

# Towards user-friendliness in proof assistants: automated strategies via algebraic effects and handlers

Proof assistants provide a framework for modelling and verification of theories, as well as trust-worthy software. However, such power is usually only available to experts. We propose a new approach, based on algebraic effects and handlers, to integrate different automated proof strategies that enables newcomers to take advantage of proof assistants without a more in-depth understanding of underlying theory. Our approach gives beginner users an effect system, a handful of effectful strategies (tactics, proof search and a SMT solver) and their handlers under a shared interface, while advanced users can extend our system with new effects and new handlers. Lastly, we prototype the system as a library in Agda. While our prototype is minimal, it shows how easily proofs can be carried on so long as the user has the correct intuition. We believe our system empowers non-experts and has the potential to bring verified software to many relevant industries, such as finance.

Additional Key Words and Phrases: effect handlers, proof assistants, user experience

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## 1 VERIFICATION FOR THE MASSES

The usability of proof assistants such as Coq, Agda, Isabelle/HOL and Twelf is hard to measure: all of them require significant domain knowledge that frequently matches a deeper understanding of Type Theory, the theory that enables such assistants to exist. Coq and Isabelle/HOL are more commonly applied outside their niche, and their programming style is sharply different from mainstream programming and even functional programming. Granted, they come with their own integrated development environment (IDE), which may make it easier for students and newcomers. To our knowledge, no usability study has been held for a proof assistant, however there exists studies that account for an informal “usability” criteria – they appear to measure how fast users can familiarise themselves with the system or IDE.

Folklore within the proof assistants community seems to show that typical users find Coq hard to use due to a large library of tactics and difficult readability of the code after its completion. Many Coq learning materials suggest users to add comments for structural induction cases and inductive hypothesis. As mentioned, no user studies were conducted to attest such hypotheses.

It also has to be granted that proof assistants are relatively new technology and there are no guidelines or even an intuition on how such systems should perform or what features should be available to the users. The closest model we have are proofs by hand, however, following that model for proof assistants has two main pitfalls: (a) proofs by hand are also not widely taught in mathematics classes during school years; and (b) proofs by hand and automated proof

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writing employ fundamentally different paradigms, where the latter requires axioms and formulations to be encoded explicitly, commonly known as a “pedantic proof style”.

### 1.1 Empowering users of different levels of expertise

To popularise software verification, we must make proof writing less of a burden to the user. Currently, only specialists can write proofs for their programs, even though the working programmer understands the domain and invariants of their projects – otherwise they would not be able to write any software. *Our hypothesis is that programmers are well-equipped to derive and prove properties of the software they write, but they lack the mathematical maturity and vocabulary to carry out a formal proof.* We have evidence that students fail to produce well-made proofs due to the lack of mathematical maturity, even though they do understand the subject matter at hand.

In our approach, we propose an (algebraic) effects and handlers view of such proofs, based on prior work developed by the Andromeda proof assistant. Here, our users will program as they would normally and invoke a proof environment as an effect to prove certain properties as they go. Given that the proof environment is *just* an effect, we envision that different proof styles (e.g. Agda-style dependent types, SMT solver, proof search) can be “composed” under a shared interface, a proof object that can manipulate itself while enquiring different automated proof engines.

The reasons we employ algebraic effects and handlers are manifold:

- (1) as proofs cannot be fully automated, all approaches that try to automate the process (e.g. proof search, SMT solver) may either be non-deterministic or never find a solution. Therefore, the system should be able to handle “impure” computations and errors. Algebraic effects and handlers have well-defined semantics and provide a simple interface for performing effects. With them, we avoid indiscriminate effects that are often error-prone and intricate effects machinery such as monad transformers;
- (2) its semantics accommodate composition of arbitrary effects and the composition of multiple handlers, which means users have the ability to weaken more general strategies into specific ones while maintaining the original untouched;
- (3) it has well-defined semantics, and it is the main feature of many new research languages, such as Koka, Eff, Frank and Effekt, what suggests a greater potential.

### 1.2 Goals and Contributions

The present project intends to land itself as a compiler-level implementation into the Juvix<sup>1</sup> programming language. We design Juvix with focus on working programmers familiar with strongly-typed functional programming.

*Goals.* The project exists to enable users to write safe, verified code without investing time into learning dependent types and theorem proving. The project’s goals are presented as follows:

- (1) make software verification more accessible to programmers familiar with functional programming;
- (2) foster knowledge sharing between domain experts and regular programmers;
- (3) lower the barrier to entrance into software verification to non-experts.

