

# MatGeo Presentation - Problem 12.493

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## Question

Characteristic equation of the matrix with eigenvalue  $\lambda$  is

$$\mathbf{A} = \begin{pmatrix} 2 & \sqrt{2} \\ \sqrt{2} & 1 \end{pmatrix} \quad (0.1)$$

(a)  $\lambda^2 + 3\lambda + 4 = 0$

(c)  $\lambda^2 - 3\lambda = 0$

(b)  $\lambda^2 + 3\lambda - 2 = 0$

(d)  $\lambda^2 + 3\lambda = 0$

## Solution

→ This problem statement can be solved via the following two methods:

1) The characteristic equation can be given by

$$\text{char}(\mathbf{A}) \implies |\mathbf{A} - \lambda \mathbf{I}| = 0 \quad (0.2)$$

$$\begin{vmatrix} 2 - \lambda & \sqrt{2} \\ \sqrt{2} & 1 - \lambda \end{vmatrix} = 0 \quad (0.3)$$

$$(\lambda - 2)(\lambda - 1) - 2 = 0 \quad (0.4)$$

$$\lambda^2 - 3\lambda = 0 \quad (0.5)$$

## Solution

2) Another method to find characteristic equation for a  $2 \times 2$  matrix is

$$\text{char}(\mathbf{M}_{2 \times 2}) \implies \lambda^2 - \text{trace}(\mathbf{M})\lambda + \det(\mathbf{M}) = 0 \quad (0.6)$$

For the given matrix  $\mathbf{A}$ , we have

$$\text{trace}(\mathbf{A}) = 3 \qquad \det(\mathbf{A}) = 0 \quad (0.7)$$

From (6) and (7), we get

$$\text{char}(\mathbf{A}) \implies \lambda^2 - 3\lambda = 0 \quad (0.8)$$

→ Therefore, (c)  $\lambda^2 - 3\lambda = 0$  is the correct option.