The Effect of Redundant Degrees of Freedom on Manipulator's Kinematic Characteristics

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ABSTRACT
In order to verify the necessity and effectiveness of redundant degrees of freedom (DOF) of anthropomorphic manipulators, the effects of redundant DOFs on manipulator's kinematic characteristics are quantitatively studied. Firstly, the manipulator configurations with 6, 7 and 8 DOFs as well as optimal position and posture structure are introduced. Kinematic models of the three manipulators with the same maximum working radius, the same joint motion range are established. Workspace, Jacobian condition number and manipulability of the three manipulators are obtained respectively through numerical simulations, and their kinematic characteristics are analyzed from three aspects. Then, the distribution of relative manipulation capabilities in the workspaces and the optimal workspace of the three manipulators are obtained based on the concept of global relative manipulability. Through comparing and analysis, the effects of redundant DOFs on workspace, movement flexibility and manipulability of manipulators are obtained, which can provide theoretical basis for the configuration design of anthropomorphic manipulators.

CCS Concepts
• Applied computing→Computer-aided design

Keywords
Anthropomorphic manipulator; redundant degrees of freedom; kinematic characteristics; global relative manipulability; optimum workspace.

1. INTRODUCTION
With the transformation from the closed working environment to the semi-structured one where human and machine coexist, anthropomorphic manipulator has become a research focus in the robot field [1]. The redundant manipulators can avoid the interior singular configuration, obstacles and prevent joint overruns, so most of the anthropomorphic manipulators are 7 DOFs or 8 DOFs.

RCAE 2018, December 26–28, 2018, Beijing, China

ACM ISBN 978-1-4503-6102-6/18/12…$15.0
https://doi.org/10.1145/3303714.3303718

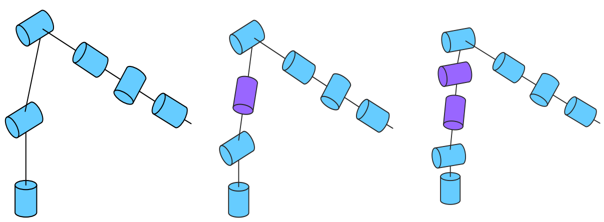
It's easily understood that the increasing of redundant DOFs would make the control difficult and the structure complex, so how to choose the suitable DOFs is very important for anthropomorphic manipulator design.

Anthropomorphic manipulator's configurations have been intensively investigated. Feng et al. [2] used an 8 DOFs manipulator to mimic human arm and optimized its structure with comfort index. Tondu [3] thought the scapula plays an important role in the shoulder movement and proposed a 9-DOF anthropomorphic manipulator by adding the clavicle joint on the basis of the shoulder joint. Referring to the structure and DOFs of human arm, Zacharias et al. [4] proposed two new 7-DOF manipulator configurations and analyzed their flexibility. Kuhlemann [5] studied the impact of the 7th DOF on manipulator's flexibility with LRB iiwa 7 and KR10 R900 robot, but ignoring the influence of the dimensions and joint motion ranges. Zhou [6] proposed the selection criteria of 7 DOFs and 8 DOFs manipulators with the consideration of the flexibility, singularity, obstacle avoidance and complexity of inverse solution. Combining the movement mechanism of the human arm, Zhao [7] gave the optimal configuration of 7 DOFs manipulator according to the analysis of six different manipulator configurations.

At present, there are few studies on the selection of joint numbers of anthropomorphic manipulators, which causes a big trouble in the design of anthropomorphic manipulators. Focusing on the above problem, this paper establishes the kinematic models of three manipulators with the same maximum working radius, the same joint motion range but different redundant DOFs, then analyzes the effects of the redundant DOFs on the kinematic characteristics of three manipulators, the results could provide a theoretical basis for the configuration design and structure design of anthropomorphic manipulators.

