Project 1

Radar Imaging

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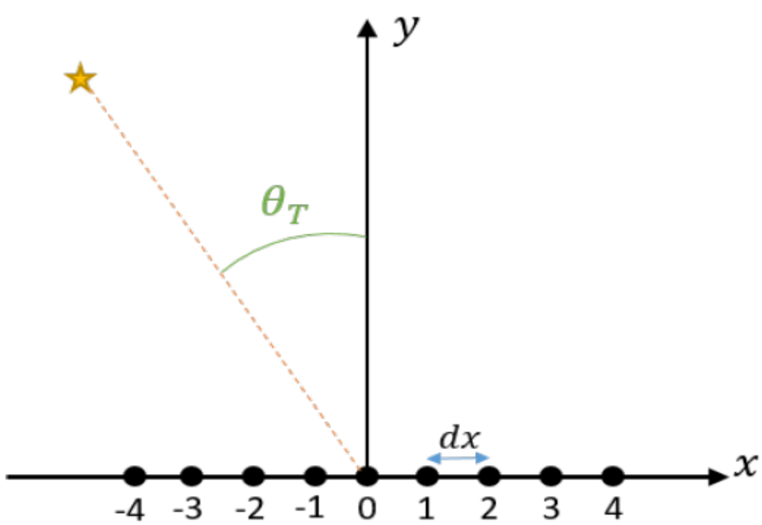
Introduction

What is happening in the scenario?

- There is a target (yellow star in the figure) that sends a signal toward an array of antennas.
- The antennas are arranged in a straight line with equal spacing (dx) between them, forming the **(ULA) Uniform Linear Array**.
- Each antenna receives the signal from the target, and the challenge is to use the received signals to estimate the **direction of arrival (DoA)** of the target.
- The system works in the **monostatic configuration** (each antenna both transmits and receives).
- In this project, we need to calculate both the **angular position** and the **distance** of the target relative to the array, using mathematical formulas and simulations. In the final part, we'll use **MIMO** technology to evaluate the antenna system's performance.
- For each part of the project, I will explain the goals and discuss the approach taken.



I. System Geometry

In this part, the goal is to estimate the angular position of a target using a Uniform Linear Array (ULA). The ULA consists of a line of equally spaced antennas, with each antenna receiving the signal emitted by a single target. By analyzing the phase differences between the signals received at each antenna, we can calculate the estimation angle position.

Key parameters:

- Frequency (f_0) : 77 GHz, commonly used in radar systems.
- Wavelength (λ): c/f_0, where c is the speed of light.
- Antenna spacing (dx) : Set to $\frac{\lambda}{2}$ or $\frac{\lambda}{4}$ for better angular resolution and to avoid aliasing.
- Array length (L): Derived to achieve the desired angular resolution $(\Delta\theta = 2°)$.

$$L = \frac{\lambda}{\Delta\theta}$$

The number of antennas (N) is calculated as:

$$N = \frac{L}{dx} + 1$$

II. Direction of Arrival (DoA) Estimation

First, we start by considering the target's angle θT as 20 degrees and convert it into radians, which will be needed for the estimation.

We have transmitted signal $e^{j2\pi f_0 t_0}$, also we know

$$f_0 = \frac{\sin(\theta_T)}{\lambda}$$

Once we receive the signal, the next step is to **demodulate** it. Demodulation involves removing the carrier frequency $(\boldsymbol{f0})$ by multiplying the received signal by $\boldsymbol{e}^{-j2\pi f0t}$.

Mathematical Background

Now, we want to estimate the true angle using the **Fast Fourier Transform (FFT)**. The FFT converts the demodulated signal from the time domain into the frequency domain, allowing us to analyze the signal's frequency components and accurately estimate the angle.

2. Array Design and DoA Estimation:

What should be the spacing between antennas?

