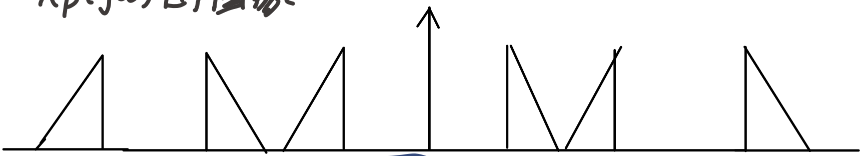
7.26 $x_r(t)=x(t)$ 时

对于采样 $P(j\omega)=\frac{2\pi}{T}\sum_{k=-\infty}^{\infty}\delta(\omega-k\frac{2\pi}{T})$

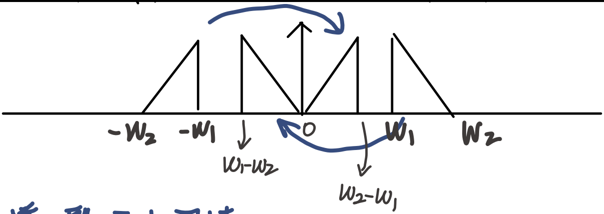
∴ $X_p(j\omega)=\frac{1}{2\pi}\{X(j\omega)*P(j\omega)\}$

$=\frac{1}{T}X(j(\omega-k\frac{2\pi}{T}))$

$X_p(j\omega)$ 的图像：



当



考虑频谱搬运上图情况， $2\omega_1-\omega_2<\frac{2\pi}{T}-\omega_2<\omega_2$

当 $2\omega_1-\omega_2\geq 0$ 则 $0\leq\frac{2\pi}{T}-\omega_2\leq 2\omega_1-\omega_2$.

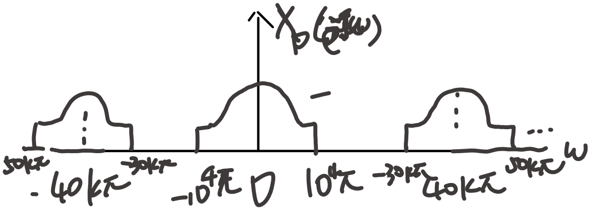
∴ 满足不混叠最大周期 $T=\frac{2\pi}{\omega_2}$.

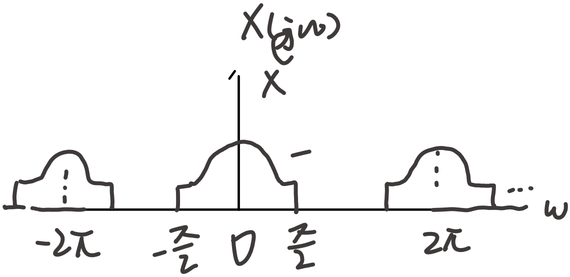
∴ $A=T$. $\omega_b=\frac{2\pi}{T}$. $\omega_a=\omega_b-\omega_1$

7.29 $f_s=\frac{1}{Ts}=20kHz$

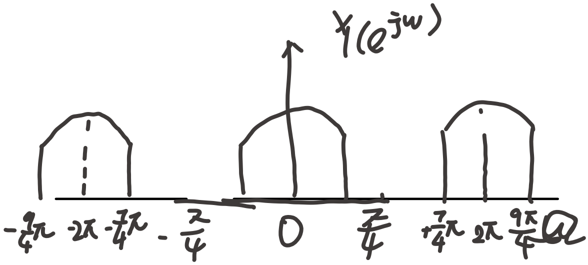
∴ $\omega_s=2\pi f_s=40\pi kHz$

∴ $X(j\omega)$ 和 $X_p(j\omega)$ 波形为.

