9.8.1

EE25BTECH11064 - Yojit Manral

Question:

The line $x + 3y = 0$ is the diameter of the circle $x^2 + y^2 - 6x + 2y = 0$.

Solution:

→ The given circle can be expressed as

$$\mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{1}$$

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}, f = 0 \tag{2}$$

→ Also, the center of the circle is

$$\mathbf{c} = -\mathbf{V}^{-1}\mathbf{u};\ |\mathbf{V}| \neq 0 \tag{3}$$

$$\mathbf{c} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} \tag{4}$$

→ The given line can be expressed as

$$\mathbf{x} = \mathbf{h} + \kappa\mathbf{m};\ \kappa \in R \tag{5}$$

$$\mathbf{h} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} \tag{6}$$

→ If the line is a diameter of the circle, the center of the circle must lie on the line

$$\mathbf{c} = \mathbf{h} + \lambda\mathbf{m}\ \ \exists\ \lambda \in R \tag{7}$$

$$\begin{pmatrix} 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \lambda\begin{pmatrix} 3 \\ -1 \end{pmatrix} \tag{8}$$

$$\lambda = 1 \in R \tag{9}$$

$\implies$ Center of circle, $\mathbf{c}$ lies on the line $\implies$ The line is a diameter of the circle

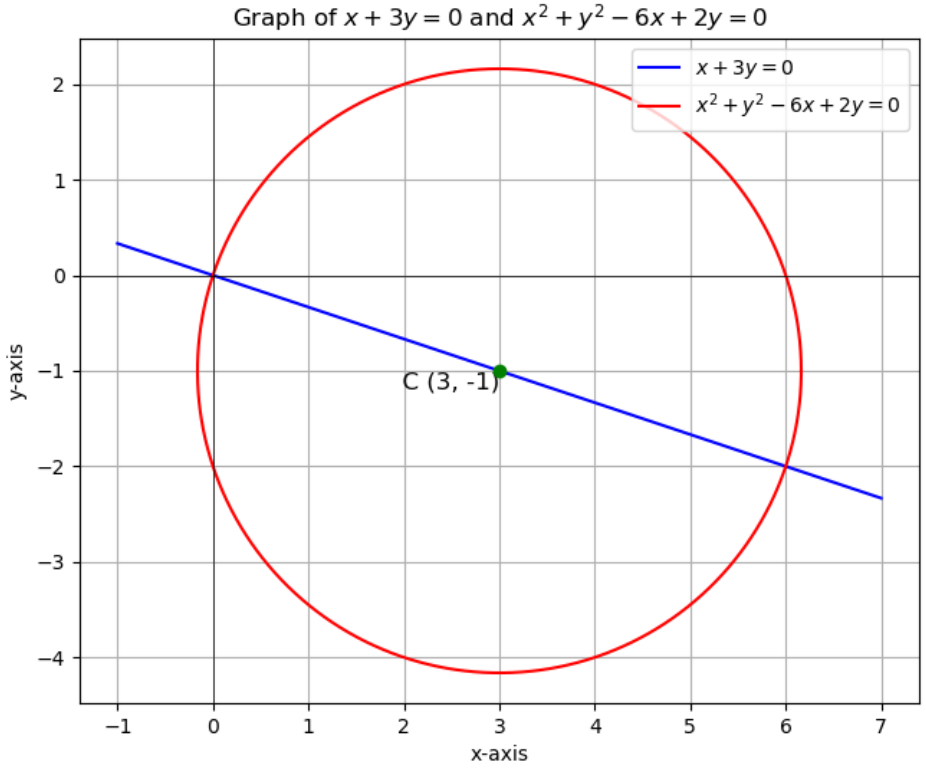

Fig. 0: Plot of given line and circle