CSCI 5521 (002) Hw3-Written

Asal Shavandi

TOTAL POINTS

70 / 70

QUESTION 1

1 Q1 30 / 30

✓ **+ 30 pts** **Everything correct**

- 5 pts Forget to update of $$w_h$$ based on the second term of the objective function:$$\triangle w_h^{'} = -\eta \cdot 2w_{h}$$, the final update of $$w_{h}$$ should be the sum of updates from both terms

+ 2 pts Correct use of the chain rule for updating $$v_h$$ based on the first term in the objective function: $$\triangle v_{h} = -\eta \frac{\partial E}{\partial v_{h}} = -\eta \sum\limits_{t} \frac{\partial E}{\partial y^t} \frac{\partial y^t}{\partial \alpha^t} \frac{\partial \alpha^t}{\partial v_h} $$

+ 3 pts Update $$v_{h}$$ with the first term (a), correct derivative for $$ \frac{\partial E}{\partial y^t} = \sum\limits_{t} \frac{y^t-r^t}{y^t(1-y^t)}$$

+ 2 pts Update $$v_{h}$$ with the first term (b), correct derivative for $$ \frac{\partial y^t}{\partial \alpha^t} = y^t(1-y^t)$$

+ 2 pts Update $$v_{h}$$ with the first term (c), correct derivative for $$ \frac{\partial \alpha^t}{\partial v_h} = z_h^t$$

+ 3 pts Correct update of $$v_h$$ based on the second term of the objective function: identify that the update is independent on the regularization term (the second term in the objective function E) / Directly using the first term as the final solution

+ 3 pts Correct use of the chain rule for updating $$w_h$$ based on the first term in the objective function: $$\triangle w_{h} = -\eta \frac{\partial E}{\partial w_{h}} = -\eta \sum\limits_{t} \frac{\partial E}{\partial y^t} \frac{\partial y^t}{\partial \alpha^t} \frac{\partial \alpha^t}{\partial z_h^t} \frac{\partial z_h^t}{\partial w_h}$$

+ 2 pts Update $$w_{h}$$ with the first term (a), correct derivative for $$ \frac{\partial E}{\partial y^t} = \sum\limits_{t} \frac{y^t-r^t}{y^t(1-y^t)}$$

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+ 2 pts Update $$w_{h}$$ with the first term (c), correct derivative for $$ \frac{\partial \alpha^t}{\partial z_h^t} = v_h$$

+ 4 pts Update $$w_{h}$$ with the first term (d), correct derivative for $$ \frac{\partial z_h^t}{\partial w_h}$$: $$ \frac{\partial z_h^t}{\partial w_h}=x^t$$ if $$w_{h}^T x^t+w_{0} > 0$$ and 0 otherwise

+ 5 pts Correct update of $$w_h$$ based on the second term of the objective function:$$\triangle w_h^{'} = -\eta \cdot 2w_{h}$$, the final update of $$w_{h}$$ should be the sum of updates from both terms

- 1 pts Arithmetic error

+ 0 pts Unattempted/incorrect

QUESTION 2

Q2 30 pts

2.1 (a) 15 / 15

✓ **+ 15 pts** **Everything correct**

+ 7.5 pts Correct vector for figure (2) (1, 1,0) and the scaled versions

+ 4 pts Partial credit for figure (2) (e.g., one of the coefficients is missing/incorrect)

+ 7.5 pts Correct vector for figure (3) (-1, -1, -1) and the scaled versions

+ 4 pts Partial credit for figure (3) (e.g., one of the coefficients is missing/incorrect)

+ 0 pts Incorrect/unattempted

2.2 (b) 15 / 15

✓ **+ 15 pts Everything correct**

+ 7.5 pts Correct W coefficients (stacking of vectors from part (a))

+ 4 pts Partial credit for W (incorrect coefficient for one/more of the W matrix)

+ 7.5 pts Correct v vector (-1.5, 1, 1) (AND operation)

+ 4 pts Partial credit for v (incorrect coefficient for one/more of the v vectors)

+ 0 pts No attempt/completely incorrect

QUESTION 3

3 Q3 10 / 10

✓ **+ 10 pts Everything correct**

+ 1 pts Reasonable reporting of the validation set accuracy over different hidden layers

+ 1 pts Reasonable reporting of the test set accuracy over selected hidden layers

+ 1 pts Correct 2D Plot (Correct Formula)

+ 1 pts Correct 2D Plot (Correct Distribution)

+ 1 pts Correct 3D Plot (Correct Formula)

+ 1 pts Correct 3D Plot (Correct Distribution)

+ 2 pts Correct Explanation. (Specify that some data may need to be mapped to higher dimensions for separation, and compare the results)

+ 2 pts Code Submission Compensation (selected as long as problem attempted)

