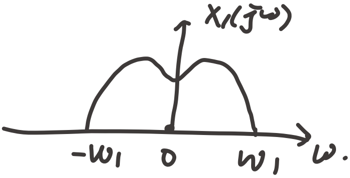
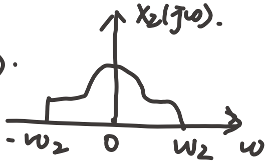
7.6 (a) ∵ $X_1(j\omega)$



$X_2(j\omega)$.



$$x_1(t) \times x_2(t) \xrightarrow{\mathcal{F}} \frac{1}{2\pi}\left[X_1(j\omega) * X_2(j\omega)\right].$$

由 Nyquist 采样定理 $\omega_s > 2\omega_m$.

ω_m 指待采样信号的带限.

∴ $X_1(j\omega) * X_2(j\omega)$ 易知带限于 $\omega_m = \omega_1+\omega_2$

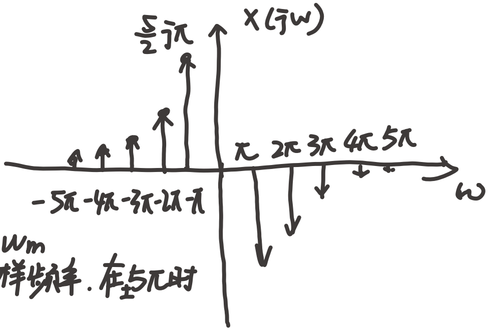
∴ $\omega_s = \frac{2\pi}{T} > 2(\omega_1+\omega_2)$

∴ $T < \frac{\pi}{\omega_1+\omega_2}$. 最大采样间隔为 $\frac{\pi}{\omega_1+\omega_2}$.

7.8 (a) $x(t) = \sum_{k=0}^{5} \left(\frac{1}{2}\right)^k \sin k\pi t$

∴ $X(j\omega) = \mathcal{F}[x(t)] = \sum_{k=0}^{5} \left(\frac{1}{2}\right)^k \frac{\pi}{j}\left[\delta(\omega-k\pi) - \delta(\omega+k\pi)\right]$

作出其图像



则 $\omega_m = 5\pi$

$T=0.2$ 时

$\omega_s = \frac{2\pi}{T} = 10\pi = 2\omega_m$

不满足 Nyquist 采样频率. 在 $\pm5\pi$ 时会产生混叠

(b) 经过 $w_c = \frac{\pi}{T} = 5\pi$ 滤波时，$\pm 5\pi$ 冲激被去除。

则 $G(jw) = \sum_{k=0}^{4} (\frac{1}{2})^k \frac{\pi}{j} [\delta(w-k\pi) - \delta(w+k\pi)]$

$\therefore g(t) = \sum_{k=0}^{4} (\frac{1}{2})^k \sin k\pi t$.

7.10 (a). 错误 $x(t)$ 为门信号、频谱带宽无限

不可能不发生混叠

(b). $x(t)$ 带限于 w_0 要求 $w_s > 2 \cdot w_0$

则 $\frac{2\pi}{T} > 2w_0$ $T < \frac{\pi}{w_0}$ 正确

(c) 正确 对单边频谱 T仍应满足 $T < \frac{2\pi}{w_0}$

可刚好不混叠

7.15 $X(e^{jw}) = 0$ 在 $\frac{3\pi}{7} \le |w| \le \pi$,

$\therefore x[n]$ 带限于 $w_m = \frac{3\pi}{7}$

由 $w_s = \frac{2\pi}{N} > 2 \cdot \frac{3\pi}{7}$

$\therefore N < \frac{7}{3}$

$\therefore N$ 取整，则 $N=2$.

7.19 $x[n] = \frac{\sin w_1 n}{\pi n} \xrightarrow{\mathcal{F}} \begin{cases} 1 & |w| < w_1 \\ 0 & \text{else} \end{cases}$

(a) $w_1 \le \frac{3\pi}{5}$, 插入2个零值点、

设插值后为 $z[n]$

则 $z[n] \xrightarrow{\mathcal{F}} \begin{cases} 1 & |\omega| < \frac{\omega_1}{3} \\ 0 & \text{else} \end{cases}$

此时. $\frac{\omega_1}{3} \leq \frac{\pi}{5}$. 完全处于 $H(e^{j\omega})$ 滤波范围内.

$\therefore w[n] = z[n] = x[3n] = \frac{\sin(\frac{\omega_1}{3}n)}{\pi n}$

$\therefore y[n] = w[5n] = \frac{\sin\frac{5}{3}\omega_1 n}{5\pi n}$

(b). 插值后. $z[n] \xrightarrow{\mathcal{F}} \begin{cases} 1 & |\omega| < \frac{\omega_1}{3} \\ 0 & \text{else} \end{cases}$

$\frac{\omega_1}{3} > \frac{\pi}{5}$ 部分被滤去

$\therefore$ 对 $w[n]$ $W(e^{j\omega}) = \begin{cases} 1 & |\omega| \leq \frac{\pi}{5} \\ 0 & \text{else.} \end{cases}$

$\therefore w[n] = \frac{\sin\frac{\pi}{5}n}{\pi n}.$

$y[n] = \frac{\sin \pi n}{5\pi n}$

7.21 $\because T = 10^{-4}$ $\therefore \omega_s = \frac{2\pi}{T} = 2\pi\, 10^4$ rad/s

若要满足采样定理 $\omega_s > 2\omega_m$ 才可恢复信号

即 $\omega_m < \pi \cdot 10^4$ rad/s

(a) $x(t)$ 带限于 $\omega_m = 5000\pi < 10^4\pi$ 可恢复.

(b) $x(t)$ 带限于 $\omega_m = 15000\pi > 10^4\pi$ 不可恢复

(c). $Im\{X(j\omega)\}$ 未知, 不能判断

(d) $x(t) \in R$ 带限于 5000π，类似于 (a) 可恢复

(e) $x(t) \in R$ 则 $X(j\omega)$ 为偶信号 $x(t)$ 带限于 15000π 不可恢复

$> \frac{1}{2}\omega_s$

(f) $\mathcal{F}^{-1}\{X(j\omega) * X(j\omega)\}$ 带限于 $2\omega_m$; $\omega_m = 7500\pi$

$< \frac{1}{2}\omega_s$ 可恢复

(g) $x(t)$ 负频率未知，无法判断

7.22 $\because$ $x_1(t)$ 带限于 $\omega_1 = 1000\pi$

$x_2(t)$ 带限于 $\omega_2 = 2000\pi$

两者均为关于纵轴对称的

由卷积性质 $x_1(t) * x_2(t) \xrightarrow{\mathcal{F}} X_1(j\omega) \cdot X_2(j\omega)$

其显然带限于 $\omega_m = \min\{\omega_1, \omega_2\} = 1000\pi$

想恢复 $y(t)$ 则 $\omega_s = \frac{2\pi}{T} > 2\omega_m = 2000\pi$.

$\therefore T < \frac{1}{1000} s = 1ms.$

采样周期小于 1ms. 可保证恢复 $y(t)$