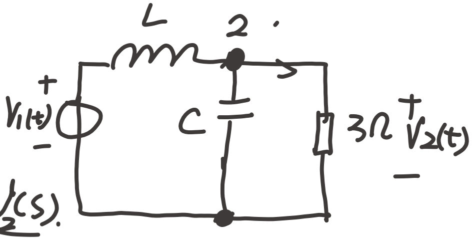
专题1: 10-1-1




$$\begin{cases} V_2(s) = U_c(s) \\ \dfrac{V_2(s)-V_1(s)}{sL} = CsU(s) + \dfrac{U_2(s)}{3} \end{cases}$$

我是根据节压法得到的

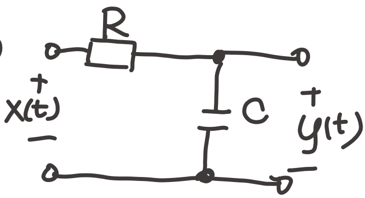
$$U_2(s)\left(\frac{1}{sL}+Cs+\frac{1}{3}\right) = \frac{U_1(s)}{sL}$$

$$\frac{U_2(s)}{U_1(s)} = \frac{\frac{1}{sL}}{\frac{1}{sL}+Cs+\frac{1}{3}} = \frac{1}{1+Cs^2L+\frac{1}{3}sL}$$

$$= \frac{1}{1+s+s^2}$$

$$\begin{cases} \frac{1}{3}L=1 \\ CL=1 \end{cases} \quad \therefore \begin{cases} L=3 \\ C=\frac{1}{3} \end{cases}$$

10-1-3 既考了方程 又考响应和零极点图




(1). $X(s)+i_C(s)R=Y(s)$

$i_C(s) = CsU_c(s)$

$= CsY(s)$

$\therefore X(s) - RCsY(s) = Y(s) \Rightarrow$ 写成?

(2) $H(s) = \dfrac{Y(s)}{X(s)} = \dfrac{1}{1+RCs} = \dfrac{1}{1+2s} = \dfrac{1}{2}\cdot\dfrac{1}{\frac{1}{2}+s}$

$x(t)u(t) = u(t) + 2\cos 2t\, u(t)$

$\mathcal{F}(x(t)u(t)) = \dfrac{1}{s} + 2\dfrac{s}{s^2+4}$

$$= \frac{s^2+4+2s}{s(s^2+4)} \qquad \frac{1}{2}\cdot\frac{1}{\frac{1}{2}+j2} =$$

$H(j\omega) = \frac{1}{2}\cdot\frac{1}{\frac{1}{2}+j\omega}$ 对一个复指数周期信号

(3) 用正弦稳态去做 $\quad = \frac{1}{1+2j\omega}$

$$x(t) = 1+2\,\frac{e^{2jt}+e^{-2jt}}{2} = 1+e^{2jt}+e^{-2jt}$$

$$|H(j2)| = \frac{1}{2}\cdot\frac{1}{\sqrt{\frac{1}{4}+4}} = \frac{1}{2}\cdot\frac{1}{\sqrt{\frac{17}{4}}} = \frac{1}{\sqrt{17}}.$$

$$\because H(j\omega) = \frac{1-2j\omega}{1+4\omega^2} \qquad |H(j0)| = 1.$$

$$\varphi(j2) = \arctan\left(\frac{1-4j}{1+16}\right) = \arctan(-4) = -75.96°.$$

$$\therefore y(t) = 1\,|H(j0)| + \frac{2}{\sqrt{17}}\cos\left(\cancel{2t-75.96°}\right)$$

$2t-\arctan 4$ 不用对t单独加

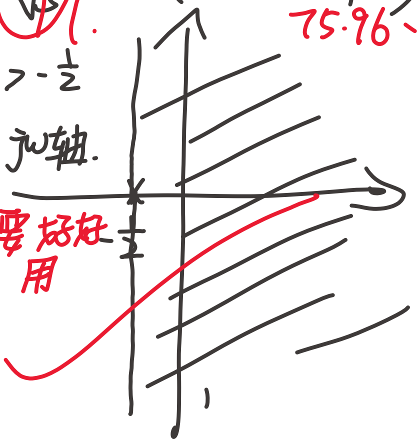
$$= 1+\frac{2}{\sqrt{17}}\cos\left(2t-15\cancel{\times}92°\right)$$

$75.96$、

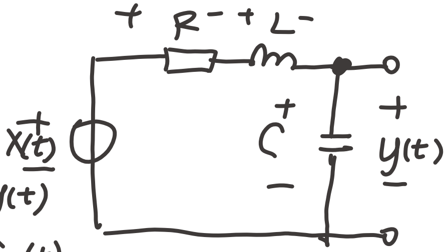
(4). ROC $\quad Re\{s\} > -\frac{1}{2}$

稳定. 因为包含 $j\omega$ 轴.

