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ACORD—an adaptive corner detector for planar curves

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Abstract

In contrast to the conventional method of smoothing a curve at multiple scales and integrating the information at various scales, a technique for smoothing a curve adaptively based on the roughness present in the curve is suggested. The procedure does not require smoothing at all levels of detail and it does not require construction of complete scale space map and representation of the map by tree. The procedure has been applied on a number of digital curves and the results have been compared with those of the recent work. © 2002 Pattern Recognition Society. Published by Elsevier Science Ltd. All rights reserved.

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1. Introduction

Corner detection is an important research area in computer vision. Corner points are important features that may be used at the subsequent level of computer vision such as object recognition, stereo matching, motion estimation. Corner (dominant) points are points at which the curvature is a local maximum and is above a threshold. Following Attneave's famous observation [1] that information about a curve is concentrated at the corner points, many algorithms have been developed for corner detection. An overview of these algorithms can be found in Ref. [2]. Each of these algorithms, except that of Sanker–Sharma [3], needs one or more input parameters. These parameters determine the degree of smoothing that is to be applied to a curve. Each value of the input parameter corresponds to a specific degree of smoothing. Larger values of the parameter analyze a curve at coarser levels of detail imparting a higher degree of smoothing to a curve and smaller values of parameter analyze a curve at finer levels of detail imparting lower degree smoothing to the curve. These procedures are single scale smoothing techniques that do not take into account the information about a curve at various levels of detail. Moreover, tuning the input parameter is a *fundamental problem of scale*. Too large a parameter may miss many fine features and too small a parameter may produce many unwanted corner points. The value of the input parameter is chosen generally on the basis of a trial-and-error process when no a priori information about the feature size of a curve is known. If a curve consists of features of multiple size (large and small curvature regions co-exist) it is necessary to use different input parameters for regions with different curvature. Shankar–Sharma's algorithm does not need any input parameter and it takes into account various levels of detail of a curve, but it fails to operate successfully on curves that consist of features of multiple size. Teh and Chin [2] introduced a dominant point detection algorithm that need no input parameter and remains reliable even when features of multiple size are present on a curve. But the procedure is known to produce many unwanted dominant points in the presence of noise.

Scale space filtering is a multi scale approach to detect and locate significant local events such as zero-crossings, extreme curvature points. Witkin proposed this in his work scale space filtering [4]. To smooth a one-dimensional signal Witkin convolved it with the Gaussian kernel treating the parameter of the kernel as a continuous scale parameter. The zero crossings of the curvature of the smoothed signal were located making the parameter vary continuously. The arc

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length is shown along the x-axis (horizontal) and the scale parameter along the y-axis (vertical). The image on the xy half plane showing the location of the zero-crossings over scales is called scale space image. Mokhtarian and Mackworth [5] extended this work to two-dimensional shapes. Asada and Brady [6] used the concept of scale space filtering to extract primitives such as corners, ends, smooth joins, crank, bump/dent from the bounding contour of planar shapes. They make a scale space analysis of the behavior of these primitives making the parameter of the Gaussian kernel vary in one octave, corresponding to multiplying by 2. A tree representation showing the movement of the positions of the local positive maxima and negative minima in the first and second derivatives of the Gaussian smoothed curve is constructed making the parameter of the kernel vary. The primitives are detected and located in a process of parsing the tree. Using the location of the primitives at each scale as a set of knot points they have made a polygonal approximation, circular spline approximation, cubic spline approximation and B-spline approximation of the planar shapes. Saint-Marc et al. [7] suggested an adaptive smoothing leading to the construction of scale space image without using the Gaussian kernel. The smoothing is done using a decaying exponential window that is a function of a smoothing parameter k and a measure of signal discontinuity. The underlying concept of the procedure is to keep the window size of the smoothing kernel constant and to apply the kernel iteratively on a signal. They construct two types of scale space map. In one case they keep the parameter k fixed and use the number of iterations as the scale parameter. Here the parameter k determines the magnitude of the edges (corners) to be preserved during the smoothing process. They call this scale-space Gaussian scale space. The other scale space map that they construct has been referred to as the adaptive scale space. Here the number of iterations to be performed is held constant and the parameter k is varied. Without making an attempt to give a multi scale interpretation of the scale space map they detect corners on planar shapes for different values of the parameter k. They also show the application of the procedure to edge detection from gray level image and range image segmentation. Meer et al. [8] suggested a method to detect dominant points by first determining the optimal scale of a Gaussian-like convolution multiple scale representation of the boundary. Then, a measure of optimality, which is directly proportional to the total curvature of the boundary, is defined. The optimal scale is determined such that the difference in the measure of optimality between two successive scales is minimal. The corners are detected at the optimal scale. The procedure does not take into account the information about the curve at various levels of detail. Rattarangsi and Chin [11] use the concept of scale space filtering to design a method of corner detection that takes into account various levels of detail of a curve. They make an analysis of the scale space behavior of different contour models. The extreme curvature points are detected and located by convolving a curve with the Gaussian kernel with varying window size. The scale space map shows the location of the extreme curvature points over scales. The scale space map is converted into a tree representation. The tree is interpreted using stability criteria and corners are detected and located.