2. KINEMATICS MODELS OF THREE MANIPULATORS

2.1 Configurations of Three Manipulators
By combing the 3R optimal position structure and the 3R optimal posture structure, a 6-DOF manipulator configuration with the optimal flexible workspace can be got, shown in Fig.1(a). Based on the 6-DOF manipulator configuration, adding kinematic pairs can obtain redundant manipulators' configuration. The 7-DOF manipulator configuration is got from the 6-DOF manipulator by adding a rotary joint after the second joint, shown in Fig. 1(b). The 8-DOF manipulator shown in Fig. 1(c) is also based on the 6-DOF robot by adding a rotary pair and a pitch pair after the second joint sequentially. The redundant DOFs of these three types of arms is 0, 1, and 2 respectively.



(a) 6-DOF manipulator (b) 7-DOF manipulator (c) 8-DOF manipulator

Figure 1. Configurations of three manipulators

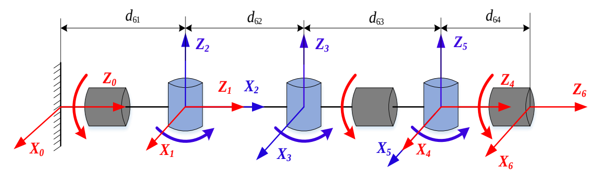
2.2 Kinematic Models of Three Manipulators

According to the three manipulator configurations, their kinematics models are established with D-H method. The established D-H coordinate frames are shown in Fig.2 (a), Fig.2 (b) and Fig.2 (c). The D-H parameters of the three manipulators are shown in table 1.

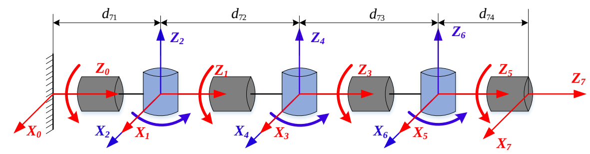
The forward kinematic model of the three manipulators can be expressed in the form of

$$ {}^{n}_{0}T = {}^{1}_{0}T\,{}^{2}_{1}T\,{}^{3}_{2}T \cdots {}^{n-1}_{n-2}T\,{}^{n}_{n-1}T, \quad n = 6, 7, 8 $$

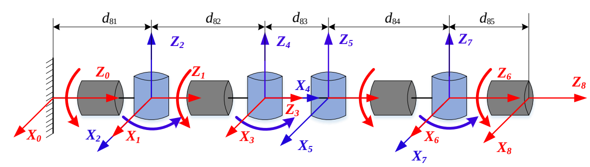
Where, ${}^{i}_{i\text{-}1}T$ is the homogeneous transform matrix from frame $\{i\}$ to frame $\{i\text{-}1\}$.



(a) D-H model of 6 DOFs manipulator



(b) D-H model of 7 DOFs manipulator



(c) D-H model of 8 DOFs manipulator

Figure 2. D-H frames of three manipulators

Table 1. D-H parameters of three manipulators

i	6-DOF manipulator				7-DOF manipulator				8-DOF manipulator			
	a_{i-1}	α_{i-1}	d_i	θ_i	a_{i-1}	α_{i-1}	d_i	θ_i	a_{i-1}	α_{i-1}	d_i	θ_i
1	0	0	d_{61}	θ_1	0	0	d_{71}	θ_1	0	0	d_{81}	θ_1
2	0	90	0	θ_2	0	90	0	θ_2	0	90	0	θ_2
3	d_{62}	0	0	θ_3	0	-90	d_{72}	θ_3	0	-90	d_{82}	θ_3
4	0	-90	d_{63}	θ_4	0	90	0	θ_4	0	90	0	θ_4
5	0	90	0	θ_5	0	-90	d_{73}	θ_5	d_{83}	0	0	θ_5
6	0	-90	d_{64}	θ_6	0	90	0	θ_6	0	-90	d_{84}	θ_6
7					0	-90	d_{74}	θ_7	0	90	0	θ_7
8									0	-90	d_{85}	θ_8

3. THE KINEMATIC CHARACTERISTICS SIMULATION ANALYSES OF THREE MA NIPULATORS

3.1 Movements Performance Indices

Flexibility and manipulability are very important indices to reflect the motion dexterity of manipulators. In this paper, the Jacobian matrix condition number, manipulability [8] and global relative manipulability are selected as the evaluation indices to evaluate three manipulator's kinematic performances.

