

ENGINEERING ECONOMICS SUPPLEMENT

PROBLEMS AND SOLUTIONS FROM TEST PAPERS

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Sessional Portions

1st:

2nd:

*Practices
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Regulation Board*

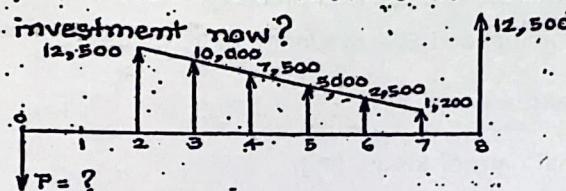
Chapter 1: Time Value of Money

1.1 Present Worth, Annual Worth and Future Worth

Sessional Test (Feb 2014) 1st Sessional

- Q) Wonder La Entertainment Company is evaluating an investment that will provide the following returns at the end of each of the following years: Year 2, Rs 12,500; Year 3, Rs 10,000; Year 4, Rs 7,500; Year 5, Rs 5,000; Year 6, Rs 2,500; Year 7, Rs 1,200; and Year 8, Rs 12,500. Wonder La believes that it should earn an annual rate of 9 percent on this investment. How much it can pay for this investment now?

Answer:



Determining the present worth,

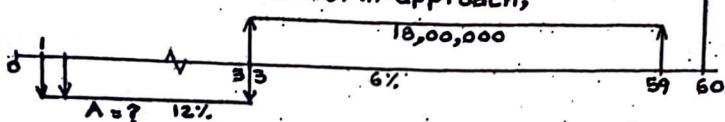
$$P = \left[\left[12,500 - 2,500 \left(\frac{A}{G}, 9\%, 5 \right) \right] \left(\frac{P}{A}, 9\%, 5 \right) \right] \left(\frac{P}{F}, 9\%, 1 \right) + \\ 1,200 \left(\frac{P}{F}, 9\%, 7 \right) + 12,500 \left(\frac{P}{F}, 9\%, 8 \right)$$
$$P = 28,295.71 + 656.4 + 6,273.75 = 35,225.8$$

Sessional Test (Feb 2014) 1st sessional

- Q) You plan to retire 33 years from now. You expect that you will live 27 years after retiring. You want to have enough money upon reaching retirement age to withdraw Rs 1,80,000 from the account at the beginning of each year you expect to live, and yet still have Rs 25,00,000 left in the account at the time of your expected death (60 years from now). You plan to accumulate the retirement fund by making equal annual deposits at the end of each year for the next 33 years. You expect that you will be able to earn 12% per year on your deposits.

However, you only expect to earn 6% per year on your investment after you retire since you will choose to place the money in less risky investments. What equal annual deposits must you make each year to reach your retirement goal?

Answer: Using Annual Worth approach,



$$P_{33} = 25,00,000 \left(\frac{I}{F, 6\%, 27} \right)^{0.2074} + 18,00,000 + 18,00,000 \left(\frac{P}{A, 6\%, 26} \right)^{13.0032}$$

$$= 5,18,500 + 18,00,000 + 23,40,576 = 30,39,076$$

$$A = 30,39,076 \left(\frac{A}{F, 12\%, 33} \right)^{0.0029} = 8,813.3$$

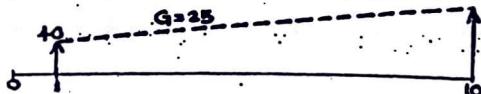
Sessional Test (April 2014) 2nd Sessional

Q) Arvind, a young industrial engineer, prepared an economic analysis for some equipment to replace one production worker. The analysis showed that the present worth of benefits (of employing one less production worker) just equalled the present worth of the equipment costs, based on a 10-year useful life for the equipment. It was decided not to purchase the equipment.

A short time later, the production workers won a new 3-year Union contract that granted them an immediate Rs 40 per hour wage increase, plus an additional Rs 2.5 per hour wage increase in each of the two subsequent years. Assume that in each and every future year, an Rs 25 per hour wage increase will be granted.

Arvind has been asked to revise his earlier economic analysis. The present worth of benefits of replacing one production employee will now increase. Assuming an interest rate of 8%, the justifiable cost of the automation equipment (with a 10-year useful life) will increase by how much? Assume the plant operates a single 8-hour shift, 250 days per year.

Answer:



$$\text{Per hour wage, PW} = \left[40 + 25 \left(\frac{A}{G}, i = 8\%, 10 \right) \right] \left(\frac{P}{A}, 8\%, 10 \right)$$

$$= [40 + 96.75] \times 6.710 = \text{Rs } 917.5925 \text{ per hour}$$

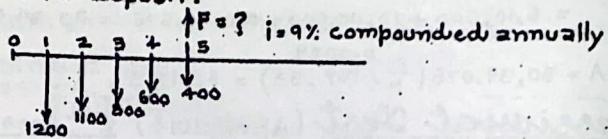
$$\therefore \text{Total Cost} = 917.5925 \times 8 \times 250 = \text{Rs } 18,35,185$$

Rs 18,35,185 is the increased justifiable cost of the equipment

University Test (July 2014) Make-up

- Q.) Five annual deposits in the amounts of Rs 1,200, Rs 1,000, Rs 800, Rs 600, and Rs 400 are made into a fund that pays interest at a rate of 9% compounded annually. Determine the amount in the fund immediately after the fifth deposit.

Answer:



$$A = A_1 + G \left(\frac{A}{G} \cdot i, n \right)$$

$$= 1200 - 200 \left(\frac{A}{G} \cdot 9, 5 \right) = 1200 - 200(1.828) = 1200 - 365.6 = 834.4$$

We need to calculate the future value in order to calculate the fund available at the end of fifth deposit.

$$A = A_1 + G \left[\frac{(1+i)^n - 1}{i(1+i)^n - i} \right]$$

$$F = A \left[\frac{(1+i)^n - 1}{i} \right]$$

$$F = A \left(\frac{F}{A} \cdot i, n \right)$$

$$= 834.4 \left(\frac{F}{A} \cdot 9, 5 \right)$$

$$= 834.4 (5.985) = 4993.88$$

University Test (May 2014)

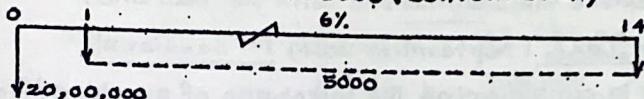
- Q.) Polymer Moulding, Inc., is considering two processes for manufacturing storm drains. Plan A involves conventional injection moulding at a cost of \$2 million. The cost for inspecting, maintaining, and cleaning the moulds is expected to be \$5000 per year. The cost of materials for this plan is expected to be the same as for the other plan. Plan B involves using an innovative process known as virtual engineered composites wherein a floating mould uses an operating system that constantly adjusts the water pressure around the mould, and the chemicals entering the process. The first cost to tool the

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floating mould is only \$25,000, but because of the newness of the process, personnel and product-reject costs are expected to be higher than those for a conventional process. The company expects the operating costs to be \$45,000 per year for the first 8 years and then to decrease to \$10,000 per year thereafter. At an interest rate of 6% per year, which process should the company select on the basis of an annual worth analysis over a 14-year study period?

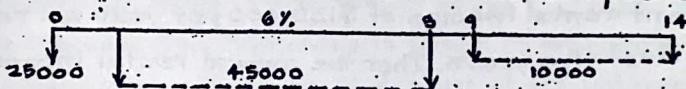
Answer: Study period given is 14 years

Plan A - Older Process (conventional)



$$EAC = 20,00,000 \left(\frac{0.1076}{P}, 6\%, 14 \right) + 5,000 = 2,20,200$$

Plan B - Virtual Engineered Composites



$$\begin{aligned} EAC &= 25000 \left(\frac{A}{P}, i, 14 \right) + 45000 \left(\frac{P}{A}, i, 8 \right) \left(\frac{A}{P}, i, 14 \right) + 10000 \left(\frac{F}{A}, i, 6 \right) \left(\frac{A}{F}, i, 14 \right) \\ &= 25000(0.1076) + 45000(6.210)(0.1076) + 10000(6.975)(0.0476) \\ &= 36,078.92 \end{aligned}$$

∴ Plan B should be selected.

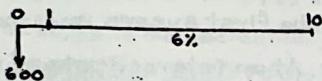
University Test (May 2011)

- (Q) As proprietor of your own business, you are considering the option of purchasing a new high-efficiency machine to replace older machines currently in use. You believe that the new technology can be used to replace four of the older machines, each with a current market value of \$600. The new machine will cost \$3000 and will save the equivalent of 10,000 kW-hr of electricity per year over the older machines. After a period of 10 years neither option (new or old) will have any market value. If you use a before tax MARR of 6% and

pay \$0.075 per kilowatt-hour, would you replace the old machines today with the new one?

Answer:

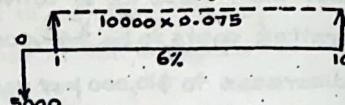
Defender (+m/s in no.)



EUAC (Defender)

$$= 600 \left(\frac{A}{P}, 6\%, n=10 \right) \times 4 = 326.16$$

challenger



EUAC (Challenger)

$$= 5000 \left(\frac{A}{P}, 6\%, n=10 \right) - 10000 \times 0.075 \\ = -70.5$$

4 machines of Defender can be replaced by one challenger

Yes, replace the old machine with the new one.

Sessional Test (September 2014) 1st sessional

Q1 Your firm is considering the purchase of an old office building with an estimated remaining service life of 25 years. The tenants have recently signed long-term leases, which leads you to believe that the current rental income of \$150,000 per year will remain constant for the first five years. Then the annual rental income will increase by 10% for every five-year interval over the remaining asset life. You estimate that operating expenses including income taxes will be \$45,000 for the first year and that they will increase by \$3000 each year thereafter. You estimate that razing the building and selling the lot on which it stands will realize a net amount of \$50,000 at the end of the 25-year period. If you had the opportunity to invest your money elsewhere and thereby earn interest at the rate of 12% per annum. What would be the maximum amount you would be willing to pay for the building and lot at the present time?

Data given:

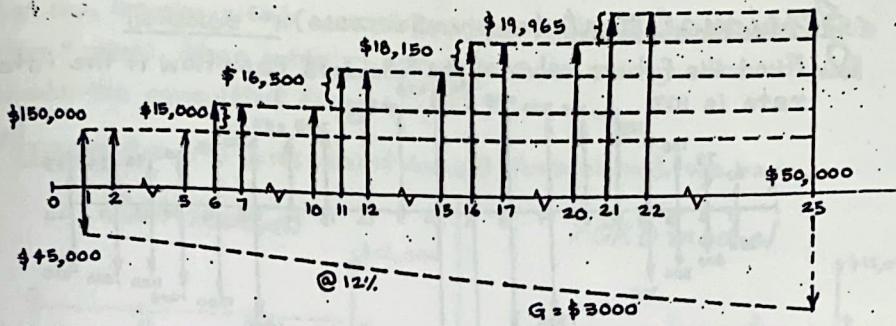
$$n = 25 \text{ years}$$

Current rental income = \$150,000 per year

O & M Costs = \$45,000 for first year increasing by \$3000 thereafter

Salvage value = \$50,000

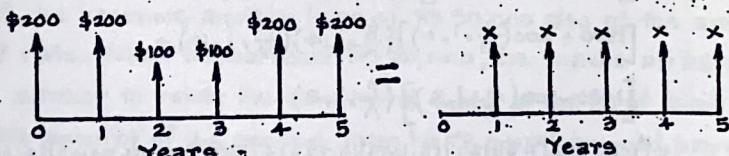
MARR = 12%



$$\begin{aligned}
 FW(\text{at } 12\%) &= \left[\$150,000 \left(\frac{F}{A}, 12\%, 25 \right) + \$15,000 \left(\frac{F}{A}, 12\%, 20 \right) + \$16,500 \left(\frac{F}{A}, 12\%, 15 \right) + \right. \\
 &\quad \$18,150 \left(\frac{F}{A}, 12\%, 10 \right) + \$19,965 \left(\frac{F}{A}, 12\%, 5 \right) + \$50,000 - \\
 &\quad \left. [45,000 + \$3000 \left(\frac{A}{G}, 12\%, 25 \right) \left(\frac{F}{A}, 12\%, 25 \right)] \right] \\
 P &= \$7,83,639.69
 \end{aligned}$$

Sessional Test (September 2014) 1st Sessional

- (Q) The two cash flow transactions shown in the accompanying cash flow diagram are said to be equivalent at 10% interest compounded annually. Find the unknown X value that satisfies the equivalence.



Answer: Determining the Present Worth for First Cash Flow Diagram, we have

$$\begin{aligned}
 PW &= 200 + 200 \left(\frac{P}{F}, 10, 1 \right) + 100 \left(\frac{P}{F}, 10, 2 \right) + 100 \left(\frac{P}{F}, 10, 3 \right) + 200 \left(\frac{P}{F}, 10, 4 \right) + 200 \left(\frac{P}{F}, 10, 5 \right) \\
 &= 200 + 200(0.9091) + 100(0.8265) + 100(0.7513) + 200(0.6830) + 200(0.6209) \\
 &= 200 + 181.82 + 82.65 + 75.13 + 136.6 + 124.18 \\
 &= 800.38
 \end{aligned}$$

Using Annual Worth for substituting the two CFD, we have

$$X = PW * \left(\frac{A}{P}, 10, 5 \right)$$

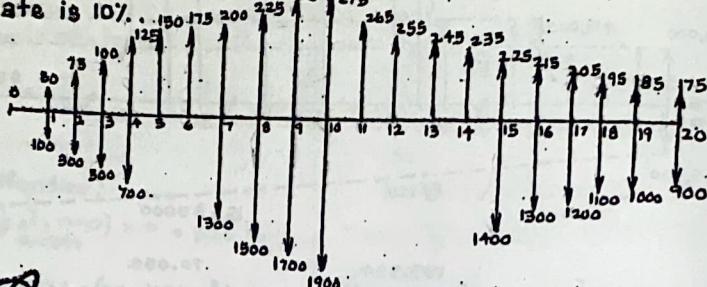
$$= 800.38 (0.2638)$$

$$= 211.14$$

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- Sessional Test (September 2014) 1st Sessional

Q) Find the future value of the following cash flow if the interest rate is 10%.



Answer

Cash Inflow

$$F_{\text{Inflow}} = \left[50 + 25 \left(\frac{A}{G}, 10y, 10 \right) \right] \left(\frac{F}{A}, i, 10 \right) \left(\frac{F}{P}, i, 10 \right) +$$

$$\left[265 - 10 \left(\frac{A}{G}, i, 10 \right) \right] \left(\frac{F}{A}, i, 10 \right)$$

$$= \left[50 + 25 (3.7255) \right] 15.937 * 2.594 + \left[265 - 10 (3.7255) \right] 15.937$$

$$= 5917.39 + 3629.57$$

$$= 9546.96$$

Cash Outflow

$$F_{\text{Outflow}} = \left[100 + 200 \left(\frac{A}{G}, i, + \right) \right] \left(\frac{F}{A}, i, 4 \right) \left(\frac{F}{P}, i, 20 \right)$$

$$\left[1300 + 200 \left(\frac{A}{G}, i, + \right) \right] \left(\frac{F}{A}, i, 4 \right) \left(\frac{F}{P}, i, 10 \right) +$$

$$\left[1400 - 100 \left(\frac{A}{G}, i, 6 \right) \right] \left(\frac{F}{A}, i, 6 \right)$$

$$= \left[100 + 200 (1.3812) \right] (8.1699)(6.728) + \left[1300 + 200 (1.3812) \right] (4.641)(2.594) + \left[1400 - 100 (2.2256) \right] (7.718)$$

$$= 8024.10 + 18975.97 + 9089.03$$

$$= 36089.1$$

$$\therefore \text{Net F} = F_{\text{Inflow}} - F_{\text{Outflow}} = 9546.96 - 36089.1 = -26542.14$$

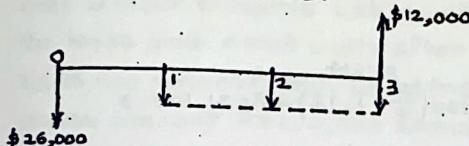
- Sessional Test (October 2014) 2nd Sessional

Q) A consulting engineering firm is considering two models of SUVs for the company principals. A GM model will have a first cost of \$25,000, an operating cost of \$2,000; and a salvage value of \$12,000 after 3 years. A FORD model will have a first cost of

\$29,000, an operating cost of \$1200, and a \$15,000 resale value after 4 years. At an interest rate of 15% per year, which model should the consulting firm buy?