Rattarangsi and Chin analyzed the entire curve at various levels of detail. Every region of the curve is given the same amount of smoothing irrespective of the nature of the curve in that region. But we believe that the degree of smoothing on a region should depend on the nature of the curve. Since different regions have different levels of detail, the amount of smoothing to be imparted should depend on the level of details of the curve in the region. The region that has higher roughness should be subjected to more smoothing and vice versa.

Mokhtarian and Suomela [9] compute the curvature on the curve at a high scale, detect the corners and track the corners through multiple lower scale to improve localization. A refinement is applied to this output to remove close corners.

Lindeberg [10] addresses the problem of how to select appropriate scales for image analysis. A heuristic principle is proposed stating that the local extrema over scales of different combinations of normalized scale invariant derivatives are the likely candidates to correspond to interesting structures. The methodology involves two stages. At the first stage, features at coarse scales are detected and at the second stage, they are localized at the finer scales.

In this paper, we suggest a method for determining the degree of smoothing in the various regions of a curve based on the local nature of the curve. The more noise a curve has, the more smoothing is required before the actual curvature can be determined. The noise present on a curve is reflected by the rapid variation along the curve over a comparatively small region. This rapid variation exhibits itself in form of roughness on the curve. At rough regions there is a high variation in curvature and vice versa. So we propose to measure the variation in the roughness of a curve using *variance of curvature.* In general, different regions of the curve have different roughness and so different amount of smoothing is required at different regions of a curve. We will use Gaussian filter for smoothing the curve. Since the window size of the Gaussian filter determines the amount of smoothing the filter can impart, we will determine the actual window size the filter should have at each point of the curve. In an attempt to find out the actual window size of the filter, we make use of the variance of curvature. As the window size of the filter is increased the variability in curvature measured by the variance of curvature is supposed to increase, since the larger window size of filter imparts higher smoothing to the curve. We will start with the smallest window size and continually increment the window size until the variance of curvature starts decreasing. Since the degree of smoothing applied to a region is determined based on the roughness of the curve in the region so this approach is known as *adaptive smoothing.*

2. Discrete curvature

To compute the curvature at a point (x_i, y_i) of a digital curve, we take the first- and second-order finite differences of x_i and y_i as in [9]

$$\Delta x_i = \frac{x_{i+1} + x_{i-1}}{\sqrt{(x_{i+1} + x_{i-1})^2 + (y_{i+1} + y_{i-1})^2}}, \quad (1)$$

$$\Delta y_i = \frac{y_{i+1} + y_{i-1}}{\sqrt{(x_{i+1} + x_{i-1})^2 + (y_{i+1} + y_{i-1})^2}}, \quad (2)$$

$$\Delta^2 x_i = \frac{\frac{x_{i+1}-x_i}{\sqrt{(x_{i+1}-x_i)^2+(y_{i+1}-y_i)^2}} - \frac{x_i-x_{i-1}}{\sqrt{(x_i-x_{i-1})^2+(y_i-y_{i-1})^2}}}{\frac{1}{2}\sqrt{(x_{i+1}-x_{i-1})^2+(y_{i+1}-y_{i-1})^2}}, \quad (3)$$

