推导：从 $f(t)=a_0+\sum_{n=1}^{\infty}(a_n\cos n\omega_1 x+b_n\sin n\omega_1 x)$

因为 $C_n\cos(n\omega_1 x+\varphi_n)$ 若 $\varphi_n=0$

$$=C_n(\cos n\omega_1 x\cdot\cos\varphi_n-\sin n\omega_1 x\cdot\sin\varphi_n)$$

$$\begin{cases}C_n\cos\varphi_n=a_n.\\-C_n\sin\varphi_n=b_n\end{cases}\quad\therefore a_n^2+b_n^2=C_n^2.$$

令 $\cos n\omega_1 x=\frac{1}{2}(e^{jn\omega_1 x}+e^{-jn\omega_1 x})$

$$\sin n\omega_1 x=\frac{1}{2j}(e^{jn\omega_1 x}-e^{-jn\omega_1 x})$$

$$\therefore f(t)=a_0+\sum_{n=1}^{\infty}a_n\frac{1}{2}(e^{jn\omega_1 x}+e^{-jn\omega_1 x}+\frac{1}{j}e^{jn\omega_1 x}-\frac{1}{j}e^{-jn\omega_1 x})$$

$$=a_0+\sum_{n=1}^{\infty}\frac{a_n-jb_n}{2}e^{jn\omega_1 x}+\frac{a_n+jb_n}{2}e^{-jn\omega_1 x}$$

只有复指数可以展 孩子，e^{-t} 就别拿出来给人笑话了.

其实部(幅值是实偶)、

$$\therefore F(n\omega_1)=\frac{a_n-jb_n}{2}.\quad F(-n\omega_1)=\frac{a_n-jb_n}{2}$$

令 $a_n=F(n\omega_1)$

$$\therefore f(t)=\sum_{-\infty}^{\infty}a_n e^{jn\omega_1 t}$$

$$\frac{a_n-jb_n}{2}=\frac{2}{2T}\int_{-\frac{T}{2}}^{\frac{T}{2}}f(t)\cos n\omega_1 t\,dt+\frac{2}{2jT}\int_{-\frac{T}{2}}^{\frac{T}{2}}f(t)\sin n\omega_1 t\,dt.$$

$$=\frac{1}{T}\int_T f(t)(\cos n\omega_1 t-j\sin n\omega_1 t)\,dt$$

$$\Rightarrow a_n=\frac{1}{T}\int_T f(t)e^{-jn\omega_1 t}dt.$$ 系数由来

$T \to \infty \quad \frac{1}{T} = \frac{\Delta\omega}{2\pi} \to 0 \quad ; \quad n\omega_1 \to \omega$ 连续

$$\Rightarrow a_n T = \int_{-\infty}^{+\infty} f(t) e^{-j\omega t} dt = F(j\omega)$$

对于 $f(t) = \sum_{-\infty}^{+\infty} a_n e^{+jn\omega_0 t}$

$T \to \infty$ 时 $\quad a_n = \lim_{T \to \infty} \frac{1}{T} \int_{-\infty}^{+\infty} f(t) e^{-j\omega t} dt$

$$f(t) = \lim_{\omega \to 0} \sum_{-\infty}^{+\infty} \frac{\Delta\omega}{2\pi} \int_{-\infty}^{+\infty} f(t) e^{-j\omega t} dt \; e^{+jn\omega_0 t}$$

$$= \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(j\omega) \cdot e^{+j\omega t} d\omega.$$

$$= \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(j\omega) e^{+j\omega t} d\omega$$

有理分式的FT 靠对偶性了

$\frac{1}{t} \overset{\mathcal{F}}{\longleftrightarrow} ?$

$\because \ \mathrm{Sgn}(t) \overset{\mathcal{F}}{\longleftrightarrow} \frac{2}{j\omega}$ 对偶

$x(t) \overset{\mathcal{F}}{\longleftrightarrow} X(j\omega)$

$X(t) \overset{\mathcal{F}}{\longleftrightarrow} 2\pi x(-\omega)$

$$\frac{2}{jt} \overset{\mathcal{F}}{\longleftrightarrow} 2\pi \,\mathrm{Sgn}(-\omega)$$

$$\frac{1}{t} \overset{\mathcal{F}}{\longleftrightarrow} \frac{\pi}{j} \mathrm{sgn}(-\omega)$$

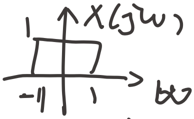
$$\int_{-\infty}^{+\infty} \frac{1}{(a^2+\omega^2)^2} d\omega = \frac{\pi}{2a^3}$$ 虽回到信数，但有专属信号的处理：

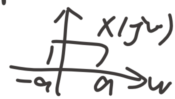
$$e^{-|\alpha|t} \overset{\mathcal{F}}{\longleftrightarrow} \frac{2a}{\omega^2+a^2}$$

$$\int_{-\infty}^{+\infty} (e^{-|\alpha|t})^2 dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{4a^2}{(\omega^2+a^2)^2} d\omega$$

$$\frac{1}{a} = \frac{4a^2}{2\pi}\int_{-\infty}^{+\infty} \frac{1}{(\omega^2+a^2)^2} d\omega$$

