5.9 从(3)可知，由共轭对称，$x[n]$为实信号

偶信号 $x_e[n]=\frac{1}{2}(x[n]+x[-n])$.

奇信号 $x_o[n]=\frac{1}{2}(x[n]-x[-n])$

$$\mathcal{F}\{x_e[n]\}=\frac{1}{2}X(e^{j\omega})+\frac{1}{2}X(e^{-j\omega})$$

$$=\frac{1}{2}X(e^{j\omega})+\frac{1}{2}X^*(e^{j\omega})=Re\{X(e^{j\omega})\}$$

同理 $\mathcal{F}\{x_o[n]\}=jIm\{X(e^{j\omega})\}$

$$=j(\sin\omega-\sin 2\omega)$$

$$=j\left(\frac{e^{j\omega}-e^{-j\omega}}{2j}-\frac{e^{2j\omega}-e^{-2j\omega}}{2j}\right)$$

$$=\frac{1}{2}(e^{j\omega}-e^{-j\omega}-e^{2j\omega}+e^{-2j\omega})$$

$\therefore x_o[n]=\frac{1}{2}\{x[n]-x[-n]\}=\mathcal{F}^{-1}\{jIm\{X(e^{j\omega})\}\}$

$$=\frac{1}{2}\{\delta[n+1]-\delta[n-1]-\delta[n+2]+\delta[n-2]\}$$

$\because n>0 \quad x[n]=0$

$n=-1 \quad x[-1]=1 \qquad n=-3 \cdot x[n]=0$.

$n=-2 \quad x[-2]=-1 \qquad$ 只有 $x[0], x[-1], x[-2]$ 非0

由 $\frac{1}{2\pi}\int_{-\pi}^{\pi}|X(e^{j\omega})|^2 d\omega=3=\sum_{n=0}^{2}|x[n]|^2$

$1+1+x^2[0]=3 \qquad x[0]>0 \quad$ 则 $x[0]=1$

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega = 3 = \sum |x(n)|^2$$

$$j\,\mathrm{Im}\{X(e^{j\omega})\} = \mathcal{F}\{x_o(n)\} = \mathcal{F}\left\{\frac{x_{od}[n]+x_{od}[-n]}{2}\right\}$$

$$\mathcal{F}\left(\frac{e^{j\omega}+e^{-j\omega}}{2j} - \frac{e^{2j\omega}-e^{-2j\omega}}{2j}\right) \quad \mathcal{F}\left(\frac{x_{od}[n]+x_{od}[-n]}{2}\right)$$

$$x_{od}[n] = \frac{1}{2}\left(\delta[n+1] - \delta[n-1] - \delta[n-2] + \delta[n+2]\right)$$

$$\frac{x[n]+x[-n]}{2} = \frac{1}{2}\cdots$$

$x[0]+x[0] = ?$ 计不了.

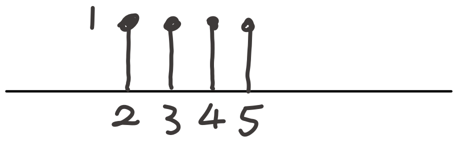
$x[1]+x[-1] = x[-1] = 1$

$x[2]+x[-2] = x[-2] = 1$

$\wedge\ \boxed{x[0]=1}$ $x[3]=0$ 算的

$\cdot \quad x[n] = \delta[n] + \delta[n+1] - \delta[n+2]$

5 21(a)



$$x[n] = \delta[n-2] + \delta[n-3] + \delta[n-4] + \delta[n-5]$$

$$\therefore X(e^{jw}) = e^{-2jw} + e^{-3jw} + e^{-4jw} + e^{-5jw}$$

用冲激串. 而非门函数

(c) $u[-n-2] = \begin{cases} 1 & n \le 2 \\ 0 & \text{else.} \end{cases}$

$$x[n] = \frac{1}{9}\delta[n-2] + \frac{1}{3}\delta[n-1] + \left(\frac{1}{3}\right)^{-n} u[-n]$$

$$X(e^{jw}) = \mathcal{F}\{x[n]\} = \frac{1}{9}e^{-2jw} + \frac{1}{3}e^{-jw} + \sum_{n=0}^{-\infty}\left(\frac{1}{3}\right)^{-n} u[-n] e^{-jwn}$$

