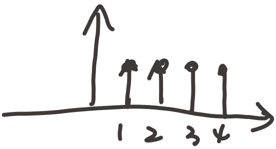
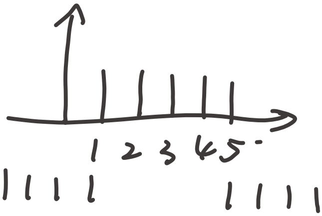
6-1-1



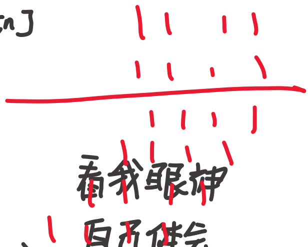


$\boxed{4+5-1}=8.$

6-1-2 $h[n]=\delta[n]+\delta[n-1]+\delta[n-2]+\delta[n-3]$

$x[n]$, $h[n]$

不进位法.

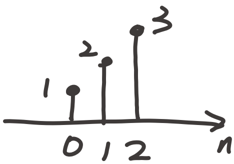


看我眼神

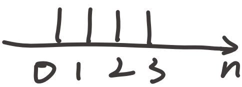
自己体会

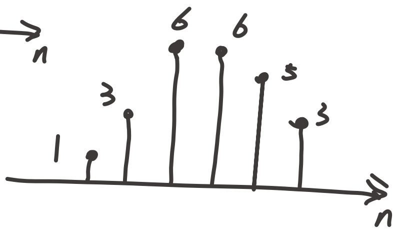
$\boxed{1\ 2\ 3\ 4\ 3\ 2\ 1}$

6-1-3 $f_1[n]=$



$f_2[n]$

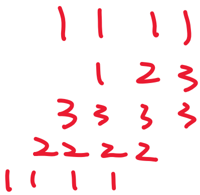




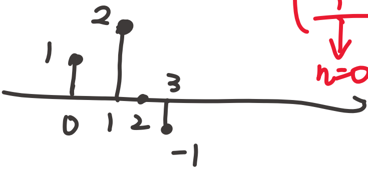
$$f[0]=\sum_{k=-\infty}^{+\infty} f_1[k]\cdot f_2[0-k]=1$$

$f[1]=3.$ $f[2]=6$ $f[3]=6$

$f[4]=5$ $f[5]=3$ $f[6]=0$



6-1-4 $f[n]$



3 6 6 5 3

$n=0$ $n=5$.

$h[n]$

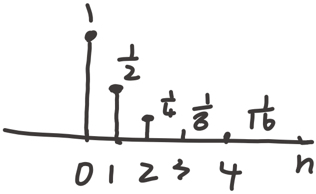
$$y[n]=\sum_{k=-\infty}^{+\infty} f[k]\,h[n-k]$$

$y[-1]=2$ $\quad y[0]=4.$

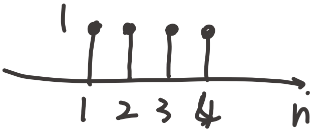
$y[1]=2$ $\quad y[2]=2$

$y[3]=0$ $\quad y[4]=-2.$

6-1-5 $x_1[n]=\left(\frac{1}{2}\right)^n\left(u[n]-u[n-5]\right)$



$x_2[n]$.



用竖式牛逼.

$$y[n]=x_1[n]*x_2[n]=\sum_{k=-\infty}^{+\infty} x_1[k]\,x_2[n-k]$$

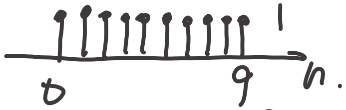
$y[0]=1$ $\quad y[1]=\frac{3}{2}$ $\quad y[2]=1\frac{3}{4}$ $\quad y[3]=1\frac{7}{8}$

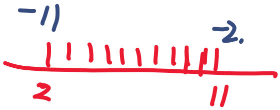
$y[4]=\frac{15}{16}.$ $\quad y[5]=\frac{7}{16}$ $\quad y[6]=\frac{3}{16}$ $\quad y[7]=\frac{1}{16}$

$y[8]=0$

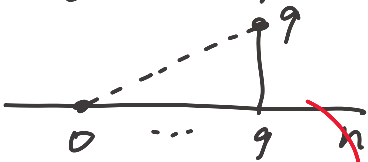
6-1-6.

