# Word Embeddings

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# Overview

## Recap on Word Vectors

# Vector Semantics

"The meaning of a word is its use in the language." Wittgenstein, 1953.

1. Vector semantics combines two intuitions:
   - **Distributional approach**: define a word by the contexts it occurs into.
   - **Vectorization**: use vectors to represent word meaning.
2. **Feature engineering** for NLP: word vectors are used as features for other tasks.
3. (Word) vectors are usually referred as (word) **embeddings** in modern neural network literature.

# Co-occurrences

| | | |
|---|---|---|
| …ound and sonic power of a [new electric | guitar | played through] a guitar amp has play… |
| …[Some electric | guitar | models feature] piezoelectric pickups… |
| …[Playing | guitar | with a] pick produces a bright sound … |
| …ings, he is known for [playing fretless | guitar | in his] performances… |
| …the neck of [a classical | guitar | is too] wide and the normal position … |
| …t in the centre of Bristol [playing the | piano | , I was] punched in the head while, a… |
| …r in Houston, Texanstagram [playing the | piano | in his] flooded home after Hurrican H… |
| … some supplies, he stopped to [play the | piano | that was] sitting in knee-high water … |
| …te and one black, who [played classical | piano | together]… |
| …The [first electric | pianos | from the] late 1920s used metal strin… |
| …technologies, for example [the electric | car | and the] integration of mobile commun… |
| …study had each driver of [each electric | car | drive unimpeded], perform a task whil… |
| …Honda to commence testing of [their new | car | and the] American was no doubt more t… |
| …mary design considerations for [the new | car | were "safety] innovations, performanc… |
| …would be possible if almost [all private | cars | requiring drivers], which are not in … |
| … who donate to groups [providing private | school | scholarships have] written pieces att… |
| … that students participating [in private | school | choice programs] graduate high school… |
| …s in the establishment of this [new high | school | , named the] Gavirate Business School… |
| …Anna heads into her [final high | school | year before] university wanting somet… |
| … but he can prevent them from [playing at | school] | |

# Word-Context matrix

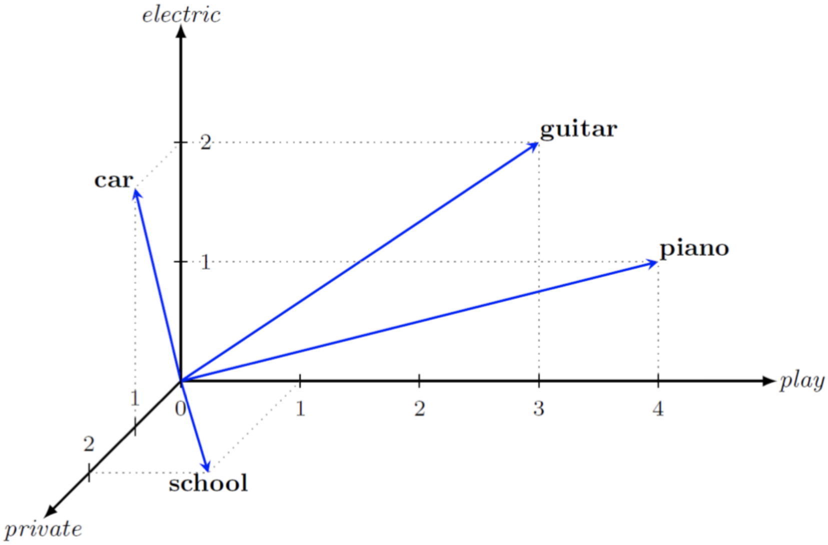
- We have a set of words $V$ and a set of contexts they occur into $C$, taken from our corpus of documents. $X$ in this case is a $|V| \times |C|$ matrix with word occurrences in contexts.
- The most intuitive context are co-occurrences with other words in $V$, within a certain **window**. In this case, $X$ would be a $|V| \times |V|$ matrix.

