Rendering on the GPU

Projects 1-8

1. Hello World
2. Transformations
3. Shading
4. Textures
5. Render Buffers
6. Environment Mapping
7. Shadow Mapping
8. Tessellation

Graphics API

OpenGL®

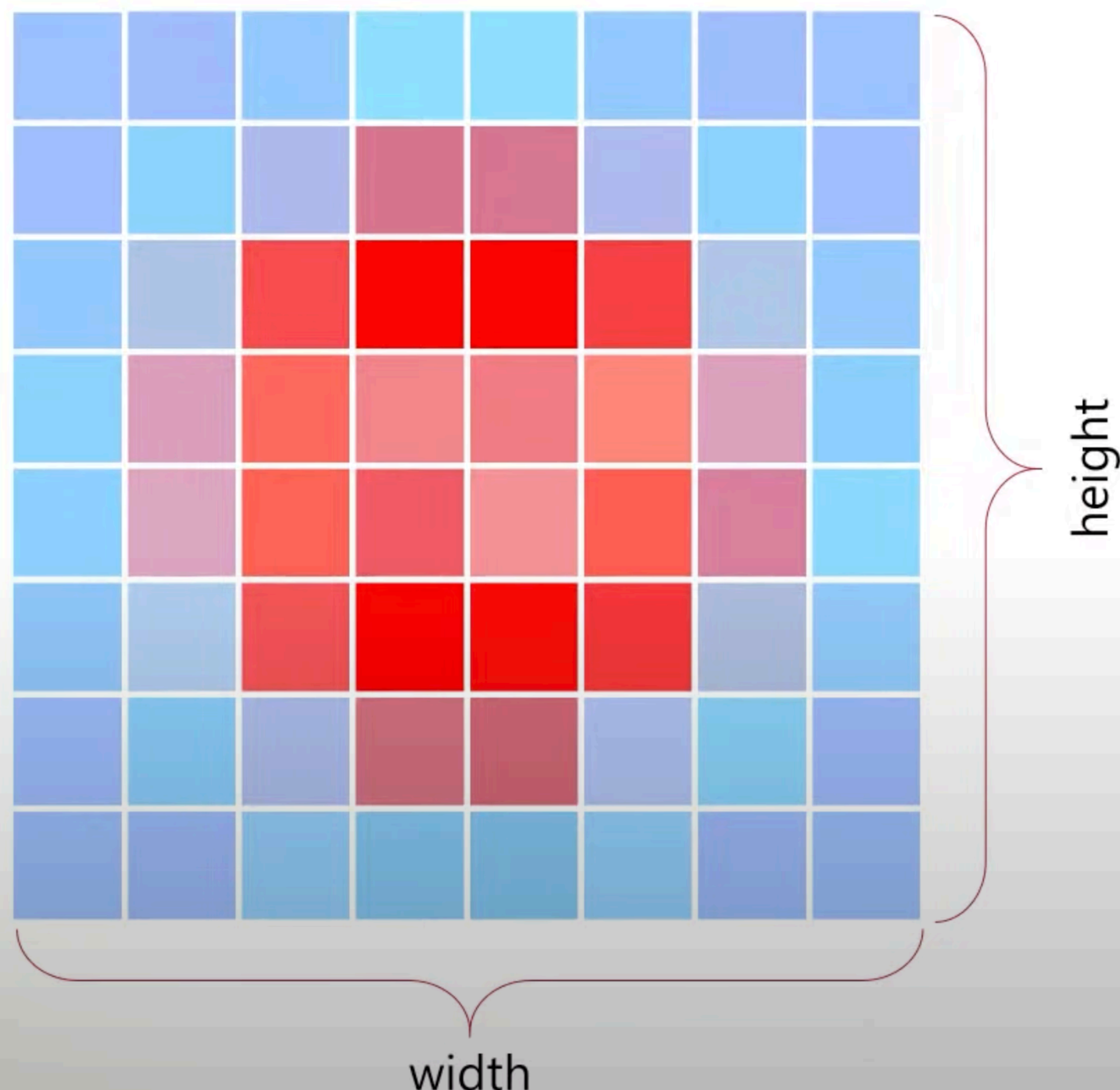
Metal

Microsoft DirectX

Vulkan™

Raster Images

Raster Images

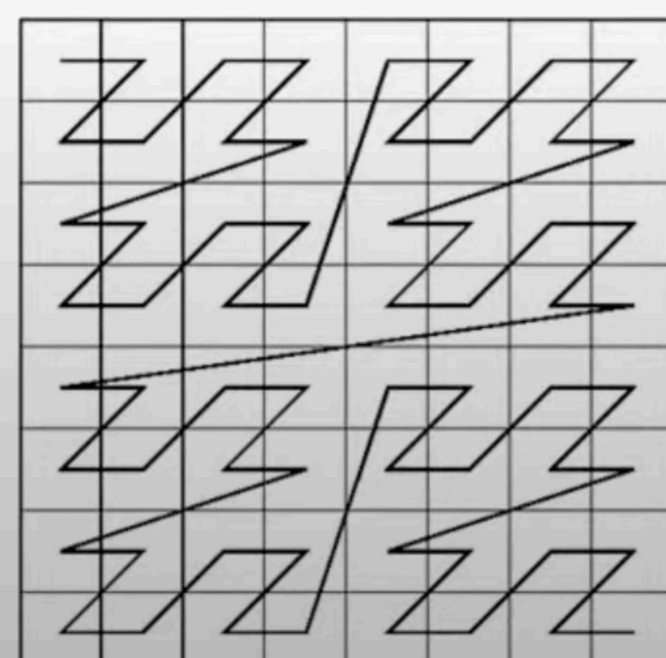


Raster Image In Memory

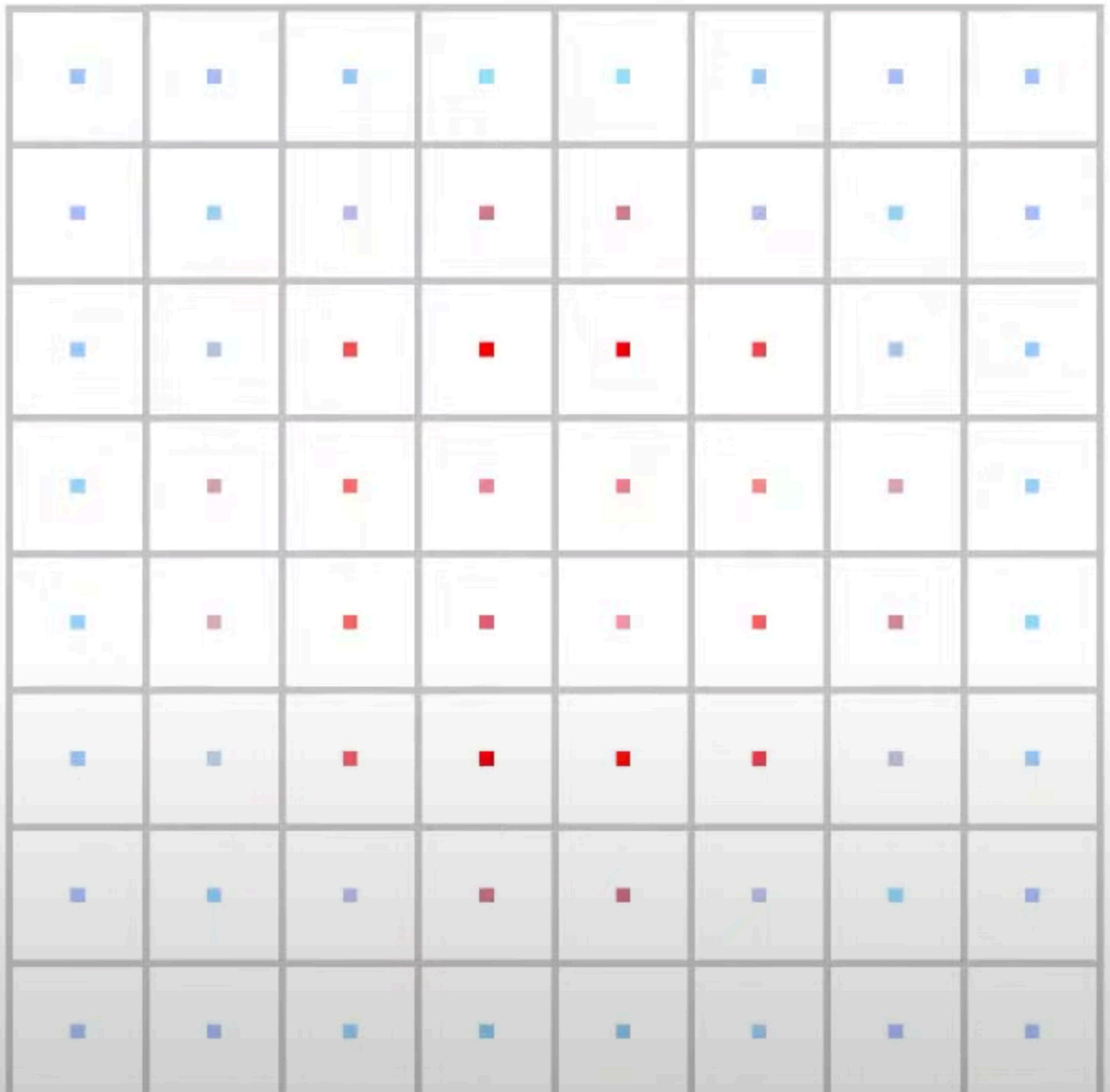
- Interleaved

  RGB RGB RGB RGB RGB RGB
  RGB RGB RGB RGB RGB RGB
  RGB RGB RGB RGB RGB RGB

- Scanline order
- Swizzled order



Raster Images

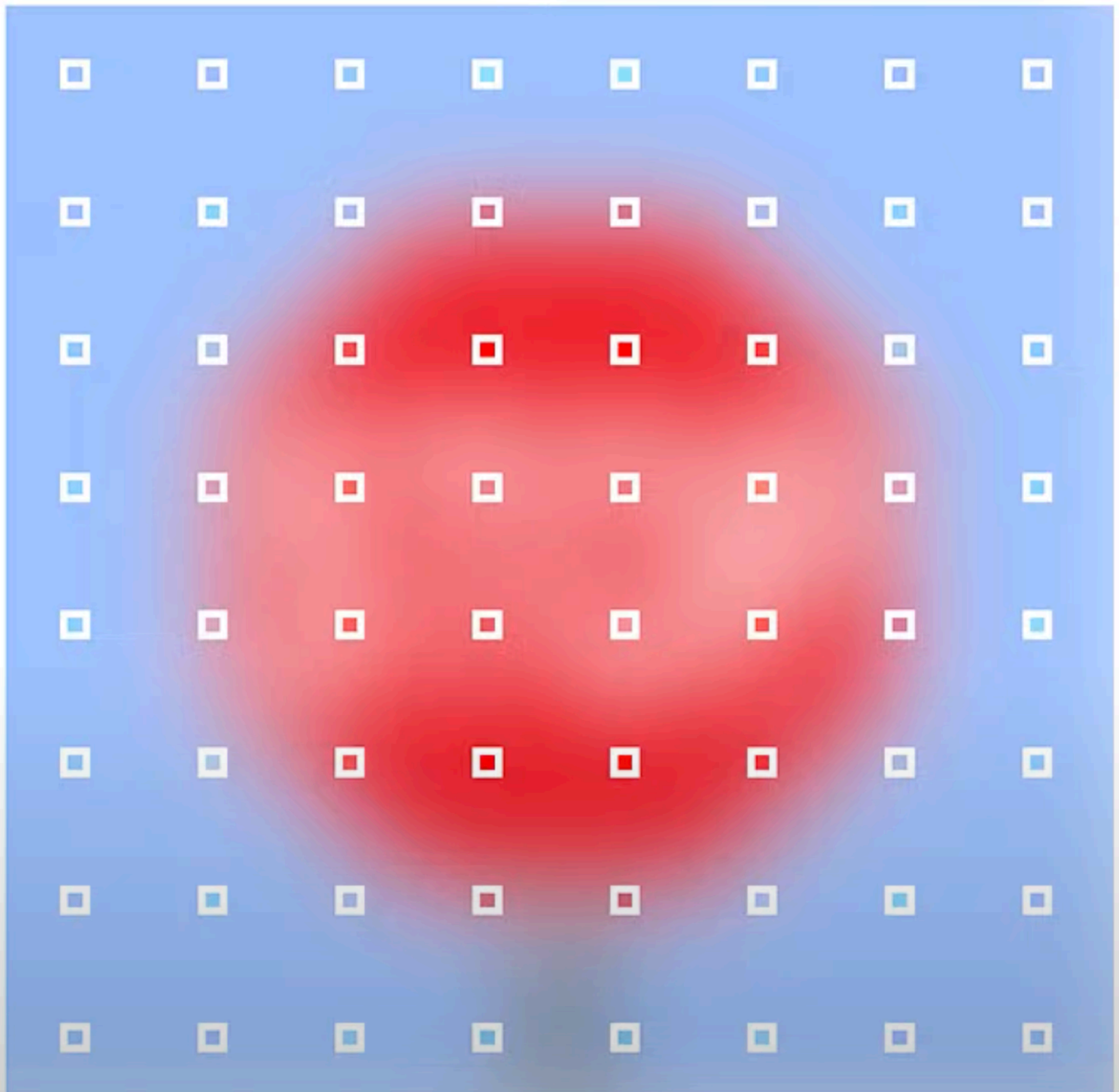


“A pixel is not a little square!”

Raster Images

"A pixel is not a little square!"

Raster Images



“A pixel is not a little square!”

Alpha α

Opacity

fully transparent

$\alpha = 0$

Alpha

For each pixel:

- Color & alpha
- **RGBA**

8-bits per channel ➔ 32 bits

Raster Image Storage

- Interleaved

  RGBA RGBA RGBA RGBA RGBA RGBA
  RGBA RGBA RGBA RGBA RGBA RGBA
  RGBA RGBA RGBA RGBA RGBA RGBA

Vectors & Matrices

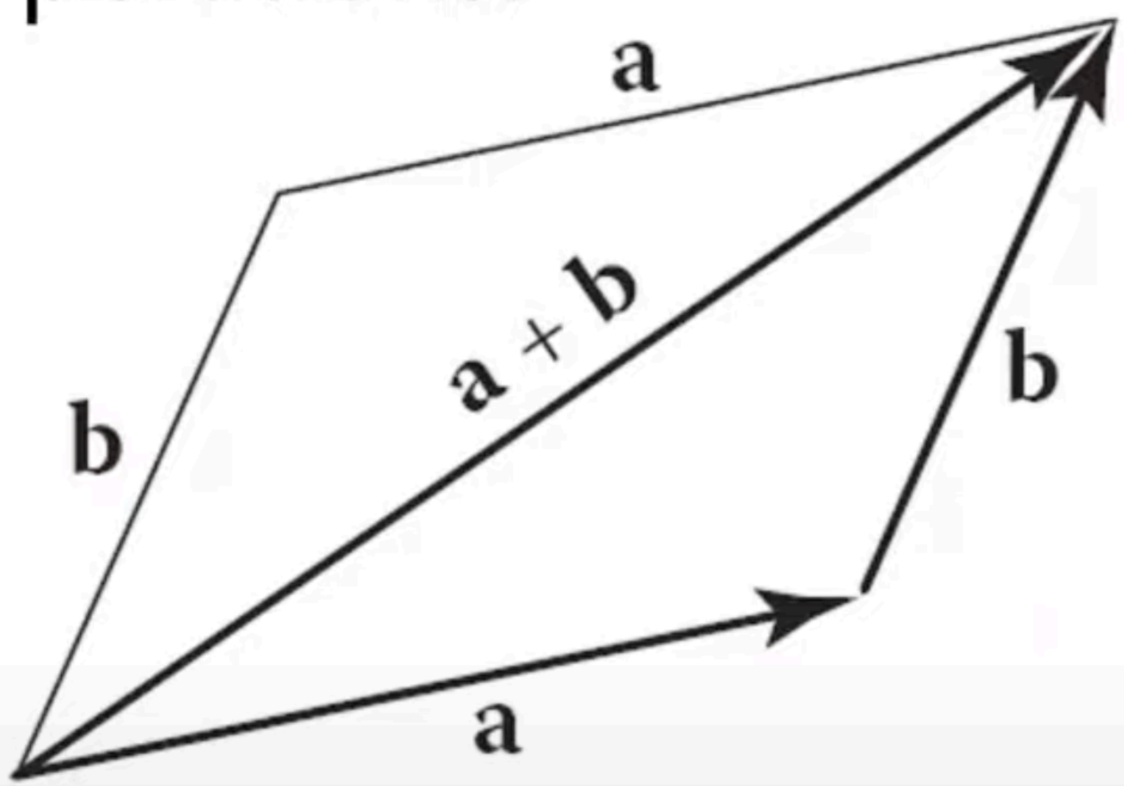
Vectors

Notation:

$$\mathbf{a} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$$

Vectors

Operations:

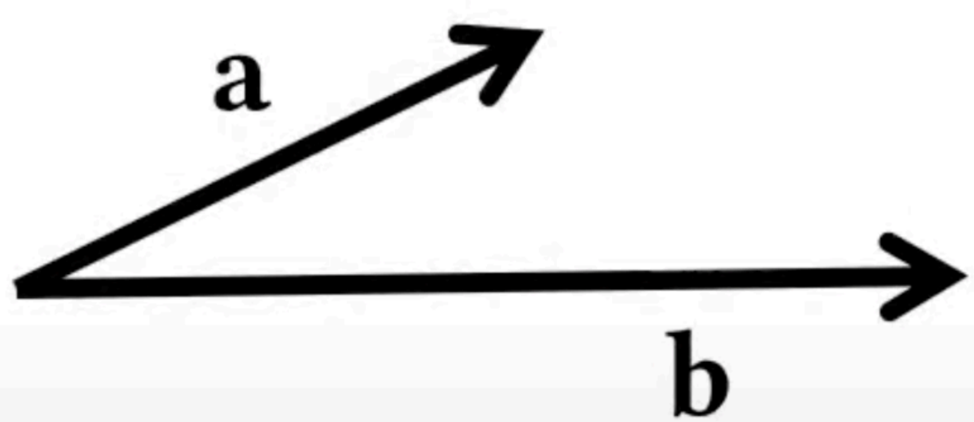


$$\mathbf{a} + \mathbf{b} = \begin{bmatrix} a_x + b_x \\ a_y + b_y \\ a_z + b_z \end{bmatrix}$$

Vectors

Operations:

dot product

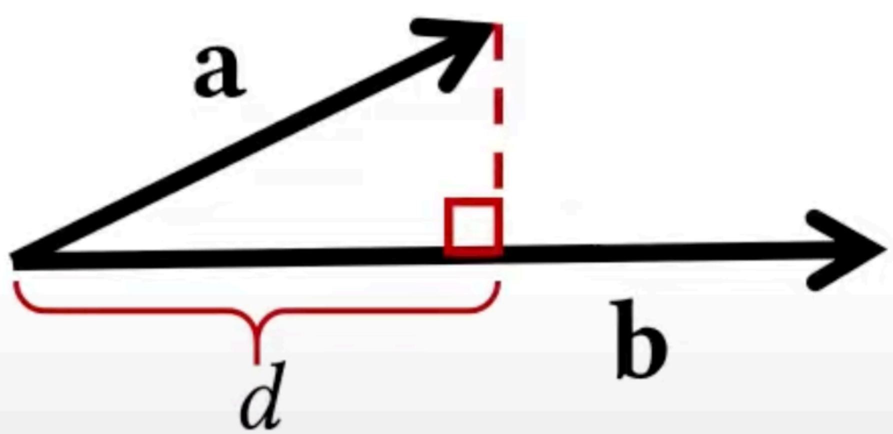


$$\mathbf{a} \bullet \mathbf{b} = a_x b_x + a_y b_y + a_z b_z$$

Vectors

Operations:

dot product



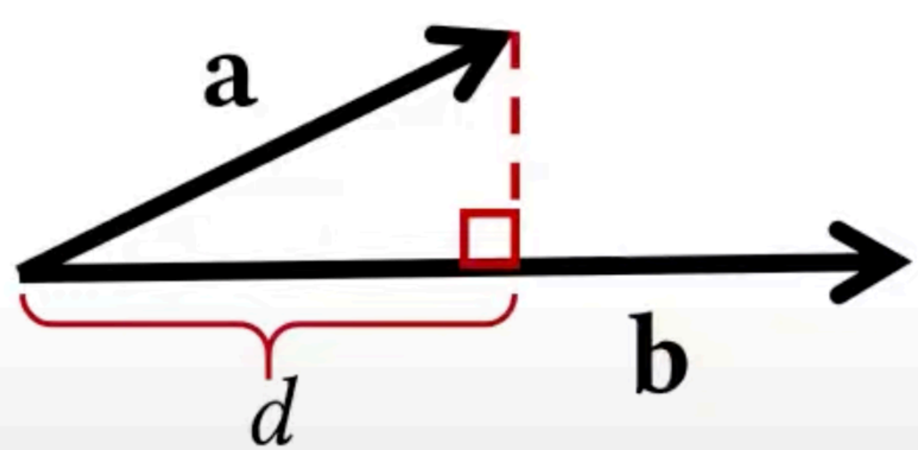
$$\mathbf{a} \bullet \mathbf{b} = a_x b_x + a_y b_y + a_z b_z$$

$$\text{if } |\mathbf{b}| = 1, \quad d = \mathbf{a} \bullet \mathbf{b}$$

Vectors

Operations:

dot product



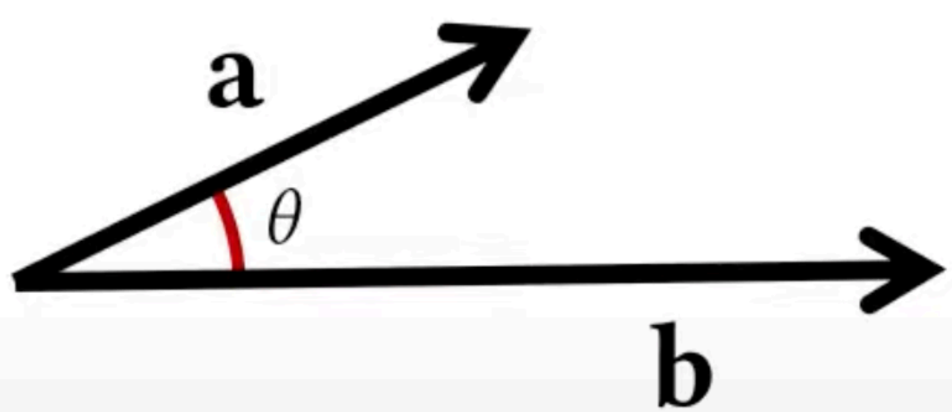
$$\mathbf{a} \bullet \mathbf{b} = a_x b_x + a_y b_y + a_z b_z$$

$$d = (\mathbf{a} \bullet \mathbf{b}) \;/\; |\mathbf{b}|$$

Vectors

Operations:

dot product

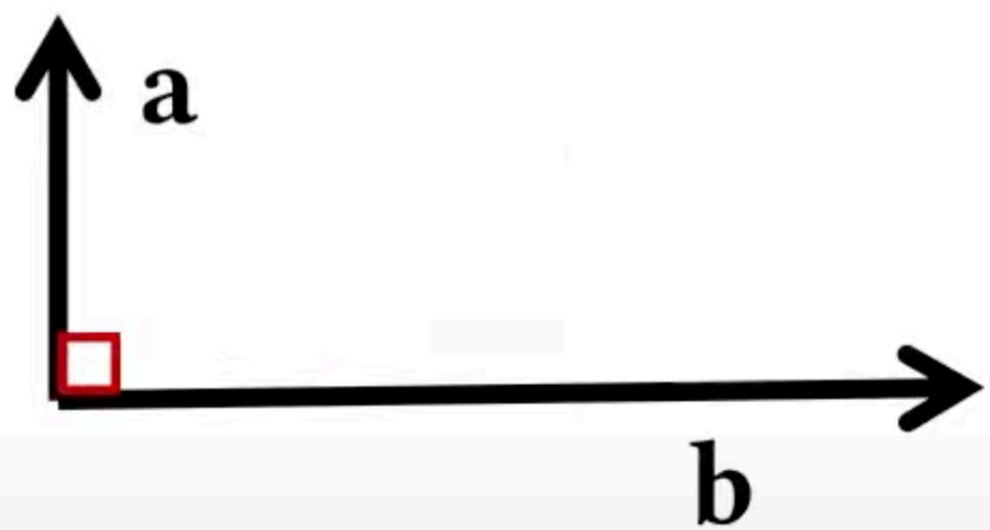


$$\mathbf{a} \bullet \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta$$

Vectors

Operations:

dot product



$$\mathbf{a} \bullet \mathbf{b} = 0$$

Vectors

Operations:

dot product

$$\mathbf{a} \bullet \mathbf{b} = \mathbf{b} \bullet \mathbf{a}$$

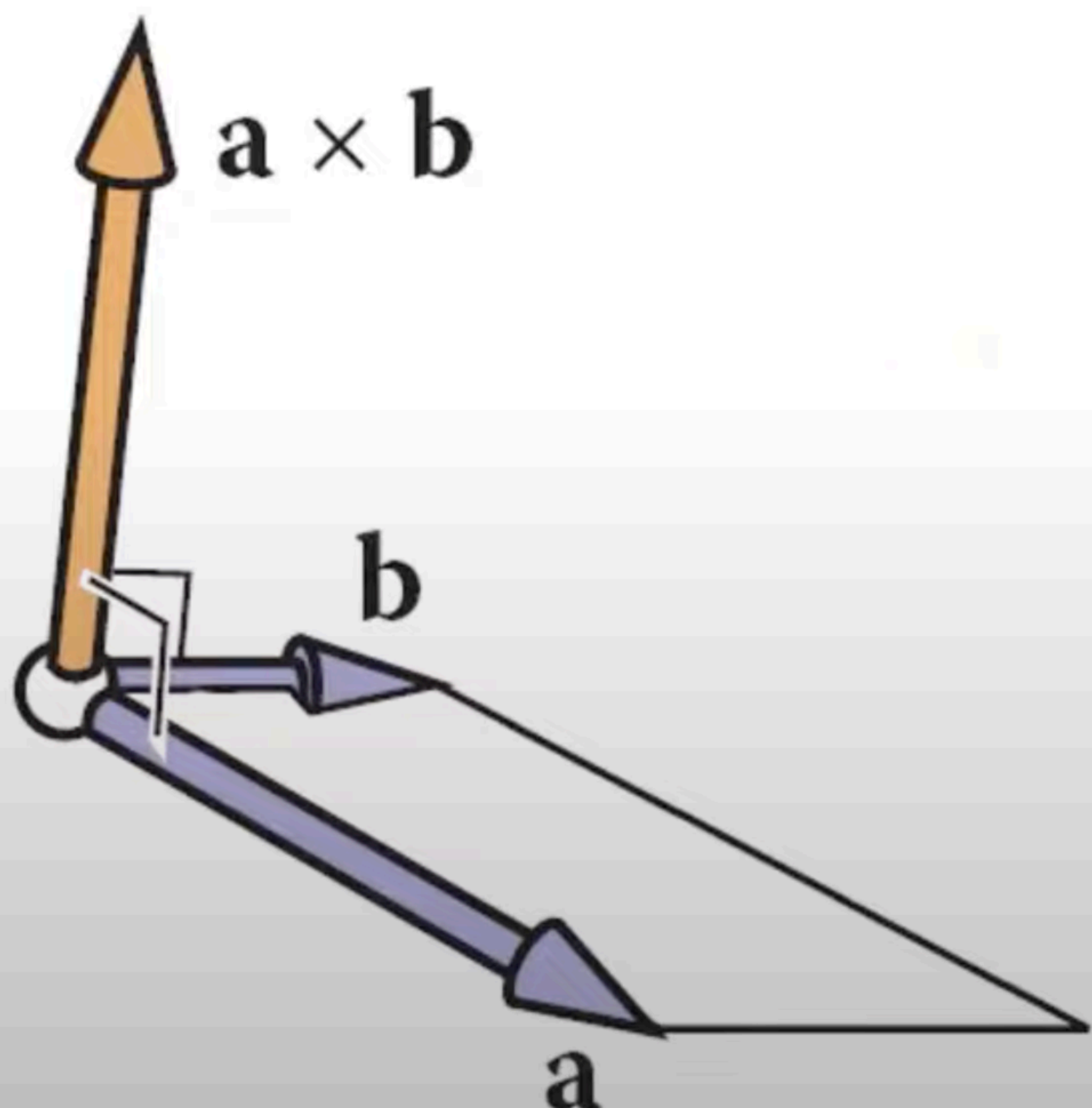
$$\mathbf{a} \bullet (\mathbf{b} \bullet \mathbf{c}) = (\mathbf{a} \bullet \mathbf{b}) \bullet \mathbf{c}$$

$$k\mathbf{a} \bullet \mathbf{b} = \mathbf{a} \bullet k\mathbf{b} = k\,(\mathbf{a} \bullet \mathbf{b})$$

