MatGeo Presentation - Problem 11.2.5

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Question

In $\triangle ABC$, **D**, **E** and **F** are, respectively, the mid-points of sides AB, BC and CA. Show that $\triangle ABC$ is divided into four congruent triangles by joining **D**, **E**, and **F**.

Solution

$\rightarrow$ Given that

$$\mathbf{D} = \frac{\mathbf{A} + \mathbf{B}}{2} \qquad \mathbf{E} = \frac{\mathbf{B} + \mathbf{C}}{2} \qquad \mathbf{F} = \frac{\mathbf{C} + \mathbf{A}}{2} \tag{0.1}$$

$\rightarrow$ From (1), it follows that

$$\mathbf{A} = \mathbf{D} + \mathbf{F} - \mathbf{E} \qquad \mathbf{B} = \mathbf{E} + \mathbf{D} - \mathbf{F} \qquad \mathbf{C} = \mathbf{F} + \mathbf{E} - \mathbf{D} \tag{0.2}$$

$\rightarrow$ From (2), we get that

$$\text{In } \triangle FAD \text{ and } \triangle DEF \tag{0.3}$$

$$\mathbf{A} - \mathbf{D} = \mathbf{F} - \mathbf{E} \text{ (Side 1)} \tag{0.4}$$

$$\mathbf{A} - \mathbf{F} = \mathbf{D} - \mathbf{E} \text{ (Side 2)} \tag{0.5}$$

$$\mathbf{D} - \mathbf{F} \text{ is common to both} \tag{0.6}$$

$$\triangle FAD \cong \triangle DEF (\text{SSS criterion}) \tag{0.7}$$

Solution

$$\text{In } \triangle DBE \text{ and } \triangle DEF \tag{0.8}$$
$$\mathbf{B} - \mathbf{E} = \mathbf{D} - \mathbf{F} \text{ (Side 1)} \tag{0.9}$$
$$\mathbf{B} - \mathbf{D} = \mathbf{E} - \mathbf{F} \text{ (Side 2)} \tag{0.10}$$
$$\mathbf{E} - \mathbf{D} \text{ is common to both} \tag{0.11}$$
$$\triangle DBE \cong \triangle DEF (\text{SSS criterion}) \tag{0.12}$$

$$\text{In } \triangle ECF \text{ and } \triangle DEF \tag{0.13}$$
$$\mathbf{C} - \mathbf{F} = \mathbf{E} - \mathbf{D} \text{ (Side 1)} \tag{0.14}$$
$$\mathbf{C} - \mathbf{E} = \mathbf{F} - \mathbf{D} \text{ (Side 2)} \tag{0.15}$$
$$\mathbf{F} - \mathbf{E} \text{ is common to both} \tag{0.16}$$
$$\triangle ECF \cong \triangle DEF (\text{SSS criterion}) \tag{0.17}$$

$\rightarrow$ From (7), (12), and (17), we know that $\triangle ABC$ is divided into four congruent triangles

Solution

$$\triangle FAD \cong \triangle DBE \cong \triangle ECF \cong \triangle DEF \tag{0.18}$$

