Robert Maloy
11/8/17

Discrete Mathematics

5.4 #6 Suppose that $f_0, f_1, f_2 \dots$ is a sequence defined as follows:

$f_0 = 5, \quad f_1 = 16$

$f_k = 7f_{(k-1)} - 10f_{(k-2)}$

Prove $f_n = 3\cdot 2^n + 2\cdot 5^n$

$P(0) \Rightarrow f_0 = 3\cdot 2^0 + 2\cdot 5^0$

$f_0 = 5$

$P(1) \Rightarrow f_1 = 3\cdot 2^1 + 2\cdot 5^1$

$f_1 = 3\cdot 2 + 2\cdot 5$

$f_1 = 6 + 10$

$f_1 = 16$

Let $k \geq 1$ be an integer, and suppose $f_i = 3\cdot 2^i + 2\cdot 5^i$ for all integers i with $0 \leq i \leq k$.

$$
\begin{aligned}
f_{k+1} &= 7f_k - 10f_{k-1} \\
&= 7(3\cdot 2^k + 2\cdot 5^k) - 10(3\cdot 2^{k-1} + 2\cdot 5^{k-1}) \\
&= 21\cdot 2^k + 14\cdot 5^k - 30\cdot 2^{k-1} + 20\cdot 2^{k-1} \\
&= 21\cdot 2^k + 14\cdot 5^k - 15\cdot 2^k + 4\cdot 5^k \\
&= (21-15)\cdot 2^k + (14-4)\cdot 5^k \\
&= 6\cdot 2^k + 10\cdot 5^k \\
&= 3\cdot 2\cdot 2^k + 2\cdot 5\cdot 5^k \\
&\boxed{= 3\cdot 2^{k+1} + 2\cdot 5^{k+1} \leftarrow \text{Proof.}}
\end{aligned}
$$

8.1 #3 The congruence modulo 3 relation T, is defined from
Z to Z as follows: $\forall$ integers m, n:

$$m\,T\,n \Leftrightarrow 3 \mid (m-n)$$

a. $3 \mid (10-1) \Rightarrow 3 \mid 9 \Rightarrow 3$ Yes
$3 \mid (1-10) \Rightarrow 3 \mid -9 \Rightarrow -6$ ~~Yes~~ No.
Is $(2,2) \in T$? No, $3 \nmid 0$.
Is $(8,1) \in T$? yes.

b. $n > 3$, gives $n\,T\,0$. $(4,5,6,7,8)$
c. $n > 4$ gives a defined $n\,T\,1$. ~~$(4,5,6,7,8)$~~ $(5,6,7,8,9)$
d. $n > 5$; $(6,7,8,9,10)$

#11. $A = \{3,4,5\}$; $B = \{4,5,6\}$
S = "divides" relation ↓

$$x\,S\,y \Leftrightarrow x \mid y.$$

State explicitly which ordered pairs are in S and S^{-1}

$S = \{(4,4), (5,5), (3,6)\}$
$S^{-1} = \{(4,4), (5,5)\}$

#17. $A = \{2,3,4,5,6,7,8\}$, define a relation T on A as follows:

$$x\,T\,y \Leftrightarrow 3 \mid (x-y)$$

~~$\{(2,4), (2,6), (2,8), (3,6), (4,8),$~~ $T = \{(2,5), (2,8), (3,6), (4,7), (5,2), (5,8), (6,3), (7,4), (8,5)\}$

1.3

#4. Let $G = \{-2, \boxed{0}, 2\}$ and $H = \{4, \boxed{6}, 8\}$ and define a relation V from G to H as follows:

For all $(x, y) \in G \times H$.

↳ $(x,y) \in V$ means $\frac{x-4}{4}$ = integer.

a. $\frac{(2)-(6)}{4} = \frac{-4}{4} = -1 \to$ yes, $2 V 6$ b/c -1 is an integer.

a. $\frac{(-2)-(-6)}{4} = \frac{4}{4} = 1 \to$ yes.

a. Yes, $(0,6) \in V$.

a. yes, $(2,4) \in V$.

b. $V = \{(-2,4), (-2,6), (-2,8),$
$(0,4), (0,6), (0,8),$
$(2,4), (2,6), (2,8)\}$

c. Domain: $\{-2, 0, 2\}$
Codomain: $\{4, 6, 8\}$

d. $\{-2, 0, 2, 4, 6, 8\}$