12.389

EE25BTECH11064 - Yojit Manral

Question:
Characteristic equation of the matrix with eigenvalue λ is

$$\mathbf{A} = \begin{pmatrix} 2 & \sqrt{2} \\ \sqrt{2} & 1 \end{pmatrix} \tag{1}$$

(a) $\lambda^2 + 3\lambda + 4 = 0$ (b) $\lambda^2 + 3\lambda - 2 = 0$ (c) $\lambda^2 - 3\lambda = 0$ (d) $\lambda^2 + 3\lambda = 0$

Solution:
$\longrightarrow$ This problem statement can be solved via the following two methods:

1) The characteristic equation can be given by

$$char(\mathbf{A}) \implies |\mathbf{A} - \lambda\mathbf{I}| = 0 \tag{2}$$

$$\begin{vmatrix} 2-\lambda & \sqrt{2} \\ \sqrt{2} & 1-\lambda \end{vmatrix} = 0 \tag{3}$$

$$(\lambda - 2)(\lambda - 1) - 2 = 0 \tag{4}$$

$$\lambda^2 - 3\lambda = 0 \tag{5}$$

2) Another method to find characteristic equation for a 2×2 matrix is

$$char(\mathbf{M}_{2\times 2}) \implies \lambda^2 - trace(\mathbf{M})\lambda + det(\mathbf{M}) = 0 \tag{6}$$

For the given matrix $\mathbf{A}$, we have

$$trace(\mathbf{A}) = 3 \qquad det(\mathbf{A}) = 0 \tag{7}$$

From (6) and (7), we get

$$char(\mathbf{A}) \implies \lambda^2 - 3\lambda = 0 \tag{8}$$

$\longrightarrow$ Therefore, *(c)* $\lambda^2 - 3\lambda = 0$ is the correct option.