差不多. 正弦稳态要好好用



10-1-4




(1) $\begin{cases} y(t)=u_C(t) \\ x(t)=u_R(t)+u_L(t)+y(t) \\ i_C(t)=C\dfrac{du_C(t)}{dt}=i_L(t) \\ u_L(t)=L\dfrac{di_L(t)}{dt} \end{cases}$

$10^{-1}\cdot 10^{-4}$

$$\therefore x(t)=R\cdot C\frac{dy(t)}{dt}+LC\frac{d^2y(t)}{dt}+y(t)$$

变换域. $X(s)=RCs\,Y(s)+LCs^2Y(s)+Y(s)$

$$\therefore H(s)=\frac{Y(s)}{X(s)}=\frac{1}{RCs+LCs^2+1}$$

(2) $$H(s)=\frac{1}{R\cdot 0.1m\,s+10^{-5}s^2+1}$$ 别给自己找一堆麻烦,

非实数极点. 则 $\Delta=(R\cdot 0.1m)^2-4\ 10m<0$

$=R^2\ 0.01m-40m<0$

$R^2<4000$

算对了是凑巧

$$H(s)=\frac{1}{R10^{-4}s+10^{-5}s^2+1}$$

$$=\frac{10^5}{s^2+10Rs+10^5}$$

$\Delta=100R^2-4\times10^5<0$

$R^2-4\times10^3<0$

$\therefore 0<R<\sqrt{4000}$.

$$H(s)=\frac{10^5}{(s-s_1)(s-s_2)}$$

$$s=\frac{-10R\pm\sqrt{100R^2-4\times10^5}}{2}$$

这样证明是不够的

① 对实信号. $x(t)=x^*(t)$.

$$X(s)=X^*(s^*)$$

也就是 如 $S_1$是零点 $X(s_1)=0$

$$X^*(s_1^*)=0=X(s_1^*)$$

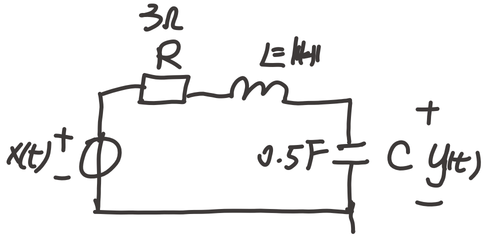
因为首先0是个实值，共轭是其本身. $s_1^*$也为零点.

$S_1$是极点. ∴ $X(s_1)=\infty=X^*(s_1^*)=\infty$

∞的含义是距离上的无穷远，所以共轭肯定也在无穷远处。∴ $S_1^*$也是极点.

说明了共轭之道理是：零极点总共轭存在(对实信号)

10-1-6 $R=3$




(1). $i_{C(t)}=C\dfrac{dy(t)}{dt}$

$u_{L(t)}=L\dfrac{di_C(t)}{dt}$

$=LC\dfrac{d^2y(t)}{dt^2}$.

$V_C(0^-)=1$.

$V_C'(0^-)=2$.

∵ $x(t)=i_C(t)R+u_L(t)+y(t)$

变换域. $X(s)=RCsY(s)+LCs^2Y(s)+Y(s)$.

∴ $H(s)=\dfrac{Y(s)}{X(s)}=\dfrac{1}{RCs+LCs^2+1}$

代入 $H(s)=\dfrac{1}{1.5s+0.5s^2+1}=\dfrac{2}{s^2+3s+2}$

$$= 2\cdot\left(\frac{1}{s+1}-\frac{1}{s+2}\right) \quad ROC \quad Re(s)>-1$$

$$\therefore h(t)=2\cdot(e^{-t}-e^{-2t})u(t)$$

(2)