+ 0 pts Unattempted _only_

(1) $E(w,v|X) = -\sum_{t=1}^{N}\left(r^t \log y^t + (1-r^t)\log(1-y^t)\right) + \sum_{h=1}^{H} \|w_h\|_2^2$

$y^t = \text{Sigmoid}\left(\sum_{h=1}^{H} V_h z_h^t + V_{h0}\right) \qquad z_h^t = \text{relU}(w_h^T x^t + w_{h0})$

$\Delta V_h = -\eta \frac{\partial E}{\partial V_h}, \quad \frac{\partial E}{\partial V_h} = \frac{\partial E}{\partial y^t} \cdot \frac{\partial y^t}{\partial V_h} \qquad \Delta V_{h0} = -\eta \frac{\partial E}{\partial V_{h0}}, \quad \frac{\partial E}{\partial V_{h0}} = \frac{\partial E}{\partial y^t} \cdot \frac{\partial y^t}{\partial V_{h0}}$

$$\frac{\partial E}{\partial V_h} = -\sum_{t=1}^{N}\left(\frac{r^t}{y^t} - \frac{1-r^t}{1-y^t}\right) \cdot y^t(1-y^t) \cdot z_h^t = -\sum_{t=1}^{N}(r^t - y^t) z_h^t$$

$$\frac{\partial E}{\partial V_{h0}} = -\sum_{t=1}^{N}\left(\frac{r^t}{y^t} - \frac{1-r^t}{1-y^t}\right) \cdot y^t(1-y^t) \cdot 1 = -\sum_{t=1}^{N}(r^t - y^t)$$

$$\Rightarrow \Delta V_h = \eta \sum_{t=1}^{N}(r^t - y^t) z_h^t, \qquad \Delta V_{h0} = \eta \sum_{t=1}^{N}(r^t - y^t)$$

$$\Delta w_h = -\eta \frac{\partial E}{\partial w_h}, \quad \frac{\partial E}{\partial w_h} = \frac{\partial E}{\partial y^t} \cdot \frac{\partial y^t}{\partial z_h^t} \cdot \frac{\partial z_h^t}{\partial w_h}$$

$$\frac{\partial E}{\partial w_h} = \begin{cases} -\sum_{t=1}^{N}\left(\frac{r^t}{y^t} - \frac{1-r^t}{1-y^t}\right) \cdot y^t(1-y^t) \cdot V_h^t \cdot x^t + \sum_{h=1}^{H} 2w_h & \text{if } w_h^T x^t + w_{h0} > 0 \\ -\sum_{t=1}^{N}\left(\frac{r^t}{y^t} - \frac{1-r^t}{1-y^t}\right) \cdot y^t(1-y^t) \cdot V_h^t \cdot 0 + \sum_{h=1}^{H} 2w_h & \text{otherwise} \end{cases}$$

$$\Rightarrow \frac{\partial E}{\partial w_h} = \begin{cases} -\sum_{t=1}^{N}(r^t - y^t) \cdot V_h^t \cdot x^t + \sum_{h=1}^{H} 2w_h & \text{if } w_h^T x^t + w_{h0} > 0 \\ + \sum_{h=1}^{H} 2w_h & \text{otherwise} \end{cases}$$

$$\Delta w_h = \begin{cases} \eta \sum_{t=1}^{N}(r^t - y^t) V_h^t \cdot x^t - \eta \sum_{h=1}^{H} 2w_h \\ -\eta \sum_{h=1}^{H} 2w_h \end{cases} \qquad \Delta w_{hj} = \begin{cases} \eta \sum_{t=1}^{N}(r^t - y^t) V_h^t x_j^t - 2\eta w_{hj} \\ -2\eta w_{hj} \end{cases}$$

$$\Delta w_{h0} = \begin{cases} \eta \sum_{t=1}^{N}(r^t - y^t) V_h^t \cdot 1 & \text{if } w_h^T x^t \cdot w_{h0} > 0 \\ 0 & \text{otherwise} \end{cases}$$

1 Q1 30 / 30

✓ **+ 30 pts** **Everything correct**

- 5 pts Forget to update of $$w_h$$ based on the second term of the objective function:$$\triangle w_h^{'} = -\eta \cdot 2w_{h}$$, the final update of $$w_{h}$$ should be the sum of updates from both terms

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+ 2 pts Update $$v_{h}$$ with the first term (b), correct derivative for $$ \frac{\partial y^t}{\partial \alpha^t} = y^t(1-y^t)$$

