac{2\sin\omega\, e^{j\omega} - 4\sin\frac{\omega}{2} e^{-\frac{1}{2}j\omega}}{j\omega^2}$$

$$\therefore a_k = \frac{1}{T} \tilde{X}(j\omega) \Big|_{\omega = k\frac{2\pi}{3}}$$

$$= \frac{1}{3} \frac{2\sin(\frac{2}{3}k\pi) e^{j\frac{2}{3}k\pi} - 4\sin\frac{k}{3}\pi\, e^{-\frac{1}{3}jk\pi}}{j(\frac{2}{3}\pi k)^2}$$

$k=0$. $a_0 = \frac{1}{3}\int_{-2}^{1} x(t)\,dt = \frac{1}{3}\ 6\ \frac{1}{2} = 1$

$$\therefore a_k = \begin{cases} 3\dfrac{2\sin(\frac{2}{3}k\pi)e^{j\frac{2}{3}k\pi} - 4\sin\frac{k}{3}\pi e^{-\frac{1}{3}jk\pi}}{j4\pi k^2} & (k \neq 0) \\ 1 & k=0 \end{cases}$$

(4) $T=2$. $w=\frac{2\pi}{T}=\pi$ 以冲激串形式表示

当$k\neq 0$时

$$x(t)=\sum_{k=-\infty}^{+\infty}\{\delta(t-k2)-2\delta[t-2k-1]\}$$

$$X(jw)=\mathcal{F}[x(t)]=\sum_{k=-\infty}^{+\infty}\left(e^{-2kjw}-2e^{-(2k-1)jw}\right)$$

取 $\hat{x}(t)$ 为周期内信号，取 $(-1,1]$.

$$\hat{X}(jw)=\frac{1}{2}\left(e^{-2\cdot 0jk}-2e^{-jk\pi}\right)$$

$$=\frac{1}{2}(1-2e^{-jk\pi})=\frac{1}{2}-e^{-jk\pi}$$

$$=\frac{1}{2}-(-1)^k$$

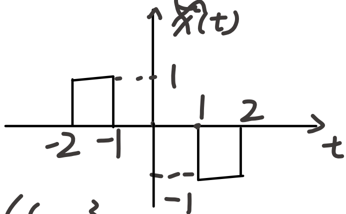
当$k=0$. $a_0=\frac{1}{2}\int_0^2 x(t)dt=\frac{1}{2}(1-2)=-\frac{1}{2}$

$$=\frac{1}{2}-(-1)^0$$

$\therefore a_k=\frac{1}{2}-(-1)^k$ (无论 $k$ 取值)

(5) 取 $\tilde{x}(t)$ 作为

单周期内信号 $\tilde{x}(t)$ 为




周期为 $T=6$ $\quad \tilde{x}'(t) = E_g(2t)*(\delta(t+\frac{3}{2})-\delta(t-\frac{3}{2}))$

$k=0$ 时

$$a_0 = \frac{1}{6}\int_{-3}^{3} x(t)\,dt = 0$$

$k\neq 0$ 时. $\tilde{x}(t)$ 求导 $\tilde{x}'(t)$ 如上所示. 为冲激序列

$$\because \delta(t-t_0) \overset{\mathcal{F}}{\longleftrightarrow} e^{-j\omega t_0}.$$

$$\hat{X}(j\omega) = \frac{2\sin\frac{\omega}{2}}{\omega}\left(e^{j\omega\frac{3}{2}} - e^{-j\omega\frac{3}{2}}\right)$$

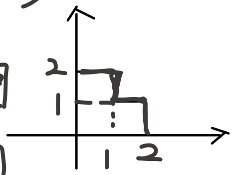
$$= \frac{2\sin\frac{\omega}{2}\cos\frac{3}{2}\omega}{\omega}$$

$$\therefore a_k = \frac{1}{6}\tilde{X}(j\cdot\frac{\pi}{3}k) = \frac{2\sin\frac{k\pi}{6}\cos\frac{k\pi}{2}}{k\pi} \quad k\neq 0$$

$$a_k = \begin{cases} 0 & (k=0) \\ \dfrac{2\sin\frac{k\pi}{6}\cos\frac{k\pi}{2}}{k\pi} & (k\neq 0) \end{cases}$$

$w-$

(6) $T=3$. 取 $\tilde{x}(t)$ 信号为右图




$$\tilde{x}(t) = E_g(2t)*(2\delta(t-\frac{1}{2}) - \delta(t-\frac{3}{2}))$$

$$\mathcal{F}[\tilde{x}(t)] = \frac{2\sin\frac{\omega}{2}}{\omega}\left(2e^{-\frac{1}{2}j\omega} - e^{-\frac{3}{2}j\omega}\right) = \tilde{X}(j\omega)$$

