

MAD 2104: Proof Guidelines

Rules:

- Copy the statement of the theorem to be proved
- Mark the beginning of your proof with “proof” (or “pf”).
- Your proof should be self-contained.
Explain the meaning of each variable used within the body of the proof.
- Write in complete, grammatically correct sentences.
Symbols and abbreviations can be used, but should be incorporated into sentences.
- Keep the reader informed about the status of each statement
The reader should always know whether something has been assumed or is still to be deduced. If something is assumed, preface it with a word like “suppose”. If it is still to be shown use a phrase like “we must show”.
- Justify each assertion
Use phrases like “by definition of...” and “by theorem...” at each step.
- Include little words to make logic clear
Example: “then”, “thus”, “hence”, “therefore”, “it follows that”
- Display equations and inequalities
Generally display equations and inequalities on separate lines to increase readability. Don’t skip steps and leave plenty of space- don’t be stingy with paper!

Mistakes to avoid:

- Arguing from examples
A general statement cannot be proved by showing it to be true for some special cases.
- Using the same letter to mean two different things
Consider this example:
Suppose m and n are any odd integers.
Then $m=2k+1$ and $n=2k+1$ for some integer k .
This implies that $m=n$! It follows that the rest of the proof only applies for m and n that are equal to each other.
- Jumping to a conclusion
Don’t allege the truth of something without giving an adequate reason.
Example:
Suppose m and n are even integers. Then $m=2r$ and $n=2s$ for some integers r and s . Then $m+n=2r+2s$, so $m+n$ is even.
This is missing the step that $2r+2s=2(r+s)$.
- Circular reasoning
This is to assume what is to be proved. Example:
Suppose m and n are any odd integers. When any odd integers are multiplied, their product is odd. Hence, mn is odd.

- Confusion between what is known and what is to be shown
 Example: if you want to show that mn is odd and write
 There exists s such that $mn=2s+1$.
 Instead, say “we must show...” first.
- Use of “any” rather than “some”
 Sometimes these words can be used interchangeably. For example,
 these two sentences mean the same thing:
 Suppose m is any odd integer.
 Suppose m is some odd integer.
 Here is an example where they are not interchangeable:
 Suppose m is a particular but arbitrarily chosen odd integer. Then
 $m=2s+1$ for any integer s .
 In this case, s cannot be just “any” integer- there is only one possible value
 for s .
- Misuse of the word “if”
 “Because” is really the word that’s meant:
 Suppose p is prime. If p is prime, p cannot be written as a product of
 two smaller numbers.