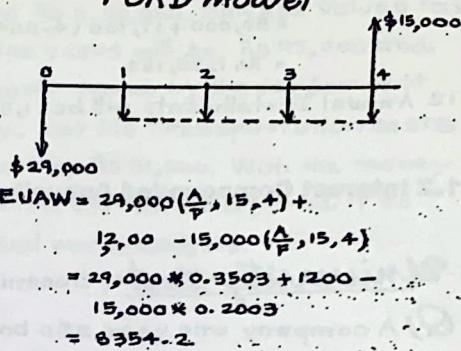
Answer: with $i = 15\%$, using Annual Worth Method we have,

GM model



$$\begin{aligned} \text{EUAW} &= 26,000 \left(\frac{A}{P}, 15, 3 \right) + 2,000 - \\ &\quad 12,000 \left(\frac{A}{P}, 15, 3 \right) \\ &= 26,000 * 0.4380 + 2,000 - \\ &\quad 12,000 * 0.2880 \\ &= 9932 \end{aligned}$$

FORD model



$$\begin{aligned} \text{EUAW} &= 29,000 \left(\frac{A}{P}, 15, 4 \right) + \\ &\quad 12,000 - 15,000 \left(\frac{A}{P}, 15, 4 \right) \\ &= 29,000 * 0.3503 + 1200 - \\ &\quad 15,000 * 0.2003 \\ &= 8354 - 2 \end{aligned}$$

∴ Select FORD model

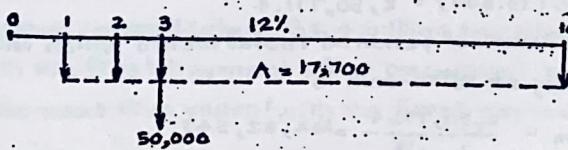
University Test (January 2013) Make-up

Q) A man arranges to repay Rs 1,00,000 bank loan in 10 equal annual installments at 12% interest per year. Immediately after his third payment, he borrows another loan of Rs 50,000 also at the same interest rate. When he borrows Rs 50,000, he made an agreement with the banker to repay the remaining debt of the first loan and the entire amount of the second loan in 12 equal annual payments. The first of 12 payments would be made one year after the receipt of the second loan. Compute the amount of each of 12 payments.

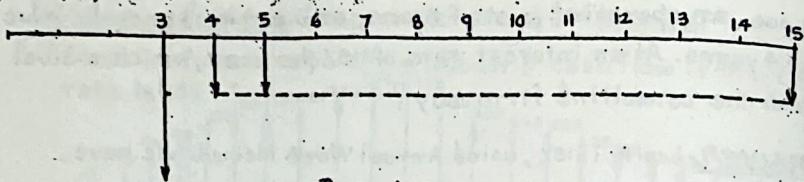
Answer: Rs 1,00,000 is to be paid back in 10 equal installments at 12% interest

$$1,00,000 \left(\frac{A}{P}, 12, 10 \right) = 1,00,000 (0.1770) = \text{Rs } 17,700$$

Installment Amount per year is Rs 17,700



Total repayment schedule is as given in CFD below:



$$= 50,000 + 17,700 \left(\frac{P}{A}, i, 7 \right)$$

$$= 50,000 + 17,700 (4.564)$$

$$= \text{Rs } 1,30,782$$

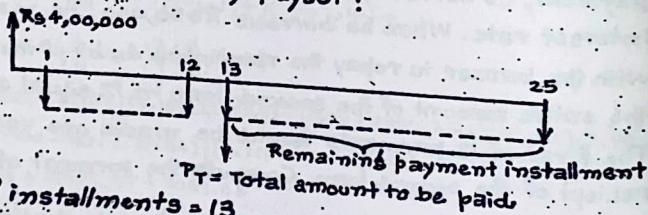
$$12 \text{ Annual Installments will be } 1,30,782 \left(\frac{A}{P}, i, 12 \right) = \text{Rs } 21,108.3$$

1.2 Interest Compounded Annually

University Test (December 2014) Make-up

Q) A company one year ago borrowed Rs 4,00,000 to pay for a new machine tool, agreeing to repay the loan in 25 monthly equal payments at an annual nominal interest rate of 12% compounded monthly. The company now wants to pay off completely the remaining loan amount, which will be paid during the next installment period (i.e. at 13th installment). How much would this payment be, assuming no penalty costs for early payout?

Answer:



Remaining no. of installments = 13

Future Value of remaining installments at 25th month

$$F_{25} = A \left(\frac{F}{A}, i = 1\%, n = 13 \right)$$

$$A = 4,00,000 \left(\frac{1}{P}, i = 1\%, 25 \right) = 4,00,000 \times 0.0454 = 18,160$$

$$F_{25} = 18,160 (13.809) = 2,50,771.4$$

Present Value at 13th month of pending installments which will be paid off completely at 13th month is

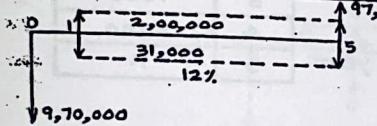
$$P_{\text{pending}} - 13m = \frac{2,50,771.4}{1.01^{12}} = \text{Rs } 2,22,547$$

University Test (Dec 2014) make-up

Q) A company engaging in selling of laboratory equipment estimates that profit from sales should increase by Rs 2,00,000 per year if a mobile demonstration unit is built. A large unit with sleeping accommodation for the driver will cost Rs 9,70,000 while a smaller unit without sleeping cabin will be Rs 6,30,000. Salvage values for the large and small units after five years will be, Rs 97,000 and Rs 35,000 respectively. Lodging costs saved by the larger unit should amount Rs 1,10,000 annually, but its transportation costs will exceed those of the smaller unit by Rs 31,000. With the money at 12% should a mobile demonstration unit be built? And if so which size is preferable? Use annual worth analysis.

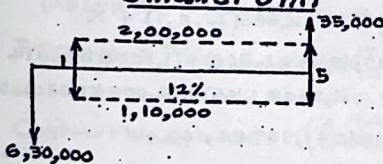
Answer:

Large Unit



$$\begin{aligned}
 AW &= -\left(9,70,000 \left(\frac{A}{P}, i, 5\right) + 31,000\right) + 2,00,000 + \\
 &\quad 97,000 \left(\frac{F}{P}, i, 5\right) \\
 &= -(9,70,000(0.2774) + 31,000) + 2,00,000 + \\
 &\quad 97,000(0.1846) \\
 &= -82,171.8
 \end{aligned}$$

Smaller Unit



$$\begin{aligned}
 AW &= -6,30,000 \left(\frac{A}{P}, i, 5\right) - 1,10,000 + \\
 &\quad 2,00,000 + 35,000 \left(\frac{F}{P}, i, 5\right) \\
 &= -6,30,000(0.2774) - 1,10,000 + \\
 &\quad 2,00,000 + 35,000(0.1846) \\
 &= -78,301
 \end{aligned}$$

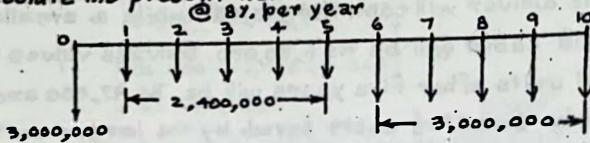
Sessional Test (February 2015) 1st Sessional

Q) A local newspaper headline blared "Bo Smith Signs for \$30 Million". The article revealed that, on April 1, 2002, Bo Smith, the former record breaking running back from Football University, signed a \$30 million package with the Nebraska Lions. The terms of the contract were \$3 million immediately, \$2.4 million per year for the first five years (with the first payment after one year), and \$3 million per year for the next five years (with the first payment at the end of

¹²
year six). If the interest rate is 8% compounded annually, what is Bo's contract worth at the time of contract signing?

Answer:

To find the worth of the amount for Bo's contract, we need to calculate the present worth which is at Time zero.



$$PW = 3,00,000 + 2,400,000 \left(\frac{P}{A}, 8\%, 5 \right) + \left[3,00,000 \left(\frac{P}{A}, 8\%, 5 \right) \times \left(\frac{F}{P}, 8\%, 5 \right) \right]$$

$$= \$ 20,734,618$$

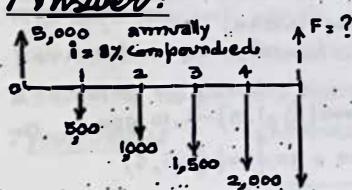
- Sessional Test (Feb 2015) 1st sessional

- Q) A debt of \$5000 can be repaid, with interest at 8% compounded annually, by the following payments.

The payment at the end of the fifth year is shown as X. How much is X?

Year	Payments (\$)
1	500
2	1000
3	1500
4	2000
5	X

Answer:



$$5,000 \left(\frac{F}{P}, 8, 5 \right) = 5,00 \left(\frac{F}{P}, 8, 4 \right) + 1,000 \left(\frac{F}{P}, 8, 3 \right) \\ + 1,500 \left(\frac{F}{P}, 8, 2 \right) + 2,000 \left(\frac{F}{P}, 8, 1 \right) + X$$

$$5,000 \times 1.469 = (5,00 \times 1.360) + (1,000 \times 1.260) + (1,500 \times 1.166) + (2,000 \times 1.080) + X$$

$$7,345 = 6,80 + 1,260 + 1,791 + 2,160 + X$$

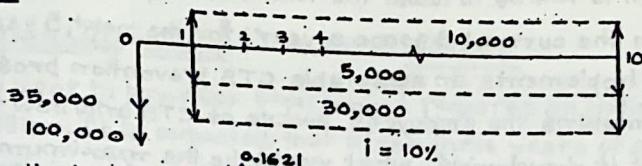
$$X = 1,496$$

Sessional Test (April 2015) 2nd sessional

- Q) An industrial firm is considering purchasing several programmable controllers and automating the company's manufacturing operations. It is estimated that the equipment will initially cost \$100,000 and the labor to install it will cost \$35,000. A service contract to maintain the equipment will cost \$5,000 per year. Trained service personnel will have to be hired at an annual salary of \$30,000. Also estimated is an approximate \$10,000 annual income-tax savings (cash inflow). How much will this investment in equipment and services have to increase

the annual revenues after taxes in order to break even? The equipment is estimated to have an operating life of 10 years, with no salvage value because of obsolescence. The firm's MARR is 10%.

Answer:



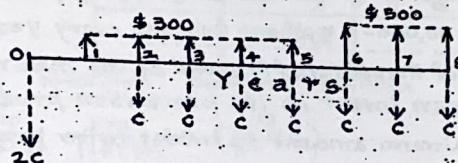
$$\text{Annual Worth, } AW = 135000 \left(\frac{A}{P}, 10\%, 10 \right) + 25000$$

$$= 21964.5 + 25000 = \$46,964.5/\text{year}$$

∴ Income should increase by \$46,964.5 in order to break even.

Sessional Test (April 2015) 1st Sessional (make-up)

Q) For the CFD below, determine the value of C which makes the net worth equal to zero with interest rate of 10%.



Answer:

Using the Present Worth (PW) method,

$$\text{Revenue} \rightarrow PW_{\text{revenue}} = 300 \left(\frac{A}{P}, 10\%, 5 \right) + 500 \left(\frac{P}{F}, 10\%, 3 \right) \left(\frac{A}{P}, 10\%, 5 \right)$$

$$= 3140.5$$

$$\text{Cost} \rightarrow PW_{\text{cost}} = 2C + C \left(\frac{P}{A}, 10\%, 7 \right) \left(\frac{A}{P}, 10\%, 1 \right)$$

$$= 6.425C$$

Substituting the revenue & cost value i.e.

$$PW_{\text{revenue}} = PW_{\text{cost}}$$

$$3140.5 = 6.425C \quad \therefore C = 488.79$$

Sessional Test (April 2015) 1st Sessional (make-up)

Q) Fairmount Textile has a plant in which employees have been having trouble with Carpal Tunnel Syndrome (CTS) (inflammation of the nerves that pass through the carpal tunnel, a tight space at the base of the palm), resulting from long-term repetitive activities, such as years of sewing operations. It seems as if 15 of the employees

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working in this facility developed signs of CTS over the last five years. Deep South, the company's insurance firm, has been increasing Fairmont's liability insurance steadily because of this CTS problem. Deep South is willing to lower the insurance premiums to \$16000 a year (from the current \$30000 a year) for the next 5 years. If Fairmont implements an acceptable CTS prevention program that includes making the employees aware of CTS and how to reduce chances of it developing, what would be the maximum amount that Fairmont should invest in the CTS prevention program to make this program worthwhile? The firms interest rate is 12% compounded annually.

Answer:

Deep South insurance firm is willing to reduce the insurance payments from \$30,000 to \$16,000. If the reduction takes place, the firm will have a benefit $\$30,000 - \$16,000 = \$14,000$ every year.

Present worth of these benefits will be the maximum amount, the firm can invest in its prevention program.

Therefore, the maximum amount to invest in the prevention program will be

$$P = \$14,000 \left(\frac{1}{A}, 12\%, 5 \right)$$
$$= \$50,467$$

Sessional Test (April 2015) 2nd Sessional (make-up)

Q) A manufacturing plant requires an initial investment of \$10 million. It is expected that a supplemental investment of \$4 million will be needed every 3 years to update the plant. The plant is expected to start producing goods 2 years after the initial investment is made (at the start of the third year). Revenues of \$5 million per year are expected to begin to flow at the start of the fourth year. Annual operating costs are expected to be \$2 million per year. The plant has a 15-year life. Determine the net annual worth of the cash flows.

Answer:



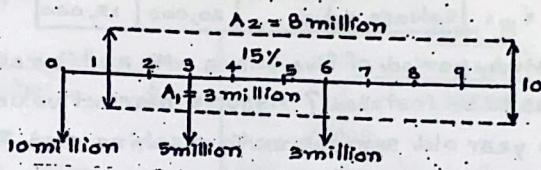
In millions,

$$AW = -10\left(\frac{A}{P}, i, 15\right) - 4\left(\frac{A}{F}, i, 3\right) - 2\left(\frac{F}{A}, i, 13\right)\left(\frac{A}{F}, i, 15\right) + 5\left(\frac{F}{A}, i, 12\right)\left(\frac{A}{F}, i, 15\right)$$
$$= -4.0897 + 3.3322$$
$$= -0.757$$

University Test (July 2015) Make Up

- (Q) A new plant to produce steel tubing requires an initial investment of \$10 million. It is expected that after three years of operation an additional investment of \$5 million will be required; and, after six years of operation, another investment of \$3 million. Annual operating costs will be \$3 million and annual revenues will be \$8 million. The life of the plant is 10 years. If the interest rate is 15% per year, compounded annually, what is the NPV of this plant?

Answer:



$$NPV = -10m - \frac{5m}{(1+i)^3} - \frac{3m}{(1+i)^6} + 5m\left(\frac{P}{A}, i, 10\right)$$
$$= -10m - \frac{5m}{1.15^3} - \frac{3m}{1.15^6} + 5m(5.019) = 10.51 \text{ million}$$

University Test (May 2015)

- (Q) Compare the interest earned from an investment of \$1000 for 15 years at 15% per annum simple interest, with the amount of interest that could be earned if these funds were invested for 15 years at 15% per year, compounded annually.

Answer:

$$P = \$1000 \quad n = 15 \text{ years.} \quad i = 15\%, \text{ pa}$$

Simple Interest

$$F = P * i * n$$
$$= 1000 * 0.15 * 15$$
$$= 2250$$

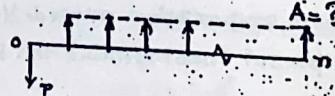
Compound Interest

$$F = P(1+i)^n$$
$$= 1000 * (1+0.15)^{15}$$
$$= 8137.06$$

Capital Recovery Cost (definition): Interest = 8137 - 1000 = 7137

$$CR = A = P\left(\frac{A}{P}, i, n\right) - F\left(\frac{A}{F}, i, n\right)$$

$$A = P \left[\frac{i(1+i)^n}{(1+i)^n - 1} \right]$$



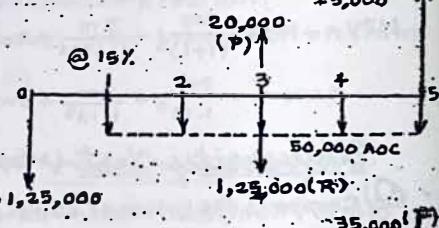
- Q) The Smith Novelty Company, a mail-order firm, wants to install an automatic mailing system to handle product announcements and invoices. The firm has a choice between two different types of machines. The two machines are designed differently, but have identical capacities and do exactly the same job. The semiautomatic machine (model A) costs Rs 1,25,000 and will last for three years, while the fully automatic machine (model B) will cost Rs 1,50,000 and lasts for four years. The expected cash flows for the two machines, including maintenance, and salvage value are as follows:

In Rs.	Model A	Model B
Annual operating costs	50,000	40,000
Salvage Value	20,000	15,000

Using a study period of five years, with an interest rate of 15%, which model has to be installed? Assume market value equal to Rs 25,000 for a two year old semiautomatic machine and Rs 35,000 for one year old fully automatic machine.

