MAD 2104: Practice Test 1

1. Rewrite the following using variables:

"Given any two real numbers, there is a real number in between."

$\forall$ real numbers a and B, $\exists$ a real number c s.t. $a<c<b$.

2. Let A={c, d, f, g}, B={f, j}, and C={d, g}. Answer yes or no for the following questions:

a. Is $B \subseteq A$? NO b. Is $C \subseteq A$? Yes

c. Is C a proper subset of A? Yes (A contains elts. not in C)

3. Consider the statement forms $(p \vee q) \vee (p \wedge r)$ and $(p \vee q) \wedge r$.
Fill in the truth table showing each form:

p	q	r	$p \vee q$	$p \wedge r$	$(p \vee q) \vee (p \wedge r)$	$(p \vee q) \wedge r$
T	T	T	T	T	T	T
T	T	F	T	F	T	F
T	F	T	T	T	T	T
T	F	F	T	F	T	F
F	T	T	T	F	T	T
F	T	F	T	F	T	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

Are they logically equivalent? NO

4. Use De Morgan's law to write the negation of: x < 2 or x > 5.

$x \not< 2$ and $x \not> 5$

$x \geq 2$ and $x \leq 5$ or $2 \leq x \leq 5$

5. Write the negation of: "If today is New Year's Eve then tomorrow is January."

Today is New Year's Eve and tomorrow is not January

6. For the statement "If the decimal expansion of *r* is terminating then r is rational", give the

a. Inverse

If the decimal expansion of r is not terminating, then r is not rational.

b. Contrapositive

If r is not rational, then the decimal expansion of r is not terminating.

7. Construct a truth table for: $(p \to r) \leftrightarrow (q \to r)$.

p	q	r	$p \to r$	$q \to r$	$(p \to r) \leftrightarrow (q \to r)$
T	T	T	T	T	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T

8. Write the truth table for the following argument. You should have a column for each premise and for the conclusion. Circle the critical rows.

$p \to q \vee r$
$\sim q \vee \sim r$
$\therefore \sim p \vee \sim r$

p	q	r	$\sim p$	$\sim q$	$\sim r$	$q \vee r$	$p \to q \vee r$	$\sim q \vee \sim r$	$\sim p \vee \sim r$
T	T	T	F	F	F	T	T	F	
T	T	F	F	F	T	T	T	T	T
T	F	T	F	T	F	T	T	T	F
T	F	F	F	T	T	F	F	T	
F	T	T	T	F	F	T	T	F	
F	T	F	T	F	T	T	T	T	T
F	F	T	T	T	F	T	T	T	T
F	F	F	T	T	T	F	T	T	T

Is the argument valid? NO

9. Let P(x) be the predicate "x > 1/x". What is the truth set of P(x) if the domain of x is $\mathbb{R}$.

$x > 1$ or $-1 < x < 0$

10. Rewrite the following:

a. "All rectangles are quadrilaterals"

∀ rectangles x, x is a quadrilateral.

b. "Some sets have 16 subsets"

∃ a set x such that x has 16 subsets.

11. Consider the statement "∀ real numbers x, if $x^2 \geq 1$ then $x > 0$."

a. Write the negation of the statement.

∃ a real number x s.t. $x^2 \geq 1$ and [redacted] $x \leq 0$.

b. Write the converse of the statement.

∀ real numbers x, if $x > 0$ then $x^2 \geq 1$.