2.5.32

EE25BTECH11064 - Yojit Manral

Question:
Show that the points $(7, 10)$, $(-2, 5)$ and $(3, 4)$ are vertices of an isosceles right triangle.

Solution:

Points	Name
$\begin{pmatrix}7\\10\end{pmatrix}$	Point **A**
$\begin{pmatrix}-2\\5\end{pmatrix}$	Point **B**
$\begin{pmatrix}3\\4\end{pmatrix}$	Point **C**

TABLE 0: List of Points

→ We have $\triangle ABC$ with the given points. Then,

$$\|\mathbf{B}-\mathbf{A}\|^2 = \begin{pmatrix}-9 & -5\end{pmatrix}\begin{pmatrix}-9\\-5\end{pmatrix} = 106 \implies \|\mathbf{B}-\mathbf{A}\| = \sqrt{106} \tag{1}$$

$$\|\mathbf{C}-\mathbf{B}\|^2 = \begin{pmatrix}5 & -1\end{pmatrix}\begin{pmatrix}5\\-1\end{pmatrix} = 26 \implies \|\mathbf{C}-\mathbf{B}\| = \sqrt{26} \tag{2}$$

$$\|\mathbf{A}-\mathbf{C}\|^2 = \begin{pmatrix}4 & 6\end{pmatrix}\begin{pmatrix}4\\6\end{pmatrix} = 52 \implies \|\mathbf{A}-\mathbf{C}\| = \sqrt{52} \tag{3}$$

$$\|\mathbf{B}-\mathbf{A}\|^2 + \|\mathbf{C}-\mathbf{B}\|^2 = 26 + 52 = 78 \neq 106 = \|\mathbf{A}-\mathbf{C}\|^2 \tag{4}$$

→ From (1), (2) and (3), we know that the sides are different lengths, or $\triangle ABC$ is not isosceles. From (4), we prove that $\triangle ABC$ is not right-angled as well.
$\implies$ The $\triangle ABC$ is neither isosceles nor right-angled.

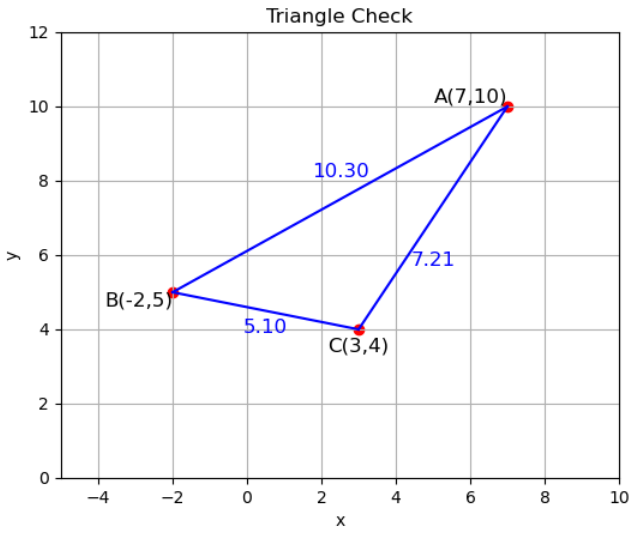

Fig. 0: Plot of $\triangle ABC$