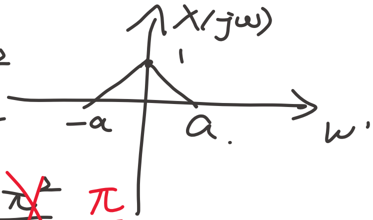
$$f(t) \sum_{n} \delta(t-nT) =$$

3-2-1 $\quad Sa(t) = \pi \dfrac{\sin t}{\pi t} \xrightarrow{\mathcal{F}} \pi$ 

$$Sa(at) = \frac{\sin at}{at} = \frac{\pi}{a}\,\frac{\sin at}{\pi t} \xrightarrow{\mathcal{F}} \frac{\pi}{a}$$ 

$$\int_{-\infty}^{+\infty} Sa^2(at)\, e^{j\omega t}\, dt = \mathcal{F}[\,Sa^2(at)\,].$$

$$\therefore \int_{-\infty}^{+\infty} Sa^2(at)\, dt = X(j0)$$

$$\because \mathcal{F}[\,Sa^2(at)\,] = \frac{\pi^2}{a^2}$$ 

$$\therefore \int_{-\infty}^{+\infty} Sa^2(at)\, dt = \cancel{\frac{\pi^2}{a^2}}\ \frac{\pi}{a}$$

注：虽 $Sa(at) \xrightarrow{\mathcal{F}} \frac{a}{\pi} G_{2a}(\omega)$.

但 $Sa^2(at) \xrightarrow{\mathcal{F}} \frac{1}{2\pi}\left[\frac{\pi}{a}G_{2a}(\omega) * \frac{\pi}{a}G_{2a}(\omega)\right]$

$$= \frac{1}{2\pi}\boxed{\frac{\pi^2}{a}\,\Delta_{4a}(\omega)} = \frac{\pi}{a}\Delta_{4a}(\omega)$$

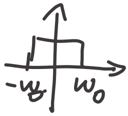
$$\therefore H(j0) = \frac{\pi}{a}\,\Delta_{4a}(0) = \frac{\pi}{a}.$$

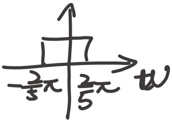
(狼狈翻书)

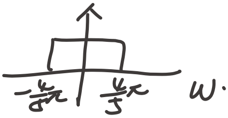
$$\frac{1}{2\pi}\cdot 2a\frac{\pi^2}{a^2}\Delta_{4a}(\omega).$$

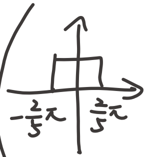
$$\underline{\frac{\pi}{a}\cdot\Delta_{4a}(\omega)}$$ 3-2-2看懂了

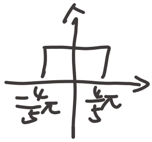
3-2-2. $\int_{-\infty}^{\infty} \frac{\sin\frac{2}{5}\pi t \sin\frac{4}{5}\pi t}{t^2\pi^2} dt$

· $\frac{\sin\omega_0 t}{\pi t} \xrightarrow{\mathcal{F}}$ 

· $\frac{\sin\frac{2}{5}\pi t}{\pi t} \xrightarrow{\mathcal{F}}$ 

$\frac{\sin\frac{4}{5}\pi t}{\pi t} \xrightarrow{\mathcal{F}}$ 

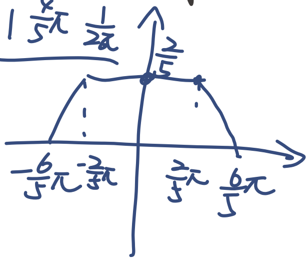
∵ 原式 $= \frac{1}{2\pi}\Big($ 

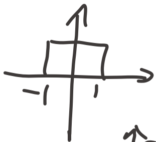
$*$ 

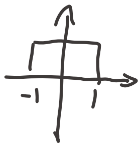
$\Big)\Big|_{w=0}$

$=$ 梯形高 $\times \frac{1}{2\pi} = \frac{4}{5}\pi\ \frac{1}{2\pi}$

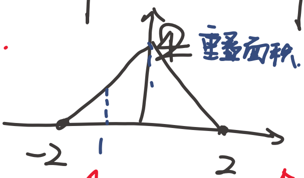
高 $=$ min 矩宽 $(\frac{4}{5}\pi)$.

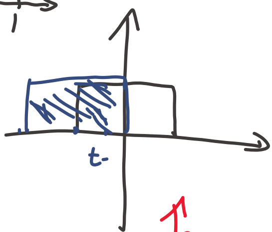


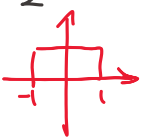
证明 

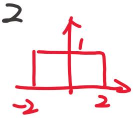
$*$ 

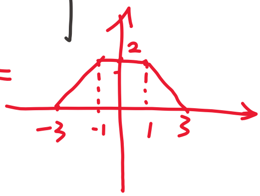
模型.