$$\Delta^2 y_i = \frac{\frac{y_{i+1}-y_i}{\sqrt{(x_{i+1}-x_i)^2+(y_{i+1}-y_i)^2}} - \frac{y_i-y_{i-1}}{\sqrt{(x_i-x_{i-1})^2+(y_i-y_{i-1})^2}}}{\frac{1}{2}\sqrt{(x_{i+1}-x_{i-1})^2+(y_{i+1}-y_{i-1})^2}} \quad (4)$$

and the curvature measure is given by

$$\kappa = \Delta x \Delta^2 y - \Delta y \Delta^2 x. \quad (5)$$

3. Present method

In the existing multi scale smoothing technique, each point of a curve is subjected to various degrees of smoothing. At each level of smoothing the extreme curvature points are detected. In this approach it is essential to smooth the curve at all levels of detail, to construct a scale space map, to convert the map into a tree representation and to interpret the tree to detect and locate the corners. The presented method avoids the construction of the complete scale space map. We also avoid tree representation and interpretation. So instead of smoothing a curve at all levels of detail we determine the level of smoothing required to be imparted to each point based on the roughness of the curve in the neighborhood of the point. The roughness of a curve generally varies from point to point. So the degree of smoothing is determined adaptively based on the nature of the curve in the neighborhood of a point. Each point will usually require a different degree of smoothing. So we call this procedure *adaptive smoothing*.

The central problem is to determine the roughness of a curve in the neighborhood of a point. We propose to take the *variance of curvature* in the neighborhood of point as a measure of roughness. The higher the variance is, the rougher the curve is in the neighborhood of a point. In order to determine curvature it is necessary to smooth the curve. The curve is smoothed using the digital Gaussian filter coefficients. The digital Gaussian filter coefficients for window size $w = 3$ are given by

$$c_{-1} = 0.2236, \quad c_0 = 0.5477 \quad \text{and} \quad c_1 = 0.2236. \quad (6)$$

The digital Gaussian filter coefficients for window size higher than $w = 3$ are obtained by repeated convolution of these coefficients with themselves.

We take three points, namely, $(x_{i\pm1}, y_{i\pm1})$ and (x_i, y_i) and using

$$X_i = c_{-1}x_{i-1} + c_0 x_i + c_1 x_{i+1} \quad \text{and}$$

$$Y_i = c_{-1}y_{i-1} + c_0 y_i + c_1 y_{i+1}. \quad (7)$$

We get the smooth coordinates (X_i, Y_i) corresponding to the input point (x_i, y_i). The same technique can be applied to find (x_{i-1}, y_{i-1}) and (x_{i+1}, y_{i+1}). We calculate the curvature at the point (x_i, y_i) using the formula (5) after replacing (x, y) by (X, Y). The differences Δ and Δ^2 can be obtained from formulas (1)–(4) after replacing (x, y) by (X, Y). We propose to denote the curvature at the ith point for window size w by $\kappa_{i,w}$. We compute the curvature value at three points (x_i, y_i), $(x_{i\pm1}, y_{i\pm1})$ and compute the variance of these curvatures using

$$(\sigma_{i,w})^2 = \Sigma(\kappa_{i,w})^2/w - (\Sigma\kappa_{i,w}/w)^2. \quad (8)$$

The smoothed coordinates X_i, Y_i with window size $w=5$ are given by

$$X_i = c_{-2}x_{i-2} + c_{-1}x_{i-1} + c_0 x_i + c_1 x_{i+1} + c_2 x_{i+2},$$

$$Y_i = c_{-2}x_{i-2} + c_{-1}x_{i-1} + c_0 x_i + c_1 x_{i+1} + c_2 x_{i+2}. \quad (9)$$

The curvature values are computed using the formula (5) for five points, namely, $(x_{i\pm2}, y_{i\pm2})$, $(x_{i\pm1}, y_{i\pm1})$ and (x_i, y_i). The variance of curvature over these five points surrounding (x_i, y_i) is given by

$$(\sigma_{i,5})^2 = \Sigma(\kappa_{i,5})^2/w - (\Sigma\kappa_{i,5}/w)^2. \quad (10)$$