*Contributions.* While the project is a work-in-progress, this paper provides three important contributions that lay out the full extent of the work, which are presented as follows:

<sup>1</sup>Juvix is a dependently-typed programming language for smart contracts. More information at [juvix.org](http://juvix.org)

- (1) a specification of a user interface for automated proofs based on algebraic effects and handlers that permits user-defined effects for proof strategies, and also user-defined handlers for existing effects (Section 2);
- (2) a minimal prototype in Agda that validates the feasibility of such system (Section 3);
- (3) a short technical account of the effort required to materialise such a user interface into proof assistants (Section 3.1).

## 2 INTRODUCING WITCH

In this section, we introduce our approach, named Witch<sup>2</sup>. We define a *witch* as an assistant tool for theorem provers that congregates many different strategies for proof automation. Our specification for a *witch* for Juvix uses algebraic effects and handlers as the means to congregate strategies as effects.

*Algebraic Effects and Handlers.* Algebraic effects were introduced by <sup>1</sup>citation, and their handlers were introduced later on by <sup>2</sup>citation. An algebraic effect is defined by an effect signature that comprises a set of operations, and handlers that denote said operations. A handler may interpret operations at the compiler-level, which was formally named co-model, but colloquially called “top-level handler”. Handlers are the most general form to denote a set of operation, and more specific classes of handlers also exist.

*The Essence of Witch.* In Figure 1, we sum up our *witch* approach. As for the syntax, we use operation `{ params }` via `handler` that is an improvement over `handler(operation, params)`, since effect handling is similar to function application where it carries effect information as well. The user defines data types and functions as usual, and then uses a *witch* to prove properties concerning said definitions. The Examples section in Figure 1 shows it in action. The examples are over-simplified to enable a clearer communication of our proposal – we avoid more complex properties and complicated proofs as initial examples. The proof of commutativity of addition under natural numbers and of associativity of list concatenation are shown, and use the three main effects: Solver, Proof Search and Tactics. In the proof assistant literature, there exists no precise definition of commonly used terms “solver”, “proof search” and “tactics”. All said terms are used in different communities to mean “automatic strategy to construct a term under certain constraints”. Here, however, we opt for a pragmatic differentiation:

- The Solver effect is used for integration with external engines via IO; we believe it suffices to say, e.g. SMT, as handlers should implement internal strategies to choose between the so-called logics within a solver. If the black-box approach to solvers presents itself a challenge, an specialisation of the handler is possible, e.g. operation `{ params }` via `Z3.QF-UFLIA`.
- The Proof Search effect is used for library-level algorithms; users may choose to implement their own algorithms using a limited set of meta-programming constructs that are handled at top-level<sup>3</sup>.
- The Tactics effect represent strategies that simplifies the proof at least one step, and may not complete all proof goals. This style is mainly employed in the proof assistant Coq. While we do not foresee as many tactics implemented as in Coq, we believe famous tactics such as `omega`, `ring`, `Admitted` are useful to users of all levels of expertise.

<sup>2</sup>Our tool’s name is a play after assistant tools colloquially called “wizards”. There is no consensus of what a wizard is or what are exactly the tasks it is supposed to help users. Wizards seem to be used mostly for multiple-step and/or configuration features, however. We went for the name “witch” to align it to the idea of assistant tools, while dodging the overloaded, yet nebulous terminology.

<sup>3</sup>The meta-programming constructs are operations of the Typechecker effect whose handlers are not available for the user.

- Lastly, the Error effect represents feedback messages to the user, since any of the strategies may fail. Error has two operations, throw and trace: the former notifies the user a strategy has failed, while the latter registers<sup>4</sup> completed sub-goals during the strategy’s attempt to complete the proof.

*Witch’s Architecture.* A *witch* sits between the typechecker and the interactive mode of a proof assistant. It uses constructs provided by the typechecker and notifies the interactive mode with regards to the state of the current strategy attempted. The user may manipulate proof handlers by composing or writing their own.

*Downsides of our Approach.* There are two main styles for writing proofs within a proof assistant: external verification and internal verification. The latter concerns itself with code that is *correct-by-construction*, meaning that only valid states can be encoded and it carries all information necessary to make proofs trivial; while the former concerns itself with proofs about code that does not carry any additional information for proving purposes. Witch’s primary focus is on external verification. We believe that newcomers will struggle to write proof-aware code, since it departs strongly from mainstream programming, even within the functional style. We expect users to write their code as they normally would do and then prove necessary properties. Witch does not support writing correct-by-construction code. As proofs are erased during compilation-time in Juvix, Witch could be extended to be indexed by proof objects and parameterised over any type, such as  $\{A: \text{Type}\} \rightarrow x\ y: A \rightarrow \text{⚡} (x * y \equiv y * x)$  where we make sure that  $*$  over  $A$  is commutative. However, it is unclear whether having user’s code under a monad-like structure leads to well-structured code.

### 3 AGDA’S WITCH

*Why Agda?* Agda provides higher-level meta-programming constructs, and its coding style is a reminiscent of the ML family of functional programming languages.