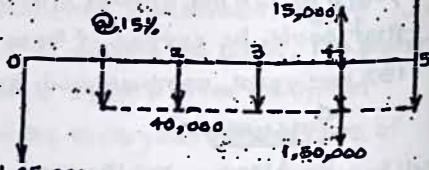
Answer: MODEL A'

$$PW(at 15\%) = 1,25,000 + 50,000 \left(\frac{1}{1,15\%, 5} \right) + 0.6575 \\ 105,000 \left(\frac{1}{1,15\%, 3} \right) - 25,000 \left(\frac{1}{1,15\%, 5} \right) \\ = 3,49,207.5$$



MODEL B'

$$PW(at 15\%) = 1,50,000 + 40,000 \left(\frac{1}{1,15\%, 5} \right) - 0.5718 \\ 15,000 \left(\frac{1}{1,15\%, 4} \right) - 715,000 \left(\frac{1}{1,15\%, 5} \right) \\ - 35,000 \left(\frac{1}{1,15\%, 5} \right) \\ = 3,43,971$$



∴ Model B' is selected

University Test (May 2015) Make-up

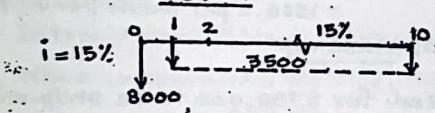
- Q) In 2000, Bell Atlantic and GTE merged to form a giant telecommunications corporation named Verizon Communications. As expected, some equipment incompatibilities had to be rectified, especially for long

distance and international wireless and video services. One item had two suppliers - a U.S. firm(A) and an Asian firm(B). Approximately 3000 units of this equipment were needed. Estimates for vendors A and B are given for each unit. Determine which vendor should be selected, if the MARR is 15% per year.

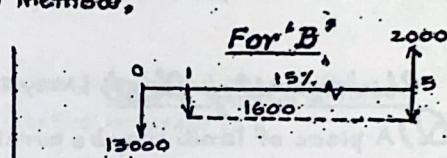
	A	B
Initial Cost, \$	-8000	-13000
Annual Cost, \$	-3500	-1600
Salvage Value, \$	0	2000
Life, Years	10	5

Answer: Using Annual Worth Methods,

For 'A'



$$\begin{aligned} AW_A &= -8000 \left(\frac{A}{P}, i, 10 \right) - 3500 \\ &= -8000(0.1993) - 3500 \\ &= 5094.4. \end{aligned}$$



$$\begin{aligned} AW_B &= -13000 \left(\frac{A}{P}, i, 5 \right) - 1600 + \\ &\quad 2000 \left(\frac{F}{P}, i, 5 \right) \\ &= -13000(0.2983) - 1600 + 2000(0.15) \\ &= -5163.1 \end{aligned}$$

∴ Select 'A'

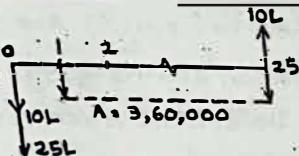
Note: PW, FW or IRR (incremental) methods can also be adopted.

University Test (May 2015) Make-up

Q) Sunbelt Corporation, an investment company, is considering a 50-unit apartment complex in a growing area near Tucson, Arizona. Since the long-term growth potential of the town is excellent, it is believed that the company could average 85% full occupancy for the complex, each year. If the following financial data are reasonably accurate estimates, determine the minimum monthly rent that should be charged if a 12% rate of return is desired:

- Land investment cost Rs 10 lakh
- Building investment cost Rs 25 lakh
- Annual upkeep cost Rs 1,50,000
- Property taxes and insurance = 6% of total initial investment
- Study period of 25 years
- Salvage value = Only land cost can be recovered in full

18 Answer: Using Annual Worth Analysis



$$\begin{aligned} \text{Total Cost} &= \text{Cost of (land + building)} \\ &= 10L + 25L = 35L \end{aligned}$$

$$\begin{aligned} \text{Property taxes \& insurance} &> 6\% \text{ of total investment} \\ &= 6\% \text{ of } 35L = 2,10,000 \text{ per year} \end{aligned}$$

$$\begin{aligned} \text{Annual Worth of cost} &= \text{Annual Upkeep cost} + \text{Property taxes \& insurance} \\ &= \text{Rs } 1,80,000 (\text{given}) + \text{Rs } 2,10,000 \\ &= \text{Rs } 3,60,000 / \text{year} \end{aligned}$$

$$\begin{aligned} AW &= 35,00,000 \left(\frac{1}{P}, i, 25 \right) - 10,00,000 \left(\frac{1}{P}, i, 25 \right) + 3,60,000 \\ &= 35,00,000 (0.1275) - 10,00,000 (0.0075) + 3,60,000 \\ &= 7,98,750 / \text{year} \end{aligned}$$

$$\text{Minimum monthly rental should be equal to } = \frac{7,98,750}{0.85 \times 12 \times 50}$$

= 1566.2 per month per unit

University Test (May 2015) Make-up.

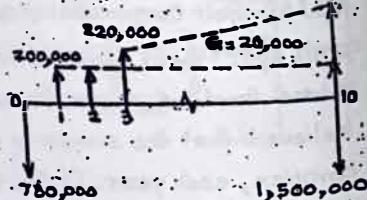
- Q) A piece of land may be purchased for \$780,000 to be strip-mined for the underlying coal. Annual net income will be \$200,000 per year for first two years after which it increases by a gradient of \$20,000 with respect to previous year, till 10 years. At the end of the 10 years, the surface of the land will be restored as required by a federal law on strip mining. The reclamation will cost \$1.5 million more than the resale value of the land after it is restored. Using a 12% interest rate and present worth method of comparison, determine whether the project is desirable.

Answer:

$$\begin{aligned} PW &= -780,000 - \frac{1,500,000}{1.12^2} + 200,000 \left(\frac{1 - 1.640}{1 - 1.12} \right) \\ &\quad + [220,000 + 20,000 \left(\frac{1}{G}, i, 8 \right)] \left(\frac{1 - 1.13}{1 - 1.12} \right) \end{aligned}$$

$$\begin{aligned} &= -780,000 - 482,959.8 + 338,600 + 558349.62 \\ &= -366,610.2 \end{aligned}$$

∴ Not desirable.



University Test (May 2015) ¹⁹ Make-up

Q) Upon the birth of his first child, Dean Jones decided to establish a savings account to partly pay for his son's education. He plans to deposit \$20 per month in the account, beginning when the boy is 13 months old. The savings and loan association has a current interest policy of 12% per annum, compounded monthly.

- Assuming no change in the interest rate, how much will be in the savings account when Dean Jones son becomes 16 years old?
- If the interest rate increases to 15% per annum from the 37th month onwards, what would be the accumulated amount in the account when Dean Jones son becomes 16 years old?

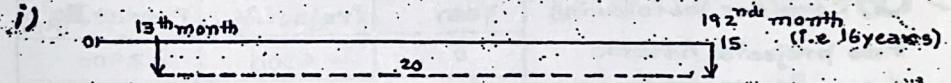
Answer:

Interest rate (given) = 12% per annum, compounded monthly

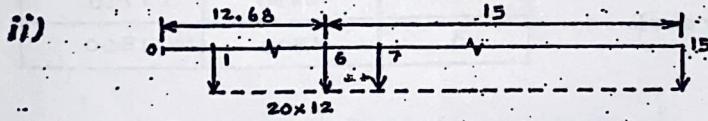
When compounded monthly, the effective interest is

$$(1 + \frac{0.12}{12})^{12} - 1 = 12.68\% \text{ per annum}$$

Total deposit made in a year = 12 months * amount deposited per month
 $= 12 * \$20 = \240



$$F = A \left[\frac{(1+i)^n - 1}{i} \right] \Rightarrow 240 \left[\frac{(1+0.1268)^{15} - 1}{0.1268} \right] \Rightarrow \$9452$$



$$\begin{aligned} F &= 240 \left[\frac{(1+0.1268)^6 - 1}{0.1268} \right] (1+0.15)^9 + 240 \left[\frac{(1+0.15)^9 - 1}{0.15} \right] \\ &= 6970.23 + 4028.6 \\ &= \$10998.8 \end{aligned}$$

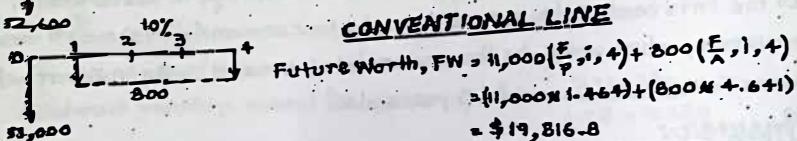
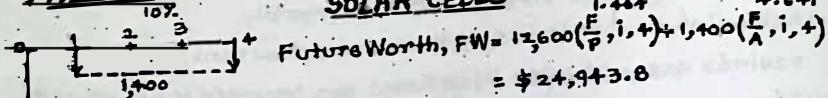
University Test (November 2014)

Q) A remotely located air sampling station can be powered by solar cells or by running an electric line to the site and using conventional power. Solar cells will cost \$12,600 to install and will have a useful life of four years with no salvage value. Annual costs for inspection, cleaning, and other costs are expected to be \$1,400. A new power line

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will cost \$11,000 to install, with power costs expected to be \$800 per year. Since the air sampling project will end in four years, the salvage value of the line is considered to be zero. At an interest rate of 10% per year, which alternative should be selected on the basis of a future worth analysis?

Answer:



∴ Select Conventional Line

University Test, (July 2014) Make-up

- Q) Consider the following two projects. Assume interest rate of 12% and Select the best alternative using present worth method.

Year	Project A (\$)	Project B (\$)
0	-4500	-2900
1	2610	1210
2	2930	1720
3	2300	1500

Answer:

Present Worth (PW) is the amount at time zero

$$PW_A = -4500 + 2610 \left(\frac{P}{F}, 12, 1 \right) + 2930 \left(\frac{P}{F}, 12, 2 \right) + 2300 \left(\frac{P}{F}, 12, 3 \right)$$

$$= -4500 + 2610(0.8929) + 2930(0.7972) + 2300(0.7118)$$

$$= 1803.405$$

Project A

$$PW_B = -2900 + 1210 \left(\frac{P}{F}, 12, 1 \right) + 1720 \left(\frac{P}{F}, 12, 2 \right) + 1500 \left(\frac{P}{F}, 12, 3 \right)$$

$$= -2900 + 1210(0.8929) + 1720(0.7972) + 1500(0.7118)$$

$$= 619.29$$

Project B

∴ Project A is selected

University Test (July 2015) Make-up

- Q) An engineer is thinking of starting a part-time consulting business next September 5, on his 40th birthday. He expects the business will require an initial cash outlay of \$5,000, to come from his savings, and will cost \$500 per year to operate; the business ~~ought~~ to generate \$2,000 per year in cash receipts. During the 20 years that he expects to operate the business, he plans to deposit the annual net proceeds in a bank each year, at an interest rate of 12% per year, compounded annually. When he retires, on his 60th birthday, the engineer expects to invest whatever proceeds plus interest he then has from the business in a long-term savings plan that pays 15% per year, compounded annually. What is the maximum amount he could withdraw from the savings plan each year during his retirement and still have the funds last 15 years?

Answer:

Note: To calculate the amount at the end of every year, students can either calculate (Present worth or Future worth) and then convert to annual worth.

$$FW_1 = ?$$

$$FW_1 = -5000 \left(\frac{F}{P}, 12, 20 \right) + 1500 \left(\frac{F}{A}, 12, 20 \right)$$

$$= -5000 (9.646) + 1500 (72.052)$$

$$= 59,848$$

$$AW = FW_1 \left(\frac{A}{P}, i, 15 \right)$$

$$= 59848 \left(\frac{A}{P}, 15, 15 \right)$$

$$= 59848 (0.1710)$$

$$= 10,234$$

- (Q) A road building contractor has received a major highway construction contract that will require 50,000 m³ of crushed stone each year for 5 years. The needed stone can be obtained from a quarry for \$5.80 per m³. As an alternative, the contractor has decided to try and purchase the quarry. He believes that if he owned the quarry, the stone would only cost him \$4.301 per m³. He thinks, he could resell the quarry at the end of 5 years for \$40,000. If the contractor uses a 6% interest rate, how much would he be willing to pay for the quarry?

Answer:

Life = 5 years

$$\text{Yearly requirement} = 5000 \text{ m}^3$$

If stone is purchased, the cost will be, $5000 \times \$5.8 = \$290,000$

If quarry is purchased, the cost will be, $5000 \times \$4.301 = \$215,050$

To find the initial cost of quarry that makes the purchase of quarry feasible, we equate these two costs in the following equation.

$$AW_{\text{stone purchase}} = AW_{\text{quarry purchase}}$$

$$290000 = P \left(\frac{0.2374}{0.06} - 40000 \left(\frac{0.1714}{0.06} \right) \right) + 215050$$

$$290000 = P(0.2374) + 221906$$

$$\therefore P = \$286832.4$$

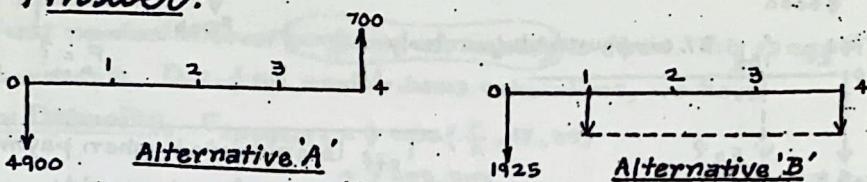
∴ Initial cost of quarry $\leq \$286832.4$

University Test (Jan 2015) Make-up

- (Q) A 50 HP motor is required to drive a pump to remove water from a tunnel. Two alternatives are under consideration. Alternative A calls for the construction of an electric motor along with power lines at a total cost of \$4,900. The salvage value of this equipment after four years is \$700. Alternative B calls for purchase of a diesel pump set at a cost of \$1,925 with no salvage value at the end of four years of its life. The cost of diesel per hour is \$0.5 and maintenance cost of \$0.55 per hour. Cost of wages chargeable when the diesel pump set runs is \$2 per hour. How many hours of operation per year, the two machines have to run so that the two alternatives

incur equal annual cost. If the number of hours of operation is 100 per year which alternative is more economical if the interest rate is 12% per year?

Answer:



(i) Let x be the no. of hours used in a year

$$\text{Cost of diesel} = 0.5/\text{hr}$$

$$\text{Maintenance cost} = 0.55/\text{hr}$$

$$\text{Wages} = 2/\text{hr}$$

$$-700\left(\frac{A}{P}, i, n\right) + 4900\left(\frac{A}{P}, i, n\right) = 1925\left(\frac{A}{P}, i, n\right) + 3.05x$$

$$3.05x = 1466.64 - 633.71$$

$$x = \frac{832.9}{3.05} = 273.08 \text{ hours/year}$$

(ii) If the number of hours of operation is 100 per year

$$-700(0.2092) + 4900\left(\frac{A}{P}, i, 4\right) = 1925\left(\frac{A}{P}, i, 4\right) + 3.05x$$

$$-146.44 + 4900 = 1925 + 3.05x \quad (x = 100)$$

$$-146.44 + 4900 = 1925 + 305$$

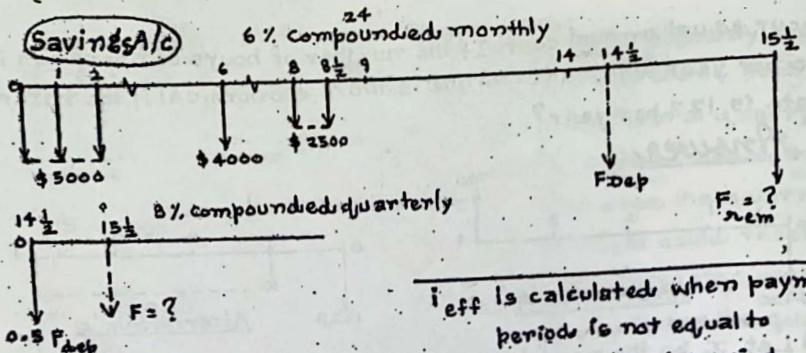
$$4153.56 = 2230$$

Alternative B is preferable

1.3 Nominal and Effective Interests

Sessional Test (February 2015) 1st Sessional

- Q) Georgi Rostov deposits \$5,000 for the three years (starting from now) in a savings account that pays a 6% interest compounded monthly. At the end of sixth year, he deposits \$4000. Two years after the \$4000 deposit, he makes another two equal deposits amount of \$2,500 semi-annually. Six years after the previous deposit, half of the accumulated fund is transferred to a fund that pays 8% interest compounded quarterly. How much money will be accumulated in each account after one year of this transfer?



i_{eff} is calculated when payment period is not equal to compounding period

Amount Accumulated

$$i_{\text{eff}} (\text{Annual}) = \left(1 + \frac{0.06}{m}\right)^m - 1 = \left(1 + \frac{0.06}{12}\right)^{12} - 1 = 0.0617$$

$$i_{\text{eff}} (\text{Semi-annual}) = \left(1 + \frac{0.06}{2}\right)^2 - 1 = 0.03037$$

Amount at 14th year

$$= 3000 \left(\frac{F}{P}, i = 6.17, 3\right) \left(\frac{F}{P}, i = 12, 12\right) + 4000 \left(\frac{F}{P}, i = 8, 5\right) + 2500 \left(\frac{F}{P}, i = 6.17, 6\right) + \\ 2500 \left(\frac{F}{P}, i = 3.037, 11\right)$$

$$= 3000 \left[\frac{(1 + 0.0617)^3 - 1}{0.0617} \right] (1 + 0.0617)^{12} + 4000 (1 + 0.0617)^5 +$$

$$2500 (1 + 0.0617)^6 + 2500 (1 + 0.03037)^{11}$$

$$= 32706.5 + 4457.1 + 3280.4 + 3474.28 = 46,218.9$$

$$\text{At } 14.5 \text{ years} \quad 46,218.9 (1.03037) = 47,622$$

$$\frac{1}{2} \text{ of the amount} = 23,811$$

$$\text{At } 15 \frac{1}{2} \text{ years} \quad \text{In Savings A/c} = 27,623.42 (1.03037)^{\frac{1}{2}} \\ = 29,358.6$$

$$\text{Interest} \quad \left(1 + \frac{0.08}{4}\right)^4 - 1 = 8.243\%$$

$$27,623.42 (1 + 0.08243) = 25774.9$$

Sessional Test (April 2015) 1st Make-Up

- 1) A series of monthly cash flows is deposited into an account that earns 12% nominal interest compounded monthly. Each monthly deposit is equal to \$200. The first monthly deposit occurred on June 1, 1998 and the last monthly deposit will be on January 1, 2005. The account (the series of monthly deposits, 12% nominal interest, and monthly compounding) also has equivalent quarterly withdrawals from

it. The first quarterly withdrawal is equal to \$5000 and occurred on October 1, 1998. The last \$5000 withdrawal will occur on January 1, 2005. How much remains in the account after the last withdrawal?

