12.597

EE25BTECH11064 - Yojit Manral

Question:

The value of p such that the vector $\begin{pmatrix}1\\2\\3\end{pmatrix}$ is an eigenvector of the matrix $\begin{pmatrix}4 & 1 & 2\\ p & 2 & 1\\ 14 & -4 & 10\end{pmatrix}$ is ___ .

Solution:

→ If the vector is an eigenvector for the matrix, it satisfies

$$\mathbf{Ax} = \lambda\mathbf{x} \tag{1}$$

$$\begin{pmatrix}4 & 1 & 2\\ p & 2 & 1\\ 14 & -4 & 10\end{pmatrix}\begin{pmatrix}1\\2\\3\end{pmatrix} = \lambda\begin{pmatrix}1\\2\\3\end{pmatrix} \quad \exists\lambda \in R \tag{2}$$

$$\begin{pmatrix}12\\ p+7\\ 36\end{pmatrix} = \lambda\begin{pmatrix}1\\2\\3\end{pmatrix} \tag{3}$$

→ Putting $\lambda = 12$ to satisfy (3), we get

$$\begin{pmatrix}12\\ p+7\\ 36\end{pmatrix} = \begin{pmatrix}12\\24\\36\end{pmatrix} \tag{4}$$

$$\implies p + 7 = 24 \tag{5}$$

$$\implies p = 15 \tag{6}$$