$R_{10}[n]=$





$nR_{10}[n]=$



但如果取翻转

$$y[9]=\sum_{k=-\infty}^{+\infty} kR_{10}[k]\,R_{10}[9-k]$$

$$=\cancel{n}+\cancel{n}+1+\cdots+\cancel{n}+9$$

$$=10n+\sum_{k=1}^{9}k=\cancel{10n}+\boxed{45}$$

10 序列

$$\boxed{R_{10}[n]=\begin{cases}1 & 0\sim 9\\ 0 & \text{其他}\end{cases}}$$

6-1-7. $\alpha>1$ $h[n]=\sum_{n=\infty}^{+\infty}\{u[k]-u[k-N]\}\,\alpha^{n-k}u[n-k]$

$$=\sum_{k=0}^{N}\alpha^{n-k}u[n-k]$$

却忘讨论 $\alpha=1$

① $n<0$ $h[n]=0$

② $0<n<N$ $h[n]=\sum_{k=n}^{N}\alpha^{n-k}=\frac{(1-\alpha^N)}{1-\alpha}$

③ $n>N$ $h[n]=\sum_{k=0}^{N}\alpha^{n-k}=\frac{\alpha^{-k}(1-\alpha^N)}{1-\alpha}$

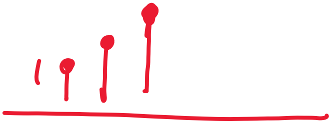
实际 α 大于 or 小于1不重要

$\alpha=1$ $u[n]-u[n-N]*u[n]$

$$=nu[n]-u[n-N]\,u[n-N]+\sum_{k=0}^{N}\delta[k]$$ n-

$=$

$$u[n] * \alpha^n u[n] = \sum \alpha^m = \begin{cases} \dfrac{1-\alpha^{n+1}}{1-\alpha} u[n] & \alpha \neq 1 \\ \boxed{(n+1)u[n]} & \end{cases}$$



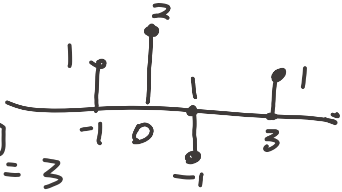
不同点在于 $n=0$ 处取1 故+1

6-2-1. $X(e^{j0}) = \sum_{-\infty}^{+\infty} x[n]$

$= 11$

$\int_{-\pi}^{\pi} X(e^{j\omega}) d\omega = 2\pi x[0] = 8\pi.$

6-2-2 (1) $x[n]$



$X(e^{j0}) = \sum_{-\infty}^{\infty} x[n] = 3$

$X(e^{j\pi}) = \sum_{n=-\infty}^{+\infty} x[n] e^{-j\pi n}$ $\quad T = \frac{2\pi}{\pi} = 2.$

$= \sum_{n=-\infty}^{+\infty} x[n] \left[\cos(\pi n) + j \sin \pi n\right]$

$= \sum_{n=-\infty}^{+\infty} x[n] (-1)^n. = -1+2+1-1 = 1.$

$\therefore X(e^{j0}) \cdot X(e^{j\pi}) = 3$

(2) $\int_{-\pi}^{\pi} |X(e^{jw})|^2 dw = 2\pi \sum_{n=-\infty}^{+\infty} |x[n]|^2$

$= 2\pi(1+4+1+1) = 14\pi$

6-2-3 $\sum_{n=-\infty}^{+\infty} x[n] = X(e^{j0}) =$

令 $g[n] = n\, x[n]$

$\mathcal{F}\{g[n]\} = G(e^{jw}) = j\dfrac{d\,X(e^{jw})}{dw}$

$\sum_{-\infty}^{+\infty} n\, x[n] = G(e^{j0}) = j \cdot \dfrac{X(e^{jw})}{dw}\Big|_{jw=0}$

6-3-1

A $x[-n] \xrightarrow{\mathcal{F}} \sum_{n=-\infty}^{+\infty} x[-n] e^{-j\omega n}$

$= \sum_{n=-\infty}^{+\infty} x[-n] e^{-j(-\omega)(-n)} = X(e^{-j\omega})$

B $x[\frac{n}{2}] \xrightarrow{\mathcal{F}} \sum_{n=-\infty}^{+\infty} x[\frac{n}{2}] e^{-j\omega n}$

$= \sum_{n=-\infty}^{+\infty} x[\frac{n}{2}] e^{-(2j\omega)\frac{n}{2}} = X(e^{2j\omega})$

X

C 也无系数

6-3-2 $x[n] = n\alpha^n u[n+2]$ 不太懂午

$x[n] \xrightarrow{\mathcal{F}} \sum_{-\infty}^{+\infty} n\alpha^n u[n+2]$

$= \sum_{n=-2}^{+\infty} n\alpha^n$

X

$x[n] = n\alpha^n u[n] + (-1)\alpha^{-1}\delta[n+1] + (-2)\alpha^{-2}\delta[n+2]$

