5.12 记 $x[n]=\frac{\sin(\frac{\pi}{4}n)}{\pi n}$

$h[n]=\frac{\sin(w_c n)}{\pi n}$ 记其为系统函数，

视 $h[n]$ 为滤波器，但保证 $X(e^{jw})$ 的频段可以全部通过.

由 $\frac{\sin Wn}{\pi n} \xrightarrow{\mathcal{F}} \begin{cases}1 & |w|<W\\ 0 & \text{else.}\end{cases}$

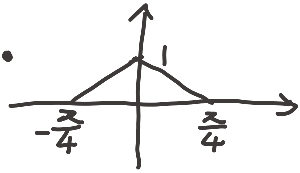
， $\frac{\sin(w_c n)}{\pi n} \xrightarrow{\mathcal{F}} \begin{cases}1 & |w|<w_c\\ 0 & \text{else.}\end{cases}$

由相乘性质

$$x[n]\,x[n]=\frac{1}{2\pi}X(e^{jw})*X(e^{jw})$$

$$=\frac{1}{2\pi}\begin{cases}1 & |w|<\frac{\pi}{4}\\ 0 & \text{else}\end{cases} * \begin{cases}1 & |w|<\frac{\pi}{4}\\ 0 & \text{else}\end{cases}$$

$=\frac{1}{2\pi}\cdot$



$\because$ 要求 $x^2[n]$ FT后频段可全通过

$\therefore \frac{\pi}{4}\le w_c\le\pi$

5.19 (a) 进行 DTFT、得

$$Y(e^{jw})-\frac{1}{6}e^{-jw}Y(e^{jw})-\frac{1}{6}e^{-2jw}Y(e^{jw})=X(e^{jw})$$

由 $H(e^{jw})=\frac{Y(e^{jw})}{X(e^{jw})}=\frac{1}{1-\frac{1}{6}e^{-jw}-\frac{1}{6}e^{-2jw}}$

记 $z=e^{-jw}$

$$H(e^{jw})=\frac{1}{1-\frac{1}{6}z-\frac{1}{6}z^2}=\frac{1}{-\frac{1}{6}(z+3)(z-2)}$$

$$=-6\,\frac{1}{5}\left(\frac{1}{z-2}-\frac{1}{z+3}\right)=+\frac{6}{5}\frac{1}{2-e^{jw}}-\frac{6}{5}\frac{1}{3-e^{-jw}}$$

$$=\frac{6}{5}\,\frac{1}{2}\,\frac{1}{1-\frac{1}{2}e^{-jw}}-\frac{6}{5}\,\frac{1}{3}\,\frac{1}{1-\frac{1}{3}e^{-jw}}$$

$\therefore h[n]=\mathcal{F}^{-1}[H(e^{jw})]=\left[\frac{3}{5}\left(\frac{1}{2}\right)^n-\frac{2}{5}\left(\frac{1}{3}\right)^n\right]u[n]$

5.26 (a) $\because X(e^{jw})=Re\{X(e^{jw})\}+j\,Im\{X(e^{jw})\}$

发现 $X_2(e^{jw})=Re\{X_1(e^{jw})\}$ (当 $-\frac{\pi}{3}\le w\le\frac{\pi}{3}$)

取 $x_1[n]$ 的偶部 $x_{1e}[n]=\frac{x[n]+x[-n]}{2}=Ev\{x_1[n]\}$

$Re\{X_1(e^{jw})\}$ 填充了 $X_2(e^{jw})$ 一个周期的 $\frac{1}{3}$

$$\therefore X_2(e^{jw})=Re\{X_1(e^{jw})\}+Re\{X_1(e^{j(w-\frac{2}{3}\pi)})\}+Re\{X_1(e^{j(w+\frac{2}{3}\pi)})\}$$

$\because x_2[n] = \mathcal{F}[x_2(e^{jw})]$

$= \mathcal{E}v\{x_1[n]\}(1 + e^{j\frac{2}{3}\pi n} + e^{-j\frac{2}{3}\pi n})$.

(b) 在一个周期（选定 $-\frac{2}{3}\pi < w < \frac{4}{3}\pi$）

$$X_3(e^{jw}) = Im\{X_1(e^{j(w-\pi)})\}$$

由频移性质 $x_3[n] \xrightarrow{\mathcal{F}} X_3(e^{jw})$ 也是虚奇函数

$$x_3[n] = Od\{x_1[n]\}e^{j\pi n}$$

5.35 (a) 转换为频域

$$Y(e^{jw}) - a e^{-jw} Y(e^{jw}) = b \cdot X(e^{jw}) + e^{-jw} X(e^{jw})$$

$$H(e^{jw}) = \frac{Y(e^{jw})}{X(e^{jw})} = \frac{1 - ae^{-jw}}{b + e^{-jw}}$$

$$|H(e^{jw})| = \frac{\sqrt{(1 - a\cos w)^2 + (a\sin w)^2}}{\sqrt{(b + \cos w)^2 + (\sin w)^2}} = 1$$

