MatGeo Presentation - Problem 4.3.55

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Question

Find the vector equation of the plane passing through the points $\mathbf{R}\,(2, 5, -3)$, $\mathbf{S}\,(-2, -3, 5)$ and $\mathbf{T}\,(5, 3, -3)$.

Solution

Points	**Name**
$\begin{pmatrix} 2 \\ 5 \\ -3 \end{pmatrix}$	Point **R**
$\begin{pmatrix} -2 \\ -3 \\ 5 \end{pmatrix}$	Point **S**
$\begin{pmatrix} 5 \\ 3 \\ -3 \end{pmatrix}$	Point **T**

Table: List of Points

$\rightarrow$ We can write the equation for the required plane as

$$\mathbf{n}^T\mathbf{x} = c \tag{0.1}$$

$\rightarrow$ Also, **R**, **S** and **T** satisfy this equation. Hence

Solution

$$\mathbf{n}^T\mathbf{R} = c \tag{0.2}$$

$$\mathbf{n}^T\mathbf{S} = c \tag{0.3}$$

$$\mathbf{n}^T\mathbf{T} = c \tag{0.4}$$

$\rightarrow$ From (1), (2), (3) and (4), we get

$$\mathbf{n}^T\begin{pmatrix}\mathbf{R} & \mathbf{S} & \mathbf{T}\end{pmatrix} = c\begin{pmatrix}1 & 1 & 1\end{pmatrix} \tag{0.5}$$

$\rightarrow$ Using transpose on both sides, we get

$$\begin{pmatrix}\mathbf{R} & \mathbf{S} & \mathbf{T}\end{pmatrix}^T\mathbf{n} = c\begin{pmatrix}1\\1\\1\end{pmatrix} \tag{0.6}$$

$$\mathbf{n} = c\left(\begin{pmatrix}\mathbf{R} & \mathbf{S} & \mathbf{T}\end{pmatrix}^T\right)^{-1}\begin{pmatrix}1\\1\\1\end{pmatrix} \tag{0.7}$$

Solution

$$= c\left(\begin{pmatrix} 2 & -2 & 5 \\ 5 & -3 & 3 \\ -3 & 5 & -3 \end{pmatrix}^T\right)^{-1}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{0.8}$$

$$= c\begin{pmatrix} 2 & 5 & -3 \\ -2 & -3 & 5 \\ 5 & 3 & -3 \end{pmatrix}^{-1}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{0.9}$$

$$= \frac{c}{56}\begin{pmatrix} -6 & 6 & 16 \\ 19 & 9 & -4 \\ 9 & 19 & 4 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{0.10}$$

$$= \frac{c}{7}\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \tag{0.11}$$

Solution

$\rightarrow$ From (11), we get the value of

$$\mathbf{n} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \tag{0.12}$$

$\rightarrow$ From (2) and (12), we get

$$c = \begin{pmatrix} 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 2 \\ 5 \\ -3 \end{pmatrix} = 7 \tag{0.13}$$

Solution

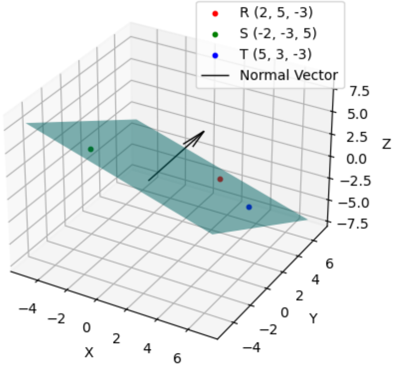

Figure: Plot of plane $\mathbf{n}^T\mathbf{x} = c$

File: points.c

```
#include <stdio.h>

int main() {
  FILE *fp;

  // -------------------
  // Question 4.3.55
  // -------------------


  fp = fopen("points.dat", "w");
  fprintf(fp, "%d,%d,%d\n", 2, 5, -3); // R
  fprintf(fp, "%d,%d,%d\n", -2, -3, 5); // S
  fprintf(fp, "%d,%d,%d\n", 5, 3, -3); // T
  fclose(fp);
  return 0;
}
```

File: call_c.py

```
import subprocess

# Compile the C program
subprocess.run(["gcc", "points.c", "-o", "points"])

# Run the compiled C program
result = subprocess.run(["./points"], capture_output=True, text=True)

# Print the output from the C program
print(result.stdout)
```

File: plot.py

```
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

# Define the points
R = np.array([2, 5, -3])
S = np.array([-2, -3, 5])
T = np.array([5, 3, -3])
normal = np.array([2, 3, 4])
# Create a grid to plot the plane
x = np.linspace(-5, 7, 10)
y = np.linspace(-5, 7, 10)
X, Y = np.meshgrid(x, y)

# Equation of the plane: Ax + By + Cz = D
A, B, C = normal
D = np.dot(normal, R)

# Solve for Z
Z = (D - A * X - B * Y) / C

# Plotting the plane and the points R, S, T
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')

# Plot the points
ax.scatter(*R, color='r', label='R (2, 5, -3)', s=10)
ax.scatter(*S, color='g', label='S (-2, -3, 5)', s=10)
ax.scatter(*T, color='b', label='T (5, 3, -3)', s=10)
```

File: plot.py

```
# Plot the plane
ax.plot_surface(X, Y, Z, alpha=0.5, rstride=100, cstride=100, color='c')

# Plot the normal vector starting from point R
ax.quiver(0, 0, 0, normal[0], normal[1], normal[2], color='k', length=1, linewidth=1, label='Normal␣Vector'
      )

# Labels and legend
ax.set_xlabel('X')
ax.set_ylabel('Y')
ax.set_zlabel('Z')
ax.legend()

# Display plot
plt.show()
```