Vectors

Operations:

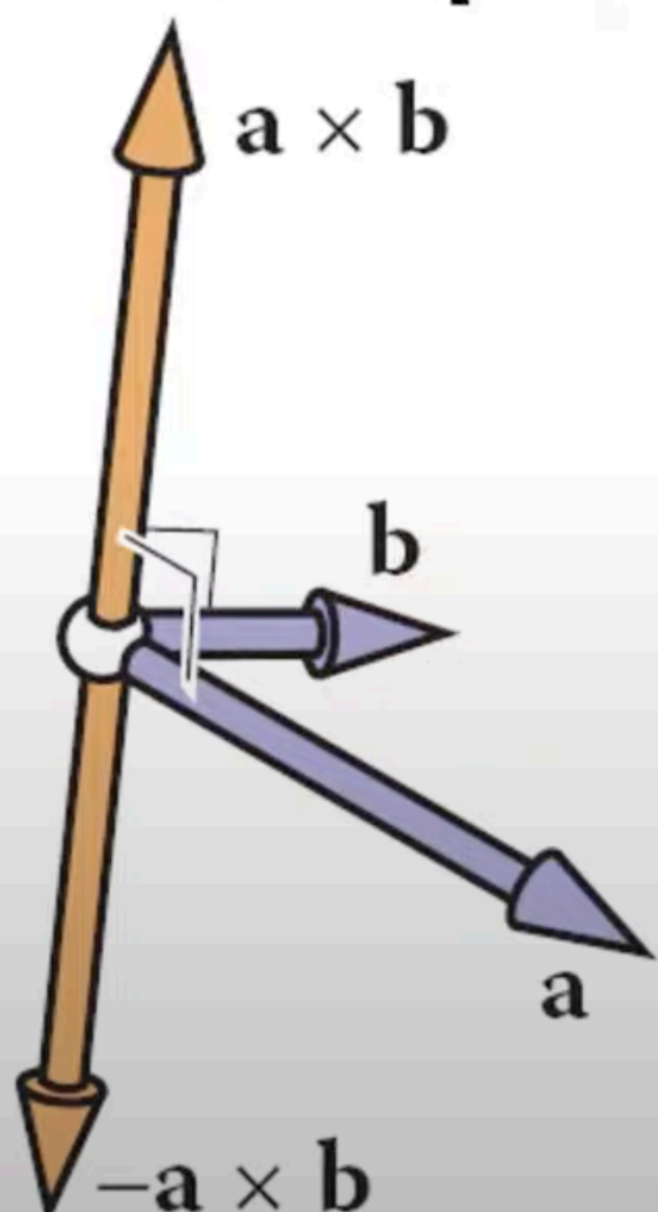
cross product



Vectors

Operations:

cross product



Vectors

Operations:

cross product

$\mathbf{a} \times \mathbf{b} = -(\mathbf{b} \times \mathbf{a})$

$(k\mathbf{a}) \times \mathbf{b} = k(\mathbf{a} \times \mathbf{b})$

$\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$

Matrices

Notation:

$$\mathbf{A} = \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \end{bmatrix}$$

$$\mathbf{A}\,\mathbf{B} \neq \mathbf{B}\,\mathbf{A}$$

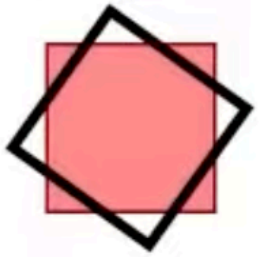
$$\mathbf{c} = \mathbf{A}\,\mathbf{b}$$

Transformations

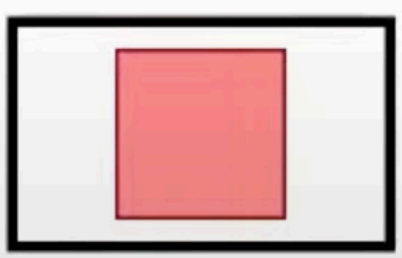
Affine Transformations

- Translation

- Rotation



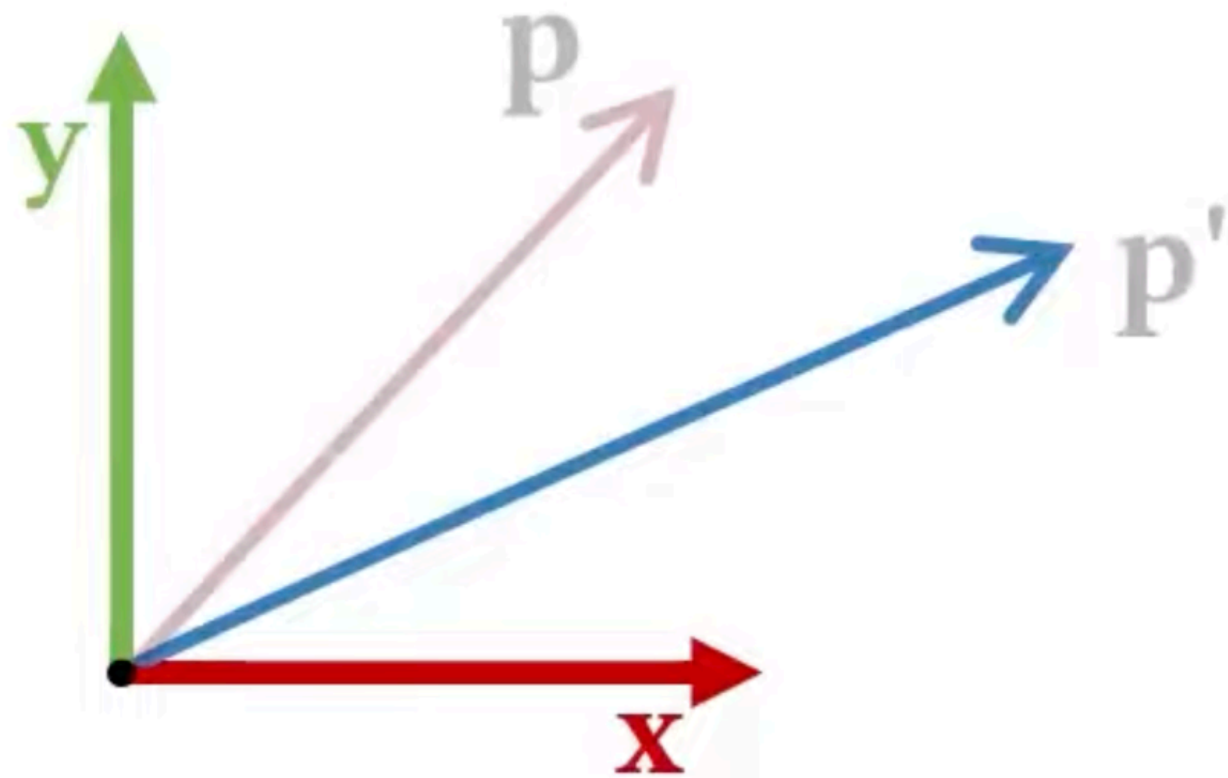
- Scale



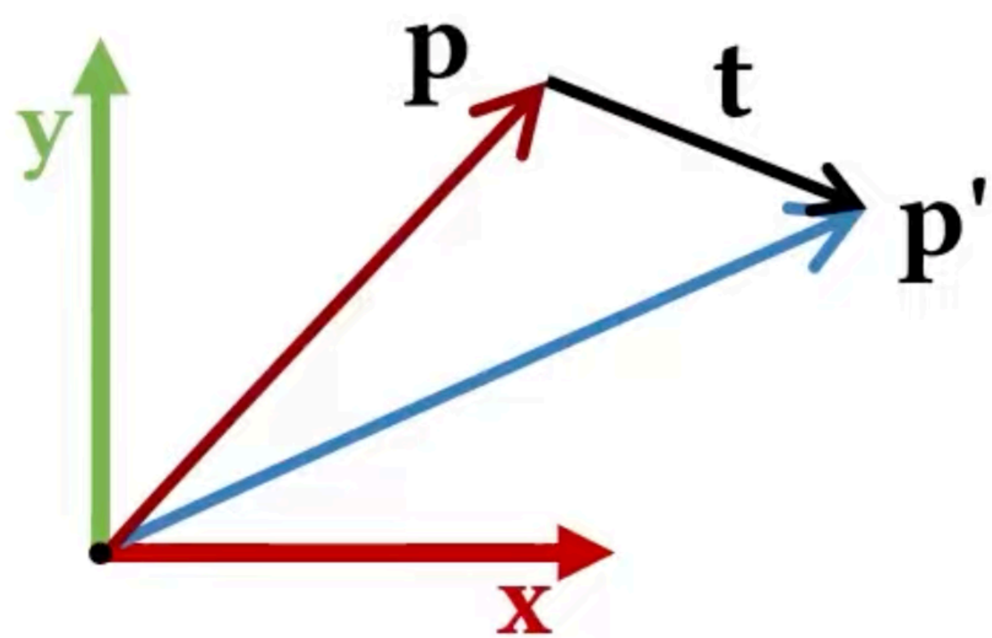
- Skew

2D Affine Transformations

Translation



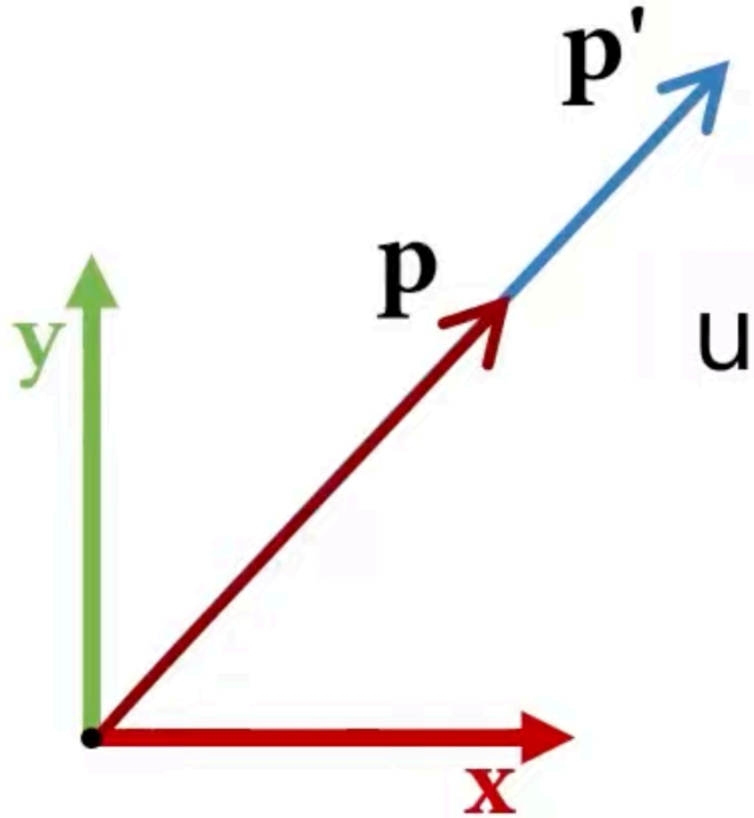
Translation



$$\mathbf{p'} = \mathbf{p} + \mathbf{t}$$

$$\begin{bmatrix} p'_x \\ p'_y \end{bmatrix} = \begin{bmatrix} p_x + t_x \\ p_y + t_y \end{bmatrix}$$

Scale

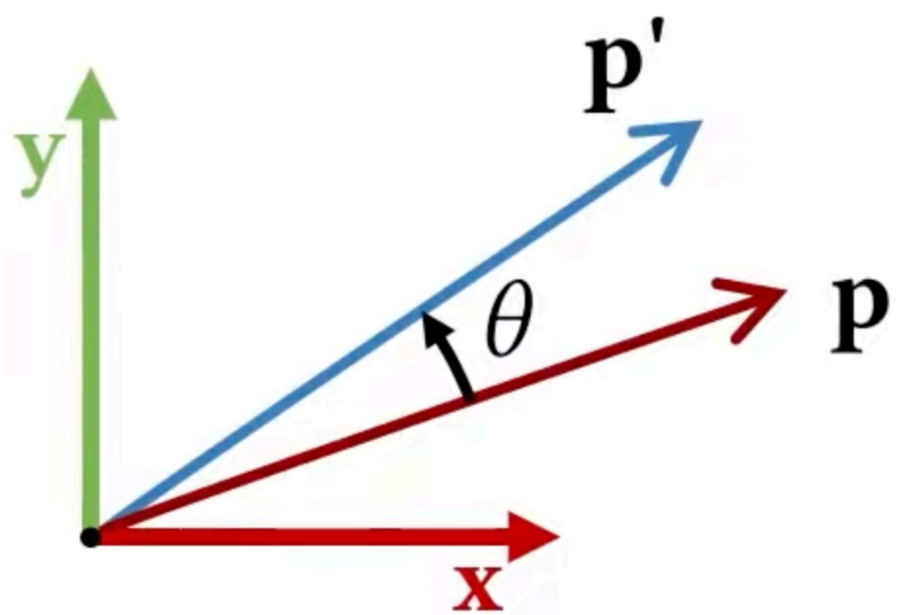


uniform scale

$$\mathbf{p}' = s\,\mathbf{p}$$

$$\begin{bmatrix} p'_x \\ p'_y \end{bmatrix} = \begin{bmatrix} s\,p_x \\ s\,p_y \end{bmatrix}$$

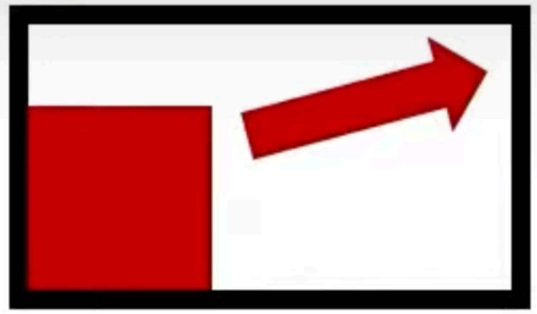
Rotation



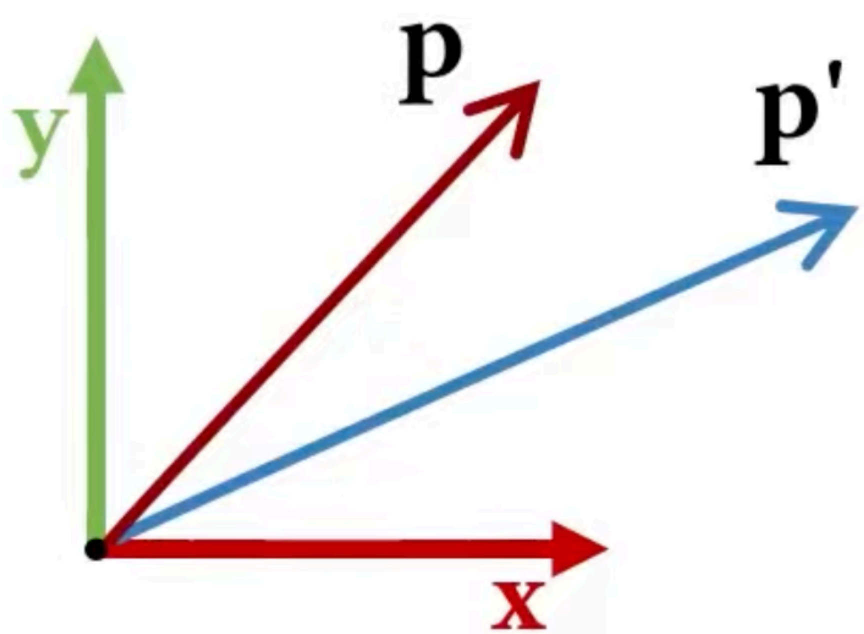
$$\begin{bmatrix} p'_x \\ p'_y \end{bmatrix} = p_x \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} + p_y \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}$$

$$\begin{bmatrix} p'_x \\ p'_y \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} p_x \\ p_y \end{bmatrix}$$

Scale



Non Uniform Scale



$$\begin{bmatrix} p'_x \\ p'_y \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} p_x \\ p_y \end{bmatrix} = \begin{bmatrix} s_x p_x \\ s_y p_y \end{bmatrix}$$

$$\mathbf{p}' = \mathbf{S}\,\mathbf{p}$$

Any 2x2 Matrix

- Singular value decomposition (SVD)

$$\mathbf{M} = \mathbf{U}\,\mathbf{S}\,\mathbf{V}^{\mathrm{T}}$$

orthogonal (rotation)

diagonal (scale)

orthogonal (rotation)

Any Series of Rotation

$$\mathbf{p}' = \underbrace{\mathbf{R\,S\,R\,S\,R\,S\,R\,S\,R}}\,\mathbf{p}$$

$$\mathbf{p}' = \mathbf{M}\,\mathbf{p}$$

Any Series of Rotation

$$\mathbf{p}' = \underbrace{\mathbf{R}\,\mathbf{S}\,\mathbf{R}\,\mathbf{S}\,\mathbf{R}\,\mathbf{S}\,\mathbf{R}\,\mathbf{S}\,\mathbf{R}}\,\mathbf{p}$$

$$\mathbf{p}' = \mathbf{M}\,\mathbf{p}$$

Translation?

$$\mathbf{p}' = \mathbf{M}\,\mathbf{p} + \mathbf{t}$$

$$\mathbf{p}' = \mathbf{M}_2\,(\,\mathbf{M}_1\,\mathbf{p} + \mathbf{t}\,)$$

Homogeneous Coordinates

$$\mathbf{p}' = \mathbf{p} + \mathbf{t}$$

$$\mathbf{p}' = \mathbf{T}\,\mathbf{p}$$

$$\begin{bmatrix} p'_x \\ p'_y \end{bmatrix} = \begin{bmatrix} p_x + t_x \\ p_y + t_y \end{bmatrix}$$

$$\begin{bmatrix} p'_x \\ p'_y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ 1 \end{bmatrix}$$

Homogeneous Coordinates

$$\mathbf{p}' = \underbrace{\mathbf{T\,S\,R\,T\,S\,R\,S\,R\,S\,R\,T}}\,\mathbf{p}$$

$$\mathbf{p}' = \mathbf{M\,p}$$

$$\begin{bmatrix} p'_x \\ p'_y \\ 1 \end{bmatrix} = \begin{bmatrix} a & c & e \\ b & d & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ 1 \end{bmatrix}$$

3D Affine Transformations

Affine Transformations

2D

$$\begin{bmatrix} a & c & e \\ b & d & f \\ 0 & 0 & 1 \end{bmatrix}$$

3x3

3D

$$\begin{bmatrix} a & d & g & j \\ b & e & h & k \\ c & f & i & l \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4x4

homogeneous coordinates

Scale

2D

$$\begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3x3

3D

$$\begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4x4

homogeneous coordinates

Translation

2D

$$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

3x3

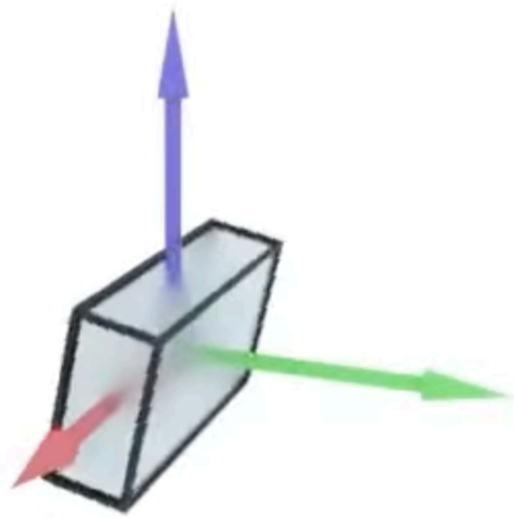
3D

$$\begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

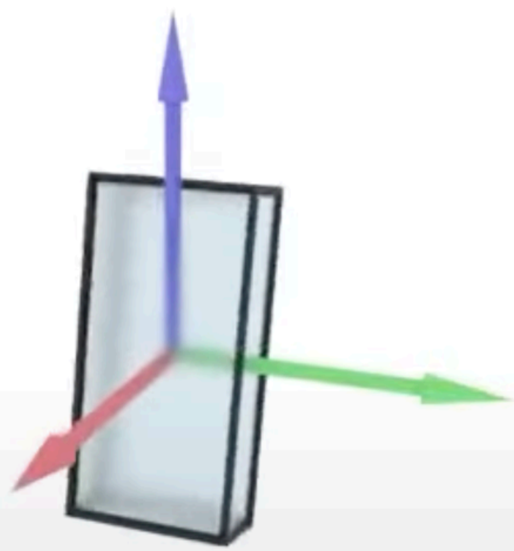
4x4

homogeneous coordinates

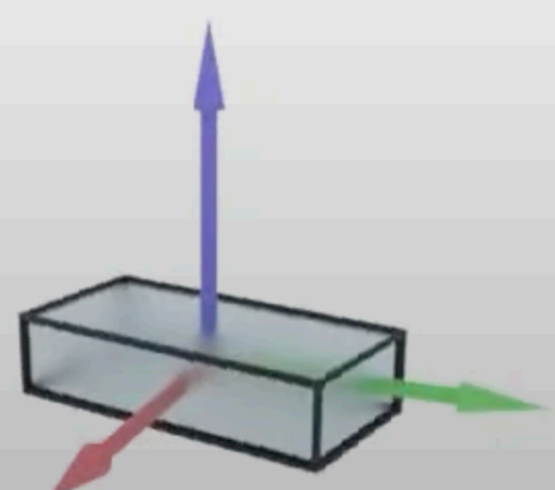
Rotation



$$R_x : \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$$R_y : \begin{bmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$$R_z : \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotation

$$\mathbf{R_z}(\alpha)\,\mathbf{R_y}(\beta)\,\mathbf{R_x}(\gamma)\,\mathbf{p}$$

$$\neq \mathbf{R_y}(\beta)\,\mathbf{R_z}(\alpha)\,\mathbf{R_x}(\gamma)$$

$$\neq \mathbf{R_x}(\gamma)\,\mathbf{R_z}(\alpha)\,\mathbf{R_y}(\beta)$$

$$\neq \mathbf{R_z}(\alpha)\,\mathbf{R_x}(\gamma)\,\mathbf{R_y}(\beta)$$

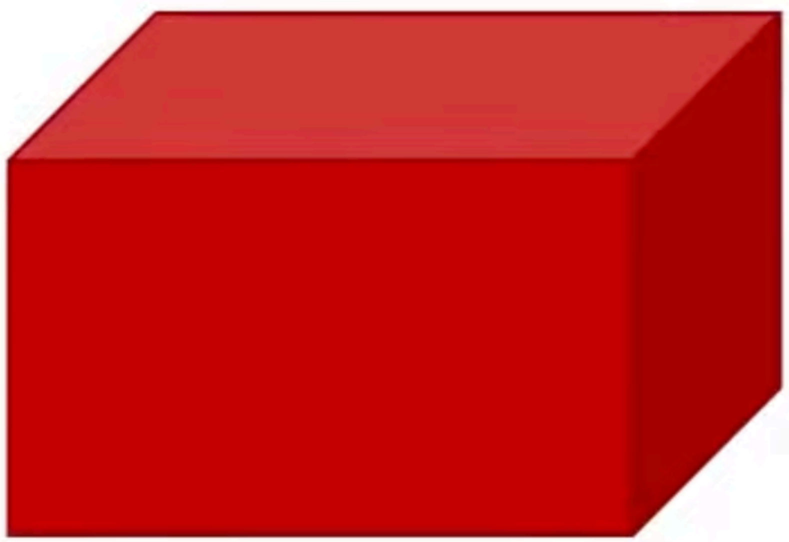
$$\neq \mathbf{R_y}(\beta)\,\mathbf{R_x}(\gamma)\,\mathbf{R_z}(\alpha)$$