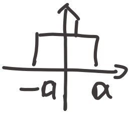
$=$ 



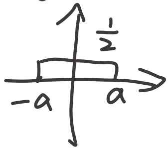


$*$ 

$=$ 

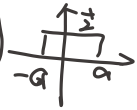
3-2-3  $\xrightarrow{\mathcal{F}} \int_{-a}^{a} e^{-j\omega t}dt = \frac{e^{j\omega a}-e^{-j\omega a}}{j\omega} = \frac{2\sin a\omega}{\omega}$

① $a>0$

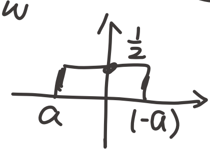
$\because \frac{\sin a\omega}{\omega} = \frac{1}{2}\,\frac{2\sin a\omega}{\omega} \xrightarrow{\mathcal{F}^{-1}}$ 

偶函

$\therefore \int_{0}^{\infty}\frac{\sin a\omega}{\omega}d\omega = \frac{1}{2}\overset{2\pi}{}\int_{-\infty}^{+\infty}\frac{\sin a\omega}{\omega}d\omega\Big|_{t=0}$

$= \pi$  $\Big|_{t=0} = \pi\,\frac{1}{2}$

② $a<0$. $\frac{\sin a\omega}{\omega} = -\frac{\sin(-a)\omega}{\omega}$.

$\xrightarrow{\mathcal{F}^{-1}}$ 

原式 $= \pi\int_{-\infty}^{+\infty}-\frac{\sin(-a)\omega}{\omega}d\omega\Big|_{t=0} = -\frac{1}{2}\pi$.

3-2-7 (1) $\mathscr{F}(f(t)) \triangleq \int_{-\infty}^{+\infty} f(t) e^{-j\omega t} dt$

$$= \int_{-1}^{0} (t+1) e^{-j\omega t} dt + \int_{0}^{1} e^{-j\omega t} dt$$

$$= \int_{-1}^{0} e^{-j\omega t} dt + \int_{0}^{1} e^{-j\omega t} dt + \int_{-1}^{0} t e^{-j\omega t} dt$$

$$= \frac{2\sin\omega}{\omega} + \left.\frac{te^{-j\omega t}}{-j\omega}\right|_{-1}^{0} - \left.\frac{e^{-j\omega t}}{(-j\omega)^2}\right|_{-1}^{0}$$

$$= \frac{2\sin\omega}{\omega} + \frac{e^{+j\omega}}{-j\omega} - \frac{1-e^{j\omega}}{-\omega^2}$$

结挖坑. X(0) 不就直流分量

$$\int_{-\infty}^{+\infty} f(t) dt = S = 1.5$$

(2) $\int_{-\infty}^{+\infty} F(\omega) d\omega = 2\pi f(0) = 2\pi$

3-2-8 直接做 (3)

$x(t) = \delta(t-1) * 4\Delta_4(t)$ 记 $g(t) = 4\Delta_4(t)$ 为 一个偶函数

$X(j\omega) = \mathscr{F}[x(t)] = \mathscr{F}[\delta(t-1) * g(t)] = e^{-j\omega} G(j\omega)$

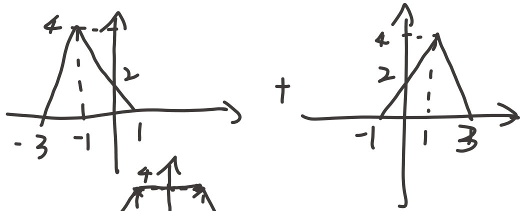
其中 $G(j\omega)$ 是结实偶函数. $e^{-j\omega} G(j\omega) = \cos\omega G(j\omega) - j\sin\omega G(j\omega)$

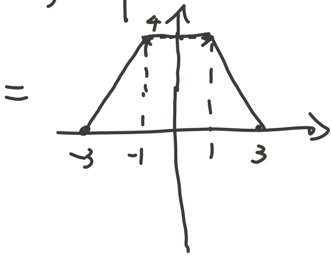
$\therefore Re[X(j\omega)] = \cos\omega\, G(j\omega) = \frac{e^{j\omega}+e^{-j\omega}}{2} G(j\omega)$

$\therefore \mathscr{F}^{-1}\{Re[X(j\omega)]\} = \frac{1}{2}\delta(t+1) * g(t) + \frac{1}{2}\delta(t-1) * g(t)$

图像为.

还是算扎实





$$\mathcal{F}^{-1}\{Re[X(j\omega)]\} = \mathcal{F}^{-1}\left\{\frac{X(j\omega)+X^*(j\omega)}{2}\right\}$$

$$\parallel$$

$$\mathcal{F}^{-1}\{\mathcal{F}\{Ev\{x(t)\}\}\}$$

$x(t)$实信号

利用共轭对称
这样方便
一点

$$= \frac{x(t)+x^*(-t)}{2}$$

$$= \frac{x(t)+x(-t)}{2}$$

3-5-1 $Re\{H(j\omega)\} = \cos\omega = \frac{e^{j\omega}+e^{-j\omega}}{2}$

又: $Re\{H(j\omega)\} = \mathcal{F}\{\mathcal{E}v[h(t)]\}$

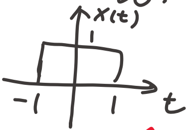
∴ $\mathcal{E}v[h(t)] = \frac{1}{2}\delta(t+1) + \frac{1}{2}\delta(t-1)$

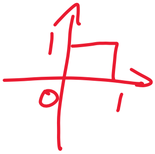
$h(t) + h(-t) = \delta(t+1) + \delta(t-1)$

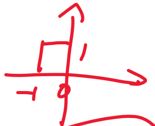
∴ $h(t) = \delta(t-1)$ 问题在于因果LTI 忽略 $\delta(t+1)$

