4.12.1

EE25BTECH11064 - Yojit Manral

Question:

For which values of a and b does the following pair of linear equations have an infinite number of solutions?

$$2x + 3y = 7$$
$$(a - b)x + (a + b)y = 3a + b - 2$$

Solution:

→ The equation of the lines can be written as

$$\mathbf{n_1}^T\mathbf{x} = c_1 \tag{1}$$
$$\mathbf{n_2}^T\mathbf{x} = c_2 \tag{2}$$

where,

$$\mathbf{n_1} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \tag{3}$$
$$\mathbf{n_2} = \begin{pmatrix} a - b \\ a + b \end{pmatrix} \tag{4}$$
$$c_1 = 7 \tag{5}$$
$$c_2 = 3a + b - 2 \tag{6}$$

→ For two lines to have infinite solutions,,

$$\mathbf{n_1} = \alpha\mathbf{n_2} \tag{7}$$
$$c_1 = \alpha c_2 \tag{8}$$
$$\implies c_2\mathbf{n_1} = c_1\mathbf{n_2} \tag{9}$$

→ Substituting the values from (3), (4), (5), and (6) in (9)

$$(3a + b - 2)\begin{pmatrix} 2 \\ 3 \end{pmatrix} = 7\begin{pmatrix} a - b \\ a + b \end{pmatrix} \tag{10}$$
$$\begin{pmatrix} 6a + 2b \\ 9a + 3b \end{pmatrix} - \begin{pmatrix} 7a - 7b \\ 7a + 7b \end{pmatrix} = \begin{pmatrix} -a + 9b \\ 2a - 4b \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix} \tag{11}$$
$$\implies \begin{pmatrix} -1 & 9 \\ 2 & -4 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix} \tag{12}$$

→ Using row transformations

$$\begin{pmatrix} -1 & 9 \\ 2 & -4 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow -R_1} \begin{pmatrix} 1 & -9 \\ 2 & -4 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \tag{13}$$
$$\xrightarrow{R_2 \leftrightarrow R_2 - 2R_1} \begin{pmatrix} 1 & -9 \\ 0 & 14 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -4 \\ 14 \end{pmatrix} \tag{14}$$
$$\xrightarrow{R_2 \leftrightarrow (1/14)R_2} \begin{pmatrix} 1 & -9 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -4 \\ 1 \end{pmatrix} \tag{15}$$
$$\xrightarrow{R_1 \leftrightarrow R_1 + 9R_2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \end{pmatrix} \tag{16}$$
$$\implies \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \end{pmatrix} \implies a = 5 \text{ and } b = 1 \tag{17}$$

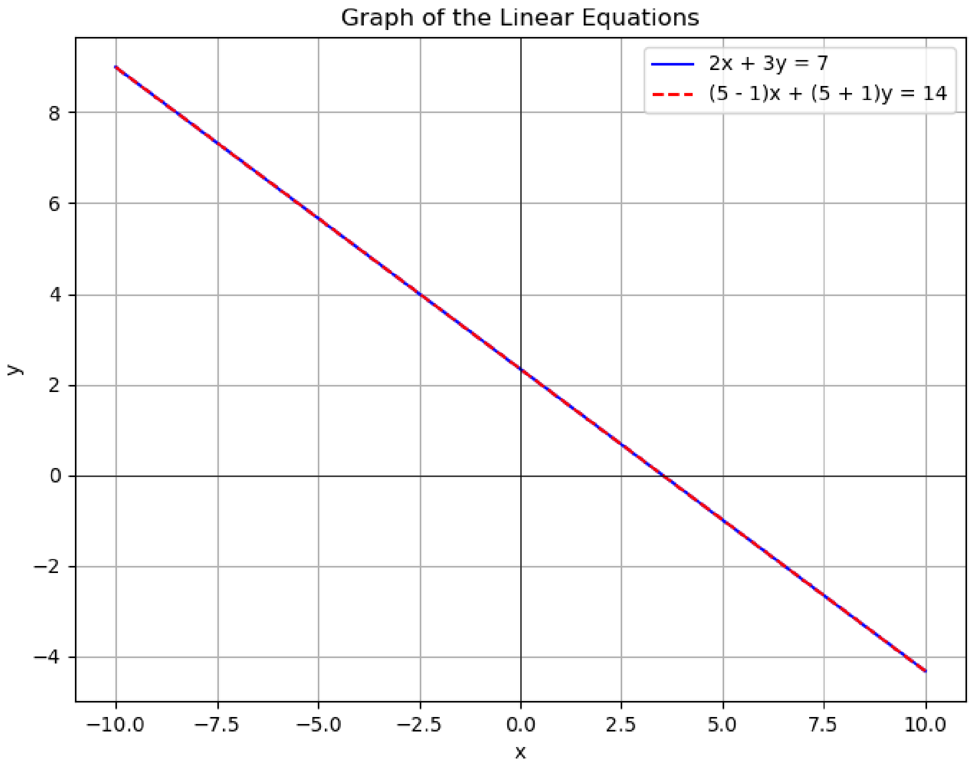

Fig. 0: Plot of the given lines