$$\neq \mathbf{R_x}(\gamma)\,\mathbf{R_y}(\beta)\,\mathbf{R_z}(\alpha)$$

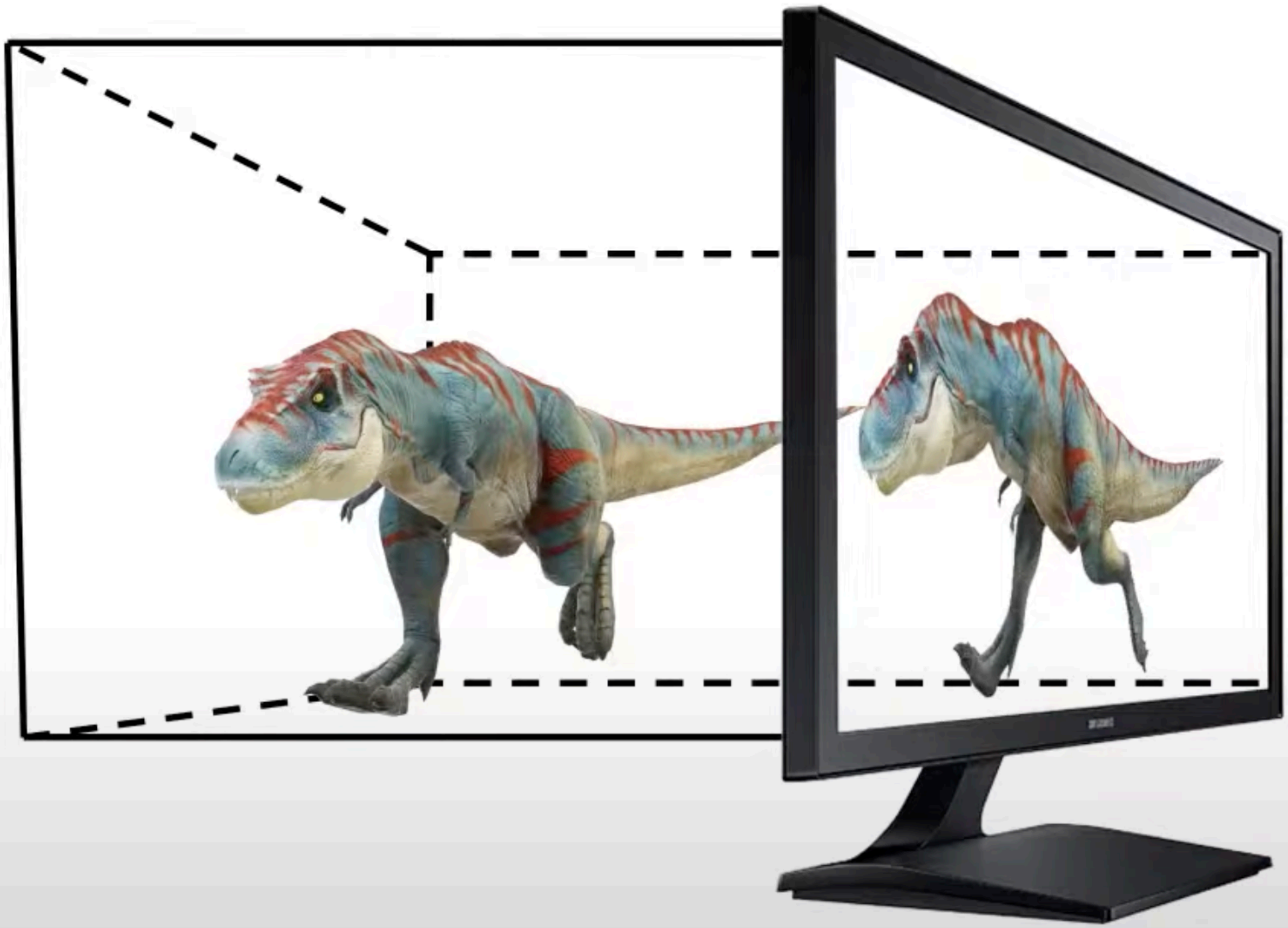
Viewing

Projection Transformations

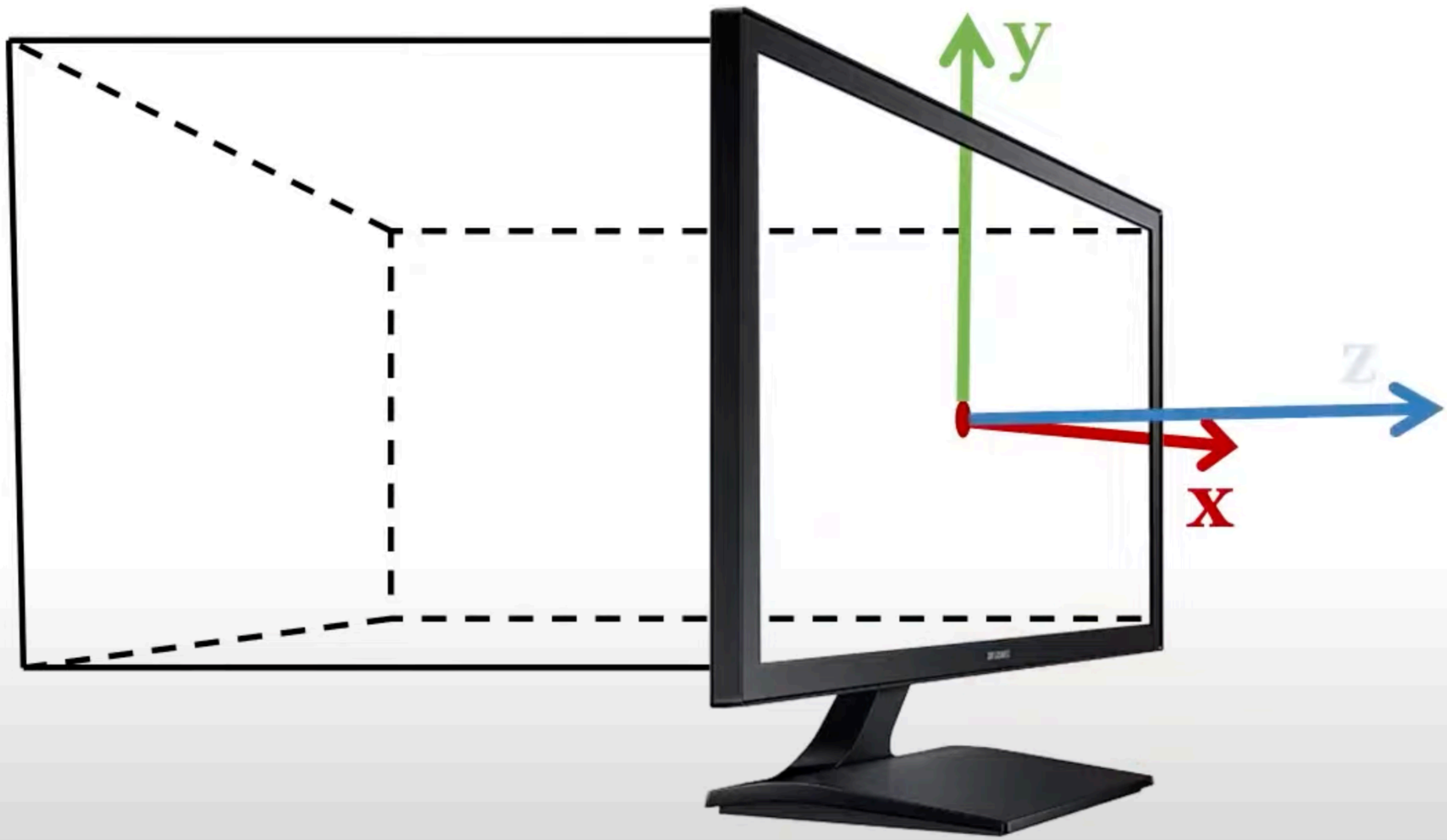
Viewing



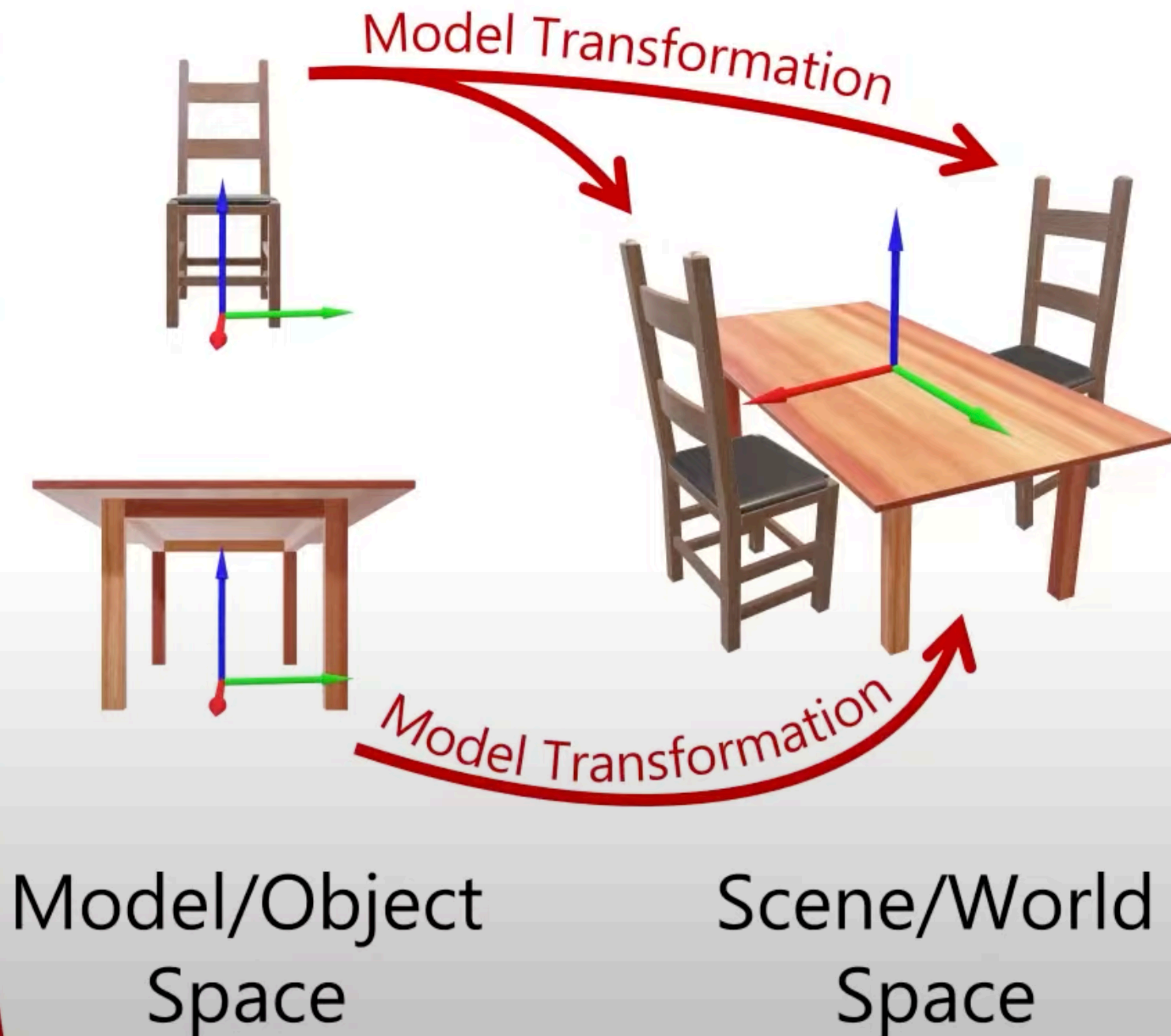
Viewing



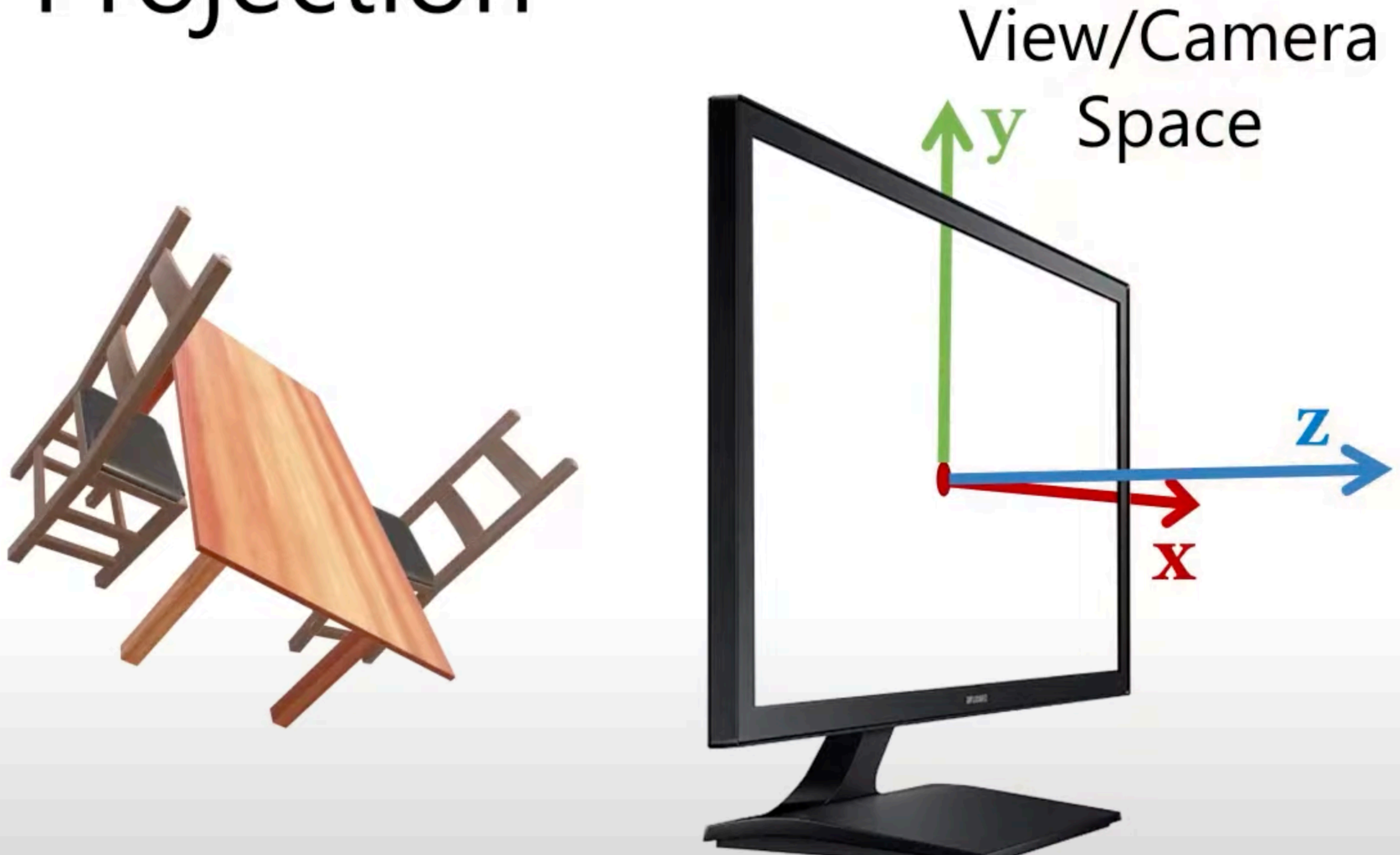
Viewing



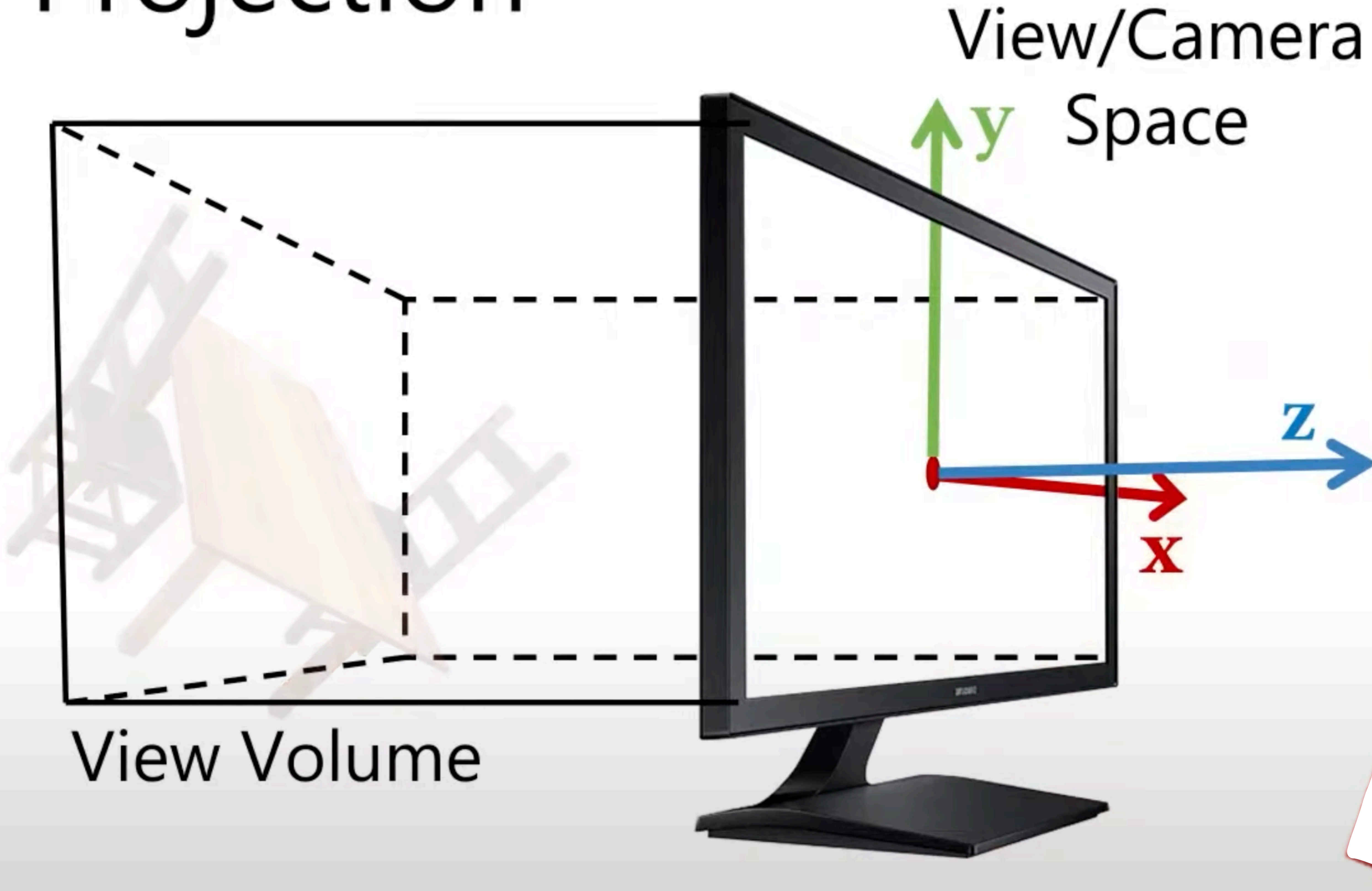
Viewing Transformations



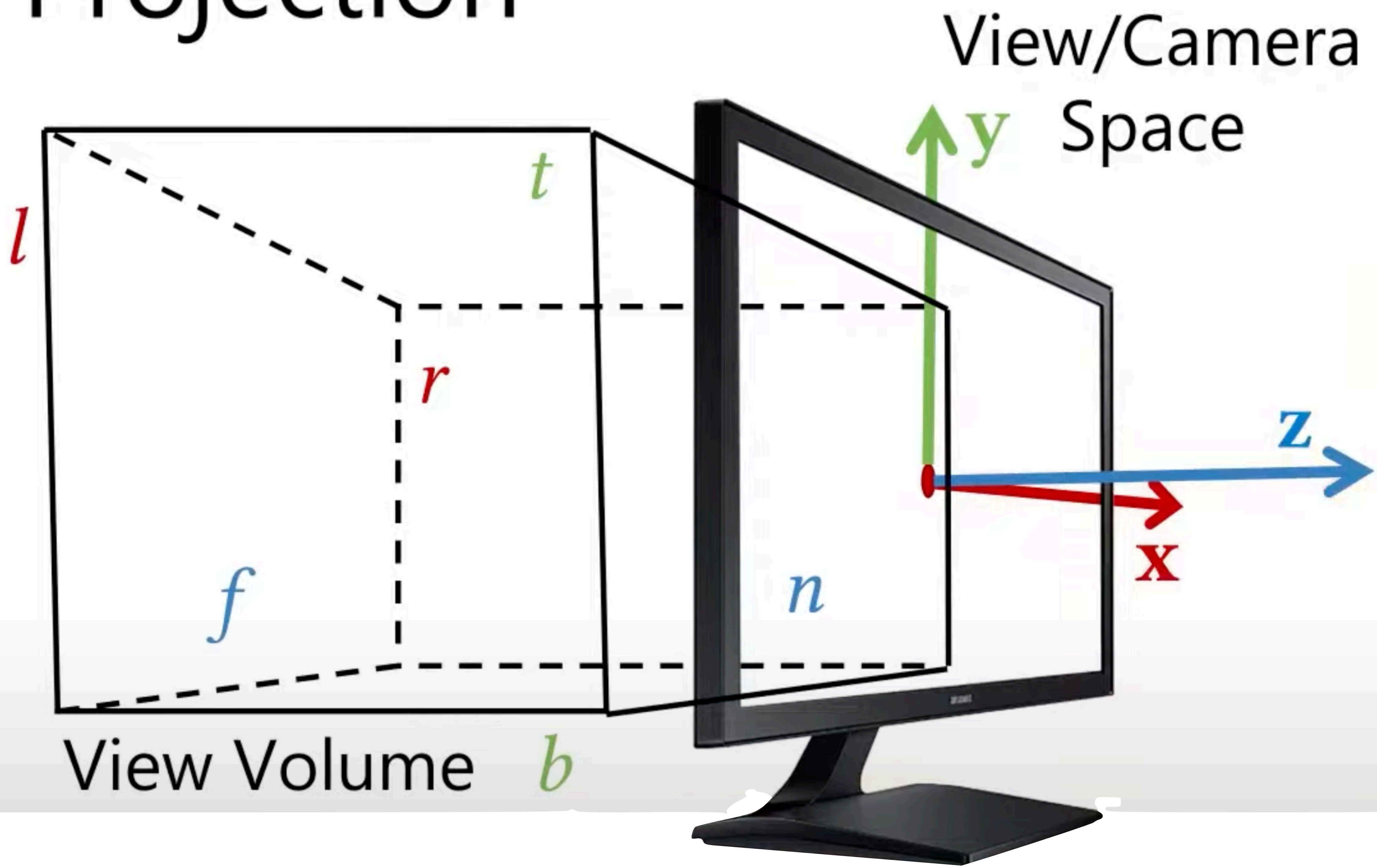
Projection



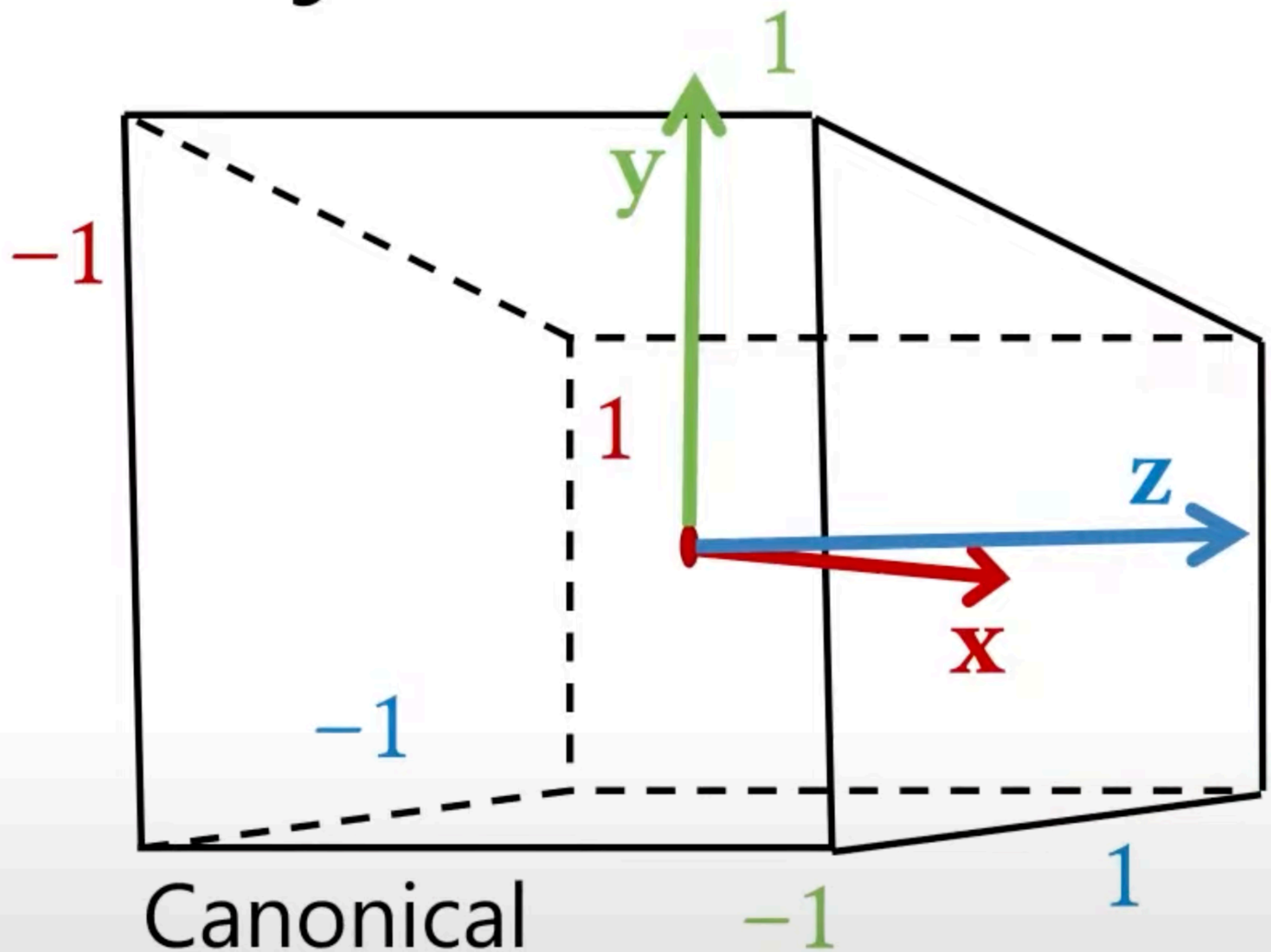
Projection



Projection

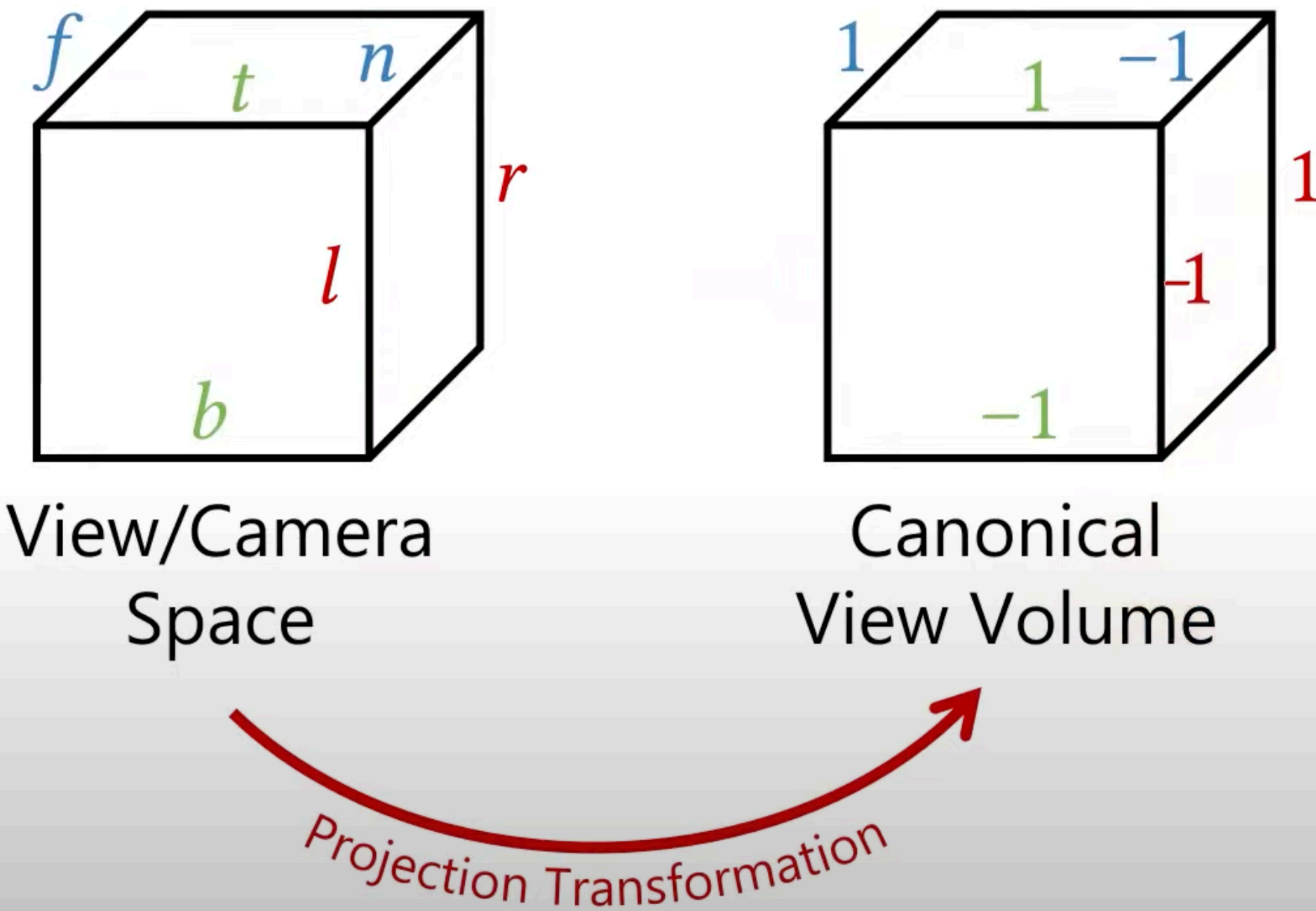


Projection



Canonical
View Volume

Orthographic Projection



Orthographic Projection

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} ? & 0 & 0 & ? \\ 0 & ? & 0 & ? \\ 0 & 0 & ? & ? \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Orthographic Projection

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & \frac{-2l}{r-l} - 1 \\ 0 & \frac{2}{t-b} & 0 & \frac{-2b}{t-b} - 1 \\ 0 & 0 & \frac{2}{f-n} & \frac{-2n}{f-n} - 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

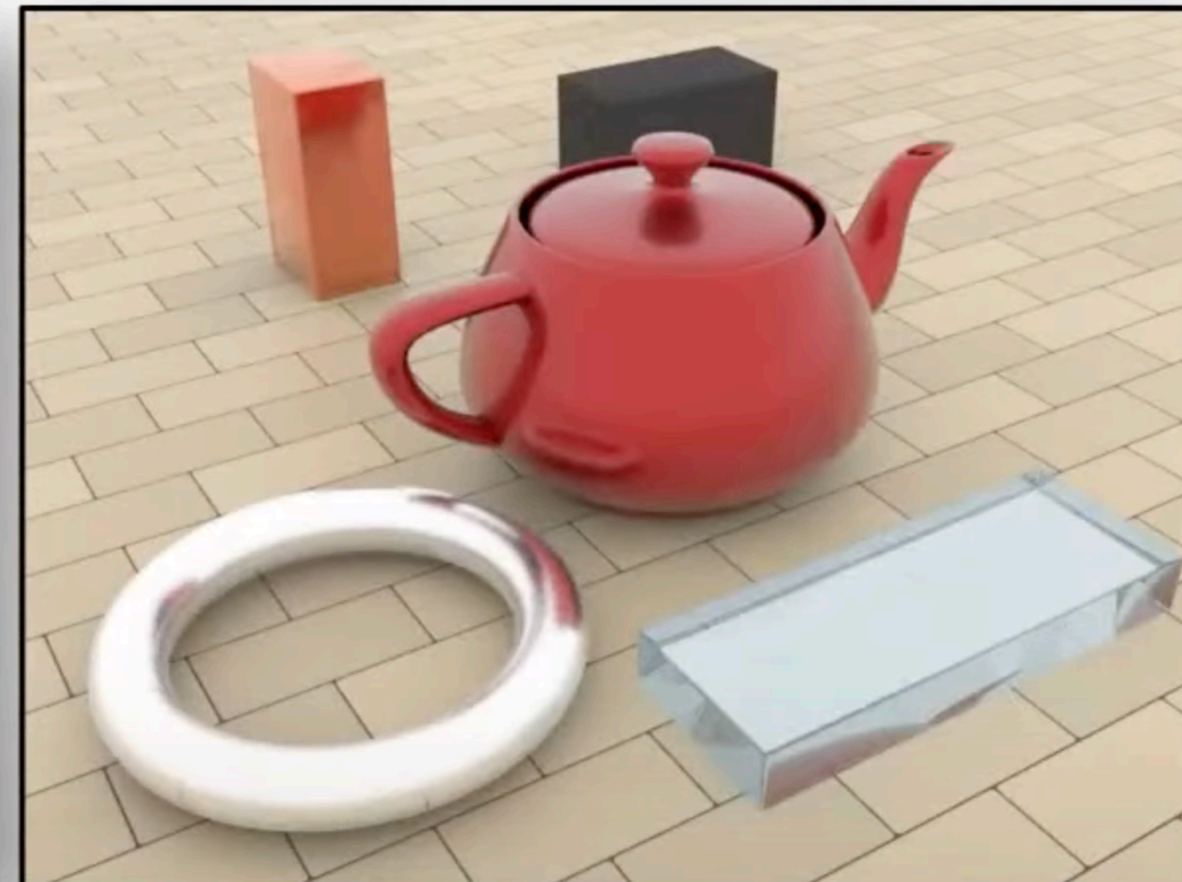
$$\begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{2}{f-n} & -\frac{f+n}{f-n} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Projection

