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Importance of Statistical Measures in Digital Image Processing

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***Abstract*— This paper presents the comprehensive study of the various statistical measures and their application in digital image processing at root level. We have simulated the majority of statistical measures and reviewed their existing applications. Also we have explored and proposed their importance in some more research area of digital image processing. We have done their comparative analysis with the help of MATLAB simulation to ease the selection of statistical parameter for a specific image processing technique like image enhancement, denoising, restoration, edge detection etc.**

***Keywords*— Statistical measures, noise, filtering.**

I. Introduction

Statistics is the study of the collection, organization, analysis, and interpretation of data. It deals with all aspects of this, including the planning of data collection in terms of the design of surveys and experiments.

The various statistical measures [1, 7] are mean, mode, median, variance, standard deviations, covariance, skewness and kurtosis. All of these measures are used in a wide range of scientific and social research, including: biostatistics, computational biology, computational sociology, network biology, social science, sociology and social research etc.

In this paper we have studied various type of statistical measure in respect to image processing and simulated all of these. Main purpose is to highlight the application of these measures in the various fields of digital image processing like, image enhancement, image restoration, image denoising, and edge detection etc. at the basic level and ease the selection of statistical parameter for a specific image processing technique.

The organization of the paper is as follows. Following the brief introduction proposed statistical model is discussed in section II. From section III to section XIII details of various statistical parameters is discussed. Section XIV discusses about analysis and simulation results followed by concluding remarks in section XV.

II. Proposed Statistical Model

Although research has already been done on few of these measures at quite advance level, we have proposed a simple statistical model in Fig.1 for image processing to optimize it features.

The proposed statistical model consists of the following steps:

1. *Statistical analysis of input image, $I_i(x, y)$*: In this step statistical analysis of input image is done using various measures like mean, mode, median, variance, standard deviation, covariance, skewness, kurtosis etc.
2. *Selection of statistical measure:* Depending upon the requirements in output optimized image, the statistical parameter is chosen.
3. *Image Filtering:* Image filtering is done using the filter based on the statistical parameter selected in previous step.

Depending upon requirements, for the image filtering we can choose from a very basic filter to any multiparameter complex filtering. In the next few sections we have discussed the statistical analysis of an image using various statistical measures.

III. Mean

Mean [1, 2] is most basic of all statistical measure. Means are often used in geometry and analysis; a wide range of means have been developed for these purposes. In contest of image processing filtering using mean is classified as spatial filtering and used for noise reduction. In this section we have discussed about various type of mean and analysed their use for removing various type of noise in image processing.

A. Arithmetic Mean

The arithmetic mean filter [2], also known as averaging filter, operates on an sliding ‘m×n’ window by calculating the average of all pixel values within the window and replacing the center pixel value in the destination image with the result. Its mathematical formulation is given as follows

$$f(x,y) = \frac{1}{mn} \sum_{(r,c)\in W} g(r,c) \tag{1}$$

Where ‘g’ is the noisy image, f(x,y) is the restored image, and ‘r’ and ‘c’ are the row and column coordinates respectively, within a window ‘W’ of size ‘m×n’ where the operation takes place.

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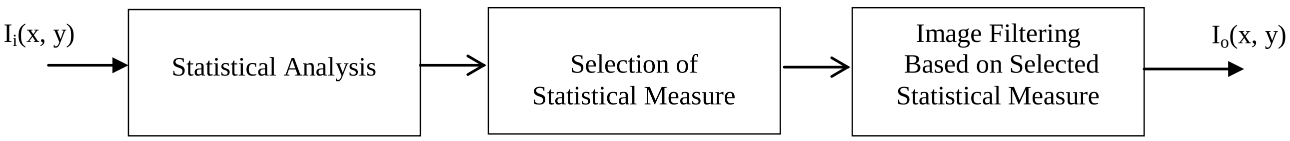

Figure 1 Proposed Statistical Model for Image Optimization

The arithmetic mean filter causes a certain amount of blurring (proportional to the window size) to the image, thereby reducing the effects of noise and local variations. It can be used to reduce noise of different types, but works best for Gaussian, uniform, or Erlang noise. Fig. 2 (C) shows the image after arithmetic mean filtering of Gaussian noise added image.