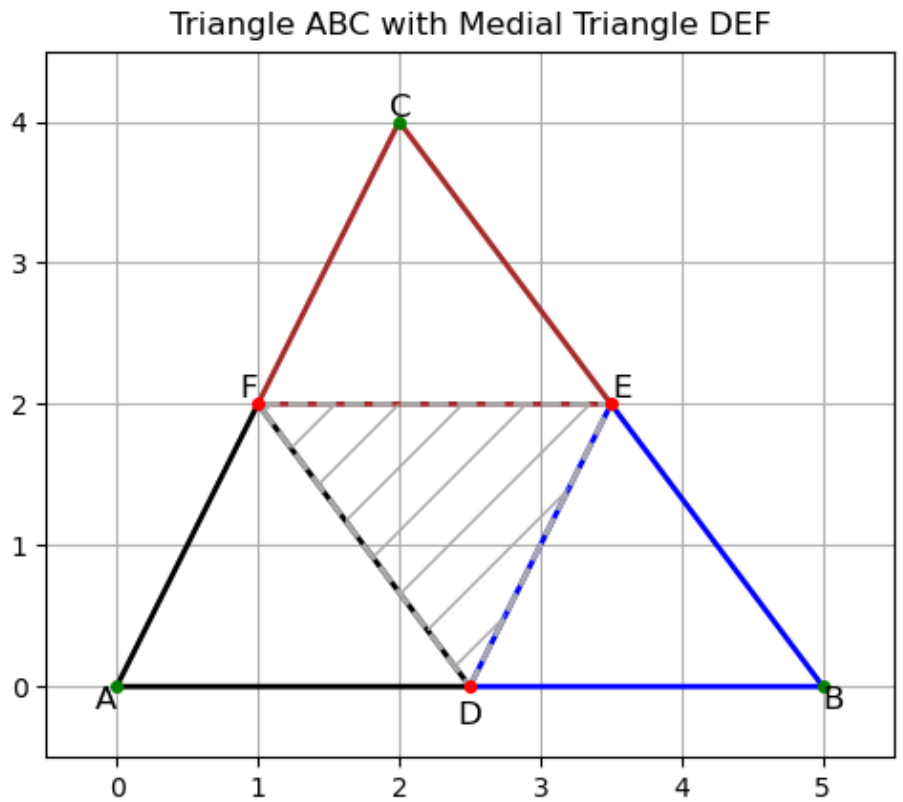

Figure: Plot of $\triangle ABC$ and its medial triangle $\triangle DEF$

File: plot.py

```
import matplotlib.pyplot as plt
import numpy as np

# Coordinates of the vertices of triangle ABC
A = np.array([0, 0])
B = np.array([5, 0])
C = np.array([2, 4])
# Midpoints of the sides
D = (A + B) / 2
E = (B + C) / 2
F = (C + A) / 2

# Plot the triangle ABC
fig, ax = plt.subplots()
triangle1 = plt.Polygon([A, D, F], fill=None, edgecolor='black', linewidth=2, zorder=2)
triangle2 = plt.Polygon([B, E, D], fill=None, edgecolor='blue', linewidth=2, zorder=2)
triangle3 = plt.Polygon([C, F, E], fill=None, edgecolor='brown', linewidth=2, zorder=2)
triangle4 = plt.Polygon([D, E, F], fill=None, edgecolor='darkgray', hatch='/', linestyle='--', linewidth=2,
       zorder=2)

# Plotting the triangle ABC and medial triangle DEF
ax.add_patch(triangle1)
ax.add_patch(triangle2)
ax.add_patch(triangle3)
ax.add_patch(triangle4)
```

File: plot.py

```
# Plot the points A, B, C, D, E, F
ax.plot(A[0], A[1], 'go', ms=4) # A
ax.plot(B[0], B[1], 'go', ms=4) # B
ax.plot(C[0], C[1], 'go', ms=4) # C
ax.plot(D[0], D[1], 'ro', ms=4) # D
ax.plot(E[0], E[1], 'ro', ms=4) # E
ax.plot(F[0], F[1], 'ro', ms=4) # F

# Labels for points
ax.text(A[0], A[1], 'A', fontsize=12, ha='right', va='top')
ax.text(B[0], B[1], 'B', fontsize=12, ha='left', va='top')
ax.text(C[0], C[1], 'C', fontsize=12, ha='center', va='bottom')
ax.text(D[0], D[1]-0.1, 'D', fontsize=12, ha='center', va='top')
ax.text(E[0], E[1], 'E', fontsize=12, ha='left', va='bottom')
ax.text(F[0], F[1], 'F', fontsize=12, ha='right', va='bottom')

# Title and showing the plot
ax.set_aspect('equal')
ax.grid()
plt.xlim(-0.5, 5.5)
plt.ylim(-0.5, 4.5)
plt.title("Triangle␣ABC␣with␣Medial␣Triangle␣DEF")
plt.show()
```