If the variance $(\sigma^2_{i,5})$ does not exceed the variance $(\sigma^2_{i,3})$ then the curvature value $\kappa_{i,5}$ is the curvature at the point (x_i, y_i), otherwise, the window size has to be increased to $w = 7$. The window size is incremented in this way and for each window size w, the Gaussian filter coefficients are computed by convolving the filter coefficients are for window size $w-2$ with those for $w=3$. These filter coefficients are used to compute the smoothed coordinates X and Y using the formulae

$$X_i = \sum_{j=-\frac{w-1}{2}}^{\frac{w-1}{2}} c_j x_{i+j}, \quad Y_i = \sum_{j=-\frac{w-1}{2}}^{\frac{w-1}{2}} c_j y_{i+j} \quad (11)$$

The curvature value at the point (x_i, y_i) using

$$(\sigma_{i,w})^2 = \sum_{-\frac{w-1}{2}}^{\frac{w-1}{2}} (\kappa_{i,w})^2/w - \sum_{-\frac{w-1}{2}}^{\frac{w-1}{2}} (\kappa_{i,w}/w)^2. \quad (12)$$

As it has been already stated, the greater the window size is, the more smoothing the Gaussian filter imparts to the curve and the less smooth the input curve is (i.e. the rougher the curve is), the higher is the value of *variance of curvature.*

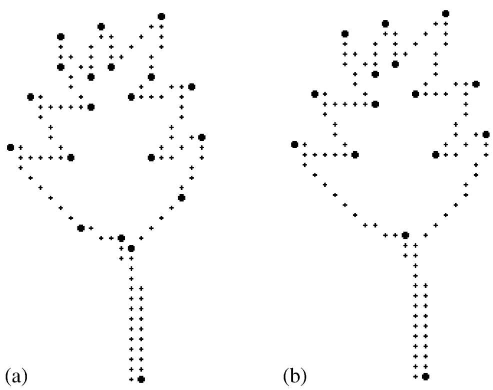
This is why when it is found that the value of curvature variance for window size w falls below the value of curvature variance for window size $w-2$, then the curve had attained the necessary amount of smoothing in the neighborhood of a point and the window size w is taken as the required window size at the point. Thus if $(\sigma_{i,w})^2 \leqslant (\sigma_{i,w-2})^2$ then the curvature at the point (x_i, y_i) is $\kappa_{i,w}$. On the other hand, if $(\sigma_{i,w})^2 > (\sigma_{i,w-2})^2$, the window size should be increased by 2 and the variance of curvature at window size w is to be compared with the variance of curvature at window size $w-2$. The maximum value of w should never exceed the length of the curve in order to avoid aliasing.

The procedure determines the amount of smoothing required in the neighborhood of each point adaptively based on the roughness of the curve. The curvature values determined by smoothing are used to detect corners on the digital curve. The local maxima of the curvature values are the corner points provided the curvature value is above a threshold. If the curvature at (x_i, y_i) determined by adaptive smoothing exceeds the curvature at the points (x_{i-1}, y_{i-1}) and (x_{i+1}, y_{i+1}) then the point $p_i(x_i, y_i)$ is a corner point. The close corners are removed by merging technique.

4. Experimental results and discussion

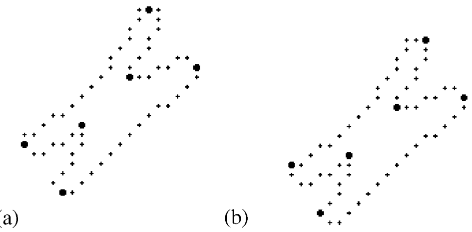
The procedure presented above has been applied on four digital curves namely, a leaf shaped curve (Fig. 1), a chromosome shaped curve (Fig. 2), a figure eight curve (Fig. 3) and a curve with four semi circles (Fig. 4). Solid circles on each curve have indicated the corners. These figures also show corner points obtained by the Rattarangsi–Chin algorithm [11]. As seen from these results, the present procedure, in most of the cases, detects more corner points than the Rattarangsi–Chin algorithm and it does not detect false corners. This is evident from the fact that if we try to make a representation of the curve using a suitable function (straight line/circles) then the approximation error in the algorithm [11] is higher than that given by the present method. As we see some new corner points have been found in Figs. 1, 3 and 4. The Rattarangsi–Chin algorithm over smooths the curve and this is why these corner points were not detected by this method.