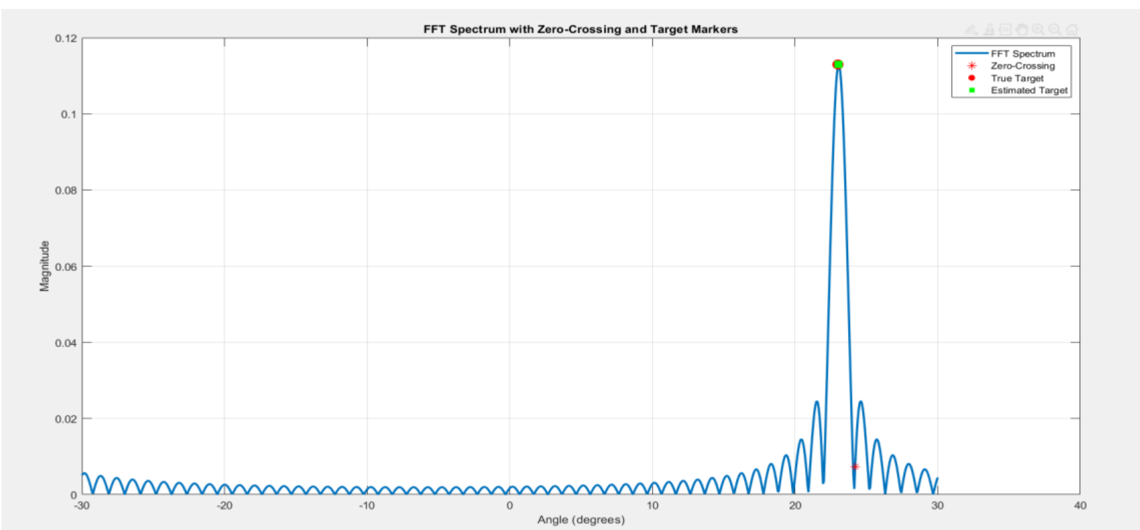
we should use($\frac{\lambda}{2}$) for 30degrees and angles less than 30 degrees and we chose($\frac{\lambda}{4}$) for angles greater than 30 degrees, to prevent aliasing and minimize grating lobes.

Can you propose a way to detect the direction of arrival?

To estimate the direction of arrival, we apply the **Fast Fourier Transform (FFT)** to the received demodulated signals. First, we define some parameters for the FFT, such as the sampling frequency, the number of samples, and the frequency spacing. then, we calculate the FFT of the received signal and use the peak frequency (f peak) to estimate the angle.

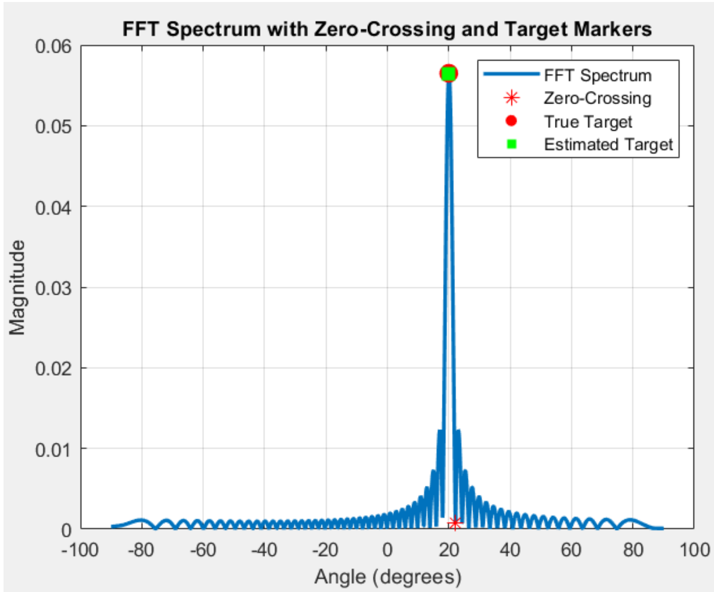
As shown in the figure below and results, when we set the true angle to 23 degrees, the estimated angle was 23.05 degrees. Additionally, as seen in the figure, the true angle (marked in red) and the estimated angle (marked in green) are very close to each other.

```
Command Window
True Angle (degrees): 23.00
Estimated Angle (degrees): 23.05
Spatial Frequency Resolution (degrees): 1.06
fx >>
```



Can you design the total array length to achieve a resolution of 2 degrees?

