EE25BTECH11064 - Yojit Manral

Question:

Solve the system of equations

$$2x + 3y = 11 (1)$$

$$2x + 4y = -24 \tag{2}$$

Hence, find the value of m for which

$$y = mx + 3 \tag{3}$$

Solution:

→ We have

$$\mathbf{n_1}^T \mathbf{x} = c_1 \qquad \qquad \mathbf{n_2}^T \mathbf{x} = c_2 \qquad \qquad \mathbf{n_3}^T \mathbf{x} = c_3$$

$$\mathbf{n_1} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \qquad \qquad c_1 = 11 \tag{4}$$

$$\mathbf{n_2} = \begin{pmatrix} 2\\4 \end{pmatrix} \qquad \qquad c_2 = -24 \tag{5}$$

$$\mathbf{n_3} = \begin{pmatrix} -m \\ 1 \end{pmatrix} \qquad c_3 = 3 \tag{6}$$

 \rightarrow To solve the system of equations, they must be concurrent at a point x

$$\begin{pmatrix} \mathbf{n_1}^T \\ \mathbf{n_2}^T \\ \mathbf{n_3}^T \end{pmatrix} \mathbf{x} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} \implies \begin{pmatrix} 2 & 3 \\ 2 & 4 \\ -m & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 11 \\ -24 \\ 3 \end{pmatrix}$$
(7)

→ Using augmented matrix

$$\begin{pmatrix} 2 & 3 & 11 \\ 2 & 4 & -24 \\ -m & 1 & 3 \end{pmatrix} \xrightarrow{R_2 \leftrightarrow R_2 - R_1} \begin{pmatrix} 2 & 0 & 116 \\ 0 & 1 & -35 \\ -m & 1 & 3 \end{pmatrix}$$
(8)

$$\frac{R_1 \leftrightarrow (1/2)R_1}{R_3 \leftrightarrow -R_3 + R_2} \begin{pmatrix} 1 & 0 & 58 \\ 0 & 1 & -35 \\ m & 0 & -38 \end{pmatrix}$$
 (9)

$$\implies x = 58, y = -35, \text{ and } mx = -38$$
 (10)

$$\implies m = -\frac{38}{58} = -\frac{19}{29} \tag{11}$$

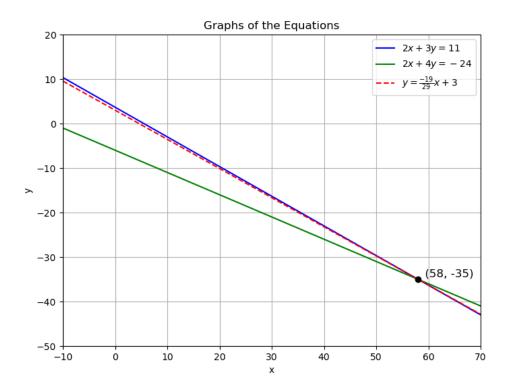


Fig. 0: Plot of the Equations