Orthographic
projection

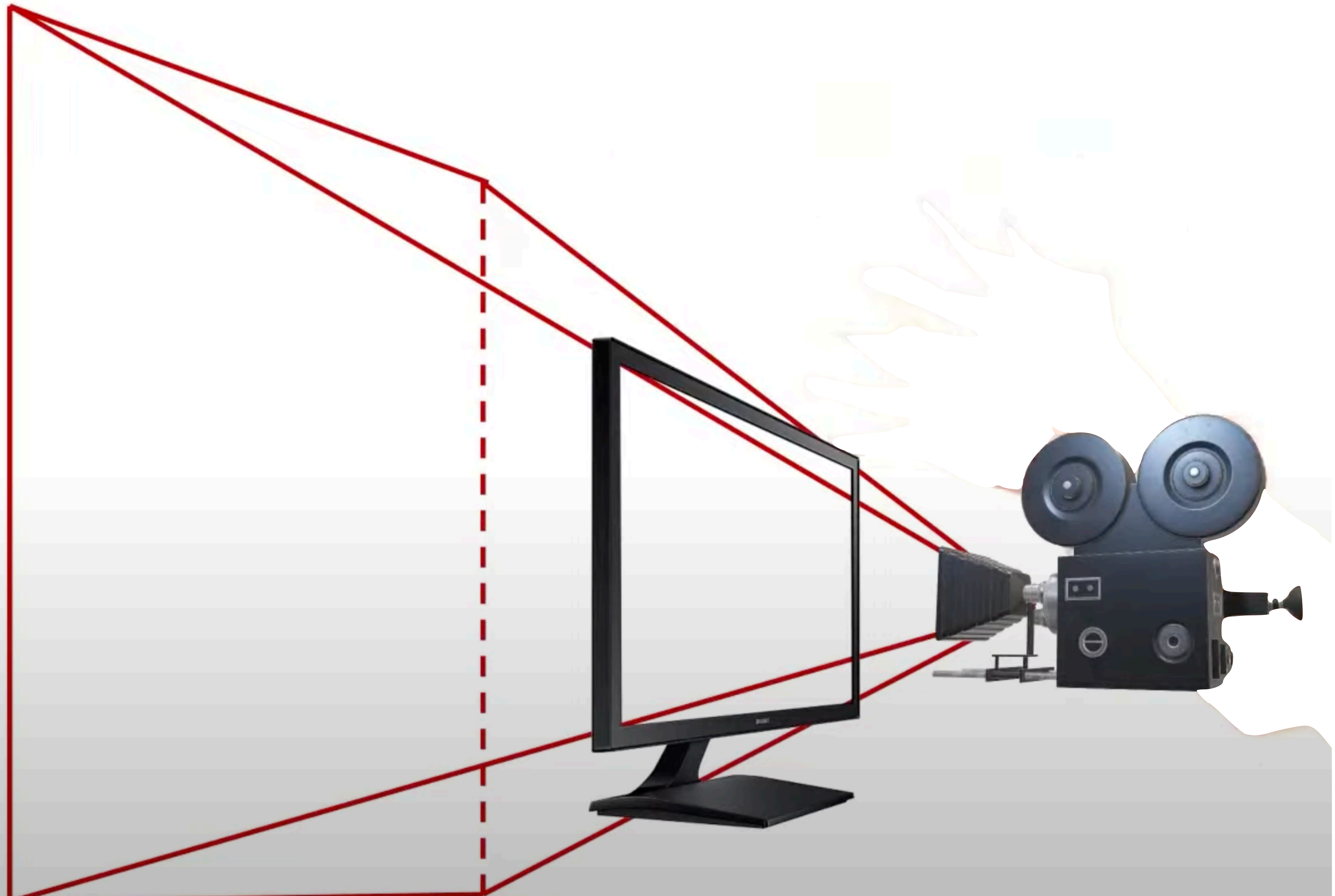
Projection

Orthographic projection

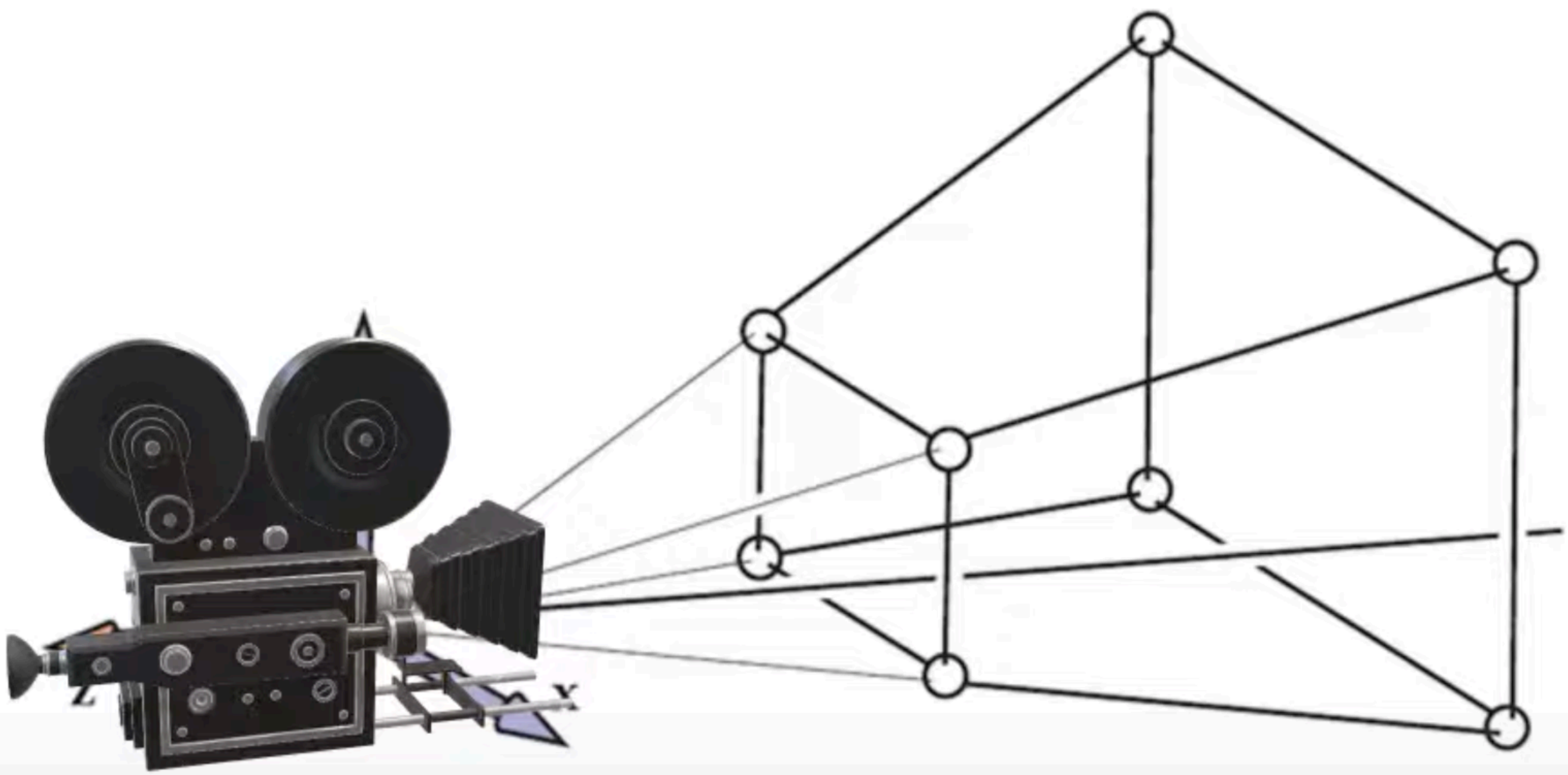


Perspective projection

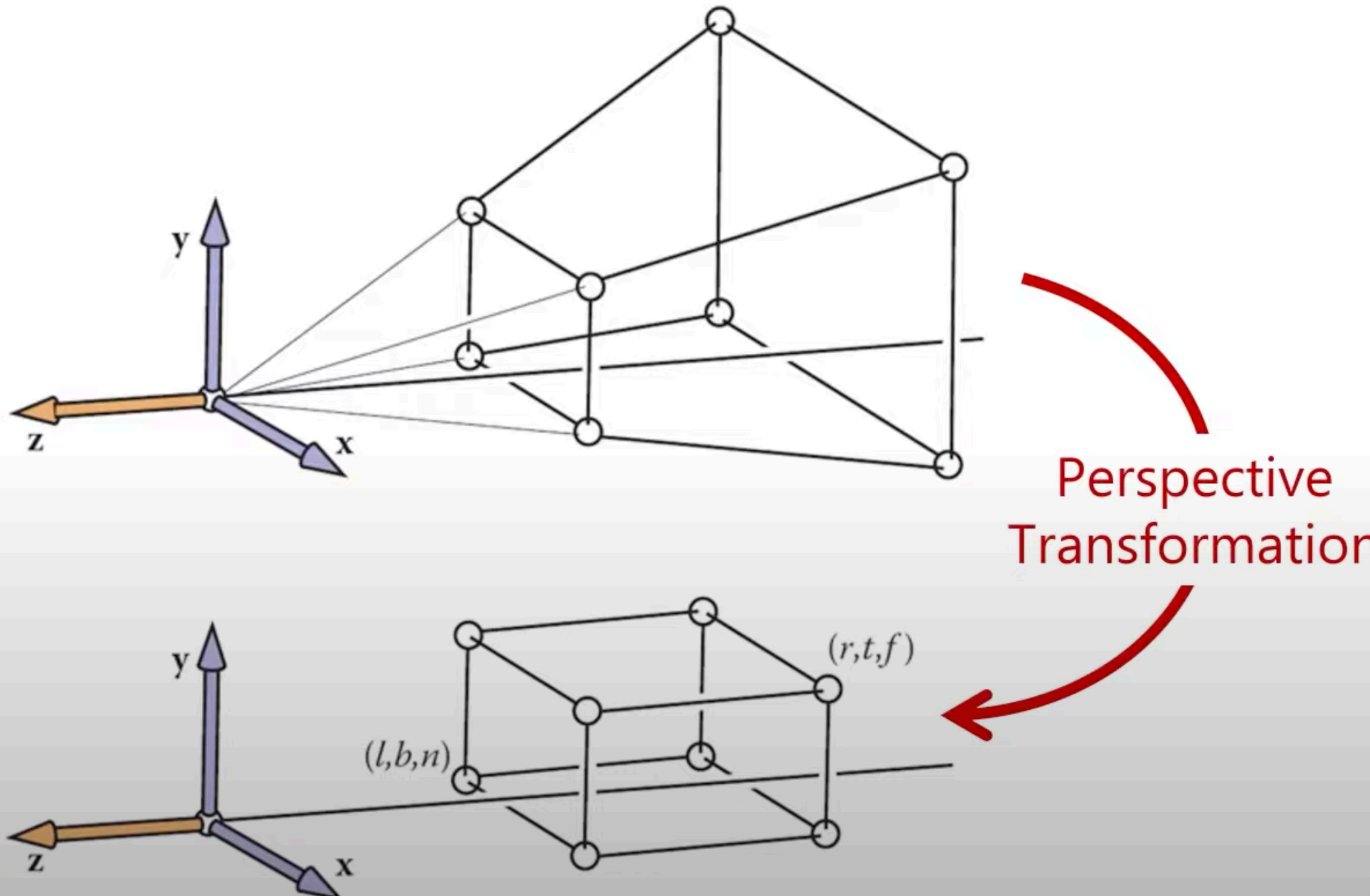
Perspective Projection



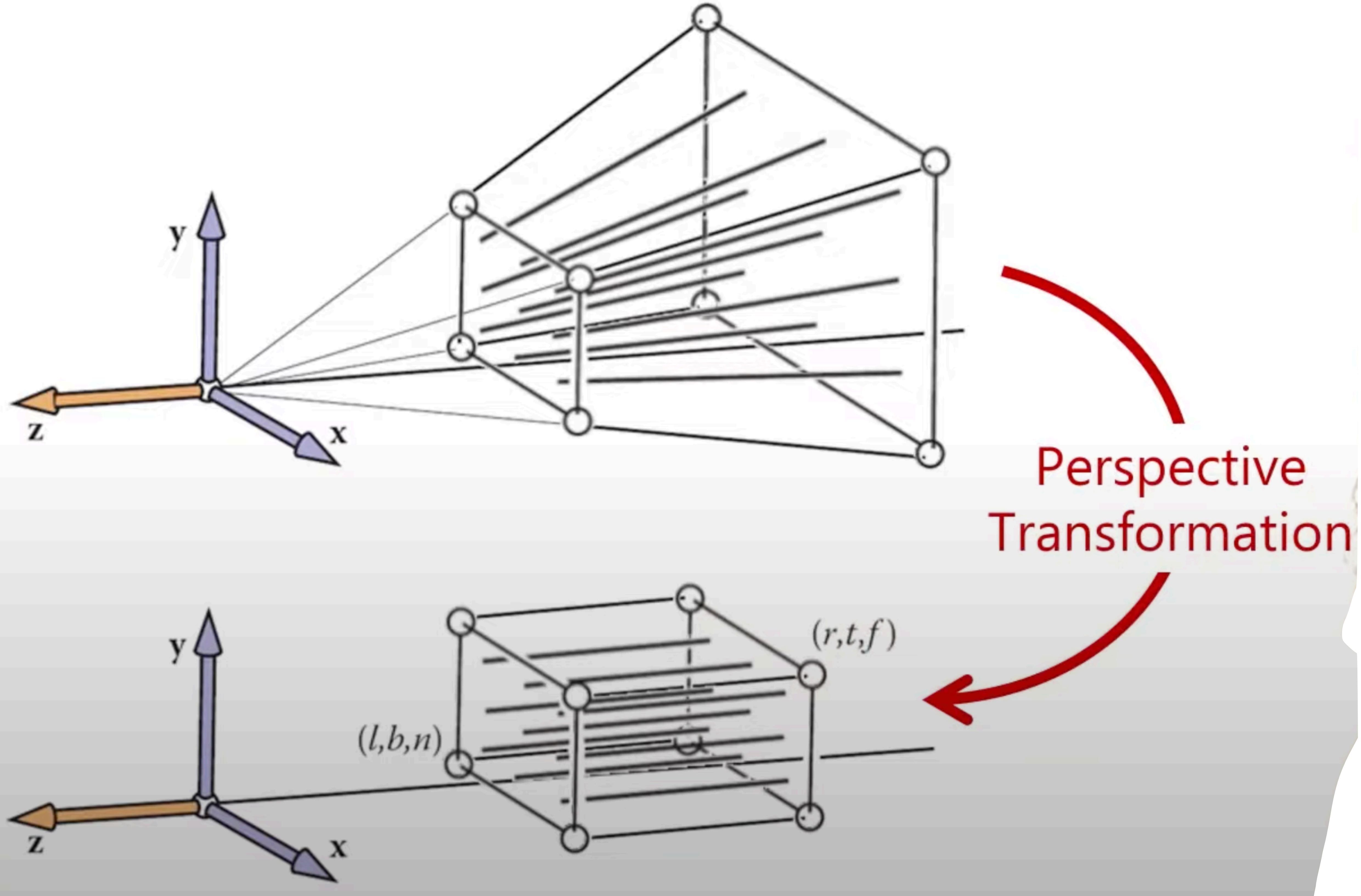
Perspective Projection



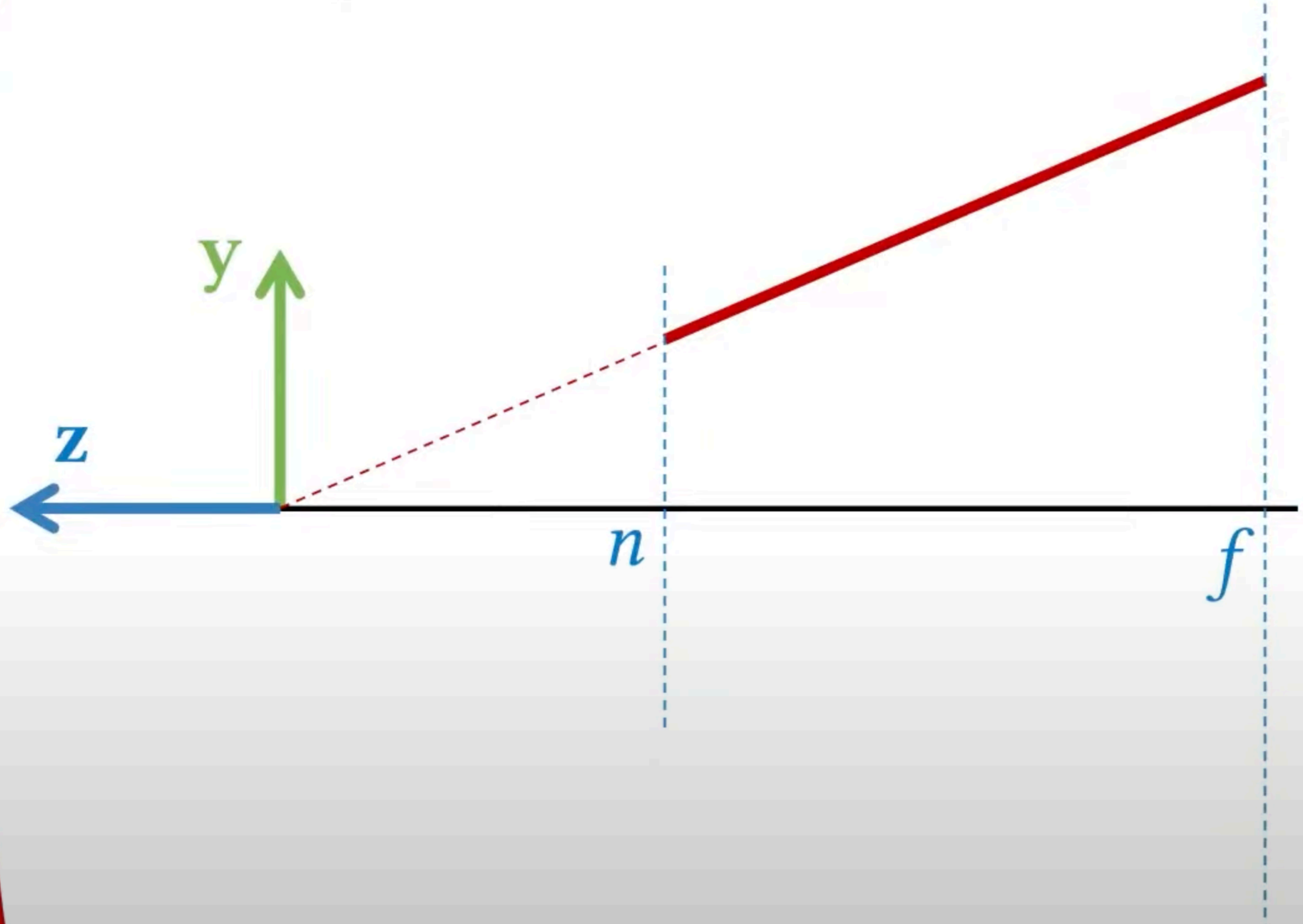
Perspective Projection



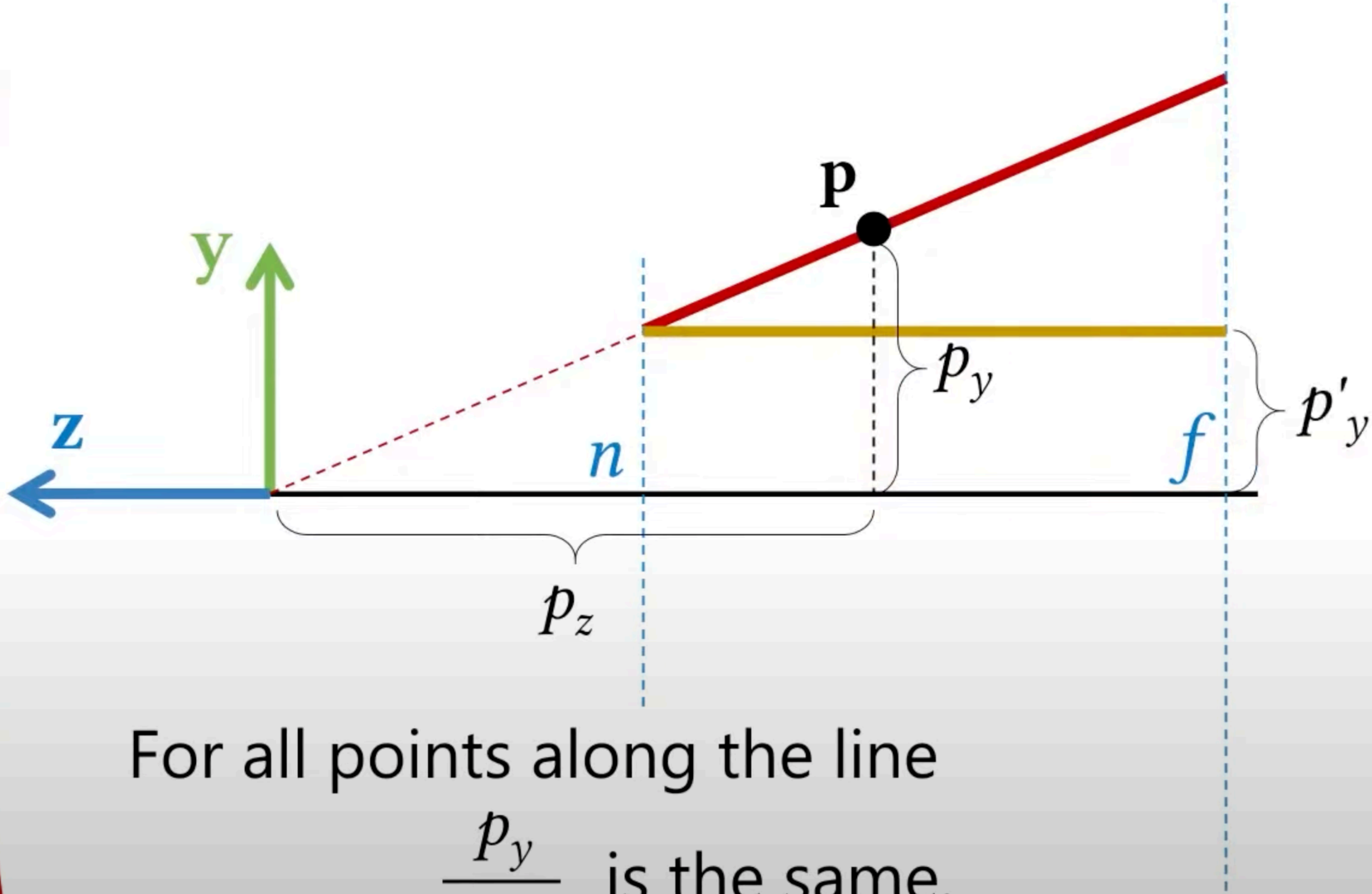
Perspective Projection



Perspective Transformation



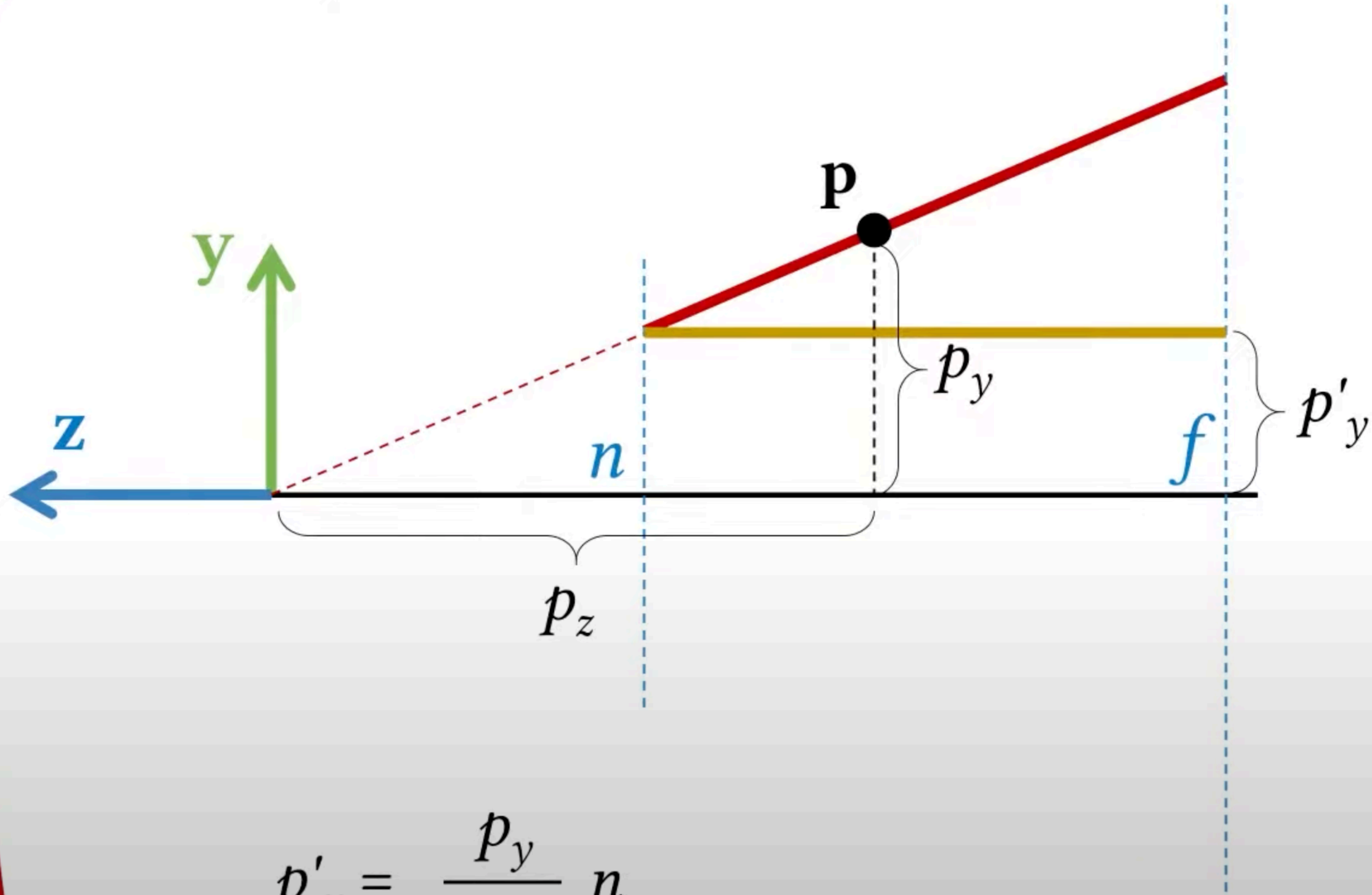
Perspective Transformation



For all points along the line

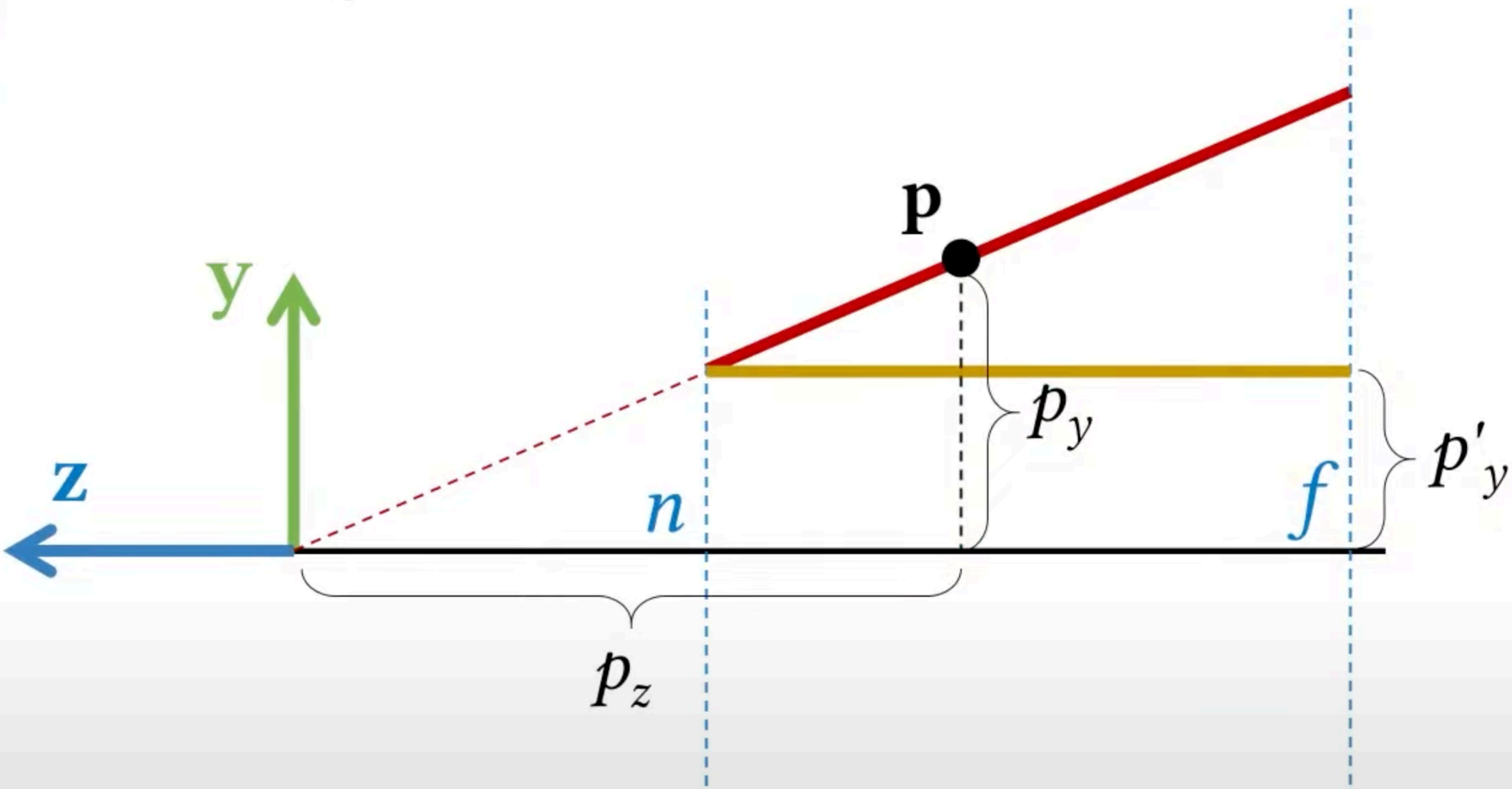
$$\frac{p_y}{p_z} \text{ is the same.}$$

Perspective Transformation



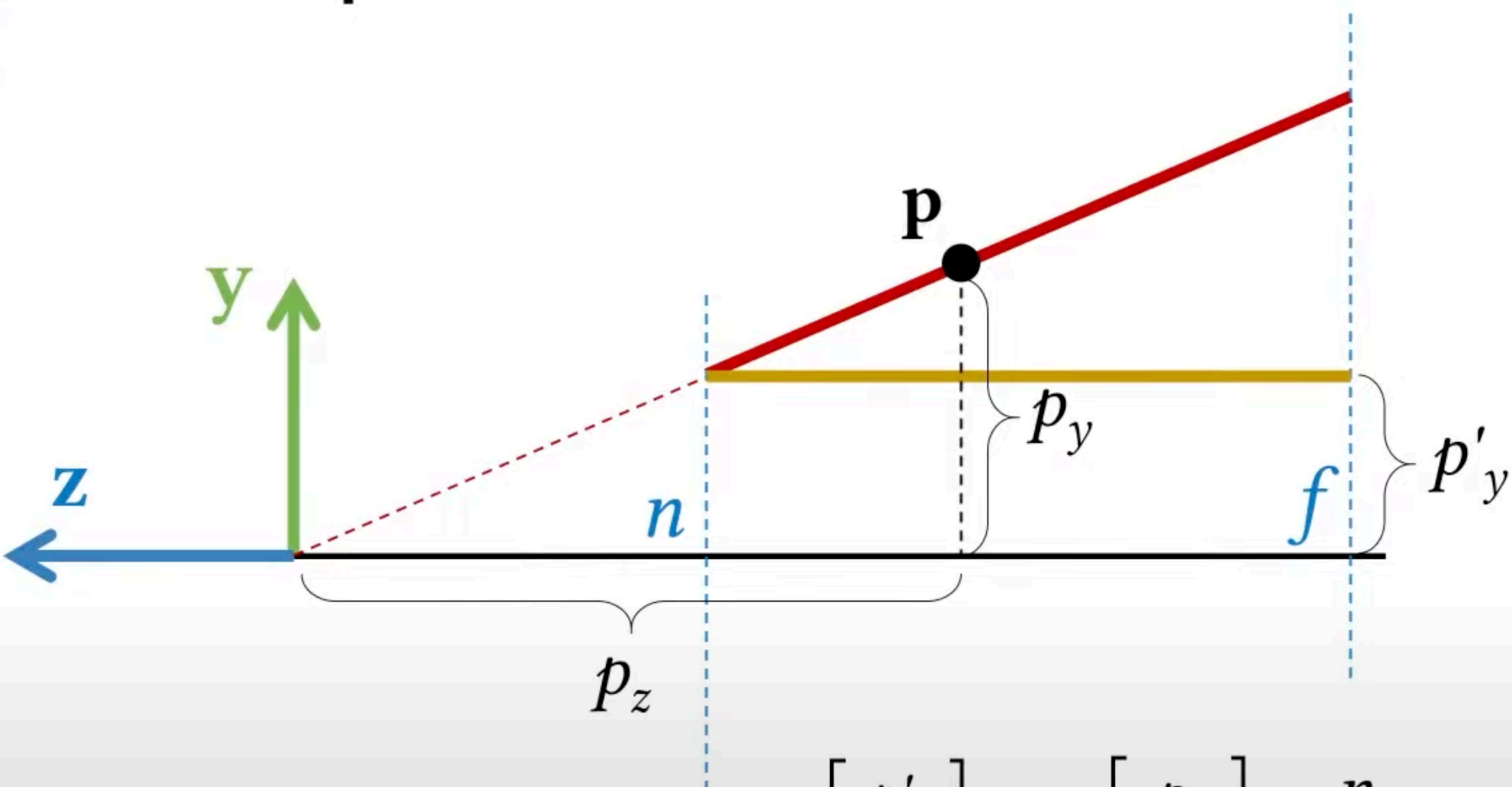
$$p'_y = \frac{p_y}{p_z}\, n$$

Perspective Transformation



$$p'_y = \frac{p_y}{p_z}\,n \qquad p'_x = \frac{p_x}{p_z}\,n$$

Perspective Transformation



$$\begin{bmatrix} p'_x \\ p'_y \end{bmatrix} = \begin{bmatrix} p_x \\ p_y \end{bmatrix} \frac{n}{p_z}$$

Homogeneous Coordinates

$$\begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix} \equiv \begin{bmatrix} \alpha\, p_x \\ \alpha\, p_y \\ \alpha\, p_z \\ \alpha \end{bmatrix}$$

$$\begin{bmatrix} p'_x \\ p'_y \\ p'_z \\ 1 \end{bmatrix} = \begin{bmatrix} n\, p_x / p_z \\ n\, p_y / p_z \\ ? \\ 1 \end{bmatrix} \equiv \begin{bmatrix} n\, p_x \\ n\, p_y \\ ? \\ p_z \end{bmatrix}$$

