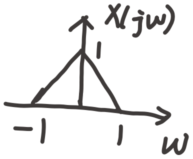
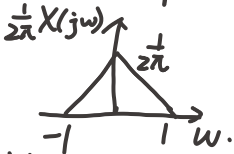
4 10 (a). 由频域微分性质·

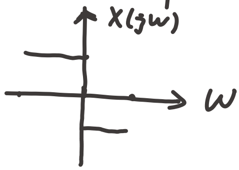
$$t\left(\frac{\sin t}{\pi t}\right)^2 \xrightarrow{\mathcal{F}} j\frac{d\mathcal{F}\left[\left(\frac{\sin t}{\pi t}\right)^2\right]}{dw}.$$

由相乘性质

又∵ $\mathcal{F}\left[\left(\frac{\sin t}{\pi t}\right)^2\right] = \frac{1}{2\pi}\begin{cases}1 & |w|<1\\ 0 & else\end{cases} * \begin{cases}1 & |w|<1\\ 0 & else\end{cases}$

$= \frac{1}{2\pi}$ 

$=$ 

∴ $t\left(\frac{\sin t}{\pi t}\right)^2 \xrightarrow{\mathcal{F}} j\cdot$ 

$$= \begin{cases}\frac{j}{-2\pi} & 0\leq w\leq 1\\ \frac{j}{2\pi} & -1\leq w<0\end{cases}$$

(b). ∵ $|x(t)|^2 = t^2\left(\frac{\sin t}{\pi t}\right)^4$

$$A = \int_{-\infty}^{+\infty} t^2\left(\frac{\sin t}{\pi t}\right)^4 dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty}|X(jw)|^2 dw$$

$$= \frac{1}{2\pi}\int_0^1\left|\frac{j}{2\pi}\right|^2 dw + \frac{1}{2\pi}\int_{-1}^0\left|\frac{j}{2\pi}\right|^2 dw$$

$$= \frac{1}{2\pi^3}$$

4.21 (a). 由 $\cos\omega_0 t=\frac{1}{2}(e^{+j\omega_0 t}+e^{-j\omega_0 t})$、

$\therefore e^{-\alpha t}\cos\omega_0 t=\frac{1}{2}e^{-(\alpha-j\omega_0)t}+\frac{1}{2}e^{-(\alpha+j\omega_0)t}$

由 $e^{-\alpha t}u(t)\xrightarrow{\mathcal{F}}\frac{1}{\alpha+j\omega}$

$\therefore \mathcal{F}\{[e^{-\alpha t}\cos\omega_0 t]u(t)\}=\frac{1}{2}\frac{1}{\alpha-j\omega_0+j\omega}+\frac{1}{2}\frac{1}{\alpha+j\omega_0+j\omega}$

(c) $\because 1\xrightarrow{\mathcal{F}}2\pi\delta(\omega)$.

$\cos\pi t\xrightarrow{\mathcal{F}}\pi[\delta(\omega+\pi)+\delta(\omega-\pi)]$.

$\therefore \mathcal{F}[x(t)]=\int_{-\infty}^{+\infty}x(t)e^{-j\omega t}dt=\int_{-1}^{1}(1+\cos\pi t)e^{-j\omega t}dt$

$=\frac{e^{j\omega}-e^{-j\omega}}{j\omega}+\int_{-1}^{1}\frac{1}{2}(e^{-j(\omega-\pi)t}+e^{-j(\omega+\pi)t})dt$

$=\frac{e^{j\omega}-e^{-j\omega}}{j\omega}+\frac{1}{2}\frac{e^{j(\omega-\pi)}-e^{-j(\omega-\pi)}}{j(\omega-\pi)}+\frac{1}{2}\frac{e^{j(\omega+\pi)}-e^{-j(\omega+\pi)}}{j(\omega+\pi)}$

$=\frac{2\sin\omega}{\omega}+\frac{\sin(\omega-\pi)}{(\omega-\pi)}+\frac{\sin(\omega+\pi)}{(\omega+\pi)}$

(e) $\because \sin\omega_0 t=\frac{e^{j\omega_0 t}-e^{-j\omega_0 t}}{2j}$ $\therefore \sin 4t=\frac{1}{2j}(e^{j4t}-e^{-j4t})$

$e^{-2t}\sin 4t=\frac{1}{2j}(e^{(j4-2)t}-e^{-(j4+2)t})$、

$\because tx(t)\xrightarrow{\mathcal{F}}j\frac{dX(j\omega)}{d\omega}$ 又 $\because \begin{cases} e^{(j4-2)t}u(t)\xrightarrow{\mathcal{F}}\frac{1}{(2-j4)+j\omega} \\ e^{-(2+j4)t}u(t)\xrightarrow{\mathcal{F}}\frac{1}{(2+j4)+j\omega}\end{cases}$

$\therefore \mathcal{F}[e^{-2t}\sin 4t\,u(t)]=\frac{4}{(2+j\omega)^2+16}$.