| | aardvark | ... | computer | data | pinch | result | sugar | ... |
|---|---|---|---|---|---|---|---|---|
| **apricot** | 0 | ... | 0 | 0 | 1 | 0 | 1 | |
| **pineapple** | 0 | ... | 0 | 0 | 1 | 0 | 1 | |
| **digital** | 0 | ... | 2 | 1 | 0 | 1 | 0 | |
| **information** | 0 | ... | 1 | 6 | 0 | 4 | 0 | |

**Figure 6.5** Co-occurrence vectors for four words, computed from the Brown corpus, showing only six of the dimensions (hand-picked for pedagogical purposes). The vector for the word *digital* is outlined in red. Note that a real vector would have vastly more dimensions and thus be much sparser.

*Credit: J&M, ch. 6.*

# Vectors



# Families of vectors

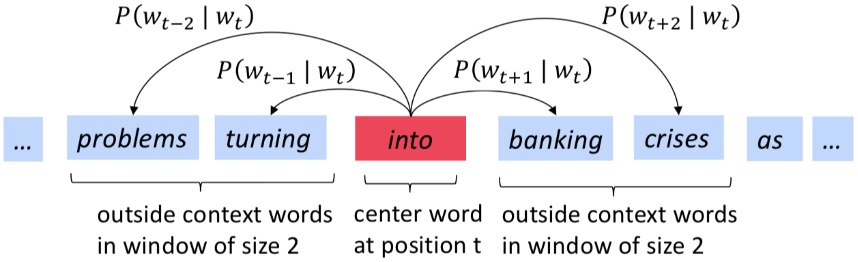
- **Sparse vectors**: many zero values and high-dimensional spaces. E.g., weighted co-occurrence matrices.
- **Dense vectors**: no zero values and comparatively smaller-dimensional spaces.
  - Dimensionality reduction (Singular Value Decomposition, Random indexing, Non-negative matrix factorization).
  - **Neural-network inspired** (Word2Vec, GloVe, BERT and many more): we start today.

**Word2Vec**

# Intuition

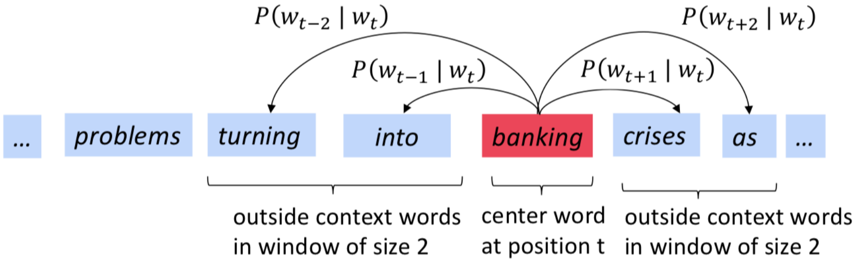
- **Word2Vec**: a framework for learning dense word vectors.
- Idea:
  1. We have a large corpus of text.
  2. We want each word in the vocabulary to be represented by a vector.
  3. We can go through the corpus and establish a *context o* for every *center/focus word c*, using a certain window/span.
  4. **We use the similarity of the word vectors c and o to calculate the probability of context words o given c.**
  5. **We keep adjusting word vectors until our predictions are good.**

