MatGeo Presentation - Problem 5.2.66

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Question

Solve the system of equations

$$2x + 3y = 11 \tag{0.1}$$
$$2x + 4y = -24 \tag{0.2}$$

Hence, find the value of m for which

$$y = mx + 3 \tag{0.3}$$

Solution

→ We have

$$\mathbf{n_1}^T\mathbf{x} = c_1 \qquad \mathbf{n_2}^T\mathbf{x} = c_2 \qquad \mathbf{n_3}^T\mathbf{x} = c_3$$

$$\mathbf{n_1} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \qquad c_1 = 11 \tag{0.4}$$

$$\mathbf{n_2} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \qquad c_2 = -24 \tag{0.5}$$

$$\mathbf{n_3} = \begin{pmatrix} -m \\ 1 \end{pmatrix} \qquad c_3 = 3 \tag{0.6}$$

→ To solve the system of equations, they must be concurrent at a point $\mathbf{x}$

$$\begin{pmatrix} \mathbf{n_1}^T \\ \mathbf{n_2}^T \\ \mathbf{n_3}^T \end{pmatrix} \mathbf{x} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} \implies \begin{pmatrix} 2 & 3 \\ 2 & 4 \\ -m & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 11 \\ -24 \\ 3 \end{pmatrix} \tag{0.7}$$

Solution

' $\rightarrow$ Using augmented matrix

$$\left(\begin{array}{cc|c} 2 & 3 & 11 \\ 2 & 4 & -24 \\ -m & 1 & 3 \end{array}\right) \xrightarrow[R_1 \leftrightarrow R_1 - 3R_2]{R_2 \leftrightarrow R_2 - R_1} \left(\begin{array}{cc|c} 2 & 0 & 116 \\ 0 & 1 & -35 \\ -m & 1 & 3 \end{array}\right) \tag{0.8}$$

$$\xrightarrow[R_3 \leftrightarrow -R_3 + R_2]{R_1 \leftrightarrow (1/2)R_1} \left(\begin{array}{cc|c} 1 & 0 & 58 \\ 0 & 1 & -35 \\ m & 0 & -38 \end{array}\right) \tag{0.9}$$

$$\xrightarrow{R_3 \leftrightarrow R_3 - mR_1} \left(\begin{array}{cc|c} 1 & 0 & 58 \\ 0 & 1 & -35 \\ 0 & 0 & -38 - 58m \end{array}\right) \tag{0.10}$$