$\therefore (te^{-2t}\sin 4t)u(t)\xrightarrow{\mathcal{F}}j\cdot\frac{d(\frac{4}{(2+j\omega)^2+16})}{d\omega}=\frac{8\omega-16j}{(-\omega^2+4j\omega+20)^2}$

纠正一个问题 $tx(t)\xrightarrow{\mathcal{F}}j\frac{dX(j\omega)}{d\omega}$，而非 $\frac{1}{j}$!!!

(g). $x_{(t)} = \begin{cases} -1 & -2 \le t \le -1 \\ t & -1 < t < 1 \\ 1 & 1 \le t \le 2. \\ 0 & \text{else}. \end{cases}$

$$\therefore X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt = \int_{-2}^{-1} -e^{-j\omega t} dt + \int_{-1}^{1} t e^{-j\omega t} dt$$

$$= \frac{e^{2j\omega} - e^{j\omega}}{j\omega} + \frac{te^{-j\omega t}}{-j\omega}\Big|_{-1}^{1} - \frac{e^{-j\omega t}}{(-j\omega)^2}\Big|_{-1}^{1} + \frac{e^{-j\omega} - e^{-2j\omega}}{j\omega} + \int_{1}^{2} e^{-j\omega t} dt.$$

$$= \frac{2j\cos 2\omega}{\omega} - \frac{2j\sin\omega}{\omega^2} = \frac{2j}{\omega}\left(\cos 2\omega - \frac{\sin\omega}{\omega}\right)$$

4.34. (a). $H(j\omega) = \frac{j\omega + 4}{(j\omega)^2 + 5j\omega + 6} = \frac{j\omega + 4}{(j\omega+2)(j\omega+3)}$

其中 $H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$　由于因果LTI. $y_{zi}(0) = 0$

$\therefore (j\omega+2)(j\omega+3)Y(j\omega) = (j\omega+4)X(j\omega)$

微分方程为. $\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = \frac{dx(t)}{dt} + 4x(t)$

(b). 当 $x(t) = \delta(t)$ 时.

对原方程微分. $\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = \delta'(t) + 4\delta(t)$

$\therefore H(j\omega) = \frac{j\omega+4}{(j\omega+2)(j\omega+3)} = \frac{2}{j\omega+2} - \frac{1}{j\omega+3}$

$\therefore h(t) = \mathcal{F}^{-1}[H(j\omega)] = 2e^{-2t}u(t) - e^{-3t}u(t)$

(c) $\because$ 单位冲激响应 $h(t) = 2e^{-2t}u(t) - e^{-3t}u(t)$

$\because$ 输入 $x(t) \xrightarrow{\mathcal{F}} \frac{1}{j\omega+4} + \frac{j}{j}\left(\frac{1}{j\omega+4}\right)^2 = \frac{1}{j\omega+4}\left(1 + \frac{1}{j\omega+4}\right)$

$\therefore Y(j\omega) = H(j\omega)X(j\omega) = \left(\frac{2}{j\omega+2} - \frac{j\cdot 1}{j\omega+3}\right)\left(\frac{1}{j\omega+4} + \frac{1}{(j\omega+4)^2}\right)$

$$= \frac{1}{j\omega+2} - \frac{1}{j\omega+4} - \left(\frac{1}{j\omega+3} - \frac{1}{j\omega+4}\right) - \frac{1}{j\omega+3}\frac{1}{(j\omega+4)^2} + \frac{2}{j\omega+2}\frac{1}{(j\omega+4)^2}$$

$$= \frac{1/2}{jw+2} + \frac{1/2}{jw+4}$$

$\therefore y(t) = \mathcal{F}^{-1}[Y(jw)] = \frac{1}{2}e^{-2t}u(t) - \frac{1}{2}e^{-4t}u(t).$

