

MatGeo Presentation - Problem 1.6.14

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Question

Prove or disprove: The straight line $5x + 4y = 0$ passes through the point of intersection of the straight lines $x + 2y - 10 = 0$ and $2x + y + 5 = 0$.

Solution

→ Let

$$\mathbf{n}_1^T \mathbf{x} = c_1$$

$$\mathbf{n}_2^T \mathbf{x} = c_2$$

$$\mathbf{n}_3^T \mathbf{x} = c_3$$

$$\mathbf{n}_1 = \begin{pmatrix} 5 \\ 4 \end{pmatrix} \qquad c_1 = 0 \qquad (0.1)$$

$$\mathbf{n}_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \qquad c_2 = 10 \qquad (0.2)$$

$$\mathbf{n}_3 = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \qquad c_3 = -5 \qquad (0.3)$$

→ The three lines are concurrent if there exists some \mathbf{x} such that

$$\begin{pmatrix} \mathbf{n}_1^T \\ \mathbf{n}_2^T \\ \mathbf{n}_3^T \end{pmatrix} \mathbf{x} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} \qquad (0.4)$$

$$\mathbf{A} \quad \mathbf{x} = \quad \mathbf{B} \qquad (0.5)$$

Solution

→ A unique \mathbf{x} exists iff the augmented matrix ($\mathbf{A}|\mathbf{B}$) is a full-rank matrix (i.e., rank 2 for a system of 2 variables).

$$(\mathbf{A}|\mathbf{B}) = \left(\begin{array}{cc|c} 5 & 4 & 0 \\ 1 & 2 & 10 \\ 2 & 1 & -5 \end{array} \right) \xrightarrow[R_2 \leftrightarrow R_2 - R_1]{R_1 \leftrightarrow (1/5)R_1} \left(\begin{array}{cc|c} 1 & 4/5 & 0 \\ 0 & 6/5 & 10 \\ 2 & 1 & -5 \end{array} \right) \quad (0.6)$$

$$\xrightarrow[R_2 \leftrightarrow (5/6)R_2]{R_3 \leftrightarrow R_3 - 2R_1} \left(\begin{array}{cc|c} 1 & 4/5 & 0 \\ 0 & 1 & 25/3 \\ 0 & -3/5 & -5 \end{array} \right) \quad (0.7)$$

$$\xrightarrow[R_1 \leftrightarrow R_1 - (4/5)R_2]{R_3 \leftrightarrow R_3 + (3/5)R_2} \left(\begin{array}{cc|c} 1 & 0 & -20/3 \\ 0 & 1 & 25/3 \\ 0 & 0 & 0 \end{array} \right) \quad (0.8)$$

$$\implies \mathbf{x} = \begin{pmatrix} -20/3 \\ 25/3 \end{pmatrix} \quad (0.9)$$

Only 2 nonzero rows → Rank of \mathbf{A} is 2 → The 3 lines are concurrent at \mathbf{x}

Solution

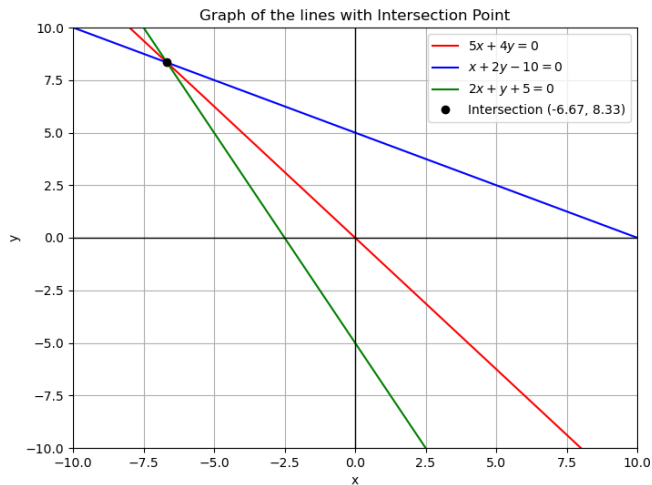


Figure: Plot of the three lines

File: points.c

```
#include <stdio.h>

int main() {
    FILE *fp;

    // -----
    // Question 4.13.68
    // -----

    fp = fopen("points.dat", "w");
    fprintf(fp, "%d,%d,%d\n", 5, 4, 0); // n1 c1
    fprintf(fp, "%d,%d,%d\n", 1, 2, 10); // n2 c2
    fprintf(fp, "%d,%d,%d\n", 2, 1, -5); // n3 c3
    fclose(fp);
    return 0;
}
```

File: call_c.py

```
import subprocess

# Compile the C program
subprocess.run(["gcc", "points.c", "-o", "points"])

# Run the compiled C program
result = subprocess.run(["./points"], capture_output=True, text=True)

# Print the output from the C program
print(result.stdout)
```

File: plot.py

```
import numpy as np
import matplotlib.pyplot as plt
from sympy import symbols, Eq, solve

# Define the symbols
x, y = symbols('x y')

# Define the equations
eq1 = Eq(5*x + 4*y, 0) #  $5x + 4y = 0$ 
eq2 = Eq(x + 2*y - 10, 0) #  $x + 2y - 10 = 0$ 

# Solve the system of two equations
solution = solve((eq1, eq2), (x, y))

# Extract the intersection point
intersection_x = solution[x]
intersection_y = solution[y]

# Define the lines
def line1(x):
    return (-5*x) / 4

def line2(x):
    return (10 - x) / 2

def line3(x):
    return (-2*x - 5)

# Generate x values
x_vals = np.linspace(-10, 10, 400)
```


File: plot.py

```
# Plot the lines
plt.figure(figsize=(8, 6))
plt.plot(x_vals, line1(x_vals), label=r'$5x_1+4y_1=0$', color='r')
plt.plot(x_vals, line2(x_vals), label=r'$x_1+2y_1-10=0$', color='b')
plt.plot(x_vals, line3(x_vals), label=r'$2x_1+y_1+5=0$', color='g')

# Mark the point of intersection
plt.plot(intersection_x, intersection_y, 'ko', label=f'Intersection_{intersection_x:.2f}_{intersection_y:.2f}')

# Set plot limits and labels
plt.xlim(-10, 10)
plt.ylim(-10, 10)
plt.axhline(0, color='black', linewidth=1)
plt.axvline(0, color='black', linewidth=1)
plt.grid(True)

# Labeling
plt.title('Graph of the lines with Intersection Point')
plt.xlabel('x')
plt.ylabel('y')

# Adding a legend
plt.legend()

# Show plot
plt.show()
```