Answer:

i = 12% nominal interest compounded monthly is given. This is equal to 1% per month. Doing the monthly basis calculations, we have

$$\text{For Deposits, } F_{\text{deposits}} = \$2100 \left(\frac{1}{1}, 17, 30 \right)$$

$$= \$255,509$$

For Withdrawals,

Withdrawals are made on quarterly basis. i.e. effective interest is to be calculated on quarterly basis.

$$F_{\text{withdrawals}} = \$5000 \left(\frac{1}{1}, 3.03\%, 26 \right)$$

$$= \$193,561$$

$$i_{\text{eff}} = \left(1 + \frac{0.12}{12} \right)^4 - 1 \rightarrow (1.01)^4 - 1 \rightarrow 3.03\%$$

∴ Amount remaining in the account on January 1, 2005

$$= \$255,509 - \$193,561$$

$$= \$61,948$$

University Test (July 2015) Make-up

Q) The ABD Company is building a new plant, whose equipment maintenance costs are expected to be \$500 the first year, \$150 the second year, \$200 the third year, \$250 the fourth year, etc., increasing by \$50 per year through the 10th year. The plant is expected to have a 10-year life. Assuming the interest rate is 12%, compounded quarterly, how much should the company plan to set aside now in order to pay for the maintenance? Draw the Cash Flow Diagram.

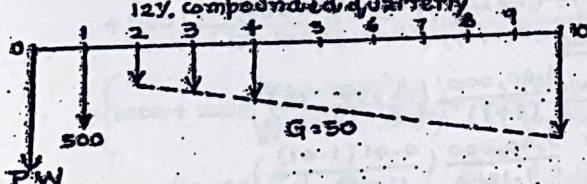
Answer:

Convert the given interest rate to annual basis since the payment is made on annual. To do this, calculate effective interest rate

$$\text{effective interest, } i_{\text{eff}} = \left(1 + \frac{0.12}{4} \right)^4 - 1$$

$$= \left(1 + \frac{0.12}{4} \right)^4 - 1 = 12.53\% \text{ per year}$$

compounded annually.



$$\text{W.R.L. } A = A_1 + G \frac{(1+i)^n - 1}{(1+i)^n - 1}$$

Calculate present worth, that gives an idea regarding the amount to be reserved by company

$$PW = \frac{500}{1+i} + \left\{ 150 + 50 \left(\frac{1}{1+i}, 1, 10 \right) \right\} \left(\frac{1}{1+i}, 1, 10 \right)$$

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Use interpolation to find the interest for $i_{\text{eff}} = 12.55\%$.

$\left[\left(\frac{A}{G}, i, q \right) \right] x_1, 12\% - y_1, 3.257$ $x_2, 15\% - y_2, 3.092$ $x, 12.55\% - y ?$ $\frac{y_2 - y_1}{y - y_1} = \frac{x_2 - x_1}{x - x_1}$ $\frac{3.092 - 3.257}{y - 3.257} = \frac{15 - 12}{12.55 - 12}$ $\frac{3.092 - 3.257}{5.455} = y - 3.257$ $y = 3.257 - 0.030 = 3.227$	$\left(\frac{P}{A}, i, q \right) 12\%, 5.328$ $15\%, 4.772$ $\frac{4.772 - 5.328}{5.455} = y - 5.328$ $y = 5.328 + \left(\frac{4.772 - 5.328}{5.455} \right)$ $= 5.328 - 0.102$ $= 5.226$
$PW = \frac{500}{1.1255} + \frac{\{150 + 50(3.227)\}(5.226)}{1.1255}$ $= 444.25 + 1445.68 = 1889.93$	

University Test (July 2015) Make-up

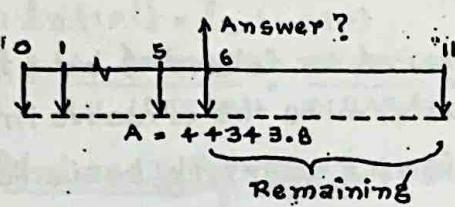
- Q) An engineering student bought a car at a local used car lot. Including tax and insurance, the total price was Rs 3,00,000. He is to pay for the car in 12 equal monthly payments, beginning with the first payment immediately (in other words, the first payment was the down payment). Nominal interest on the loan is 12%, compounded monthly. After six payments (the down payment plus five additional payments), he decides to sell the car. A buyer agrees to pay a cash amount to pay off the loan in full at the time the next payment is due and also to pay the engineering student Rs 1,00,000. If there are no penalty charges for this early payment of the loan, how much will the car costs to the new buyer?

Answer:

$$i_{\text{eff}} = \left(1 + \frac{0.12}{12}\right)^{12} - 1 = \left(1 + \frac{0.12}{12}\right)^{12} - 1 = 12.68\%$$

Price of the car = Rs 3,00,000.

$$\begin{aligned} \text{12 installment amount} &= \frac{3,00,000}{(1+i)} \left(\frac{1}{P}, 12, 12 \right) \\ &= \frac{3,00,000}{1.1268} \left(\frac{0.01(1.01)^{12}}{1.01^{12} - 1} \right) \\ &= 23655.16 \end{aligned}$$



$$\begin{aligned}
 & \text{Answer} = 23,655.2 \left(\frac{P}{A}, i, 6 \right) (1+i) + 1,000 \\
 & = 23,655.2 \left[\frac{(1+0.01)^6 - 1}{0.01 (1.01)^6} \right] (1.01) + 1,00,000 \\
 & = 1,38,464 + 1,00,000 \\
 & = 2,38,464.
 \end{aligned}$$

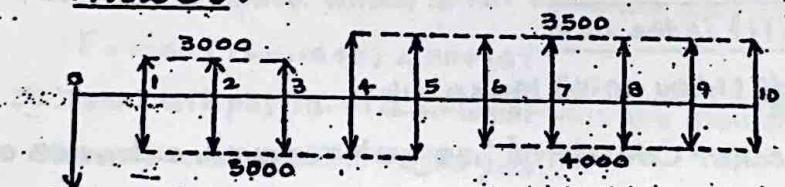
University Test (May 2015)

Q) Suppose that you have a salvage plan covering the next ten years, according to which you put aside Rs 6,000 today, Rs 5,000 at the end of every other year for the next five years, and Rs 4,000 at the end of each year for the remaining five years. As part of this plan, you expect to withdraw Rs 3,000 at the end of every year for the first three years, and Rs 3,500 at the end of every other year thereafter.

i) Draw your cash flow diagram.

ii) Determine the Net Annual Worth of the cash flow if the interest rate is 15% compounded semiannually.

Answer:



$i = 15\%$ compounded semi-annually (Given)

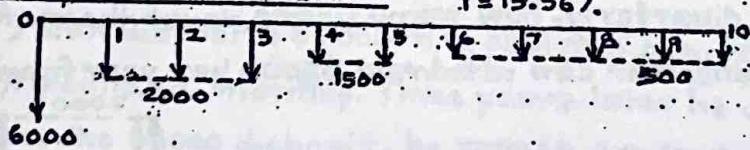
$$i_{\text{eff}} = \left(1 + \frac{r}{n} \right)^n - 1$$

$$= \left(1 + \frac{0.15}{2} \right)^2 - 1$$

$$= 15.56\%$$

Net Cash Flow Diagram

$$i = 15.56\%$$



$$\begin{aligned}
 AW &= 6000 \left(\frac{i(1+i)^n}{(1+i)^n - 1} \right) + 2000 \left(\frac{(1+i)^n - 1}{i(1+i)^n} \right) \left(\frac{i(1+i)^n}{(1+i)^n - 1} \right) + 1500 \left(\frac{(1+i)^n - 1}{i(1+i)^n} \right) \left(\frac{1}{(1+i)^n} \right) \left(\frac{i(1+i)^n}{(1+i)^n - 1} \right) \\
 &\quad + 500 \left(\frac{(1+i)^n - 1}{i(1+i)^n} \right) \left(\frac{1}{(1+i)^n} \right) \left(\frac{i(1+i)^n}{(1+i)^n - 1} \right) \\
 &= \left\{ 6000 + 2000 \left(\frac{(1+0.1556)^3 - 1}{0.1556(1.1556)^3} \right) + 1500 \left(\frac{(1.1556)^2 - 1}{0.1556(1.1556)^2} \right) \left(\frac{1}{(1.1556)^2} \right) + \right. \\
 &\quad \left. 500 \left(\frac{(1.1556)^5 - 1}{0.1556(1.1556)^5} \right) \left(\frac{1}{(1.1556)^5} \right) \right\} * \left\{ \frac{0.1556(1.1556)^{10}}{(1.1556)^{10} - 1} \right\} \\
 &= \left\{ 6000 + 2000(2.262) + 1500(1.1614)(0.648) + 500(3.308)(0.485) \right\} * (0.2) \\
 &= 2763.03 \text{ per year}
 \end{aligned}$$

University Test (May 2015) 28

- Q) The manufacturing company is having the following bids from its suppliers. You are required to determine the effective interest rates for its payment periods which is on a quarterly basis. Also, state the best deal for the manufacturer of the three bids. The details of the bids are as follows:

Bid 1	12% per year compounded daily (360 days a year)
Bid 2	13% compounded quarterly
Bid 3	7% per semi-annual compounded monthly

Answer:

- (1) Bid (1) 12% per year, compounded daily (360 days a year)

$$i_{\text{eff}} = \left(1 + \frac{0.12}{360}\right)^{360} - 1 = 0.03 \approx 3\%$$

- (2) Bid (2) 13% compounded quarterly

$$i_{\text{eff}} = \frac{0.13}{4} = 0.0325 \approx 3.25\%$$

- (3) Bid (3) 7% per semi-annual compounded monthly

$$i_{\text{eff}} = \left(1 + \frac{0.07}{6}\right)^6 - 1 = 0.035 \approx 3.5\%$$

∴ Bid (1) is the best.

University Test (May 2015) Make-up

- Q) An engineer with Becker Consulting has just received a bonus of \$10,000. If the engineer deposits it now at an interest rate of 12% per year compounded quarterly, how many years must the money accumulate before the engineer can withdraw \$2000 per year forever?

Answer:

$$i_{\text{eff}} = \left(1 + \frac{0.12}{4}\right)^4 - 1 = 12.55\%$$

Using Capitalized Cost (CC)

approach for payment of 2000

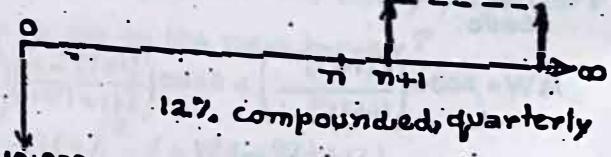
every year from $n+1$ to ∞ , we have

At ' n ' the amount accumulated should be equal to,

$$CC = \frac{A}{i} = \frac{2000}{0.1255} = 15,936.3$$

$$10,000 (1+i)^n = 15,936.3$$

$$(1+i)^n = \frac{15,936.3}{10,000} = 1.594$$



continued

$$n \ln(1+i) = \ln(1.594) \quad 29$$

$$n = \frac{\ln(1.594)}{\ln(1.1255)} = \frac{0.466}{0.11823} = 3.94 \text{ years}$$

University Test (November 2014)

- Q) Jacki obtained a new credit card from a national bank, MBNA, with a stated rate of 10% per year, compounded monthly. For a \$1,000 balance at the beginning of the year, find the effective annual rate and the total amount owed to MBNA after one year, provided no payments are made during the year.

Answer:

$$i = 10\% \text{ per year (Given)}$$

There are 12 compounding periods per year. Thus, $m = 12$ and

$$i = \frac{10}{12} = 0.833\%, \text{ per month}$$

To calculate i_{eff} , payment periods should be equal to compounding periods]

$$\begin{aligned} i_{\text{eff}} &= \left(1 + \frac{i}{m}\right)^m - 1 \\ &= \left(1 + \frac{0.833}{12}\right)^{12} - 1 \\ &\approx 0.1047 = 10.47\% \end{aligned}$$

For \$1000 balance which is not reduced during the year,

$$F = 1000 (1 + 0.1047) = 1104.67$$

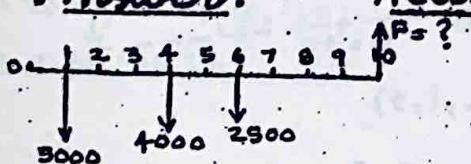
∴ Jacki will pay 10.47% or 104.67 plus the 1000 balance, for the use of bank's money during the year.

University Test (July 2014)

- Q) Mr X deposits \$5000 in a savings account that pays 6% interest, compounded monthly. Three years later he deposits \$4000. Two years later the \$4000 deposit, he makes another deposit of \$2500. Four years after the \$2500 deposit, half of the accumulated fund is transferred to a fund that pays 8% interest, compounded quarterly. How much money will be in each account 6 years after the transfer?

Answer:

ACCOUNT - I



6% compounded monthly (Given)

$$\begin{aligned} i_{\text{eff}} &= \left(1 + \frac{0.06}{12}\right)^{12} - 1 = (1 + 0.005)^{12} - 1 \\ &= 1.0616 - 1 = 0.0616 \\ &= 6.16 \end{aligned}$$

$$F_{10} = 5000(1+0.0616)^9 + 4000(1+0.0616)^6 + 2500(1+0.0616)^4$$

$$= 8562.84 + 5725.65 + 3175.29$$

$$= 17463.78$$

$\frac{F_{10}}{2} = \frac{17463.78}{2} = 8731.89$ is transferred to another account.

ACCOUNT - 1

$$F_{16} = 8731.89(1+0.0616)^6$$

$$= 12498.95$$

ACCOUNT - 2

8% interest compounded quarterly (Given)

$$r_{\text{eff}} = \left(1 + \frac{0.08}{4}\right)^4 - 1$$

$$= 8.24\%$$

$$F_6 = 8731.89(1+0.0824)^6$$

$$= 14042.19$$

University Test (May 2014)

- Q) A man decides to deposit \$50 in the bank today and to make 10 additional deposits every 6 months beginning 6 months from now, the first of which will be \$50 and increasing \$10 per deposit after that. A few minutes after making the last deposit, he decides to withdraw all the money deposited. If the bank pays 6% nominal interest compounded semi-annually, how much money will he receive?