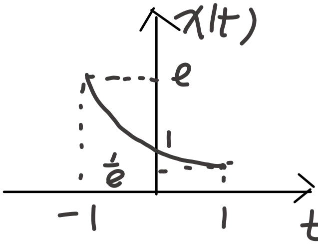
$\therefore a_k = \frac{1}{3}\tilde{X}(j\frac{2}{3}\pi) = \frac{\sin(\frac{k\pi}{3})[2e^{-j\frac{k\pi}{3}} + (-1)^k]}{k\pi}$

$a_0 = \frac{1}{3}\ 3 = 1$

$$a_k = \begin{cases} \frac{\sin(\frac{k\pi}{3})[2e^{-j\frac{k\pi}{3}} + (-1)^k]}{k\pi} & (k \neq 0) \\ 1 & k=0 \end{cases}$$

(b)(1) $T=2$, $w_0 = \pi$

记单周期信号 $\tilde{x}(t) = e^{-t}$
$(-1<t<1)$




$a_0 = \frac{1}{2}\int_{-1}^{1} e^{-t}dt = -\frac{1}{2}(e^{-t}\Big|_{-1}^{1}) = \frac{1}{2}(e - e^{-1})$

$a_k = \frac{1}{2}\int_{-1}^{1} e^{-t}\cdot e^{-jw_0kt}dt.$

$= \frac{1}{2}\int_{-1}^{1} e^{-(j\pi k+1)t}dt.$

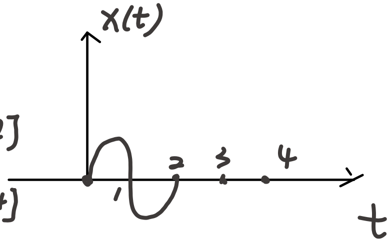
$= \frac{1}{2}\ -\left(\frac{1}{j\pi k+1}\right) e^{-(j\pi k+1)t}\Big|_{-1}^{1}$

$= \frac{1}{2(j\pi k+1)}\left(e^{+(j\pi k+1)} - e^{-(j\pi k+1)}\right)$

$$\therefore a_k = \begin{cases} \frac{1}{2}(e - \frac{1}{e}) & k = 0 \\ \frac{1}{2(j\pi k+1)}\left(e^{j\pi k+1} - e^{-(j\pi k+1)}\right). & k \neq 0 \end{cases}$$

(c) $T = 4 \quad \omega = \frac{1}{2}\pi$

$$\therefore \tilde{x}(t) = \begin{cases} \sin \pi t & t \in [0.2] \\ 0 & t \in [2.4] \end{cases}$$




$\therefore a_0 = 0$

$$a_k = \frac{1}{4}\int_0^4 \sin \pi t\, e^{-jk\pi t} dt.$$

对于 $4a_k = I = \int_0^4 \sin \pi t\, e^{-jk\pi t} dt$

$$= \sin \pi t \frac{e^{-jk\pi t}}{-jk\pi}\Big|_0^4 - \frac{\pi \cos \pi t\, e^{-jk\pi t}\Big|_0^4}{(jk\pi)^2} + \frac{-\pi^2 I}{(jk\pi)^2}$$

$$\therefore \left[1 + \frac{-\pi^2}{(jk\pi)^2}\right] I = 0 - \frac{\pi e^{-jk4\pi} - \pi e^{-jk\pi}}{(jk\pi)^2} \quad {}^{-jk}$$

$$\therefore a_k = \frac{\frac{\pi}{(jk\pi)^2}\left(e^{-jk\pi} - e^{-jk4\pi}\right)}{4(1 + \frac{1}{k^2})}$$

$$a_k = \begin{cases} 0 & k = 0 \\ \frac{\frac{\pi}{(jk\pi)^2}\left(e^{-jk\pi} - e^{-jk4\pi}\right)}{4(1 + \frac{1}{k^2})} & k \neq 0 \end{cases}$$

3.34 (b) $x(t)=\sum\limits_{k=-\infty}^{+\infty}(-1)^k\delta(t-k)$

$$=\sum_{k=-\infty}^{+\infty}\delta(t-2k)+\sum_{k=-\infty}^{+\infty}(-1)\cdot\delta(t-2k-1)$$