3.2 Manipulators' Dimensions and Motion Parameters Setting

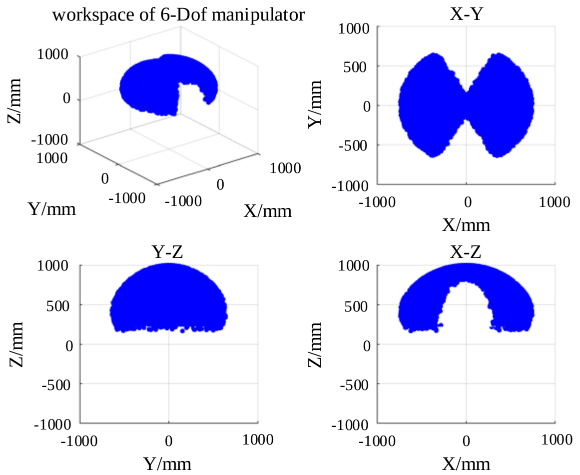
In order to get the effect law of redundant DOFs on the kinematic performance of manipulators, the three manipulators are set to have the same length, as shown in table 2. In addition, in order to eliminate the effect of the joint motion range on the motion characteristics of each manipulator, suppose the motion range of all the joints of three manipulators is [-60 °, 60 °]. It means the rest parameters of the three manipulators are consistent except for the DOFs. Based on the Monte Carlo method [9], this paper takes 200,000 joint angles to simulate the motion characteristics of three manipulators.

Table 2. Dimensions of three manipulators

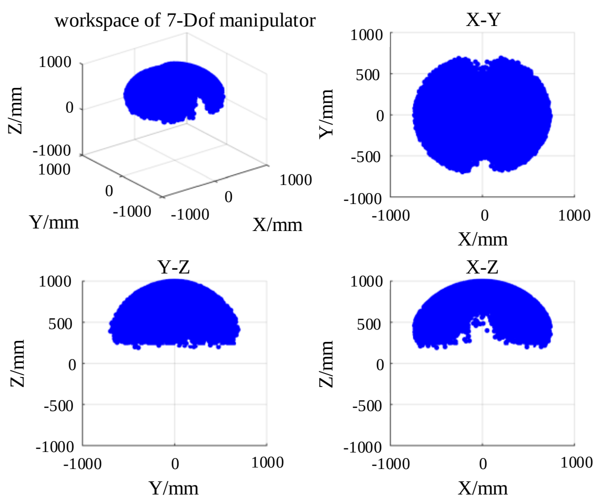
DOFs	Boom Size/mm		Forearm Size/mm	
	L_B=600		L_S=400	
6-DOF	d_{61}=200	d_{62}=400	d_{63}=300	d_{64}=100
7-DOF	d_{71}=200	d_{72}=400	d_{73}=300	d_{74}=100
8-DOF	d_{81}=200	d_{82}=300, d_{83}=100	d_{84}=300	d_{85}=100

3.3 Comparisons and Analyses of Workspaces

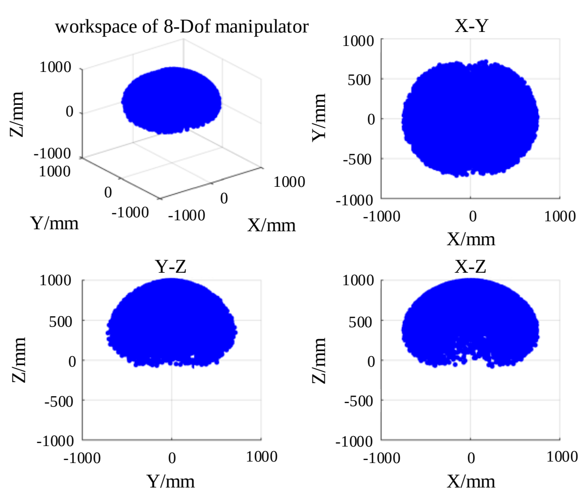
Through the simulation calculations of three manipulators, the workspaces corresponding to 6-DOF, 7-DOF and 8-DOF manipulators are got, as shown in Fig.3(a), Fig.3(b), Fig.3(c).



