MatGeo Presentation - Problem 4.7.61

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Question

Find the coordinates of the foot of the perpendicular drawn from the origin to the plane $2x - 3y + 4z - 6 = 0$

Solution

→ From the equation of a general plane,

$$\mathbf{n}^T\mathbf{x} = c \tag{0.1}$$

→ A vector perpendicular to the plane and passing through the origin can be given as

$$\mathbf{y} = \alpha\mathbf{n} \tag{0.2}$$

→ The foot of perpendicular must be the intersection of (1) and (2)...

$$\mathbf{n}^T(\alpha\mathbf{n}) = c \tag{0.3}$$

$$\alpha\mathbf{n}^T\mathbf{n} = c \tag{0.4}$$

$$\alpha = \frac{c}{\|\mathbf{n}\|^2} \tag{0.5}$$

→ Now we can find the foot of perpedicular as

$$\mathbf{x}_\perp = \alpha\mathbf{n} = \frac{c}{\|\mathbf{n}\|^2}\mathbf{n} \tag{0.6}$$

Solution

→ Given that

$$\mathbf{n} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} \implies \|\mathbf{n}\|^2 = 29 \text{ and } c = 6 \tag{0.7}$$

$$\implies \mathbf{x}_\perp = \frac{6}{29} \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} = \begin{pmatrix} 12/29 \\ -18/29 \\ 24/29 \end{pmatrix} \tag{0.8}$$

Solution

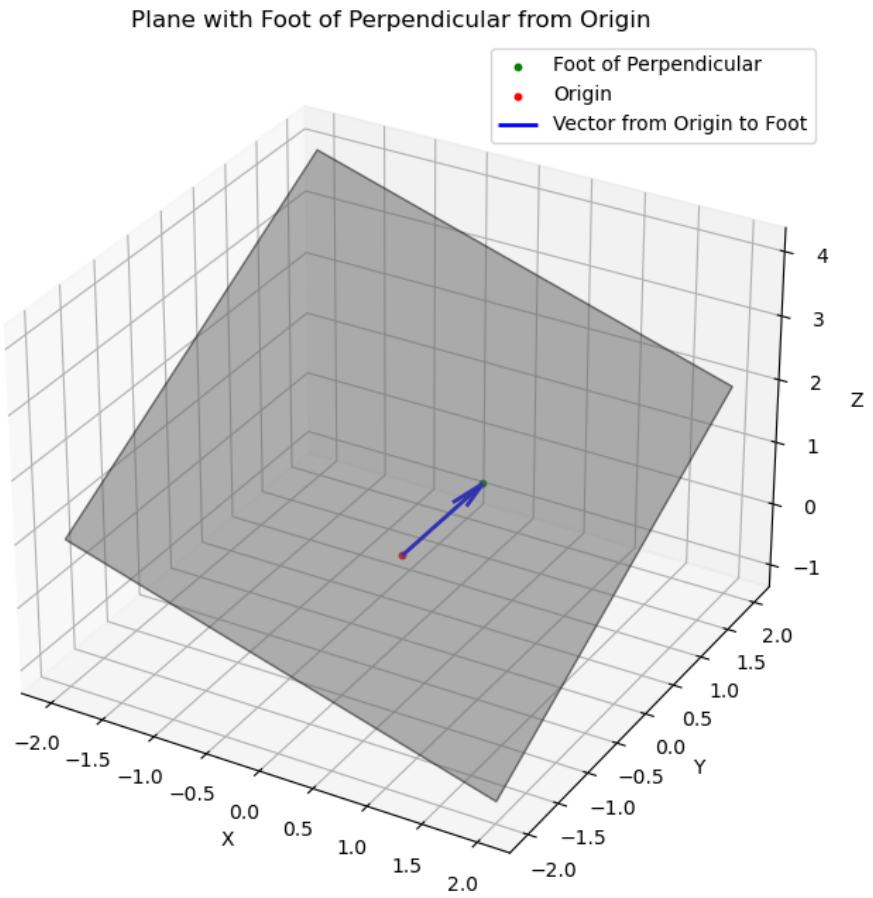

Figure: Foot of perpendicular of plane from origin

File: points.c

```
#include <stdio.h>

int main() {
  FILE *fp;

  // -------------------
  // Question 4.7.61
  // -------------------


  fp = fopen("points.dat", "w");
  fprintf(fp, "%d,%d,%d\n", 2, -3, 4); // n
  fclose(fp);
  return 0;
}
```

File: call_c.py

```
import subprocess

# Compile the C program
subprocess.run(["gcc", "points.c", "-o", "points"])

# Run the compiled C program
result = subprocess.run(["./points"], capture_output=True, text=True)

# Print the output from the C program
print(result.stdout)
```

File: plot.py

```
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

# Plane equation: 2x - 3y + 4z - 6 = 0
# The point is the foot of the perpendicular (12/29, 18/29, 24/29)
foot_of_perpendicular = np.array([12/29, 18/29, 24/29])

# Create grid for the plane
x = np.linspace(-2, 2, 100)
y = np.linspace(-2, 2, 100)
X, Y = np.meshgrid(x, y)

# Solve for Z using the plane equation 2x - 3y + 4z - 6 = 0
Z = (6 - 2*X + 3*Y) / 4

# Create a 3D plot
fig = plt.figure(figsize=(8, 8))
ax = fig.add_subplot(111, projection='3d')

# Plot the plane
ax.plot_surface(X, Y, Z, alpha=0.5, rstride=100, cstride=100, color='gray', edgecolor='k')

# Plot the foot of the perpendicular
ax.scatter(foot_of_perpendicular[0], foot_of_perpendicular[1], foot_of_perpendicular[2], color='g', s=10,
      label='Foot␣of␣Perpendicular')
```

File: plot.py

```
# Plot the origin
ax.scatter(0, 0, 0, color='r', s=10, label='Origin')

# Draw the vector from origin to foot of perpendicular
ax.quiver(0, 0, 0, foot_of_perpendicular[0], foot_of_perpendicular[1], foot_of_perpendicular[2],
          color='b', linewidth=2, label='Vector from Origin to Foot')

# Labels and Title
ax.set_xlabel('X')
ax.set_ylabel('Y')
ax.set_zlabel('Z')
ax.set_title('Plane with Foot of Perpendicular from Origin')

# Show the plot
plt.legend()
plt.show()
```