Answer: 6% nominal interest compounded semi-annually (Given)

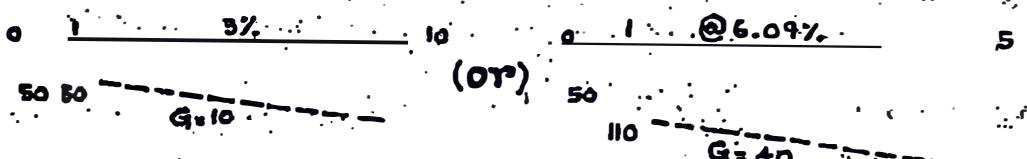
$$r_{\text{eff}} = \left(1 + \frac{0.06}{2}\right)^2 - 1$$

$$= \left(1 + \frac{0.06}{2}\right)^2 - 1 = 0.0609 = 6.09\%$$

We need to find future value in order to calculate the amount which he receives at the end of 5th/10th year

Future Value calculations can be carried out in 2 different ways.

Both are shown in the CFD diagrams illustrated below:



$$F_{10, \text{years}} = 50(1.0609)^{10} + \left\{ 50 + 10 \left(\frac{1+i}{G} \right) \right\} \left(\frac{1-(1+i)^{-n}}{1-(1+i)^{-1}} \right)$$

$$= 50(1.0609)^{10} + \left\{ 50 + 10 \left(\frac{(1+i)^n - 1}{i} \right) \right\} \left(\frac{(1+i)^n - 1}{i} \right)$$

Use interpolation in finding the values of $(\frac{A}{G}, i, 5)$ and $(\frac{F}{A}, i, 5)$.

	6%	8%	6.09%
$(\frac{A}{G}, i, 5)$	1.884	1.846	?
$(\frac{F}{A}, i, 5)$	5.637	5.867	?

Similarly, 2nd calculation

$$y = \frac{(5.867 - 5.637) 0.09}{2} + 5.637$$

$$= 5.6474$$

$$\frac{y_2 - y_1}{y - y_1} = \frac{x_2 - x_1}{x - x_1}$$
1st calculation

$$\frac{1.846 - 1.884}{y - 1.884} = \frac{8 - 6}{6.09 - 6}$$

$$y = \frac{(1.846 - 1.884) 0.09}{2} + 1.884$$

$$= 1.8823\%$$

$$F_5 = 50(1.0609)^5 + (110 + 40(1.8823)) 5.6474$$

$$= 67.2 + 1046.42$$

$$= 1113.62$$

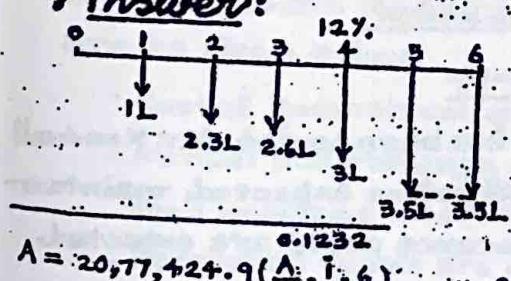
Chapter. 2 Capitalized Costs

University Test (December 2014) Make-up

- Q) Using an interest rate of 12%, what is the capitalized cost of a tunnel to transport water through the Someshwar Mountain range if the first cost is Rs 100,00,000 and the maintenance costs are expected to occur in a 6-year cycles as shown below?

Year	1	2	3	4	5	6
Maintenance Cost	1,00,000	2,30,000	2,60,000	3,00,000	3,50,000	3,50,000

Answer:



Finding the Annuity of all CF for 1 cycle

$$= 3.5 + 3.5(1.12) + 3(1.12)^2 + 2.6(1.12)^3 + 2.3(1.12)^4 +$$

$$+ 1(1.12)^5$$

$$= 20.774249 \text{ Lakhs}$$

$$= 20,77,424.9$$

$$A = 20,77,424.9 \left(\frac{1 - 1.12^6}{1 - 1.12} \right)$$

$$= 2,55,938.75$$

$$CC_{Revenue} = \frac{A}{i} \times \frac{2,55,938.75}{0.12}$$

$$= \text{Rs } 21,292,822.9$$

$$CC_{Total} = CC_{Revenue} + P$$

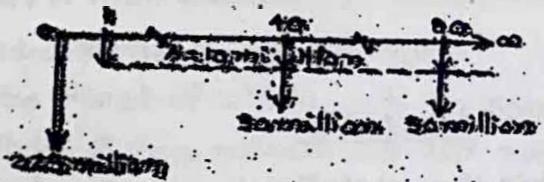
$$= \text{Rs } 1,21,32,822.9$$

32
Sessional Test (April 2013) 2nd Sessional

- Q) Two large-scale conduits are under consideration by a large Municipal Utility District (MUD). The first involves construction of a steel pipeline at a cost of Rs 225 million. Portions of the pipeline will have to be replaced every 40 years at a cost of Rs 50 million. The pumping and other operational costs are expected to be Rs 10 million per year. Alternatively, a gravity flow canal can be constructed at a cost of Rs 350 million. The maintenance costs for the canal are expected to be Rs 0.2 million per year for the first four years and then onwards increases to Rs 0.5 million per year, forever. If both conduits are expected to last forever, which should be built, at an interest rate of 10% per year?

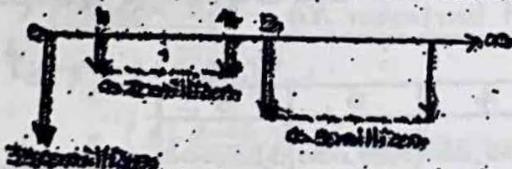
Answer: With $i = 10\%$ and Capitalized Cost Approach, we have

Alternative 1: Steel Pipe Line



$$\begin{aligned} CC &= \frac{225,000,000 + 50,000,000}{0.1} + \\ &\quad + \frac{50,000,000(1 - 1/40)}{0.1 \times 1 - 1/40} \\ &= 225,000,000 + 100,000,000 + 1,130,000 \\ &= 326,130,000 \end{aligned}$$

Alternative 2: Gravity Flow



$$\begin{aligned} CC &= 350,000,000 + \\ &\quad + \frac{300,000}{0.1} + \frac{300,000}{0.1 \times 1 - 1/40} \\ &= 350,000,000 + 2,000,000 + 2,019,040 + \\ &= 354,019,040 \end{aligned}$$

∴ Alternative 1: Steel Pipe Line is better

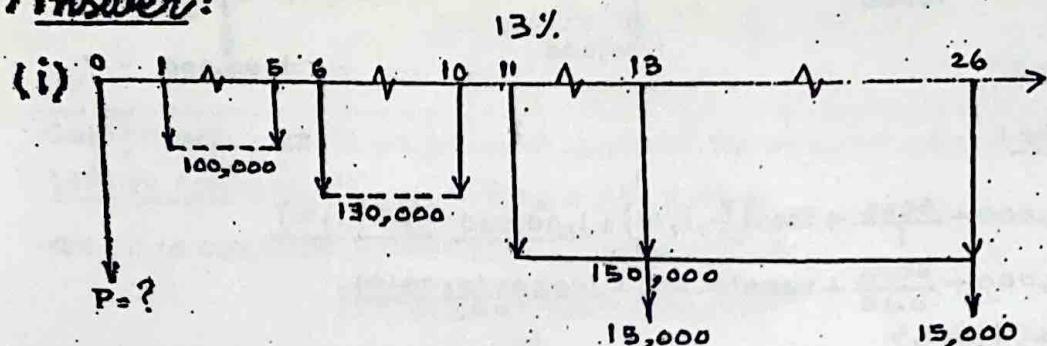
Sessional Test (April 2013) Make-up

- Mr. Mendell wants to raise money for a new building. He has been sought. Mr. Mendell would like to make a donation to cover all future expected maintenance costs for the building. These maintenance costs are expected to be \$100,000 each year for the first five years, \$130,000 each year for years 6 through 10 and \$150,000 each year after that. Also there is a rework which will be carried out for every 13 years of an amount \$15,000 (the building has an indefinite service life). Draw the cash flow diagram.

contd.

- (i) If the money is placed in an account that will pay 13% interest compounded annually, how large should the gift be?
- (ii) What is the equivalent annual maintenance cost over the infinite service life of the building?

Answer:



$$\begin{aligned}
 CC &= \frac{100,000}{i} + \frac{30,000}{i} \left(\frac{P}{F}, i, 5 \right) + \frac{20,000}{i} \left(\frac{P}{F}, i, 10 \right) + \frac{15,000}{i} \left(\frac{A}{F}, i, 13 \right) \\
 &= 769,230.8 + 125,252.3 + 45,321.3 + 3,624.2 \\
 &= 943,428.6
 \end{aligned}$$

$$(iii) AW = CC * i = 122,645.72$$

University Test (July 2015) Make-up

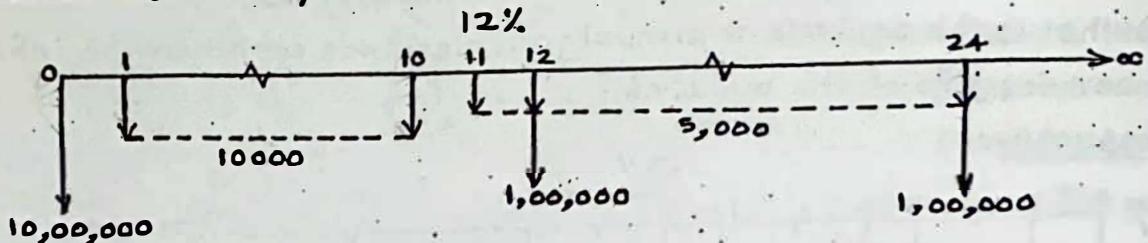
Q) The archaeologist group of Karnataka have recently found a very old temple in the midst of the dense forest of Western Ghats near Agumbe in the Someshwara forest region. The temple is in a completely destroyed condition. But because of its scenic location which is along the bank of the river followed by a few kilometres trek to Snake Abbi Falls, the local government bodies are planning to redevelop this temple and make this as a tourist spot which would definitely attract tourists from all around the country. The expected costs details of this project are as given below:

- Cost of Reconstruction work Rs 10,00,000.
- Annual Maintenance costs Rs 10,000 in the first ten years and then onwards decreases to Rs 5,000 and continues forever.
- Renovation works are expected to be Rs 1,00,000 once in every 12 years.

If the Government bodies are planning to raise a fund under the name of the temple which would meet all the listed future expenses. What should be the fund amount that is required now in order to meet these expenses take $i = 12\%$?

34

Capitalized Cost (cc) is the present worth of the amount which is recovering from year 1 to infinite



Answer:

$$\begin{aligned}
 CC &= 10,00,000 + \frac{5000}{12\%} + 5000 \left(\frac{1}{A}, i, 10 \right) + 1,00,000 \left(\frac{1}{F}, i, 12 \right) \\
 &= 10,00,000 + \frac{5000}{0.12} + 5000(5.65) + \frac{100000(0.0414)}{0.12} \\
 &= 1,104,416.67
 \end{aligned}$$

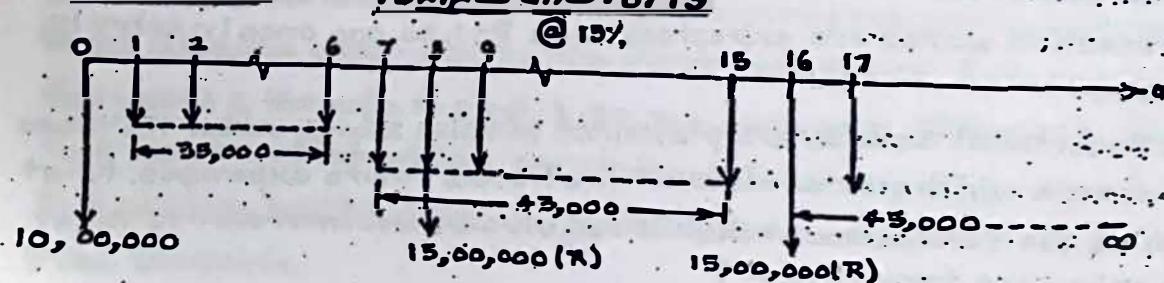
University Test (May 2015)

- Q) The ministry of tourism of Karnataka is planning for the development and maintenance of few tourism places in and around Udupi district. The few places identified are Barkur Fort, Koodlu Falls, Jain temple Karkala etc. The Government is planning to put aside a lump sum amount now into a fund account so that the money from this can be utilized for this project forever. The cost estimation of this project are as follows.

Particulars (in Rs)	Temples and Forts	Waterfalls and Beaches
Initial Cost	1,00,00,000	2,00,00,000
Annual Maintenance Cost	Rs 35,000 for the first six years and then till 15 th year Rs 43,000 and then onwards Rs 45,000	50,000 forever
Major Rework	Rs 15,00,000 once in every eight years	Once in every 15 years of Rs 30,00,000

Determine the total lump sum amount to be invested now using a capitalized cost approach with an interest rate of 15%.

Answer: Temples and Forts



$$CC_1 = 100,00,000 + \frac{35,000}{(0.15)} + \frac{8000}{(0.15)} \left(\frac{P}{F}, 15\%, 6 \right) + \frac{2000}{(0.15)} \left(\frac{P}{F}, 15\%, 15 \right)$$

$$= 102333333.33 + 23056 + 1638.6$$

$$= 10258027.93/-$$

For Rs 15,00,000

$$CC_2 = \frac{15,00,000}{0.15} \left(\frac{A}{F}, 15\%, 8 \right)$$

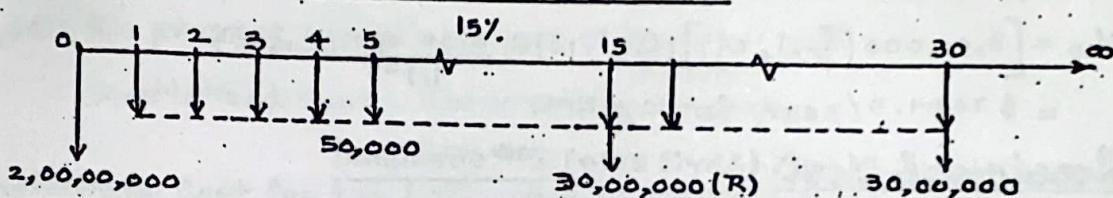
$$= 729000/-$$

Capitalized cost is the present worth of the amount where the service life is infinite.

$$CCA = CC_1 + CC_2$$

$$CCA = 102333333.33 + 729000 = 103062333.3$$

Waterfalls and Beaches



$$CC_B = 2,00,00,000 + \frac{50,000}{0.15} + \frac{30,00,000}{0.15} \left(\frac{A}{F}, 15\%, 15 \right)$$

$$= 20753333.33/-$$

Total lumpsum amount, $CC = CCA + CC_B = 123815666.6/-$

University Test (May 2015) Make-up

- Q) Using an interest rate of 12%, what is the capitalized cost of a tunnel to transport water through the Girnar Mountain range, if the first cost is \$1,000,000 and the maintenance costs are expected to occur in a 6-year cycles as shown below?

End of Year	1	2	3	4	5	6
Maintenance Cost	\$35,000	\$35,000	\$35,000	\$45,000	\$45,000	\$60,000

Answer: Determining the annual worth for Recurring cash flow in one cycle. Then dividing by i gives capitalized cost for recurring CF.

$$\therefore \text{Capitalized Cost} = P + \frac{AW_{\text{recurring}}}{i}$$

$$AW_R = 35000 \left(\frac{P}{A}, i, 3 \right) + \frac{4500 \left(\frac{P}{A}, i, 2 \right)}{(1+i)^3} + \frac{60000}{(1+i)^6}$$

$$= 84070 + 54130.9 + 30397.9$$

$$= 1,68,598.8$$

$$CC = 1,000,000 + \frac{168598.8}{0.12}$$

$$= 2,404,989.72$$

University Test (November 2014)

36

- Q) What amount should be deposited in your retirement account starting from now and continuing each year through year nine (i.e., to deposit) if you want to withdraw an amount of \$80,000 per year continuing forever beginning 30 years from now? Assume the account earns an interest rate of 10% per year.

Answer:



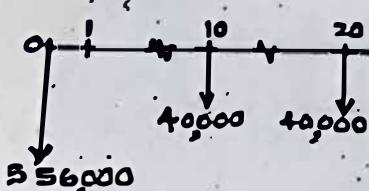
$$CC = \frac{A}{i} = \frac{80000}{0.1} = 800,000.$$

$$PW_{10} = \left[800,000 \left(\frac{P}{F}, i, 29 \right) \right] \left(\frac{A}{P}, i, 10 \right) \rightarrow \frac{800,000}{1.1^{29}} (0.1627) \\ = \$7499.3 / \text{year for 10 years.}$$

Sessional Test (April 2014) 2nd sessional

- Q) A city has developed a plan to provide for future municipal water needs. The plan proposes an aqueduct that passes through 500feet of tunnel in a nearby mountain. Two alternatives are being considered. The first proposes to build a full-capacity tunnel now for \$556,000. The second proposes to build a half-capacity tunnel now (cost=\$102,000) which should be adequate for 20 years, and then to build a second parallel half-capacity tunnel at the same cost which was 20 years ago. The maintenance cost of the tunnel lining for the full-capacity tunnel is \$40,000 every 10 years, and for each half-capacity tunnel it is \$32,000 every 10 years. The friction losses in the half-capacity tunnel will be greater than if the full-capacity tunnel were built. The estimated additional costs in the single half-capacity tunnel will be \$2000 per year; and for the two half-capacity tunnels it will be \$4000 per year. Based on capitalized cost and a 7% interest rate, which alternative should be selected?

