MatGeo Presentation - Problem 2.5.32

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Question

Show that the points $(7, 10)$, $(-2, 5)$ and $(3, 4)$ are vertices of an isosceles right triangle.

Solution

Points	Name
$\begin{pmatrix} 7 \\ 10 \end{pmatrix}$	Point **A**
$\begin{pmatrix} -2 \\ 5 \end{pmatrix}$	Point **B**
$\begin{pmatrix} 3 \\ 4 \end{pmatrix}$	Point **C**

Table: List of Points

$\rightarrow$ The equation of the sides are given as

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} -9 \\ -5 \end{pmatrix} \qquad \mathbf{C} - \mathbf{B} = \begin{pmatrix} 5 \\ -1 \end{pmatrix} \qquad \mathbf{A} - \mathbf{C} = \begin{pmatrix} 4 \\ 6 \end{pmatrix} \qquad (0.1)$$

$\rightarrow$ The medians **D**, **E** and **F** of the triangle are

$$\mathbf{D} = \frac{\mathbf{A} + \mathbf{B}}{2} = \begin{pmatrix} 5/2 \\ 15/2 \end{pmatrix} \quad \mathbf{E} = \frac{\mathbf{B} + \mathbf{C}}{2} = \begin{pmatrix} 1/2 \\ 9/2 \end{pmatrix} \quad \mathbf{F} = \frac{\mathbf{C} + \mathbf{A}}{2} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$$
$$(0.2)$$

Solution

(A) For an isosceles triangle, median to the base is also the perpendicular bisector. Using this property

$$(\mathbf{C} - \mathbf{D})^T(\mathbf{B} - \mathbf{A}) = \begin{pmatrix} 1/2 & -7/2 \end{pmatrix} \begin{pmatrix} -9 \\ -5 \end{pmatrix} = 13 \neq 0 \qquad (0.3)$$

$$(\mathbf{A} - \mathbf{E})^T(\mathbf{C} - \mathbf{B}) = \begin{pmatrix} 13/2 & 11/2 \end{pmatrix} \begin{pmatrix} 5 \\ -1 \end{pmatrix} = 27 \neq 0 \qquad (0.4)$$

$$(\mathbf{B} - \mathbf{F})^T(\mathbf{A} - \mathbf{C}) = \begin{pmatrix} -7 & -2 \end{pmatrix} \begin{pmatrix} 4 \\ 6 \end{pmatrix} = -40 \neq 0 \qquad (0.5)$$

$\rightarrow$ Since none of the sides satisfy this property, the triangle is not isosceles.

Solution

(B) For a right triangle, dot product of the perpendicular sides must be zero.

$$(\mathbf{B} - \mathbf{A})^T(\mathbf{C} - \mathbf{B}) = \begin{pmatrix} -9 & -5 \end{pmatrix} \begin{pmatrix} 5 \\ -1 \end{pmatrix} = -40 \neq 0 \qquad (0.6)$$

$$(\mathbf{C} - \mathbf{B})^T(\mathbf{A} - \mathbf{C}) = \begin{pmatrix} 5 & -1 \end{pmatrix} \begin{pmatrix} 4 \\ 6 \end{pmatrix} = 14 \neq 0 \qquad (0.7)$$

$$(\mathbf{A} - \mathbf{C})^T(\mathbf{B} - \mathbf{A}) = \begin{pmatrix} 4 & 6 \end{pmatrix} \begin{pmatrix} -9 \\ -5 \end{pmatrix} = -66 \neq 0 \qquad (0.8)$$

$\rightarrow$ Since none of the dot products is zero, no two sides are perpendicular, and the triangle is not right angled.

Solution

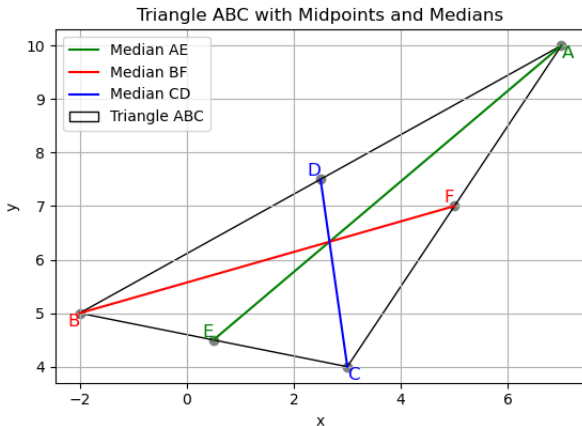$\longrightarrow$ From (A) and (B), we get that $\triangle ABC$ is not isosceles right triangle.

Figure: Plot of $\triangle$ABC

File: points.c

```c
#include <stdio.h>

int main() {
  FILE *fp;

  // -------------------
  // Question 2.5.32
  // -------------------

  fp = fopen("points.dat", "w");
  fprintf(fp, "%d,%d,%d\n", 7, 10, 0); // A
  fprintf(fp, "%d,%d,%d\n", -2, 5, 0); // B
  fprintf(fp, "%d,%d,%d\n", 3, 4, 0); // C
  fclose(fp);
  return 0;
  }
```

File: call_c.py

```python
import subprocess

# Compile the C program
subprocess.run(["gcc", "points.c", "-o", "points"])

# Run the compiled C program
result = subprocess.run(["./points"], capture_output=True, text=True)

# Print the output from the C program
print(result.stdout)
```

File: plot.py

```python
import matplotlib.pyplot as plt

# Vertices of the triangle
A = (7, 10)
B = (-2, 5)
C = (3, 4)

# Midpoints of sides
D = ((A[0] + B[0]) / 2, (A[1] + B[1]) / 2)
E = ((B[0] + C[0]) / 2, (B[1] + C[1]) / 2)
F = ((A[0] + C[0]) / 2, (A[1] + C[1]) / 2)

# Plotting the triangle and midpoints
fig, ax = plt.subplots()

# Plot the triangle ABC
triangle = plt.Polygon([A, B, C], closed=True, fill=None, edgecolor='black', label='Triangle ABC')

# Plot the medians AD, BE, CF
plt.plot([A[0], E[0]], [A[1], E[1]], color='g', label='Median AE')
plt.plot([B[0], F[0]], [B[1], F[1]], color='r', label='Median BF')
plt.plot([C[0], D[0]], [C[1], D[1]], color='b', label='Median CD')
```

File: plot.py

```python
# Plot the midpoints D, E, F
ax.scatter(*zip(A, B, C, D, E, F), color='gray')
ax.text(A[0], A[1], 'A', fontsize=12, ha='left', va='top', color='g')
ax.text(B[0], B[1], 'B', fontsize=12, ha='right', va='top', color='r')
ax.text(C[0], C[1], 'C', fontsize=12, ha='left', va='top', color='b')
ax.text(D[0], D[1], 'D', fontsize=12, ha='right', va='bottom', color='b')
ax.text(E[0], E[1], 'E', fontsize=12, ha='right', va='bottom', color='g')
ax.text(F[0], F[1], 'F', fontsize=12, ha='right', va='bottom', color='r')

# Labels and settings
ax.add_patch(triangle)
ax.set_aspect('equal', adjustable='box')
ax.set_xlabel('x')
ax.set_ylabel('y')
plt.title('Triangle ABC with Midpoints and Medians')
plt.grid(True)
plt.legend()
plt.show()
```