9.2.32

EE25BTECH11064 - Yojit Manral

Question:
Find the area of the region included between $y^2 = 9x$ and $y = x$.
Solution:
→ The given conic can be expressed with parameters

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} -\frac{9}{2} \\ 0 \end{pmatrix}, f = 0 \tag{1}$$

→ The given line can be expressed with the parameters

$$\mathbf{h} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{2}$$

→ The point of intersection of the line

$$\mathbf{L} \equiv \mathbf{x} = \mathbf{h} + \kappa\mathbf{m} \tag{3}$$

with a general conic

$$g(\mathbf{x}) = \mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{4}$$

can be given by

$$\mathbf{x_i} = \mathbf{h} + \kappa_i\mathbf{m} \tag{5}$$

where

$$\kappa_i = \frac{1}{\mathbf{m}^T\mathbf{V}\mathbf{m}}\left(-\mathbf{m}^T(\mathbf{V}\mathbf{h} + \mathbf{u}) \pm \sqrt{\left(\mathbf{m}^T(\mathbf{V}\mathbf{h} + \mathbf{u})\right)^2 - g(\mathbf{h})\left(\mathbf{m}^T\mathbf{V}\mathbf{m}\right)}\right) \tag{6}$$

→ Substituting the parameters from (1), (2) in (6), we get

$$\mathbf{x_1} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{x_2} = \begin{pmatrix} 9 \\ 9 \end{pmatrix} \tag{7}$$

→ From the figure, the area bounded by the conic $y^2 = 9x$ and the line $y = x$ is given by

$$\int_0^9 \left(3\sqrt{x} - x\right) dx = \left[2(x)^{3/2} - \frac{x^2}{2}\right]_0^9 = \frac{27}{2} \ \ units \tag{8}$$

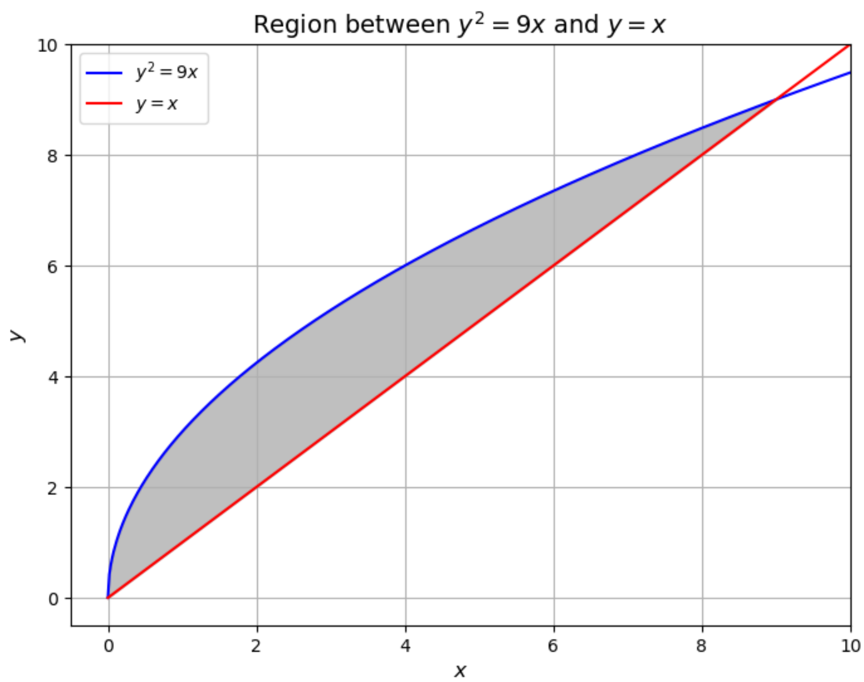

Fig. 0: Plot of $y^2 = 9x$ and $y = x$