换元.
$$= \frac{1}{9}e^{-2jw} + \frac{1}{3}e^{-jw} + \sum_{n=0}^{+\infty}\left(\frac{1}{3}\right)^{n} e^{-j(-w)n}$$

$$= \frac{1}{9}e^{-2jw} + \frac{1}{3}e^{-jw} + \frac{1}{1-\frac{1}{3}e^{jw}}$$

(h). $x[n]$两项周期 $N_1 = 6 \quad N_2 = 6$

$x[n] \quad N = 6 \quad w_0 = \frac{\pi}{3}$

$$x[n] = \frac{1}{2j}\left(e^{j\frac{5}{3}\pi n} - e^{-j\frac{5}{3}\pi n}\right) + \frac{1}{2}\left(e^{j\frac{7}{3}\pi n} + e^{-j\frac{7}{3}\pi n}\right).$$

$$\therefore X(e^{jw}) = \mathcal{F}\{x[n]\} = \frac{1}{2j}\left(\frac{1-e^{-j(w-\frac{1}{3}\pi)5}}{1-e^{-j(w-\frac{1}{3}\pi)}} - \frac{1-e^{-j(w+\frac{1}{3}\pi)5}}{1-e^{-j(w+\frac{1}{3}\pi)}}\right)$$

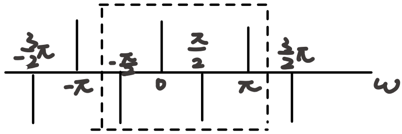
$$+\frac{1}{2}\left(\frac{1-e^{-j(w-\frac{1}{3}\pi)7}}{1-e^{-j(w-\frac{1}{3}\pi)}} + \frac{1-e^{-j(w+\frac{1}{3}\pi)7}}{1-e^{-j(w+\frac{1}{3}\pi)}}\right)$$

5.22(b) 由于 $\mathcal{F}[e^{-jwn_0}] = \delta[n-n_0]$

$\therefore x[n] = \mathcal{F}^{-1}[X(e^{jw})] = \mathcal{F}^{-1}[1+3e^{-jw}+2e^{-j2w}+(-4)e^{-j3w}+e^{-j10w}]$

$= \delta[n] + 3\delta[n-1] + 2\delta[n-2] - 4\delta[n-3] + \delta[n-10]$

(e) $X(e^{jw})$ 为



由综合公式.

$$x[n] = \frac{1}{2\pi}\int_{-\frac{3}{4}\pi}^{\frac{5}{4}\pi} X(e^{jw})e^{jwn}dw = \frac{1}{2\pi}\sum_{n=-1}^{2} e^{jwn}(-1)^n.$$

$$= \frac{1}{2\pi}\left(-e^{-j\frac{\pi}{2}n} + 1 - e^{j\frac{\pi}{2}n} + e^{j\pi n}\right)$$

$$= \frac{1}{2\pi} - \frac{1}{\pi}\cos\frac{\pi}{2}n + \cos\pi n.$$

(g) $$X(e^{jw}) = \frac{1-\frac{1}{3}e^{-jw}}{(-\frac{1}{2}e^{-jw}+1)(\frac{1}{4}e^{-jw}+1)}$$

令 $z = e^{-jw}$

$$X(e^{jw}) = \frac{1-\frac{1}{3}z}{(-\frac{1}{2}z+1)(\frac{1}{4}z+1)} = \frac{8}{3}\frac{z-3}{(z-2)(z+4)}$$

$$= \frac{8}{3}\left(-\frac{1}{6}\frac{1}{z-2}+\frac{7}{6}\frac{1}{z+4}\right)$$

$$= -\frac{4}{9}\frac{1}{e^{-jw}-2}+\frac{28}{9}\frac{1}{e^{-jw}+4} = +\frac{2}{9}\frac{1}{1-\frac{1}{2}e^{-jw}}+\frac{7}{9}\frac{1}{1+\frac{1}{4}e^{-jw}}$$