(a) workspace of 6-DOF manipulator



(b) workspace of 7-DOF manipulator



(c) workspace of 8-DOF manipulator

Figure 3. Workspaces of three manipulators

Comparative analyses of the workspaces of three manipulators are shown in table 3. According to the simulation results, compared to 6-DOF manipulator, the 7-DOF manipulator keeps the same reachable range in X and Z directions but gains an increase of 84.1 mm (6.46%) in Y direction. The 8-DOF manipulator gains increases by 21.5 mm (1.45%) in X direction, 131.6 mm (10.10%) in Y direction and 240.0 mm (28.46%) in Z direction. Therefore, when the arm length and the joint motion range are the same, increasing the redundant DOFs can increase the workspace of manipulators with different values in different directions.

Table 3 Comparisons and analyses of workspaces

F	X/[mm]		Y/[mm]		Z/[mm]	
	min	max	min	max	min	max
6-DOF	-742.8	744.7	-651.4	651.4	156.6	1000.0
7-DOF	-743.7	744.2	-692.3	694.6	157.9	1000.0
8-DOF	-753.9	755.1	-716.8	717.6	-83.4	1000.0

3.4 Comparisons and Analyses of the Flexibility and Manipulability

The condition numbers and the manipulability [10] values of three manipulators in their workspaces are calculated, as shown in table 4, where K_{min}, K_{max} and K_{ave} represent the minimum, the maximum and the average values of condition number in the workspace of a manipulator respectively, and w_{min}, w_{max} and w_{ave} indicate the minimum, the maximum and the average values of the manipulability in the workspace.

Table 4 shows that as the number of redundant DOFs increases, the condition number in its workspace of a manipulator decreases while the manipulability value rises. Compared with the 6-DOF manipulator, the average condition numbers in the workspace of the 7-DOF manipulator and the 8-DOF manipulator decrease by 37.41% and 94.21% respectively, and the average manipulability increased by 78.65% and 215.73% respectively. Consequently, the increase of redundant DOFs number can significantly improve the flexibility and manipulability.

Table.4 Condition numbers and manipulability values in the workspaces of three manipulators

F	Condition number K			Manipulability w		
	K_{min}	K_{max}	K_{ave}	w_{min}	w_{max}	w_{ave}
6-DOF	3.64	2.9e6	155.56	0	0.0538	0.0089
7-DOF	3.31	2.3e4	97.37	0	0.0828	0.0159
8-DOF	2.23	1.5e3	9.02	0	0.1221	0.0281

3.5 Comparisons and Analyses of Global Relative Manipulability

The global relative manipulability can indicate the relative size of the manipulability at each point in the workspace of a manipulator. The distribution of manipulability values in the workspace is got by counting and arranging the global relative manipulability values. The range of the global relative manipulability is dived into five sections: [0，0.2)，[0.2，0.4)，[0.4，0.6)，[0.6，0.8)，[0.8，1.0], in which the distribution curves of global relative manipulability values are shown in Fig.4.

