4.11 $\because x(3t) \xrightarrow{\mathcal{F}} \frac{1}{3}F(j\frac{w}{3})$

$h(3t) \xrightarrow{\mathcal{F}} \frac{1}{3}H(j\frac{w}{3})$.

$\therefore G(jw) = \mathcal{F}[x(3t)] \cdot \mathcal{F}[h(3t)]$

$= \frac{1}{9}F(j\frac{w}{3})H(j\frac{w}{3})$.

$Y(jw) = 1\, F(jw)\, H(jw)$

记 $F(j\frac{w}{3})H(j\frac{w}{3}) = L(jw)$.

$\therefore G(jw) = \frac{1}{9}L(jw)$

$Y(jw) = L(j3w)$

$\therefore g(t) = \frac{1}{9}l(t)$.

$y(t) = \frac{1}{3}\ L(\frac{1}{3}t)$, $y(3t) = \frac{1}{3}l(t)$.

$= 3g(t)$

$\therefore g(t) = \frac{1}{3}y(3t)$. ✓

4 22 (a) 由常见 Fourier 变换对

$$X(jw) = \frac{2\sin[3(w-2\pi)]}{(w-2\pi)} \xrightarrow{\mathcal{F}^{-1}} \begin{cases} 1 & |t|<3 \\ 0 & 其他 \end{cases} \cdot e^{-j2\pi t}$$

$x(t) = e^{-j2\pi t}[u(t+3) - u(t-3)]$

$$g(t) = x(3t) * h(3t).$$

$$G(j\omega) = \frac{1}{3} X(j\tfrac{\omega}{3}) \cdot \frac{1}{3} H(j\tfrac{\omega}{3})$$

$$= \frac{1}{9} X(j\tfrac{\omega}{3}) H(j\tfrac{\omega}{3}) = \frac{1}{9} Y(j\tfrac{\omega}{3})$$

$$Y(j\omega) = X(j\omega) H(j\omega). \quad \frac{1}{3} y(3t)$$

$$=$$

(c) $X(j\omega) = |X(j\omega)| e^{j\measuredangle X(j\omega)}$

$$= |X(j\omega)| e^{j(-3\omega)}$$

$$\therefore x(t) = \mathcal{F}^{-1}[X(j\omega)] = \frac{1}{2\pi}\int_{-1}^{1} |\omega| e^{-j(3\omega)} e^{+j\omega t} d\omega$$

$$= \frac{1}{2\pi}\int_{-1}^{0} -\omega e^{-j3\omega} e^{+j\omega t} d\omega + \frac{1}{2\pi}\int_{0}^{1} \omega e^{-j3\omega} e^{+j\omega t} d\omega$$

$$= \frac{1}{2\pi}\left\{ -\frac{e^{-j(t-3)}}{j(t-3)} - \frac{1}{(t-3)^2}\left[1-e^{-j(t-3)}\right] + \frac{e^{j(t-3)}}{j(t-3)} - \frac{1}{(t-3)^2}\left[1-e^{j(t-3)}\right] \right\}$$

$$= \frac{1}{\pi}\left\{ \frac{e^{j(t-3)} - e^{-j(t-3)}}{2j(t-3)} + \frac{-2+e^{j(t-3)}+e^{-j(t-3)}}{2(t-3)^2} \right\}$$

$$= \frac{1}{\pi}\frac{\sin(t-3)}{t-3} + \frac{1}{\pi}\frac{\cos(t-3)-1}{(t-3)^2}$$

(e) (b)图为一个关于ω的奇函数

$$x(t) = \mathcal{F}^{-1}[X(j\omega)] = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega) e^{+j\omega t} d\omega$$ (逆变换)

$$= \frac{1}{2\pi}\left\{\int_{-3}^{-2} -e^{+j\omega t} d\omega + \int_{-2}^{-1} (\omega+1) e^{j\omega t} dt + \int_{1}^{2} (\omega-1) e^{j\omega t} d\omega + \int_{2}^{3} e^{j\omega t} d\omega \right\}$$

$$= \frac{1}{2\pi}\left\{ \frac{1}{jt}\left(e^{-j3t}-e^{-j2t}\right) + \frac{e^{-j2t}}{jt} + \frac{1}{t^2}\left(e^{-jt}-e^{-j2t}\right) + \frac{e^{j2t}}{jt} + \frac{1}{t^2}\left(e^{j2t}-e^{jt}\right) + \frac{1}{jt}\left(e^{j3t}-e^{j2t}\right) \right\}$$