$a_k$

其中2个序列 $T=2$ $\therefore \omega_0=\pi$

$$\mathcal{F}\Big[\sum_{k=-\infty}^{+\infty}\delta(t-2k)\Big]=\frac{1}{2}\,1=\frac{1}{2}$$

$$\mathcal{F}\Big[\sum_{k=-\infty}^{+\infty}-\delta(t-2k-1)\Big]=-\frac{1}{2}\cdot e^{-jk\pi}$$

$$a_k=\frac{X(j\omega)}{T}\Big|_{k\omega_0}=\frac{1}{2}(1-e^{-jk\pi})=\frac{1}{2}(1-(-1)^k)$$

又对 $y(t)$ 级数. 设系数为 $b_k$.

$$y(t)=\sum_{k=-\infty}^{+\infty}b_k e^{-jk\omega_0 t}$$ 在变换域内

$$b_k=a_k\cdot\mathcal{F}[e^{-4|t|}]$$

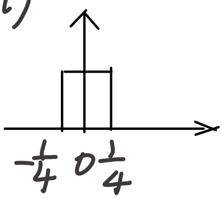
$$=\frac{1-(-1)^k}{2}\,\frac{8}{k^2\pi^2+16}$$

$$\therefore b_k=\begin{cases}\dfrac{8}{k^2\pi^2+16} & k\text{奇}\\[2mm] 0 & k\text{为偶}\end{cases}$$

(c). $T=1$. 取一个周期信号 $\tilde{x}(t)$

$\omega_0=2\pi$

$$\mathcal{F}[\tilde{x}(t)]=\frac{2\sin(\frac{\omega}{4})}{\omega}=\tilde{X}(j\omega).$$



$$x(t)=\sum_{k=-\infty}^{+\infty}(-1)^n\delta(t-n)$$ 冲激串间con

$$x(t)=\sum_{k=-\infty}^{+\infty}\delta(t-2k)+(-1)\sum_{k=-\infty}^{+\infty}\delta(t-2k-1)$$

$\downarrow$

$$a_k=\frac{1}{2}\cdot 1+(-1)\cdot\frac{1}{2}\cdot e^{j(2k+1)\pi}$$

$$=\frac{1}{2}\left[1+(-1)\cdot(-1)^{k}\right]$$

$jk\pi$

$\cos(k\pi)$

$(-1)^k$

$\because a_k = \frac{1}{T} X(j\omega)\Big|_{\omega = k\omega_0}^{k2\pi} = \frac{\sin(\frac{1}{2}k\pi)}{k\pi}$ 包络取样.

$\because b_k = a_k\ \mathcal{F}[e^{-4|t|}] = \frac{\sin\frac{k\pi}{2}}{k\pi}\ \frac{8}{4k^2\pi^2+16}$

$= \frac{\sin\frac{k\pi}{2}}{k\pi}\ \frac{2}{4+k^2\pi^2}$

$\mathcal{F}[b_k]$ 

$x(t) * h(t) = H(j\omega)\ \cancel{X(j\omega)}\ a_k.$

$a_k = \frac{b_k}{\boxed{H(j\omega)}}$ ✓.

3.43 (a) (i) $x(t)$ 若奇谐

则 $x(t) = \sum_{k=-\infty}^{+\infty} a_n e^{j(n)(2\pi/T)t}$. 其中 $n = 2k+1$ $(k \in \mathbb{Z})$

$\because x(t)$ 的 Fourier 级数系数 $a_k$.

对于 $-x(t+\frac{T}{2})$ 根据表3.1

其 Fourier 级数系数 $b_k = -a_k e^{+j\omega_0 k t_0}$

$b_k = -a_k e^{jk\pi}$

由于 $k$ 为奇数.

$\therefore e^{jk\pi} \equiv -1$. $\therefore b_k = a_k$

$\therefore x(t) = -x(t+\frac{T}{2})$ 得证.

(ii) 若已知 $x(t) = -x(t+\frac{T}{2})$

则其 Fourier 级数系数均为 $a_k$.

将 $-x(t+\frac{T}{2})$ 还原.

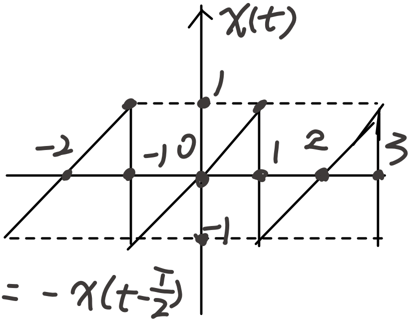
$$a_k \xrightarrow{\text{时移}\frac{T}{2}} a_k \cdot e^{-jk\frac{T}{2}\frac{2\pi}{T}} \xrightarrow{\text{反转}} -a_k e^{jk\pi}$$

要使 $a_k = -a_k e^{jk\pi}$ 则 $e^{jk\pi} = -1$.

