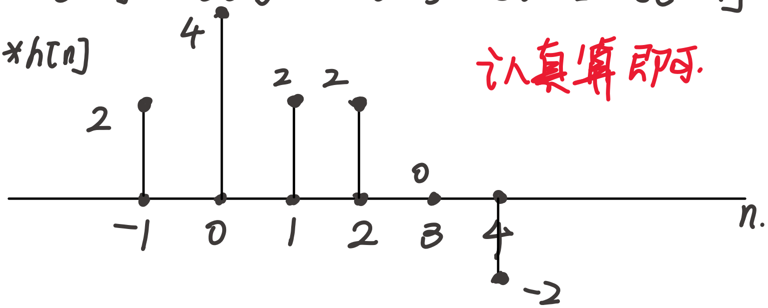
2.1 (a) $x[n] * h[n] = \{\delta[n]+2\delta[n-1]-\delta[n-3]\} * \{2\delta[n+1]+2\delta[n-1]\}$

$= 2\delta[n+1]+4\delta[n]-2\delta[n-2]+2\delta[n-1]+4\delta[n-2]-2\delta[n-4]$

$= 2\delta[n+1]+4\delta[n]+2\delta[n-1]+2\delta[n-2]-2\delta[n-4]$

$x[n]*h[n]$

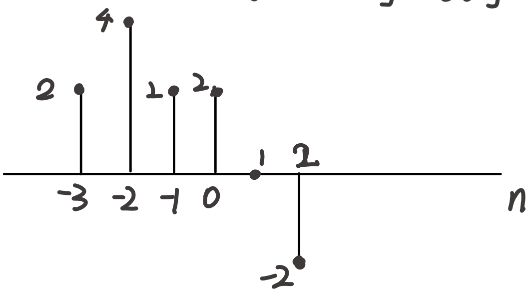


认真算即可.

(b) $y_2[n] = x[n+2]*h[n] = \{\delta[n+2]+2\delta[n+1]-\delta[n-1]\} * \{2\delta[n+1]+2\delta[n-1]\}$

$= 2\delta[n+3]+4\delta[n+2]-2\delta[n]+2\delta[n+1]+2\delta[n]-2\delta[n-2]$

$= 2\delta[n+3]+4\delta[n+2]+2\delta[n+1]+2\delta[n]-2\delta[n-2]$

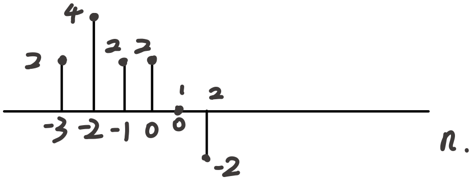


(3) $y_3[n] = x[n] * \{2\delta[n+3]+2\delta[n+1]\}$

$= \{\delta[n]+2\delta[n-1]-\delta[n-3]\} * \{2\delta[n+3]+2\delta[n+1]\}$

$= 2\delta[n+3]+4\delta[n+2]-2\delta[n]+2\delta[n+1]+4\delta[n]-2\delta[n-2]$

$= 2\delta[n+3]+4\delta[n+2]+2\delta[n+1]+2\delta[n]-2\delta[n-2]$



2.5 ∵ $x[n]=\begin{cases}1 & 0\le n\le 9\\ 0 & 其他\end{cases}$ $h[n]=\begin{cases}1 & 0\le n\le N\ (N\le 9)\\ 0 & 其他\end{cases}$

2.5考讨论
也是卷积的精髓

由卷积和定义.

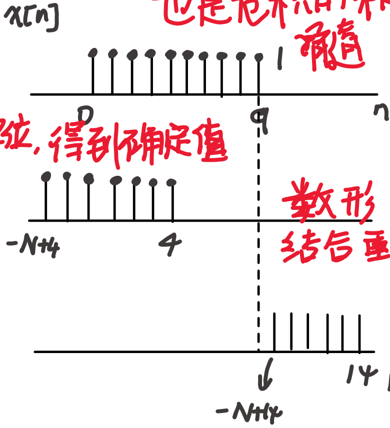
$y[4]=\sum_{k=-N}^{4} x[k]\,h[4-k]=5$

反转平移
移定的单位, 得到确定值

$h[4-k]$ 的图如右所示

可知 $-N+4\le 0$ 即 $N\ge 4$. 可满足

$y[4]=5$



数形
结合重要

又 $y[14]=\sum_{k=-N}^{14} x[k]\,h[14-k]=0$

则 $-N+14\ge 10$ $N\le 4$

由 $4\le N\le 4$ 可知 $N=4$ 夹逼.