Perspective Transformation

$$\begin{bmatrix} p'_x \\ p'_y \\ p'_z \\ 1 \end{bmatrix} = \begin{bmatrix} n\, p_x / p_z \\ n\, p_y / p_z \\ ? \\ 1 \end{bmatrix} \equiv \begin{bmatrix} n\, p_x \\ n\, p_y \\ ? \\ p_z \end{bmatrix}$$

$$\begin{bmatrix} p'_x \\ p'_y \\ p'_z \\ 1 \end{bmatrix} \equiv \begin{bmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & ? & ? \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix}$$

Perspective Transformation

$$\begin{bmatrix} p'_x \\ p'_y \\ p'_z \\ 1 \end{bmatrix} \equiv \begin{bmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & ? & ? \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix}$$

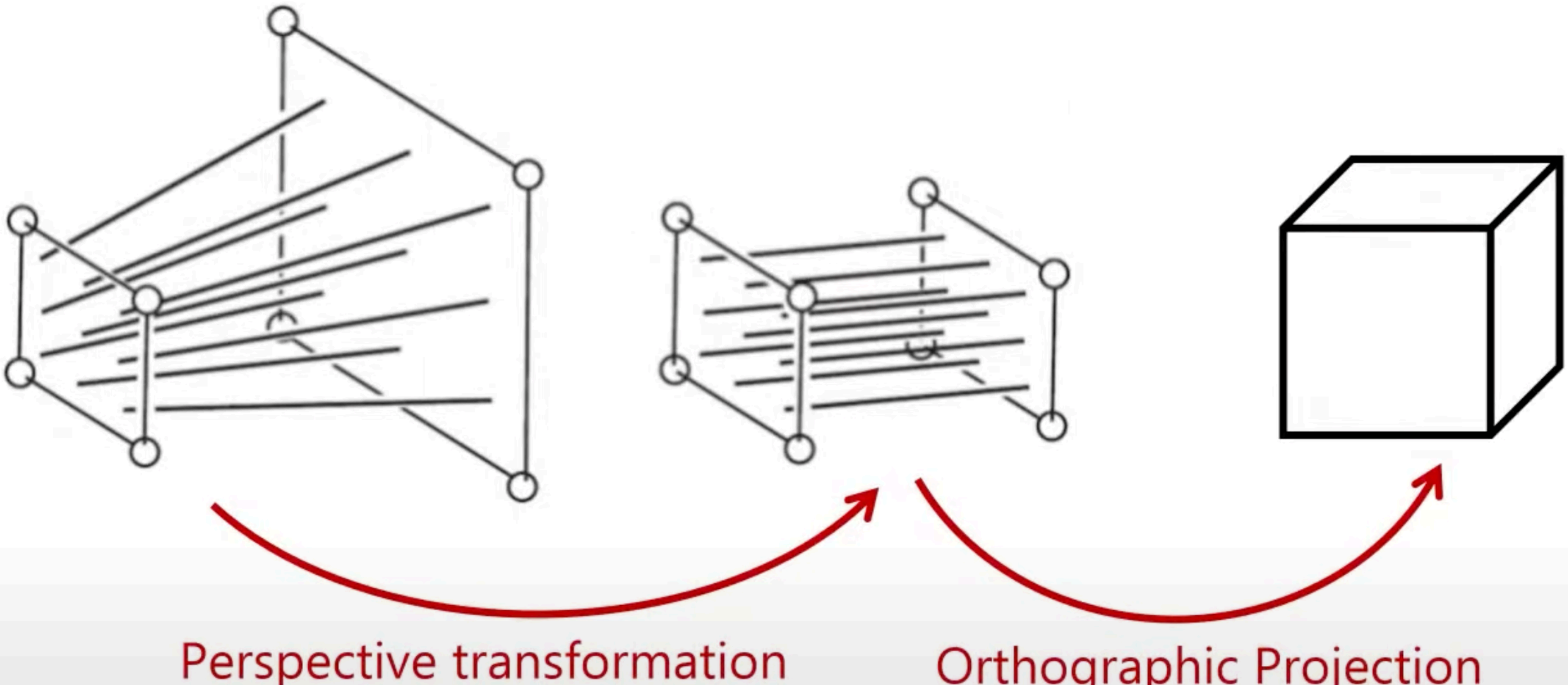
Perspective Transformation

$$\begin{bmatrix} p'_x \\ p'_y \\ p'_z \\ 1 \end{bmatrix} \equiv \begin{bmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n+f & -fn \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix}$$

$$p_z = n \quad \Rightarrow \quad \begin{aligned} p'_z &= ((n+f)\,p_z - fn)\,/\,p_z \\ &= (n+f) - fn\,/\,p_z \\ &= (n+f) - f \\ &= n \end{aligned}$$

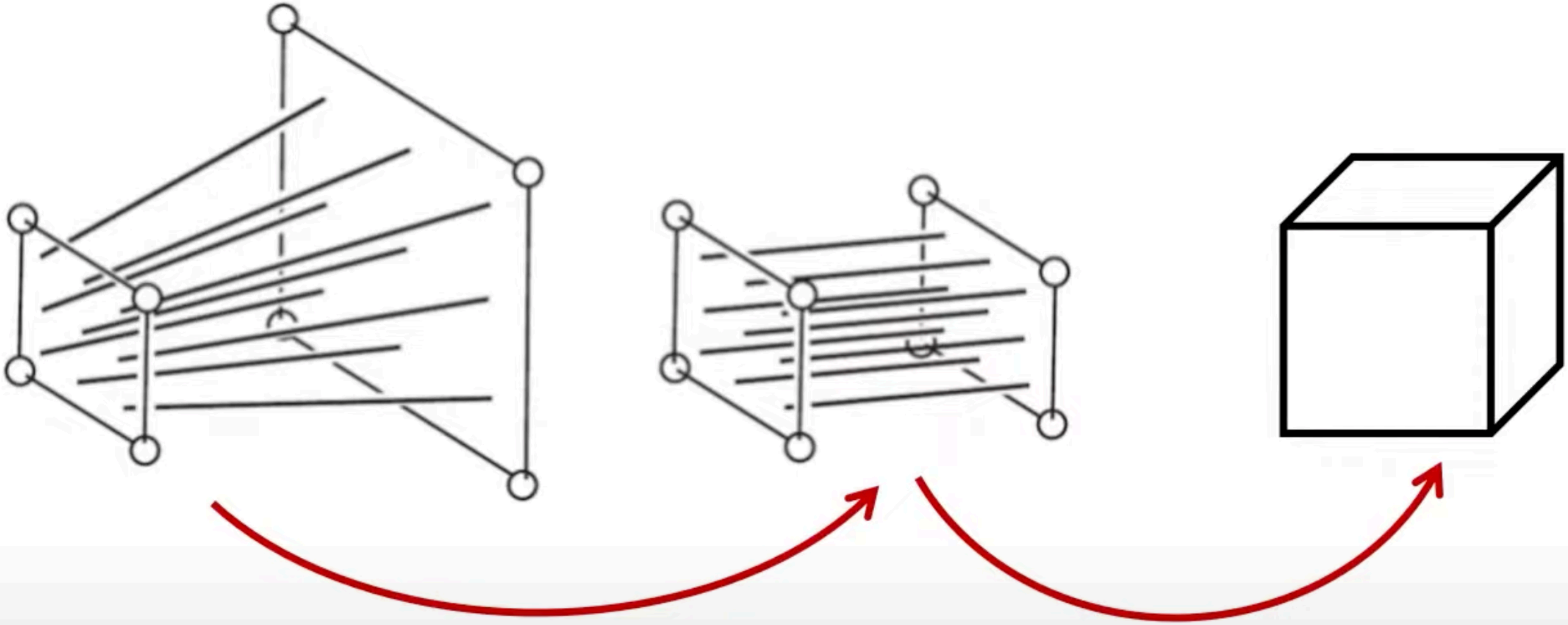
$$p_z = f \quad \Rightarrow \quad \begin{aligned} p'_z &= (n+f) - n \\ &= f \end{aligned}$$

Perspective Projection



$$
\begin{bmatrix}
n & 0 & 0 & 0 \\
0 & n & 0 & 0 \\
0 & 0 & n+f & -fn \\
0 & 0 & 1 & 0
\end{bmatrix}
$$