# Words in context



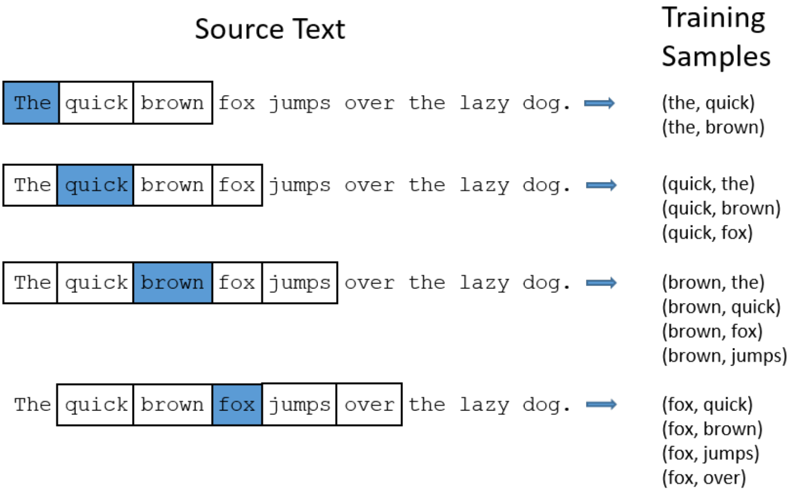

*Credit: Stanford CS224N.*

# Words in context




*Credit: Stanford CS224N.*

# Words in context as data




*Credit:* *http://mccormickml.com/2016/04/19/word2vec-tutorial-the-skip-gram-model.*

# The model

- Our task, for every $c$ (center), $o$ (context) pair, is to estimate high probabilities for:

$$p(w_o|w_c)$$

- *The model parameters are the word embeddings $w$.*
- For each word position $t = 1 \dots T$, we predict context words within a windows of size $m$, given the center word $w_t$ (at each position):

$$L(\boldsymbol{w}) = \prod_{t=1}^{T} \prod_{-m \leq j \leq m; j \neq 0} p(\boldsymbol{w}_{t+j}|\boldsymbol{w}_t)$$

- $L(\boldsymbol{w})$ is the likelihood.

# The model

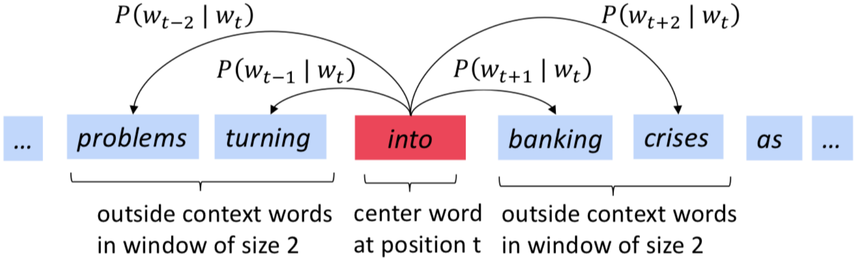
- Loss function is the negative log likelihood:

$$\mathcal{L}(\boldsymbol{w}) = -\frac{1}{T} logL(\boldsymbol{w}) = -\frac{1}{T}\sum_{t=1}^{T}\sum_{-m\leq j\leq m; j\neq 0} logp(\boldsymbol{w}_{t+j}|\boldsymbol{w}_t)$$

- *Minimizing the loss is equivalent to maximizing the likelihood.*
- How to calculate $p(\boldsymbol{w}_{t+j}|\boldsymbol{w}_t)$? Use two vectors for each word:
  - $v_w$ when $w$ is a center word
  - $u_w$ when $w$ is a context word
- Use the **softmax** (generalization of the sigmoid) to predict the probabilities of a $c$ (center), $o$ (context) pair:

$$p(o|c) = \frac{exp(u_o^T v_c)}{\sum_{w\in V} exp(u_w^T v_c)}$$

# Example




We learn to predict:

- $p(u_{problems} | v_{into})$
- $p(u_{turning} | v_{into})$
- $p(u_{banking} | v_{into})$
- $p(u_{crises} | v_{into})$
- ...

*Credit: Stanford CS224N.*

# Softmax

Exponentiation makes anything positive

$$P(o|c) = \frac{\exp(\boxed{u_o^T v_c})}{\boxed{\sum_{w \in V} \exp(u_w^T v_c)}}$$

Dot product compares similarity of *o* and *c*.
$u^T v = u.v = \sum_{i=1}^{n} u_i v_i$
Larger dot product = larger probability

Normalize over entire vocabulary
to give probability distribution

- The softmax maps any value to a probability distribution.
- It amplifies large values (*max*) but still gives non-zero probabilities to small values (*soft*).

*Credit: Stanford CS224N.*

# Training via SGD

- Parameters: our word embeddings, **two per word**.
- Usually, these vectors have length $d$ within 50-1000, thus $d \ll |V|$.
- Use gradient descent to optimize and find a minimum of the loss.

