4.13.68

EE25BTECH11064 - Yojit Manral

Question:
Prove or disprove: The straight line $5x + 4y = 0$ passes through the point of intersection of the straight lines $x + 2y - 10 = 0$ and $2x + y + 5 = 0$.

Solution:
→ Let

$$\mathbf{n_1}^T\mathbf{x} = c_1 \qquad \mathbf{n_2}^T\mathbf{x} = c_2 \qquad \mathbf{n_3}^T\mathbf{x} = c_3$$

where

$$\mathbf{n_1} = \begin{pmatrix}5\\4\end{pmatrix} \qquad c_1 = 0 \tag{1}$$

$$\mathbf{n_2} = \begin{pmatrix}1\\2\end{pmatrix} \qquad c_2 = 10 \tag{2}$$

$$\mathbf{n_3} = \begin{pmatrix}2\\1\end{pmatrix} \qquad c_3 = -5 \tag{3}$$

→ The three lines are concurrent if there exists some $\mathbf{x}$ such that

$$\begin{pmatrix}\mathbf{n_1}^T\\\mathbf{n_2}^T\\\mathbf{n_3}^T\end{pmatrix}\mathbf{x} = \begin{pmatrix}c_1\\c_2\\c_3\end{pmatrix} \tag{4}$$

$$\implies \begin{pmatrix}\mathbf{n_1} & \mathbf{n_2} & \mathbf{n_3}\end{pmatrix}^T\mathbf{x} = \begin{pmatrix}c_1\\c_2\\c_3\end{pmatrix} \tag{5}$$

$$\implies \mathbf{Ax} = \mathbf{B} \tag{6}$$

→ A unique $\mathbf{x}$ exists iff the matrix $\mathbf{A}$ is a full-rank matrix (i.e., rank 2 for a system of 2 variables).

$$\mathbf{A} = \begin{pmatrix}5 & 4\\1 & 2\\2 & 1\end{pmatrix} \xrightarrow{R_2 \leftrightarrow R_2-(1/5)R_1} \begin{pmatrix}5 & 4\\0 & 6/5\\2 & 1\end{pmatrix} \tag{7}$$

$$\xrightarrow{R_3 \leftrightarrow R_3-(2/5)R_1} \begin{pmatrix}5 & 4\\0 & 6/5\\0 & -3/5\end{pmatrix} \tag{8}$$

$$\xrightarrow{R_3 \leftrightarrow R_3+(1/2)R_1} \begin{pmatrix}5 & 4\\0 & 6/5\\0 & 0\end{pmatrix} \tag{9}$$

$$\begin{aligned}\text{Only 2 nonzero rows in the matrix in (9)} &\longrightarrow \text{Rank of the matrix } \mathbf{A} \text{ is } 2\\ &\longrightarrow \mathbf{A} \text{ is a full rank matrix}\\ &\longrightarrow \text{A unique } \mathbf{x} \text{ exists which satisfies all equations}\\ &\longrightarrow \text{The three lines are concurrent}\end{aligned}$$

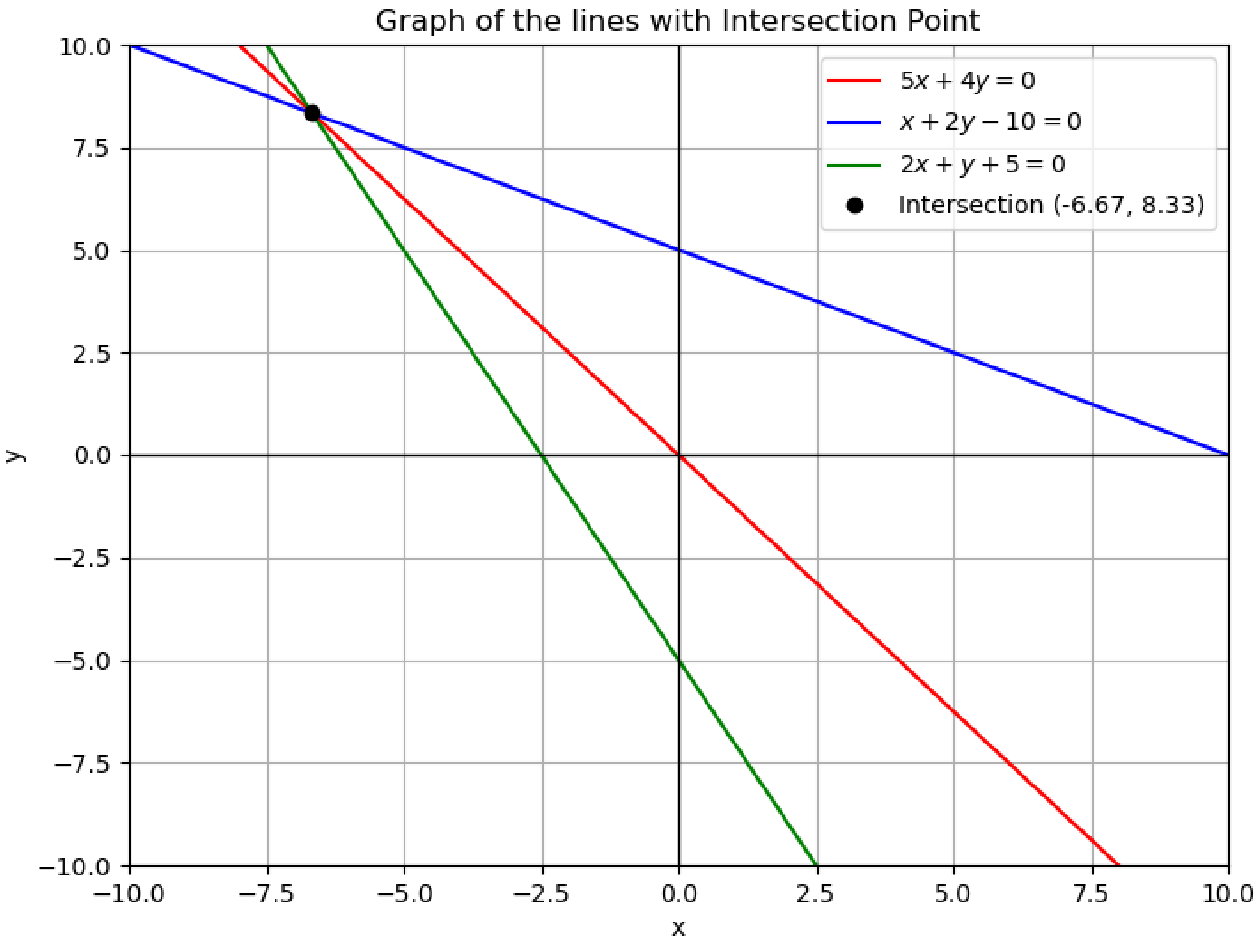

Fig. 0: Plot of the three lines