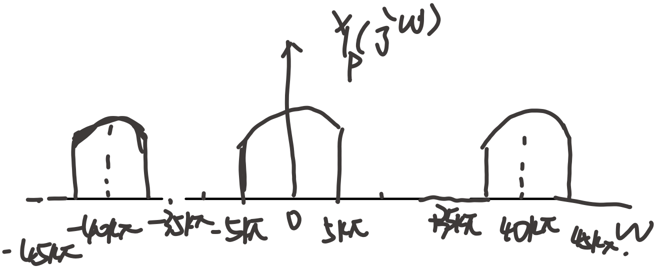


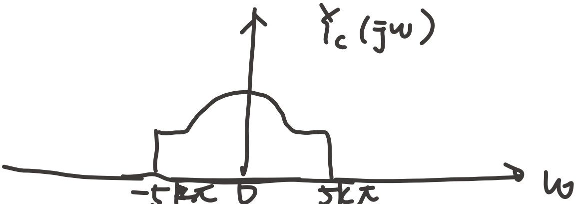


$$Y(e^{j\omega}) = H(e^{j\omega})\cancel{X}(e^{j\omega}) = X(e^{j\omega}).$$



$$\frac{\omega}{T} = \Omega$$





7.31 对 $y[n]=\frac{1}{2}y[n-1]+x[n]$

$$Y(e^{j\Omega})=\frac{1}{2}e^{-j\Omega}Y(e^{j\Omega})+X(e^{j\Omega})$$

$$\therefore H(e^{j\Omega})=\frac{1}{1-\frac{1}{2}e^{-j\Omega}}=\frac{2e^{j\Omega}}{2e^{j\Omega}-1}$$

$$\because x_p(t)=x_c(t)p(t)=x_c(t)\sum_{-\infty}^{\infty}\delta(t-nT)$$

$$\therefore x[n]=x_c(nT)$$

对 $H_c(j\omega)$ 首先 $x_c(t)$ 带限

$$\therefore X(e^{j\Omega})=\frac{1}{T}\sum_{-\infty}^{\infty}X_c\left(j\left(\frac{\Omega}{T}-\frac{2k\pi}{T}\right)\right)$$

$$Y(e^{j\Omega})=H(e^{j\Omega})X(e^{j\Omega})$$

从序列到冲激串

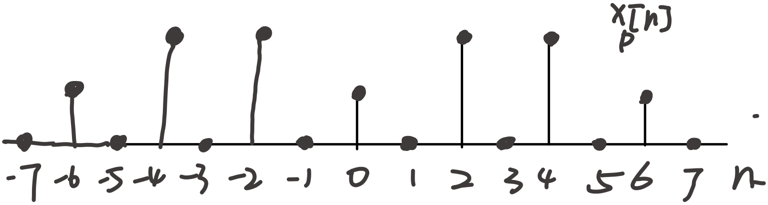
$$Y_c(j\omega)=H_c(j\omega)X_c(j\omega)$$

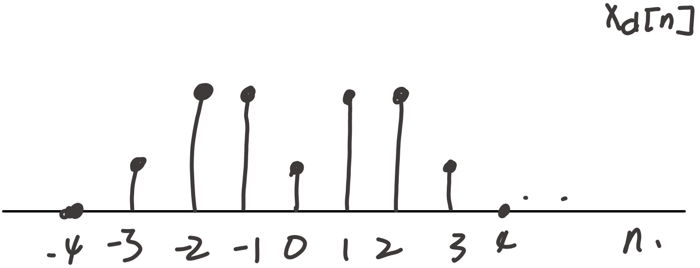
有 $H_c(j\omega)=H(e^{j\omega T})$

$\Omega=\omega T$ 则 $H_c(j\omega)=\frac{e^{j\omega T}}{e^{j\omega T}-\frac{1}{2}}$. $|\omega|<\frac{\pi}{T}$

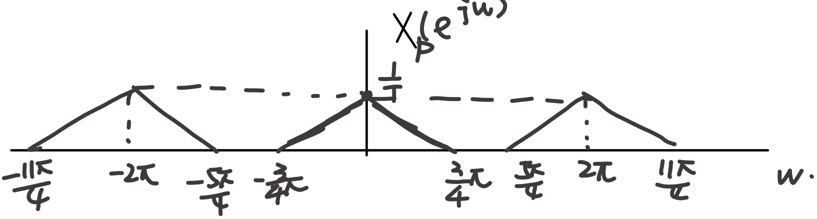
$$\therefore H_c(j\omega)=\begin{cases}\frac{e^{j\omega T}}{e^{j\omega T}-\frac{1}{2}} & |\omega|<\frac{\pi}{T}\\ 0. & |\omega|>\frac{\pi}{T}\end{cases}$$

