MatGeo Presentation - Problem 5.10.2

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Question

Balance the following chemical equation:

$$NaOH + H_2SO_4 \rightarrow Na_2SO_4 + H_2O \tag{0.1}$$

Solution

→ Let the balanced version of (1) be

$$x_1 NaOH + x_2 H_2SO_4 \rightarrow x_3 Na_2SO_4 + x_4 H_2O \tag{0.2}$$

→ This results in the following equations

$$(x_1 - 2x_3)Na = 0 \tag{0.3}$$
$$(x_1 + 4x_2 - 4x_3 - x_4)O = 0 \tag{0.4}$$
$$(x_1 + 2x_2 - 2x_4)H = 0 \tag{0.5}$$
$$(x_2 - x_3)S = 0 \tag{0.6}$$

→ Which can further be expressed as

$$(1x_1 + 0x_2 - 2x_3 + 0x_4)Na = 0 \tag{0.7}$$
$$(1x_1 + 4x_2 - 4x_3 - 1x_4)O = 0 \tag{0.8}$$
$$(1x_1 + 2x_2 + 0x_3 - 2x_4)H = 0 \tag{0.9}$$
$$(0x_1 + 1x_2 - 1x_3 + 0x_4)S = 0 \tag{0.10}$$

Solution

→ Giving us the matrix equation

$$\begin{pmatrix} 1 & 0 & -2 & 0 \\ 1 & 4 & -4 & -1 \\ 1 & 2 & 0 & -2 \\ 0 & 1 & -1 & 0 \end{pmatrix} \mathbf{x} = 0, \qquad \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \tag{0.11}$$

→ Now, (11) can be reduced as follows

$$\begin{pmatrix} 1 & 0 & -2 & 0 \\ 1 & 4 & -4 & -1 \\ 1 & 2 & 0 & -2 \\ 0 & 1 & -1 & 0 \end{pmatrix} \xrightarrow[R_3 \leftrightarrow R_3 - R_1]{R_2 \leftrightarrow R_2 - R_1} \begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 4 & -2 & -1 \\ 0 & 2 & 2 & -2 \\ 0 & 1 & -1 & 0 \end{pmatrix} \tag{0.12}$$

$$\xrightarrow{R_2 \leftrightarrow (1/4)R_2} \begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & -1/2 & -1/4 \\ 0 & 2 & 2 & -2 \\ 0 & 1 & -1 & 0 \end{pmatrix} \xrightarrow[R_4 \leftrightarrow R_4 - R_2]{R_3 \leftrightarrow R_3 - 2R_2} \begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & -1/2 & -1/4 \\ 0 & 0 & 3 & -3/2 \\ 0 & 0 & -1/2 & 1/4 \end{pmatrix} \tag{0.13}$$

Solution

$$\xrightarrow[R_1 \leftrightarrow R_1+2R_3]{R_3 \leftrightarrow (1/3)R_3} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -1/2 & -1/4 \\ 0 & 0 & 1 & -1/2 \\ 0 & 0 & -1/2 & 1/4 \end{pmatrix} \xrightarrow[R_4 \leftrightarrow R_4+(1/2)R_3]{R_2 \leftrightarrow R_2+(1/2)R_3} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1/2 \\ 0 & 0 & 1 & -1/2 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{0.14}$$

$\rightarrow$ Thus

$$x_1 = x_4, x_2 = \frac{1}{2}x_4, x_3 = \frac{1}{2}x_4 \tag{0.15}$$

$$\implies \mathbf{x} = x_4 \begin{pmatrix} 1 \\ 1/2 \\ 1/2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 1 \\ 2 \end{pmatrix} \tag{0.16}$$

by substituting $x_4 = 2$. Hence, (2) finally becomes

$$2NaOH + H_2SO_4 \rightarrow Na_2SO_4 + 2H_2O \tag{0.17}$$