MatGeo Presentation - Problem 2.5.32

EE25BTECH11064 - Yojit Manral

Question

Show that the points $(7, 10)$, $(-2, 5)$ and $(3, 4)$ are vertices of an isosceles right triangle.

Solution

Points	Name
$\begin{pmatrix} 7 \\ 10 \end{pmatrix}$	Point **A**
$\begin{pmatrix} -2 \\ 5 \end{pmatrix}$	Point **B**
$\begin{pmatrix} 3 \\ 4 \end{pmatrix}$	Point **C**

Table: List of Points

$\rightarrow$ Let

$$\angle A = \angle BAC \tag{0.1}$$
$$\angle B = \angle CBA \tag{0.2}$$
$$\angle C = \angle ACB \tag{0.3}$$

Solution

$$\mathbf{a} = \mathbf{C} - \mathbf{B} = \begin{pmatrix} 5 \\ -1 \end{pmatrix} \tag{0.4}$$

$$\mathbf{b} = \mathbf{A} - \mathbf{C} = \begin{pmatrix} 4 \\ 6 \end{pmatrix} \tag{0.5}$$

$$\mathbf{c} = \mathbf{B} - \mathbf{A} = \begin{pmatrix} -9 \\ -5 \end{pmatrix} \tag{0.6}$$

$\rightarrow$ Then the cosines of the angles are

$$\cos A = -\frac{\mathbf{b}^T\mathbf{c}}{\|\mathbf{b}\|\|\mathbf{c}\|} = \frac{66}{\sqrt{52}\sqrt{106}} = 0.889 \tag{0.7}$$

$$\cos B = -\frac{\mathbf{c}^T\mathbf{a}}{\|\mathbf{c}\|\|\mathbf{a}\|} = \frac{40}{\sqrt{106}\sqrt{26}} = 0.762 \tag{0.8}$$

$$\cos C = -\frac{\mathbf{a}^T\mathbf{b}}{\|\mathbf{a}\|\|\mathbf{b}\|} = -\frac{14}{\sqrt{26}\sqrt{52}} = -0.381 \tag{0.9}$$

Solution

$\rightarrow$ As none of the cosines are equal, $\triangle ABC$ is not isosceles. Also, since $\cos C$ is negative, $\triangle ABC$ forms an obtuse angled triangle.

$\Longrightarrow$ $\triangle ABC$ is neither isosceles nor right-angled triangle.

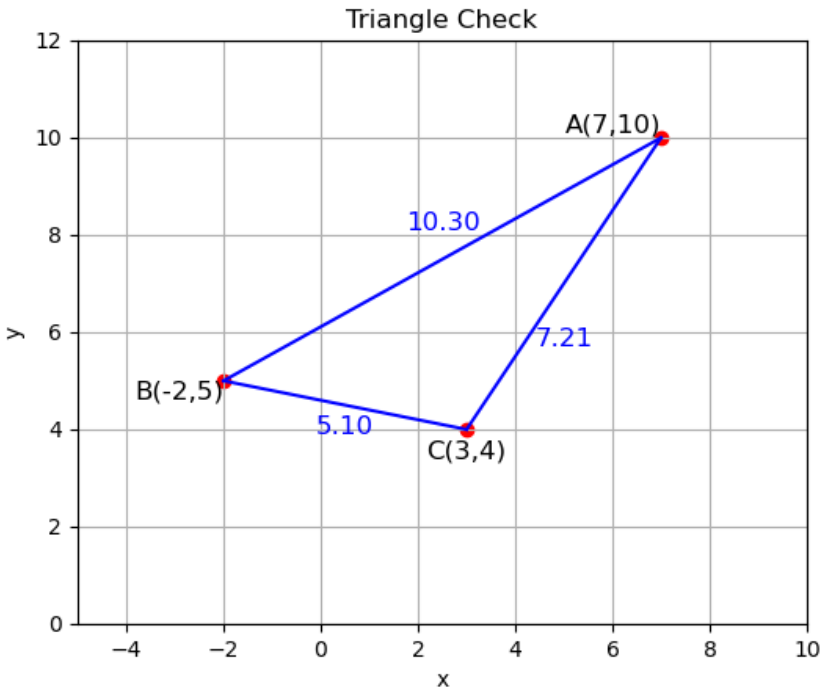

Figure: Plot of $\triangle ABC$

File: points.c

```
#include <stdio.h>

int main() {
  FILE *fp;

  // -------------------
  // Question 2.5.32
  // -------------------


  fp = fopen("points.dat", "w");
  fprintf(fp, "%d,%d,%d\n", 7, 10, 0); // A
  fprintf(fp, "%d,%d,%d\n", -2, 5, 0); // B
  fprintf(fp, "%d,%d,%d\n", 3, 4, 0); // C
  fclose(fp);
  return 0;
  }
```

File: call_c.py

```
import subprocess

# Compile the C program
subprocess.run(["gcc", "points.c", "-o", "points"])

# Run the compiled C program
result = subprocess.run(["./points"], capture_output=True, text=True)

# Print the output from the C program
print(result.stdout)
```

File: plot.py

```
import matplotlib.pyplot as plt
import numpy as np

# Points A(7,10), B(-2,5), C(3,4)
A = np.array([7, 10])
B = np.array([-2, 5])
C = np.array([3, 4])

# Function to calculate distance between two points
def distance(p1, p2):
    return np.sqrt((p2[0] - p1[0])**2 + (p2[1] - p1[1])**2)

# Calculate distances
AB = distance(A, B)
BC = distance(B, C)
CA = distance(C, A)

# Plotting the triangle
plt.figure(figsize=(6, 6))
plt.plot([A[0], B[0]], [A[1], B[1]], 'b-', label="AB")
plt.plot([B[0], C[0]], [B[1], C[1]], 'b-', label="BC")
plt.plot([C[0], A[0]], [C[1], A[1]], 'b-', label="CA")

# Annotating points
plt.text(A[0], A[1], 'A(7,10)', fontsize=12, ha='right', va='bottom')
plt.text(B[0], B[1], 'B(-2,5)', fontsize=12, ha='right', va='top')
plt.text(C[0], C[1] - 0.2, 'C(3,4)', fontsize=12, ha='center', va='top')

# Highlighting the vertices
plt.scatter([A[0], B[0], C[0]], [A[1], B[1], C[1]], color='red')
```

File: plot.py

```
# Displaying distances on the plot with offset adjustments
mid_AB = (A + B) / 2
mid_BC = (B + C) / 2
mid_CA = (C + A) / 2

# Adjusting text placement for better spacing
plt.text(mid_AB[0], mid_AB[1] + 0.6, f'{AB:.2f}', fontsize=12, color='blue', ha='center')
plt.text(mid_BC[0], mid_BC[1] - 0.6, f'{BC:.2f}', fontsize=12, color='blue', ha='center')
plt.text(mid_CA[0], mid_CA[1] - 1.3, f'{CA:.2f}', fontsize=12, color='blue', ha='center')

# Setting plot limits and labels
plt.xlim(-5, 10)
plt.ylim(0, 12)
plt.gca().set_aspect('equal', adjustable='box')
plt.xlabel('x')
plt.ylabel('y')
plt.title('Triangle␣Check')

# Show the plot
plt.grid(True)
plt.show()
```