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13 (a). 代入 $h[n]=(\frac{1}{5})^n u[n]$.

$(\frac{1}{5})^n u[n] - A(\frac{1}{5})^{n-1}u[n-1]=\delta[n]$

可知 $n=1$ 时.

$\frac{1}{5}-A=0$

$\therefore A=\frac{1}{5}$

$\frac{1}{1-\frac{1}{5}e^{-j\omega}} - A\frac{e^{-j\omega}}{1-\frac{1}{5}e^{-j\omega}}=1$

频域中.

$$\boxed{\frac{1-Ae^{-j\omega}}{1-\frac{1}{5}e^{-j\omega}}=1.}$$

(b) 已知 $h[n]-\frac{1}{5}h[n-1]=\delta[n]$

$\because$ $h[n]$ 为输入 $x[n]$ 的单位脉冲响应.

$y_1[n]=x[n]*h[n]$

对于 S_2 的脉冲响应 $g[n]$. 有 $\underbrace{\boxed{x[n]*h[n]}}_{输出\ y_2[n]=}*g[n]=x[n]$

$\therefore$ $h[n]*g[n]=\delta[n]$. 由(a)我们知

$h[n]-\frac{1}{5}h[n-1]=\delta[n]$

$\therefore$ $g[n]=\delta[n]-\frac{1}{5}\delta[n-1]$

巧妙

逆 S_2 : $g[n]$

$g[n]$ 导致了 S_2 是个逆系统

2.19 (a) $w[n]-\frac{1}{2}w[n-1]=x[n]$ ①

$y[n]-\alpha y[n-1]=\beta w[n]$ ② 对②进行时移

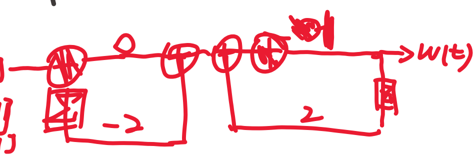
$\therefore$ $y[n-1]-\alpha y[n-2]=\beta w[n-1]$ ③

将②,③代入①

累加框图

$w[n+1]=\frac{1}{2}w[n]+x[n+1]$

$w[n]=2w[n+1]-2x[n+1]$



$$\frac{1}{\beta}y[n]-\frac{\alpha}{\beta}y[n-1]-\frac{1}{2}\left(\frac{1}{\beta}y[n-1]-\frac{\alpha}{\beta}y[n-2]\right)=x[n]$$

$$\frac{1}{\beta}y[n]-\left(\frac{\alpha}{\beta}+\frac{1}{2}\right)y[n-1]+\frac{\alpha}{2\beta}y[n-2]=x[n]$$

$$\frac{1}{\beta}y[n]=\frac{\alpha}{2\beta}y[n-2]+\left(\frac{\alpha}{\beta}+\frac{1}{2}\right)y[n-1]+x[n]$$

则满足方程组 $\begin{cases}\frac{\alpha}{2}=-\frac{1}{8}\\ \alpha+\frac{1}{2}\beta=\frac{3}{4}\end{cases} \Rightarrow \begin{cases}\alpha=-\frac{1}{4}\\ \beta=2\end{cases}$

(b) 要求单位脉冲响应 $g[n]$ 则令 $x[n]=\delta[n]$ √

$$g[n]+\frac{1}{8}g[n-2]-\frac{3}{4}g[n-1]=\delta[n]$$

特征方程为 $s^2-\frac{3}{4}s+\frac{1}{8}=0$ $(s-\frac{1}{2})(s-\frac{1}{4})=0$ $s_1=\frac{1}{2}$ $s_2=\frac{1}{4}$

∴设齐次解为 $g[n]=\left[c_1(\frac{1}{2})^n+C_2(\frac{1}{4})^n\right]u[n]$ 在差分中 令 $n=0$

$g[0]=C_1+C_2=\delta[0]=1$

则 $C_1+C_2=1$ ①

令 $n=1$ 有 $g[1]+\frac{1}{8}g[-1]-\frac{3}{4}g[0]=\delta[1]=0$

则 $\frac{1}{2}C_1+\frac{1}{4}C_2-\frac{3}{4}C_1-\frac{3}{4}C_2=0$ ② 联立①② $\begin{cases}C_1=2\\ C_2=-1\end{cases}$