$\xrightarrow{\mathcal{F}} \sum_{-\infty}^{\infty} x[n] e^{-j\omega n} = \sum_{0}^{+\infty} n\alpha^n e^{-j\omega n} - \alpha^{-1} e^{+j\omega} - 2\alpha^{-2} e^{2j\omega}$

$\because \alpha^n u[n] \xrightarrow{\mathcal{F}} \frac{1}{1-\alpha e^{-j\omega}}$

$n\alpha^n u[n+2] \xrightarrow{\mathcal{F}} j \frac{-1}{(1-\alpha e^{-j\omega})^2}$

Z变换别做了

6-3-3. $x[n]=(n-1)(\frac{1}{3})^{|n|}$

① 去绝对值 /注 0 是算正面 ☆

$$x[n]=(n-1)(\frac{1}{3})^n u[n]+(n-1)(\frac{1}{3})^{-n}u[-n-1]$$

离散的就有这一个问题，但边界却更明显. $n=0$ 对 $u[n]$ 来说是 1，须注意.

$$(\frac{1}{3})^n u[n] \xrightarrow{\mathcal{F}} \frac{1}{1-\frac{1}{3}e^{-j\omega}}$$

$$n(\frac{1}{3})^n u[n] \xrightarrow{\mathcal{F}} j\cdot\frac{d(\frac{1}{1-\frac{1}{3}e^{-j\omega}})}{d\omega}=\frac{\frac{1}{3}e^{-j\omega}}{(1-\frac{1}{3}e^{-j\omega})^2}$$

$$n(\frac{1}{3})^{-n}u[-n-1] \xrightarrow{\mathcal{F}} j\cdot\frac{d(\frac{1}{1-3e^{-j\omega}})}{d\omega}=\frac{-3e^{-j\omega}}{(1-3e^{-j\omega})^2}$$

$$X(e^{j\omega})=\frac{\frac{1}{3}e^{-j\omega}}{(1-\frac{1}{3}e^{-j\omega})^2}-\frac{1}{1-\frac{1}{3}e^{-j\omega}}+\frac{-3e^{-j\omega}}{(1-3e^{-j\omega})^2}+\frac{1}{1-3e^{-j\omega}}.$$

$$(\frac{1}{3})^{-n}u[-n-1] \xrightarrow{\mathcal{F}} -\frac{1}{1-3e^{-j\omega}}$$

$$(\frac{1}{3})^{-n}u[-n-1] \xrightarrow{\mathcal{F}} \sum_{n=-\infty}^{+\infty}(\frac{1}{3})^{-n}u[-n-1]e^{-j\omega n}$$

$$=\sum_{n=-\infty}^{-1}(\frac{1}{3})^{-n}e^{-j\omega n}=\sum_{1}^{+\infty}(\frac{1}{3})^n e^{j\omega n}.$$

$$=\sum_{1}^{+\infty}(\frac{1}{3}e^{j\omega})^n=\frac{\frac{1}{3}e^{j\omega}(1)-(\cancel{\frac{1}{3}e^{j\omega}})^n)}{1-\frac{1}{3}e^{j\omega}}$$

自己具体这样时能做.

$\lim_{n\to\infty}(\frac{1}{3}e^{j\omega})^n$. 收敛的

因为 $(\frac{1}{3})$ 收敛

$$=\frac{1}{3e^{-j\omega}-1}$$

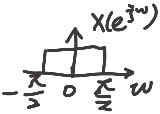
$$=\frac{-1}{1-3e^{-j\omega}}$$

6-3-4 既然分母没2个n 可以不凑

$x[n] = \frac{\sin(\frac{n\pi}{2})}{n\pi} \quad \sin(\frac{n\pi}{2})$

$\sin(\frac{n\pi}{2}) \xrightarrow{\mathcal{F}} \frac{\pi}{j}\{\delta[\omega-\frac{\pi}{2}]-\delta[\omega+\frac{\pi}{2}]\}$

$\frac{\sin\frac{n\pi}{2}}{n\pi} \xleftrightarrow{\mathcal{F}}$



注：(这是单个周期)
→ 这不是周期离散。
是非周期的
频域连续 但周期。

周期连续信号 $\xRightarrow{频域周期}$ 周期离散信号

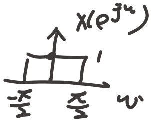
频域离散 ⇑ (周期连续信号)　　⇑ 频域离散。(周期离散信号)