$= \frac{\cos 3t}{j\pi t} + \frac{\sin t - \sin 2t}{j\pi t^2}$ （欧拉公式）

4.31 (a) (1) $x(t) = \frac{e^{jt} + e^{-jt}}{2}$ 其中 $w_{01} = 1$. $w_{02} = -1$

对输入 $e^{jw_0t} \xrightarrow{h(t)} H(jw_0)e^{jw_0t}$

对 $h_1(t)$. $y_1(t) = x(t) * h_1(t)$ $H_1(jw) = \pi\delta(w) + \frac{1}{jw}$

$= \frac{1}{2}e^{jt} \cdot H(+j) + \frac{1}{2}e^{-jt} H(-j)$ $H_1(j1) = \frac{1}{j}$

$= \frac{1}{2} e^{jt} \cdot (\frac{1}{j}) + \frac{1}{2} e^{-jt} \frac{1}{-j}$ $H_2(j\cdot -1) = \frac{1}{-j} = j$

$= \frac{e^{jt}}{2j} + \frac{1}{2} j e^{-jt}$

(2). $H_2(jw) = -2 + 5 \frac{1}{jw+2}$

$H_2(j1) = -2 + 5\frac{1}{j+2} = -j = \frac{1}{j}$

$H_2(j(-1)) = -2 + 5\frac{1}{2-j} = j$

$\because y_2(t) = x(t) * h_2(t)$

$= \frac{e^{jt}}{2j} + \frac{1}{2} j e^{-jt}$

(3) $\because$ 对偏微分性质 $X(jw) = \int_{-\infty}^{+\infty} x(t)e^{-jwt}dt$

$\frac{-jt}{\cancel{x}} x(t) \xrightarrow{\mathcal{F}} j\frac{dX(jw)}{dw} \cdot -j$ $= -jt$

令模板中 $x(t) = e^{-t}u(t)$

$$\frac{t}{j} e^{-t}u(t) \xrightarrow{\mathcal{F}} \left(\frac{1}{j\omega+1}\right)' = -\frac{j}{(j\omega+1)^2}$$

$\because h_3(t) = 2te^{-t}u(t) = 2j\,\frac{t}{j}e^{-t}u(t)$

$\therefore H_3(j\omega) = 2j \cdot \frac{-j}{(j\omega+1)^2}$ $\quad H_3(j1) = \frac{2}{(1+j)^2} = \frac{1}{j}$

$= \frac{2}{(j\omega+1)^2}$ $\quad H_3(-j1) = \frac{2}{(1-j)^2} = j$

$\therefore y_3(t) = \frac{e^{jt}}{2j} + \frac{1}{2}je^{-jt} = y_2(t) = y_1(t)$

则响应全一样

(b) 只需找到 $h_4(t)$ 使 $H_4(j\omega)$ 有 $H_4(j1) = \frac{1}{j}$, $H_4(j\cdot(-1)) = j$

不妨令 $H_4(j\omega) = \frac{2}{j\omega} \cdot \frac{1}{2} = \frac{1}{2}\mathcal{F}[sgn(t)]$

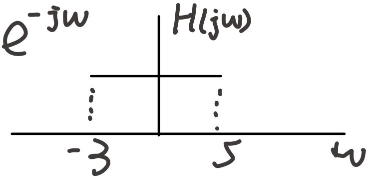
直接记

选取 $h_4(t) = \frac{1}{2}sgn(t)$ 也是同样响应

4.32 对 $h(t) = \frac{\sin 4(t-1)}{\pi(t-1)} = \frac{\sin 4t}{\pi t} * \delta(t-1)$

根据变换对 $\frac{\sin Wt}{\pi t} \xleftrightarrow{\mathcal{F}} \begin{cases} 1 & |\omega| < W \\ 0 & |\omega| > W \end{cases}$

$\therefore H(j\omega) = \begin{cases} 1 & |\omega| < 4 \\ 0 & 其他 \end{cases} \cdot e^{-j\omega}$



$H(j\omega)$相当于低通滤波器

(a) $x_1(t) = \cos(6t + \frac{\pi}{2}) = \cos[6(t+\frac{\pi}{12})]$

$= \cos 6t * \delta(t+\frac{\pi}{12})$

$X_1(j\omega) = \pi[\delta(\omega-6) + \delta(\omega+6)] \cdot e^{j\omega\frac{\pi}{12}}$

由于相乘性 $\omega = \pm 6$ 处 $H(j\omega)$ 已低通滤过

$\therefore Y_1(j\omega) = 0 \quad y_1(t) = 0$,

(b) $x_2(t) = \sum_{k=0}^{\infty} (\frac{1}{2})^k \sin(3kt)$

$\sin 3kt \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{j}\pi[\delta(\omega-3k) - \delta(\omega+3k)]$

$\therefore X_2(j\omega) = \sum_{k=0}^{\infty} (\frac{1}{2})^k \frac{\pi}{j}[\delta(\omega-3k) - \delta(\omega+3k)]$

$\because H(j\omega)$ 将滤除除 $\omega \in [-4,4]$ 外的谐波.