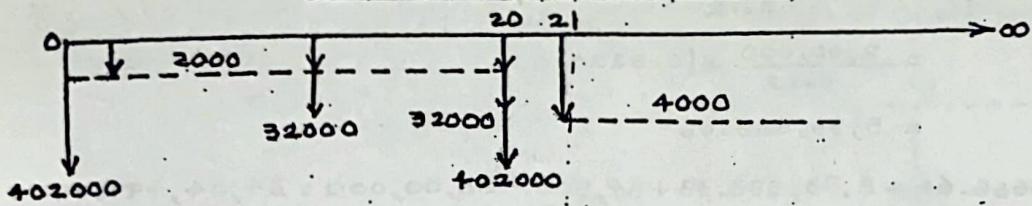
Solution: (A) Full Capacity Tunnel



$$\text{Capitalized Cost} = \$556,000$$

$$+ \left[(\$40,000 \left(\frac{A}{P}, 7\%, 10 \right)) / 0.07 \right] \\ = \$597,400$$

97
(B) First Half Capacity Tunnel



$$\begin{aligned}\text{Capitalized Cost} &= \$402000 + \left[(\$32000(0.0724))/0.07 \right] + \left[2000/0.07 \right] \\ &= \$463,700\end{aligned}$$

(c) Second Half Capacity Tunnel

20 years hence the Capitalized Cost of the second half-capacity tunnel equals the present capitalized cost of the first cost.

$$\begin{aligned}\text{Capitalized Cost} &= \$463,700 \left(\frac{P}{F}, 7\%, 20 \right)^{0.2584} \\ &= \$119,800\end{aligned}$$

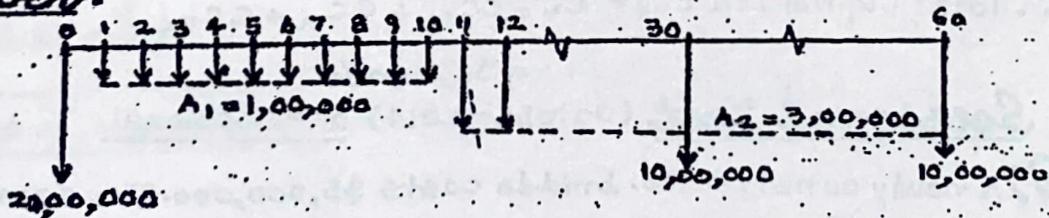
$$\begin{aligned}\text{Capitalized Cost for two half-capacity tunnels} \\ &= \$463700 + \$119800 \\ &= \$583500\end{aligned}$$

∴ Build the full-capacity tunnel

University Test (July 2014) Make-up

- Q) A newly constructed water treatment facility costs Rs 20,00,000. It is estimated that the facility will need renovation every 30 years at a cost of Rs 10,00,000. Annual repairs and maintenance costs are expected to be Rs 1,00,000 per year for first 10 years and Rs 3,00,000 thereafter. Determine the capitalized cost of the facility. Assume i = 12%.

Answer:



Drawing CFD for the two cycles of Cash Flow (i.e. 60 years)

$$CC_1 = 20,00,000$$

$$CC_2 = \frac{10,00,000}{i} \left(\frac{A}{P}, i, 30 \right)$$

$$\therefore = \frac{10,00,000}{0.12} (0.00414) = 34,500$$

$$A = F \left[\frac{i}{(1+i)^n - 1} \right]$$

$$CC_3 = \frac{A_1}{i} = \frac{1,00,000}{0.12} = 8,33,333.33$$

$$\begin{aligned} CC_4 &= \frac{A_2}{i} = \frac{(3,00,000 - 1,00,000)}{0.12} \left(\frac{P}{F}, 12, 10 \right) \\ &= \frac{2,00,000}{0.12} * (0.3220) \\ &= 5,36,666.66 \end{aligned}$$

$$CC = 5,36,666.66 + 8,33,333.33 + 34,500 + 20,00,000 = 34,04,499.99$$

University Test (May 2014)

- Q) A trust fund is to be established for three purposes:

- To provide \$ 750,000 for the construction and \$ 250,000 for the initial equipment of a small engineering laboratory.
- To pay the \$ 150,000 per year laboratory operating cost; and
- To pay for \$ 100,000 of replacement equipment every 4 years, beginning 4 years from now.

At 6% interest, how much money is required in the trust fund to provide for the laboratory and equipment and its perpetual operation and equipment replacement?

Answer:

First Cost $CC_{FC} = 7,50,000 + 2,50,000 = 10,00,000$

Laboratory Operating Cost

$$CC_{OC} = \frac{1,50,000}{0.06} = 25,00,000$$

Replacement of equipment every 4 years

$$CC_R = \frac{1,00,000 \left(\frac{A}{F}, i, 4 \right)}{0.06} = 3,81,000$$

$$\therefore \text{Total capitalized cost, } CC = CC_{OC} + CC_R + CC_{FC}$$

$$= 38,81,000$$

Sessional Test (October 2014) 2nd Sessional

Q) A newly constructed bridge costs \$5,000,000. The same bridge is estimated to need renovation every 20 years at a cost of \$1,000,000. Annual repairs and maintenance are estimated to be \$100,000 per year for first 25 years and increases to \$135,000 thereafter. If the interest rate is 5%, determine the capitalized cost of the bridge.

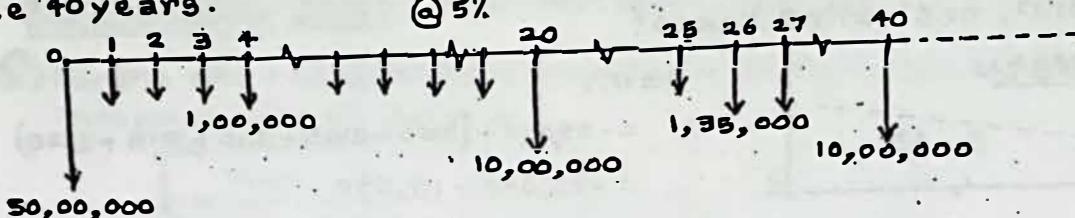
Answer:

39

Drawing Cash Flow Diagram (CFD) for two cycles of Cash Flow (CF)

i.e. 40 years.

@ 5%



$$CC = 50,00,000 + \frac{1,00,000}{0.05} + \frac{35,000}{0.05} \left(\frac{P}{F}, 5, 25 \right) + \frac{10,00,000}{0.05} \left(\frac{A}{F}, 5, 20 \right)$$

$$= 50,00,000 + 20,00,000 + 2,06,710 + 6,04,000$$

$$= 78,10,710$$

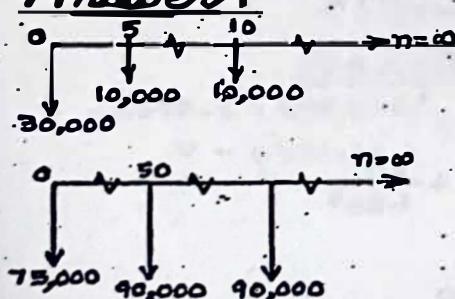
University Test (Jan 2015) Make-up

- Q) Two methods of carrying away surface run-off water from a new subdivision are being evaluated:

Method A: Dig a ditch, the first cost would be \$30,000, and \$10,000 for redditting and shaping would be required at five year intervals forever.

Method B: Lay concrete pipe. The first cost would be \$75,000 and a replacement would be required at 50 year intervals at a net cost of \$90,000 for ever. At $i = 12\%$, which method is better one?

Answer:



$$CC_A = 30,000 + \frac{10,000}{0.12} \left(\frac{A}{F}, 12, 5 \right)$$

$$= 30,000 + \frac{10,000}{0.12} (0.1574)$$

$$= 43,116.66$$

Method 'A'

$$CC_B = 75,000 + \frac{90,000}{0.12} \left(\frac{A}{F}, 12, 50 \right)$$

$$= 75,000 + \frac{90,000}{0.12} \left(\frac{1}{(1+0.12)^{50}-1} \right)$$

$$= 75,000 + \frac{90,000}{0.12} \left(\frac{0.12}{(1+0.12)^{50}-1} \right)$$

$$= 75,312.49$$

Method 'B'

Method 'A' is preferred

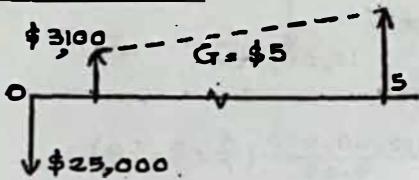
Chapter. 3 Internal Rate of Return (IRR)

Sessional Test (April 2014) 2nd Sessional

Q) A company purchases a piece of construction equipment for rental purposes. The expected income is \$3100 for the first year and then increasing at a gradient of \$5 per year, it has a useful life of 5 years.

Expenses are estimated to be \$355 annually. If the purchase price is \$25,000 and there is no salvage value, what is the prospective rate of return, neglecting taxes?

Answer:



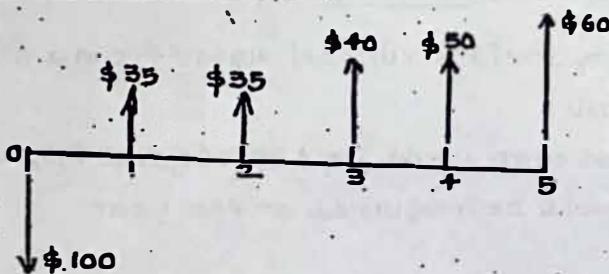
$$i = 0\%$$

$$\begin{aligned} &= -25,000 + (3100 + 3105 + 3110 + 3115 + 3120) \\ &= -25,000 + 15,550 \\ &= -9,450 \end{aligned}$$

Since revenue is less than cost,
there is no return

Sessional Test (October 2014) 2nd Sessional

- Q)



Consider the below project with given cash flows. Comment on the feasibility of the project by calculating the IRR, assume MARR as 10%.

Answer:

$$NPW = 0 \Rightarrow -100 + \frac{35}{(1+i)} + \frac{35}{(1+i)^2} + \frac{40}{(1+i)^3} + \frac{50}{(1+i)^4} + \frac{60}{(1+i)^5} \quad (\text{OR})$$

$$\begin{aligned} i = 10\% \\ &= -100 + 35\left(\frac{P}{A}, i, 2\right) + [40 + 10\left(\frac{A}{G}, i, 3\right)]\left(\frac{P}{A}, i, 3\right)\left(\frac{P}{A}, i, 2\right) \\ &= -100 + \frac{35}{1.1} + \frac{35}{1.1^2} + \frac{40}{1.1^3} + \frac{50}{1.1^4} + \frac{60}{1.1^5} \\ &= 62.2 \end{aligned}$$

$$i = 25\% = -100 + \frac{35}{1.25} + \frac{35}{1.25^2} + \frac{40}{1.25^3} + \frac{50}{1.25^4} + \frac{60}{1.25^5}$$

$$\approx 11.0208$$

$$i = 30\% = -100 + \frac{35}{1.3} + \frac{35}{1.3^2} + \frac{35}{1.3^3} + \frac{35}{1.3^4} + \frac{60}{1.3^5}$$

$$= -7.3$$

Use Interpolation to find IRR

$$\begin{bmatrix} x_1 \rightarrow 25\% & y_1 \rightarrow 11.0208 \\ x_2 \rightarrow 30\% & y_2 \rightarrow -7.3 \\ x & y \rightarrow 0 \end{bmatrix} \quad \begin{aligned} \frac{-7.3 - 11.021}{0 - 11.021} &= \frac{30 - 25}{x - 25} \\ x &= 25 + \frac{(30 - 25)(-11.021)}{-7.3 - 11.021} \\ IRR &= 28\% \end{aligned}$$

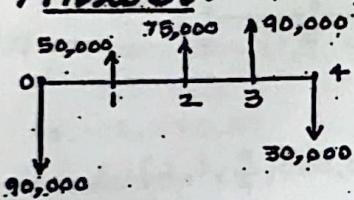
3.1 IRR

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University Test (December 2014) Make-up

- Q) For the given cash flow diagram below, determine the internal rate of return.

Year	0	1	2	3	4
Cash Flow(Rs)	-90,000	50,000	75,000	90,000	-30,000

Answer:

$$NPW=0 \Rightarrow$$

$$-90,000 - \frac{30,000}{(1+i)^4} + \frac{50,000}{(1+i)^1} + \frac{75,000}{(1+i)^2} + \frac{90,000}{(1+i)^3}$$

$$i=10\% = -90,000 - \frac{30,000}{1.1^4} + \frac{50,000}{1.1^1} + \frac{75,000}{1.1^2} + \frac{90,000}{1.1^3}$$

$$\approx 64,565.9$$

$$i=20\% = -90,000 - \frac{30,000}{1.2^4} + \frac{50,000}{1.2^1} + \frac{75,000}{1.2^2} + \frac{90,000}{1.2^3}$$

$$= 41,365.7$$

$$x, i=40\% = -90,000 - \frac{30,000}{1.4^4} + \frac{50,000}{1.4^1} + \frac{75,000}{1.4^2} + \frac{90,000}{1.4^3}$$

$$= 8,969.18$$

$$x_2, i=50\% = -90,000 - \frac{30,000}{1.5^4} + \frac{50,000}{1.5^1} + \frac{75,000}{1.5^2} + \frac{90,000}{1.5^3}$$

$$= -3,592.6$$

Use Interpolation to find IRR.

$$-3,592.6 - 8,969.18 = \frac{30-40}{x-40}$$

$$0 = 8,969.18 \quad x-40$$

$$x-4 = 7.16 \quad \therefore x = 47.76$$

$$(IRR = 47.76\%)$$

Sessional Test (April 2015) 2nd sessional

- Q) You are going to buy a new car worth \$14,500. The dealer computes your monthly payment to be \$267 for 72 months' financing. What is the dealer's rate of return on this loan transaction?

Answer: Present value of the car = 14,500 \$ (given)

$$14,500 = 267 \left(\frac{P}{A}, 1,72 \right)$$

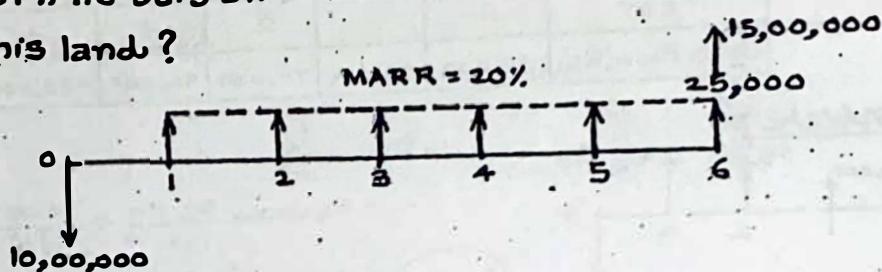
$$\left(\frac{P}{A}, 1,72 \right) = \frac{14,500}{267} = 54.307$$

$$i = \%.$$

- Sessional Test (April 2015) 2nd Sessional (make-up)

Q) Mr. Khanna is planning to invest an amount of Rs. 10,00,000. He can invest in bank and earn a return of 20% or he can invest in a land now and give this for rent. The annual rental income is Rs 25,000. He is planning to sell this land after 6 years at an amount of Rs 15,00,000. What is the return he gets on this land investment. Is it worth investing on this land?

Answer:



$$\begin{aligned} \text{PW at } i=10\% &= -10,00,000 + 25,000 \left(\frac{P}{A}, i, 6 \right) + 15,00,000 \left(\frac{P}{F}, i, 6 \right) \\ &= -10,00,000 + 25,000 (4.19) + 15,00,000 (0.5488) \\ &= -69,550 \end{aligned}$$

$$\begin{aligned} i = 5\% &= -10,00,000 + 25,000 \left(\frac{P}{A}, i, 6 \right) + 15,00,000 \left(\frac{P}{F}, i, 6 \right) \\ &= -10,00,000 + 25,000 (5.055) + 15,00,000 (0.740) \\ &= 236,375 \end{aligned}$$

Observations:

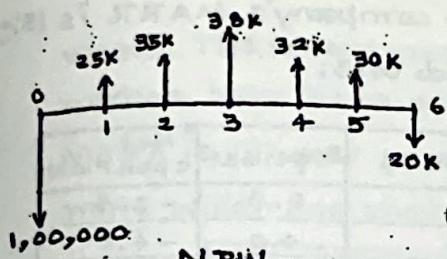
Present Value of the cash flow became negative at $i=10\%$. and positive at $i=5\%$. So, the IRR value lies between 5% and 10%. Use interpolation, to find the exact value of IRR.

$$\left[\begin{array}{cc|c} 5 & \propto & 10 \\ 236,375 & & 0 \end{array} \right] \Rightarrow \text{IRR} = 5 + \frac{236,375 - 0}{(236,375 + 69,550)} * 5 = 8.86\%$$

University Test (July 2015) Make-up

- Q) Consider the below project with given cash flows. Comment on the feasibility of the project by calculating the IRR, assume MARR as 15%.