3-5-2 $Re\{X(j\omega)\} = \frac{\sin\omega}{\omega}$

$Re\{\mathcal{E}v[x(t)]\} = \frac{1}{2}\cdot\frac{2\sin\omega}{\omega}$

$\mathcal{E}v\{x(t)\} = \frac{1}{2}$ 

$x(t) =$ 

$x(-t)$ 

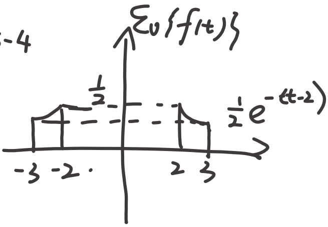
3-5-3 概念 $x[n] + x[-n] \xrightarrow{\mathcal{F}} X(e^{j\omega}) + X(e^{-j\omega})$

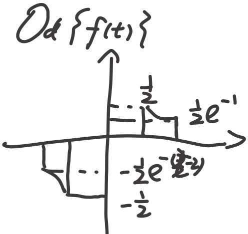
问 $X(e^{j\omega}) + X(e^{-j\omega})$ 是否为 f 的函数

当 $x(t)$ 实信号 奇 $X(e^{-j\omega}) = -X(e^{j\omega})$

$X(e^{j\omega})$ 为虚奇 → $X(e^{j\omega}) + X(e^{-j\omega}) = 0$

就不是 f 函数.

3-5-4 



3-5-5　A 没给 $x(t)$ 情况 X

B X　　C $X(j\omega)=\int_{-\infty}^{+\infty}x(t)e^{-j\omega t}dt.$

$$\int_{-\infty}^{+\infty}x^*(t)e^{-j\omega t}dt = \int_{-\infty}^{+\infty}x^*(t)(e^{j\omega t})^*dt = X^*(-j\omega).$$

D. 实信号有共轭对称 X

3-5-6 送分

3-5-10 $f(t)=\sum_{-\infty}^{\infty} C_n e^{jk\frac{2\pi}{T}t}$

$$C_n=\frac{1}{T}\int_T f(t)e^{-jn\omega_0 t}dt$$

记 $E_v\{f(t)\} = \frac{f(t)+f(-t)}{2}.$　主要是 $f(t)\xrightarrow{-t}f(-t)$ 不会影响 $\boxed{T=\frac{2\pi}{\omega_0}}$

$$D_n = \underbrace{\boxed{\frac{1}{T}\int_T}}_{\text{抽象表达}} \frac{f(t)+f(-t)}{2} e^{-jn\omega_0 t}dt.$$

$$=\frac{1}{2T}\int_T f(t)e^{-jn\omega_0 t}dt+\frac{1}{2T}\int_T f(-t)e^{-jn\omega_0 t}dt$$

$$=\frac{1}{2}C_n+\frac{1}{2}C_n$$

$Re\{f(t)\} = \frac{f(t)+f^*(t)}{2}$ — 正负消虚部

$E_n=\frac{1}{2}C_n.+\frac{1}{2}FS\{f^*(t)\}\to F_n$

$$F_n = \frac{1}{T}\int_T f^*(t)\, e^{-j\omega t}\, dt.$$

$$= \frac{1}{T}\int_T f^*(t)\left[e^{-j(-\omega)t}\right]^* dt$$

$$= C_{-n}^*. \qquad \therefore E_n = \frac{1}{2} C_n + \frac{1}{2} C_{-n}^*$$

3-5-13不难. 别想太多 周期是没变的

3-6-1 $H(j\omega)=\dfrac{1-e^{-(j\omega+1)}}{j\omega+1}=\dfrac{Y(j\omega)}{X(j\omega)}$

令 $x(t)=u(t)$

$X(j\omega)=\frac{1}{j\omega}+\pi\delta(\omega)$ $\quad Y(j\omega)\to G(j\omega)$

$\therefore G(j\omega)=\dfrac{1-e^{-1}\cdot e^{-j\omega}}{j\omega+1}\left[\frac{1}{j\omega}+\pi\delta(\omega)\right]$

$=e^{-1}(e^{1}-e^{-j\omega})$

$H(j\omega)=\dfrac{1}{j\omega+1}-\dfrac{e^{(-j\omega-1)}}{j\omega+1}$ 必须拆项

$=\dfrac{1}{j\omega+1}-e^{-1}\cdot\dfrac{e^{-j\omega}}{j\omega+1}$ → 时延

$h(t)=e^{-t}u(t)-e^{-1}\delta(t-1)*e^{-t}u(t)$

$=e^{-t}u(t)-\frac{1}{e}e^{-t+1}u(t-1)$

$\therefore y(t)=h(t)*u(t)$

$=1-e^{-t}-\frac{1}{e}\cdot e^{-t}u(t)*\delta(t-1)*u(t-1)$

$=1-e^{-t}-\frac{1}{e}\quad e^{-t}u(t)*u(t-2)$

$u(t-2)\xrightarrow{\mathcal{F}}e^{-2j\omega}\left[\frac{1}{j\omega}+\pi\delta(\omega)\right]$

$e^{-t}u(t)\xrightarrow{\mathcal{F}}\frac{1}{j\omega+1}$

$\mathcal{F}\left[e^{-t}u(t)*u(t-2)\right]=e^{-2j\omega}\cdot\dfrac{1}{(j\omega+1)j\omega}+\dfrac{e^{-2j\omega}\pi\delta(\omega)}{j\omega+1}$

$=e^{-2j\omega}\left(\frac{1}{j\omega}-\frac{1}{j\omega+1}\right)+$ 下.