非周期连续 $\xRightarrow{频域周期}$ 非周期离散信号

别记其他的

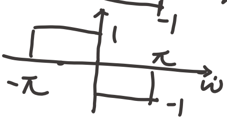
记 $\frac{\pi}{j}[\delta(\omega-\frac{\pi}{2})-\delta(\omega+\frac{\pi}{2})]$

$\therefore x[n] \xrightarrow{\mathcal{F}} \frac{1}{2\pi}\ \frac{\pi}{j}\{\delta[\omega-\frac{\pi}{2}] - \delta[\omega+\frac{\pi}{2}]\} *$



$= \frac{1}{2j}$

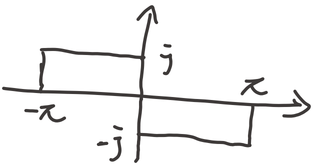
$= \frac{+j}{2}$



叫做. $\frac{j}{2}\left(G_\pi(\omega+\frac{\pi}{2}) - G_\pi(\omega-\frac{\pi}{2})\right)$

6-3-5.

对一个周期.



$\frac{1}{\pi}$.

$\frac{-j}{\pi} j.$

$G_\pi * [(\delta(\omega-\frac{\pi}{2})-\delta(\omega+\frac{\pi}{2})]$

$\frac{\pi}{j}$

$H(e^{j\omega}) = j[G_\pi(\omega+\frac{\pi}{2}) - G_\pi(\omega-\frac{\pi}{2})]$

$= \frac{1}{\pi} G_\pi(\omega) * \frac{\pi}{j}[\delta(\omega-\frac{\pi}{2}) - \delta(\omega+\frac{\pi}{2})]$

$\because G_\pi(\omega) \xrightarrow{\mathcal{F}} \frac{\sin(\frac{\pi n}{2})}{\pi n}$

$$\frac{\pi}{j}\left[\delta(\omega-\tfrac{\pi}{2})-\delta(\omega+\tfrac{\pi}{2})\right] \xrightarrow{\mathcal{F}^{-1}} \sin(\tfrac{\pi}{2}n).$$

$$h[n] = \mathcal{F}^{-1}\left\{\frac{1}{2\pi} G_{\pi}(\omega) * \frac{\pi}{j}(\quad)\right\}$$

$$= 2\cdot\frac{\sin\frac{\pi n}{2}}{\pi n}\sin\frac{\pi}{2}n$$

笑 想复杂了 但 n 为偶 $h[n]=0$.

n 为奇 $\sin\frac{\pi n}{2}=(-1)^n$

$$h[n] = 2\cdot\frac{(-1)^n}{\pi n}(-1)^n = \frac{2}{\pi n}.$$

没错. 还好, 但可以更容易.

不如直接逆变换.

$$h[n] = \frac{1}{2\pi}\int_{2\pi} -j\,\mathrm{sgn}(\omega)\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}(-j)\int_0^{\pi} e^{j\omega n} d\omega + \frac{-j}{2\pi}\int_{-\pi}^{0}(-1)e^{j\omega n} d\omega$$

$$= \frac{-j}{2\pi}\,\frac{e^{j\omega n}\big|_0^{\pi}}{jn} + \frac{+j}{2\pi}\,\frac{e^{j\omega n}\big|_{-\pi}^{0}}{jn}.$$

$$= \frac{-((-1)^n-1)}{2\pi n} + \frac{1-(-1)^n}{2\pi n} = \frac{1-(-1)^n}{\pi n} = \begin{cases}\frac{2}{\pi n} & 奇\\ 0 & 偶\end{cases}$$

6-3-6 $y[-2]=0$ $y[-1]=1$

$y[0]=1$ $y[1]=2$. $y[2]=4$ $y[3]=4\ldots$

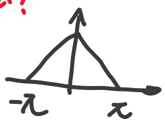
6-3-9. (1) $y_1[n] = x[n]e^{j\pi n}$.

$$e^{j\pi n} \xrightarrow{\mathcal{F}} \sum_{n=-\infty}^{+\infty} e^{j\pi n} e^{-j\omega n} = \sum_{-\infty}^{+\infty} e^{-j(\omega+\pi)n}.$$

$$= \delta(\omega+\pi)$$ 为什么?

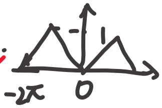
这个2π是无用

$$y_1[n] \xrightarrow{\mathcal{F}} \cancel{\frac{1}{2\pi}}\, \delta(\omega+\pi) *$$



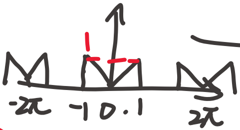
π.

