5.2.66

EE25BTECH11064 - Yojit Manral

Question:
Solve the system of equations

$$2x + 3y = 11 \tag{1}$$
$$2x + 4y = -24 \tag{2}$$

Hence, find the value of m for which

$$y = mx + 3 \tag{3}$$

Solution:
→ We have

$$\mathbf{n_1}^T\mathbf{x} = c_1 \qquad \mathbf{n_2}^T\mathbf{x} = c_2 \qquad \mathbf{n_3}^T\mathbf{x} = c_3$$

$$\mathbf{n_1} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \qquad c_1 = 11 \tag{4}$$
$$\mathbf{n_2} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \qquad c_2 = -24 \tag{5}$$
$$\mathbf{n_3} = \begin{pmatrix} -m \\ 1 \end{pmatrix} \qquad c_3 = 3 \tag{6}$$

→ To solve the system of equations, they must be concurrent at a point $\mathbf{x}$

$$\begin{pmatrix} \mathbf{n_1}^T \\ \mathbf{n_2}^T \\ \mathbf{n_3}^T \end{pmatrix}\mathbf{x} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} \implies \begin{pmatrix} 2 & 3 \\ 2 & 4 \\ -m & 1 \end{pmatrix}\mathbf{x} = \begin{pmatrix} 11 \\ -24 \\ 3 \end{pmatrix} \tag{7}$$

→ Using augmented matrix

$$\left(\begin{array}{cc|c} 2 & 3 & 11 \\ 2 & 4 & -24 \\ -m & 1 & 3 \end{array}\right) \xrightarrow[R_1 \leftrightarrow R_1 - 3R_2]{R_2 \leftrightarrow R_2 - R_1} \left(\begin{array}{cc|c} 2 & 0 & 116 \\ 0 & 1 & -35 \\ -m & 1 & 3 \end{array}\right) \tag{8}$$

$$\xrightarrow[R_3 \leftrightarrow -R_3 + R_2]{R_1 \leftrightarrow (1/2)R_1} \left(\begin{array}{cc|c} 1 & 0 & 58 \\ 0 & 1 & -35 \\ m & 0 & -38 \end{array}\right) \tag{9}$$

$$\xrightarrow{R_3 \leftrightarrow R_3 - mR_1} \left(\begin{array}{cc|c} 1 & 0 & 58 \\ 0 & 1 & -35 \\ 0 & 0 & -38 - 58m \end{array}\right) \tag{10}$$

→ For the system of equations to be consistent, we must have

$$-38 - 58m = 0 \implies m = -\frac{19}{29} \tag{11}$$

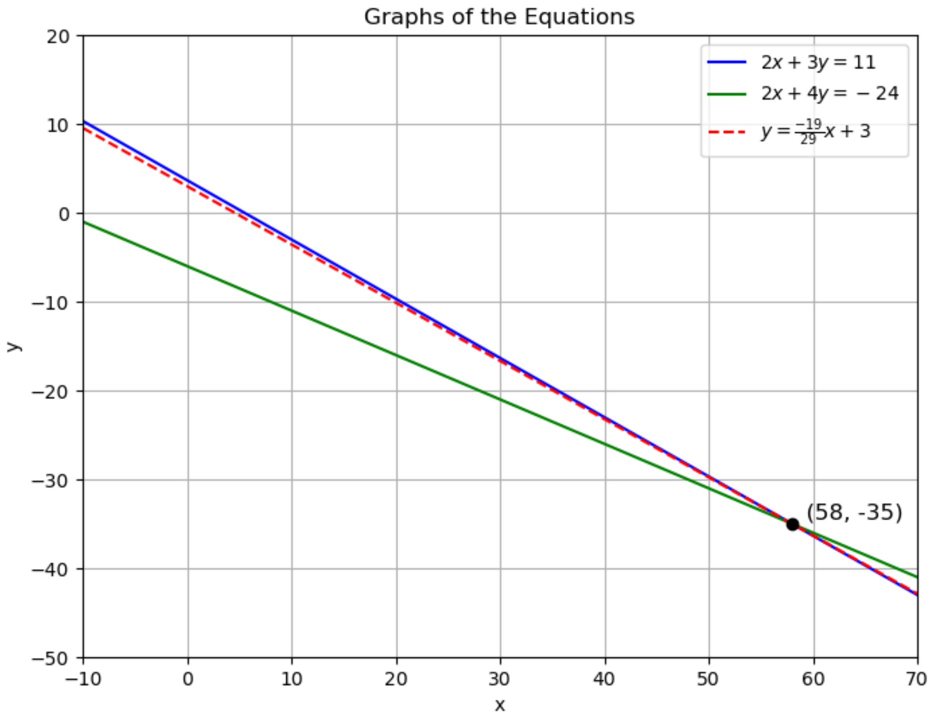

Fig. 0: Plot of the Equations