4.35(a) 将 $H(jw)$ 写成实+虚部形式

$$|H(jw)| = \frac{|a-jw|}{|a+jw|} = \frac{\sqrt{a^2+w^2}}{\sqrt{a^2+w^2}} = 1$$

$$\measuredangle H(jw) = \arg(a-jw) - \arg(a+jw) = -\arctan\frac{w}{a} - \arctan\frac{w}{a} = -2\arctan\frac{w}{a}$$

(b) 在LTI中 $H(jw) = \frac{Y(jw)}{X(jw)} = \frac{a-jw}{a+jw}$

$\therefore$

$$= -1 + \frac{2a}{a+jw}$$

$\therefore h(t) = \mathcal{F}^{-1}[H(jw)] = -\delta(t) + 2a e^{-at}u(t).$

$a=1$ 时 $h(t) = -\delta(t) + 2e^{-t}u(t)$

$\because |H(jw)| = 1 \quad \measuredangle H(jw) = -2\arctan w$

则 $|Y(jw)| = |X(jw)|$

$\measuredangle Y(jw) = \measuredangle X(jw) - 2\arctan w$

$w_1 = \pm\frac{\sqrt{3}}{3}$ 则 $2\arctan w_1 = \frac{\pi}{3}$. $\quad w_3 = \sqrt{3}$

$w_2 = \pm 1$ 则 $2\arctan w_2 = \frac{\pi}{2}$ $\quad 2\arctan w_3 = \frac{2}{3}\pi$

由于 $x(t)$ 项均为复指数函数

$$y(t) = x(t) * h(t)$$

$$= \sum_{k=-\infty}^{+\infty} a_k e^{jkw_0t} \cdot H(jw)\Big|_{w=kw_0}$$

$$= \cos\left(\frac{\sqrt{3}}{3}t - \frac{\pi}{3}\right) + \cos\left(t - \frac{\pi}{2}\right) + \cos\left(\sqrt{3}t - \frac{2}{3}\pi\right)$$

$$H(j\omega)=\frac{Y(j\omega)}{X(j\omega)}=\frac{j\omega+4}{6-\omega^2+5j\omega-}$$

$$\therefore\quad (6-\omega^2+5j\omega)Y(j\omega)=(j\omega+4)X(j\omega)$$

$$\frac{d^2y(t)}{dt}+5\frac{dy(t)}{dt}+6y(t)=\frac{dx(t)}{dt}+4x(t)$$

$$h(t)=\mathcal{F}^{-1}(\cdot H(j\omega))=\mathcal{F}^{-1}\left[\frac{j\omega+4}{(j\omega+2)(j\omega+3)}\right]$$

$$=(2e^{-2t}-e^{-3t})u(t)$$

$$x(t)*h(t)\overset{\mathcal{F}}{\Rightarrow}\frac{1}{4+j\omega}-j\frac{d\frac{1}{4+j\omega}}{d\omega}$$

$$Y(j\omega)=\frac{1}{4+j\omega}-j\frac{-j}{(4+j\omega)^2}$$

$$=\left[\frac{1}{4+j\omega}-\frac{1}{(4+j\omega)^2}\right]\left(\frac{2}{j\omega+2}-\frac{1}{j\omega+3}\right)$$

$$=\frac{1}{4+j\omega}\frac{2}{j\omega+2}-\frac{1}{(4+j\omega)^2}\frac{2}{j\omega+2}-\frac{1}{(4+j\omega)(3+j\omega)}+\frac{1}{(4+j\omega)^2(j\omega+3)}$$

$$=\frac{1}{2+j\omega}-\cancel{\frac{1}{4+j\omega}}-\frac{1}{3+j\omega}+\cancel{\frac{1}{4+j\omega}}-\frac{1}{4+j\omega}\left(\frac{2}{j\omega+4\;j\omega+2}-\frac{1}{j\omega+4\;j\omega+3}\right)$$

$$-\frac{1}{2+j\omega}-\cancel{\frac{1}{3+j\omega}}-\frac{1}{2}\left(\frac{1}{j\omega+2}-\frac{1}{j\omega+4}\right)\quad\left(\frac{1}{j\omega+2}-\cancel{\frac{1}{j\omega+4}}-\frac{1}{j\omega+3}+\cancel{\frac{1}{j\omega+4}}\right)$$

$$+\cancel{\frac{1}{j\omega+3}}-\frac{1}{j\omega+4}$$

$$=\frac{1}{2}\frac{1}{j\omega+2}+\frac{1}{2}\frac{1}{j\omega+4}$$

$-\frac{2}{\sqrt{u+4}}$

$\hbar \quad \int^{2}$