B. Geometric Mean

The geometric mean [2] filter is a variation of the arithmetic mean filter and is primarily used on images with Gaussian noise. This filter is known to retain image detail better than the arithmetic mean filter. Its mathematical formulation is as follows:

$$f(x,y) = \left[\prod_{(r,c)\in W} g(r,c) \right]^{1/mn} \tag{2}$$

In this case each restored pixel is given by the product of the pixel in the sub image window raised to the power '1/mn'. Fig. 2 (D) shows the image after geometric mean filtering of Gaussian noise added image.

C. Harmonic Mean

The harmonic mean filter [2] is yet another variation of the arithmetic mean filter and is useful for images with Gaussian or salt noise. Black pixels (pepper noise) are not filtered. The filter's mathematical formulation is as follows:

$$f(x,y) = \frac{mn}{\sum_{(r,c)\in W} \left(1 / g(r,c) \right)} \tag{3}$$

Fig. 2 (E) shows the image after harmonic mean filtering of Gaussian noise added image.

D. Contraharmonic Mean

The contra-harmonic mean filter [2] is another variation of the arithmetic mean filter and is primarily used for filtering salt or pepper noise (but not both). Images with salt noise can be filtered using negative values of R, whereas those with pepper noise can be filtered using positive values of R.

The filter's mathematical formulation is

$$f(x,y) = \frac{\sum_{(r,c)\in W} \left(g(r,c) \right)^{R+1}}{\sum_{(r,c)\in W} \left(g(r,c) \right)^{R}} \tag{4}$$

Where R is the order of filter. Fig. 2 (H) shows image after Contraharmonic mean filtering of image with Salt & Pepper noise for R=1 Fig. 2 (I) R=-1.

IV. MEDIAN

Median [2] is measure of intensity level of pixel which is separating the high intensity value pixels from lower intensity value pixels. It is a type of order-statistic filter. The most popular and useful of the rank filters is the median filter. It works by selecting the middle pixel value from the ordered set of values within the 'm×n' neighbourhood 'W' around the reference pixel. If 'mn' is an even number (which is not common), the arithmetic average of the two values closest to the middle of the ordered set is used instead. Mathematically,

$$\tilde{f}(x,y) = median\{ g(r,c) \mid (r,c) \in W \} \tag{5}$$

There have been many variants, extensions, and optimized implementations of the median filter proposed in the literature. This filter simply sorts all values within a window, finds the median value, and replaces the original pixel value with the median value. It is commonly used for salt and pepper noise. . Fig. 2 (J) shows image after median filtering of image with Gaussian noise. Fig. 2 (K) shows image after median filtering of image with Salt & Pepper noise.

V. MAX AND MIN FILTERS

These are another type of rank-statistics filter [2]. The min and max filters also work on a ranked set of pixel values. Contrary to the median filter, which replaces the reference pixel with the median of the ordered set, the min filter, also known as the zeroth percentile filter, replaces it with the lowest value instead.

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A quick way to get rid of salt noise in an image is to use the min filter, which simply takes the minimum value of a window when the values are ordered. Mathematically

$$\tilde{f}(x,y)=min\{g(r,c)\,|\,(r,c)\in W\} \tag{6}$$

Similarly, the max filter, also known as the 100th percentile filter, replaces the reference pixel within the window with the highest value that is

$$\tilde{f}(x,y)=max\{g(r,c)\,|\,(r,c)\in W\} \tag{7}$$

The max filter is used for filtering pepper noise, similar to the technique of the min filter. Fig. 2 (L) shows image after min filtering of image with Salt & Pepper noise. Fig. 2 (M) shows image after max filtering of image with Salt & Pepper noise

VI. MID-POINT FILTER

The midpoint filter [2] calculates the average of the highest and lowest pixel values within a window, thereby combining order statistics and averaging into one filter. It is used to reduce Gaussian and uniform noise in images. Mathematically,

$$\tilde{f}(x,y)=\frac{1}{2}\left[max\{g(r,c)\,|\,(r,c)\in W\}+min\{g(r,c)\,|\,(r,c)\in W\}\right] \tag{8}$$

Fig. 2 (N) shows image after mid-point filtering of image with Salt & Pepper noise.