则 $k = 2n+1$ $(n \in Z)$. ∴ $k$ 为奇数.

即级数中仅有奇数次谐波存在

∴ $x(t)$ 奇谐时



(b) 由 (a) 中奇谐信号结论

$x(t) = -x(t+\frac{T}{2})$ 可还原 $0<t<1$ 时另半边

信号 $x(t) = -x(t+\frac{T}{2}) = -x(t-\frac{T}{2})$

即 $x(t) = -x(t+1) = -x(t-1)$ 图像如上所示

记 $x(t)$ 一个周期内信号为 $\tilde{x}(t)$ $\omega_0 = \pi$

可知 $\tilde{x}(t) = t$ $(-1<t<1)$

记 $x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$

$$a_0 = \int_{-1}^{1} x(t)dt = 0.$$

$$a_k = \frac{1}{2}\int_{-1}^{1} x(t) e^{-jk\omega_0 t} dt$$ 通式

$$= \frac{1}{2}\int_{-1}^{1} t e^{-jk\pi t} dt =$$

$$\frac{1}{2}\left[\frac{te^{-jk\pi t}}{-jk\pi}\Big|_{-1}^{1} - \frac{e^{-jk\pi t}}{(-jk\pi)^2}\Big|_{-1}^{1}\right]$$

$$= \frac{1}{2}\left[\frac{e^{-jk\pi}+e^{+jk\pi}}{-jk\pi} - \frac{e^{-jk\pi}-e^{jk\pi}}{-k^2\pi^2}\right].$$

$$= \frac{1}{2}\left[\frac{2\cos(k\pi)}{-jk\pi} + \frac{-2j\sin(k\pi)}{k^2\pi^2}\right].$$

$$= j\frac{\cos k\pi}{k\pi} = \frac{(-1)^k\cdot j}{k\pi}.$$

$$\therefore a_k = \begin{cases} 0 & k=0 \\ \frac{(-1)^k j}{k\pi} & k\neq 0. \end{cases}$$ ✓

3.45 (a). $\because Ev\{x(t)\} = \frac{x(t)+x(-t)}{2}$

$$Od\{x(t)\} = \frac{x(t)-x(-t)}{2}.$$ 凑组合

$$\therefore Ev\{x(t)\} = \frac{2a_0}{2} + 2\sum_{k=1}^{\infty} B_k \underbrace{\cos(k\omega_0 t)}_{\frac{e^{jk\omega_0 t}+e^{-jk\omega_0 t}}{2}}$$ 展开

$\frac{1}{2}B_k$

$$Od\{x(t)\} = \sum_{k=1}^{+\infty}(-2)C_k \sin(k\omega_0 t)$$ $a_0$消了.

$$\therefore a_0 = a_0 \quad a_k = a_{-k} = B_k \quad (k>0).$$

$\because Od\{x(t)\} = -2\sum_{k=1}^{+\infty} C_k \cdot \frac{e^{jkw_0} - e^{-jkw_0}}{2j}$

$$- \sum_{k=-\infty}^{+\infty} \beta_k \cdot e^{jkw_0 t}$$

$$\beta_0 = 0 \quad \beta_k = -C_k j \quad (k>0)$$

(b) $a_k = a_{-k} \quad \beta_k = -\beta_{-k}$

(c) $x(t) = a_0 + 2\sum_{k=1}^{+\infty}\left(B_k \cos\frac{2\pi kt}{3} - C_k \sin\frac{2\pi kt}{3}\right)$

$z(t) = d_0 + 2\sum_{k=1}^{+\infty}\left(E_k \cos\frac{2\pi kt}{3} - F_k \sin\frac{2\pi kt}{3}\right)$

其中 $x(t)$ 周期为 $T_x=3$. $z(t)$ 则 $T_z=3$, -

~~#~~次谐波

观察 $y(t)$ 发现不存在 $x(t)$ 的奇分量.

$y(t)$ 奇分量为 $F_k$. 偶分量为系数 $B_k + \frac{1}{2}E_k$

系数

$\therefore y(t) = x(t) + \frac{1}{2}Ev(z(t)) - Od(z(t))$

$$= x(t) + \frac{1}{2}z(+t) + 3a_0 + \frac{Z}{2}d_0$$

$\because a_0 = \frac{1}{3}\int_T x(t)dt = \frac{1}{3}$

$d_0 = \frac{1}{3}\int_T z(t)dt = 0.$

$\because y(t) = x(t) + \frac{1}{2}z(1+t) + 1$

图象如下