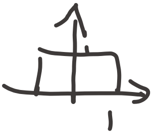
8-1-1

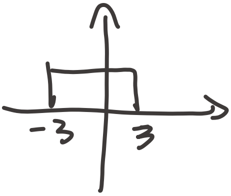
$$e(t) = E\,H(j0) + a_1 H(j\omega_0) e^{j\omega_0 t} + a_{-1} H(-j\omega_0) e^{-j\omega_0 t}$$

8-1-2  $\xrightarrow{\mathcal{F}}$ $\dfrac{2\sin\omega}{\omega}$

把其看作 $H(j\omega)$

$\therefore\ y(t) = e^{j\omega_0 t} H(j\omega_0)$

$= e^{j\omega_0 t}\dfrac{2\sin\omega_0}{\omega_0}$ 要 $\underline{\omega_0 = k\pi}$ $(k=0.1.2\cdots)$

8-1-3 

$$h(t) = \frac{\sin 3t}{\pi t} * \delta(t-2)$$

$$H(j0) = [u(3) - u(-3)] = 1$$

保住直流.

$$H(j2) = [u(5) - u(-1)]e^{-j4} = e^{-j4}$$

$$H(j(-2)) = [u(1) - u(-5)]e^{j4} = e^{j4}$$

$$\cos 2t * h(t) = \frac{1}{2} H(j2) e^{j2t} + \frac{1}{2} H(-j2) e^{j(-2t)}$$

$$= \frac{1}{2} e^{j(2t-4)} + \frac{1}{2} e^{j4} e^{j(-2t)} = \frac{1}{2}\cdot 2\cos(2t-4)$$

$y(t) = \cos(2t-4)+1$

3-1-4 $\quad h(t)=\frac{1}{\pi t}$

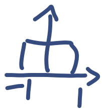
$$(-jt)^n x(t) \overset{\mathcal{F}}{\longleftrightarrow} \frac{d^n X(j\omega)}{d\omega} \quad \times$$

$$x(t)=1 \overset{\mathcal{F}}{\longleftrightarrow} 2\pi\delta(\omega)$$

$$\left(\frac{t}{j}\right)^{-1} = \frac{j}{t} \overset{\mathcal{F}}{\longleftrightarrow} 2\pi \cdot u(\omega)$$

$$\frac{1}{\pi t} \overset{\mathcal{F}}{\longleftrightarrow} -j\,\mathrm{sgn}(\omega)$$

$\frac{\sin t}{\pi t}$



$\frac{2\sin\omega}{\omega} \longleftrightarrow$

$$\frac{j}{t} \overset{\mathcal{F}}{\longleftrightarrow} \pi\,\mathrm{sgn}(\omega)$$

考对偶. 即. $\quad x(t) \overset{\mathcal{F}}{\longleftrightarrow} 2\pi x(-\omega)$

对于 $\quad \mathrm{sgn}(t) \overset{\mathcal{F}}{\longleftrightarrow} \frac{2}{j\omega}$

$$\frac{j}{\pi 2}\,\mathrm{sgn}(t) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{\pi\omega} \cdot \frac{j}{\pi 2}$$

对偶性知

$$\frac{1}{\pi t} \overset{\mathcal{F}}{\longleftrightarrow} \frac{2\pi}{2\pi} \cdot j\,\mathrm{sgn}(-\omega)$$

$$= -j\,\mathrm{sgn}(\omega)$$

$H(j\omega) = -j\,\mathrm{sgn}(\omega)$

$H(j200\pi) = -j \quad H(j\,{-200\pi}) = j$

$$\varphi(j200\pi) = -\frac{\pi}{2} \qquad \varphi(j{-200\pi}) = +\frac{\pi}{2}$$

$$\cos 200\pi t \xrightarrow{h(t)} \frac{1}{2}\left(e^{j200\pi t}(-j) + e^{j(-200\pi)t}(j)\right)$$

系统等效 $H(j200\pi)$ 为个复数

$$= \frac{1}{2}\left(\frac{\cancel{+}e^{j200\pi t}}{j} + \frac{\cancel{+}e^{j-200\pi t}}{-j}\right)$$

$$= -\cancel{\ \ }\ \sin 200\pi t.$$

8-1-5 $H(j\omega) = \dfrac{1}{j\omega+1}$

$\therefore h(t) = e^{-t}u(t)$

$H(j0) = 1$

$H(j1) = \dfrac{1}{j+1} = \dfrac{\sqrt{2}}{2}e^{-j45^\circ}$ $\quad |H(j1)|$ $\quad \varphi(j1) = -45^\circ$

$\therefore f(t) \xrightarrow{h(t)} 1 + \frac{\sqrt{2}}{2}\cos(t-45^\circ).$

8-1-6 $= \delta'(t) * \overline{e^{-2t}u(t) * u(t)}$

$= e^{-2t}u(t) * \delta(t)$

$h(t) = \delta(t) \overset{\mathcal{F}}{\longleftrightarrow} H(j\omega) = 1$

$\therefore = e^{-2t}u(t)$ 有无$u(t)$?

