6 15 (a). 进行傅立叶变换.

$$(j\omega)^2 Y(j\omega) + 4(j\omega)Y(j\omega) + 4Y(j\omega) = X(j\omega).$$

$$\therefore H(j\omega) = \frac{1}{(j\omega)^2 + 4j\omega + 4}$$

$$= \frac{1}{4\left[\left(\frac{j\omega}{2}\right)^2 + 2\frac{j\omega}{2} + 1\right]}$$

$$= \frac{1}{4}\frac{1}{\left[\left(\frac{j\omega}{2}\right)^2 + 2\frac{j\omega}{2} + 1\right]}$$

则 $\xi = 1$. 临界阻尼

(b) $$H(j\omega) = \frac{7}{5(j\omega)^2 + 4(j\omega) + 5}$$

$$= \frac{1}{\frac{5}{7}(j\omega)^2 + \frac{4}{7}(j\omega) + \frac{5}{7}}$$

$$= \frac{1}{\frac{5}{7}\left[(j\omega)^2 + \frac{4}{5}(j\omega) + 1\right]}$$

则 $\xi = \frac{2}{5}$. 欠阻尼

(c). $$H(j\omega)=\frac{1}{(j\omega)^2+20(j\omega)+1}$$

$$=\frac{1}{(j\omega)^2\cdot 2\ 10\ (j\omega)+1}$$

则 $\xi=10$ 过阻尼

(d) $$H(j\omega)=\frac{7+3(j\omega)}{5(j\omega)^2+4j\omega+5}$$

由于系统阻尼系数只与单位冲激响应有关

与输入其他信号无关

$$\therefore H(j\omega)=\frac{\frac{7}{5}+\frac{1}{15}(j\omega)}{(j\omega)^2+\frac{4}{5}(j\omega)+1}$$

$\xi=\frac{2}{5}$ 欠阻尼

6 17 (a) DTFT 的结果为

$$Y(e^{j\omega})+e^{-j\omega}Y(e^{j\omega})+\frac{1}{4}e^{-2j\omega}Y(e^{j\omega})=X(e^{j\omega})$$

$$\therefore H(e^{j\omega})=\frac{1}{1+e^{-j\omega}+\frac{1}{4}e^{-2j\omega}}$$

其中 $\begin{cases} -2r\cos\theta = 1 \\ r^2 = \frac{1}{4} \end{cases}$

其中 $r \in (0.1)$ $\theta \in [0, \pi]$

则 $r = \frac{1}{2}$ $\theta = \pi$

是振荡型的

(b). $H(e^{j\omega}) = \dfrac{1}{1 - e^{-j\omega} + \frac{1}{4}e^{-2j\omega}}$

$\begin{cases} -2r\cos\theta = -1 \\ r^2 = \frac{1}{4} \end{cases}$ $\therefore r = \frac{1}{2}$ $\theta = 0$

无振荡

6.27 (a) $H(j\omega) = \dfrac{Y(j\omega)}{X(j\omega)} = \dfrac{1}{j\omega + 2}$ $|H(j\omega)| = \dfrac{1}{\sqrt{\omega^2 + 4}}$

$20\log_{10}|H(j\omega)| = -20\log_{10}\left|\dfrac{1}{H(j\omega)}\right| = -10\log(\omega^2+4)$

$\sphericalangle H(j\omega) = -\sphericalangle \dfrac{1}{H(j\omega)} = -\arctan\frac{\omega}{2}.$

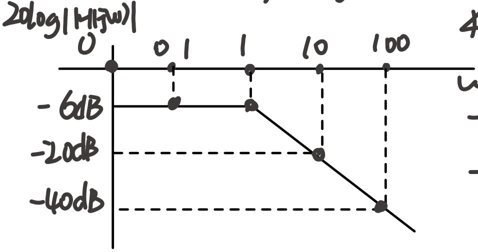
当 $\omega \ll 1$ $-10\log_{10}(\omega^2+4) \approx -20\log_{10}2 = -6dB$

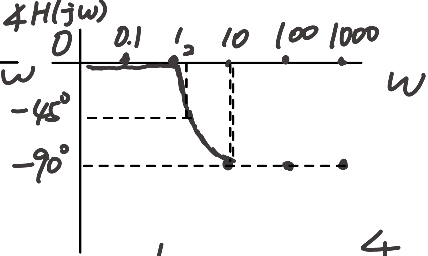
$\sphericalangle H(j\omega) \approx 0^\circ$