Perspective Projection



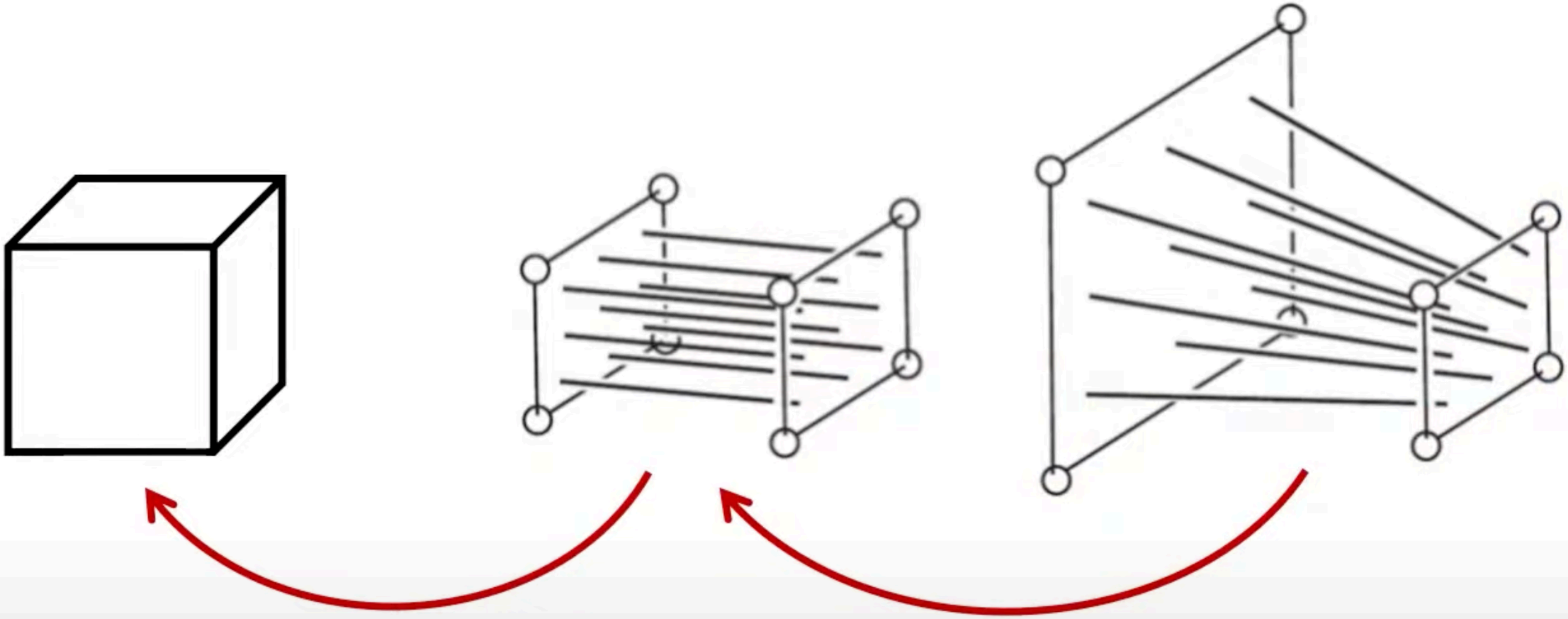
Perspective transformation

$$\begin{bmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n+f & -fn \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Orthographic Projection

$$\begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{2}{f-n} & -\frac{f+n}{f-n} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Perspective Projection

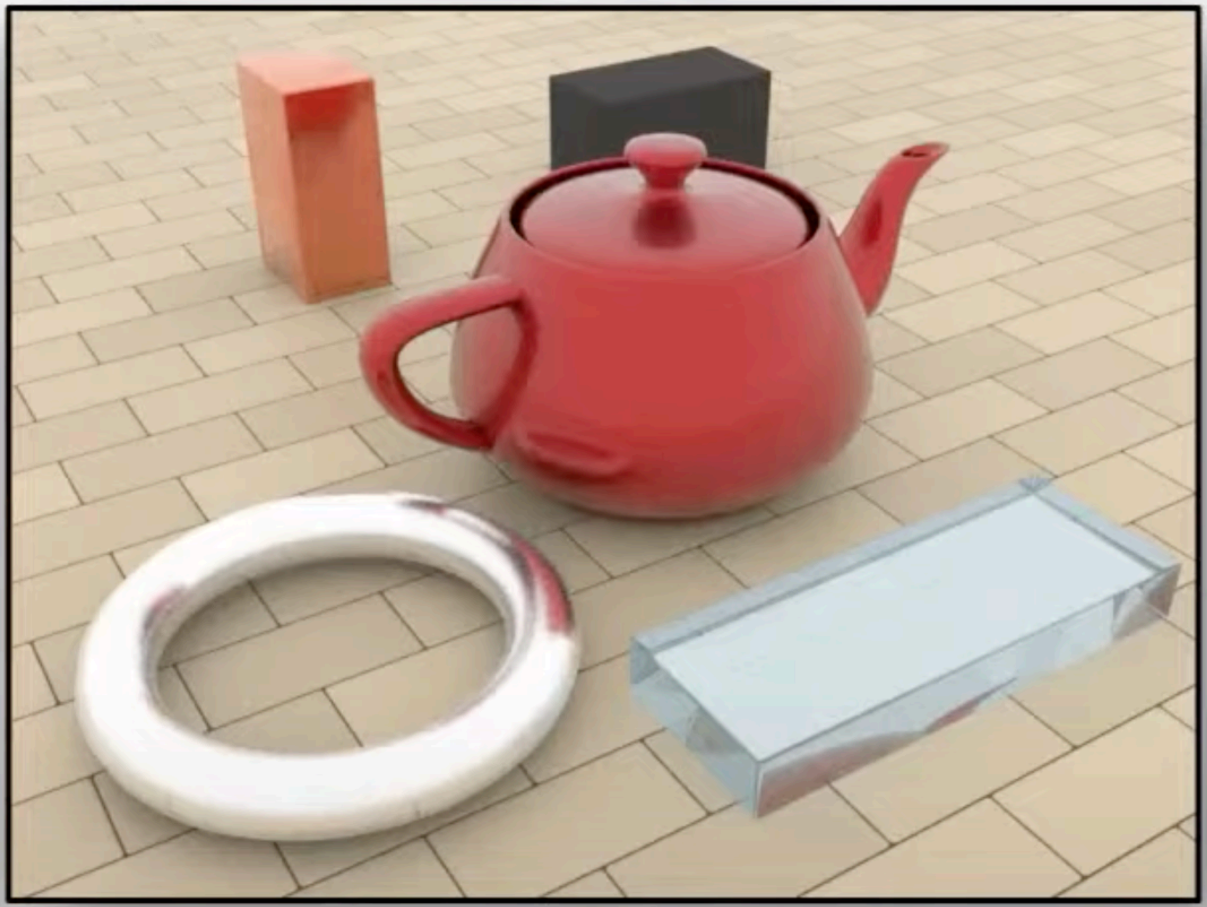


$$\begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{2}{f-n} & -\frac{f+n}{f-n} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n+f & -fn \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Projection

Orthographic projection



Perspective projection

Rendering Algorithms

Rendering Algorithms

Rendering Algorithms

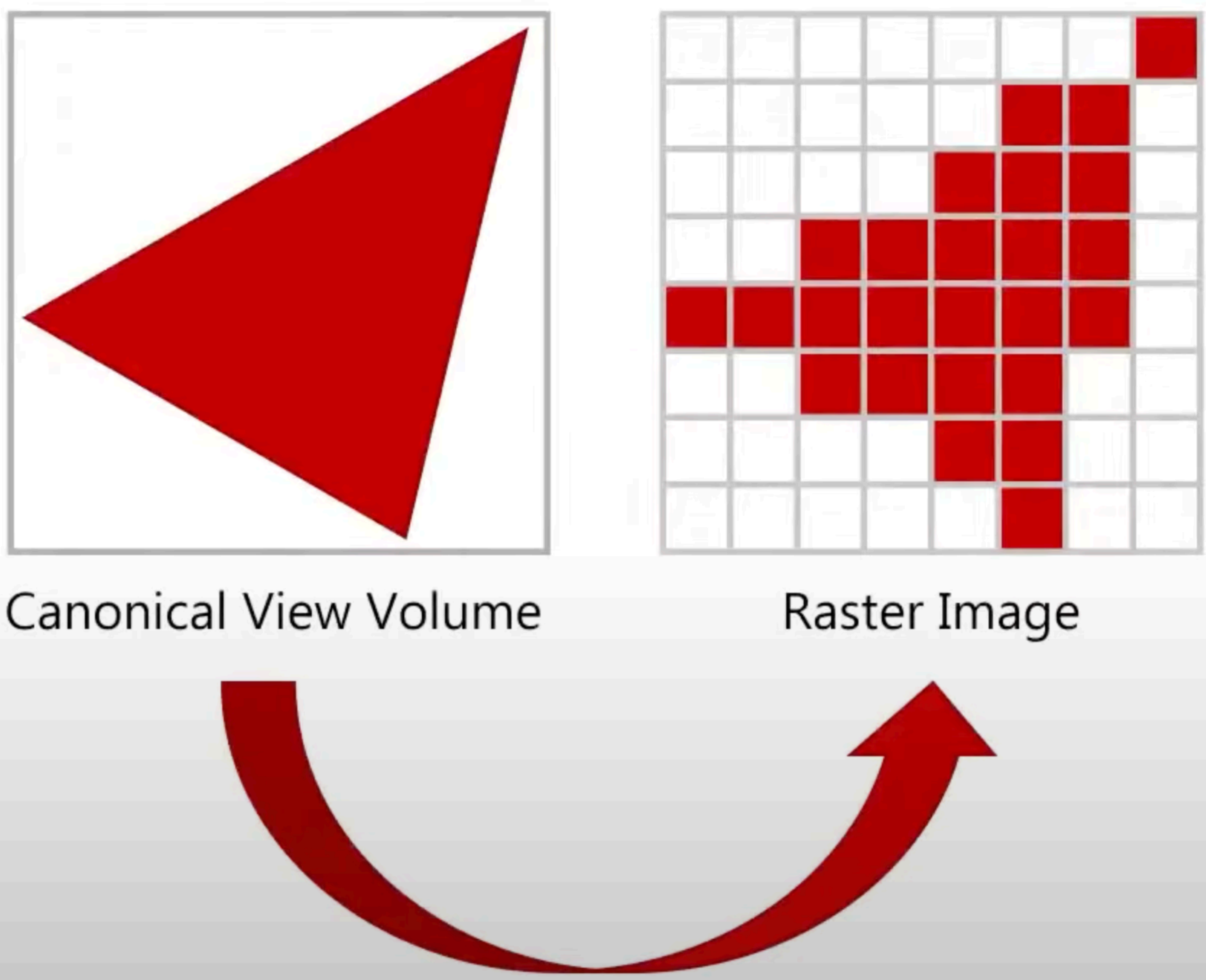
- **Rasterization**
  - Painter's Algorithm
  - Z-Buffer Rasterization
  - A-Buffer Rasterization
  - REYES
- **Ray Tracing**

Rendering Algorithms

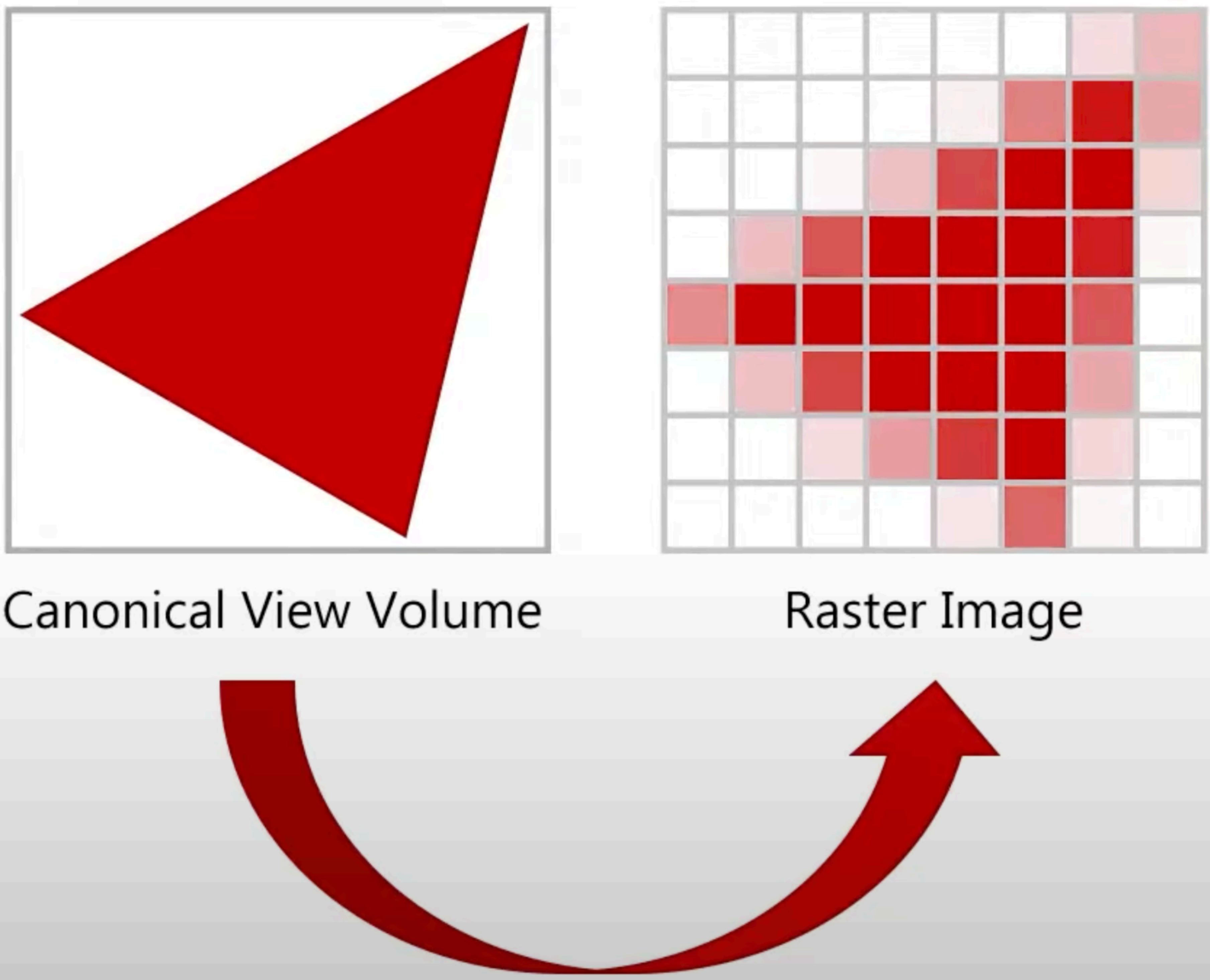
- **Rasterization**
  - Painter’s Algorithm
  - Z-Buffer Rasterization
  - A-Buffer Rasterization
  - REYES
- **Ray Tracing**
  - Ray Casting
  - Path Tracing

  ...

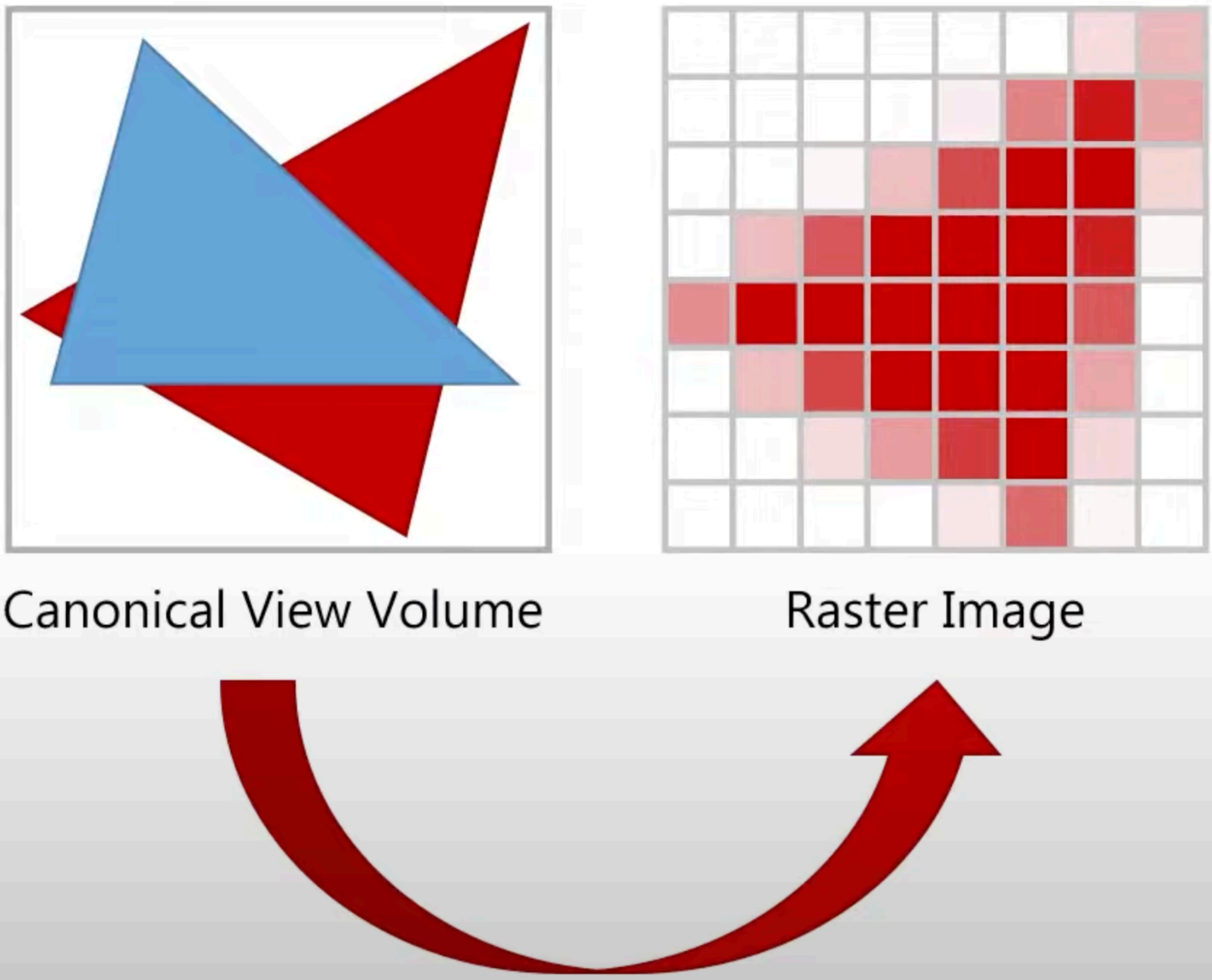
Rasterization



Rasterization



Rasterization

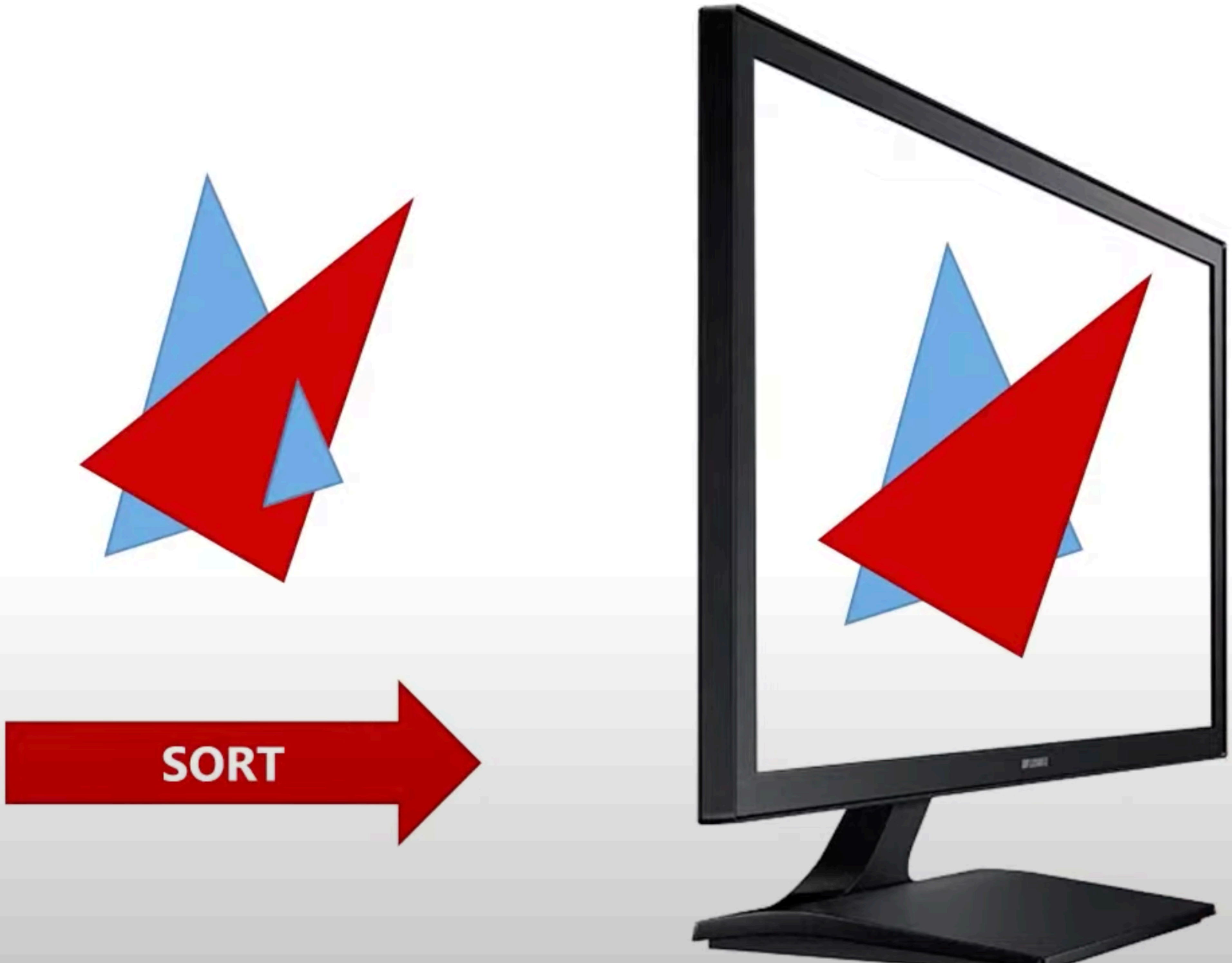


Painter's Algorithm

Painter's Algorithm

Painter's Algorithm

Painter's Algorithm

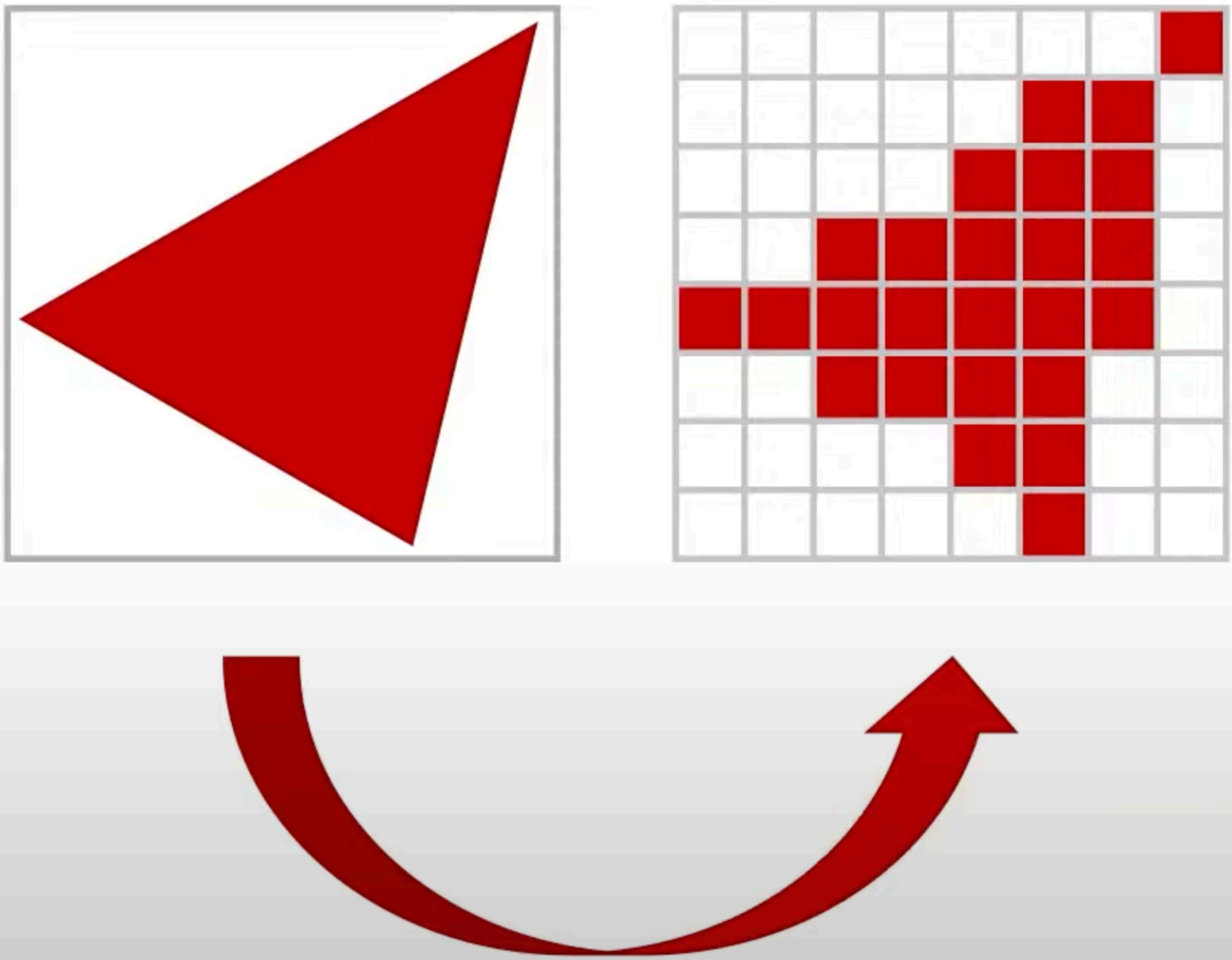


Painter’s Algorithm

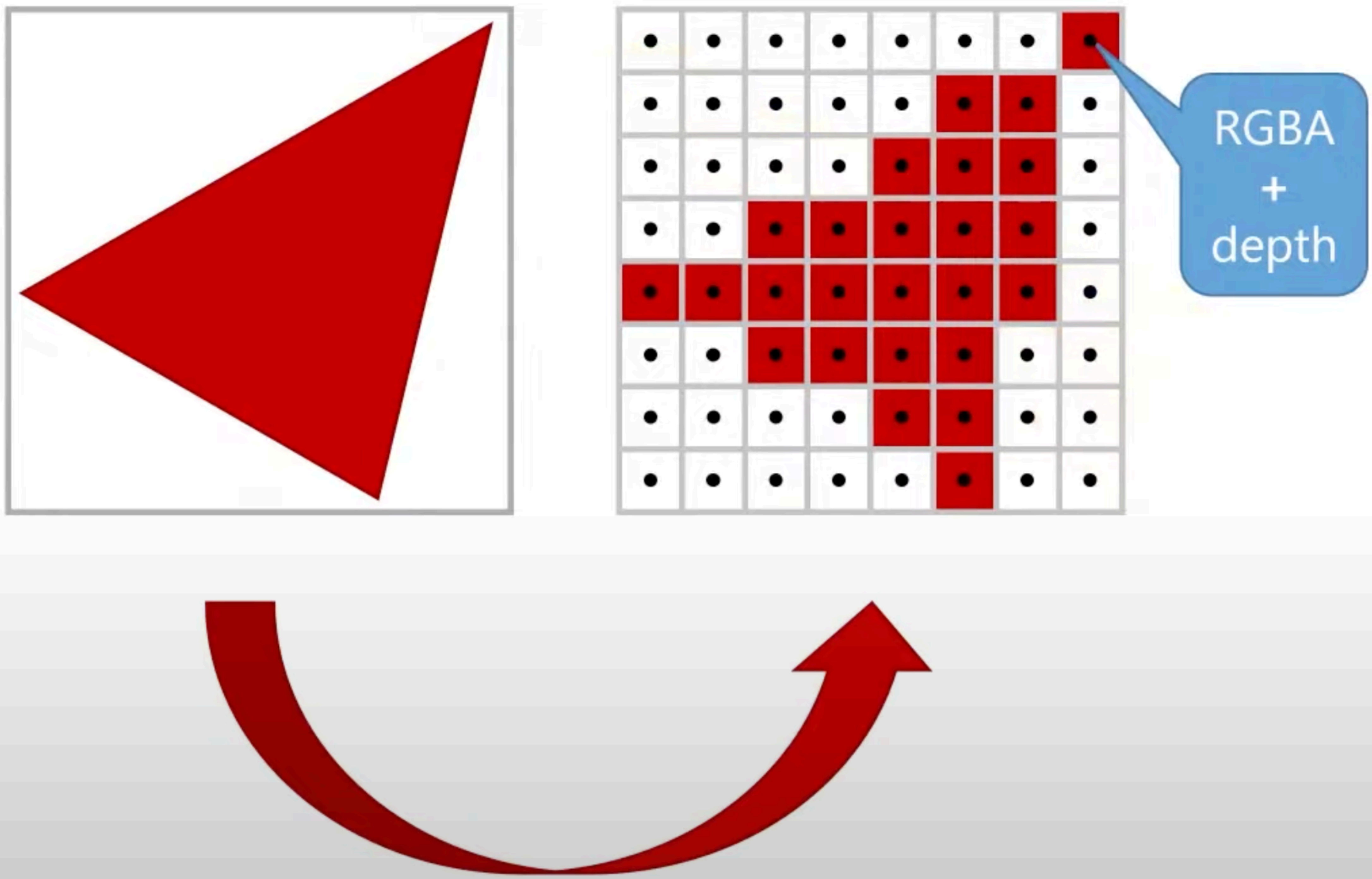
- × Needs sorting
- × Cannot handle intersecting geometry