We note that in Rattarangsi–Chin algorithm the curve is smoothed by varying the window size from its minimum value of $w = 3$ to its maximum value which is equal to the length of the curve. But in the present technique, the window size is increased as long as the variance of curvature increases. Whenever the variance of curvature at window size w falls below the variance of curvature at window size $w-2$, we get the required window size for smoothing at a point. At any point of the curves that have been used for experiment, the window size is always far below its maximum possible value. Thus in the present technique the



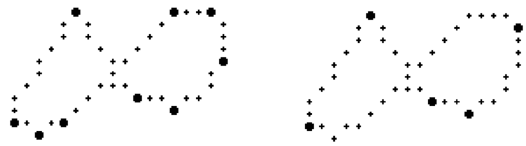
(a) (b)

Fig. 1. The leaf-shaped curve. Bold solid circles indicate the corners. (a) Present method, (b) Rattarangsi–Chin method.



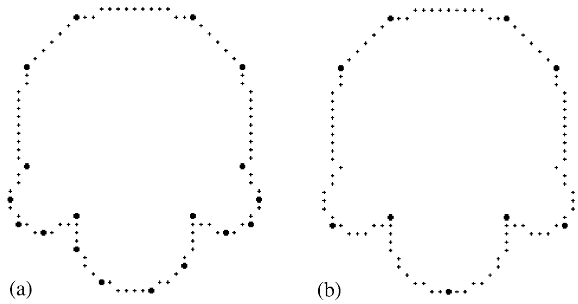
(a) (b)

Fig. 2. The chromosome shaped curve. Bold solid circles indicate the corners. (a) Present method, (b) Rattarangsi–Chin method.



(a) (b)

Fig. 3. The figure eight curve. Bold solid circles indicate the corners. (a) Present method, (b) Rattarangsi–Chin method.



(a) (b)

Fig. 4. A curve with four semi-circles. Bold solid circles indicate the corners. (a) Present method, (b) Rattarangsi–Chin method.

computation involved in smoothing the curve is lower than that involved in the Rattarangsi–Chin algorithm. The additional computation involved in the present method is the computation of variance of curvature. But this is the cost of adaptive smoothing. This cost is compensated by the improved experimental results found in the present technique. The corner detector was also found to be robust with respect to Gaussian noise $\sigma < 2$. For higher values of σ the performance of the corner detector degrades.

5. Conclusion

In contrast to the existing methods of smoothing an entire curve at various levels of detail in this paper we have suggested an adaptive method of corner detection. This procedure does not require construction of the complete scale space map and it is also not necessary to convert the map into a tree representation. The procedure has been applied on a number of digital curves and the experimental results have been compared with existing work.

6. Summary

Corner detection is an important research area in computer vision. Corner points are important features that may be used at the subsequent level of computer vision such as object recognition, stereo matching, motion estimation. Corner (dominant) points are points at which the curvature is a local maximum and is above a threshold. There exist single parameter procedures as well as multi-scale approach to detect corners on planar curves. The fundamental problem in corner detection is parameter tuning. Too large a parameter may miss many fine features and too small a parameter may produce many unwanted corner points. In this paper, we suggest a method for determining the degree of smoothing in the various regions of a curve based on the local nature of the curve. The more noise a curve has, the more smoothing is required before the actual curvature can be determined. The noise present on a curve is reflected by the rapid variation along the curve over a comparatively small region. This rapid variation exhibits itself in the form of roughness on the curve. At rough regions there is a high variation in curvature and vice versa. So we propose to measure the variation in the roughness of a curve using *variance of curvature*. In general, different regions of the curve have different roughness and so a different amount of smoothing is required at different regions of a curve. We will use Gaussian filter for smoothing the curve. Since the window size of the Gaussian filter determines the amount of smoothing the filter can impart, we will determine the actual window size the filter should have at each point of the curve. In an attempt to find out the actual window size of the filter, we make use of the variance of curvature. As the window size of the filter is increased the variability in curvature measured by the variance of curvature is supposed to increase, since the larger window size of filter imparts higher smoothing to the curve. We start with the smallest window size and continually increment the window size until the variance of curvature starts decreasing. Since the degree of smoothing applied to a region is determined based on the roughness of the curve in the region so this approach is known as *adaptive smoothing*. This procedure does not require construction of the complete scale space map and it is also not necessary to convert the map into a tree representation. The procedure has been applied on a number of digital curves and the experimental results have been compared with existing work.

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