∴单位脉冲响应 $g[n]=\left\{2(\frac{1}{2})^n-(\frac{1}{4})^n\right\}u[n]$

$0<t<1$

2.22 (b) $x(t)=u(t)-2u(t-2)+u(t-5)$ $h(t)=e^{2t}u(1-t)$

∵ $y(t)=x(t)*h(t)=[u(t)-2u(t-2)+u(t-5)]*e^{2t}u(1-t)$

$=\left\{[u(t)-u(t-2)]-[u(t-2)-u(t-5)]\right\}*e^{2t}u(1-t)$

$$=\int_0^2 e^{2(t-\tau)}u[1-(t-\tau)]d\tau-\int_2^5 e^{2(t-\tau)}u[1-(t-\tau)]d\tau$$

①对前一项. $t-1<0$ 时

-s)

$$\int_0^2 h(t-\tau)d\tau=\int_0^2 e^{2(t-\tau)}d\tau=-\frac{1}{2}(e^{2(t-2)}-e^{2t}).$$

$1<t<3$时 $t-1$作为下限

$$\int_0^2 h(t-\tau)d\tau = \int_{t-1}^2 e^{2(t-\tau)}d\tau = -\frac{1}{2}(e^{2(t-2)} - e^2)$$

② 对后一项 $\int_2^5 h(t-\tau)d\tau = \int_2^5 e^{2(t-\tau)}u[\tau-(t-1)]d\tau.$

当 $t-1<2$ 即 $t<3$时 $\int_2^5 h(t-\tau)d\tau = \int_2^5 e^{2(t-\tau)}d\tau$

$$= -\frac{1}{2}(e^{2(t-5)} - e^{2(t-2)})$$

当 $2<t-1<5$ $3<t<6$

$$\int_2^5 h(t-\tau)d\tau = \int_{t-1}^5 e^{2(t-\tau)}d\tau = -\frac{1}{2}(e^{2(t-5)} - e^2).$$

1). 当 $t<1$时

$$x(t)*h(t) = -\frac{1}{2}(e^{2(t-2)} - e^{2t}) - (-\frac{1}{2})(e^{2(t-5)} - e^{2(t-2)})$$

$$= \frac{1}{2}[e^{2t} - 2e^{2(t-2)} + e^{2(t-5)}]$$

2) 当 $1<t<3$

$$x(t)*h(t) = -\frac{1}{2}(e^{2(t-2)} - e^2) + \frac{1}{2}(e^{2(t-5)} - e^{2(t-2)})$$

$$= \frac{1}{2}(e^{2(t-5)} - e^2)$$

3) $3<t<6$

$$x(t)*h(t) = -\frac{1}{2}(e^{2(t-5)} - e^2) = \frac{1}{2}(e^2 - e^{2(t-5)})$$

4) $t>6$ $x(t)*h(t) = 0$

(c) $x(t)=\sin\pi t[u(t)-u(t-2)]$

$h(t)=2[u(t-1)-u(t-3)]$

$$x(t)*h(t)=2\int_{-\infty}^{+\infty}\sin\pi\tau[u(\tau)-u(\tau-2)][u(t-\tau-1)-u(t-\tau-3)]d\tau$$

$$=2\int_{-\infty}^{+\infty}\sin\pi\tau\, u(\tau)u(t-\tau-1)d\tau-2\int_{-\infty}^{+\infty}\sin\pi\tau\, u(\tau)u(t-\tau-3)d\tau$$

$$-2\int_{-\infty}^{+\infty}\sin\pi\tau\, u(\tau-2)u(t-\tau-1)d\tau+2\int_{-\infty}^{+\infty}\sin\pi\tau\, u(\tau-2)u(t-\tau-3)d\tau$$

分别记为①②,③,④式

①式: 当 $t-1>0$ $t>1$ 否则无取值, 等效为乘 $u(t-1)$.

有 $① = 2\int_0^{t-1}\sin\pi\tau\, d\tau=-\frac{2}{\pi}(\cos\pi(t-1)-1)=\frac{2}{\pi}[1-\cos\pi(t-1)]$

②式 当 $t-3>0$ $t>3$. 才有取值, 等效为乘 $u(t-3)$.

$② = 2\int_0^{t-3}\sin\pi\tau\, d\tau=\frac{2}{\pi}[1-\cos\pi(t-3)]$

③式. 当 $t-1>2$, $t>3$ 才有取值 等效为乘 $u(t-3)$

$③ = 2\int_2^{t-1}\sin\pi\tau\, d\tau=\frac{2}{\pi}(\cos 2\pi-\cos\pi(t-1))=\frac{2}{\pi}(1-\cos\pi(t-1))$

④式 当 $t-3>2$. $t>5$才有取值, 等效为乘 $u(t-5)$.