Z-Buffer Rasterization

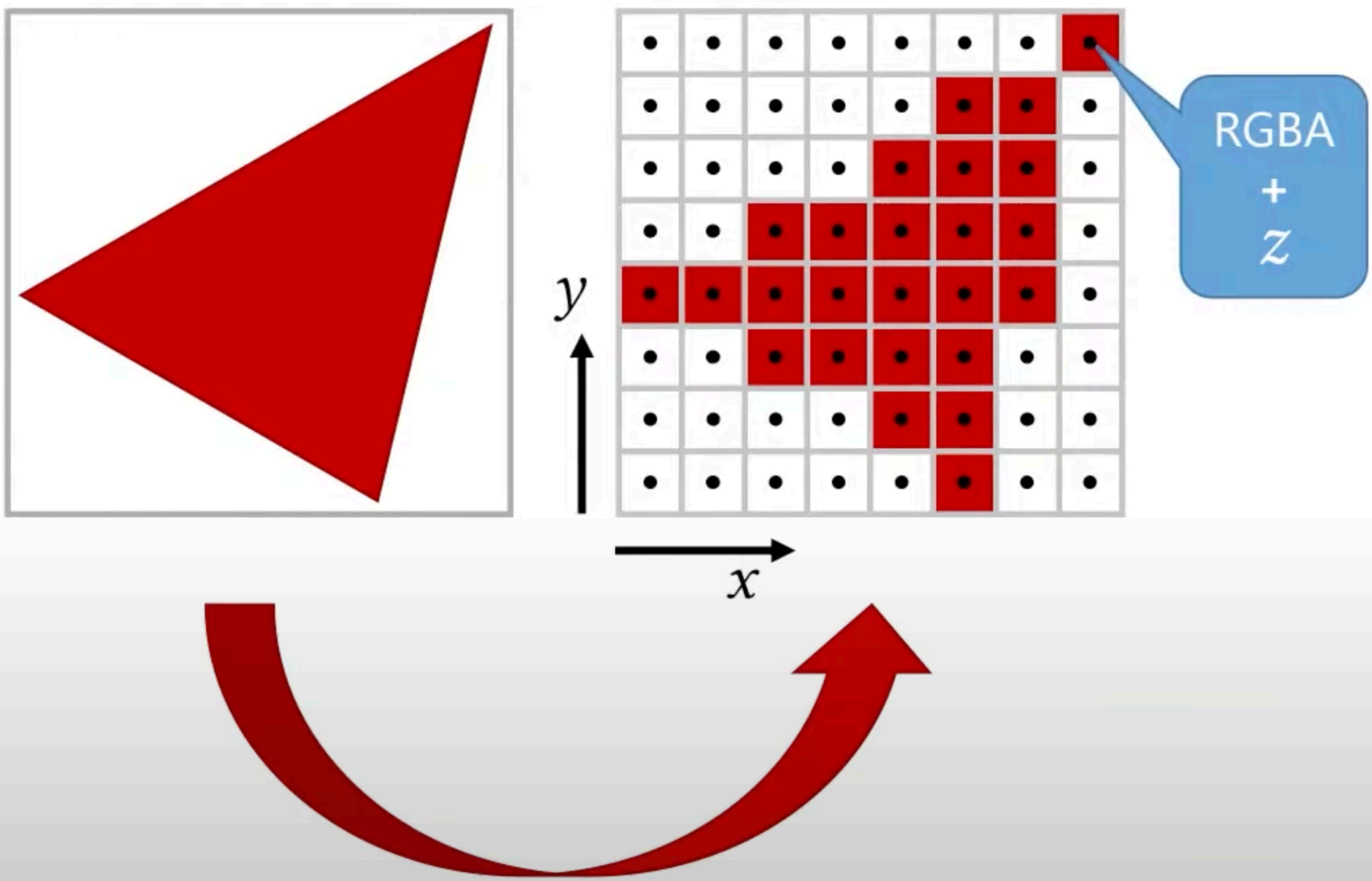
Z-Buffer Rasterization



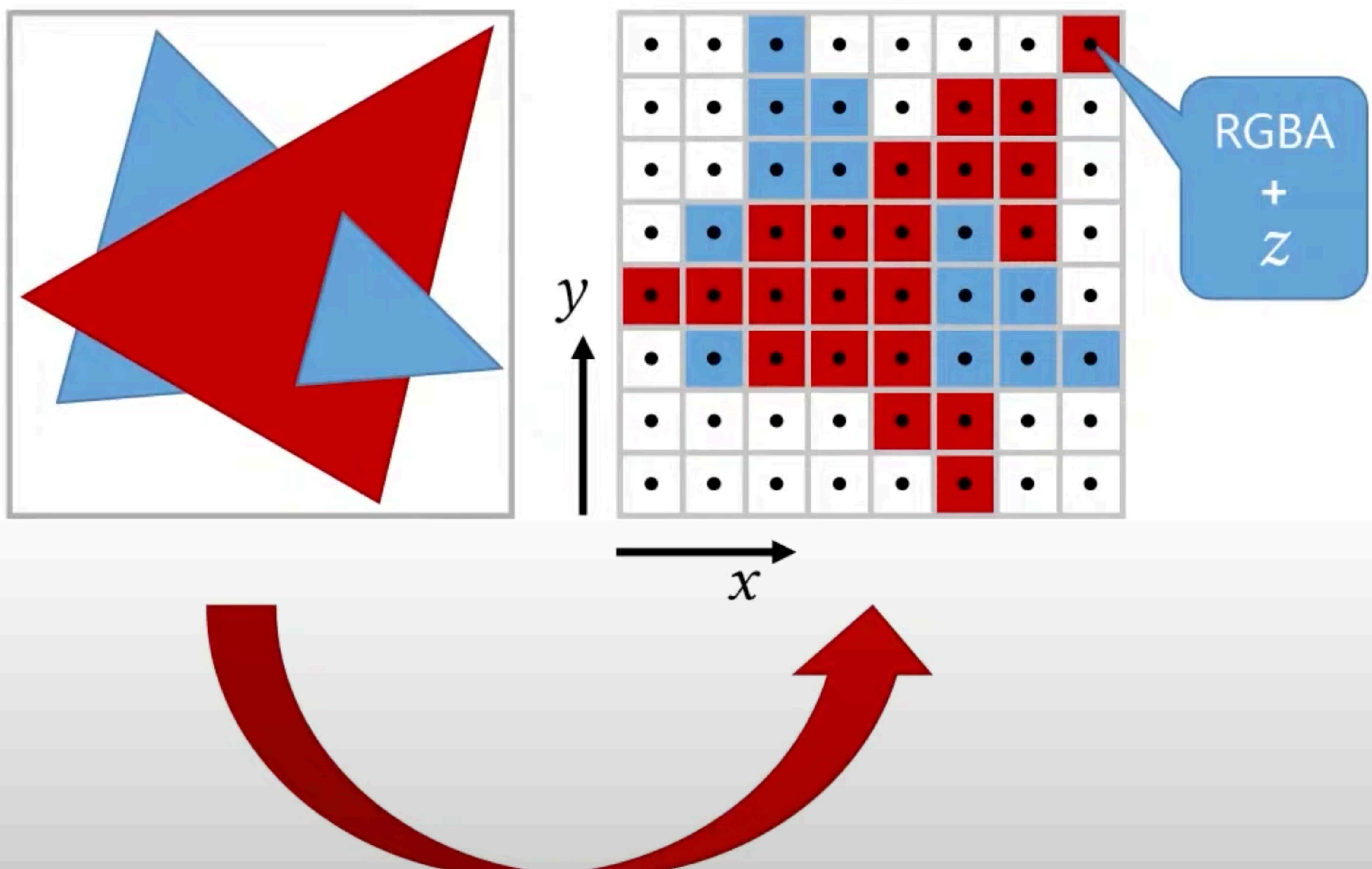
Z-Buffer Rasterization



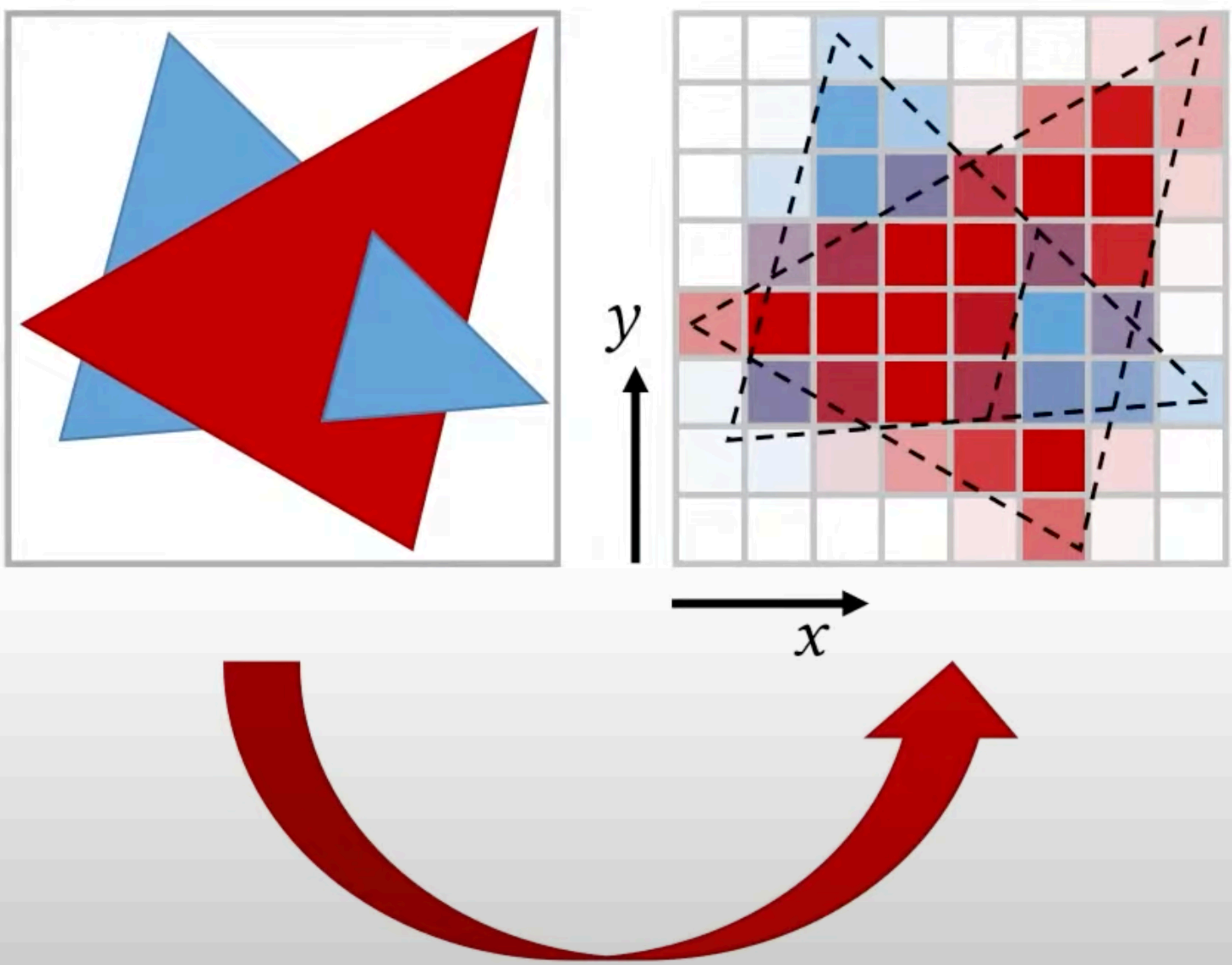
Z-Buffer Rasterization



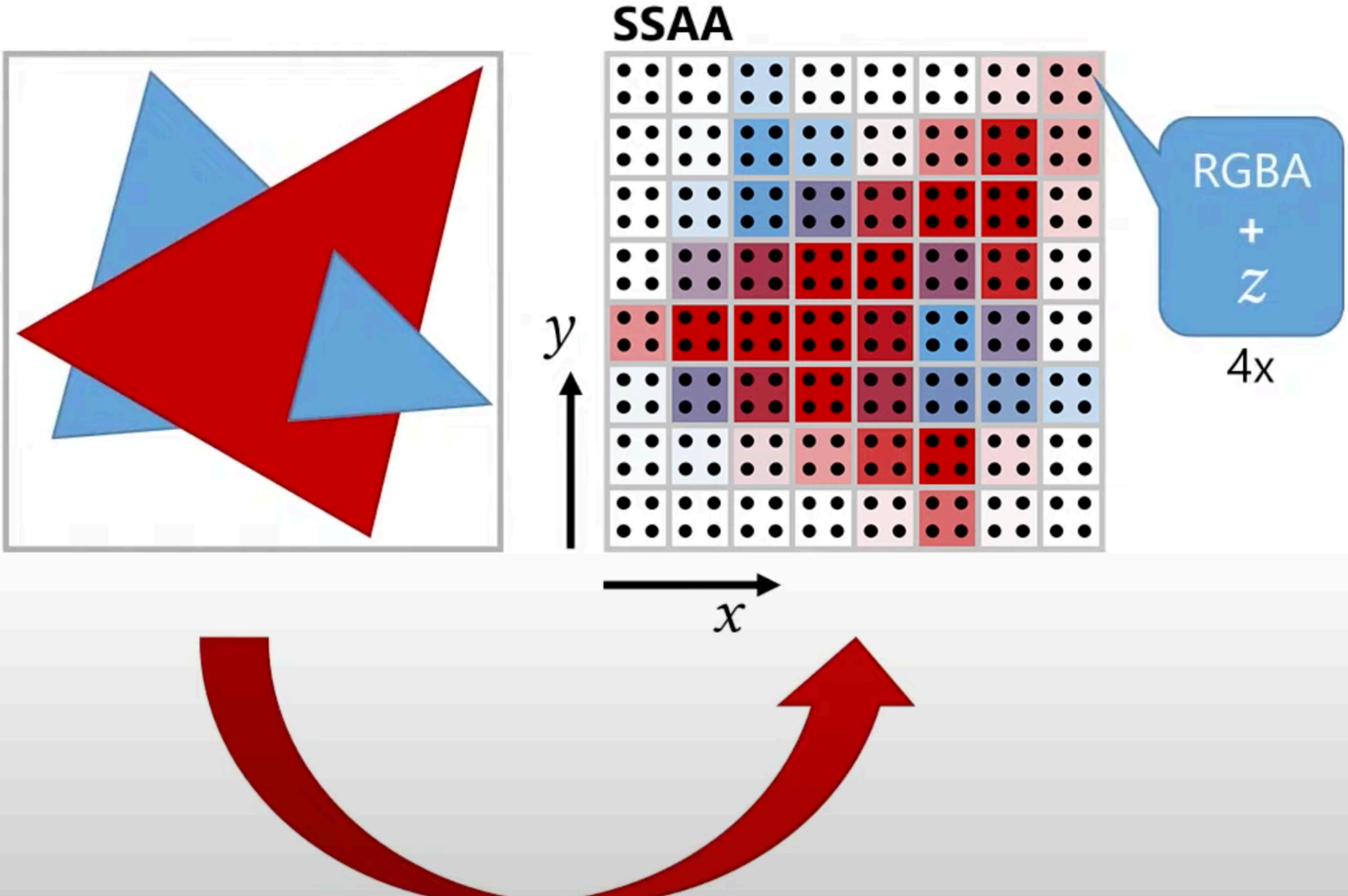
Z-Buffer Rasterization



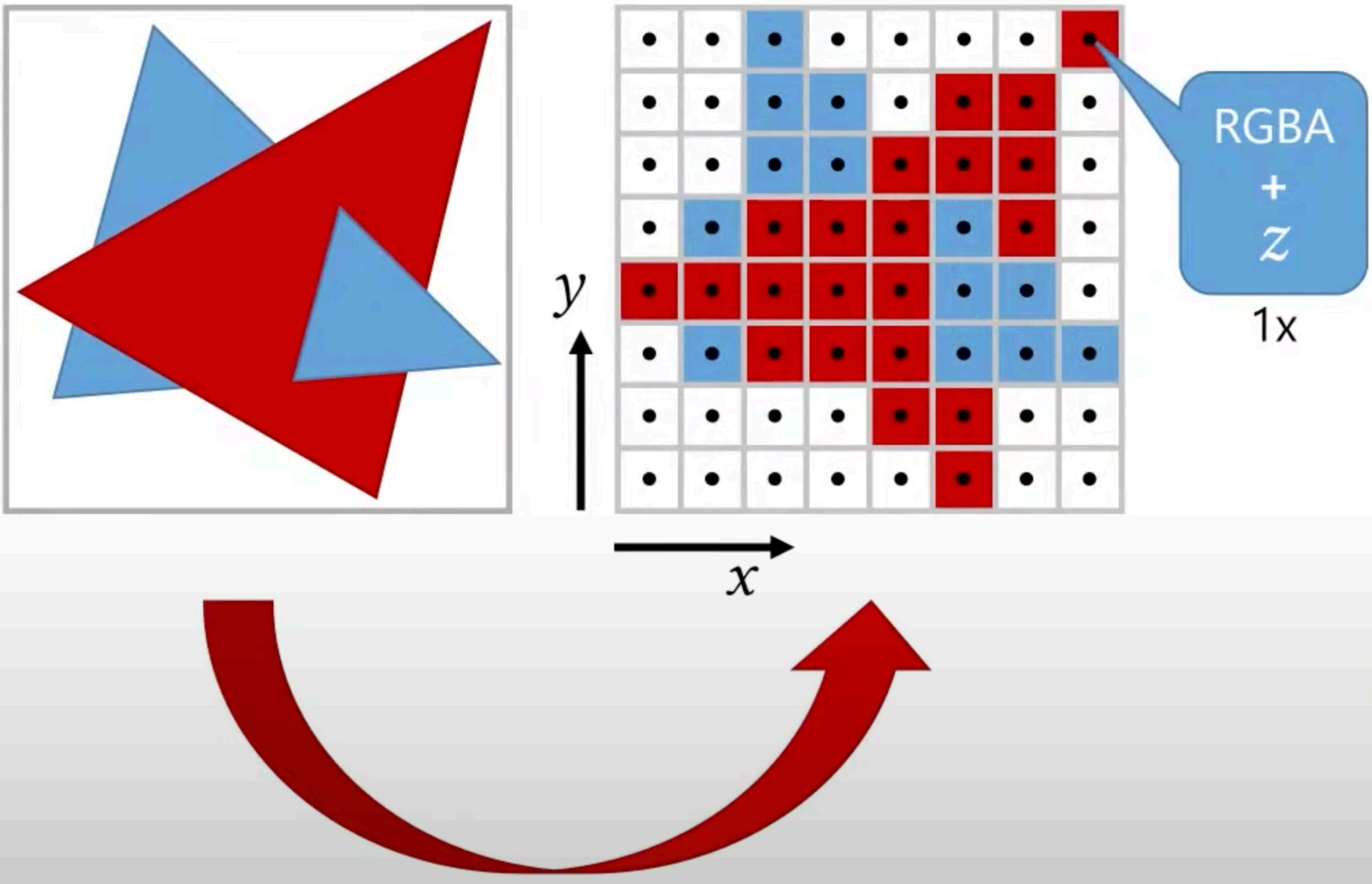
Z-Buffer Rasterization



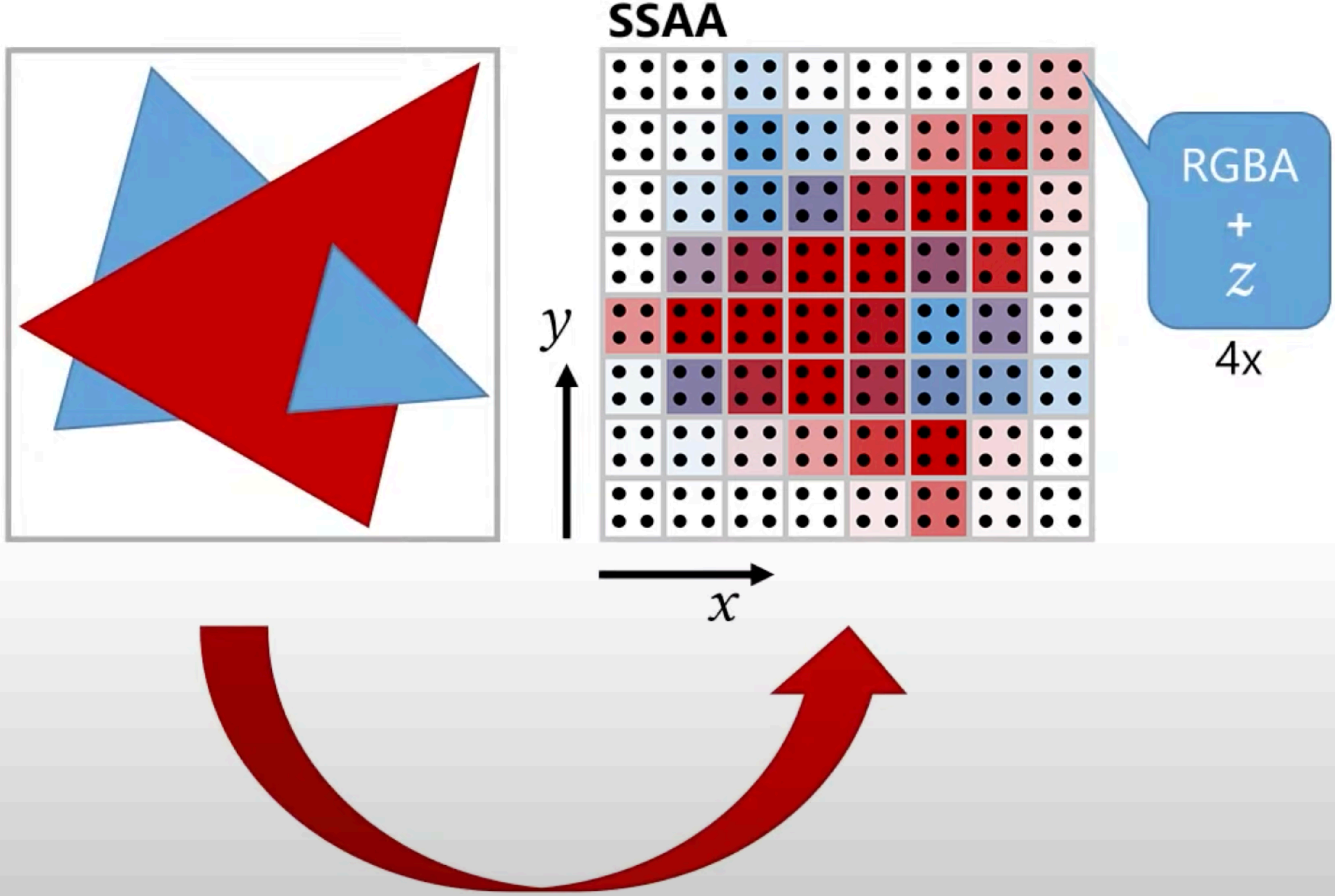
Z-Buffer Rasterization



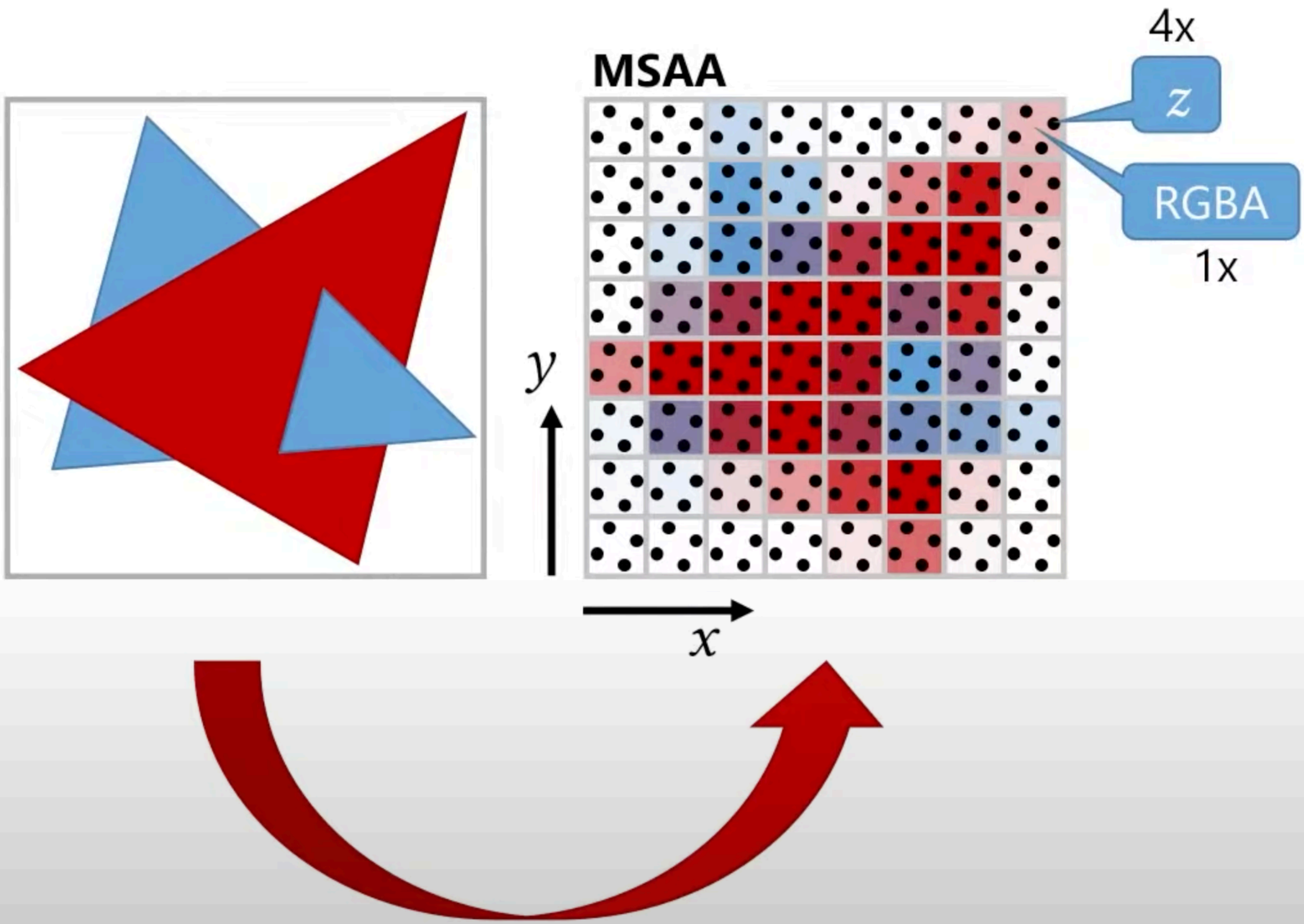
Z-Buffer Rasterization



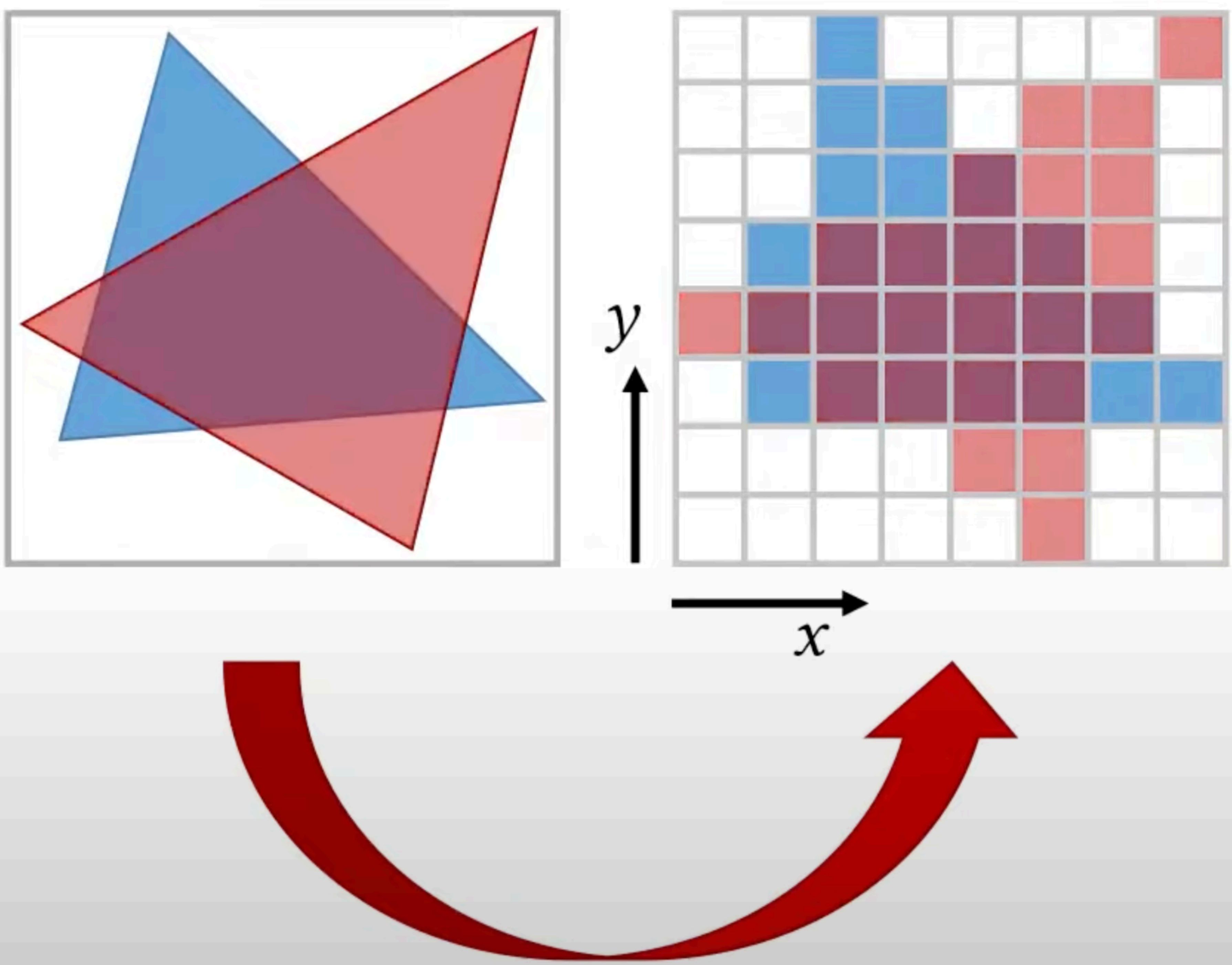
Z-Buffer Rasterization



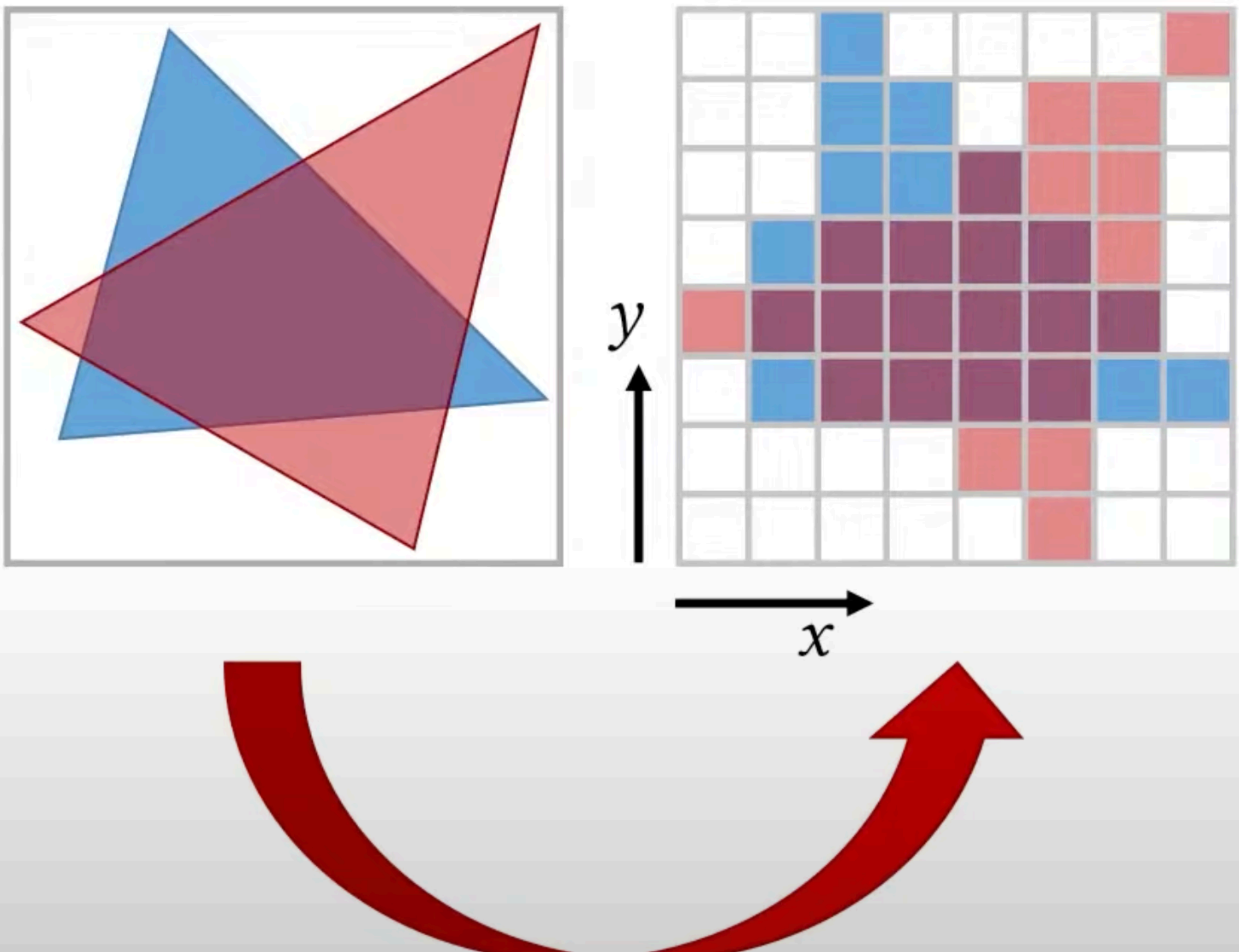
Z-Buffer Rasterization



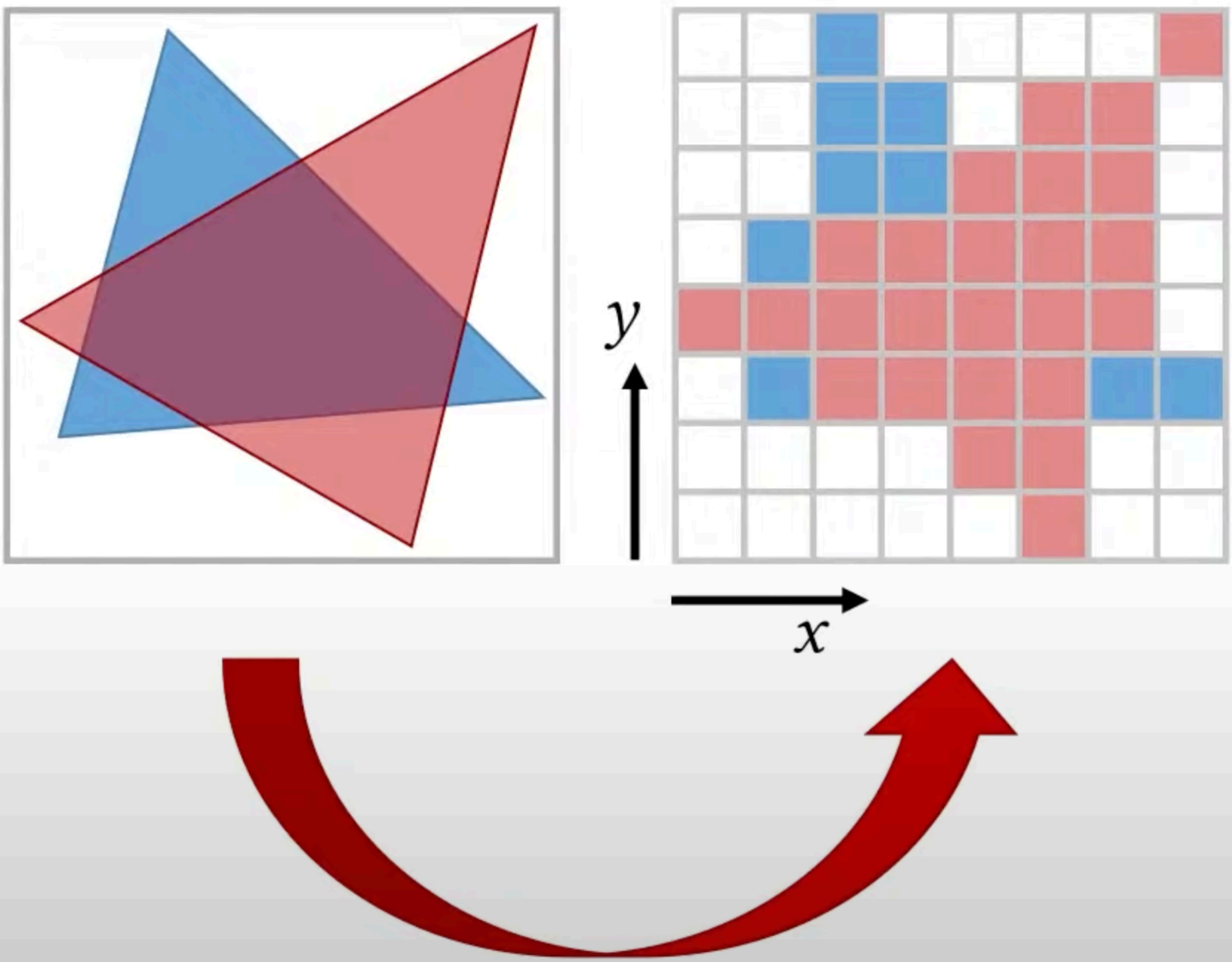