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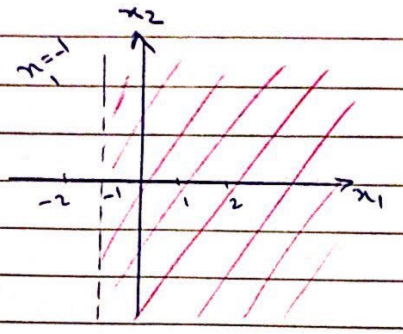
(2)

a)

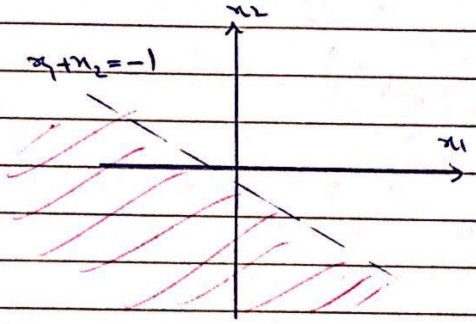


$x_1 = -1 \quad x_1 + 1 = 0$

$y = \delta(x_1 + 1)$ ✓

or $y = \delta(-x_1 - 1)$

$\Rightarrow w = [w_0, w_1, w_2]^T = [1, 1, 0]^T$



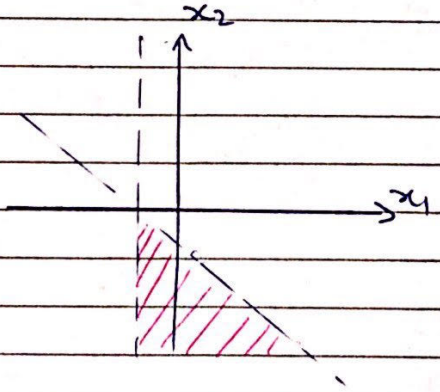
$x_1 + x_2 + 1 = 0$

$y = \delta(x_1 + x_2 + 1)$

or $y = \delta(-x_1 - x_2 - 1)$ ✓

$\Rightarrow w = [-1, -1, -1]^T$

b)



Output is 1 only if we AND two figures from a

$z_1 = \delta(x + 1) \quad , \quad z_2 = \delta(-x_1 - x_2 - 1)$

$\rightarrow y = \delta(z_1 + z_2 - 1.5)$

$$w = \begin{bmatrix} 1 & 1 & 0 \\ -1 & -1 & -1 \end{bmatrix}^T \qquad v = [-1.5 \quad 1 \quad 1]^T$$

2.1 (a) **15 / 15**

✓ **+ 15 pts Everything correct**

+ **7.5 pts** Correct vector for figure (2) (1, 1,0) and the scaled versions

+ **4 pts** Partial credit for figure (2) (e.g., one of the coefficients is missing/incorrect)

+ **7.5 pts** Correct vector for figure (3) (-1, -1, -1) and the scaled versions

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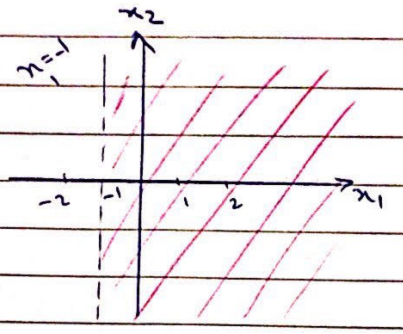
(2)

a)



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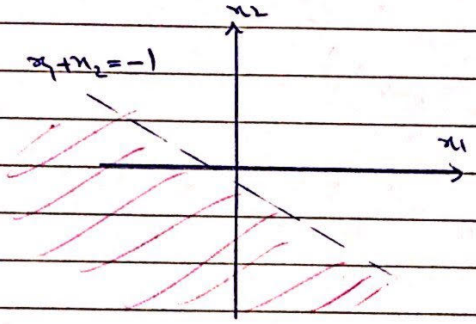
$\Rightarrow w = [w_0, w_1, w_2]^T = [1, 1, 0]^T$



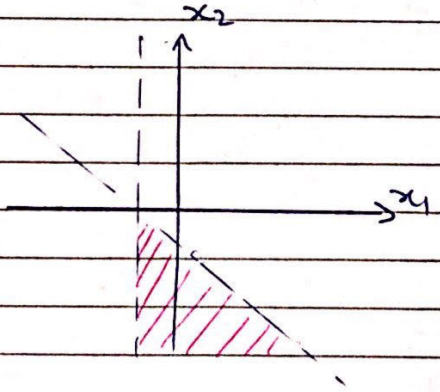
$x_1 + x_2 + 1 = 0$

$y = \delta(x_1 + x_2 + 1)$

or $y = \delta(-x_1 - x_2 - 1)$ ✓

$\Rightarrow w = [-1, -1, -1]^T$

b)



