1.1.6.14

EE25BTECH11064 - Yojit Manral

Question:
Points $\mathbf{A}(3,1)$, $\mathbf{B}(12,-2)$ and $\mathbf{C}(0,2)$ cannot be the vertices of a triangle.

Solution:

Points	Name
$\begin{pmatrix}3\\1\end{pmatrix}$	Point **A**
$\begin{pmatrix}12\\-2\end{pmatrix}$	Point **B**
$\begin{pmatrix}0\\2\end{pmatrix}$	Point **C**

TABLE 0: List of Points

→ Any 3 points form a triangle if the rank of co-linearity matrix is not equal to 1, in which case they become collinear. For the rank of a matrix to be 1, the number of rows with non-zero entries should be 1 in row echelon form.
→ The co-linearity matrix is given by,

$$\begin{pmatrix}\mathbf{B}-\mathbf{A} & \mathbf{C}-\mathbf{A}\end{pmatrix}^T = \begin{pmatrix}9 & -3\\-3 & 1\end{pmatrix} \tag{1}$$

$$\begin{pmatrix}9 & -3\\-3 & 1\end{pmatrix} \xrightarrow{R_2 \leftrightarrow R_1+3R_2} \begin{pmatrix}9 & -3\\0 & 0\end{pmatrix} \tag{2}$$

→ The above matrix is in the row echelon form. Rank of the matrix in echelon form is the number of non-zero rows. Hence, rank of the above matrix is 1.
$\implies$ The given 3 points A, B, C are collinear. Thus, they cannot be part of a triangle.

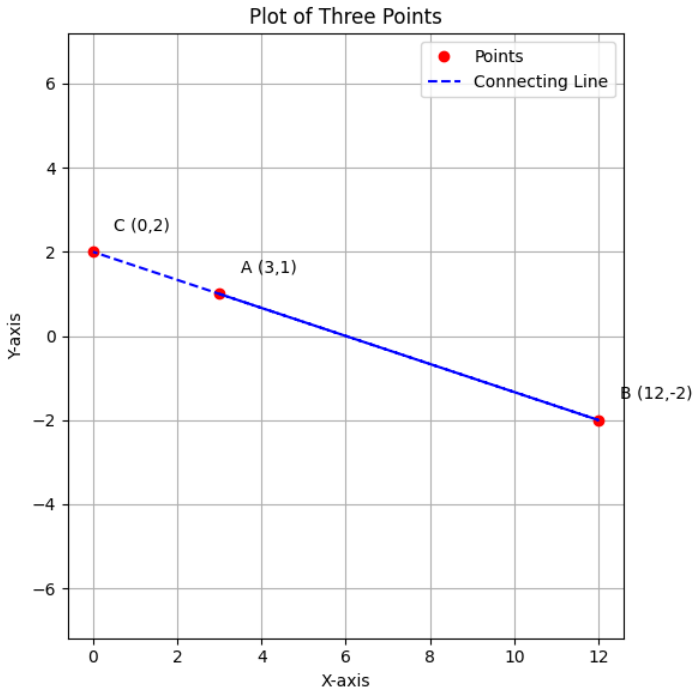

Fig. 0: Plot of the points A, B, C