VII. THE ALPHA-TRIMMED MEAN FILTER

The alpha-trimmed filter [2] uses another combination of order statistics and averaging, in this case an average of the pixel values closest to the median, after the 'D' lowest and 'D' highest values in an ordered set have been excluded. The rationale behind this filter is to allow its user to control its behaviour by specifying the parameter: 'D' for 'D'=0, the filter behaves as a regular arithmetic mean filter; for D= (mn-1)/2 it is equivalent to the median filter. It is used in cases where the image is corrupted by more than one type of noise, for example, salt and pepper (where the median filter performs well) and Gaussian (where the arithmetic mean filter shows satisfactory performance). The mathematical description of the alpha-trimmed filter is as follows:

$$\tilde{f}(x,y)=\frac{1}{(mn-2D)}\sum_{(r,c)\in W} g(r,c) \tag{9}$$

Where 'D' is the number of pixel values excluded at each end of the ordered set, which can range from 0 to (mn-1)/2. Fig. 2 (P) shows image after Alpha trimmed filtering of image with Gaussian, Salt & Pepper noise.

VIII. MODE

In statistics [1], the mode is the value that occurs most frequently. In image processing filter will each pixel value by its most common neighbour. This is a particularly useful filter for classification procedures where each pixel corresponds to an object which must be placed into a class; in remote sensing, for example, each class could be some type of terrain, crop type, water, etc. Fig. 3 (A) shows image after mid-point filtering of image with Salt & Pepper noise. Mathematically,

$$Mean-Mode=3(Mean-Median) \tag{10}$$

IX. STANDARD DEVIATION

It is a most widely used measure of variability or diversity used in statistics. In terms of image processing it shows how much variation or "dispersion" exists from the average (mean, or expected value). A low standard deviation indicates that the data points tend to be very close to the mean, whereas high standard deviation indicates that the data points are spread out over a large range of values. Mathematically standard deviation is given by

$$\tilde{f}(x,y)=\sqrt{\frac{1}{mn-1}\sum_{(r,c)\in W}\left(g(r,c)-\frac{1}{mn-1}\sum_{(r,c)\in W} g(r,c)\right)^2} \tag{11}$$

A standard deviation filter calculates the standard deviation and assigns this value to the center pixel in the output map. As it has capability in measuring the variability, it can be used in edge sharpening, as intensity level get changes at the edge of image by large value.

Standard deviation filters [10] can be useful for radar images. The interpretation of radar images is often difficult: you cannot rely on spectral values because of back scatter (return of the pulse sent by the radar). This often causes a lot of 'noise'. By using a standard deviation filter, you may be able to recognize some patterns.

Figure 2 (A)Original Image (B) Image with Gaussian noise of mean 0 & variance 0.005 (C) Image after arithmetic mean filtering of (B) with 3*3 kernel size (D) Image after Geometric mean filtering of (B) with 3*3 kernel size (E) Image with Salt & Pepper noise of density 0.01 (F) Image after Harmonic mean filtering of (B) with 3*3 kernel size (G) Image after Harmonic mean filtering of (E) with 3*3 kernel size (H) Image after Contraharmonic mean filtering of (E) with 3*3 kernel size with R=1 with 3*3 kernel size (I) Image after Contraharmonic mean filtering of (E) with 3*3 kernel size with R=-1(J) Image after median filtering of (B) with 3*3 kernel size (K) Image after Median filtering of (E) with 3*3 kernel size (L) Image after Min Filtering of (E) with 3*3 kernel size (M) Image after Max Filtering of (E) with 3*3 kernel size (N) Image after Midpoint Filtering of (B) with 3*3 kernel size (O) Image with Gaussian noise of mean 0 & variance 0.005, Salt & Pepper noise of density 0.01 (P) Image after Alpha trimmed filtering of (O) with 3*3 kernel size

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Fig. 3 (B) Image after standard deviation filtering of Fig. 2 (A) with 3*3 kernel size.