Output is 1 only if we AND two figures from a

$z_1 = \delta(x + 1) \quad , \quad z_2 = \delta(-x_1 - x_2 - 1)$

$\rightarrow y = \delta(z_1 + z_2 - 1.5)$

$$w = \begin{bmatrix} 1 & 1 & 0 \\ -1 & -1 & -1 \end{bmatrix}^T \qquad v = [-1.5 \quad 1 \quad 1]^T$$

2.2 (b) **15 / 15**

✓ **+ 15 pts Everything correct**

+ **7.5 pts** Correct W coefficients (stacking of vectors from part (a))

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+ **7.5 pts** Correct v vector (-1.5, 1, 1) (AND operation)

+ **4 pts** Partial credit for v (incorrect coefficient for one/more of the v vectors)

+ **0 pts** No attempt/completely incorrect

(3) My validation accuracies based on number of hidden units:

4	hidden units:	0.844 ~ 85%
8	" :	0.895
16	" :	0.918
20	" :	0.897
24	" :	0.901

So apparently 16 hidden units works the best among other numbers with highest validation accuracy. at first it starts to get better, but higher than 16 will give us lower accuracy.
My test accuracy with 16 hidden units is 0.917

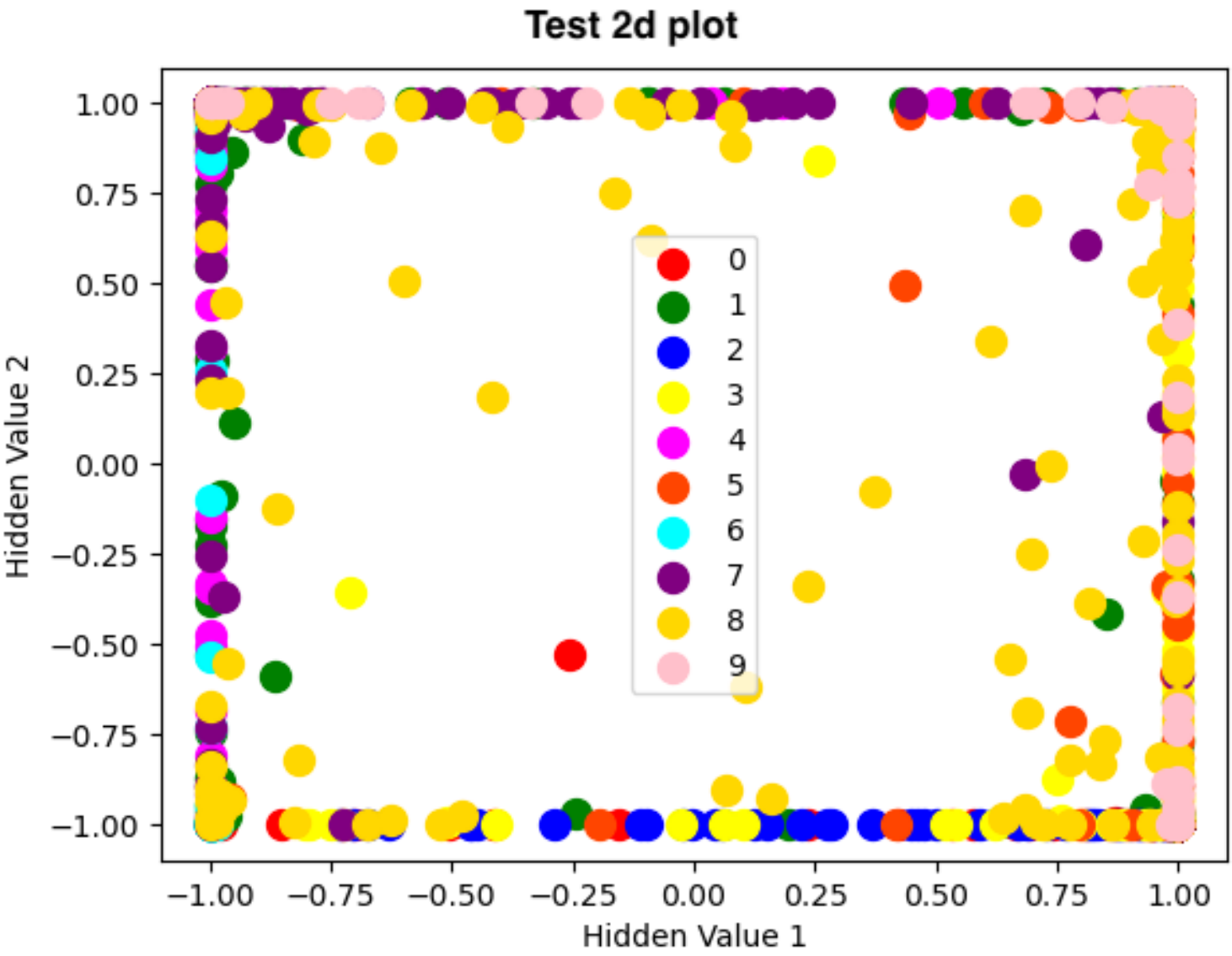
Then we can decide to use 16 hidden units.

c) Overall by Looking at the 3d plots we get a better understanding of class distributions.