Project Life (Years)	Cash Flow (in Rs.)
0	- 1,00,000
1	25,000
2	35,000
3	38,000
4	32,000
5	30,000
6	- 20,000



$$NPW = 0 \Rightarrow -1,00,000 + \frac{25,000}{1+i} + \frac{35,000}{(1+i)^2} + \frac{38,000}{(1+i)^3} + \frac{32,000}{(1+i)^4} + \frac{30,000}{(1+i)^5} - \frac{20,000}{(1+i)^6}$$

i = 12%

$$NPW = -1,00,000 + 22,321.43 + 27,901.8 + 27,047.7 + 20,366.6 \\ + 17,022.8 - 10,132.6 \\ = 4,497.73$$

i = 15%

$$NPW = -1,00,000 + 21,739.13 + 26,465.03 + 24,985.6 + 18,296.1 + 14,915.3 - 8,646.55 \\ = -2,245.39$$

Interpolation.

$$\left[\begin{array}{ll} x_1 \rightarrow 12\% & y_1 \rightarrow 4,497.73 \\ x_2 \rightarrow 15\% & y_2 \rightarrow -2,245.39 \\ x \rightarrow ? & y \rightarrow 0 \end{array} \right] \Rightarrow \frac{y_2 - y_1}{y - y_1} = \frac{x_2 - x_1}{x - x_1} \\ - \frac{-2,245.39 - 4,497.73}{0 - 4,497.73} = \frac{15 - 12}{x - 12} \\ x \Rightarrow IRR = 14\%.$$

Since MARR (15%) > IRR (14%), not feasible.

University Test (May 2015)

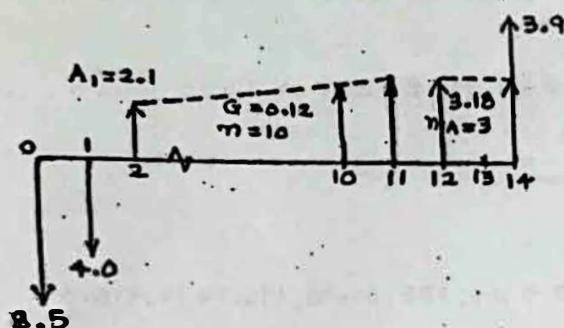
- Q) Champion Chemical Corporation is planning to expand one of its propylene manufacturing facilities. Land costing \$3.5 million must be purchased to build a plant. The building, which needs to be expanded, costs \$5 million. At the end of the first year, the company needs to spend about \$4 million on equipment and other start-up costs. Once the building becomes operational, it will generate revenue in the amount of \$3.5 million during the first operating year. This will increase by 0.2 million every year with respect to previous year revenue for the next 9 years. After 10 years, the sales revenue will stay constant for further years before the operation is phased out (it will have a project life of 13 years after construction). The expected salvage value of the land at the end of the project's life would be about \$2 million, the building about \$1.4 million, and the equipment about \$500,000. The annual operating and maintenance costs are estimated to be approximately 40% of the sales revenue each year.

Continued..

44

What is the IRR for this investment? If the company's MARR is 15%, determine whether the investment is a good one.

Answer:



$$NPW = -8.5 - \frac{4}{(1+i)^1} +$$

$$\left[2.1 + 0.12 \left(\frac{A}{G} + \frac{m}{G} + 1 \right) \right] \frac{(P_A, i, 10)}{(1+i)^1} + \\ \left[3.06 \left(\frac{F_A}{A}, i, 3 \right) + 3.9 \right]$$

$$(1+i)^{14}$$

$$(i=15\%) = -8.5 - \frac{4}{1.15} + \left[2.1 + 0.12(3.383) \right] \frac{5.019}{1.15} + \frac{[3.06(3.472) + 3.9]}{1.15^{14}}$$

$$= -8.5 + 3.48 + 10.94 + 2.10$$

$$= 1.01$$

$$NPW = -8.5 - \frac{4}{1.2} + \frac{[2.1 + 0.12(3.074)] 4.192}{1.2} + \frac{[3.06(3.640) + 3.9]}{1.2^{14}}$$

$$(i = 20\%) = -8.5 - 3.33 + 8.62 + 1.18$$

$$= -2.030$$

Interpolation

$$\begin{bmatrix} x_1 \rightarrow 15\% & y_1 \rightarrow -0.98 \\ x_2 \rightarrow 20\% & y_2 \rightarrow -2.030 \\ x \rightarrow ? & y \rightarrow 0 \end{bmatrix} \Rightarrow$$

$$\frac{y_2 - y_1}{y - y_1} = \frac{x_2 - x_1}{x - x_1} \Rightarrow \frac{-2.030 - (-0.98)}{0 - 15\%} = \frac{20 - 15}{x - 15}$$

$$3.009 = \frac{5}{x - 15}$$

$$x = 15 + \frac{5}{3.009}$$

Since $IRR > MARR$, Investment is Worth

3.2 Incremental IRR

Sessional Test (April 2015) 2nd Sessional

Q) A metal plating company is considering four different methods for recovering byproduct heavy metals from a manufacturing site's liquid. continued...

waste. The investment costs and incomes associated with each method have been estimated. All methods have an 8-year life. The MARR is 10% per year. If the methods are mutually exclusive, determine which one method should be selected.

Method	First Cost Rs	Salvage Value, Rs	Annual Income Rs/Year	IRR for each alternative, %
A	-30,000	1,000	4,000	2.20
B	-36,000	2,000	7,000	12.00
C	-41,000	3,000	8,000	11.40
D	-53,000	-2,000	10,500	11.10

Use Incremental ROR evaluation method for the analysis purpose.

Answer:

Arrange the options in ascending order and then perform Incremental ROR analysis for B, C, and D.

B-C	B	C	C-B
First cost	-36,000	-41,000	-5,000
Salvage Value	-2,000	3,000	-5,000
Annual Income	7,000	8,000	1,000

n=8 years

MARR = 10%

$$NPW = 0 \Rightarrow -5000 - 1500\left(\frac{1}{F}, i, 8\right) + 1000\left(\frac{1}{A}, i, 8\right)$$

$$\textcircled{i=10\%} \quad -5000 - 1500(0.4665) + 1000(5.335) \\ = -364.75$$

$$NPW = 0 \Rightarrow -5000 - 1500(0.6768) + 1000(6.463)$$

$$\textcircled{i=5\%} \quad = +447.8$$

∴ IRR is between 5-10% which is lesser than MARR. B is better.

B-D	B	D	D-B
First Cost	-36,000	-53,000	-17,000
Salvage Value	2,000	-2,000	-4,000
Annual Income	7,000	10,500	3,500

$$NPW = 0 \Rightarrow -17,000 - 4,000\left(\frac{1}{F}, i, 8\right) + 3,500\left(\frac{1}{A}, i, 8\right)$$

$$\textcircled{i=10\%} \quad = -17,000 - 4,000(0.4665) + 3,500(5.335) \\ = -193.5$$

$$NPW = 0 \Rightarrow -17,000 - 4,000(0.6768) + 3,500(6.463)$$

$$= 2913.3$$

By Interpolation, we have

$$\frac{y_2 - y_1}{y - y_1} = \frac{x_2 - x_1}{x - x_1} \Rightarrow \frac{2913.3 - (-193.5)}{0 - (-193.5)} = \frac{5-10}{x-10}$$

$$16.05 * (x-10) = -5$$

$x = 9.68\%$ is lesser than MARR

∴ Alternative B is the best

University Test (November 2014)

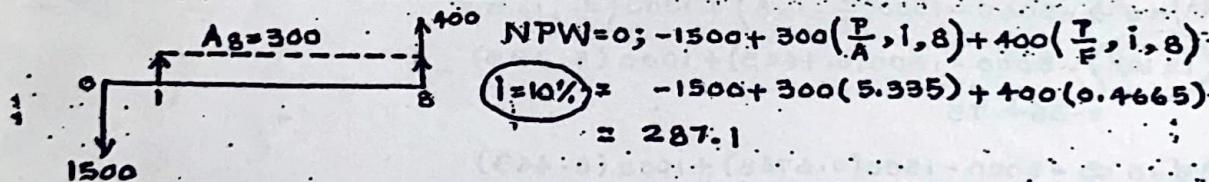
- Q) A company is considering for the purchase of new equipment. The company representatives have agreed to use the average of the corporate MARR values, which results in $MARR = 13.5\%$. Use Incremental ROR analysis to determine which equipment would be the best economic choice.

	M/c 1	M/c 2	M/c 3
First cost (in \$)	-5,000	-6,500	-15,000
Annual operating cost (in \$)	-3,300	-3,200	-1,400
Salvage Value (in \$)	500	900	1,000
Life in years	8	8	8

Answers:

Arrange the alternatives in ascending order and then calculate the Incremental IRR for the alternatives.

Incremental Cash Flow (M/c 2 - M/c 1)



$$NPW = 0; -1500 + 300(4.487) + 400(0.3269) = -23.14$$

Interpolation:

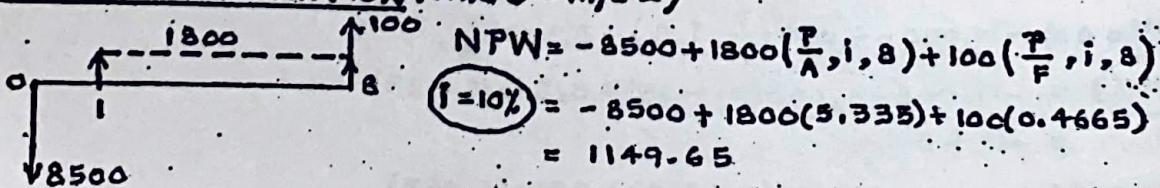
$$\left[\frac{y_2 - y_1}{y - y_1} = \frac{x_2 - x_1}{x - x_1} \right] \Rightarrow \frac{-23.14 - 287.1}{0 - 287.1} = \frac{15-10}{x-10}$$

$$x = 10 + \frac{5 * 287.1}{287.1} = 14.62\%$$

$$x = 14.62\% > MARR$$

∴ Machine 2 is selected

Incremental Cash Flow (M/c 3 - M/c 2)



$$NPW = -8500 + 1800 \left(\frac{P}{A}, i, 8 \right) + 100 \left(\frac{P}{F}, i, 8 \right)$$

$$i=15\% = -8500 + 1800(4.487) + 100(0.3269)$$

$$= -390.71$$

+7

$$\left. \begin{aligned} & - \frac{390.71 - 1149.65}{0 - 1149.65} = \frac{15-10}{x-10} \\ & x = 10 + \frac{5 + 1149.65}{1540.36} \\ & = 13.732\% > MARR (13.5\% \text{ (Given)}) \end{aligned} \right\}$$

Interpolation

∴ Machine 3 is the best option of the three machines

University Test (July 2014) Make-up.

- Q) The cash flows for two mutually exclusive alternatives are given as follows:

With the knowledge that both alternatives are revenue projects, which project should be selected at $MARR = 10\%$?

Answer:

$$MARR = 10\%$$

n	B-1	B-2
0	-\$ 3000	-\$ 12000
1	\$ 1350	\$ 4200
2	\$ 1800	\$ 6225
3	\$ 1500	\$ 6330
IRR	25%	17.43%

n	(Incremental CF)		
	B1	B2	B2-B1
0	-3000	-12000	-9000
1	1350	4200	2850
2	1800	6225	4425
3	1500	6330	4830

$$PW = -9000 + 2850 \left(\frac{P}{F}, 1, 1 \right) + 4425 \left(\frac{P}{F}, 1, 2 \right) + 4830 \left(\frac{P}{F}, 1, 3 \right) = 0$$

Δ IRR = 15% [(@ 15%, PW becomes zero)]

since $\Delta IRR_{B2-B1} = 15\% > 10\%$. and $IRR_{B2} > 10\%$.

Select B2

University Test (May 2014)

+8

- Q) Don Garlits is a landscaper. He is considering the purchase of a new commercial lawn mower, either, the Atlas or the Zippy. The minimum attractive rate of return is 6%, and the table provides all the necessary information for the two machines.

	Atlas	Zippy
Initial Cost (\$)	6,700	16,900
Annual operating cost (\$)	1,500	1,200
Annual Benefits (\$)	4,000	4,500
Salvage Value (\$)	1,000	3,500
Useful life in years	3	6
IRR	12%	8%

Use incremental rate of return analysis to decide which machine to purchase.

Answer:

Since IRR of both alternatives is greater than MARR, both alternatives qualify for incremental analysis.

The incremental cash flow B-A is as shown in table above.

$$NPW = 0$$

$$-10200 + 800\left(\frac{1}{1+0.06}, 1, 6\right) + 5700\left(\frac{1}{1+0.06}, 1, 3\right) + 2500\left(\frac{1}{1+0.06}, 1, 6\right) = 0$$

$$1 = 6\%$$

$$= -10200 + 800(4.917) + 5700(0.8396) + 2500(0.7050)$$

$$= 281.82$$

$$1 = 6\% \quad -10200 + 800(4.623) + 5700(0.7938) + 2500(0.6302) = 0$$

$$= -401.44$$

By Interpolation

$$6\% + \frac{281.82 - 0}{281.82 - (-401.44)} \times (8 - 6)$$

$$\Delta IRR = 6.82\% > MARR$$

∴ Select Zippy

University Test (January 2015) Make-up

- Q) Consider the following set of mutually exclusive alternatives. Which project is preferred, based on the rate of return on the incremental investment. Assume MARR = 15%. (Use formulae for calculation).

n	D1	D2	D3
0	-2,000	-1,000	-3,000
1	1,500	800	1,500
2	1,000	500	2,000
3	800	500	1,000
Project IRR	34.47	40.76	24.81

Answer:

Since the Project IRR is greater than MARR, all 3 projects are considered for incremental analysis.

Comparing D1 & D2

n	D1 - D2
0	-1000
1	700
2	500
3	300

$$PW = -1000 + 700\left(\frac{P}{F}, i, 1\right) + 500\left(\frac{P}{F}, i, 2\right) + 300\left(\frac{P}{F}, i, 3\right) = 0$$

$$\Delta IRR_{D1-D2} = 27.61\% > MARR$$

Select D1

Comparing D1 & D3

n	D3 - D1
0	-5000
1	0
2	1000
3	200

$$PW = -5000 + 0 + 1000\left(\frac{P}{F}, i, 2\right) + 200\left(\frac{P}{F}, i, 3\right)$$

$$\Delta IRR_{D3-D1} = 8.8\% < MARR$$

Select D1

Chapter 4 Replacement Analysis4.1 Outsider and Insider Point of ViewUniversity Test

(December 2014)

Make-up

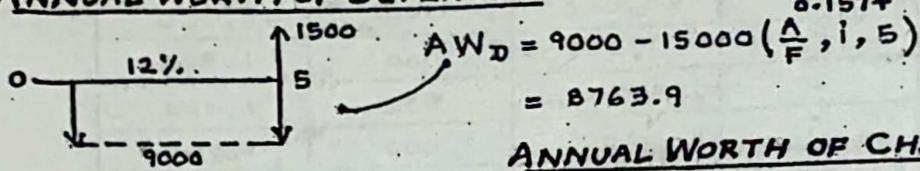
- (Q) An eight year old asset may be replaced with either of

Course of Action	Current Asset (Rs)	Challenger 1 (Rs)	Challenger 2 (Rs)
First cost	-	30000	54000
Defender Trade	-	10500	7500
Annual Cost	9000	4500	3600
Salvage Value	1500	3000	1500
Life, years	5yr	5yr	5yr

The two new assets. Current data for each alternative are given below, using the cash flow approach and interest rate of 12% per year. Determine the best course of action.

Answer:

ANNUAL WORTH OF DEFENDER



ANNUAL WORTH OF CHALLENGER 1

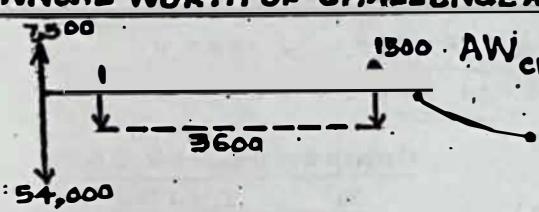
$$AW_{CH1} = (30,000 - 10500) \left(\frac{A}{P}, i, 5 \right) +$$

$$4500 - 3000 \left(\frac{A}{F}, i, 5 \right)$$

$$= 19500 \times 0.2774 + 4500 - 3000 \times 0.1574$$

$$= 9437.1$$

ANNUAL WORTH OF CHALLENGER 2



$$AW_{CH2} = (54000 - 7500) \left(\frac{A}{P}, i, 5 \right) + 3600 -$$

$$1500 \left(\frac{A}{F}, i, 5 \right)$$

$$= (46500 \times 0.2774) + 3600 - (1500 \times 0.1574)$$

$$= 16,263$$

Annual Worth of Defender is lower than the Challengers. Therefore, stay with the old machine.