Spatial frequency difference refers to the smallest angle that can be distinguished by an antenna array. In the frequency spectrum obtained after performing FFT, this minimum angle corresponds to the first point after the peak. This point is marked with an asterisk (*) in the figure:



The **angular resolution** using spatial frequency can be calculated with the following formula:

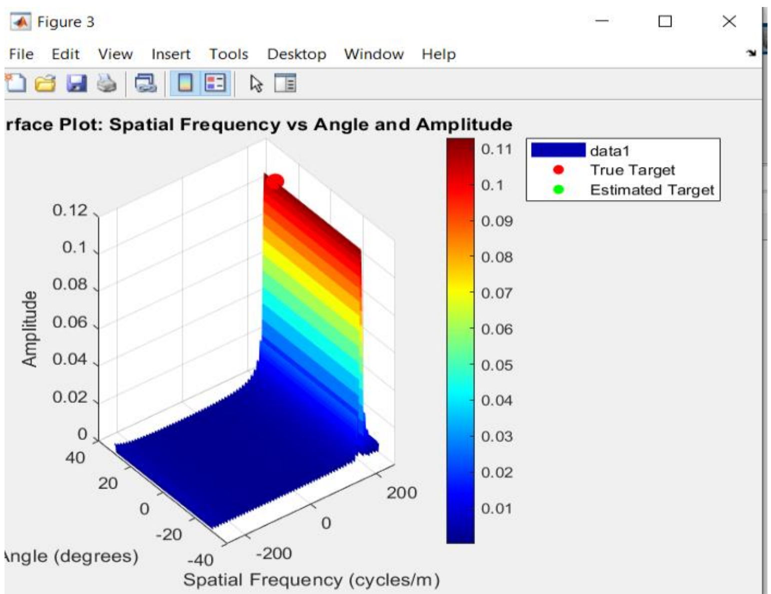
$$\Delta\theta = \arcsin\left(\frac{\Delta f \cdot \lambda}{2}\right)$$

We need to calculate the value of $\Delta\theta$, which for our example is shown, demonstrating the good performance of our algorithm. Additionally, the estimated angle is very close to the true angle. To better visualize the result, we provide a surface plot of the FFT spectrum with both the true and estimated angles.

Compare the estimated DoA with the true angular position of the target. Is it the same?

Yes, it is approximately the same, for example when we set $\theta_T = 23$, so we can see the estimation angular is 23.05 and it is very near to θ_T.

```
Command Window
  True Angle (degrees): 23.00
  Estimated Angle (degrees): 23.05
  Spatial Frequency Resolution (degrees): 1.06
fx >>
```

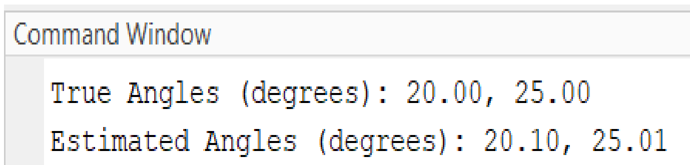


Is the resolution respected?

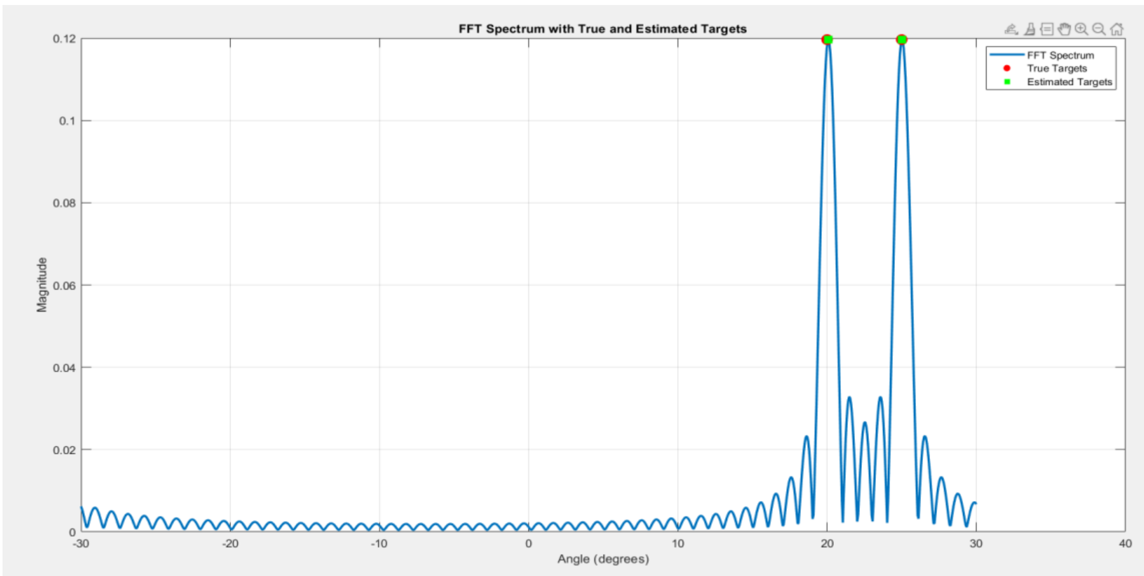
Yes, according to results ($\Delta\theta \approx 1.06$ and $\theta_T = \theta\ estimated$) the resolution is respected.