X. COVARIANCE

In statistics, covariance [3, 4, 6] is a measure of how much two random variables change together. If the greater values of one variable mainly correspond with the greater values of the other variable, and the same holds for the smaller values, i.e. the variables tend to show similar behaviour, the covariance is a positive number. In the opposite case, when the greater values of one variable mainly correspond to the smaller values of the other, i.e. the variables tend to show opposite behaviour, the covariance is negative. The sign of the covariance therefore shows the tendency in the linear relationship between the variables. Mathematically covariance between two variable x and y is given as

$$Cov(x,y)=\frac{1}{N}\sum(x_i-\bar{x})(y_i-\bar{y}) \qquad (12)$$

Covariance based filtering techniques can be used in edge sharpening, appearance based face detection and image orientation information etc. Fig. 3 (C) Image after covariance filtering of Fig. 2 (A) with 3*3 kernel size.

XI. VARIANCE

The variance [5] is a measure of how far a set of numbers is spread out. It is one of several descriptors of a probability distribution, describing how far the numbers lie from the mean (expected value). In particular, the variance is one of the moments of a distribution. In that context, it forms part of a systematic approach to distinguishing between probability distributions. While other such approaches have been developed, those based on moments are advantageous in terms of mathematical and computational simplicity. Mathematically variance is given by

$$\tilde{f}(x,y)=\frac{1}{mn-1}\sum_{(r,c)\in W}\left(g(r,c)-\frac{1}{mn-1}\sum_{(r,c)\in W}g(r,c)\right)^2 \qquad (13)$$

Variance filter can be utilized to determine edge position in image processing. Fig. 3 (D) Image after variance filtering of Fig. 2 (A) with 3*3 kernel size.

XII. SKEWNESS

In statistics, skewness [8] is a measure of the asymmetry of the probability distribution of a real-valued random variable. The skewness value can be positive or negative, or even undefined.

Qualitatively, a negative skew indicates that the tail on the left side of the probability density function is longer than the right side and the bulk of the values (possibly including the median) lie to the right of the mean. A positive skew indicates that the tail on the right side is longer than the left side and the bulk of the values lie to the left of the mean. A zero value indicates that the values are relatively evenly distributed on both sides of the mean, typically but not necessarily implying a symmetric distribution. Mathematically skewness can be given by

$$\tilde{f}(x,y)=\frac{\frac{1}{mn-1}\sum_{(r,c)\in W}\left(\frac{1}{mn-1}\sum_{(r,c)\in W}\left(g(r,c)-\frac{1}{mn-1}\sum_{(r,c)\in W}g(r,c)\right)\right)^3}{\left(\frac{1}{mn-1}\sum_{(r,c)\in W}\left(\frac{1}{mn-1}\sum_{(r,c)\in W}\left(g(r,c)-\frac{1}{mn-1}\sum_{(r,c)\in W}g(r,c)\right)\right)^2\right)^{3/2}} \qquad (14)$$

In terms of digital image processing, Darker and glossier surfaces tend to be more positively skewed than lighter and matte surfaces. Hence we can use skewness in making judgements about image surfaces. Fig. 3 (E) Image after skewness filtering of Fig. 2 (A) with 3*3 kernel size.

XIII. KURTOSIS

In statistics, kurtosis [9] is a measure of the shape of the probability distribution of a real-valued random variable. It is closely related to the fourth moment of a distribution. A high kurtosis distribution has longer, fatter tails, and often (but not always) a sharper peak. A low kurtosis distribution has shorter, thinner tails, and often (but not always) a more rounded peak. Mathematically kurtosis is given as follows

$$\tilde{f}(x,y)=\frac{\frac{1}{mn-1}\sum_{(r,c)\in W}\left(\frac{1}{mn-1}\sum_{(r,c)\in W}\left(g(r,c)-\frac{1}{mn-1}\sum_{(r,c)\in W}g(r,c)\right)\right)^4}{\left(\frac{1}{mn-1}\sum_{(r,c)\in W}\left(\frac{1}{mn-1}\sum_{(r,c)\in W}\left(g(r,c)-\frac{1}{mn-1}\sum_{(r,c)\in W}g(r,c)\right)\right)^2\right)^{2}} \qquad (15)$$

In digital image processing kurtosis values are interpreted in combination with noise and resolution measurement. High kurtosis values should go hand in hand with low noise and low resolution. Fig. 3 (F) Image after kurtosis filtering of Fig. 2 (A) with 3*3 kernel size.

