12.389

EE25BTECH11064 - Yojit Manral

Question:

$$x + 2y + z = 4$$
$$2x + y + 2z = 5$$
$$x - y + z = 1$$

The system of algebraic equations given above has

(a) a unique solution of $(x = 1, y = 1, z = 1)$.
(b) only the two solutions of $(x = 1, y = 1, z = 1)$ and $(x = 2, y = 1, z = 0)$.
(c) infinite number of solutions.
(d) no feasible solution.

Solution:

→ The above equations can be written in matrix form as

$$\begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \\ 1 \end{pmatrix} \tag{1}$$

$$\mathbf{A} \quad \mathbf{x} = \mathbf{B} \tag{2}$$

→ Using the augmented matrix $(\mathbf{A}|\mathbf{B})$

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 2 & 1 & 2 & 5 \\ 1 & -1 & 1 & 1 \end{array}\right) \xrightarrow[R_3 \leftrightarrow R_3 - R_1]{R_2 \leftrightarrow R_2 - 2R_3} \left(\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & 3 & 0 & 3 \\ 0 & -3 & 0 & -3 \end{array}\right) \xrightarrow[R_1 \leftrightarrow R_1 - (2/3)R_2]{R_3 \leftrightarrow R_3 + R_2} \left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 3 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right) \tag{3}$$

$$\implies x + z = 2 \text{ and } y = 1 \tag{4}$$

→ So, the system of equations has infinite solutions given by the intersection of the planes $x + z = 2$ and $y = 1$.
→ Therefore, *(c) infinite number of solutions.* is the correct option.

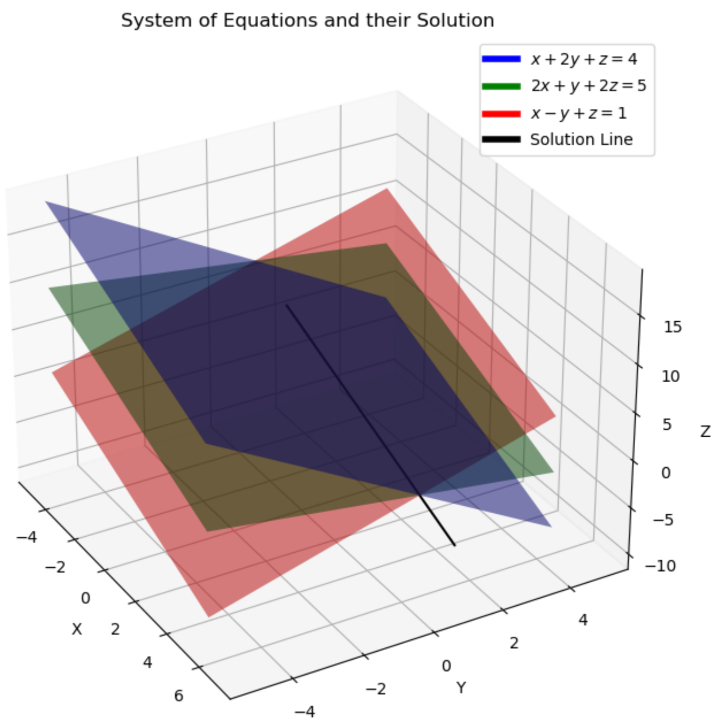

Fig. 4: Plot of system of equations and solution line