7.35 (a) x_p 图象(插0)

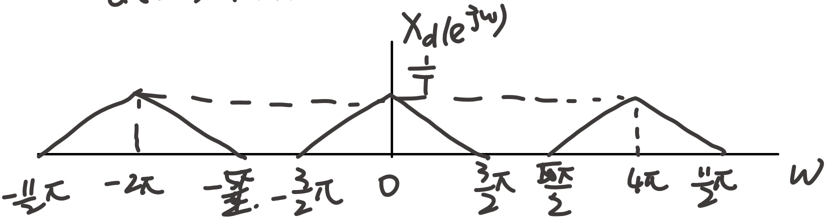




(b) $X_p(e^{jw})$ (采样)



$X_d(e^{jw})$ 内插



7.38 $x(t)=A+B\cos[(2\pi/T)t+\theta]$

最高频率 $\omega_m=\frac{2\pi}{T}$.

$\therefore f_s=\frac{\omega_s}{2}$ $\therefore \Delta=\frac{1}{2}f_M=\pi/\omega_M$

尺度变换后 $y(t)=x(at)$ $0<a<1$

$Y(j\omega)$ 带宽为 $(-a\omega_M, a\omega_M)$ $f_s=a\frac{\omega_m}{2}$

$T+\Delta=1/f_s=\pi/a\omega_M$

$\therefore \alpha=\frac{\Delta}{T+\Delta}$.

7.42 (a) $s[n]=s(nT_0)$

$S(j\omega)=X(j\omega)(1+\alpha e^{-j\omega T_0})$

采样后 $S(e^{j\Omega})$ 是 $S(j\omega)$ 以 $\frac{2\pi}{T_0}$ 为周期延拓.

对 $H(e^{j\Omega})$ $Y(e^{j\Omega})=H(e^{j\Omega})S(e^{j\Omega})$

要使 $y_c(t)\propto x(t)$ 则应消除 $1+\alpha e^{-j\omega T_0}$ 影响

令 $H(e^{j\Omega})=\frac{1}{1+\alpha e^{-j\Omega}}$ 逆变换. 有

$y[n]+\alpha y[n-1]=s[n]$

$\therefore y[n]+\alpha y[n-1]=x[n]+\alpha x[n-1]$

(b) $T=T_0<\frac{\pi}{\omega_m}$ 满足采样定理 为使 $y_c(t)\propto x(t)$.

则 $A=1$ (单位增益

(C) $\because T_0 \in (\frac{\pi}{\omega_M}, \frac{2\pi}{\omega_M})$，取$T=\frac{T_0}{2}$ 延拓变大可避免混叠 根据频谱缩放关系

$A=2$

7.52

(a) 对时限 $x(t)$，$x(t)$为非周期的

$p(t)=\sum_{-\infty}^{+\infty}\delta(t-kT)$ $p(t)*x(t)$ 相当于时域的 $x(t)$ 周期延拓

$\hat{x}(t) \xrightarrow{\mathcal{F}} \hat{X}(j\omega)=P(j\omega)X(j\omega)$

$\therefore \hat{x}(t)=\mathcal{F}^{-1}\{\hat{X}(j\omega)\}=p(t)*x(t)$

(b) $x(t)$时限，$\because \hat{X}(j\omega)=X(j\omega)P(j\omega)$

$P(j\omega)=\sum_{-\infty}^{\infty}\delta(\omega-k\omega_0)$，$\mathcal{F}\{\hat{x}(t)w(t)\}=\frac{1}{2\pi}\hat{X}(j\omega)*W(j\omega)$

$W(j\omega)=\int_{-\frac{\pi}{\omega_0}}^{\frac{\pi}{\omega_0}}\omega_0 e^{-j\omega t}dt=2\operatorname{sinc}\frac{\omega}{\omega_0}$

可以恢复出 $x(t)$

(c) $x(t)$在$|t|\geq\pi/\omega_0$时不限时，则$\hat{x}(t)$包含$x(t)$在$|t|\geq\pi/\omega_0$信息，产生时域混叠 无法恢复频域