2.9.15

EE25BTECH11064 - Yojit Manral

Question:
If the points $\mathbf{A}\,(2,0)$, $\mathbf{B}\,(6,1)$, and $\mathbf{C}\,(p,q)$ form a triangle of area 12 square units(positive only) and

$$2p + q = 10 \tag{1}$$

then find the values of p and q.
Solution:

Points	Name
$\begin{pmatrix}2\\0\end{pmatrix}$	Point **A**
$\begin{pmatrix}6\\1\end{pmatrix}$	Point **B**
$\begin{pmatrix}p\\q\end{pmatrix}$	Point **C**

TABLE 0: List of Points

→ The are of the given △ABC can be given by

$$Area(\text{ABC}) = \frac{1}{2}\left|\begin{pmatrix}2 & 6 & p\\0 & 1 & q\\1 & 1 & 1\end{pmatrix}\right| \tag{2}$$

$$2 \times Area(\text{ABC}) = 2 \times \left|\begin{pmatrix}1 & q\\1 & 1\end{pmatrix}\right| - 6 \times \left|\begin{pmatrix}0 & q\\1 & 1\end{pmatrix}\right| + p \times \left|\begin{pmatrix}0 & 1\\1 & 1\end{pmatrix}\right| \tag{3}$$

$$= 2(1-q) - 6(0-q) + p(0-1) \tag{4}$$

$$= 2 + 4q - p \tag{5}$$

$$Area(\text{ABC}) = 12 \tag{6}$$

$$|4q - p + 2| = 24 \tag{7}$$

$$4q - p = \pm 24 - 2 \tag{8}$$

→ From (1) and (8), we get

$$\begin{pmatrix}2 & 1\\-1 & 4\end{pmatrix}\begin{pmatrix}p\\q\end{pmatrix} = \begin{pmatrix}10\\\pm 24 - 2\end{pmatrix} \tag{9}$$

$$\begin{pmatrix}p\\q\end{pmatrix} = \begin{pmatrix}2 & 1\\-1 & 4\end{pmatrix}^{-1}\begin{pmatrix}10\\\pm 24 - 2\end{pmatrix} \tag{10}$$

$$= \frac{1}{9}\begin{pmatrix}4 & -1\\1 & 2\end{pmatrix}\begin{pmatrix}10\\\pm 24 - 2\end{pmatrix} \tag{11}$$

$$\begin{pmatrix}p\\q\end{pmatrix} = \begin{pmatrix}2\\6\end{pmatrix} \text{ or } \begin{pmatrix}p\\q\end{pmatrix} = \begin{pmatrix}22/3\\-14/3\end{pmatrix} \tag{12}$$

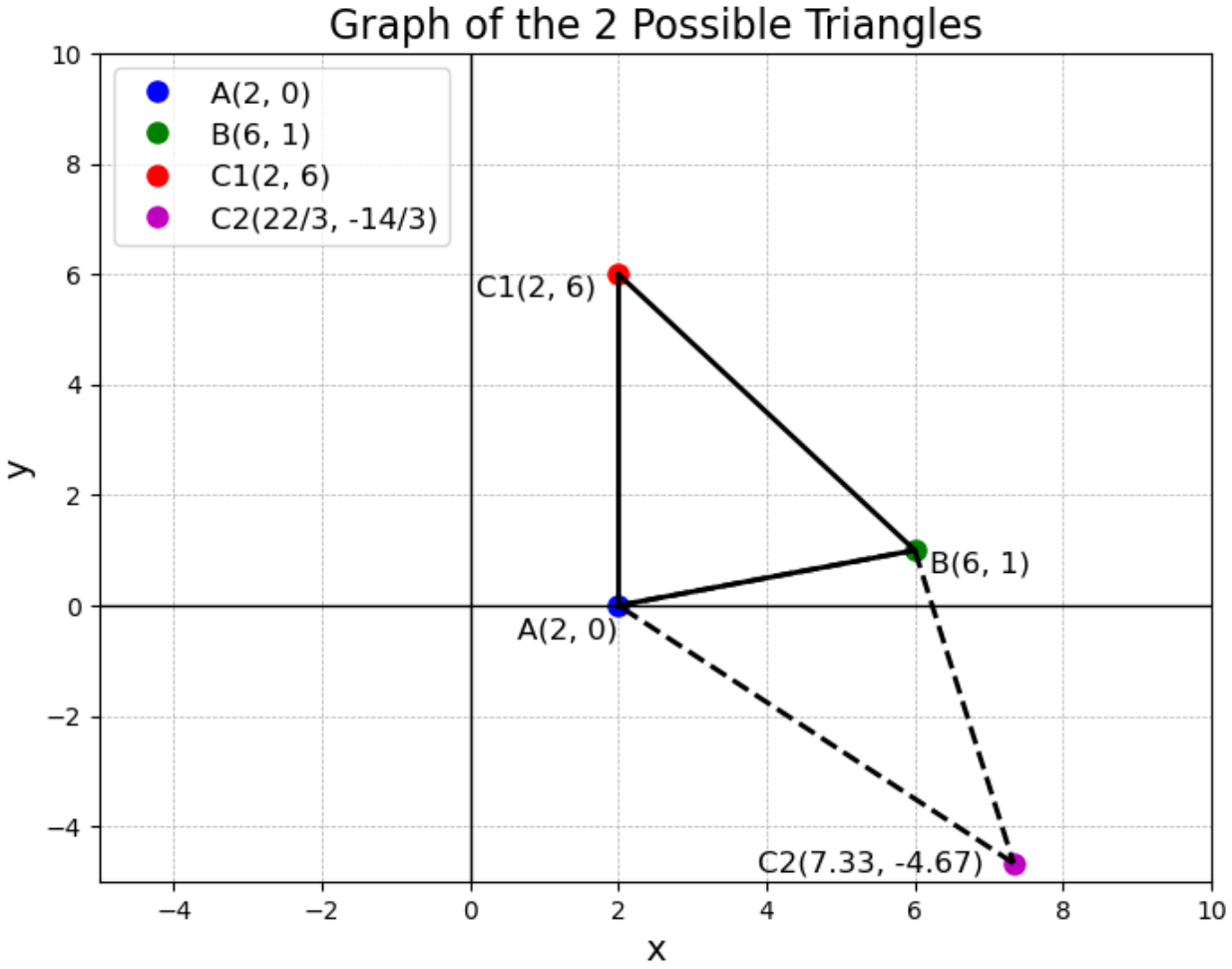

Fig. 0: Plot of points and triangles