Repeat the simulation with more than one target in the scene. What happens?

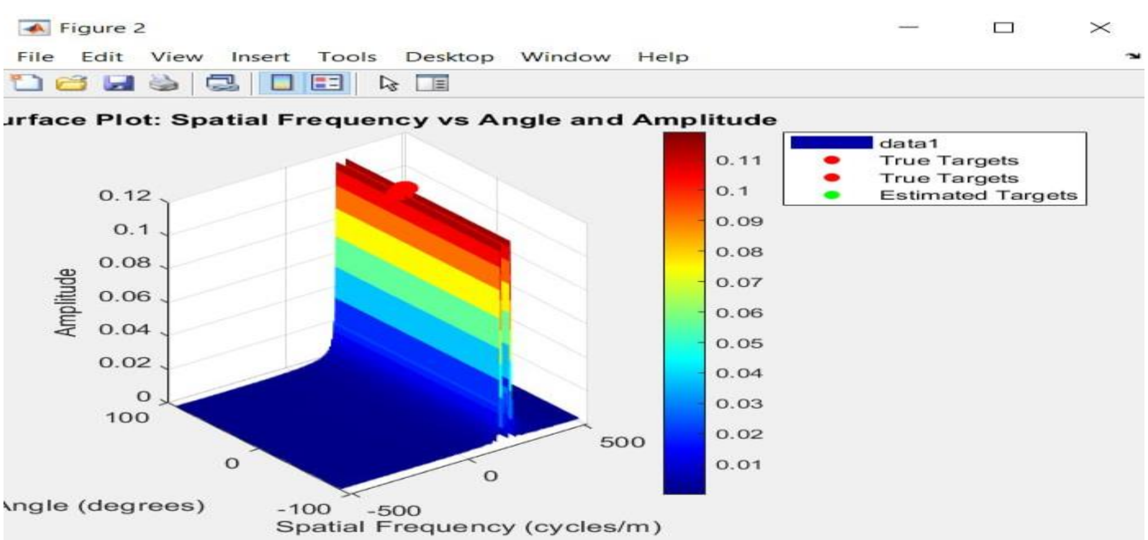
In this case, we repeat the previous steps for two targets, so we need to set two different angles, like θT=20 degrees and θT=25 degrees as shown in the code, we sum the received signals from both targets. And you can see the results:


Command Window

```
True Angles (degrees): 20.00, 25.00
Estimated Angles (degrees): 20.10, 25.01
```