2d plots allow us to visualize classes around 2 hidden nodes but in some regions its really hard to distinguish different classes.

However, 3d plots with 3 different hidden nodes, we get a better visualization by comparing it to 2d plot, we no longer have datapoints from different classes stacked upon each other and instead they are easily predicted now!

I guess at the end we can make better predictions by doing 3 hidden units compare to 2 hidden units and doing the plots just helped us to visualize this property.


Test 2d plot
Hidden Value 2
Hidden Value 1
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−1.00
−0.75
−0.50
−0.25
0.00
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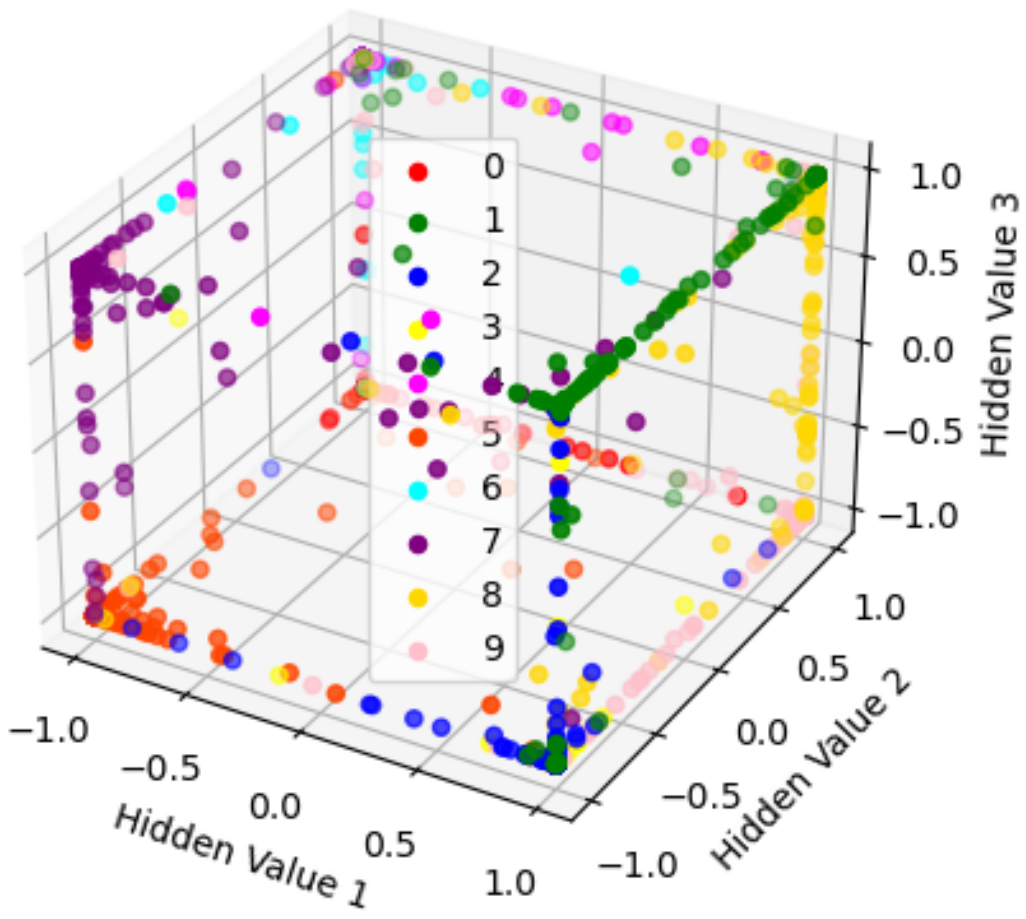
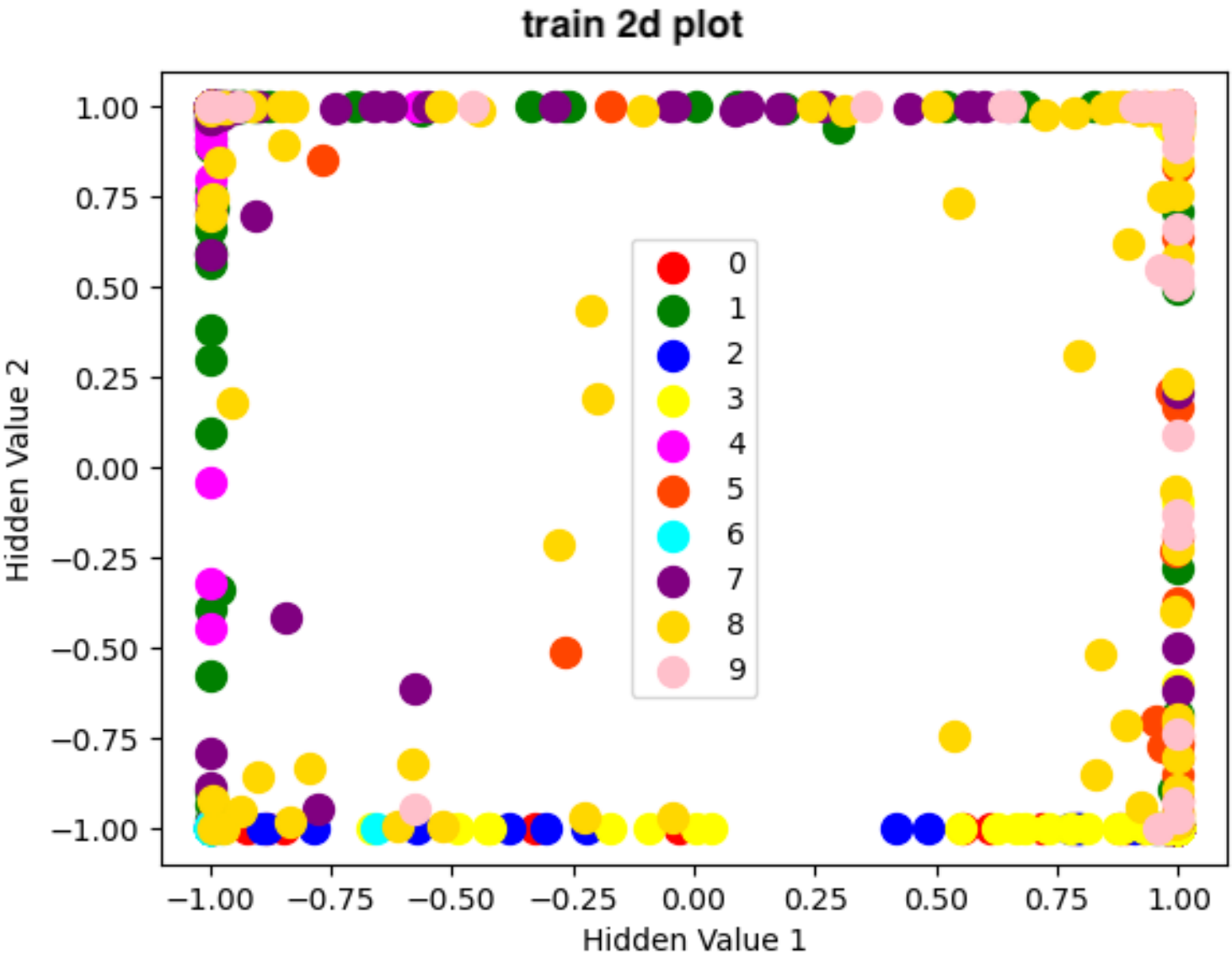
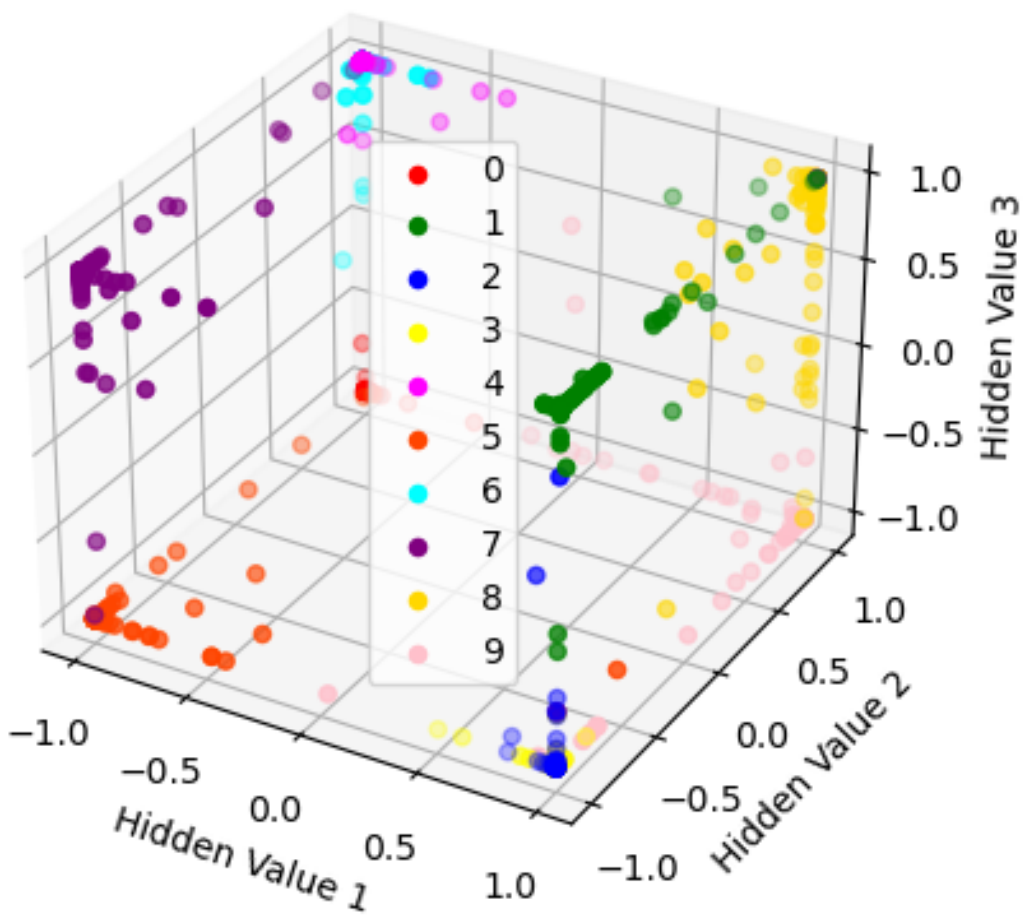
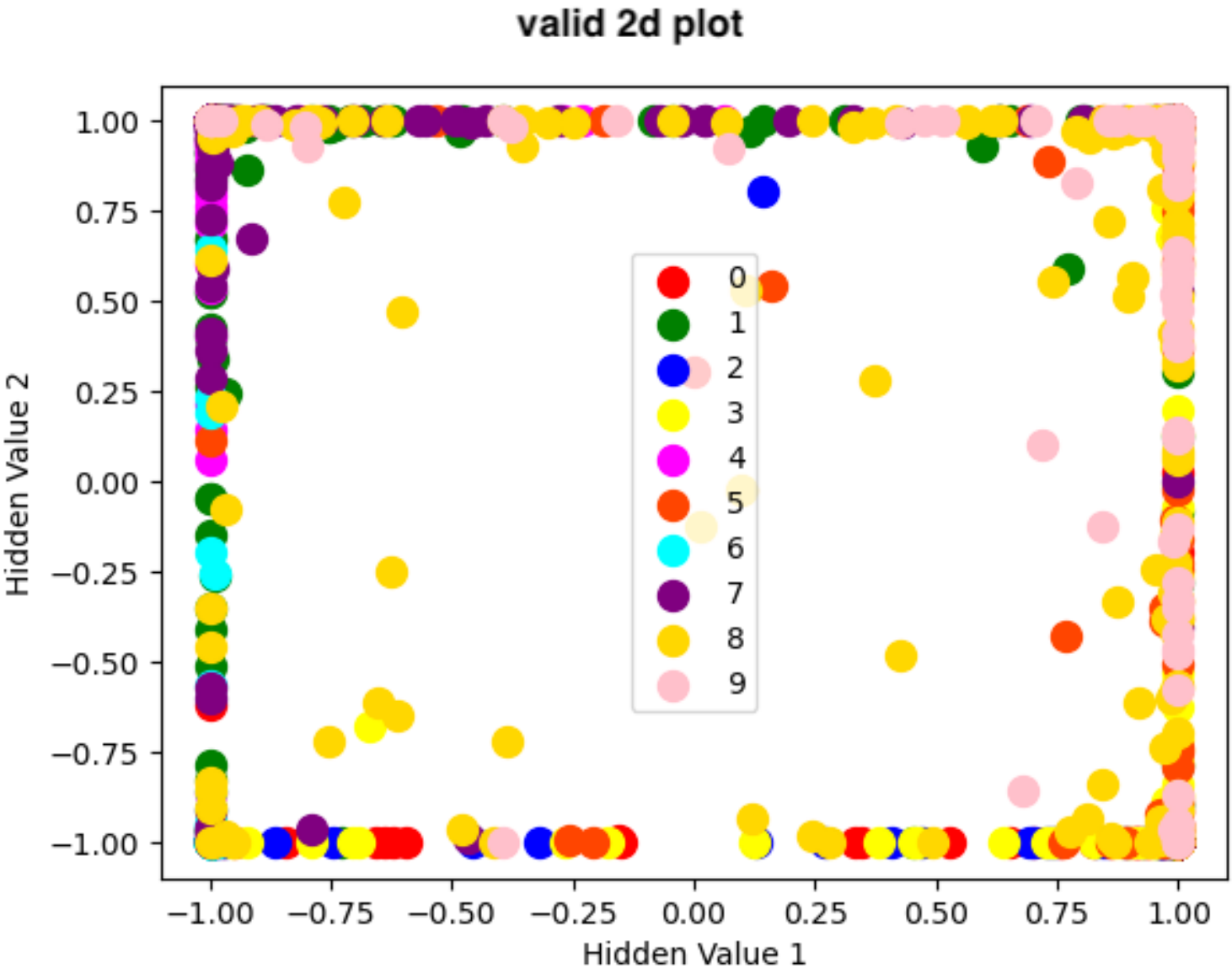
0.75
1.00
0
1
2
3
4
5
6
7
8
9


