# 5.5.16

EE25BTECH11064 - Yojit Manral

**Question:**

$$\mathbf{A} = \begin{pmatrix} 2 & -3 & 5 \\ 3 & -2 & -4 \\ 1 & 1 & -2 \end{pmatrix} \tag{1}$$

Find $\mathbf{A}^{-1}$. Use it to solve the given system of equations

$$2x - 3y + 5z = 11 \tag{2}$$
$$3x - 2y - 4z = -5 \tag{3}$$
$$x + y - 2z = -3 \tag{4}$$

**Solution:**

→ Using the properties of inverses

$$\mathbf{A}\mathbf{A}^{-1} = \mathbf{I}_{3\times3} \tag{5}$$

$$\begin{pmatrix} 2 & -3 & 5 \\ 3 & -2 & -4 \\ 1 & 1 & -2 \end{pmatrix}\mathbf{A}^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{6}$$

→ Now, using augmented matrix

$$\left(\begin{array}{ccc|ccc} 2 & -3 & 5 & 1 & 0 & 0 \\ 3 & -2 & -4 & 0 & 1 & 0 \\ 1 & 1 & -2 & 0 & 0 & 1 \end{array}\right) \xrightarrow{R_1 \leftrightarrow (1/2)R_1} \left(\begin{array}{ccc|ccc} 1 & -3/2 & 5/2 & 1/2 & 0 & 0 \\ 3 & -2 & -4 & 0 & 1 & 0 \\ 1 & 1 & -2 & 0 & 0 & 1 \end{array}\right) \tag{7}$$

$$\xrightarrow{R_2 \leftrightarrow R_2 - 3R_1} \left(\begin{array}{ccc|ccc} 1 & -3/2 & 5/2 & 1/2 & 0 & 0 \\ 0 & 5/2 & -23/2 & -3/2 & 1 & 0 \\ 1 & 1 & -2 & 0 & 0 & 1 \end{array}\right) \tag{8}$$

$$\xrightarrow{R_3 \leftrightarrow R_3 - R_1} \left(\begin{array}{ccc|ccc} 1 & -3/2 & 5/2 & 1/2 & 0 & 0 \\ 0 & 5/2 & -23/2 & -3/2 & 1 & 0 \\ 0 & 5/2 & -9/2 & -1/2 & 0 & 1 \end{array}\right) \tag{9}$$

$$\xrightarrow{R_3 \leftrightarrow R_3 - R_2} \left(\begin{array}{ccc|ccc} 1 & -3/2 & 5/2 & 1/2 & 0 & 0 \\ 0 & 5/2 & -23/2 & -3/2 & 1 & 0 \\ 0 & 0 & 7 & 1 & -1 & 1 \end{array}\right) \tag{10}$$

$$\xrightarrow{R_2 \leftrightarrow (2/5)R_2} \left(\begin{array}{ccc|ccc} 1 & -3/2 & 5/2 & 1/2 & 0 & 0 \\ 0 & 1 & -23/5 & -3/5 & 2/5 & 0 \\ 0 & 0 & 7 & 1 & -1 & 1 \end{array}\right) \tag{11}$$

$$\xrightarrow{R_3 \leftrightarrow (1/7)R_3} \left(\begin{array}{ccc|ccc} 1 & -3/2 & 5/2 & 1/2 & 0 & 0 \\ 0 & 1 & -23/5 & -3/5 & 2/5 & 0 \\ 0 & 0 & 1 & 1/7 & -1/7 & 1/7 \end{array}\right) \tag{12}$$

$$\xrightarrow{R_1 \leftrightarrow R_1 + (3/2)R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & -22/5 & -2/5 & 3/5 & 0 \\ 0 & 1 & -23/5 & -3/5 & 2/5 & 0 \\ 0 & 0 & 1 & 1/7 & -1/7 & 1/7 \end{array}\right) \tag{13}$$

$$\xrightarrow{R_2 \leftrightarrow R_2 + (23/5)R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & -22/5 & -2/5 & 3/5 & 0 \\ 0 & 1 & 0 & 2/35 & -9/35 & 23/35 \\ 0 & 0 & 1 & 1/7 & -1/7 & 1/7 \end{array}\right) \tag{14}$$

$$\xrightarrow{R_1 \leftrightarrow R_1 + (22/5)R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 8/35 & -1/35 & 22/35 \\ 0 & 1 & 0 & 2/35 & -9/35 & 23/35 \\ 0 & 0 & 1 & 1/7 & -1/7 & 1/7 \end{array}\right) \tag{15}$$

$$\implies \mathbf{A}^{-1} = \frac{1}{35}\begin{pmatrix} 8 & -1 & 22 \\ 2 & -9 & 23 \\ 5 & -5 & 5 \end{pmatrix} \tag{16}$$

