1.1.6.14

EE25BTECH11064 - Yojit Manral

Question:
Find the vector equation of the plane passing through the points $\mathbf{R}\,(2,5,-3)$, $\mathbf{S}\,(-2,-3,5)$ and $\mathbf{T}\,(5,3,-3)$.

Solution:

Points	Name
$\begin{pmatrix}2\\5\\-3\end{pmatrix}$	Point $\mathbf{R}$
$\begin{pmatrix}-2\\-3\\5\end{pmatrix}$	Point $\mathbf{S}$
$\begin{pmatrix}5\\3\\-3\end{pmatrix}$	Point $\mathbf{T}$

TABLE 0: List of Points

→ We can write the equation for the required plane as

$$\mathbf{n}^T\mathbf{x} = c \tag{1}$$

→ Also, $\mathbf{R}$, $\mathbf{S}$ and $\mathbf{T}$ satisfy this equation. Hence

$$\mathbf{n}^T\mathbf{R} = c \tag{2}$$
$$\mathbf{n}^T\mathbf{S} = c \tag{3}$$
$$\mathbf{n}^T\mathbf{T} = c \tag{4}$$

→ From (1), (2), (3) and (4), we get

$$\mathbf{n}^T\begin{pmatrix}\mathbf{R} & \mathbf{S} & \mathbf{T}\end{pmatrix} = c\begin{pmatrix}1 & 1 & 1\end{pmatrix} \tag{5}$$

→ Using transpose on both sides, we get

$$\begin{pmatrix}\mathbf{R} & \mathbf{S} & \mathbf{T}\end{pmatrix}^T\mathbf{n} = c\begin{pmatrix}1\\1\\1\end{pmatrix} \tag{6}$$

$$\mathbf{n} = c\left(\begin{pmatrix}\mathbf{R} & \mathbf{S} & \mathbf{T}\end{pmatrix}^T\right)^{-1}\begin{pmatrix}1\\1\\1\end{pmatrix} \tag{7}$$

$$= c\left(\begin{pmatrix}2 & -2 & 5\\5 & -3 & 3\\-3 & 5 & -3\end{pmatrix}^T\right)^{-1}\begin{pmatrix}1\\1\\1\end{pmatrix} \tag{8}$$

$$= c\begin{pmatrix}2 & 5 & -3\\-2 & -3 & 5\\5 & 3 & -3\end{pmatrix}^{-1}\begin{pmatrix}1\\1\\1\end{pmatrix} \tag{9}$$

$$= \frac{c}{56}\begin{pmatrix}-6 & 6 & 16\\19 & 9 & -4\\9 & 19 & 4\end{pmatrix}\begin{pmatrix}1\\1\\1\end{pmatrix} \tag{10}$$

$$= \frac{c}{7}\begin{pmatrix}2\\3\\4\end{pmatrix} \tag{11}$$

→ From (11), we get the value of

$$\mathbf{n} = \begin{pmatrix}2\\3\\4\end{pmatrix} \tag{12}$$

→ From (2) and (12), we get

$$c = \begin{pmatrix} 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 2 \\ 5 \\ -3 \end{pmatrix} = 7 \tag{13}$$

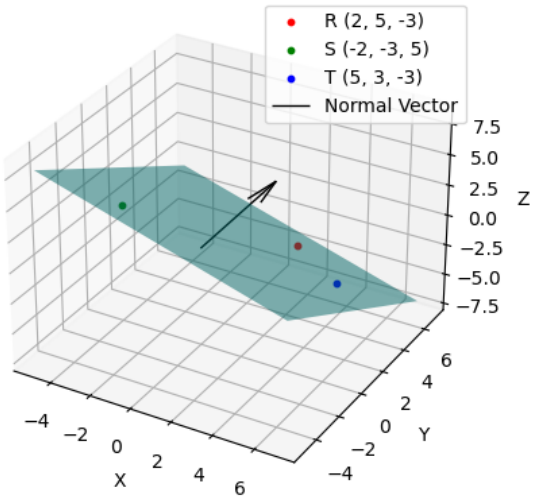

Fig. 0: Plot of plane $\mathbf{n}^T\mathbf{x} = c$