2.5.32

EE25BTECH11064 - Yojit Manral

Question:
Show that the points $(7, 10)$, $(-2, 5)$ and $(3, 4)$ are vertices of an isosceles right triangle.
Solution:

Points	Name
$\begin{pmatrix}7\\10\end{pmatrix}$	Point **A**
$\begin{pmatrix}-2\\5\end{pmatrix}$	Point **B**
$\begin{pmatrix}3\\4\end{pmatrix}$	Point **C**

TABLE 0: List of Points

→ The equation of the sides are given as

$$\mathbf{B}-\mathbf{A}=\begin{pmatrix}-9\\-5\end{pmatrix} \qquad \mathbf{C}-\mathbf{B}=\begin{pmatrix}5\\-1\end{pmatrix} \qquad \mathbf{A}-\mathbf{C}=\begin{pmatrix}4\\6\end{pmatrix} \tag{1}$$

→ The medians **D**, **E** and **F** of the triangle are

$$\mathbf{D}=\frac{\mathbf{A}+\mathbf{B}}{2}=\begin{pmatrix}5/2\\15/2\end{pmatrix} \qquad \mathbf{E}=\frac{\mathbf{B}+\mathbf{C}}{2}=\begin{pmatrix}1/2\\9/2\end{pmatrix} \qquad \mathbf{F}=\frac{\mathbf{C}+\mathbf{A}}{2}=\begin{pmatrix}5\\7\end{pmatrix} \tag{2}$$

(a) For an isosceles triangle, median to the base is also the perpendicular bisector. Using this property

$$(\mathbf{C}-\mathbf{D})^T(\mathbf{B}-\mathbf{A})=\begin{pmatrix}1/2 & -7/2\end{pmatrix}\begin{pmatrix}-9\\-5\end{pmatrix}=13\neq 0 \tag{3}$$

$$(\mathbf{A}-\mathbf{E})^T(\mathbf{C}-\mathbf{B})=\begin{pmatrix}13/2 & 11/2\end{pmatrix}\begin{pmatrix}5\\-1\end{pmatrix}=27\neq 0 \tag{4}$$

$$(\mathbf{B}-\mathbf{F})^T(\mathbf{A}-\mathbf{C})=\begin{pmatrix}-7 & -2\end{pmatrix}\begin{pmatrix}4\\6\end{pmatrix}=-40\neq 0 \tag{5}$$

→ Since none of the sides satisfy this property, the triangle is not isosceles.

(b) For a right triangle, dot product of the perpendicular sides must be zero.

$$(\mathbf{B}-\mathbf{A})^T(\mathbf{C}-\mathbf{B})=\begin{pmatrix}-9 & -5\end{pmatrix}\begin{pmatrix}5\\-1\end{pmatrix}=-40\neq 0 \tag{6}$$

$$(\mathbf{C}-\mathbf{B})^T(\mathbf{A}-\mathbf{C})=\begin{pmatrix}5 & -1\end{pmatrix}\begin{pmatrix}4\\6\end{pmatrix}=14\neq 0 \tag{7}$$

$$(\mathbf{A}-\mathbf{C})^T(\mathbf{B}-\mathbf{A})=\begin{pmatrix}4 & 6\end{pmatrix}\begin{pmatrix}-9\\-5\end{pmatrix}=-66\neq 0 \tag{8}$$

→ Since none of the dot products is zero, no two sides are perpendicular, and the triangle is not right angled.

⟶ From proofs (a) and (b) above, we can conclude that $\triangle ABC$ is not an isosceles right triangle.

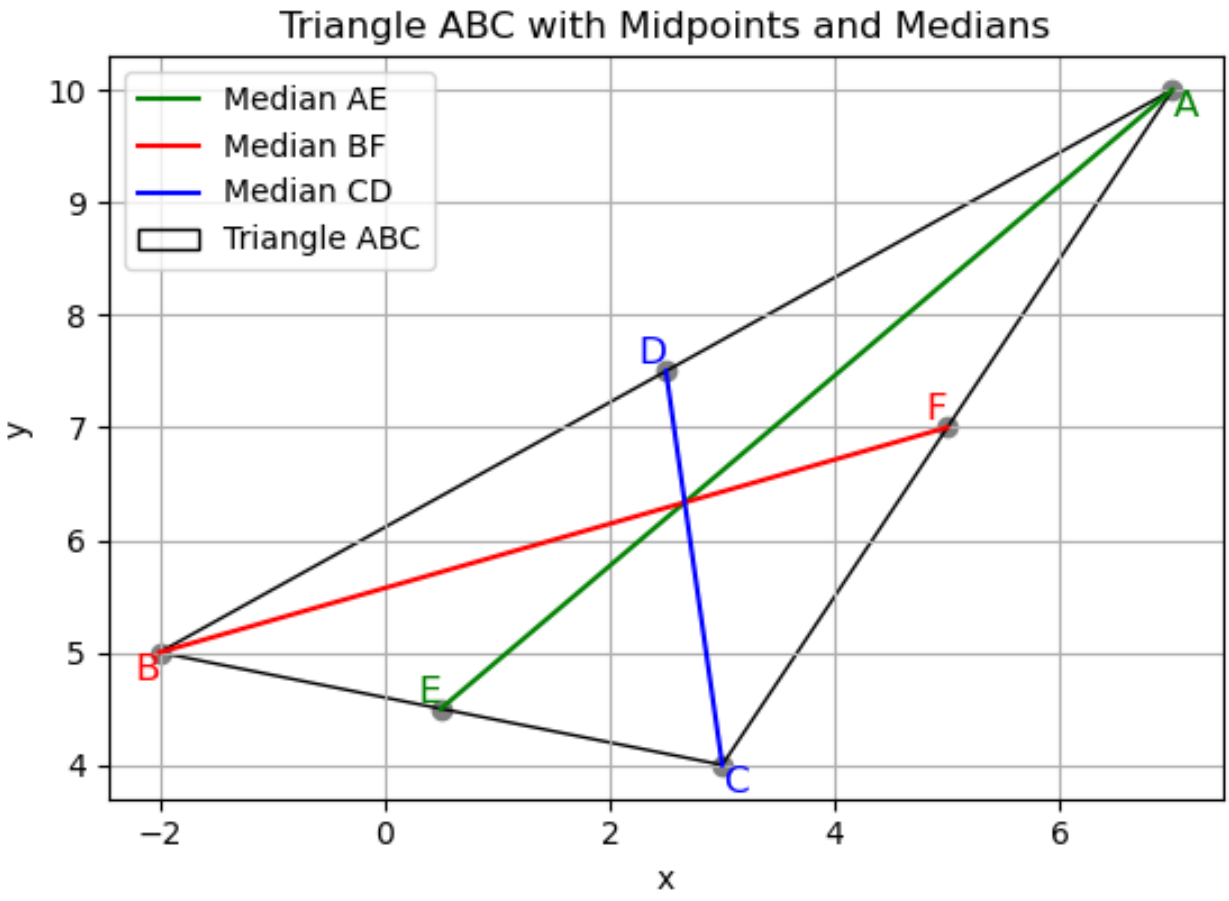

Fig. 2: Plot of △ABC