Z-Buffer Rasterization



Z-Buffer Rasterization



Z-Buffer Rasterization



Z-Buffer Rasterization

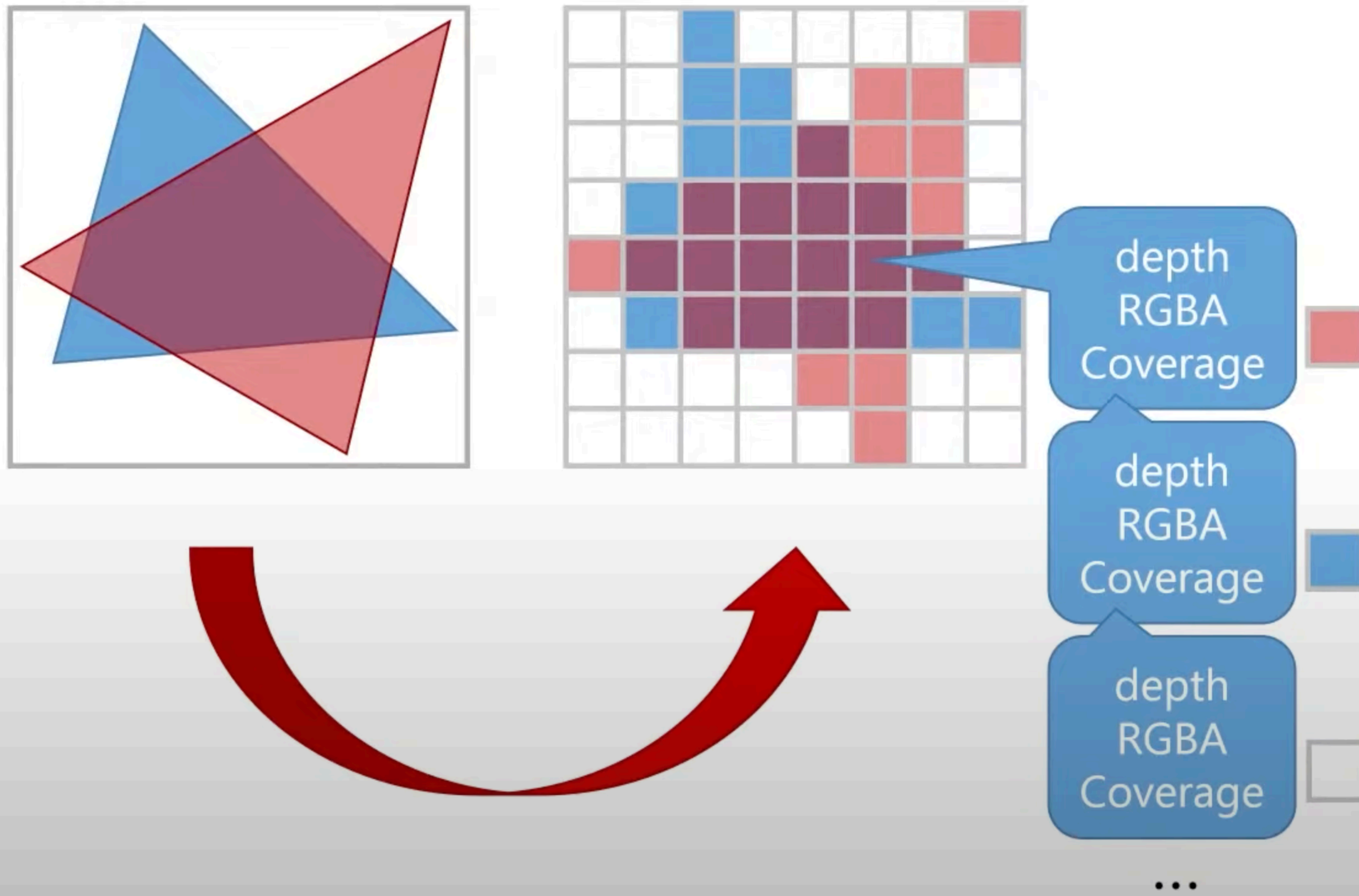
✓ Can handle intersecting geometry

× Needs sorting for transparency

A-Buffer Rasterization

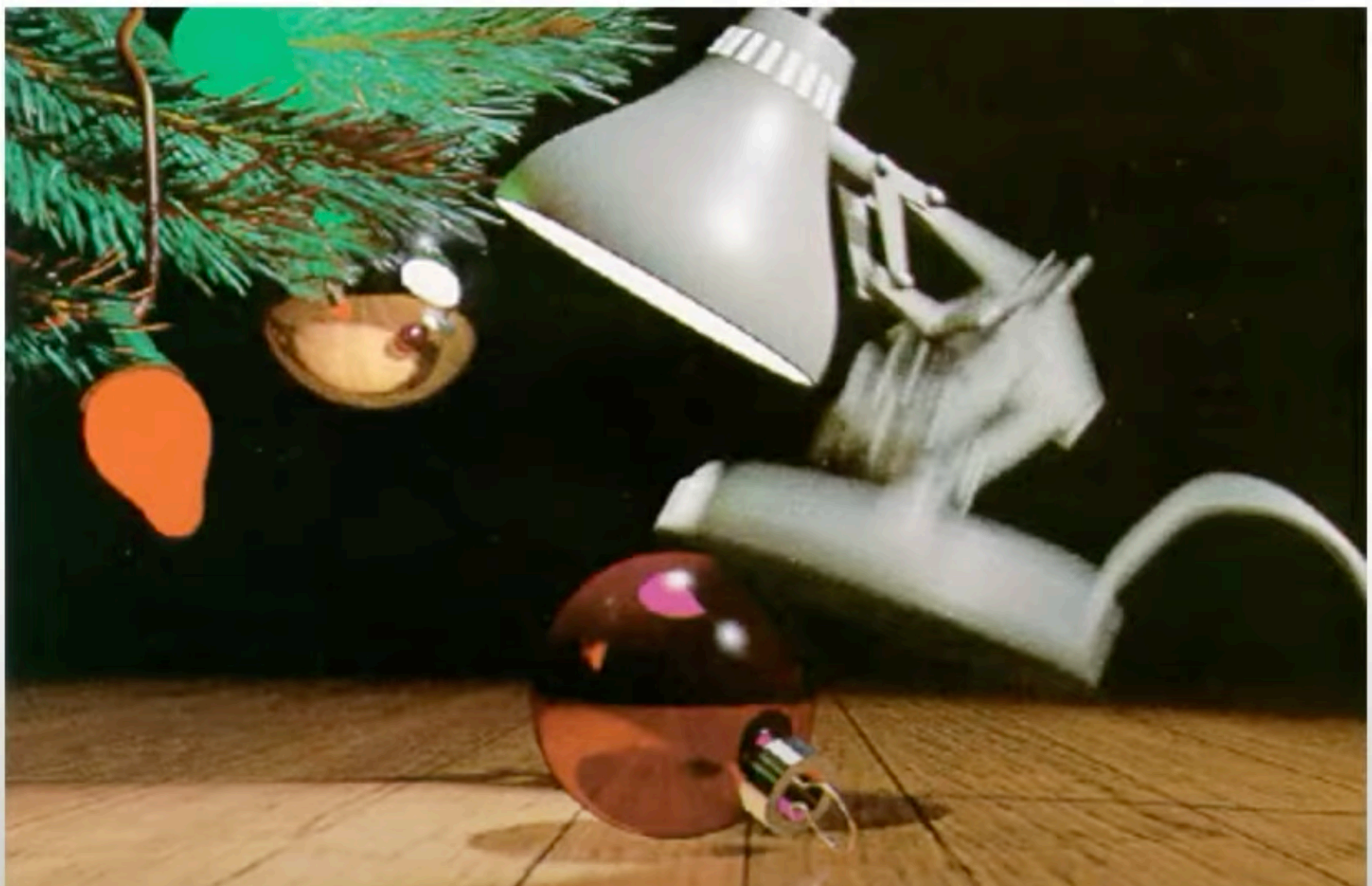
- ✓ Can handle intersecting geometry
- ✓ Supports order-independent transparency
- × Requires more (dynamic) memory

A-Buffer Rasterization



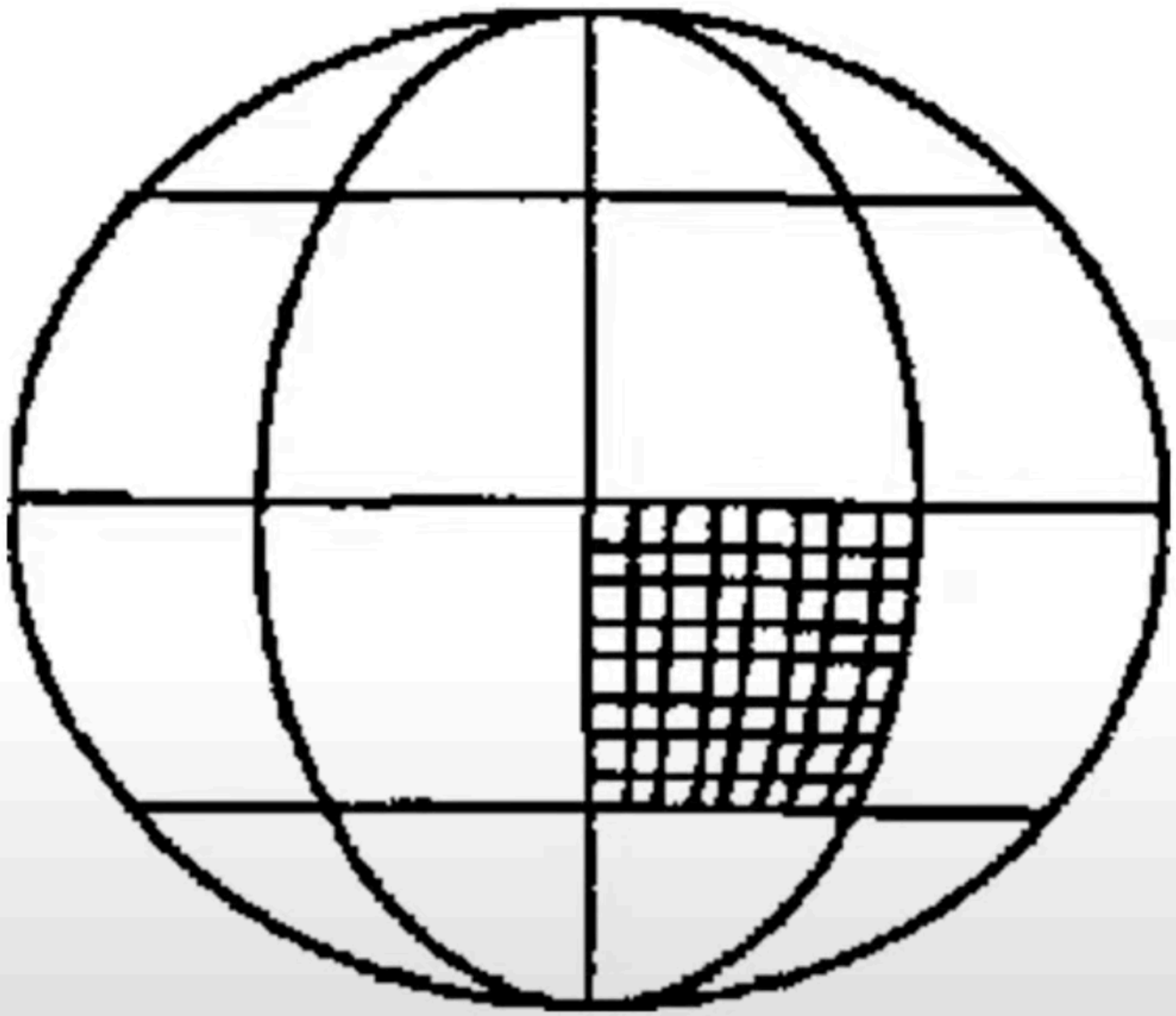
REYES

Renders **E**verything **Y**ou **E**ver **S**aw



1986 Pixar Christmas Card by John Lasseter and Eben Ostby

REYES



A sphere diced into micropolygons

REYES

The Lion King © Disney