MatGeo Presentation - Problem 5.5.16

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Question

$$\mathbf{A} = \begin{pmatrix} 2 & -3 & 5 \\ 3 & -2 & -4 \\ 1 & 1 & -2 \end{pmatrix} \tag{0.1}$$

Find $\mathbf{A}^{-1}$. Use it to solve the given system of equations

$$2x - 3y + 5z = 11 \tag{0.2}$$

$$3x - 2y - 4z = -5 \tag{0.3}$$

$$x + y - 2z = -3 \tag{0.4}$$

Solution

$\rightarrow$ Using the properties of inverses

$$\mathbf{A}\mathbf{A}^{-1} = \mathbf{I}_{3\times 3} \tag{0.5}$$

$$\begin{pmatrix} 2 & -3 & 5 \\ 3 & -2 & -4 \\ 1 & 1 & -2 \end{pmatrix} \mathbf{A}^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{0.6}$$

$\rightarrow$ Now, using augmented matrix

$$\left(\begin{array}{ccc|ccc} 2 & -3 & 5 & 1 & 0 & 0 \\ 3 & -2 & -4 & 0 & 1 & 0 \\ 1 & 1 & -2 & 0 & 0 & 1 \end{array}\right) \xrightarrow{R_1 \leftrightarrow (1/2)R_1} \left(\begin{array}{ccc|ccc} 1 & -3/2 & 5/2 & 1/2 & 0 & 0 \\ 3 & -2 & -4 & 0 & 1 & 0 \\ 1 & 1 & -2 & 0 & 0 & 1 \end{array}\right) \tag{0.7}$$

$$\xrightarrow{R_2 \leftrightarrow R_2 - 3R_1} \left(\begin{array}{ccc|ccc} 1 & -3/2 & 5/2 & 1/2 & 0 & 0 \\ 0 & 5/2 & -23/2 & -3/2 & 1 & 0 \\ 1 & 1 & -2 & 0 & 0 & 1 \end{array}\right) \tag{0.8}$$

Solution

$$\xrightarrow{R_3 \leftrightarrow R_3 - R_1} \left(\begin{array}{ccc|ccc} 1 & -3/2 & 5/2 & 1/2 & 0 & 0 \\ 0 & 5/2 & -23/2 & -3/2 & 1 & 0 \\ 0 & 5/2 & -9/2 & -1/2 & 0 & 1 \end{array}\right) \tag{0.9}$$

$$\xrightarrow{R_3 \leftrightarrow R_3 - R_2} \left(\begin{array}{ccc|ccc} 1 & -3/2 & 5/2 & 1/2 & 0 & 0 \\ 0 & 5/2 & -23/2 & -3/2 & 1 & 0 \\ 0 & 0 & 7 & 1 & -1 & 1 \end{array}\right) \tag{0.10}$$

$$\xrightarrow{R_2 \leftrightarrow (2/5) R_2} \left(\begin{array}{ccc|ccc} 1 & -3/2 & 5/2 & 1/2 & 0 & 0 \\ 0 & 1 & -23/5 & -3/5 & 2/5 & 0 \\ 0 & 0 & 7 & 1 & -1 & 1 \end{array}\right) \tag{0.11}$$

$$\xrightarrow{R_3 \leftrightarrow (1/7) R_3} \left(\begin{array}{ccc|ccc} 1 & -3/2 & 5/2 & 1/2 & 0 & 0 \\ 0 & 1 & -23/5 & -3/5 & 2/5 & 0 \\ 0 & 0 & 1 & 1/7 & -1/7 & 1/7 \end{array}\right) \tag{0.12}$$

Solution

$$\xrightarrow{R_1 \leftrightarrow R_1 + (3/2)R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & -22/5 & -2/5 & 3/5 & 0 \\ 0 & 1 & -23/5 & -3/5 & 2/5 & 0 \\ 0 & 0 & 1 & 1/7 & -1/7 & 1/7 \end{array}\right) \tag{0.13}$$

$$\xrightarrow{R_2 \leftrightarrow R_2 + (23/5)R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & -22/5 & -2/5 & 3/5 & 0 \\ 0 & 1 & 0 & 2/35 & -9/35 & 23/35 \\ 0 & 0 & 1 & 1/7 & -1/7 & 1/7 \end{array}\right) \tag{0.14}$$

$$\xrightarrow{R_1 \leftrightarrow R_1 + (22/5)R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 8/35 & -1/35 & 22/35 \\ 0 & 1 & 0 & 2/35 & -9/35 & 23/35 \\ 0 & 0 & 1 & 1/7 & -1/7 & 1/7 \end{array}\right) \tag{0.15}$$

$$\Longrightarrow \mathbf{A}^{-1} = \frac{1}{35}\begin{pmatrix} 8 & -1 & 22 \\ 2 & -9 & 23 \\ 5 & -5 & 5 \end{pmatrix} \tag{0.16}$$