不够常规，你这么做不是不可以，但显然不专业.

时移

$(1-e^{-t})u(t)-\frac{1}{e}\cdot e^{-(t-1)}u(t-1)*u(t)$

$$= (1-e^{-t})u(t) - e^{1}\cdot\left[1-e^{-(t-1)}\right]u(t-1)$$

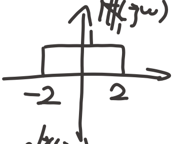
不如 $u(t) * e^{-t}u(t) * \delta(t-1)$

$$(1-e^{-t})u(t) * \delta(t-1).$$

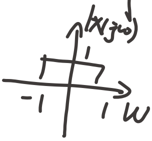
最方便.

3-6-2. $h_1(t) = \dfrac{\sin 2t}{\pi t}$ $\quad h_2(t) = 2\pi\dfrac{\sin t}{\pi t}\cdot\dfrac{\sin 2t}{\pi t}$.

$\because \dfrac{\sin 2t}{\pi t} \xrightarrow{\mathcal{F}}$



$h_2(t) \xrightarrow{\mathcal{F}}$



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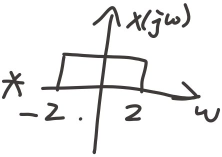
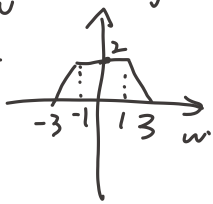
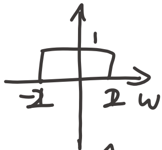


$= H_2(j\omega)$



$H(j\omega) = H_2(j\omega)\cdot H_1(j\omega)$

$=$



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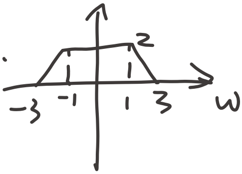

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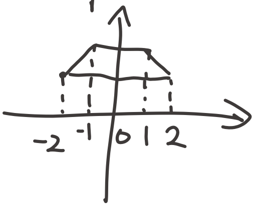

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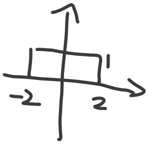

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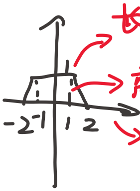


长 - 短

高为矩函数

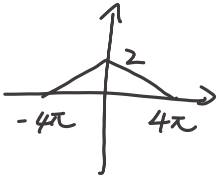
长 + 短.

$$h(t) = \frac{\sin 2t}{\pi t} + 2\pi\,\frac{\sin\frac{1}{2}t}{\pi t}\,\frac{\sin\frac{3}{2}t}{\pi t}$$

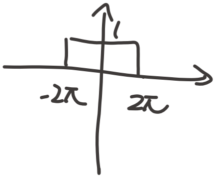
还挺恶心

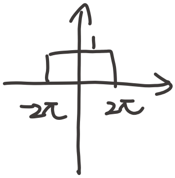
3-6-4. 模：增益 相位：时延

(1) $\varphi(\omega) = -\frac{1}{4}\omega$

$H(j\omega) =$ 

$e^{-\frac{1}{4}j\omega}$.

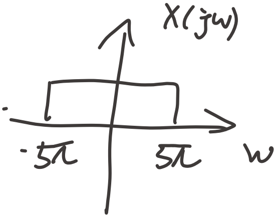
$=$ 

$*$ 

$\cdot e^{-\frac{1}{4}j\omega}$

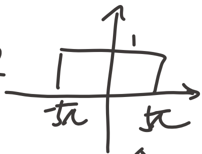
$\therefore h(t) = 2\pi\left(\frac{\sin 2\pi t}{\pi t}\right)^2 \cdot \delta(t-\frac{1}{4})$.

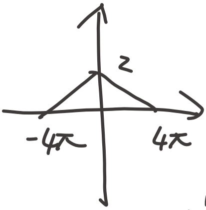
(2) $Sa(5\pi t) = \frac{\sin 5\pi t}{5\pi t}$.

$e(t) = 2\,\frac{\sin 5\pi t}{\pi t} \xrightarrow{\mathcal{F}} 2\cdot$ 

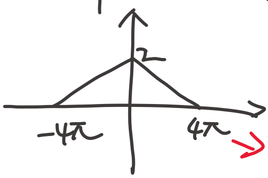
$r_{zs}(t) = e(t) * h(t)$

$R_{zs}(j\omega) = E(j\omega)\cdot H(j\omega)$

$= 2$ 



$e^{-\frac{1}{4}j\omega}$.

$= 2\cdot$ 

$e^{-\frac{1}{4}j\omega}$ $\to 0 * 0$

$\because r_{zs}(t) = 2\cdot\left(\frac{\sin 2\pi t}{\pi t}\right)^2$ $\delta(t-\frac{1}{4})$

3-7-1 $y(t) = \int_{t-1}^{\infty} x(\tau-3)\, e^{t-\tau} d\tau$

$\int_{t-1}^{\infty} x(\tau-3)\, e^{t-\tau-1}\, u\{-(\tau-(t-1))\}\, d\tau$

麻烦看下题目让你求什么再做吧.

$x(\tau-3) \to \delta(\tau-3)$ 单位冲激!!!

$\therefore h(t) = \int_{t-1}^{\infty} \delta(\tau-3)\, e^{t-\tau} d\tau$

当 $t-1 \leq 3$ τ可取到3

$h(t) = e^{t-3}$

$t-1 > 3$ τ不取3 $\Rightarrow h(t) = 0$.