$④ = 2\int_3^{t-2}\sin\pi\tau\, d\tau=\frac{2}{\pi}[-1-\cos\pi(t-2)]$

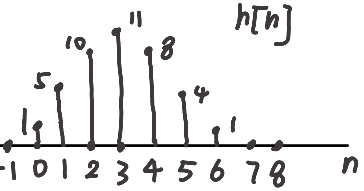
原式 $=\frac{2}{\pi}[1-\cos\pi(t-1)]u(t-1)-\frac{2}{\pi}[1-\cos\pi(t-3)]u(t-3)$

$-\frac{2}{\pi}(1-\cos\pi(t-1)]u(t-3)+\frac{2}{\pi}[1-\cos\pi(t-2)]u(t-5)$

2.24 (a) 在离散LTI系统中

$h_2[n]=u[n]-u[n-2]=\delta[n]+\delta[n-1]$

$h_2[n]*h_2[n]=\delta[n]+2\delta[n-1]+\delta[n-2]$



对单位脉冲响应 $h[n]$.

$$h[n]=\delta[n]*h_1[n]*h_2[n]*h_2[n]$$
$$=\delta[n]*h_1[n]*\{\delta[n]+2\delta[n-1]+\delta[n-2]\}$$
$$=\delta[n]*\{h_1[n]+2h_1[n-1]+h_1[n-2]\}$$
$$=h_1[n]+2h_1[n-1]+h_2[n-2]$$

$h[0]=h_1[0]+2h_1[-1]+h_1[-2]=h_1[0]=1$

$h[1]=h_1[1]+2h_1[0]+h_1[-1]=5 \Rightarrow h_1[1]=3$

$h[2]=h_1[2]+2h_1[1]+h_1[0]=10 \Rightarrow h_1[2]=3$

$h[3]=h_1[3]+2h_1[2]+h_1[1]=11 \Rightarrow h_1[3]=2$

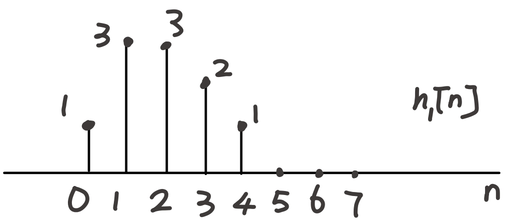
$h[4]=h_1[4]+2h_1[3]+h_1[2]=8 \Rightarrow h_1[4]=1$

$h[5]=h_1[5]+2h_1[4]+h_1[3]=4 \Rightarrow h_1[5]=0$

$h[6]=h_1[6]+2h_1[5]+h_1[4]=1 \Rightarrow h_1[6]=0$

$h[7]=h_1[7]+2h_1[6]+h_1[5]=0 \Rightarrow h_1[7]=0$

$h[8]=0$



(b) 系统工作图示为

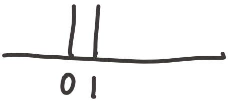
$x[n]\rightarrow \boxed{h_1[n]} \rightarrow \boxed{h_2[n]} \rightarrow \boxed{h_2[n]} \rightarrow y[n]$

$x[n]=\delta[n]-\delta[n-1]$

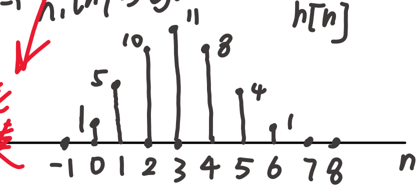
2 24 应回味一下 $h_2[n] = u[n]-u[n-2] = \delta[n]+\delta[n-1]$

$\delta[n] \to h_1[n] \to h_2[n] \to h_2[n] \to$ ______ ?

可以先经过 $h_2[n] \to h_2[n] \to h_1[n] \to y[n]$



函数化的交换



$h_2[n]*h_2[n] = \delta[n]+2\delta[n-1]+\delta[n-2]$,

$h_1[n]$ $\underbrace{\delta[n]+2\delta[n-1]}+\delta[n-2] =$

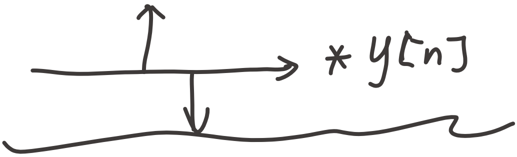
一个一个来

$y[0]=0 = h_1[0]$. $1 = 1 \Rightarrow h_1[0]=1$

⋯

可画出 $h_1[n]$ 了.