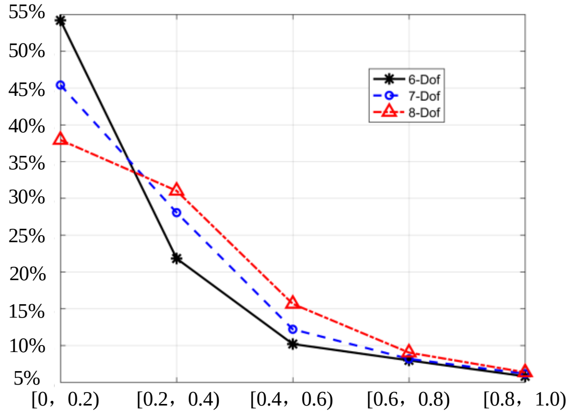


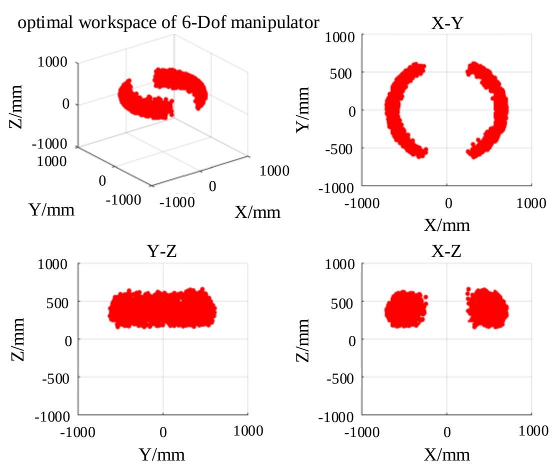
Figure 4. Distribution curves of global relative manipulability

Table 5 shows the frequency distribution of the global relative manipulability of the three manipulators in each interval, which shows that as the redundant DOFs increase, the distribution ratio of the global relative manipulability decreases in the [0,0.2) section, but increases in [0.8,1.0] section. Hence the increase of redundant DOFs can improve the distribution of manipulability in the workspace of a manipulator.

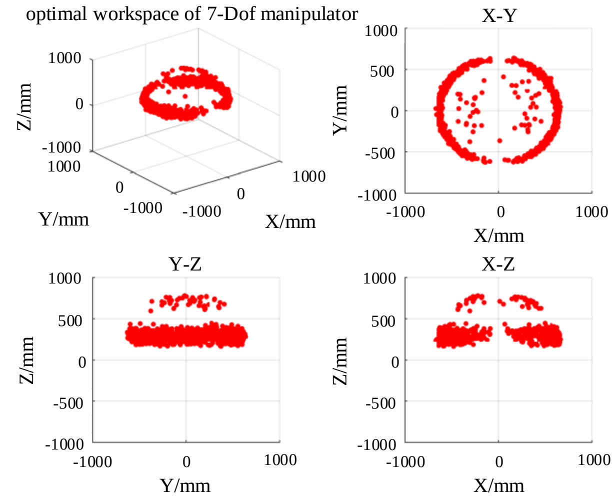
Table.5 The distribution frequency of global relative manipulability

	The frequency distribution of global relative manipulability (%)				
	[0,0.2)	[0.2,0.4)	[0.4,0.6)	[0.6,0.8)	[0.8,1]
6-DOF	54.2	21.8	10.2	8.0	5.8
7-DOF	45.4	28.1	12.1	8.2	6.2
8-DOF	37.9	31.0	15.7	9.0	6.4

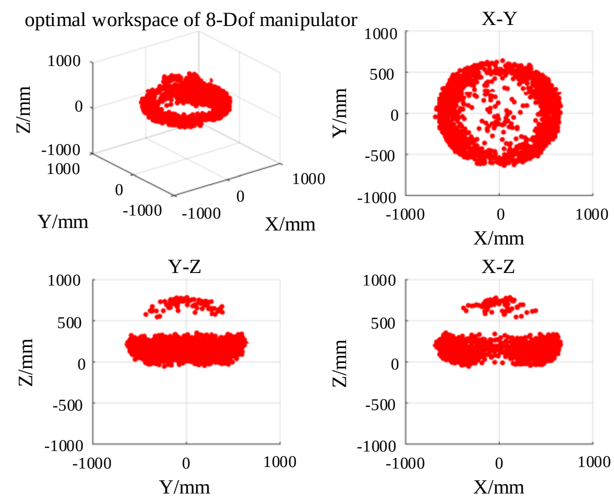
Fig.5(a), Fig.5(b) and Fig.5(c) show the optimal workspaces of the three manipulators by drawing the points corresponding to ui ⩾ 0.8. It can be seen that the distribution of the optimal workspace of a manipulator changes with the increase of redundant DOFs numbers.