3-7-3 $y(t) = 2\int_{-\infty}^{\infty} \sin(\tau-2)\, x(t-\tau) d\tau$.

$y(t) = 2\sin(t-2) * x(t)$.

$h(t) = 2\sin(t-2) = 2\sin t * \delta(t-2)$.

$H(j\omega) = 2\cdot e^{-2j\omega}\, \frac{\pi}{j}\, [\delta(\omega-1) - \delta(\omega+1)]$

$$= -2je^{-2j\omega}\pi[\delta(\omega-1)-\delta(\omega+1)]$$

3-7-4

$$x(-2t) \xrightarrow{\mathcal{F}} \frac{1}{2}X(j\cdot -\frac{\omega}{2})$$

$$h(-2t) \xrightarrow{\mathcal{F}} \frac{1}{2}H(j\cdot -\frac{\omega}{2})$$

$$\therefore \quad x(-2t) * h(-2t) \xrightarrow{\mathcal{F}} \frac{1}{4}X(j(-\frac{\omega}{2}))H(j(-\frac{\omega}{2}))$$

$$r(t) \xrightarrow{\mathcal{F}} X(j\omega)\ H(j\omega) = R(j\omega)$$

$$R(j(-\frac{\omega}{2})) = X(j(-\frac{\omega}{2}))H(j(-\frac{\omega}{2})) = 4\mathcal{F}[x(-2t) * h(-2t)]$$

$$\because \ R(j(-\frac{\omega}{2})) = 2\mathcal{F}[r(-2t)]$$

$$= 4\mathcal{F}[x(-2t) * h(-2t)]$$

$$x(-2t) * h(-2t) = \frac{1}{2}r(-2t)$$

这种题没什么技巧.你这么做已经是常规了

3-7-5 $\quad Y(j\omega) = X(j\omega)\cdot H(j\omega) = \dfrac{1}{j\omega+1}$

$$\mathcal{F}[f(2t)] = \frac{1}{2}F(\tfrac{1}{2}j\omega) \quad \mathcal{F}[h(2t)] = \frac{1}{2}H(j\tfrac{\omega}{2}).$$

$$\therefore \quad \text{原式} \xrightarrow{\mathcal{F}} \frac{1}{4}F(\tfrac{1}{2}j\omega)H(\tfrac{1}{2}j\omega)$$

$$y(2t) \xrightarrow{\mathcal{F}} \frac{1}{2}F(\tfrac{1}{2}j\omega)H(\tfrac{1}{2}j\omega)$$

$$= 2\mathcal{F}[h(2t) * f(2t)]$$

$$= \frac{1}{2} \frac{1}{j\frac{\omega}{2}+1}.$$

$$\therefore \mathcal{F}[h(2t)*f(2t)] = \frac{1}{4} \frac{1}{j\frac{\omega}{2}+1}$$

$$f(2t) * h(2t) = \frac{1}{4} \cdot e^{-\frac{1}{2}t}_{-2t} u(t).$$

3-76

①

$$f(-t) * h(-t) = \int_{-\infty}^{+\infty} f(-\tau) \cdot h(t-(-\tau))\, d\tau.$$

$$\overset{\tau'=-\tau}{=} \int_{+\infty}^{-\infty} f(\tau') \cdot h(t-\tau')\, d(-\tau')$$ 注意换元.

上下限以及d( )都应换

$$= \int_{-\infty}^{+\infty} f(\tau') \cdot h(t-\tau')\, d\tau'$$

$$= y(t).$$ → 算错 $y(-t)$ 从频域去分析.

② $$f'(t) * h'(t) = f(t) * h(t) * \cancel{\delta'(t)} * \delta'(t)$$

$$= y''(t)$$

③ $$f(3t) * h(3t) = \int_{-\infty}^{+\infty} f(3\tau)\, h(3(t-\tau))\, d\tau.$$

$$\tau' = 3\tau. = \int_{-\infty}^{+\infty} \frac{1}{3} f(\tau')\, h(3t-\tau')\, d\tau'$$

$$= \frac{1}{3} y(\cancel{3}t).$$

① 换元有点问题 $$\int_{-\infty}^{+\infty} f(-\tau)\, h(-(t-\tau))$$

$\tau' = -\tau$ $\quad = \int_{+\infty}^{-\infty} f(\tau') h(\boxed{-t}+\tau') d(-\tau')$

$= y(-t)$ 卷积互变量.

也可以频域. $f(-t) \xrightarrow{\mathcal{F}} \frac{1}{|-1|} F(j(-w))$

$h(-t) \xrightarrow{\mathcal{F}} H(-jw)$

$\therefore y(-t) \xrightarrow{\mathcal{F}} F(-jw) H(-jw)$. ✓

3-8-① 证一下帕塞瓦尔(连续)

$$\int_{-\infty}^{+\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} |X(jw)|^2 dw.$$

$\because \int_{-\infty}^{+\infty} |x(t)|^2 dt = \int_{-\infty}^{+\infty} x(t) x^*(t) dt$

$= \int_{-\infty}^{+\infty} x(t) \frac{1}{2\pi} \int_{-\infty}^{+\infty} X^*(jw) e^{-jwt} dw\, dt$

$= \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} x(t) X^*(jw) e^{-jwt} dt\, dw$