XIV. SIMULATION RESULTS

We have done MATLAB simulation for the filtering techniques based on different statistical measures. Most of them are already used by many researchers for different application in digital image processing.

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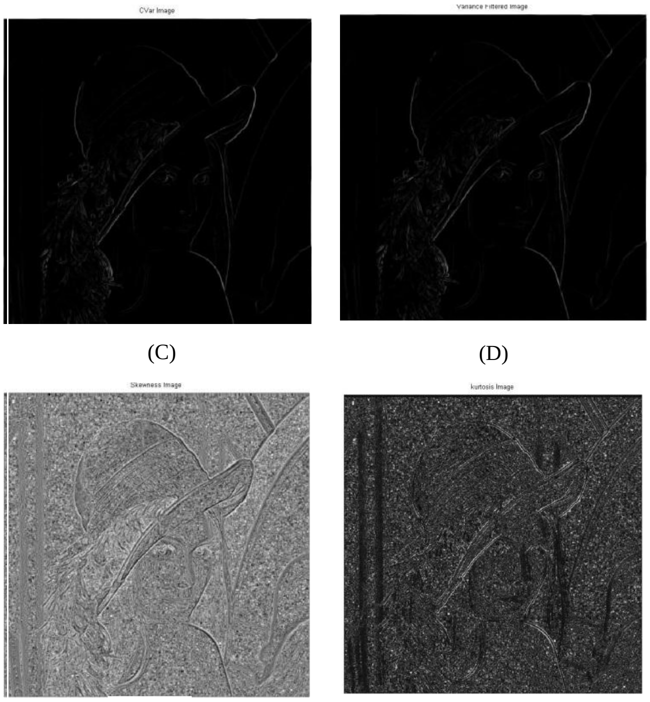
Our simulation results show that each statistical measure has significant importance depending upon requirements. So selection of statistical measures is very important and should be done wisely. By using simulated results as reference it is very easy to select the statistical parameter before going for a complex image processing technique. Like from simulation results we concluded that arithmetic mean, geometric mean, harmonic mean all three can be used for removing Gaussian noise but geometric mean preserve the image details better. Contra-harmonic mean filter can be used to remove salt or pepper noise for different values of R. Median is commonly used for salt and pepper noise. Max and min filter is used for filtering pepper and salt noise respectively. Mid-point can be used to reduce Gaussian and uniform noise in images. Alpha-trimmed filter can be used to remove more than once type of noise. Mode is basically used for image classification. Standard deviation based filter can used in radar for pattern recognition. Covariance and variance based filter can be used for edge sharpening, appearance based face detection and edge positioning. Skewness can be used for image surface recognition and kurtosis for image resolution.

XV. CONCLUSION

In this paper we have discussed the details of various statistical measures in reference to digital image processing. We have presented a statistical model for selecting the statistical measure properly before going for a complex image processing technique. In the proposed model criteria of the output requirement is taken into account while selecting the statistical measure. MATLAB simulations are done based on various statistical measures filtering technique to prove the use of each at root level. Hence we can conclude that proposed statistical model is can be used as pre-processing model for various digital image processing technique to improve the effectiveness of complex image processing technique in the next levels.

(A) (B)



(C) (D)

(E) (F)

Fig. 3 (A) Image after Mode filtering of Fig. 2 (E) with 3*3 kernel size. (B) Image after standard deviation filtering of Fig. 2 (A) with 3*3 kernel size. (C) Image after Covariance filtering of Fig. 2 (A) with 3*3 kernel size. (D) Image after variance filtering of Fig. 2 (A) with 3*3 kernel size. (E). Image after skewness filtering of Fig. 2 (A) with 3*3 kernel size. (F). Image after kurtosis filtering of Fig. 2 (A) with 3*3 kernel size

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