$h_1[n]$ 是冲激串

(b).  $* \; y[n]$ —

响应 $y[n] = x[n] * h[n]$

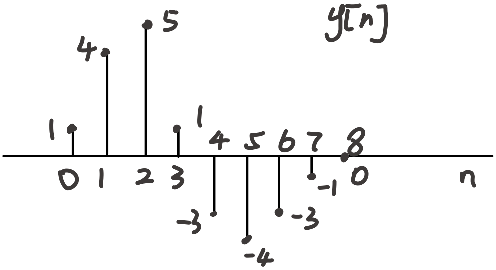
$= h[n] - h[n-1]$

$y[0] = h[0] = 1.\quad y[1] = 4 \quad y[2] = 5$

$y[3] = 1 \quad y[4] = -3 \quad y[5] = -4 \quad y[6] = -3$

$y[7] = -1 \quad y[8] = 0$

∴ $y[n]$ 为右图所示



有一些没学的系统断性方法 .

2.28(a) $h[n] = (\frac{1}{5})^n u[n]$

① 因果性: ∵ $n<0 \quad h[n]=0$ 是因果的

② 稳定性 ∵ $\sum_{k=-\infty}^{+\infty} h[k] = \lim_{k\to\infty} \frac{1-(\frac{1}{5})^k}{1-\frac{1}{5}} < \infty$. 是稳定的

(c) $h[n] = (\frac{1}{2})^n u[-n]$

① 因果性 $n<0 \quad h[n] \neq 0$ 是非因果的

② 稳定性 ∵ $\sum_{k=-\infty}^{+\infty} h[k] = \sum_{k=0}^{+\infty} 2^n = \infty$ 不是稳定的.

$\sum_{-\infty}^{0} u[-n](\frac{1}{2})^n$

$\sum (\frac{1}{2})^{-n} = \boxed{2^n}$

(g) $h[n] = n(\frac{1}{3})^n u[n-1]$

① 因果性 $n<0 \quad h[n]=0$ 是因果的

② 稳定性 ∵ $\sum_{k=-\infty}^{+\infty} n(\frac{1}{3})^n u[n-1] = \sum_{k=1}^{\infty} k(\frac{1}{3})^k$

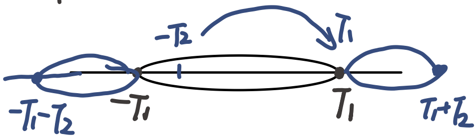
$\lim_{k\to\infty} \sqrt[k]{k(\frac{1}{3})^k} = \sqrt[k]{k}\ \frac{1}{3} = \frac{1}{3} < 1$ 由级数审敛性 级数收敛.

∴ 是稳定的

2 44(a). $x(\tau)=\begin{cases} x(\tau) & |t|\le T_1 \\ 0 & |t|>T_1 \end{cases}$

$h(\tau)=\begin{cases} h(\tau) & |t|\le T_2 \\ 0 & |t|>T_2 \end{cases}$ $\quad h(-\tau)=\begin{cases} h(-\tau) & |t|\le T_2 \\ 0 & |\tau|>T_2 \end{cases}$

$h(t-\tau)=\begin{cases} h(t-\tau) & |t-\tau|\le T_2 \\ 0 & |t-\tau|>T_2 \end{cases}$



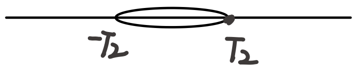
要使 $x(t)*h(t)=0$

t应满足(t作常数)

$$\int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau.$$

$$=\int_{t}^{+\infty} x(\tau)h(t-\tau)d\tau$$

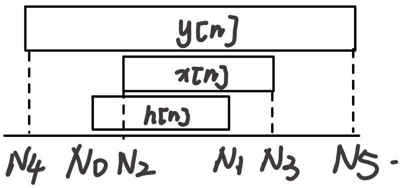
$$+\int_{-\infty}^{-t} x(\tau)h(t-\tau)d\tau.$$



在这2个区间 .$|t|$增大后
非0区间不再产生交集

$T_3=t=T_1+T_2$

(b)(i) 如右图所示:



$x[n]=x[n]\{u[n-N_2]-u[n-N_3-1]\}$

$h[n]=h[n]\{u[n-N_0]-u[n-N_1-1]\}$

$$y[n]=x[n]*h[n]=\sum_{k=-\infty}^{+\infty} x[k]\cdot h[n-k]=\sum_{k=N_2}^{N_3} x[k]\cdot h[n-k]$$

$h[n-k]\neq 0$ 则 $N_0\le n-k\le N_1$

$\therefore n-N_1\le k\le n-N_0$

$\because n-N_1\le N_3$.且 $n-N_0\ge N_2$ 时 $x[k]h[n-k]\neq 0$

$\because n<N_0+N_2$ 或 $n>N_1+N_3$ 时 $x[k]h[n-k]=0$

$\therefore N_4=N_0+N_2 \quad N_5=N_1+N_3$

(ii) $M_y = M_h+M_x-1$