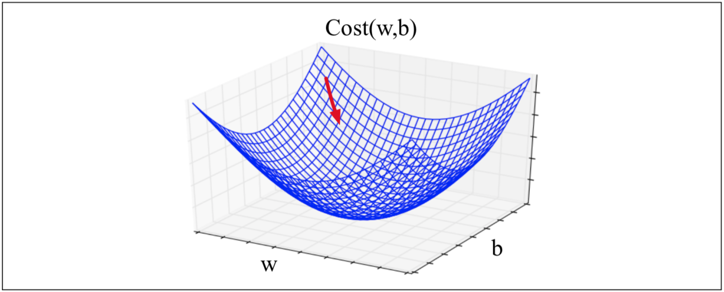



**Figure 5.4** Visualization of the gradient vector in two dimensions $w$ and $b$.

Credit: J&M, ch. 5.

# Training via SGD

- Let us ignore for a moment the normalization term $\frac{1}{T}$ and the external summations, which are straightforward.
- Let us take the first (partial) derivative w.r.t. $v_c$ (similarly, you can do this for $u_o$):

$$\begin{aligned}
\frac{\partial}{\partial v_c} log \frac{exp(u_o^T v_c)}{\sum_{w \in V} exp(u_w^T v_c)} &= u_o - \sum_{x \in V} \frac{exp(u_x^T v_c)}{\sum_{w \in V} exp(u_w^T v_c)} \cdot u_x \\
&= u_o - \sum_{x \in V} p(x|c) \cdot u_x
\end{aligned}$$

- Thus the derivative w.r.t. the central word vector $v_c$ is the vector for the current context word $u_o$, minus the weighted average of the model's current representations of other possible contexts.

# Final words on Word2Vec

- After having trained the model, we typically use the vectors $v_w$ or the average of $v_w$ and $u_w$. We use two vectors as a kind of trick to make the derivation (and thus training) simpler.
- What we discussed is called the **Skip-gram** model.
- Alternatively, we can predict the center word using the context words: this is called the Continuous Bag Of Words model (**CBOW**).

# GloVe

# So far

- So far we have seen count-based approaches to word vectors:
  1. Make use of corpus statistics.
  2. Very fast training (just count..).
  3. *Sensitive to large counts.*
  4. *Mostly only capture word similarity.*
- And approaches based on prediction tasks (Word2Vec):
  1. Can capture more complex patterns.
  2. Generate better performance as features for other tasks.
  3. *Do not make use of corpus statistics.*

# Intuition

- **GloVe key idea: capture ratios of co-occurrence probabilities as linear meaning components in a vector space.**

| | $x$ = solid | $x$ = gas | $x$ = water | $x$ = random |
|---|---|---|---|---|
| $P(x\|\text{ice})$ | large | small | large | small |
| $P(x\|\text{steam})$ | small | large | large | small |
| $\frac{P(x\|\text{ice})}{P(x\|\text{steam})}$ | large | small | ~1 | ~1 |

*Credit: Stanford CS224N.*

# Intuition

- **GloVe key idea: capture ratios of co-occurrence probabilities as linear meaning components in a vector space.**
- Learn a log-linear model as follows:

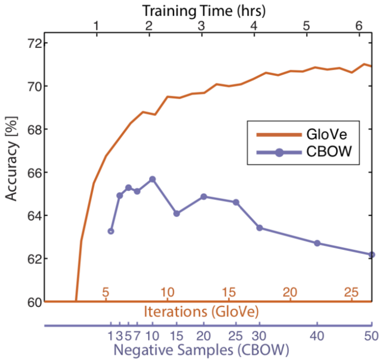
$$w_i \cdot w_j = log\big(P(i|j)\big)$$

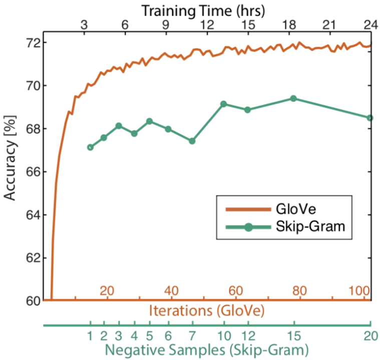
- Able to capture vector differences for ratios:

$$w_x \cdot (w_a - w_b) = log\left(\frac{P(x|a)}{P(x|b)}\right)$$

- Check the (excellent) paper for more details.