微分. $0.5\frac{d^2y(t)}{dt}+1.5\frac{dy(t)}{dt}+y(t)=x(t)$

(3) $y(t)=y_{zi}(t)+y_{zs}(t)$

$$=y_{zi}(t)+x(t)*h(t)$$

$\because x(t)=e^{-3t}u(t)$. $X(s)=\frac{1}{s+3}$ ROC: $Re(s)>-3$

$$\therefore y_{zs}(t)=2\mathcal{L}^{-1}\left\{\frac{1}{s+3}\left(\frac{1}{s+1}-\frac{1}{s+2}\right)\right\}$$

$$=2\mathcal{L}^{-1}\left[\frac{1}{2}\left(\frac{1}{s+1}-\frac{1}{s+3}\right)-\frac{1}{s+2}+\frac{1}{s+3}\right]$$

$$=2\cdot\left\{\frac{1}{2}(e^{-t}-e^{-3t})-e^{-2t}+e^{-3t}\right]u(t)$$

$$=\left[e^{-t}-e^{-3t}-2e^{-2t}+2e^{-3t}\right]u(t)$$

$$=\underline{\left(e^{-t}-2e^{-2t}+e^{-3t}\right)u(t)}$$

$$y''_{zi}(t)+3y'_{zi}(t)+2y_{zi}(t)=0$$

对单边L变换. $y'(t)\overset{\mathcal{L}}{\longleftrightarrow} sY(s)+y(0_-)$

$$y''(t)\overset{\mathcal{L}}{\longleftrightarrow} s^2Y(s)+\underline{sy(0^-)}+\underline{y'(0^-)}.$$

$$-(2+s)$$

$\therefore s^2Y(s)+1+2s+3(sY(s)\cancel{+}1)+2Y(s)=0$ ?为什么不对

$$Y(s)(s^2+3s+2)+1+\cancel{2s+3}^{\,s+5}=0$$

$$Y(s)=\frac{\cancel{-2s-4}\ +s+5}{(s+2)(s+1)}$$

$$\therefore Y(s)=\frac{-2}{s+1} \quad \therefore\ y_{zi}(t)=-2e^{-t}u(t)$$

$$y(t)=-2e^{-t}u(t)+\left(e^{-t}\cancel{-2e^{-2t}}+e^{-3t}\right)u(t)$$

$$=\cancel{\left(-e^{-t}-2e^{-2t}+e^{-3t}\right)}u(t)$$

$$y_{zi}(t)=Ae^{-2t}+Be^{-t}.$$

这个解 $y_{zi}(t)$ 的方法是没问题的。顺便证明为什么会是那样。

$$\int_{0^-}^{+\infty}y'(t)\,\cancel{e^{-st}}\,dt=$$

$$=\frac{-3}{s+2}+\frac{+4}{s+1} \Rightarrow y_{zi}(t)=(-3e^{-2t}+4e^{-t})u(t)$$

$$(+5e^{-t}-5e^{-2t}+e^{-3t})u(t)$$

$$y'(t)\overset{UL}{\longleftrightarrow}sY(s)\ominus y(0^-).$$

$$\underbrace{\int_{0^-}^{+\infty}y'(t)e^{-st}dt}_{\text{我们只能求的.}}=\underbrace{\int_{0}^{+\infty}y'(t)e^{-st}dt}_{\text{我们实际要的}}+y(0^-)$$

$$\int_{0^-}^{+\infty}e^{-st}dy(t)=y(t)e^{-st}\Big|_{0^-}^{+\infty}+s\int_{0^-}^{+\infty}y(t)e^{-st}dt$$

$$= -y(0^-) + s\int_{0^-}^{+\infty} y(t)e^{-st}\,dt = -y(0^-) + sY(s)$$

而 $\int_{0^-}^{+\infty} e^{-st} y''(t)\,dt = \int_{0^-}^{+\infty} e^{-st}\,dy'(t)$

$$= e^{-st}y'(t)\Big|_{0^-}^{+\infty} + s\int_{0^-}^{+\infty} e^{-st}\,dy(t)$$

$$= -y'(0^-) + s\left( y(t)e^{-st}\Big|_{0^-}^{+\infty} + s\int_{0^-}^{+\infty} e^{-st}y(t)\,dt \right)$$

$$\underline{= -y'(0^-) + s[-y(0^-)] + s^2Y(s)}$$

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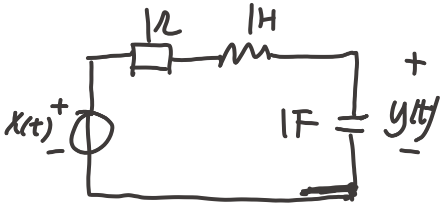
10-1-7 滤波特性