$\because 1 - 2a\cos w + a^2 = b^2 + 2b\cos w + 1$

$\therefore b = -a$

5.50 (a) 由变换对

(i) $\alpha^n u[n] \xrightarrow{FT} \dfrac{1}{1-\alpha e^{-j\omega}}$

$$x[n] = \left(\frac{1}{2}\right)^n u[n] - \frac{1}{4}\left(\frac{1}{2}\right)^n u[n] * \delta[n-1].$$

由卷积性质.

$$X(e^{j\omega}) = \mathcal{F}\{x[n]\} = \frac{1}{1-\frac{1}{2}e^{-j\omega}} - \frac{1}{4} e^{-j\omega} \frac{1}{1-\frac{1}{2}e^{-j\omega}}$$

$$= \frac{1}{1-\frac{1}{2}e^{-j\omega}} \left(1-\frac{1}{4}e^{-j\omega}\right).$$

$$Y(e^{j\omega}) = \mathcal{F}\{y[n]\} = \frac{1}{1-\frac{1}{3}e^{-j\omega}}$$

$\therefore$ 频率响应 $H(e^{j\omega}) = \dfrac{Y(e^{j\omega})}{X(e^{j\omega})} = \dfrac{1-\frac{1}{2}e^{-j\omega}}{\left(1-\frac{1}{3}e^{-j\omega}\right)\left(1-\frac{1}{4}e^{-j\omega}\right)}$

$$= -\frac{2}{1-\frac{1}{3}e^{-j\omega}} + \frac{3}{1-\frac{1}{4}e^{-j\omega}}.$$

单位脉冲 $h[n] = \mathcal{F}^{-1}[H(e^{j\omega})] = 3\left(\frac{1}{4}\right)^n u[n] - 2\left(\frac{1}{3}\right)^n u[n].$

(ii) $\because H(e^{j\omega}) = \dfrac{Y(e^{j\omega})}{X(e^{j\omega})} = \dfrac{1-\frac{1}{2}e^{-j\omega}}{\left(1-\frac{1}{3}e^{-j\omega}\right)\left(1-\frac{1}{4}e^{-j\omega}\right)}$

$$\therefore \left(\frac{1}{12}e^{-2j\omega} - \frac{7}{12}e^{-j\omega} + 1\right) Y(e^{j\omega}) = \left(1-\frac{1}{2}e^{-j\omega}\right) X(e^{j\omega})$$

逆变换到时域，得到微分方程

$$\frac{1}{12}\frac{d^2y(t)}{dt^2} - \frac{7}{12}\frac{dy(t)}{dt} + y(t) = -\frac{1}{2}\frac{dx(t)}{dt} + x(t)$$

(b) $x[n] = (n+2)\left(\frac{1}{2}\right)^n u[n]$

$y[n] = \left(\frac{1}{4}\right)^n u[n]$

则 $Y(e^{jw}) = \dfrac{1}{1-\frac{1}{4}e^{-jw}}$

$\because Y'(e^{jw}) = 1 - \dfrac{1}{1-(-\frac{1}{2})e^{-jw}}$

$$= \frac{(+\frac{1}{2})e^{-jw}}{1+\frac{1}{2}e^{-jw}}$$

$$X(e^{jw}) = 2\,\frac{1}{1-\frac{1}{2}e^{-jw}} + \frac{1}{j}\,\frac{d\left(\frac{1}{1-\frac{1}{2}e^{-jw}}\right)}{dw}$$

$$= \frac{2}{1-\frac{1}{2}e^{-jw}} + \frac{1}{j}\left(\frac{1}{1-\frac{1}{2}e^{-jw}}\right)^2 \left(-\frac{1}{2}\right)(-j)e^{-jw}$$

$$= \frac{2}{1-\frac{1}{2}e^{-jw}} + \frac{1}{2}\,\frac{e^{-jw}}{(1-\frac{1}{2}e^{-jw})^2}$$

$$= \frac{2-e^{-jw}+\frac{1}{2}e^{-jw}}{(1-\frac{1}{2}e^{-jw})^2} = \frac{2-\frac{1}{2}e^{-jw}}{(1-\frac{1}{2}e^{-jw})^2}$$

$$\because H(e^{jw}) = \frac{Y(e^{jw})}{X(e^{jw})} = \frac{(1-\frac{1}{2}e^{-jw})^2}{2(1-\frac{1}{4}e^{-jw})^2}$$

$$\because 对\ Y'(e^{jw}) = \frac{\frac{1}{2}e^{-jw}}{1+\frac{1}{2}e^{-jw}}$$

$$\therefore X'(e^{jw}) = \frac{Y'(e^{jw})}{H(e^{jw})}$$

$$= \frac{e^{-jw}\cdot(1-\frac{1}{4}e^{-jw})^2}{(1+\frac{1}{2}e^{-jw})(1-\frac{1}{2}e^{-jw})^2}$$

$$= \frac{3}{8}\frac{1}{1+\frac{1}{2}e^{-jw}} + \frac{3}{8}\frac{1}{1-\frac{1}{2}e^{-jw}} - \frac{1}{8}\frac{1}{(1-\frac{1}{2}e^{-jw})^2}.$$

$$\therefore x[n] = +\frac{3}{8}(\frac{1}{2})^{n-1}u[n] + \frac{3}{8}(\frac{1}{2})^{n-1}u[n] - \frac{1}{8}n(\frac{1}{2})^{n-1}u[n]$$