Solution

$\rightarrow$ From (2), (3) and (4), we get

$$\underbrace{\begin{pmatrix} 2 & -3 & 5 \\ 3 & -2 & -4 \\ 1 & 1 & -2 \end{pmatrix}}_{\mathbf{A}} \underbrace{\begin{pmatrix} x \\ y \\ z \end{pmatrix}}_{\mathbf{x}} = \underbrace{\begin{pmatrix} 11 \\ -5 \\ -3 \end{pmatrix}}_{\mathbf{B}} \tag{0.17}$$

$\rightarrow$ So, we compute

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{B} \tag{0.18}$$

$$\mathbf{x} = \frac{1}{35} \begin{pmatrix} 8 & -1 & 22 \\ 2 & -9 & 23 \\ 5 & -5 & 5 \end{pmatrix} \begin{pmatrix} 11 \\ -5 \\ -3 \end{pmatrix} \tag{0.19}$$

$$\mathbf{x} = \begin{pmatrix} 27/35 \\ -2/35 \\ 13/7 \end{pmatrix} \tag{0.20}$$

Solution

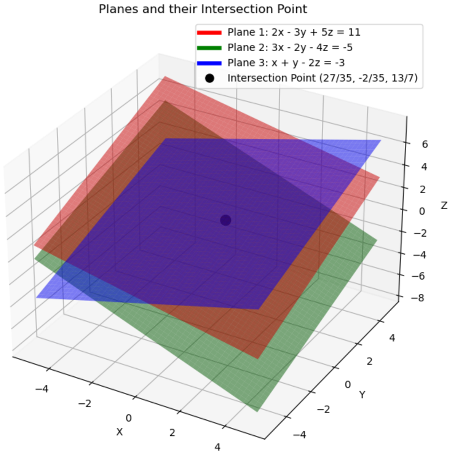

Figure: Plot of system of equations

File: points.c

```
#include <stdio.h>

int main() {
  FILE *fp;

  // -------------------
  // Question 5.5.16
  // -------------------


  fp = fopen("points.dat", "w");
  fprintf(fp, "%d,%d,%d\n", 2, -3, 5); // A
  fprintf(fp, "%d,%d,%d\n", 3, -2, -4); // B
  fprintf(fp, "%d,%d,%d\n", 1, 1, -2); // C
  fprintf(fp, "%f,%f,%f\n", 27/35, -2/35, 13/7); // P
  fclose(fp);
  return 0;
}
```

File: call_c.py

```python
import subprocess

# Compile the C program
subprocess.run(["gcc", "points.c", "-o", "points"])

# Run the compiled C program
result = subprocess.run(["./points"], capture_output=True, text=True)

# Print the output from the C program
print(result.stdout)
```

File: plot.py

```
import numpy as np
import matplotlib.pyplot as plt

# Plane functions:
def plane1(x, y):
    return (11 - 2*x + 3*y) / 5

def plane2(x, y):
    return -(3*x - 2*y + 5) / 4

def plane3(x, y):
    return (x + y + 3) / 2

# Create grid
x = np.linspace(-5, 5, 50)
y = np.linspace(-5, 5, 50)
X, Y = np.meshgrid(x, y)

Z1 = plane1(X, Y)
Z2 = plane2(X, Y)
Z3 = plane3(X, Y)

# Intersection point
x_int = 27/35
y_int = -2/35
z_int = 13/7

# Plotting
fig = plt.figure(figsize=(10, 8))
ax = fig.add_subplot(111, projection='3d')
```

File: plot.py

```
ax.plot_surface(X, Y, Z1, alpha=0.5, color='red', label='2x - 3y + 5z = 11')
ax.plot_surface(X, Y, Z2, alpha=0.5, color='green', label='3x - 2y - 4z = -5')
ax.plot_surface(X, Y, Z3, alpha=0.5, color='blue', label='x + y - 2z = -3')

# Plot the intersection point
ax.scatter(x_int, y_int, z_int, color='black', s=100, label='Intersection Point')

# Labels and title
ax.set_xlabel('X')
ax.set_ylabel('Y')
ax.set_zlabel('Z')
ax.set_title('Planes and their Intersection Point')

# Legend (manual, since plot_surface doesn't handle labels well)
from matplotlib.lines import Line2D
custom_lines = [Line2D([0], [0], color='red', lw=4),
                Line2D([0], [0], color='green', lw=4),
                Line2D([0], [0], color='blue', lw=4),
                Line2D([0], [0], marker='o', color='w', markerfacecolor='black', markersize=10)]
ax.legend(custom_lines, ['Plane 1: 2x - 3y + 5z = 11',
                         'Plane 2: 3x - 2y - 4z = -5',
                         'Plane 3: x + y - 2z = -3',
                         'Intersection Point (27/35, -2/35, 13/7)'])

plt.show()
```