8-1-7 $H(j2\pi)=1$ $H(j0)=2$

$H(j4\pi)=0$

$\therefore X(t)=0.5H(j0)\cancel{+}H(j2)\frac{1}{\pi}\sin 2\pi(t)$

$+H(j4)\,0.5\cos(4\pi t)$

$=1\cancel{+}1\cdot\frac{1}{\pi}\sin(2\pi t)$

$=1\cancel{+}\frac{1}{\pi}\sin 2\pi t$

由图.

8-1-8 $\because f(t)=3+5\cos t+5\sin 3t+8\cos 8t$

$H(j0)=6$ $H(j1)=5e^{j\frac{\pi}{8}}$ $H(j3)=3e^{-\frac{3}{8}\pi}$

$H(j8)=0$

$\therefore y(t)=3H(j0)+5\cos t\,H(j1)+5\sin 3t\,H(j3)$

$=3\times 6+25\cos(t-\frac{\pi}{8})+15\sin(3t-\frac{3}{8}\pi)$

8-1-9. $x(t)=1+\cos(\frac{3}{2}\pi t+\theta)+\sin(4\pi t+\theta)$

$H(j0)=0$ $H(j\frac{3}{2}\pi)=\frac{j\frac{3}{2}\pi}{3\pi}=\frac{j}{2}=\frac{1}{2}e^{j\frac{\pi}{2}}$

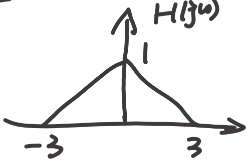
$H(j4\pi)=0$

$\therefore y(t)=H(j0)+\cos(\frac{3}{2}\pi t+\theta)\,H(j\frac{3}{2}\pi)$

$$= 0 + \frac{1}{2}\cos\left(\frac{3}{2}\pi t + \frac{\pi}{2}\right)$$

$$= -\frac{1}{2}\sin\left(\frac{3}{2}\pi t + \theta\right)$$

8-1-10. $H(j\omega)$



$\because |\omega| > 3\,rad/s$ 时 $H(j\omega) = 0$.

$$f(t) \xrightarrow{h(t)} 3 + e^{-j\frac{\pi}{2}}e^{jt} + e^{+j\frac{\pi}{2}}e^{-jt}$$

$$+ e^{-j\pi}e^{j2t} + e^{+j\pi}e^{-j2t}$$

$$+ e^{-j\frac{3}{2}\pi}e^{j3t} + e^{+j\frac{3}{2}\pi}e^{-j3t}$$

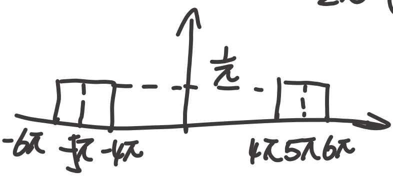
$$= 3 + 2\cos\left(t - \frac{\pi}{2}\right) + 2\cos(2t - \pi)$$

$$+ 2\cos\left(3t - \frac{3}{2}\pi\right)$$

$$= 3 + 2\sin t + (-2)\cos 2t + (-2)\sin 3t$$

8-1-12 (1) $H(j\omega) = 1 + \frac{1}{2\pi}\cdot\left[\delta(\omega - 5\pi) + \delta(\omega + 5\pi)\right] *$

$\therefore H(j\omega) =$



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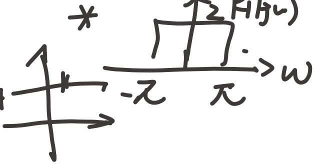