Test 3d plot
0
1
2
3
4
5
6
7
8
9
−1.0
−0.5
0.0
0.5
1.0
Hidden Value 1
Hidden Value 2
Hidden Value 3


train 2d plot
Hidden Value 2
Hidden Value 1
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
0
1
2
3
4
5
6
7
8
9


train 3d plot
0
1
2
3
4
5
6
7
8
9
−1.0
−0.5
0.0
0.5
1.0
Hidden Value 1
Hidden Value 2
Hidden Value 3


valid 2d plot
Hidden Value 2
Hidden Value 1
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
0
1
2
3
4
5
6
7
8
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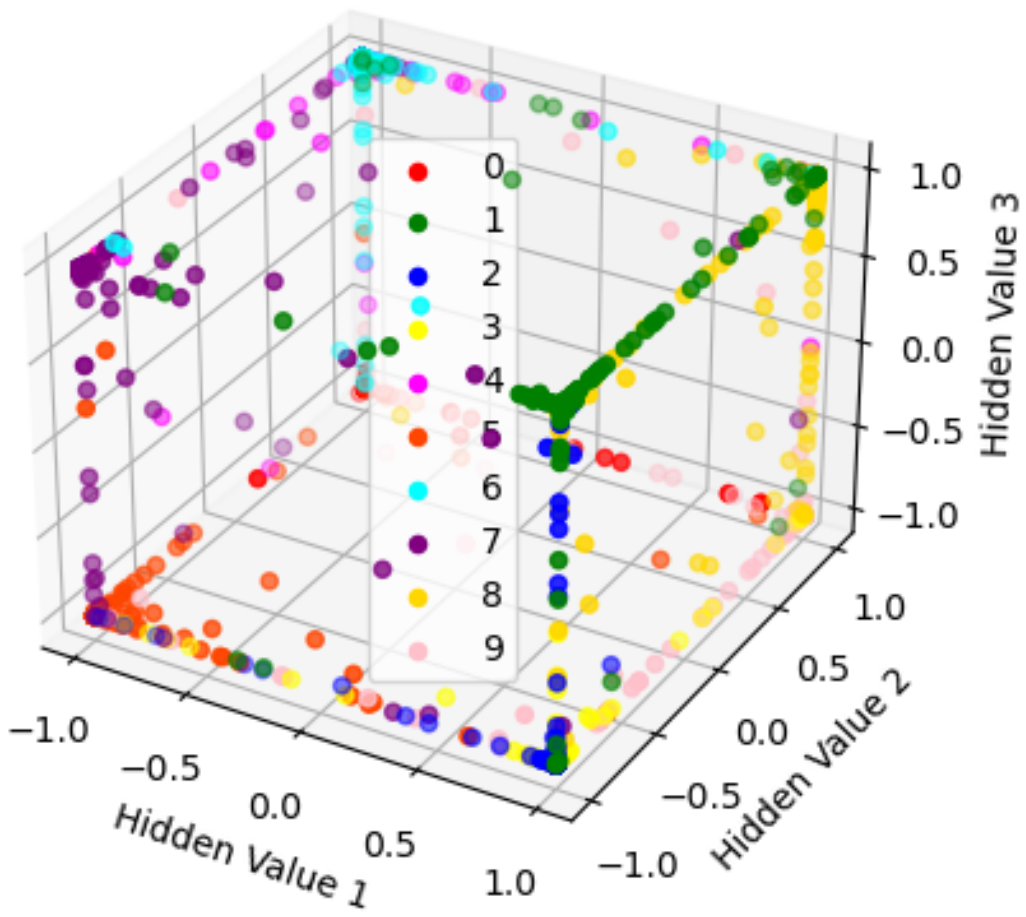
valid 3d plot
0
1
2
3
4
5
6
7
8
9
−1.0
−0.5
0.0
0.5
1.0
Hidden Value 1
Hidden Value 2
Hidden Value 3

3 Q3 10 / 10

✓ **+ 10 pts** **Everything correct**

+ 1 pts Reasonable reporting of the validation set accuracy over different hidden layers

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