由 $a^n u[n] \xrightarrow{\mathcal{F}} \frac{1}{1-ae^{-jw}}$ $|a|<1$

∴ $x[n] = \left(\frac{1}{2}\right)^n u[n] \left(+\frac{2}{9}\right) + \frac{7}{9}\left(-\frac{1}{4}\right)^n u[n]$

5.23 (a). ∵ $X(e^{jw}) = \sum_{n=-3}^{7} x[n]e^{-jwn}$

$$X(e^{j0}) = \sum_{n=-3}^{7} x[n] = -1+1+2+1+1+2+1-1$$
$$= 6$$

(c). 由综合公式对比 $x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{jw})e^{jwn}dw$

∴ $\int_{-\pi}^{\pi} X(e^{jw})dw = 2\pi\cdot x[n]\Big|_{n=0}$

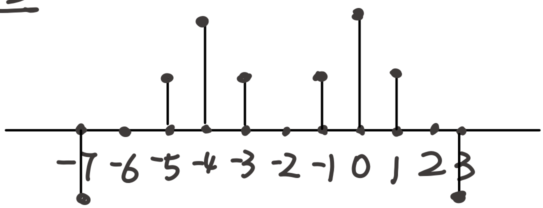
$= 2\pi x[0] = 4\pi$

(e) 由 $x[n]$ 为实信号

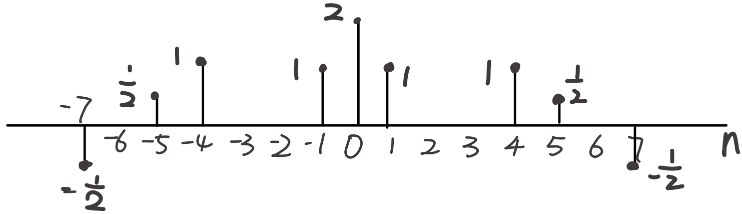
∴ $Re\{X(e^{jw})\} = \mathcal{F}\left[\frac{x[n]+x[-n]}{2}\right] = \mathcal{F}\{x_e[n]\}$

∴ 要求 $\frac{x[n]+x[-n]}{2}$

∵ $x[-n]$ 图像为



∵ 信号 $x_e[n]=\frac{1}{2}(x[n]+x[-n])$ 为·



5.24 (a). $x[n]$ 为实偶信号 $X(e^{jw})$ 为实偶的.

则 1 不满足 2 满足 3: 时移后 $X(e^{jw})$ 可为偶信号 符合.

∵ $\int_{-\pi}^{\pi}X(e^{jw})dw=2\pi x[0]\neq0$ 不满足4

5 满足(时域离散 频域周期)

$X(e^{j0})=\sum_{n=-1}^{5}x[n]\neq0$. 不满足6.

(b) $x[n]$ 为实奇信号 则 $X(e^{jw})$ 为虚奇的

∴ 1 满足 2 不满足 3 满足, 时移后 $X(e^{jw})$ 可为偶信号

4. $\int_{-\pi}^{\pi}X(e^{jw})dw=2\pi x[0]=0$ 满足

5. 满足.

6 $X(e^{j0})=x[-1]+x[1]=0$ 满足

(g) $x[n]$ 只为实信号 则 1, 2, 均不满足, 3 满足

$X(e^{jw})$ 有实.虚部非零

4 $\int_{-\pi}^{\pi}X(e^{jw})dw=2\pi x[0]=2\cdot2\pi\neq0$ 不满足

5. 满足.

6. $X(e^{j0}) = \sum_{n=-4}^{4} x[n] e^{-j\omega n} \Big|_{\omega=0}$

$= -1-1+2-1+1=0$ 、满足

(n) $x[n]$ 为实偶信号, 类比 (a). 1 不符 2 符合 3 符合

4 ∵ $2\pi x[0] = 0$ 满足

5 $X(e^{j\omega})$ 周期

6 $X(e^{j0}) = \sum_{n=-6}^{6} x[n] = 2 \neq 0$ 不符合