(3). $f(t) = 1 + \cos 10\pi t + \cos 5\pi t$

$H(j10\pi) = \cancel{0}\ 1$ $\quad H(j5\pi) = \frac{1}{\pi}\ 2$

$\therefore f(t) \xrightarrow{h(t)} \frac{1}{\pi}\cancel{\cos}\,5\pi t$

8-1-16 离散之正弦稳态

$$h[n] = \frac{1}{3}\left(\delta[n] + \delta[n-1] + \delta[n-2]\right)$$

$$H(z) = \frac{1}{3}\left(1 + z^{-1} + z^{-2}\right) \quad \text{ROC}: 8\text{平面}$$

$$H(e^{j\omega}) = \frac{1}{3}\left(1 + e^{-j\omega} + e^{-2j\omega}\right)$$

$$H(e^{j\frac{2}{3}\pi}) = \frac{1}{3}\left(1 + e^{-j\frac{2}{3}\pi} + e^{-j\frac{4}{3}\pi}\right)$$

$$= \frac{1}{3}\left(1 + (-\tfrac{1}{2}) + j\tfrac{\sqrt{3}}{2} + (-\tfrac{1}{2}) + j(-\tfrac{\sqrt{3}}{2})\right)$$

$$= \frac{1}{3}\left(1 - 1 + 0\right) = 0.$$

、 $H(e^{j0}) = \frac{1}{3} \cdot 3 = 1$

∴ $y[n] = 1$

8-1-20

(1) $H(z) = \dfrac{Az}{z-1} = \dfrac{A}{1-z^{-1}}$ $\quad h[n] = A u[n]$

$\lim\limits_{k\to\infty} h(k) = \lim\limits_{z\to\infty} H(z) = 4$

对z变换 $\lim\limits_{k\to\infty} h[n] = \lim\limits_{z\to 1} (z-1)H(z) = A = 4$

(2) $f[n] = (-3)^n u[n]$

$$F(z) = \frac{1}{1+3z^{-1}}$$

$$Y(z) = F(z)\cdot H(z) = \frac{1}{1+3z^{-1}} \cdot \frac{A}{1-z^{-1}}$$

$$= A\,\frac{1}{4}\left(\frac{1}{1-z^{-1}} + \frac{3}{1+3z^{-1}}\right)$$

$$y[n] = \frac{1}{4}\,4\cdot\left(u[n] + 3(-3)^n u[n]\right)$$

$$= u[n]\left(1+3(-3)^n\right)$$

(3)

$$f[n] = 3\cos\left(\pi n + \frac{\pi}{6}\right)$$

$$H(e^{j\omega}) = \frac{4}{1-e^{-j\omega}} + 4\pi\sum_{-\infty}^{+\infty}\delta(\omega-2k\pi)$$

$$H(e^{j\pi}) = \frac{4}{1-e^{-j\pi}} = \frac{4}{2} = 2.$$

$\varphi(e^{j\pi}) = 0$ 为什么 $H(e^{j\omega})$ 存在

$$y_f[n] = 3\cdot 2\cos\left(\pi n + \frac{\pi}{6}\right) = 6\cos\left(\pi n + \frac{\pi}{6}\right)$$

因为 $\lim\limits_{z\to 1}(z-1)H(z)$ 存在. 先这样记着

正确的解法是: $3\cos\left(\pi n+\frac{\pi}{6}\right)$ 不需要变成 $\boxed{e^{j\omega}}$ 因为不收敛

找到离散 LTI 系统中的特征函数

$$=\frac{3}{2}\left(e^{j\left(\pi n+\frac{\pi}{6}\right)}+e^{-j\left(\pi n+\frac{\pi}{6}\right)}\right)$$