#### 3.1 Technical Effort Required

Although Witch is implementable at the library level, it requires that proof assistants to provide a set of off-the-shelf features. The most important feature is type-level meta-programming. Witch is only possible given the proof assistant’s ability to manipulate its own proof terms to generate new variables, holes, goals, among others and manipulate terms and types in a proof-carrying fashion. Secondly, the proof assistant should feature an interface with the outside world. Although it is not inherently a requirement, its absence means that no solver will be integrated, diminishing the proposed value of the tool. Lastly, proof assistants must support algebraic effects and handlers. If there is no built-in solution, a library-level solution is possible – as we did in Agda. However, a solid implementation may not be trivial to achieve. In Agda, we translate effect handling into optimised continuation-passing style that inlines all applications. This strategy also works at compiler-level if necessary. There exist many strategies to efficiently implement algebraic effects and handlers, but we went for the one that seemed the most straight-forward method to implement it in Agda.

## 4 RELATED AND FUTURE WORK

### 4.1 Comparison with Similar Approaches

*Andromeda proof assistant.* As mentioned, this project was inspired by the Andromeda proof assistant. It follows the LCF tradition of theorem provers, where there exist two distinct languages. One inhabits a safe nucleus that verifies

<sup>4</sup>Internally, the Typechecker effect should have a tree that stores all currently saved traces.

hypotheses, on the other hand, the second hand comprises a polymorphic  $\lambda$  calculus where users implement their theories and relay the verification to the nucleus.

*Meta-F\**.  $F^*$  is a dependently typed language with (monadic) effects and liquid types. For tactics,  $F^*$  uses a library called *Meta-F\**, where tactics are considered an effect that uses meta-programming provided by the typechecker to manipulate proof objects. Agda’s *Witch* implementation is heavily inspired by *Meta-F\**.

*Mtac and Cybele*. The idea of proof strategies happening within an effectful structure was first introduced in *Cybele* and later extended into *Mtac*. *Mtac* extends Coq’s powerful main language, providing the user with a dependently-typed tactic language.

## 4.2 Improving Witch Further

While it is hard to say whether we have achieved our goals before user evaluations, *Witch* steps towards a future where proof assistants are friendlier towards users, particularly newcomers. The present work is in its infancy, and there are many issues concerning *Witch*’s ability to guide newcomers towards correct-by-construction software, which constitutes the main improvement we intend for *Witch*. We hypothesise about the possibility of using proof reconstruction, therefore relocating *Witch* to an assistant that lives solely within the interactive mode – as opposed to a framework for proof automation in user code.

In parallel with the usability work, we plan to develop a formalisation of *Witch* as a dependent typed language with effectful, dependently typed operations and effect handler, based on previous work on fibred algebraic effects.

# Understanding Witch



## Algebraic Effects and Handlers — quick and dirty intro

- ✧ an effect contains a list of effectful operations,  $\Sigma$ ;
- ✧ each operation in  $\Sigma$  takes a list of params  $p^k$ , where  $k$  is the length of the list;
- ✧ a handler deals with  $\Sigma + 1$ , i.e. all operations in  $\Sigma$  and returning pure values;
- ✧ internally, for each operation in  $\Sigma$ , a handler receives  $p^k$  and a continuation.
- ✧ a handler may choose to call the continuation zero, one or many times.

Syntax `operation { params } via handler`



## Examples

```
data N : Type where
  0 : N
  s : N → N

data [_] : Type → Type where
  0 : ∀ {A: Type} → [A]
  _:: : ∀ {A : Type} → A → [A] → [A]
```

✧ regular Haskell-style datatype and function definitions

✧ Witch's effect wrapper is represented by

```
+comm : ∀ (x y : N) → ✧ x + y ≡ y + x
+comm 0 y = tactic { 0 + y ≡ y + 0 } via Ring
+comm (s x) y = search { x + (s y) ≡ suc (x + y) } via Backtracking

++assoc : ∀ {A : Set} (x y z : [A]) → ✧ (x ++ y) ++ z ≡ x ++ (ys ++ zs)
++assoc 0 y z = solve { (0 ++ y) ++ z ≡ 0 ++ (ys ++ zs) } via SMT
++assoc (x :: xs) y z rewrite ++assoc xs y z = pure refl
```



## Strategy as an Effect

```
Solver      > solve
Proof Search > search
Tactics     > tactic
Error       > throw; trace
```



Main Effects and their Operations

- ✧ Strategies may fail, so Error is a built-in effect; however, Error is only supposed to be called within a handler;
- ✧ Handlers may trace errors to provide better hints to the users;
- ✧ Solver models the concept of an external engine accessed via IO, while Proof Search, of an algorithm that can be implemented as a library.
- ✧ Tactics models strategies that take a step, but may not finish the proof.



## Witch's Architecture, briefly