(1) $x(t) = i_C(t)\cdot R + u_L(t) + y(t)$

$u_L(t) = LC\dfrac{d^2y(t)}{dt^2}$

$i_C(t) = C\dfrac{dy(t)}{dt}$



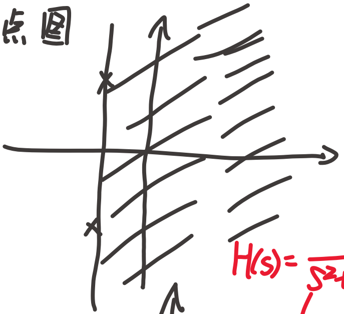

变换域 $X(s) = CsRY(s) + LCs^2Y(s) + Y(s)$

$$\therefore H(s) = \frac{Y(s)}{X(s)} = \frac{1}{1 + CRs + LCs^2}$$

$$= \frac{1}{1 + s + s^2}$$

$s^2 + s + 1 = 0 \qquad s = \dfrac{-1 \pm \sqrt{3}i}{2}$

极点图



因果系统

ROC: $\text{Re}\{s\} > -\frac{1}{2}$

$$H(j\omega) = \frac{1}{1 + j\omega + (-\omega^2)}$$

$$H(s) = \frac{1}{s^2+s+1} = \frac{1}{(s+\frac{1}{2})^2+\frac{3}{4}}$$ 没必要

$$|H(j\omega)| = \frac{1}{\cancel{\sqrt{(\frac{1}{2})^2+(\omega-\frac{\sqrt{3}}{2})^2}}\sqrt{(\frac{1}{2})^2+(\omega+\frac{\sqrt{3}}{2})^2}}$$

(2)

当 $j\omega = -\frac{1}{2}$.


当 $\omega = \pm\frac{\sqrt{3}}{2}$ 时. $|H(j\omega)|$ 最大

是~~带~~通的. 低通的

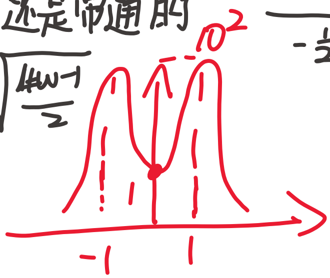
(3). $R = 10^{-2}\Omega$

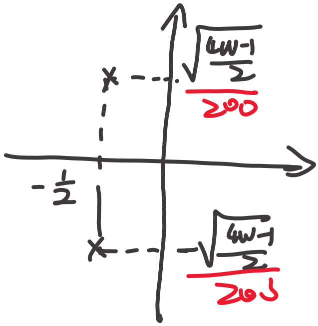
$$H(s) = \frac{1}{1 + 10^{-2}s + s^2} = \frac{1}{100s^2 + s + 100}$$

$$s = \frac{-1 \pm \sqrt{1-4w}}{200}$$

$|H(j\omega)|$ 还是带通的

$$\omega = \sqrt{\frac{4w-1}{2}}$$



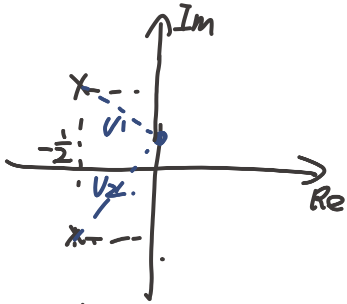

$$\frac{1}{100s^2+s+100}$$

决定对10-1-7重新梳理

$H(s)=\frac{1}{s^2+s+1}$ 得到之后，为了从零极点图得到幅频特性图，不妨用H(s)代数寻找极值

$H(s)=\frac{1}{(s+\frac{1}{2})^2+\frac{3}{4}}$ $\quad Re(s) > -\frac{1}{2}$

再代入 $s=j\omega$.




$$H(j\omega)=\frac{1}{(j\omega+\frac{1}{2})^2+\frac{3}{4}}$$

$$=\frac{1}{-\omega^2+1+j\omega}$$

$$|H(j\omega)|=\frac{1}{\sqrt{\omega^2+(1-\omega^2)^2}}=\frac{1}{\sqrt{\omega^4-\omega^2+1}}$$ 当 $\omega^2=\frac{1}{2}$ 时.

即 $\omega=\pm\frac{\sqrt{2}}{2}$ 时 $|H(j\omega)|_{max}$.

