2.5.32

EE25BTECH11064 - Yojit Manral

Question:
Show that the points $(7, 10)$, $(-2, 5)$ and $(3, 4)$ are vertices of an isosceles right triangle.

Solution:

Points	Name
$\begin{pmatrix}7\\10\end{pmatrix}$	Point **A**
$\begin{pmatrix}-2\\5\end{pmatrix}$	Point **B**
$\begin{pmatrix}3\\4\end{pmatrix}$	Point **C**

TABLE 0: List of Points

→ Let

$$\mathbf{a} = \mathbf{C} - \mathbf{B} = \begin{pmatrix}5\\-1\end{pmatrix} \tag{1}$$

$$\mathbf{b} = \mathbf{A} - \mathbf{C} = \begin{pmatrix}4\\6\end{pmatrix} \tag{2}$$

$$\mathbf{c} = \mathbf{B} - \mathbf{A} = \begin{pmatrix}-9\\-5\end{pmatrix} \tag{3}$$

$$\angle A = \angle BAC \tag{4}$$

$$\angle B = \angle CBA \tag{5}$$

$$\angle C = \angle ACB \tag{6}$$

→ Then the cosines of the angles are

$$\cos A = -\frac{\mathbf{b}^T\mathbf{c}}{\|\mathbf{b}\|\,\|\mathbf{c}\|} = \frac{66}{\sqrt{52}\,\sqrt{106}} = 0.889 \tag{7}$$

$$\cos B = -\frac{\mathbf{c}^T\mathbf{a}}{\|\mathbf{c}\|\,\|\mathbf{a}\|} = \frac{40}{\sqrt{106}\,\sqrt{26}} = 0.762 \tag{8}$$

$$\cos C = -\frac{\mathbf{a}^T\mathbf{b}}{\|\mathbf{a}\|\,\|\mathbf{b}\|} = -\frac{14}{\sqrt{26}\,\sqrt{52}} = -0.381 \tag{9}$$

→ As none of the cosines are equal, △ABC is not isosceles. Also, since $\cos C$ is negative, △ABC forms an obtuse angled triangle.
$\implies$ △ABC is neither isosceles nor right-angled triangle.

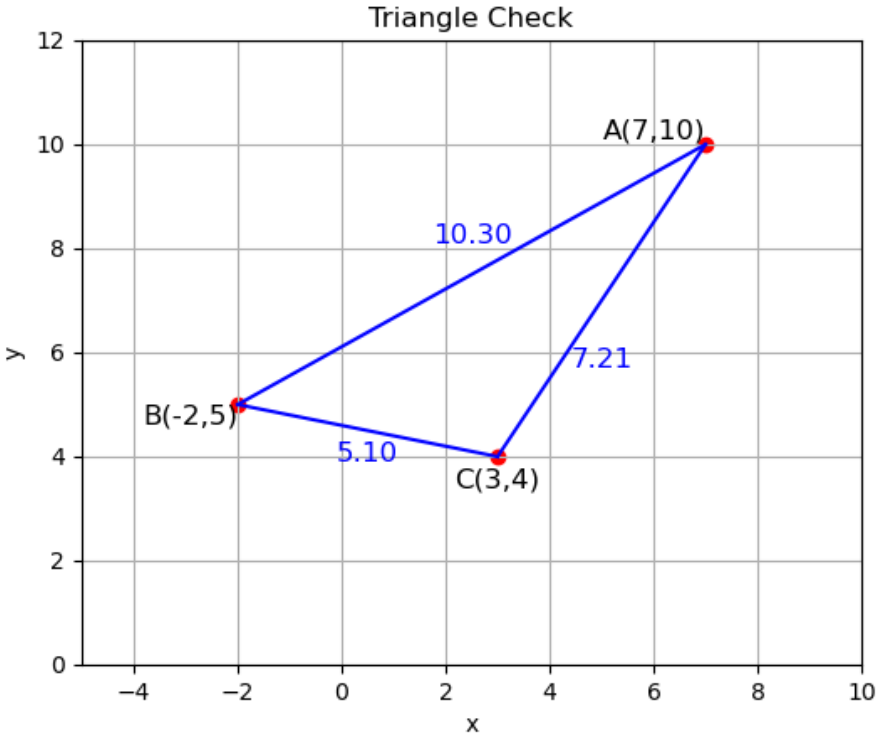

Fig. 0: Plot of △ABC