(a) Optimal workspace of 6-DOF manipulator



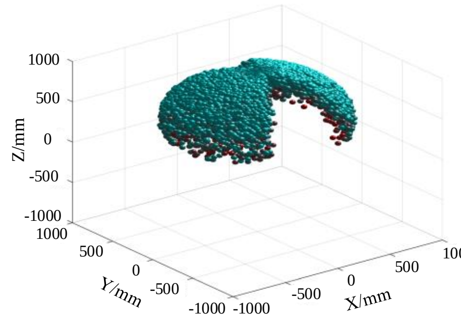
(b) Optimal workspace of 7-DOF manipulator



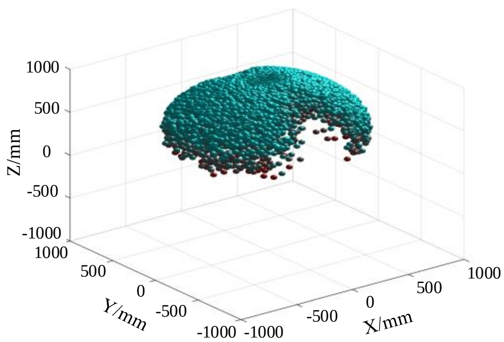
(c) Optimal workspace of 8-DOF manipulator

Figure 5. The optimal workspaces of three manipulators

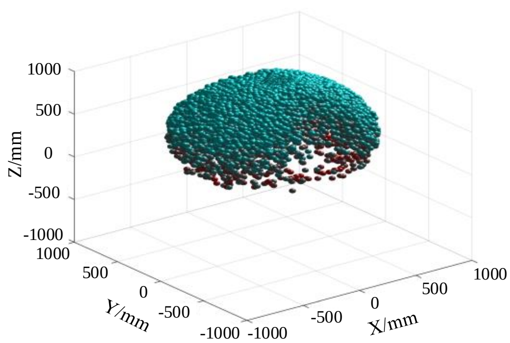
Fig.6(a), Fig.6(b) and Fig.6(c) show the differences of global relative manipulability of each point in the workspace corresponding to three manipulators. The color in the figure indicates the big or small of the global relative manipulability, and the light color means smaller global relative manipulability, while the deep color means bigger global relative manipulability. With the increase of redundant DOFs numbers, the distribution of the flexible workspace of the manipulators changes as the workspace becomes bigger. The redundant DOFs can improve the kinematics characteristics of a manipulator, whereas it reduces the stiffness and motion accuracy and increases the control difficulty so as to multiply the research and development cost. Therefore, the redundant DOFs number should be determined reasonably according to the task workspace, so the suitable manipulator's configuration and joint allocation can be selected.



(a) Distribution of flexible workspace of 6-DOF manipulator



(b) Distribution of flexible workspace of 7-DOF manipulator



(c) Distribution of flexible workspace of 8-DOF manipulator

Figure 6. Distribution of flexible workspace of three manipulators

4. CONCLUSIONS

In this paper, the configurations of three anthropomorphic manipulators with the optimal position structure and posture structure are given, and their kinematic models are established by D-H method. The three manipulators have the same maximum working radius and the same motion range of each joint. The effects of redundant DOFs on the motion characteristics of the three manipulators are analyzed quantitatively. Such following conclusions can be got:

(1) The workspace of a manipulator can be enlarged with the increase of redundant DOFs, but the increase degrees are different in different directions.

(2) With the increase of the number of redundant DOFs, the Jacobian matrix condition number decreases, while the manipulability and the flexibility increase and the distribution of the global relative manipulability is modified.

(3) The optimal workspaces of 6-DOF, 7-DOF and 8-DOF manipulators are got. If the numbers of redundant DOFs are different, the optimal workspaces and the workspace distributions corresponding to the manipulators are different too.

The paper can provide the theoretical basis for the determining of the DOFs numbers and the configuration of anthropomorphic manipulators.

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