$k=0$. $\sin 3kt = 0$ 注意 $\mathcal{F}(\sin\omega_0 t) = \frac{\pi}{j}($ 有 j. $-\omega_0$ $+\omega_0$ 1 1 $)$

$k=1$ $\delta(\omega-3)$ 与 $\delta(\omega+3)$ $\omega_1 = 3$, $\omega_2 = -3$

在 $[-4,4]$ 内 有取值

但 $k=2$ 之后 (包含 $k=2$) 的频谱滤去

$\therefore H(j\omega)X_2(j\omega) = \frac{1}{j2}\pi[\delta(\omega-3) - \delta(\omega+3)]\ e^{-j\omega}$

$\therefore y_2(t) = \frac{1}{2j}\sin 3t \quad \delta(t-1)$

$= \frac{1}{2j}\sin[3(t-1)]$

(c). $x_3(t) = \frac{\sin[4(t+1)]}{\pi(t+1)} = \frac{\sin 4t}{\pi t} * \delta(t+1)$

$X_3(j\omega) = \begin{cases} 1 & |\omega| < 4 \\ 0 & \text{其他} \end{cases} \cdot e^{j\omega 1}$ 卷积性质

$$H(j\omega) = \begin{cases} 1 & |\omega|<4 \\ 0 & \text{其他} \end{cases} e^{-j\omega}$$

$$Y_3(j\omega) = X_3(j\omega)\cdot H(j\omega) = \begin{cases} 1 & |\omega|<4 \\ 0 & \text{其他} \end{cases} \quad 1$$

$$\therefore y_3(t) = \mathcal{F}^{-1}[Y_3(j\omega)] = \frac{\sin 4t}{\pi t}.$$

(d) $$x_4(t) = \left(\frac{\sin 2t}{\pi t}\right)^2 = \frac{\sin 2t}{\pi t} \quad \frac{\sin 2t}{\pi t}.$$

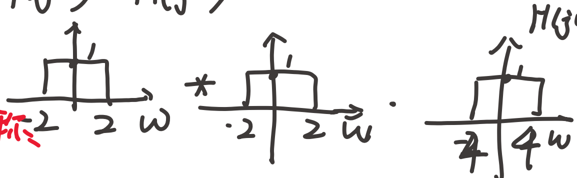
$$\therefore X_4(j\omega) = \frac{1}{2\pi} \mathcal{F}\left(\frac{\sin 2t}{\pi t}\right) * \mathcal{F}\left(\frac{\sin 2t}{\pi t}\right)$$

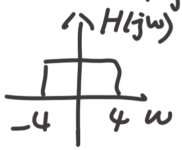
时乘 => 频卷 } 对偶
频乘 => 时卷 }

$$= \frac{1}{2\pi} \begin{cases} 1 & |\omega|<2 \\ 0 & |\omega|>2 \end{cases} * \begin{cases} 1 & |\omega|<2 \\ 0 & |\omega|>2 \end{cases}$$

$$\therefore Y_4(j\omega) = X_4(j\omega)\cdot H(j\omega)$$

门卷积：如门宽一样 中轴对称 则可以为三角信号.

$= \frac{1}{2\pi}$  $e^{-j\omega}$

$$= \frac{1}{2\pi} \begin{cases} (\omega+4) & -4<\omega<0 \\ -\omega+4 & 0<\omega<4 \end{cases}$$  $e^{-j\omega}$

H(jw)相乘无效果

$$= 2\pi \begin{cases} \omega+4 & -4<\omega<0 \\ -\omega+4 & 0<\omega<4 \end{cases} e^{-j\omega}$$

$$= X_4(j\omega)e^{-j\omega}$$

$$\therefore y(t) = x_4(t) * \delta(t-1) = \left\{\frac{[\sin 2(t-1)]}{\pi(t-1)}\right\}^2$$