MatGeo Presentation - Problem 9.8.1

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Question

The line $x + 3y = 0$ is the diameter of the circle $x^2 + y^2 - 6x + 2y = 0$.

Solution

→ The given circle can be expressed as

$$\mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{0.1}$$

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}, f = 0 \tag{0.2}$$

→ Also, the center of the circle is

$$\mathbf{c} = -\mathbf{V}^{-1}\mathbf{u};\ |\mathbf{V}| \neq 0 \tag{0.3}$$

$$\mathbf{c} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} \tag{0.4}$$

→ The given line can be expressed as

$$\mathbf{x} = \mathbf{h} + \kappa\mathbf{m};\ \kappa \in R \tag{0.5}$$

$$\mathbf{h} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} \tag{0.6}$$

→ If the line is a diameter of the circle, the center of the circle must lie on the line

Solution

$$\mathbf{c} = \mathbf{h} + \lambda \mathbf{m} \;\; \exists \, \lambda \in R \tag{0.7}$$

$$\begin{pmatrix} 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -1 \end{pmatrix} \implies \lambda = 1 \in R \tag{0.8}$$

$\implies$ **c** lies on the line $\implies$ The line is a diameter of the circle

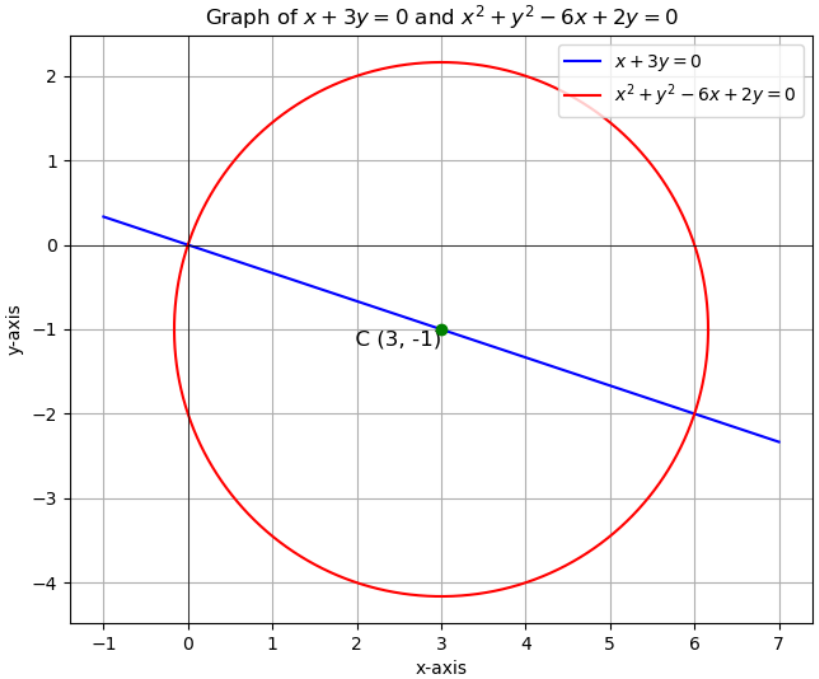

Figure: Plot of given line and circle

File: plot.py

```
import numpy as np
import matplotlib.pyplot as plt

# Define the equation of the line
def line(x):
    return -x / 3
# Create x values for the line
x_vals = np.linspace(-1, 7, 400)
y_vals = line(x_vals)

# Circle equation
theta = np.linspace(0, 2 * np.pi, 400)
x_circle = 3 + np.sqrt(10) * np.cos(theta)
y_circle = -1 + np.sqrt(10) * np.sin(theta)

# Plot the line and the circle and the center
plt.figure(figsize=(8, 8))
plt.plot(x_vals, y_vals, label=r'$x + 3y = 0$', color='blue') # Line equation
plt.plot(x_circle, y_circle, label=r'$x^2 + y^2 - 6x + 2y = 0$', color='red') # Circle equation
plt.scatter(3, -1, color='green', zorder=5) # Green dot for the center
plt.text(3, -1, '  C (3, -1)', fontsize=12, verticalalignment='top', horizontalalignment='right')

# Setting up the plot
plt.axhline(0, color='black',linewidth=0.5)
plt.axvline(0, color='black',linewidth=0.5)
plt.gca().set_aspect('equal', adjustable='box') # To ensure the circle looks like a circle
plt.legend()
plt.title("Graph of $x + 3y = 0$ and $x^2 + y^2 - 6x + 2y = 0$")
plt.xlabel("x-axis")
plt.ylabel("y-axis")
plt.grid(True)
plt.show()
```