当 $\omega = 10$. $\quad -10\lg(\omega^2+4) = -20dB$

$\omega = 100 \quad -10\lg(\omega^2+4) = -40dB$

$\omega > 10 \quad \measuredangle H(j\omega) \approx -90^\circ$





(b) $\tau(\omega) = -\dfrac{d\measuredangle H(j\omega)}{d\omega} = \dfrac{1}{\frac{\omega^2}{4}+1} = \dfrac{4}{\omega^2+4}$

(c). $Y(j\omega) = H(j\omega)\cdot X(j\omega) \qquad X(j\omega) = \dfrac{1}{j\omega+1}$

$$= \frac{1}{(j\omega+1)(j\omega+2)}$$

$$= \frac{1}{j\omega+1} - \frac{1}{j\omega+2}$$

(d) $y(t) = \mathcal{F}^{-1}\left(\dfrac{1}{j\omega+1} - \dfrac{1}{j\omega+2}\right)$

$$= (e^{-t} - e^{-2t})u(t)$$

6.42 (b). $H_1(e^{jw}) = \dfrac{1+\frac{1}{2}e^{-jw}}{1+\frac{1}{4}e^{-jw}}$

$$= 1 + \frac{\frac{1}{4}e^{-jw}}{1+\frac{1}{4}e^{-jw}}$$

$$= 2 - \frac{1}{1+\frac{1}{4}e^{-jw}}$$

∴ 单位冲激响应 $h_1[n] = 2\delta[n] - (\frac{1}{4})^n u[n]$

单位阶跃频率响应 $G_1(e^{jw}) = \dfrac{1+\frac{1}{2}e^{-jw}}{1+\frac{1}{4}e^{-jw}}\ \mathcal{F}\{u[n]\}$

$$u[n] \xrightarrow{\mathcal{F}} \frac{1}{1-e^{-jw}} + \sum_{-\infty}^{+\infty}\pi\delta(w-2\pi k)$$

由于 $G_1(e^{jw})$ 变换回时域时，在$[-\pi,\pi]$仅做一个周期积分，故可不管。

$$\therefore G_1(e^{jw}) = \frac{1+\frac{1}{2}e^{-jw}}{1+\frac{1}{4}e^{-jw}} \cdot \frac{1}{1-e^{-jw}}$$

$$= \left(2-\frac{1}{1+\frac{1}{4}e^{-jw}}\right)\frac{1}{1-e^{-jw}}$$

$$= 2\,\frac{1}{1-e^{-jw}} - \frac{1}{(1+\frac{1}{4}e^{-jw})(1-e^{-jw})}$$

阶跃响应 $\therefore g_1[n] = \mathcal{F}^{-1}[G_1(e^{jw})]$

$$= 2u[n] - \frac{1}{5}\left(\frac{1}{1+\frac{1}{4}e^{-jw}} + \frac{4}{1-e^{-jw}}\right)$$

$$= \frac{6}{5} + \frac{4}{5}\left(-\frac{1}{4}\right)^{n+1} u[n]$$

$$H_2(e^{jw}) = \frac{\frac{1}{2} + e^{-jw}}{1+\frac{1}{4}e^{-jw}}$$

$$= \frac{4(1 + \frac{1}{4}e^{-jw}) - \frac{7}{2}}{1+\frac{1}{4}e^{-jw}}$$

$$= 4 - \frac{7}{2}\,\frac{1}{1+\frac{1}{4}e^{-jw}}$$

冲激响应

$$\therefore h_2[n] = 4\delta[n] - \frac{7}{2}\left(\frac{1}{4}\right)^n u[n].$$

$$\because G_2(e^{jw}) = H_2(e^{jw})\,\frac{1}{1-e^{-jw}}$$

$$= \left(4 - \frac{7}{2}\,\frac{1}{1+\frac{1}{4}e^{-jw}}\right)\left(\frac{1}{1-e^{-jw}}\right)$$

阶跃响应

$$\therefore g_2[n] = 4u[n] - \frac{7}{2}\,\frac{1}{5}\left(\left(-\frac{1}{4}\right)^n u[n] + 4u[n]\right)$$

$$= 4u[n] - \frac{7}{10}\left(\frac{1}{4}\right)^n u[n] - \frac{14}{5}u[n]$$

$$= \left\{\frac{6}{5} + \frac{14}{5}\left(-\frac{1}{4}\right)^{n+1}\right\}u[n]$$