$|H(j\pm\frac{\sqrt{2}}{2})|=\frac{2}{\sqrt{3}}$ 其实 $|H(j\omega)|=\frac{1}{V_1V_2}$ 可以定性知道幅频，但不可以定量。定量老老实实搞函数

(3). $H(j\omega)=\frac{1}{-\omega^2+1+0.01j\omega}$ 可以先定性

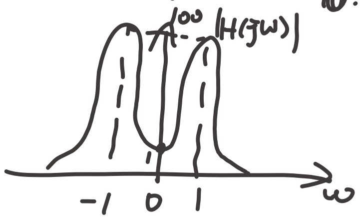
发现 $\omega\to\infty$ $\quad |H(j\omega)|\to 0$

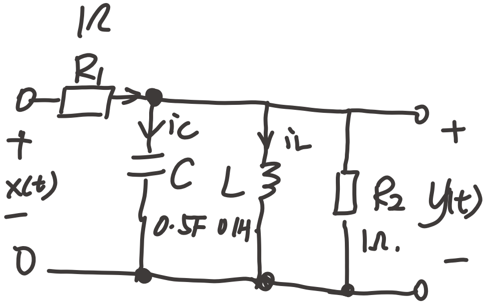
$\omega\to 0$ $\quad |H(j\omega)|\to 1$

$|H(j\omega)|=\frac{1}{\sqrt{(-\omega^2+1)^2+10^{-4}\omega^2}}$ 还是令 $\omega^2=(-2+10^{-4})$

$\omega=\sqrt{\frac{2-10^{-4}}{2}}\approx 1$

此时 $H(j\omega)$ 会 $\frac{1}{0.01} = 100$




10-1-13.(1)

$$y(t) = L\frac{di_L(t)}{dt}$$

$$i_L(t) = \frac{1}{L}\int y(t)dt$$

$$C\frac{dy(t)}{dt} + i_L(t) + \frac{y(t)}{R_2} = \frac{x(t)-y(t)}{R_1}$$

$$\because\ i_L(s) = \frac{1}{s}\cdot\frac{1}{L}Y(s)$$

$$CsY(s) + \frac{1}{sL}Y(s) + \frac{1}{R_2}Y(s) = \frac{X(s)-Y(s)}{R_1}$$

$$\therefore\ Y(s)\left(0.5s + \frac{10}{s} + 2\right) = X(s)$$

$$\frac{Y(s)}{X(s)} = H(s) = \frac{1}{0.5s + \frac{10}{s} + 2} = \frac{2s}{s^2+4s+20}$$

$$= \frac{2s}{(s+2)^2+16} = \frac{2s}{(s+2)^2+4^2}$$

ROC.

$Re(s) > -2$

稳定.

$$\therefore\ \cos(4t)u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{4}{s^2+4^2}$$

$$\sin(4t)u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{s}{s^2+4^2}.$$

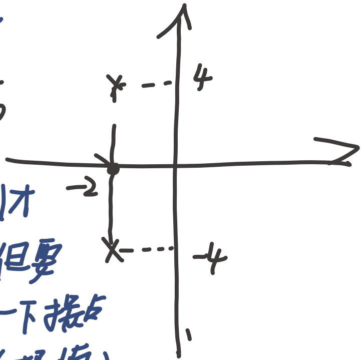
又: $e^{-2t}x(t) \xleftrightarrow{\mathcal{L}} X(s+2)$

$$h(t) = \mathcal{L}^{-1}\left[\frac{2\boxed{(s+2)}-4}{(s+2)^2+4}\right]$$ 形式不动

$$= e^{-2t}(2\cos 4t - \sin 4t)u(t)$$

留s.可自推.

(3) $$H(j\omega) = \frac{2j\omega}{-\omega^2+4j\omega+20}$$



$$= \frac{2j\omega}{(j\omega+2)^2+4}$$

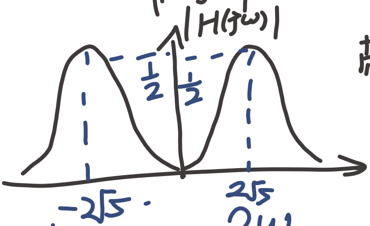
这种相比刚才其实不好算 但要算一下拐点(极值)

当 $\omega \to 0$ $|H(j\omega)| \to 0$

$\omega \to \infty$ $|H(j\omega)| \to 0$

$0<\omega<\infty$. $|H(j\omega)| \neq 0$

∴



带通的

$$|H(j\omega)| = \frac{2\omega}{\sqrt{(20-\omega^2)^2+16\omega^2}} = \frac{2\omega}{\sqrt{400-40\omega^2+\omega^4+16\omega^2}}$$

$$= \frac{2}{\sqrt{\frac{400}{\omega^2}-24+\omega^2}} \le \frac{2}{\sqrt{16}} = \frac{1}{2}$$

∵ $\frac{400}{\omega^2}+\omega^2-24 \geq 16$.

$|\omega| = 2\sqrt{5}$ 时 取.

10-2-2 $R_1=30\Omega$ $R_2=10\Omega$