- The output of FFT spectrum for two targets

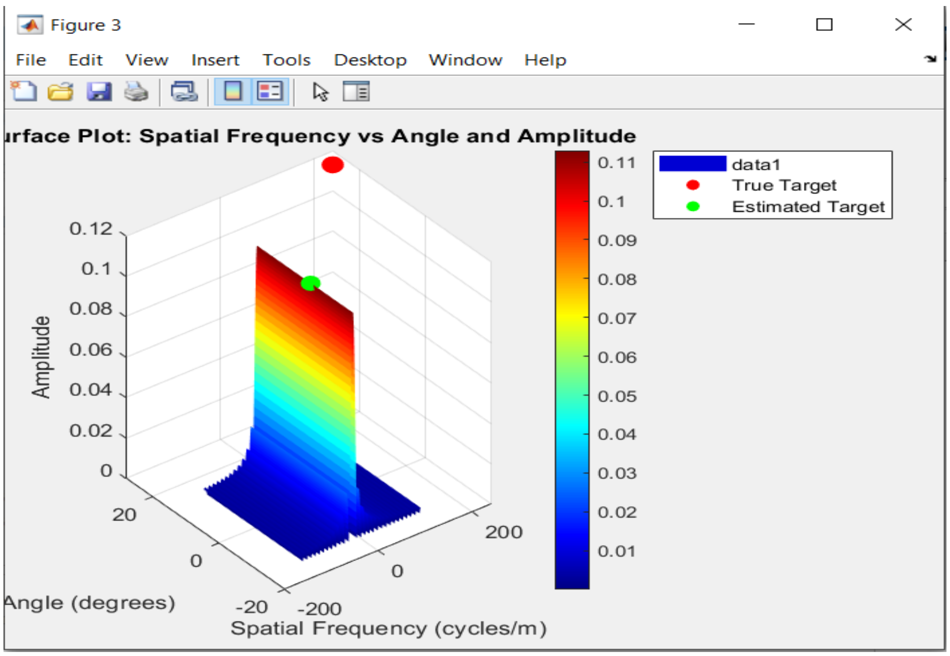


- Surface plot of FFT spectrum for two targets

What happens if you increase the spacing between antennas (dx)?

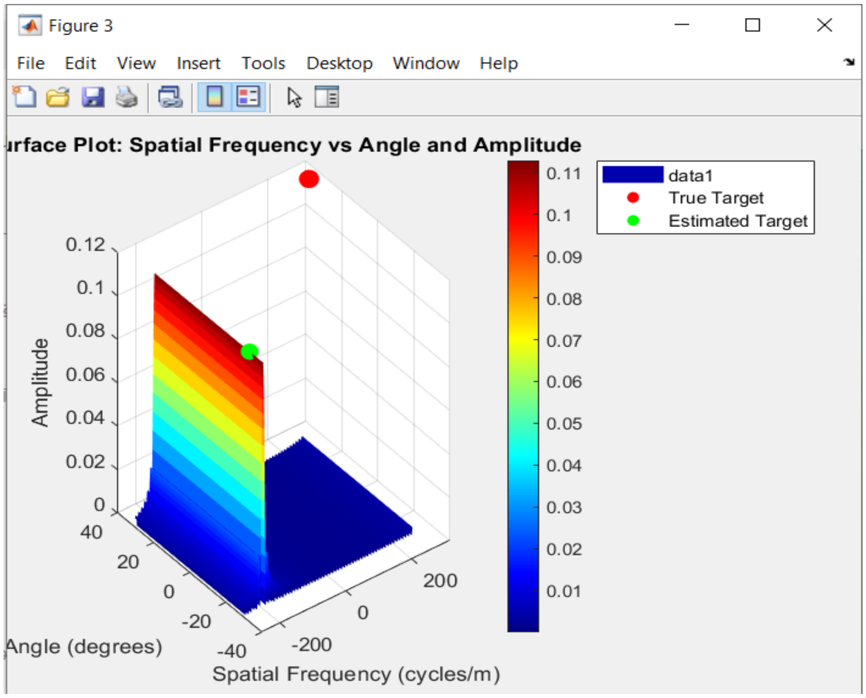
As we discussed earlier, we learned that choosing $\frac{\lambda}{2}$ is better for angles less than 30 degrees. So, in this case, when we set distance between antennas = λ and the true angle to 28 degrees, the resolution is not maintained. You can see the results below:

```
Command Window
True Angle (degrees): 28.00
Estimated Angle (degrees): -1.75
Spatial Frequency Resolution (degrees): 1.01
fx >>
```



As we know, for angles greater than 30 degrees, we should set $\frac{\lambda}{4}$. To answer this question, we fix $\frac{\lambda}{2}$, and you can see the results below:

```
Command Window
True Angle (degrees): 38.00
Estimated Angle (degrees): -22.62
Spatial Frequency Resolution (degrees): 1.06
fx >>
```



As you can see from the results, the true angle does not match the estimated angle, and in both examples, the resolution is not maintained.

