MatGeo Presentation - Problem 1.6.14

EE25BTECH11064 - Yojit Manral

Question

For which values of a and b does the following pair of linear equations have an infinite number of solutions?

$$2x + 3y = 7$$
$$(a - b)x + (a + b)y = 3a + b - 2$$

Solution

→ The equation of the lines can be written as

$$\mathbf{n_1}^T\mathbf{x} = c_1 \tag{0.1}$$
$$\mathbf{n_2}^T\mathbf{x} = c_2 \tag{0.2}$$

where,

$$\mathbf{n_1} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \tag{0.3}$$
$$\mathbf{n_2} = \begin{pmatrix} a-b \\ a+b \end{pmatrix} \tag{0.4}$$
$$c_1 = 7 \tag{0.5}$$
$$c_2 = 3a+b-2 \tag{0.6}$$

Solution

→ For two lines to have infinite solutions,,

$$\mathbf{n_1} = \alpha \mathbf{n_2} \tag{0.7}$$

$$c_1 = \alpha c_2 \tag{0.8}$$

$$\implies c_2 \mathbf{n_1} = c_1 \mathbf{n_2} \tag{0.9}$$

→ Substituting the values from (3), (4), (5), and (6) in (9)

$$(3a + b - 2) \begin{pmatrix} 2 \\ 3 \end{pmatrix} = 7 \begin{pmatrix} a - b \\ a + b \end{pmatrix} \tag{0.10}$$

$$\begin{pmatrix} 6a + 2b \\ 9a + 3b \end{pmatrix} - \begin{pmatrix} 7a - 7b \\ 7a + 7b \end{pmatrix} = \begin{pmatrix} -a + 9b \\ 2a - 4b \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix} \tag{0.11}$$

$$\implies \begin{pmatrix} -1 & 9 \\ 2 & -4 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix} \tag{0.12}$$

Solution

$\rightarrow$ Using row transformations

$$\begin{pmatrix} -1 & 9 \\ 2 & -4 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow -R_1} \begin{pmatrix} 1 & -9 \\ 2 & -4 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \tag{0.13}$$

$$\xrightarrow{R_2 \leftrightarrow R_2 - 2R_1} \begin{pmatrix} 1 & -9 \\ 0 & 14 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -4 \\ 14 \end{pmatrix} \tag{0.14}$$

$$\xrightarrow{R_2 \leftrightarrow (1/14)R_2} \begin{pmatrix} 1 & -9 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -4 \\ 1 \end{pmatrix} \tag{0.15}$$

$$\xrightarrow{R_1 \leftrightarrow R_1 + 9R_2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \end{pmatrix} \tag{0.16}$$

$$\implies \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \end{pmatrix} \implies a = 5 \text{ and } b = 1 \tag{0.17}$$

Solution

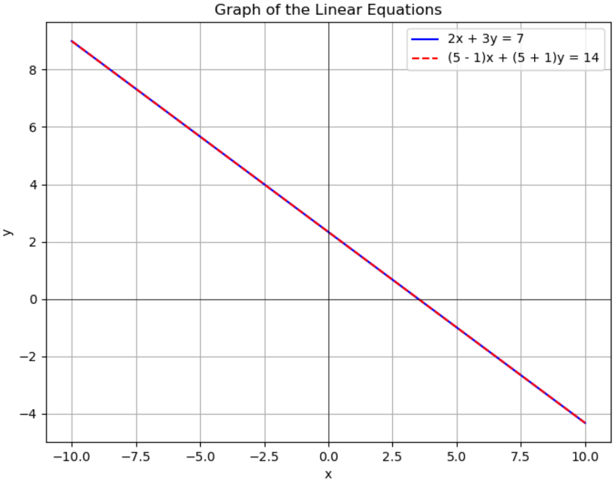

Figure: Plot of the lines

File: plot.py

```
import numpy as np
import matplotlib.pyplot as plt

# Set the values of a and b
a = 5
b = 1

# Define the equations
def line_eq1(x):
    return (7 - 2*x) / 3 # y = (7 - 2x) / 3

def line_eq2(x, a, b):
    return (3*a + b - 2 - (a - b)*x) / (a + b) # y = (3a + b - 2 - (a - b)x) / (a + b)

# Generate x values
x_vals = np.linspace(-10, 10, 400)

# Calculate the corresponding y values for both lines
y_vals_eq1 = line_eq1(x_vals)
y_vals_eq2 = line_eq2(x_vals, a, b)
```

File: plot.py

```
# Plot the lines
plt.figure(figsize=(8, 6))
plt.plot(x_vals, y_vals_eq1, label="2x␣+␣3y␣=␣7", color="blue")
plt.plot(x_vals, y_vals_eq2, label=f"({a}␣-␣{b})x␣+␣({a}␣+␣{b})y␣=␣{3*a␣+␣b␣-␣2}", color="red", linestyle="
      --")

# Add labels and title
plt.xlabel('x')
plt.ylabel('y')
plt.title('Graph␣of␣the␣Linear␣Equations')
plt.axhline(0, color='black',linewidth=0.5)
plt.axvline(0, color='black',linewidth=0.5)

# Add a legend
plt.legend()

# Show the plot
plt.grid(True)
plt.show()
```