

MatGeo Presentation - Problem 8.2.32

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Question

Find the conic equation that satisfies the given conditions: ends of the major axis $(0, \pm 5)$, ends of the minor axis $(\pm 1, 0)$.

Solution

→ The equation for any conic having directrix $\mathbf{n}^T \mathbf{x} = c$ and eccentricity e is given by

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (0.1)$$

$$\mathbf{V} = \|\mathbf{n}\|^2 \mathbf{I} - e^2 \mathbf{n} \mathbf{n}^T \quad (0.2)$$

→ As the major axis is along the Y - axis

$$\mathbf{n} = \mathbf{e}_2 \implies \mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 1 - e^2 \end{pmatrix} \quad (0.3)$$

→ Also, as the center of the ellipse is at origin

$$\mathbf{C} = 0 \implies \mathbf{u} = 0 \quad (0.4)$$

→ Let \mathbf{P} and \mathbf{Q} be points on the ellipse

$$\mathbf{P} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \mathbf{Q} = \begin{pmatrix} 0 \\ 5 \end{pmatrix} \quad (0.5)$$

Solution

→ Then, **P** and **Q** satisfy (1)

$$\mathbf{P}^T \mathbf{V} \mathbf{P} + 2\mathbf{u}^T \mathbf{P} + f = 0 \qquad \mathbf{Q}^T \mathbf{V} \mathbf{Q} + 2\mathbf{u}^T \mathbf{Q} + f = 0 \quad (0.6)$$

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 - e^2 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = -f \quad \begin{pmatrix} 0 & 5 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 - e^2 \end{pmatrix} \begin{pmatrix} 0 \\ 5 \end{pmatrix} = -f \quad (0.7)$$

$$1 = -f \qquad 25(1 - e^2) = -f \quad (0.8)$$

$$f = -1 \qquad \implies e^2 = 1 + \frac{f}{25} = \frac{24}{25} \quad (0.9)$$

→ Thus, we get the equation of the conic as

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 1 - e^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1/25 \end{pmatrix}, \mathbf{u} = 0, \text{ and } f = -1 \quad (0.10)$$

$$\implies \mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 1/25 \end{pmatrix} \mathbf{x} - 1 = 0 \quad (0.11)$$

Solution

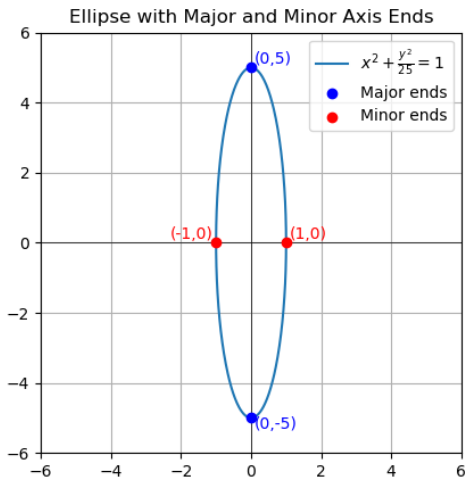


Figure: Plot of required conic

File: points.c

```
#include <stdio.h>

int main() {
    FILE *fp;

    // -----
    // Question 8.2.32
    // -----

    fp = fopen("points.dat", "w");
    fprintf(fp, "%d,%d,%d\n", 0, 5, 0); // A
    fprintf(fp, "%d,%d,%d\n", 1, 0, 0); // B
    fprintf(fp, "%d,%d,%d\n", 0, -5, 0); // C
    fprintf(fp, "%d,%d,%d\n", -1, 0, 0); // D
    fclose(fp);
    return 0;
}
```

File: call_c.py

```
import subprocess

# Compile the C program
subprocess.run(["gcc", "points.c", "-o", "points"])

# Run the compiled C program
result = subprocess.run(["./points"], capture_output=True, text=True)

# Print the output from the C program
print(result.stdout)
```

File: plot.py

```
import numpy as np
import matplotlib.pyplot as plt

# Parameters of the ellipse
a = 5 # semi-major axis
b = 1 # semi-minor axis

# Parametric equations of ellipse
theta = np.linspace(0, 2*np.pi, 400)
y = a * np.sin(theta)
x = b * np.cos(theta)

# Plot ellipse
plt.plot(x, y, label=r"$x^2+\frac{y^2}{25}=1$")

# Mark ends of the axes
plt.scatter([0, 0], [5, -5], color="blue", zorder=5, label="Major_Ends")
plt.scatter([1, -1], [0, 0], color="red", zorder=5, label="Minor_Ends")
plt.text(1.1, 0.1, "(1,0)", color="red")
plt.text(-2.3, 0.1, "(-1,0)", color="red")
plt.text(0.1, 5.1, "(0,5)", color="blue")
plt.text(0.1, -5.3, "(0,-5)", color="blue")

# Axes setup
plt.axhline(0, color="black", linewidth=0.5)
plt.axvline(0, color="black", linewidth=0.5)
plt.gca().set_aspect('equal')
plt.legend(loc='upper_right')
plt.title("Ellipse with Major-Axis and Minor-Axis Ends")
plt.xlim(-6, 6)
plt.ylim(-6, 6)
plt.grid(True)
plt.show()
```