$\rightarrow$ For the system of equations to be consistent, we must have

$$-38 - 58m = 0 \implies m = -\frac{19}{29} \tag{0.11}$$

Solution

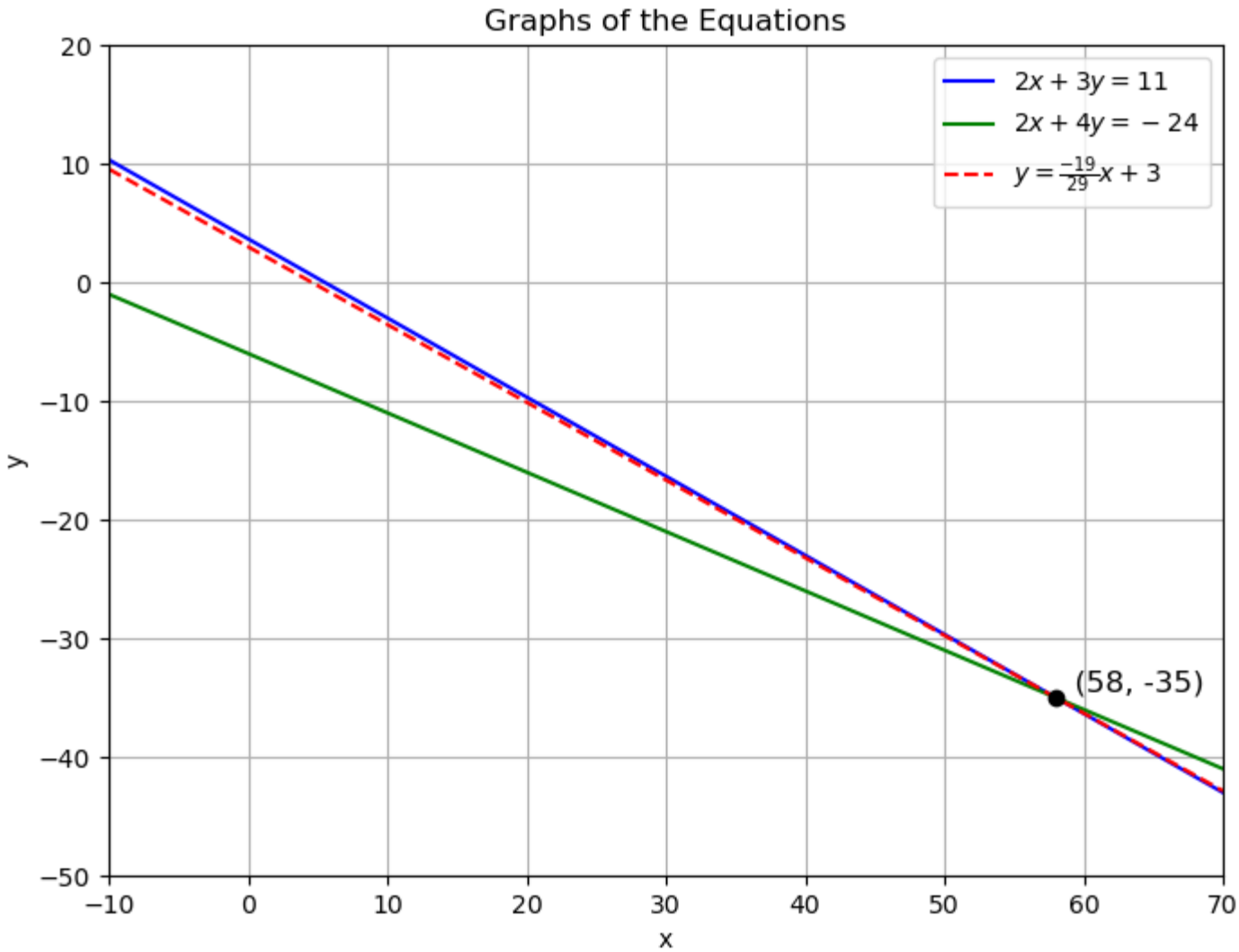

Figure: Plot of the Equations

File: plot.py

```
import numpy as np
import matplotlib.pyplot as plt

# Define the x range
x_vals = np.linspace(-10, 70, 400)

# Equation 1: 2x + 3y = 11 --> y = (11 - 2x) / 3
y1_vals = (11 - 2 * x_vals) / 3

# Equation 2: 2x + 4y = -24 --> y = (-24 - 2x) / 4
y2_vals = (-24 - 2 * x_vals) / 4

# Line y = mx + 3 where m = -19/29
m = -19 / 29
y3_vals = m * x_vals + 3

# Plotting
plt.figure(figsize=(8, 6))

# Plot the lines
plt.plot(x_vals, y1_vals, label=r'$2x␣+␣3y␣=␣11$', color='blue')
plt.plot(x_vals, y2_vals, label=r'$2x␣+␣4y␣=␣-24$', color='green')
plt.plot(x_vals, y3_vals, label=r'$y␣=␣\frac{-19}{29}x␣+␣3$', color='red', linestyle='dashed')
```

File: plot.py

```
# Mark the point (58, -35)
plt.scatter(58, -35, color='black', zorder=5)
plt.text(58, -35, f'␣␣(58,␣-35)', fontsize=12, verticalalignment='bottom')

# Set labels and title
plt.xlabel('x')
plt.ylabel('y')
plt.title('Graphs␣of␣the␣Equations')

# Show legend
plt.legend()

# Set axis limits for better viewing
plt.xlim(-10, 70)
plt.ylim(-50, 20)

# Show grid
plt.grid(True)

# Show plot
plt.show()
```