MatGeo Presentation - Problem 9.2.32

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Question

Find the area of the region included between $y^2 = 9x$ and $y = x$.

Solution

→ The given conic can be expressed with parameters

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} -\frac{9}{2} \\ 0 \end{pmatrix}, f = 0 \tag{0.1}$$

→ The given line can be expressed with the parameters

$$\mathbf{h} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{0.2}$$

→ The point of intersection of the line

$$\mathbf{L} \equiv \mathbf{x} = \mathbf{h} + \kappa \mathbf{m} \tag{0.3}$$

with a general conic

$$g(\mathbf{x}) = \mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \tag{0.4}$$

can be given by

$$\mathbf{x_i} = \mathbf{h} + \kappa_i \mathbf{m} \tag{0.5}$$

Solution

where

$$\kappa_i = \frac{1}{\mathbf{m}^T\mathbf{V}\mathbf{m}}\left(-\mathbf{m}^T\left(\mathbf{V}\mathbf{h}+\mathbf{u}\right) \pm \sqrt{\left(\mathbf{m}^T\left(\mathbf{V}\mathbf{h}+\mathbf{u}\right)\right)^2 - g\left(\mathbf{h}\right)\left(\mathbf{m}^T\mathbf{V}\mathbf{m}\right)}\right) \tag{0.6}$$

$\rightarrow$ Substituting the parameters from (1), (2) in (6), we get

$$\mathbf{x_1} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{x_2} = \begin{pmatrix} 9 \\ 9 \end{pmatrix} \tag{0.7}$$

$\rightarrow$ From the figure, the area bounded by the conic $y^2 = 9x$ and the line $y = x$ is given by

$$\int_0^9 \left(3\sqrt{x} - x\right) dx = \left[2(x)^{3/2} - \frac{x^2}{2}\right]_0^9 \tag{0.8}$$

$$= \frac{27}{2} \ \textit{units} \tag{0.9}$$

Solution

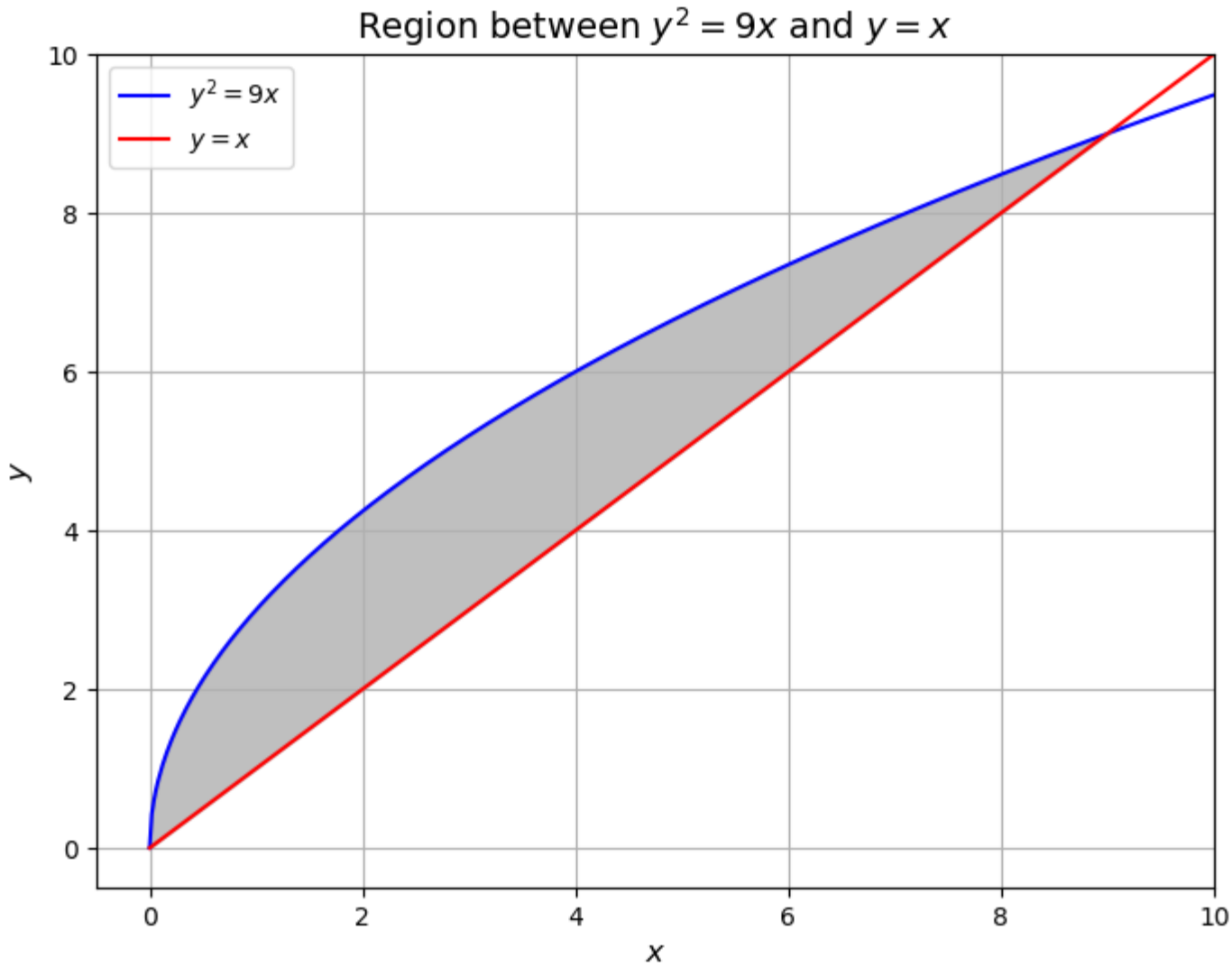

Figure: Plot of $y^2 = 9x$ and $y = x$

File: plot.py

```
import numpy as np
import matplotlib.pyplot as plt

# Define the functions
def curve1(x):
    return np.sqrt(9 * x)

def curve2(x):
    return x

# Define the x-range
x = np.linspace(0, 10, 500)

# Plot the curves
plt.figure(figsize=(8, 6))
plt.plot(x, curve1(x), label=r"$y^2 = 9x$", color="blue")
plt.plot(x, curve2(x), label=r"$y = x$", color="red")

# Fill the region between the curves
plt.fill_between(x, curve2(x), curve1(x), where=(curve2(x) <= curve1(x)), color="gray", alpha=0.5)

# Add labels and title
plt.title("Region between $y^2 = 9x$ and $y = x$", fontsize=14)
plt.xlabel("$x$", fontsize=12)
plt.ylabel("$y$", fontsize=12)
plt.legend()

# Show the plot
plt.grid(True)
plt.xlim(-0.5, 10)
plt.ylim(-0.5, 10)
plt.show()
```