2.11 (a) ∵ $x(t)=u(t)*[\delta(t-3)-\delta(t-5)]$

∴ $y(t)=x(t)*h(t)=u(t)*[\delta(t-3)-\delta(t-5)]*e^{-3t}u(t)$

老生常谈 $\boxed{u(t)*e^{-3t}u(t)}*[\delta(t-3)-\delta(t-5)]$ [交换+结合]

$=\underbrace{\int_{-\infty}^{+\infty}u(\tau)\,u(t-\tau)\,e^{-3\tau}d\tau}_{\boxed{\tau<t}} *[\delta(t-3)-\delta(t-5)]$

$=\int_{0}^{t} e^{-3\tau}d\tau *[\delta(t-3)-\delta(t-5)]$

$=+\frac{1}{3}(1-e^{-3t})*[\delta(t-3)-\delta(t-5)]$

$=\frac{1}{3}[\delta(t-3)-\delta(t-5)]-\frac{1}{3}(e^{-3t+9}-e^{-3t+15})$

现在，我们有门函数后，可视 $u(t)-u(t-t_0)$ 为门函数.

$$x(t) = u(t-3) - u(t-5) = [u(t+1) - u(t-1)] * \delta(t-4)$$

$$\cancel{X}(e^{j\omega}) = \frac{\sin\omega}{\omega\pi} e^{-4j\omega}$$

转门没什么用.
不如直接FT.

$$\mathcal{F}\{h(t)\} = \frac{1}{3+j\omega} = \int_{0}^{+\infty} e^{-(j\omega+3)t}dt$$

笑，实际不太好做

建议 $u(t)e^{-at} * u(t)e^{-bt}$ 是积分.

$$u(t-3) * e^{-3t}u(t).$$

$$= u(t)e^{-0t} * e^{-3t}u(t) * \delta(t-3)$$

$$= e^{-3j\omega}\left[\frac{1}{j\omega} + \pi\delta(\omega)\right]\frac{1}{j\omega+3}$$

筛选.

$$= e^{-3j\omega}\left\{\frac{1}{j\omega(j\omega+3)} + \pi\cdot\frac{1}{3}\delta(\omega)\right\}$$

2个 $u(t) * u(t) \Rightarrow$

(b) $\frac{dx(t)}{dt} = \delta(t-3) - \delta(t-5)$

$\therefore g(t) = [\delta(t-3) - \delta(t-5)] * e^{-3t} u(t)$

$= e^{-3t+9} u(t-3) - e^{-3t+15} u(t-5)$

(c) $\because \frac{dx(t)}{dt} = x(t) * \delta'(t)$

$\therefore g(t) = x(t) * \delta'(t) * h(t)$

$= y(t) * \delta'(t) = y'(t)$

2.16 (a) 由卷积定义

$x[n] * h[n] = \sum_{k=-\infty}^{+\infty} x[k]\, h[n-k]$

其中 $x[k] = \begin{cases} x[k] & k \geq N_1 \\ 0 & \text{其他} \end{cases}$ $h[n-k] = \begin{cases} h[n-k] & k \leq -N_2+n. \\ 0 & \text{其他} \end{cases}$

若想 $x[k]\ h[n-k] \neq 0$，则 $N_1 \leq k \leq n - N_2$ 从中可了解 n 的取值范围

也即 $N_1 + N_2 \leq n$

$\therefore$ $n < N_1 + N_2$ 时 $x[n] * h[n] = 0$ 成立. (对)

(b) (错) 反例. $h[n] = \delta[n]$ 这是恒等系统

则 $\boxed{y[n] = x[n] * \delta[n]} = x[n]$

左右各卷 $\delta(t-1)$

$y[n-1] = x[n-1] = x[n] * \delta[n-1] \neq x[n-1] * \delta[n-1]$

(c) $y(t) = x(t) * h(t) = \int_{-\infty}^{+\infty} x(\tau)\, h(t-\tau) d\tau$

(对) $y(-t) = \int_{-\infty}^{+\infty} x(\tau)\, h(-t-\tau) d\tau$

而 $x(-t)*h(-t)=\int_{-\infty}^{+\infty}x(-\tau)h(-(t-\tau))d(-\tau)\xlongequal{\tau'=-\tau}$

$\int_{-\infty}^{+\infty}x(\tau')h(-t-\tau')d\tau'\xlongequal{\tau'=\tau}\int_{-\infty}^{+\infty}x(\tau)h(-t-\tau)d\tau=f(-t)$

(d) (对) 对 $x(t)*h(t)=\int_{-\infty}^{+\infty}x(\tau)h(t-\tau)d\tau$.

$x(t)=\begin{cases}x(t) & \tau\leq T_1\\ 0 & \tau>T_1\end{cases}$ $h(t-\tau)=\begin{cases}h(t-\tau) & \tau\geq -T_2+t\\ 0 & \tau<-T_2+t\end{cases}$

要使 $x(t)*h(t)=0$

$T_1<\tau<-T_2+t$ $\therefore$ $t>T_1+T_2$.