$= \frac{1}{2\pi} \int_{-\infty}^{+\infty} X^*(jw) X(jw) dw = \frac{1}{2\pi} \int_{-\infty}^{+\infty} |X(jw)|^2 dw$

3-8-①′ 证帕塞瓦尔(级数)

$$\frac{1}{T} \int_T^{+\infty} |x(t)|^2 dt = \sum |a_k|^2.$$

$\frac{1}{T} \int_T^{+\infty} x(t) x^*(t) dt = \frac{1}{T} \int_T x(t) \cdot \sum a_k^* e^{-jk\frac{2\pi}{T}t} dt.$

$$= \sum a_k^* \frac{1}{T}\int_T^{+\infty} x(t) e^{-jk\frac{2\pi}{T}t} dt.$$

$$= \sum a_k^* a_k = \sum |a_k|^2$$

3-8-2 $\cos(7\pi t)$ $T_1 = \frac{2\pi}{7\pi} = \frac{2}{7}$

$\sin 5\pi t$ $T_2 = \frac{2\pi}{5\pi} = \frac{2}{5}$

$T = \text{lcm}(\frac{2}{7}, \frac{2}{5}) = 2.$ (基波)

$\omega_0 = \frac{2\pi}{T} = \pi$

$$x(t) = \sum a_k e^{+jk\pi t}$$

$$= 1 + \frac{e^{j7\pi t} + e^{-j7\pi t}}{2} + \frac{e^{j5\pi t} - e^{-j5\pi t}}{2j}$$

$$\int_4^8 |x(t)|^2 dt = \int_0^4 |x(t)|^2 dt = 2\int_0^2 |x(t)|^2 dt$$

$$= 4 \cdot \frac{1}{2}\int_0^2 |x(t)|^2 dt = 4\sum |a_k|^2$$

$$= 4\left(1 + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}\right)$$

$a_0 = 1$ $a_7 = a_{-7} = \frac{1}{2}$ $a_5 = \frac{1}{2j}$ $a_{-5} = -\frac{1}{2j}$

战胜西安交通大学!

$$= 8.$$

3-8-3 先复习一个变换对

$$e^{-\alpha|t|} \overset{\mathcal{F}}{\longleftrightarrow} \frac{2a}{\omega^2+a^2}$$

证明: $\mathcal{F}[e^{-\alpha|t|}] = \int_0^{+\infty} e^{-\alpha t} e^{-j\omega t} dt + \int_{-\infty}^{0} e^{\alpha t} e^{-j\omega t} dt.$

$$= \frac{1}{a+j\omega} + \frac{1}{\alpha-j\omega} = \frac{2\alpha}{\alpha^2+\omega^2}$$

对偶性质: $$2\pi e^{-\alpha|\omega|} \overset{\mathcal{F}}{\longleftrightarrow} \frac{2\alpha}{t^2+a^2}$$

$$\therefore \frac{2\pi}{2a} e^{-\alpha|\omega|} \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{t^2+a^2}$$

$$\int_{-\infty}^{\infty} \frac{1}{(a^2+t^2)^2} dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left| \frac{\pi}{a} e^{-\alpha|\omega|} \right|^2 d\omega$$

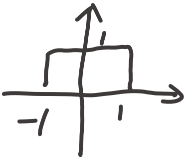
$$= \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{\pi^2}{a^2} e^{-2\alpha|\omega|} d\omega$$

$$= \frac{\pi}{a^2} \int_{0}^{+\infty} e^{-2a\omega} d\omega$$

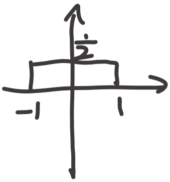
$$= \frac{\pi}{a^2} \cdot \left. \frac{e^{-2a\omega}}{-2a} \right|_0^{+\infty} = \frac{\pi}{2a^3}$$

从这个有理积分可以拓展出更多频域使用

只可惜，复变什么都没讲

3-8-4 $\displaystyle\int_0^{\infty} \frac{\sin^2 w}{w^2}\,dw$  $\xrightarrow{\mathcal{F}} \dfrac{2\sin w}{w}$.

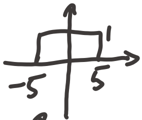
$\parallel$

$\displaystyle\frac{1}{2}\int_{-\infty}^{+\infty} \left(\frac{\sin w}{w}\right)^2 dw$.  $\xrightarrow{\mathcal{F}} \dfrac{\sin w}{w}$

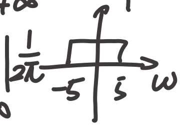
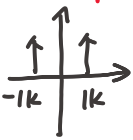
$\displaystyle= \frac{1}{2}\,2\pi\int_{-\infty}^{+\infty} |x(t)|^2 dt$

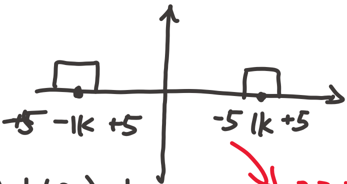
$\displaystyle= \pi\int_{-1}^{1} \frac{1}{4}\,dt = \frac{\pi}{2}$

3-8-5 $2\cos 1000t \overset{\mathcal{F}}{\longleftrightarrow} 2\pi[\delta(w+1000)+\delta(w-1000)]$