→ From (2), (3) and (4), we get

$$\underbrace{\begin{pmatrix} 2 & -3 & 5 \\ 3 & -2 & -4 \\ 1 & 1 & -2 \end{pmatrix}}_{\mathbf{A}}\underbrace{\begin{pmatrix} x \\ y \\ z \end{pmatrix}}_{\mathbf{x}} = \underbrace{\begin{pmatrix} 11 \\ -5 \\ -3 \end{pmatrix}}_{\mathbf{B}} \tag{17}$$

→ So, we compute

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{B} \tag{18}$$

$$\mathbf{x} = \frac{1}{35}\begin{pmatrix} 8 & -1 & 22 \\ 2 & -9 & 23 \\ 5 & -5 & 5 \end{pmatrix}\begin{pmatrix} 11 \\ -5 \\ -3 \end{pmatrix} \tag{19}$$

$$\mathbf{x} = \begin{pmatrix} 27/35 \\ -2/35 \\ 13/7 \end{pmatrix} \tag{20}$$

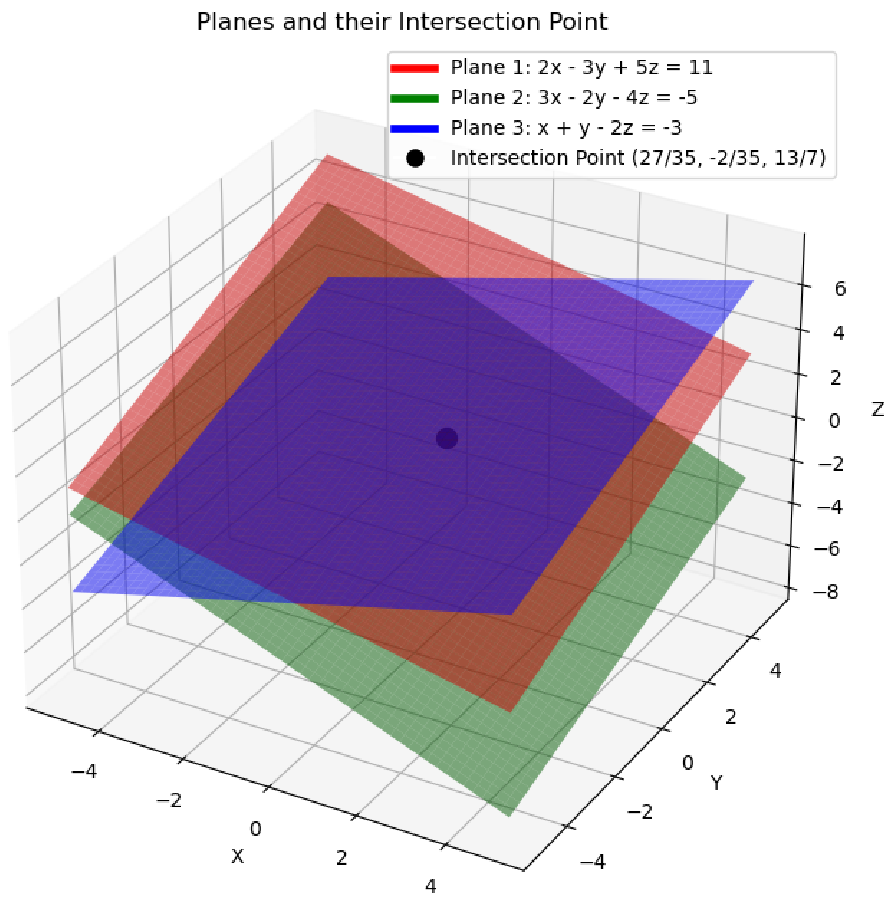


Fig. 0: Plot of system of equations