4.13.68

EE25BTECH11064 - Yojit Manral

Question:
Prove or disprove: The straight line $5x+4y=0$ passes through the point of intersection of the straight lines $x+2y-10=0$ and $2x+y+5=0$.
Solution:
$\rightarrow$ Let

$$\mathbf{n_1}^T\mathbf{x} = c_1 \qquad \mathbf{n_2}^T\mathbf{x} = c_2 \qquad \mathbf{n_3}^T\mathbf{x} = c_3$$

where

$$\mathbf{n_1} = \begin{pmatrix} 5 \\ 4 \end{pmatrix} \qquad c_1 = 0 \tag{1}$$

$$\mathbf{n_2} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \qquad c_2 = 10 \tag{2}$$

$$\mathbf{n_3} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \qquad c_3 = -5 \tag{3}$$

$\rightarrow$ The three lines are concurrent if there exists some $\mathbf{x}$ such that

$$\begin{pmatrix} \mathbf{n_1}^T \\ \mathbf{n_2}^T \\ \mathbf{n_3}^T \end{pmatrix}\mathbf{x} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} \tag{4}$$

$$\mathbf{A} \quad \mathbf{x} = \quad \mathbf{B} \tag{5}$$

$\rightarrow$ A unique $\mathbf{x}$ exists iff the matrix $\mathbf{A}$ is a full-rank matrix (i.e., rank 2 for a system of 2 variables).

$$\mathbf{A} = \begin{pmatrix} 5 & 4 \\ 1 & 2 \\ 2 & 1 \end{pmatrix} \xrightarrow[R_2 \leftrightarrow R_2 - R_1]{R_1 \leftrightarrow (1/5)R_1} \begin{pmatrix} 1 & 4/5 \\ 0 & 6/5 \\ 2 & 1 \end{pmatrix} \tag{6}$$

$$\xrightarrow[R_2 \leftrightarrow (5/6)R_2]{R_3 \leftrightarrow R_3 - 2R_1} \begin{pmatrix} 1 & 4/5 \\ 0 & 1 \\ 0 & -3/5 \end{pmatrix} \tag{7}$$

$$\xrightarrow[R_1 \leftrightarrow R_1 - (4/5)R_2]{R_3 \leftrightarrow R_3 + (3/5)R_2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \tag{8}$$

$$\begin{aligned} \text{Only 2 nonzero rows in the matrix in (8)} &\longrightarrow \text{Rank of the matrix } \mathbf{A} \text{ is 2} \\ &\longrightarrow \mathbf{A} \text{ is a full rank matrix} \\ &\longrightarrow \text{A unique } \mathbf{x} \text{ exists which satisfies all equations} \\ &\longrightarrow \text{The three lines are concurrent} \end{aligned}$$

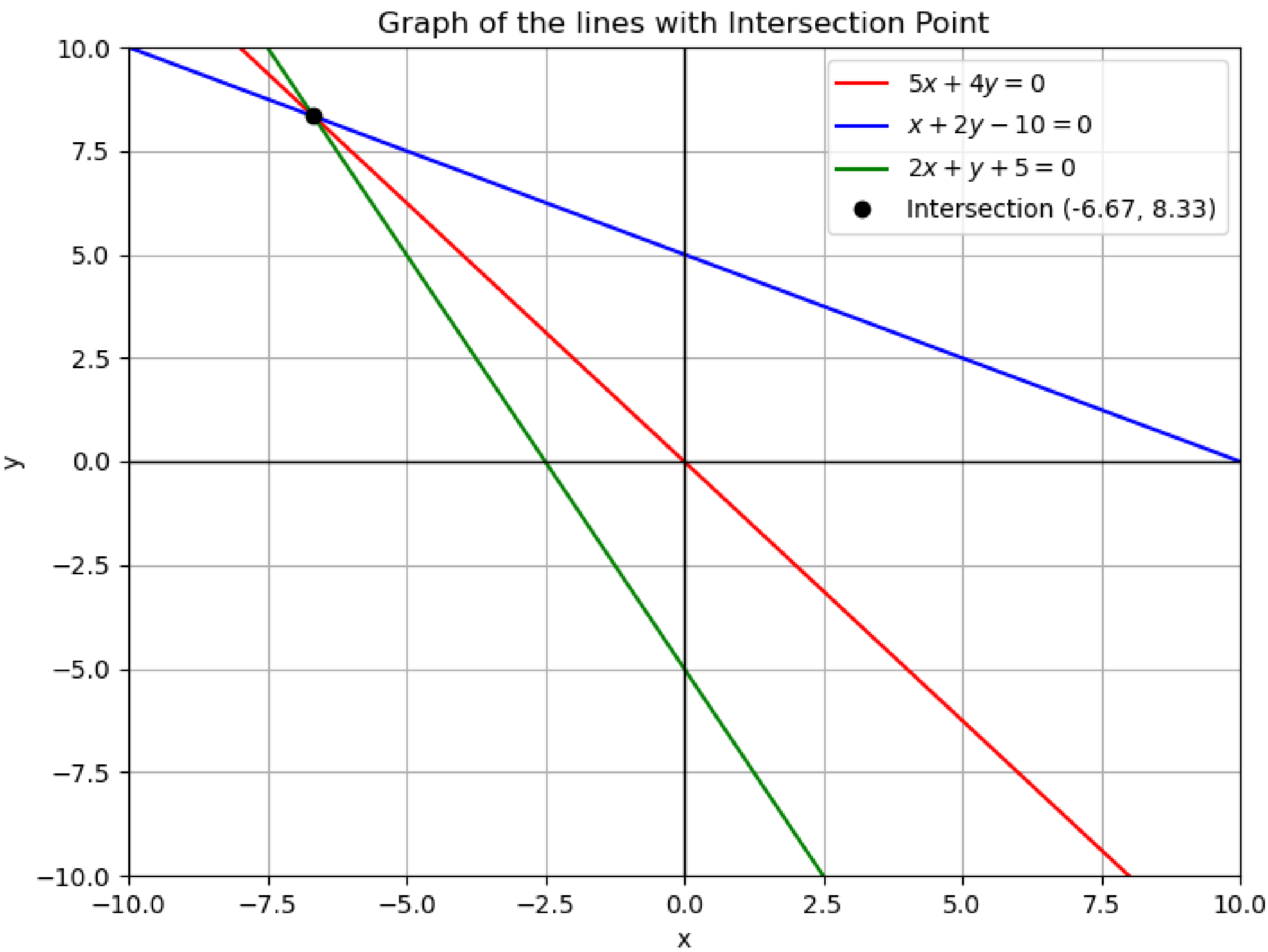

Fig. 0: Plot of the three lines