MatGeo Presentation - Problem 12.597

EE25BTECH11064 - Yojit Manral

Question

The value of p such that the vector $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ is an eigenvector of the matrix $\begin{pmatrix} 4 & 1 & 2 \\ p & 2 & 1 \\ 14 & -4 & 10 \end{pmatrix}$ is ___ .

Solution

→ If the vector is an eigenvector for the matrix, it satisfies

$$\mathbf{Ax} = \lambda\mathbf{x} \tag{0.1}$$

$$\begin{pmatrix} 4 & 1 & 2 \\ p & 2 & 1 \\ 14 & -4 & 10 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \lambda \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \quad \exists\lambda \in R \tag{0.2}$$

$$\begin{pmatrix} 12 \\ p+7 \\ 36 \end{pmatrix} = \lambda \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \tag{0.3}$$

→ Putting $\lambda = 12$ to satisfy (3), we get

$$\begin{pmatrix} 12 \\ p+7 \\ 36 \end{pmatrix} = \begin{pmatrix} 12 \\ 24 \\ 36 \end{pmatrix} \tag{0.4}$$

$$\implies p + 7 = 24 \tag{0.5}$$

$$\implies p = 15 \tag{0.6}$$