$\dfrac{\sin 5t}{\pi t} \overset{\mathcal{F}}{\longleftrightarrow}$ 

乘一个载波信号，让信号双向搬运至ω轴左右侧

∴ 原式 $= \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{+\infty}\Big|$  $* \; 2\pi$  $\Big|^2 dw$

$\displaystyle= \frac{1}{2\pi}\int_{-\infty}^{+\infty}\Big|$  $\Big|^2 dw$

$= \dfrac{1}{2\pi}(10+10) = \dfrac{10}{\pi}$

积分是2个方框面积

3-8-6 $\because 2f(t)+\frac{df(t)}{dt}=\alpha e^{-3t}u(t)$

频域内 $(2+j\omega)F(j\omega)=\alpha\cdot\frac{1}{3+j\omega}$

$\therefore F(j\omega)=\alpha\left(\frac{1}{2+j\omega}-\frac{1}{3+j\omega}\right)$

$\therefore f(t)=\alpha\left(e^{-2t}-e^{-3t}\right)u(t)$.

原式 $=2\pi\int_{-\infty}^{+\infty}|f(t)|^2dt$

$=2\pi\alpha^2\int_{0}^{+\infty}\left(e^{-4t}-2e^{-5t}+e^{-6t}\right)dt$

$=2\pi\alpha^2\left(\frac{1}{4}-\frac{2}{5}+\frac{1}{6}\right)=\frac{2\pi}{15}$.

$\alpha^2\left(\frac{15-24+10}{60}\right)=\frac{1}{15}$

$\alpha^2=\frac{4}{1}$

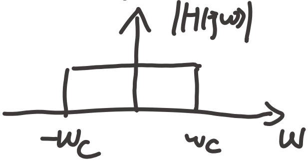
$\alpha=\pm2$.

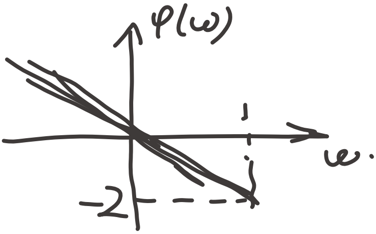
$\alpha=2$ (非负

3-8-7 (1) $H(j\omega) = \begin{cases} e^{-2j\omega} & |\omega| < \omega_c . \\ 0 & \text{else} \end{cases}$

在 $[-\omega_c, \omega_c]$ $H(j\omega) = 1 \cdot e^{j \measuredangle H(j\omega)}$

$\measuredangle H(j\omega) = -2\omega$





$$h(t) = \mathcal{F}^{-1}\{H(j\omega)\}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty} H(j\omega) e^{j\omega t} d\omega .$$

$$= \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{-2j\omega} e^{j\omega t} d\omega$$

$$= \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{j(t-2)\omega} d\omega .$$

$$= \frac{1}{2\pi} \frac{e^{j(t-2)\omega}\Big|_{-\omega_c}^{\omega_c}}{j(t-2)} = \frac{e^{j(t-2)\omega_c} - e^{-j(t-2)\omega_c}}{2\pi j(t-2)} .$$

$$= \frac{\sin[\omega_c(t-2)]}{\pi(t-2)} = \cancel{\frac{\sin \omega_c t}{\pi t}} * \delta(t-2)$$

(2) $E_{f(t)} = \lim_{T\to\infty}\int_{-T}^{T}|2e^{-t}\varepsilon(t)|^2 dt = \lim_{T\to\infty}\int_0^T 4e^{-2t}dt$

$= \lim_{T\to\infty} 4 \cdot \frac{e^{-2t}}{-2}\Big|_0^T = 2$ (J 焦耳)

(3). $E_{f(t)}=2$ $\quad E_{y(t)}=1$

能量剩一半. 固可以算y(t)再算功率, 有无依据.

$$f(t)\xrightarrow{\mathcal{F}}2\frac{1}{j\omega+1}$$

$$H(j\omega)=\begin{cases}e^{-2j\omega} & |\omega|<\omega_c\\ 0 & else\end{cases}$$

$$\therefore Y(j\omega)=\begin{cases}2\cdot\dfrac{e^{-2j\omega}}{j\omega+1} & |\omega|<\omega_c.\\ 0 & else.\end{cases}$$

$$\therefore y(t)=\mathcal{F}^{-1}(Y(j\omega))=2\cdot\delta(t-2)*e^{-t}u(t).$$

$$=2e^{-t+2}u(t-2).$$

$$\therefore E_{y(t)}=\lim_{T\to\infty}\int_{-T}^{T}4e^{-2t+4}u^2(t-2)\,dt$$

$$=\lim_{T\to\infty}\int_{2}^{T}4e^{-2t+4}\,dt$$

$$=4e^4\frac{e^{-2t}\big|_2^T}{-2}=$$

不用回y(t) $\quad |Y(j\omega)|=\dfrac{2}{\sqrt{1+\omega^2}}$

$$\therefore E=\int_{-\infty}^{+\infty}|x(t)|^2dt=\frac{1}{2\pi}\int_{-\omega_c}^{\omega_c}\frac{4}{1+\omega^2}d\omega.$$

$$=\frac{1}{2\pi}4\cdot\arctan\omega\Big|_{-\omega_c}^{\omega_c}$$

$\arctan\omega_c=\frac{\pi}{4}$ $\quad=\frac{2}{\pi}(\arctan\omega_c+\arctan\omega_c)=\frac{1}{2}\cdot 2$