$$=\frac{3}{2}\,e^{j\pi n}\,e^{\frac{\pi}{6}}+e^{-j\pi n}\,e^{-\frac{\pi}{6}}$$

$$=\frac{3}{2}\left[(-1)^n e^{\frac{\pi}{6}}+(-1)^n e^{-\frac{\pi}{6}}\right]$$

$$=3\cos\frac{\pi}{6}\cdot\boxed{(-1)^n}$$ LTI 之特征函数

$\because$ $H(z)=\dfrac{4z}{z-1}$

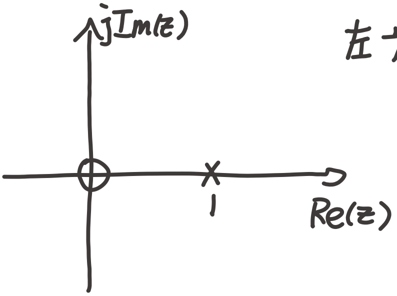
$\therefore$ $H(-1)=\dfrac{-4}{-2}=2.$

$\therefore$ $y[n]=3\cdot\frac{\sqrt{3}}{2}\cdot(-1)^n\,2=3\sqrt{3}\,(-1)^n.$

$\Rightarrow$ 答案 $6\cos(\pi k+30^\circ)$

$$=6\,\frac{\sqrt{3}}{2}\cos\pi k-6\,\frac{1}{2}\sin\pi k$$

$$=3\sqrt{3}\,(-1)^n-0=3\sqrt{3}\,(-1)^n$$



左为LTI系统零极点图

单位冲激响应 $h[n]$

$\lim\limits_{n\to\infty} h[n]=4$

若车输入 $x[n]=3\cos(\pi n+\frac{\pi}{6})$ 求正弦稳态响应 $y_s[n]$

$$\lim_{n\to\infty} h[n]=\lim_{z\to 1}(z-1)H(z)=\lim_{z\to 1}\frac{Az}{z-1}(z-1)=4$$

我已经求得 $H(z)=\dfrac{4}{1-z^{-1}}=\dfrac{4z}{z-1}$

$$H(e^{j\omega})=\frac{4}{1-e^{-j\omega}}\ ?$$

$$H(e^{j\pi})=\frac{4}{1-(-1)}=2$$

$$\varphi(e^{j\pi})=0$$

8-2-1 $e(t)=\cos t\, u(t)$

$$E(s)=\frac{s}{s^2+1}$$

稳态，对于一个复指信号
使用特征输入法

$$H(j\omega)=\frac{1}{j\omega+1}$$

$$H(j1)=\frac{1}{j+1}=\frac{\sqrt{2}}{2}e^{-j\frac{\pi}{4}}$$

一定别忘了

$$e(t)\xrightarrow{h(t)}H(j1)\cdot\cos t\, u(t)$$

$$=\frac{\sqrt{2}}{2}\cos\left(t-\frac{\pi}{4}\right)u(t)$$

8-2-2 对这种题，看其稳定状态

稳态响应趋定值，意味什么？

对于LTI系统，一般写成有理分式

$$\frac{(s-s_{01})(s-s_{02})\cdots}{(s-s_1)(s-s_2)\cdots}$$ 基本上零点由输入决定

本题输入只有一个东西

当零输入时 求出的响应只含 e^{-st} 项，并延用至 $+\infty$.

若输入是 $u(t)$，相当于积分器，对 $\sum_{n=-\infty}^{+\infty} e^{jn\omega t}$ 来说

$\int_0^{+\infty} e^{jn\omega t}dt$ 必然会得到常数值，复指数在无穷远会趋0

剩下是常数之间的线性组合

举个例子. $H(S)=\frac{1}{S+1}$

$u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{S}$ ① $\delta'(t) \overset{\mathcal{L}}{\longleftrightarrow} S$ ②

$\delta(t) \overset{\mathcal{L}}{\longleftrightarrow} 1$ ③ $tu(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{S^2}$ ④

$t^2u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{2}{S^3}$

$\lim\limits_{t\to\infty} y(t) = \lim\limits_{S\to 0} S\cancel{Y}(S)$ 发现

① $\lim\limits_{S\to 0} S\frac{1}{S}\frac{1}{S+1}=1$

② $\lim\limits_{S\to 0} S\ S\cdot\frac{1}{S+1}=0$

③ $\lim\limits_{S\to 0} S\frac{1}{S+1}=0$

④ $\lim\limits_{S\to 0} S\frac{1}{S^2}\frac{1}{S+1}$ 不存在

所以. 想输入常数稳态
须输入阶跃

8-2-3. $h[n] = 0.5^n\left(u[n] + u[n-1]\right)$

$$h[n] = (0.5)^n u[n] + (0.5)^{n-1} \cdot \frac{1}{2} u[n-1].$$

$$= (0.5)^n u[n] + \frac{1}{2} (0.5)^n u[n] * \delta[n-1]$$

$$H(z) = \frac{1}{1 - \frac{1}{2}z^{-1}} + \frac{1}{2} \frac{z^{-1}}{1 - \frac{1}{2}z^{-1}}$$

$$= \frac{1 + \frac{1}{2}z^{-1}}{1 - \frac{1}{2}z^{-1}} = \frac{z + \frac{1}{2}}{z - \frac{1}{2}} \text{ (不适)}$$

∴ 差分. $y[n] + \frac{1}{2}y[n-1] = x[n] - \frac{1}{2}x[n-1]$