# Comparison with Word2Vec




*Credit: Stanford CS224N.*

# References

- Stanford CS224N classes 1 and 2: http://web.stanford.edu/class/cs224n/index.html.
- Good tutorial http://mccormickml.com/2016/04/19/word2vec-tutorial-the-skip-gram-model.
- Original Word2Vec paper https://arxiv.org/pdf/1301.3781.pdf.
- Negative sampling paper http://papers.nips.cc/paper/5021-distributed-representations-of-words-and-phrases-and pdf.
- GloVe https://nlp.stanford.edu/pubs/glove.pdf.
- Evaluation of word embeddings: https://www.aclweb.org/anthology/D15-1036.

*Note: there is much more. Ask me if you are interested.*

# Word2Vec Expanded (optional)

# Derivation for softmax

- First, we need some notable derivatives:

$$\frac{\partial log(x)}{\partial x} = \frac{1}{x}$$
$$\frac{\partial exp(x)}{\partial x} = exp(x)$$
$$\frac{\partial f(g(x))}{\partial x} = \frac{\partial f}{\partial g} \cdot \frac{\partial g}{\partial x} \rightarrow \text{chain rule}$$

# Derivation for softmax

- We can divide in two parts:

$$\frac{\partial}{\partial v_c} log \frac{exp(u_o^T v_c)}{\sum_{w \in V} exp(u_w^T v_c)} = \frac{\partial}{\partial v_c} logexp(u_o^T v_c) - \frac{\partial}{\partial v_c} log \sum_{w \in V} exp(u_w^T v_c)$$

- First part:

$$\frac{\partial}{\partial v_c} logexp(u_o^T v_c) = u_o$$

- Second part:

$$\frac{\partial}{\partial v_c} log \sum_{w \in V} exp(u_w^T v_c) = \frac{1}{\sum_{w \in V} exp(u_w^T v_c)} \cdot \frac{\partial}{\partial v_c} \sum_{x \in V} exp(u_x^T v_c)$$

$$= \frac{\sum_{x \in V} exp(u_x^T v_c) u_x}{\sum_{w \in V} exp(u_w^T v_c)}$$

# Derivation for softmax

- Combine:

$$\begin{aligned}\frac{\partial}{\partial v_c} log \frac{exp(u_o^T v_c)}{\sum_{w \in V} exp(u_w^T v_c)} &= u_o - \frac{\sum_{x \in V} exp(u_x^T v_c) u_x}{\sum_{w \in V} exp(u_w^T v_c)} \\ &= u_o - \sum_{x \in V} p(x|c) u_x\end{aligned}$$

# Noise-contrastive estimation

Exponentiation makes anything positive

$$P(o|c) = \frac{\exp(\boxed{u_o^T v_c})}{\boxed{\sum_{w \in V} \exp(u_w^T v_c)}}$$

Dot product compares similarity of *o* and *c*.
$u^T v = u.v = \sum_{i=1}^{n} u_i v_i$
Larger dot product = larger probability

Normalize over entire vocabulary
to give probability distribution

- Normalizing over the entire vocabulary is very expensive.
- Idea: let us just **sample some negative examples** (collocations absent in the data), and train a **binary logistic regression classifier** to distinguish between positive (real) and negative (fake) pairs.
- For every center-context pair, also sample $K$ negative pairs. The center-context pair is going to be a positive datapoint, the negative pairs are negative datapoints.
- The logistic classifier then uses the same dot product of vectors as features, and a cross-entropy loss.

*Credit: Stanford CS224N.*