MAD 2104: Practice Test 2

1. Prove: There are real numbers a and b such that $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$.

2. Disprove by providing a counterexample: for all real numbers a and b, if $a < b$ then $a^2 < b^2$.

3. Prove: If a is any odd integer and b is any even integer, then $2a + 3b$ is even. *Use only the definitions of the terms and normal assumptions (basic algebraic properties, integers closed under addition, multiplication, and subraction). Do not assume any previously established properties of even and odd integers.*

4. Show the following is false: for all integers a and b, if $3|(a+b)$ then $3|(a-b)$.

5. Evaluate the following:
   (a) $28 div 5$

   (b) $43 mod 9$

6. Prove: For any odd integer n,
$$\left\lceil \frac{n^2}{4} \right\rceil = \frac{n^2+3}{4}.$$

7. Prove by contradiction: If a and b are rational numbers, $b \neq 0$, and r is an irrational number, then $a + br$ is irrational.

8. Consider the statement: For all integers m and n, if $m+n$ is even then m and n are both even or are both odd.

   (a) Write the contrapositive of the statement

   (b) Prove the original statement by contraposition.

9. Write the following using summation or product notation:

$$\frac{2}{3\cdot 4}-\frac{3}{4\cdot 5}+\frac{4}{5\cdot 6}-\frac{5}{6\cdot 7}+\frac{6}{7\cdot 8}$$

10. Prove by mathematical induction: $1+3n \leq 4^n$ for every integer $n \geq 0$.