21 (a) $x[n]=\begin{cases}\alpha^n & (n\geq 0)\\ 0 & (n<0)\end{cases}$ $h[n]=\begin{cases}\beta^n & (n\geq 0)\\ 0 & (n<0)\end{cases}$

$\therefore$ $y[n]=x[n]*h[n]=\sum_{k=-\infty}^{+\infty}x[k]h[n-k]$

对 $h[n-k]=\begin{cases}\beta^n & k\leq n\\ 0 & k>n\end{cases}$ $\therefore$ $y[n]=\sum_{k=0}^{n}\alpha^k\beta^{n-k}$

$x[k]=\begin{cases}\alpha^n & (k\geq 0)\\ 0 & (k<0)\end{cases}$ $=\beta^n\sum_{k=0}^{n}\alpha^k\beta^{-k}$

$=\beta^n\dfrac{1-(\frac{\alpha}{\beta})^{n+1}}{1-\frac{\alpha}{\beta}}u[n]$

$u[n]e^{-t}*u[n]e^{-t}$

$=\dfrac{\beta^{n+1}-\alpha^{n+1}}{\beta-\alpha}u[n]$ $\left(\begin{matrix}\alpha\neq\beta\\ 且 n\geq 0\end{matrix}\right)$

$\int_{-\infty}^{+\infty}u[\Delta]e^{-\Delta}u[n-\Delta]e^{-(n-\Delta)}d\Delta$

(c) $x[n]=\begin{cases}(-\frac{1}{2})^n & (n\geq 4)\\ 0 & 其他\end{cases}$ $h[n]=\begin{cases}4^n & (n\leq 2)\\ 0 & (n>2)\end{cases}$

大好题.

对 $y[n]=x[n]*h[n]=\sum_{k=-\infty}^{+\infty}x[k]h[n-k]$

$h[n-k]=\begin{cases}4^n & (k\geqslant -2+n)\\ 0 & 其他\end{cases}$ 则要有 $\begin{cases}k\geqslant 4\\ k\geqslant -2+n\end{cases}$

① $n-2\geqslant 4$ 下限为 $n-2$ $n\geqslant 6$

$$y[n]=\sum_{k=n-2}^{\infty}(-\tfrac{1}{2})^k\cdot 4^{n-k}$$ 考数列求和了.

$$=4^n\sum_{k=n-2}^{\infty}(-\tfrac{1}{8})^k$$

$$=4^n\lim_{k\to\infty}\frac{64(-\frac{1}{2})^n(1-(-\frac{1}{8})^k)}{1+\frac{1}{8}}\quad u[n-6]$$

$$=\frac{512}{9}(-\tfrac{1}{2})^n u[n-6]$$

② $n-2<4$ $n<6$ 下限为 4

$$y[n]=\sum_{k=4}^{\infty}(-\tfrac{1}{2})^k 4^{n-k}$$

$$=4^n\sum_{k=4}^{\infty}(-\tfrac{1}{8})^k$$

$$=4^n\lim_{k\to\infty}\frac{1-(\frac{1}{8})^k}{1+\frac{1}{8}}(-\tfrac{1}{8})^4 u[5-n]$$

$$=4^n\,\frac{8}{9}(-\tfrac{1}{8})^4 u[5-n]=\frac{4^n}{9\cdot 512}u[5-n]$$

综上所述 $y[n]=\begin{cases}\frac{1}{9}\frac{4^n}{512}u[5-n] & (n\leq 6)\\ \frac{512}{9}(-\frac{1}{2})^n u[n-6] & (n>6)\end{cases}$

$x[n] = (-\frac{1}{2})^n \cdot u[n-4]$ $h[n] = 4^n u[2-n]$

虽然变换域可以解时域问题，但变换域

和 LTI 比较吻与

$$x[n] * h[n] = \sum_{-\infty}^{+\infty} (-\tfrac{1}{2})^{\Delta} u[\Delta-4] \cdot 4^{n-\Delta} u[2-(n-\Delta)]$$

$$= \sum_{-\infty}^{+\infty} (-\tfrac{1}{2})^{\Delta}\, 4^n \cdot 4^{-\Delta}\, u[\Delta-4]\, u[\underset{\Delta-(n-2)}{\cancel{2-n+\Delta}}]$$

Δ 与 4. 与 $\boxed{n-2}$ 比较.

① $4 \geq n-2$ $n \geq 6$.

$$\sum_{\Delta=n-2}^{+\infty} (-\tfrac{1}{2})^{\Delta}\, 4^n \cdot 4^{-\Delta} \cdot 1 = 4^n \frac{(-\frac{1}{8})^{n-2}(1-\cancel{(-\frac{1}{8})^{+\infty}})}{1-(-\frac{1}{8})}$$

$(-\frac{1}{8})^{+\infty}$

$$= 4^n \frac{64\, (-\frac{1}{8})^n \cdot (1)}{\cancel{\frac{3}{2}}\ \frac{9}{8}}$$

$$= 4^n \frac{512 \cdot (-\frac{1}{8})^n}{9} u[n-6].$$

$$= \frac{512 (-\frac{1}{2})^n}{9} u[n-6]$$