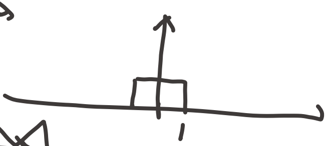
$= \cancel{\frac{1}{2}}$.



$y_2[n] \xrightarrow{\mathcal{F}}$

$\frac{1}{2}$



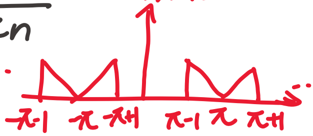


$\because (-1)^n = e^{j\pi n} = \cos\pi n + j\sin\pi n = (-1)^n$

再频移

$$h[n] = \frac{\sin n}{\pi n}$$

$y[n] =$



(2) ① 乘法器 $X(e^{j(\omega-\pi)})$

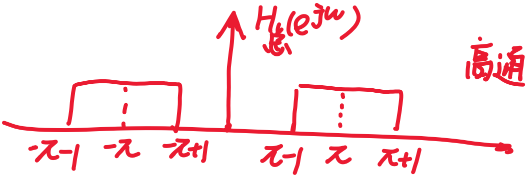
② $H(e^{j\omega})$ $X(e^{j(\omega-\pi)})H_1(e^{j\omega})$

③ 乘法器 $X(e^{j(\omega-2\pi)})H(e^{j(\omega-\pi)})$

$$= X(e^{j\omega})H(e^{j(\omega-\pi)}).$$

$$\therefore H_{总}(e^{j\omega}) = H(e^{j(\omega-\pi)}).$$

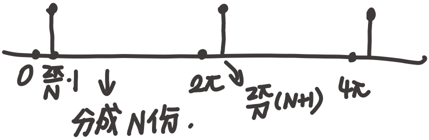
整体响应



高通.

6-4-1.

$\Downarrow$ DFT.



$$e^{j\omega n}$$

$$j$$

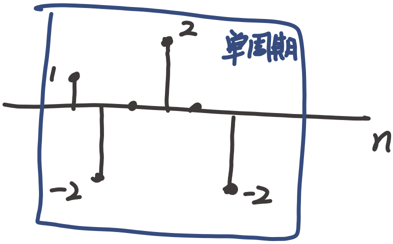
$$\frac{1}{N}\sum_{n=\langle N\rangle} x[n]e^{-jk\frac{2\pi}{N}n}$$

$$\sum_{-\infty}^{+\infty} x[n]e^{-j\omega n}$$

$$\int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}.$$

$a_1 = j \quad a_2 = -2j$ $\quad a_0 = 0$

$a_{-1} = -j \quad a_{-2} = 2j.$



$x(k) = \sum^{5}$

6-4-4. 求 关于卷积 级数.

$C_0 = \sum$

$$C_k = \sum_{l=0}^{N-1} a_l b_{k-l} = \sum_{0}^{3} a_l b_{k-l}$$

$$C_0 = \sum_{0}^{3} a_l b_{0-l}$$

$$= a_0 b_0 + a_1 b_3 + a_2 b_2 + a_3 b_1$$

C_3 易得.

$$T = \frac{2\pi}{\frac{3\pi}{5}} = \frac{10}{3} \Rightarrow T = 10.$$

$$x[n] = 1 + \frac{1}{2}\left(e^{j\frac{3\pi(n+1)}{5}} + e^{-j\frac{3\pi(n+1)}{5}}\right).$$

$$= 1 + \frac{1}{2} \cdot e^{j\frac{3\pi}{5}} e^{j\frac{3\pi}{5}} + \frac{1}{2} e^{-j\frac{3\pi}{5}} e^{-j\frac{3\pi}{5}n}$$

$a_3 = \frac{1}{2} e^{j\frac{3\pi}{5}} \quad a_0 = 1$

$a_{-3} = \frac{1}{2} e^{-j\frac{3\pi}{5}} = a_7$

$y[n] \overset{DFS}{\longleftrightarrow} b_k$.

$b_0 = H(e^{j0})\, a_0 = 1 \;\; 1 = 1$

$b_3 = H(e^{j\frac{3\pi}{5}})\, a_3 = \frac{1}{2} e^{j\frac{3\pi}{5}} \, \frac{1}{3}(1 + 2\cos\frac{3}{5}\pi)$

$b_{-3} = H(e^{-j\frac{3\pi}{5}})\, a_{-3}$

$\downarrow$

$b_7 = \frac{1}{2} e^{-j\frac{3\pi}{5}} \cdot \frac{1}{3}(1 + 2\cos\frac{3}{5}\pi)$