11.2.5

EE25BTECH11064 - Yojit Manral

Question:
In $\triangle ABC$, $\mathbf{D}$, $\mathbf{E}$ and $\mathbf{F}$ are, respectively, the mid-points of sides AB, BC and CA. Show that $\triangle ABC$ is divided into four congruent triangles by joining $\mathbf{D}$, $\mathbf{E}$, and $\mathbf{F}$.

Solution:

→ Given that

$$\mathbf{D} = \frac{\mathbf{A}+\mathbf{B}}{2} \qquad \mathbf{E} = \frac{\mathbf{B}+\mathbf{C}}{2} \qquad \mathbf{F} = \frac{\mathbf{C}+\mathbf{A}}{2} \tag{1}$$

→ From (1), it follows that

$$\mathbf{A} = \mathbf{D}+\mathbf{F}-\mathbf{E} \qquad \mathbf{B} = \mathbf{E}+\mathbf{D}-\mathbf{F} \qquad \mathbf{C} = \mathbf{F}+\mathbf{E}-\mathbf{D} \tag{2}$$

→ From (2), we get that

$$\text{In } \triangle FAD \text{ and } \triangle DEF \qquad \text{In } \triangle DBE \text{ and } \triangle DEF \qquad \text{In } \triangle ECF \text{ and } \triangle DEF \tag{3}$$

$$\mathbf{A}-\mathbf{D} = \mathbf{F}-\mathbf{E} \text{ (Side 1)} \qquad \mathbf{B}-\mathbf{E} = \mathbf{D}-\mathbf{F} \text{ (Side 1)} \qquad \mathbf{C}-\mathbf{F} = \mathbf{E}-\mathbf{D} \text{ (Side 1)} \tag{4}$$

$$\mathbf{A}-\mathbf{F} = \mathbf{D}-\mathbf{E} \text{ (Side 2)} \qquad \mathbf{B}-\mathbf{D} = \mathbf{E}-\mathbf{F} \text{ (Side 2)} \qquad \mathbf{C}-\mathbf{E} = \mathbf{F}-\mathbf{D} \text{ (Side 2)} \tag{5}$$

$$\mathbf{D}-\mathbf{F} \text{ is common to both} \qquad \mathbf{E}-\mathbf{D} \text{ is common to both} \qquad \mathbf{F}-\mathbf{E} \text{ is common to both} \tag{6}$$

$$\triangle FAD \cong \triangle DEF (\text{SSS criterion}) \qquad \triangle DBE \cong \triangle DEF (\text{SSS criterion}) \qquad \triangle ECF \cong \triangle DEF (\text{SSS criterion}) \tag{7}$$

→ From (7), we know that $\triangle ABC$ is divided into four congruent triangles

$$\triangle FAD \cong \triangle DBE \cong \triangle ECF \cong \triangle DEF \tag{8}$$

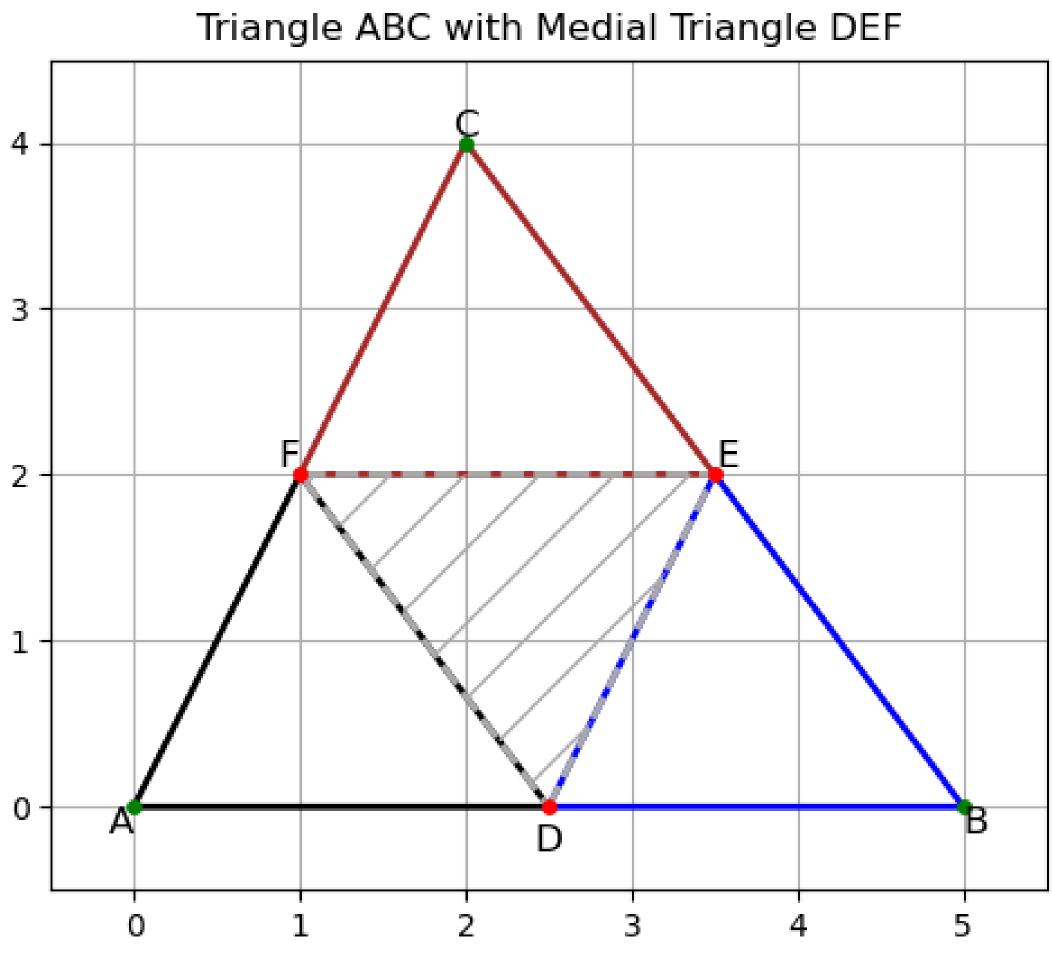

Fig. 0: Plot of $\triangle ABC$ and its medial triangle $\triangle DEF$