2D Position Estimation Using Cross-Correlation

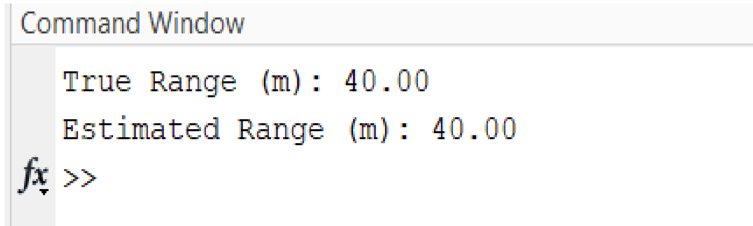
In this task, the goal is to estimate the range (distance) of a target using the Uniform Linear Array (ULA). The process involves generating a transmitted signal, adding a time delay to represent the propagation of the signal, and using the cross-correlation algorithm to calculate the distance.

Cross-Correlation Method

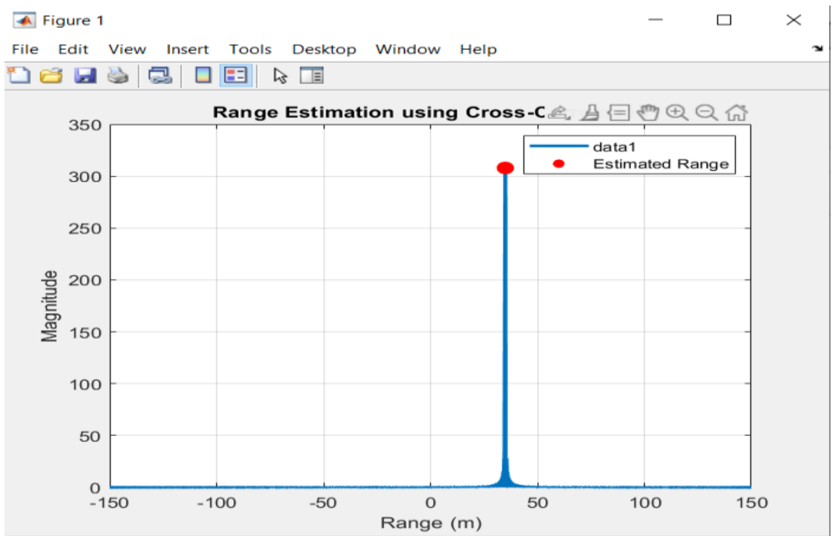
Cross-correlation is a technique used to check how similar two signals are when one is shifted over time. It helps us find the time delay between the signals, which can then be used to estimate distances or positions. To apply this method, we need both the transmitted signal and the received signal (which is delayed due to distance). The transmitted signal is:

$$g(t) = \mathrm{sinc}(Bt)e^{j2\pi f_0 t}$$

Once the time delay is found, we use it to calculate the distance to the target.



For this part of the task, just like in the angular estimation, we set the true range to 40 meters. We then see that the estimated range is also 40 meters.



- the figure of range estimation using Cross-Correlation method

1.What is the time and space resolution of such a signal?

The time and space resolution of the signal can be determined using the following equations: The **time resolution** is calculated using the formula:

$$\Delta t = \frac{1}{B}$$

where B is the bandwidth. For this task, the time resolution comes out to be 1 nano second (ns). This means that the radar can distinguish between two signals from targets that are separated by at least 1 ns in time.

$$\Delta r = \frac{c \cdot \Delta t}{2}$$

where c is the speed of light, and Δt is the time resolution. In this case, the space resolution is 0.15 meters, which indicates that the radar system can distinguish two targets that are at least 0.15 meters apart.

2.What is the expression of the received and demodulated signal at each antenna given a generic target in the scene?

The received signal at each antenna, given a generic target in the scene, is affected by time delays and phase shifts that depend on the target's position relative to the antenna. The signal, after reflection from the target, is delayed and phase-shifted accordingly. The received signal for antenna n is:

$$s_{\text{received}}\,(t,n) = g(t-\tau_n)\cdot e^{j2\pi f_0 \tau_n}$$

The **demodulated signal** is obtained by removing the carrier frequency and applying a sinc function to the time-shifted signal:

$$s_{\text{demod}}\,(t,n) = \operatorname{sinc}\big(B(t-\tau_n)\big)\cdot e^{j2\pi\frac{2d_n \sin\,\theta}{\lambda}}$$

MIMO array:

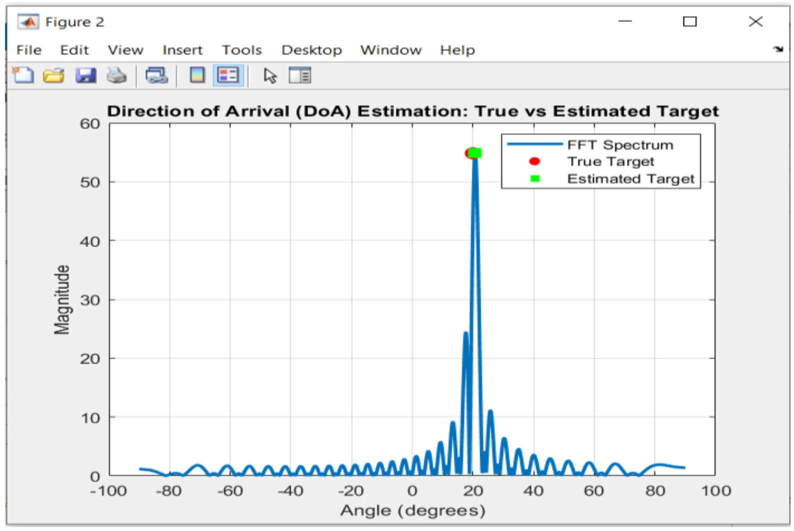
MIMO Radar helps improve the estimation of both angle and range. It provides better angular and range resolution, allowing it to detect smaller differences in distance and angle.

In this task, the goal is to estimate both the angle and range of a target using MIMO radar. At the start of the project, we replaced "**dx**" with "**dvirtual**" to represent the spacing between virtual elements and set the total number of antennas in the system to 58. Since MIMO radar works by pairing transmitting and receiving antennas, we allocated 2 transmitting antennas and 29 receiving antennas, ensuring the total number of antennas stayed at 58.

After setting up the antenna configuration, we calculated the positions of the virtual antennas to improve the radar's ability to accurately determine the target's location.

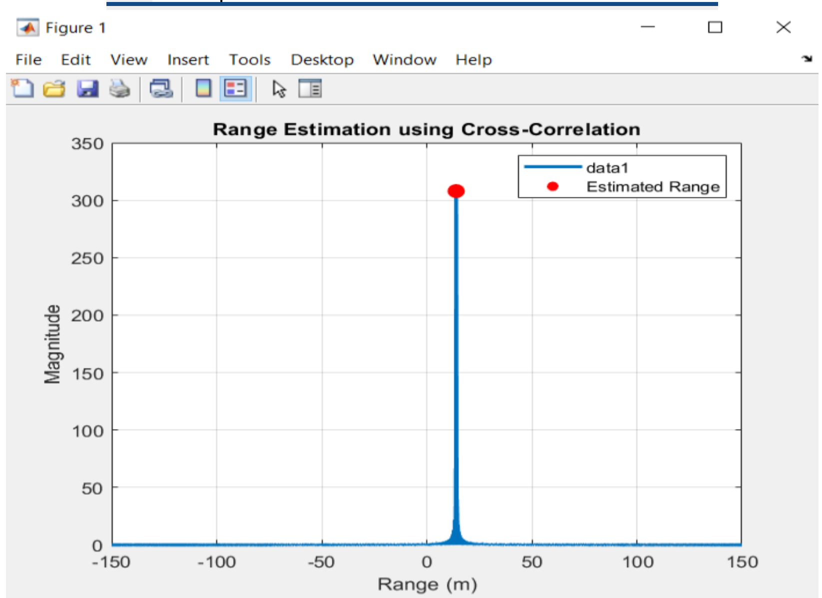
```
Command Window
  Number of virtual antennas: 58
  Number of Tx antennas: 2
  Number of Rx antennas: 29
  True Angle: 20
  Estimated Angle: 20.7836
```

- By setting 2 transmitting antennas Tx and 29 receiving antennas Rx, and with a true angle of 20 degrees, we found that the estimated angle was 20.7836, confirming the system's ability to estimate the angle.



- Output of FFT spectrum showing the true and estimated angular values in MIMO radar, with markings.

```
True Range: 14 m
Estimated Range: 14.0002 m
fx >>
```



- Similarly, based on the results and the spectrum for range estimation, after setting the true range to 14m, we found that the estimated range was very close to the true range (14.0002 m), demonstrating the accuracy of the radar's resolution.

Finally, we performed the angle and range estimation and plotted the results to visualize the performance of the MIMO radar system.