University Test (November 2014)

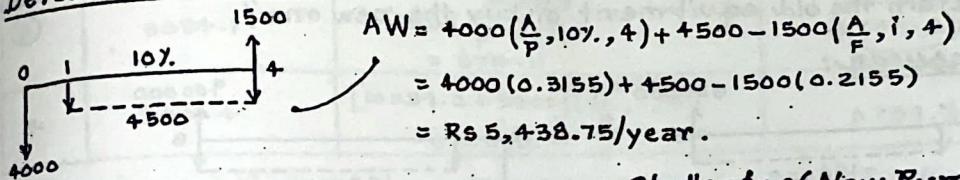
- Q) The XYZ Company owns a four year-old pump that originally cost Rs 15,000 with a life of eight years and an annual operating and maintenance cost of Rs 4,500 per year. The salvage value of this old pump at the end of its life is expected to be Rs 1500. The company is currently planning to replace this old pump with a new pump. A salesman has offered a new pump for Rs 18,000 whose annual maintenance and operating cost is Rs 1,500 in the first year and then onwards increasing at a gradient of Rs 400. The life of this new pump is 10 years and the salvage value at the end of the life is Rs 2,500. The trade-in value of the old pump is expected to be Rs 4,000. Assuming an interest rate of 10%.

i) Should the old pump be replaced with the new one?

ii) If the asset is being depreciated according to declining balance method; determine the sunk cost/capital gain, if the old pump is replaced.

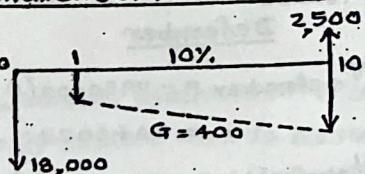
Answer: Outsider's Point of View (since lives are unequal)

Defender (Old Pump)



$$\begin{aligned}
 AW &= 18,000\left(\frac{A}{P}, 10, 10\right) + 1500 + 400\left(\frac{A}{G}, 10, 10\right) \\
 &\quad - 2,500\left(\frac{A}{F}, 10, 10\right) \\
 &= -18,000(0.1627) + 1,500 + 400(3.725) - \\
 &\quad 2,500(0.0627) \\
 &= \text{Rs } 5,761.85/\text{year}
 \end{aligned}$$

Challenger (New Pump)



\therefore Retain the old machine

(iii) Declining Balance Methods

We have,

$$R = 1 - \sqrt[t]{\frac{B_t}{P}}$$

$$\begin{aligned}
 B_t &= (1-R)^t P \\
 &= (1-0.25)^4 15000
 \end{aligned}$$

$$= \sqrt[4]{\frac{15000}{15000}}$$

$$= 1 - 0.75$$

$$= 0.25$$

$$B_4 = 4,746.1$$

\therefore Sunk Cost = Current Book Value - Current Market Value

$$= 4746.1 - 4000$$

$= 746.1 \neq$ Sunk Loss.

University Test (July 2014) Make-up

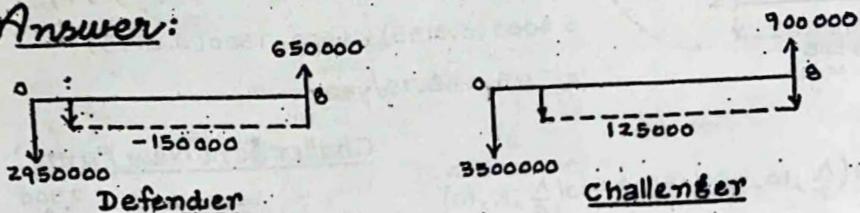
Q) A construction company has purchased a piece of construction equipment three years ago at a cost of Rs 400000. The estimated life and salvage value at the time of purchase were 12 years and Rs 850000 respectively. The annual operating and maintenance cost was Rs 150000. The construction company is now considering replacement of existing equipment with a new model available in the market.

Due to depreciation the current book value of the existing equipment is Rs 3055000. The current market value of the existing equipment is Rs 2950000. The revised estimate of salvage value and remaining life are Rs 650000 and 8 years respectively. The annual operating and maintenance cost are same as earlier.

52

The initial cost of the new model is Rs 3500000. The estimated life, salvage value and annual operating cost are 8 years, Rs 900000 and Rs 125000 respectively. If the company's MARR is 10%, should it retain the old equipment or buy the new one?

Answer:



$$AW_{defender} = -2950000 \left(\frac{A}{P}, 10, 8 \right) - 150000 + 650000 \left(\frac{A}{F}, 10, 8 \right)$$

$$= -646020.$$

$$AW_{challenger} = -3500000 \left(\frac{A}{P}, 10, 8 \right) - 125000 + 900000 \left(\frac{A}{F}, 10, 8 \right)$$

$$= -702240$$

∴ Continue with the defender.

University Test (May 2014)

- Q) With an example differentiate between outsider's point of view method and insider's point of view method, as applied to replacement analysis.

4.2 Economic Life of an Asset

University Test (July 2015) Make-up

- Q) A three year-old manufacturing process asset is being considered for early replacement. Its current market value (MV) is \$13,000. Estimated future values and Annual Operating Costs (AOC) for the next five years are given in Table above. What is the economic service life of this defender if the interest rate is 12% per year?

Answer:

n	$CR = (P-F) \left(\frac{A}{P}, i, n \right) + F$	$E_d \cdot AOC$	$CR + E_d \cdot AOC = EVAC$
1	$13000 - 9000 \left(\frac{A}{P}, 12, 1 \right) + 9000 \times 0.12$ = 5560	2500	8060

continued...

2	3918.5	$\frac{53}{[2500(1.12) + 2700] \left(\frac{1}{F}, 1, 2\right)} = 2594.35$	6512.85
3	3634.1	$9160 \left(\frac{1}{F}, 1, 3\right) = 2714.11$	6348.21
4	3861.4	$[10259.2 + 3500] \left(\frac{1}{F}, 1, 4\right) = 2678.4$	6739.6

So, Economic Life is 3 years with minimum EUAC = 6348.2 / year

University Test (May 2015)

- Q) The XYZ Company purchased a very specialized machine three years ago for Rs 25,000. This machine is not readily salable and is assumed to have a zero salvage value. Operating costs are expected to be Rs 10,000 for next year, and are expected to increase by Rs 800 per year thereafter. The company has an opportunity to replace the existing machine with another specialized one that will cost Rs 12,000. This machine has no salvage value, a useful life of 10 years, and operating costs of Rs 5,000 in the first year, with an annual increase of Rs 1,200 thereafter. If the MARR is 15%, determine the economic life of both the machines? Also suggest the best course of action.

Answer:

Concept: Economic life of an asset is the one, where for a particular value of 'n', the total cost (TC) = Capital Recovery (CR) + Annual operating Cost (AOC) is minimum.

While substituting for different values, we will have a particular pattern of behaviour where total cost will decrease upto a particular value of 'n' and after which it will start increasing.

Defender

Machine Cost 3 years ago is Rs 25,000 and zero salvage value.

Operating Cost = Rs 10,000 for next year and will increase by Rs 800 per year thereafter.

Challenger

First cost = Rs 12,000 n=10 years and zero salvage value.

Operating Cost = Rs 5000 in the first year and will increase by Rs 1200 per year thereafter.

Defender

54

$CR(i) = 0$ as there is no salvage value.

$$TC = OC(l) = 10,000 + 800 \left(\frac{A}{G}, i, n \right)$$

$$TC \text{ when } n=1; 10000 + 800(0) = 10,000$$

$$TC \text{ when } n=2; 10000 + 800(0.465) = 10,372$$

$$TC \text{ when } n=3; 10000 + 800(0.907) = 10,725$$

$$TC \text{ when } n=4; 10000 + 800(1.326) = 11,060$$

Therefore, the economic life of the defender is 1 year with a total cost of Rs 10,000.

Challenger

Year	$CR(i) = 12000 \left(\frac{A}{P}, i, n \right)$	$OC = 5000 + 1200 \left(\frac{A}{G}, i, n \right)$	TC
1	13800	5000	18800
2	7381	5558	12939
3	5286	6088	11344
4	4203	6591	10744
5	3579	7067	10646
6	3170	7516	10686
7	2884.8	7940	10824

From the above table, it can be observed that the total cost has decreased till $n=5$ and started increasing thereafter ($n=6$, onwards).

∴ Economic Life = 5 years with minimum EUAC = 10646

	Defender	Challenger
Economic Life	1	5
Annual Cost	10000	10646

∴ It is advisable to replace defender with challenger.

University Test (May 2015) Make-up

- Q) An asset purchased three years ago is now challenged by a new piece of equipment. The present market value of the defender is Rs 130000, anticipated Salvage value and Annual operating costs (AOC) for the next five years are given in table. What is the minimum cost life to be used while comparing this defender

Lifetime Years	Salvage Value	AOC
1	Rs 90,000	Rs 25,000
2	Rs 80,000	Rs 27,000
3	Rs 60,000	Rs 30,000
4	Rs 20,000	Rs 35,000
5	Rs 0	Rs 45,000

(55)

With a challenger if a 12% year return is required?

Answer: Minimum economic life is identified by calculating annual operating cost and Capital Recovery.

Economic Life

$$n=1 \text{ year}, CR = \left(P - F \right) \left(\frac{A}{P}, i, n \right) + Fi$$

$$= (130000 - 90000) \left(\frac{A}{P}, 12, 1 \right) + 90000 * 0.12$$

$$= 55600. \quad \quad \quad = 80,600$$

$$AOC = 25000$$

$$n=2 \text{ years}, CR = (130000 - 80000) \left(\frac{A}{P}, 12, 2 \right) + 80000 * 0.12$$

$$= 39185.$$

$$AOC = \left[\frac{25000}{1.12} + \frac{27000}{1.12^2} \right] \left(\frac{A}{P}, 12, 2 \right) + 25921.6$$

$$= 65106.6 \quad \quad \quad = 65106.6$$

$$n=3 \text{ years}, CR = (130000 - 60000) \left(\frac{A}{P}, 12, 3 \right) + 60000 * 0.12$$

$$= 36341$$

$$AOC = \left[\frac{25000}{1.12} + \frac{27000}{1.12^2} + \frac{30000}{1.12^3} \right] \left(\frac{A}{P}, 12, 3 \right)$$

$$= 63483.4 \quad \quad \quad = 63483.4$$

$$n=4 \text{ years}, CR = (130000 - 20000) \left(\frac{A}{P}, 12, 4 \right) + 20000 * 0.12$$

$$= 38612$$

$$AOC = \left[\frac{65199.1}{1} + \frac{35000}{1.12^4} \right] \left(\frac{A}{P}, 12, 4 \right)$$

$$= 67442.23 * 0.3292 = 28786 \quad \quad \quad = 67398$$

$\therefore n=3$ with minimum AOC = 63483.4

University Test (November 2014)

- Q) The XYZ company has an opportunity to replace the existing machine with another specialized one that will cost \$12,000. This machine has no salvage value, a useful life of 10 years, and operating costs of \$5,000 in the first year, with an annual increase of \$1,200 thereafter. Determine the economic life of the new machine if the interest rate is 10%.

Answer: Economic Life of an asset exists at a point where EUAC value ($CR + AOC$) becomes minimum.

n	Capital Recovery Cost since $F=0$, $CR = P \left(\frac{A}{P}, i, n \right)$	Equivalent Annual operating cost $AOC = 5000 + 1200 \left(\frac{A}{G}, i, n \right)$	EUAC $CR + AOC$
1	$12000 (1.100) = 13200$	$5000 + 1200 (0) = 5000$	18200
2	$12000 (0.5762) = 6914.4$	$5000 + 1200 (0.476) = 5511.2$	12485.6
3	$12000 (0.4021) = 4825.2$	$5000 + 1200 (0.937) = 6124.4$	10949.6
4	$12000 (0.3155) = 3786$	$5000 + 1200 (1.381) = 6657.2$	10443.2
5	$12000 (0.2638) = 3165.6$	$5000 + 1200 (1.810) = 7172$	10337.6
6	$12000 (0.2296) = 2755.2$	$5000 + 1200 (2.224) = 7668.8$	10424

University Test (July 2014) Make-up

- Q) Three years ago a chemical processing plant installed a system at a cost of \$20,000 to remove pollutants from waste water that is discharged into a nearby river. The present system has no present salvage value and will cost \$14500 to operate next year, with the operating cost expected to increase at the rate of \$500 per year thereafter.

A new system has been designed to replace existing system at a cost of \$10000. The new system is expected to have first year operating of \$9000 with these costs increasing at the rate of \$1000 per year. The new system is estimated to have a useful life of 12 years. The salvage values of both the system at any future time are expected to be zero. If the interest rate is 12%, conduct replacement analysis based on the economic life of the asset.

Answer: For Existing Machine

n	$(P-F) \left(\frac{C_R}{P}, i, n \right) + F_i$	Equivalent Aoc $A_i + G \left(\frac{A}{G}, i, n \right)$	EUAC
1	—	$14500 + 500 \left(\frac{A}{G}, 1, 1 \right)$ = 14500	14500
2	—	$14500 + 500 \left(\frac{A}{G}, 1, 2 \right)$ = 14736	14736
3	—	$14500 + 500 \left(\frac{A}{G}, 1, 3 \right)$ = 14963	14963

EUAC is increasing every year. \therefore Economic life of this machine is one year with minimum EUAC = 14500.

For New MACHINE

(57)

$$F=0 \text{ for } n=1, 2, \dots, n$$

n	CR $(P-F)\left(\frac{A}{P}, i, n\right) + FP$	Equivalent AOC $9000 + 1000\left(\frac{A}{G}, i, n\right)$	EUAC
1	$10000\left(\frac{A}{P}, i, 1\right) = 11200$ 1.12	$9000 + 1000\left(\frac{A}{G}, i, 1\right)$ = 9000	20200
2	$10000\left(\frac{A}{P}, i, 2\right) = 5917$ 0.5917	$9000 + 1000\left(\frac{A}{G}, i, 2\right)$ = 9471.7	15389
3	$10000\left(\frac{A}{P}, i, 3\right) = 4164$ 0.4164	$9000 + 1000\left(\frac{A}{G}, i, 3\right)$ = 9924.6	14089
4	$10000\left(\frac{A}{P}, i, 4\right) = 3292$ 0.3292	$9000 + 1000\left(\frac{A}{G}, i, 4\right)$ = 10358.9	13651
5	$10000\left(\frac{A}{P}, i, 5\right) = 2774$ 0.2774	$9000 + 1000\left(\frac{A}{G}, i, 5\right)$ = 10774.6	13549
6	$10000\left(\frac{A}{P}, i, 6\right) = 2432$ 0.2432	$9000 + 1000\left(\frac{A}{G}, i, 6\right)$ = 11172.1	13604

∴ Economic life = 5 years with minimum EUAC = 13549.

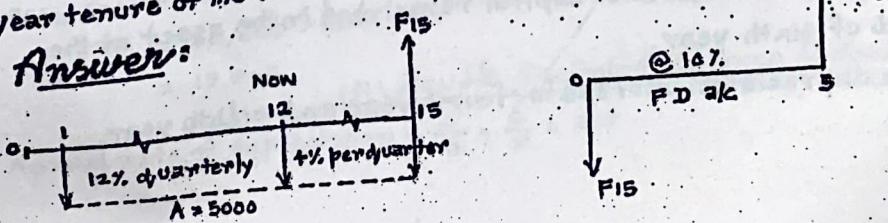
Hence existing system should be replaced with new one

Chapter.5 Depreciation

University Test (December 2014) Make-up

Q) A person is depositing an amount of RS 5,000 on a quarterly basis with an interest rate of 12% per year compounded quarterly from past three years in a saving account. He is planning to deposit the amount for another two years after which he planning to withdraw the amount from the account and will be depositing it in a FD account earning an interest rate of 10% with a maturity of five year period. What amount the person will be getting at the end of the five year tenure of the FD account?

Answer:



(58)
To find the amount at the time of maturity, calculate the future value.

$$F_{15} = 5000 \left(\frac{1}{1+i} \right)^{15} = 5000 \times 0.024 = \text{Rs } 1,00,120$$

$$\text{Final value of deposit, } F = 1,00,120 (1+0.1)^5 \\ = 1,61,244.3$$

University Test (December 2014) Make-up

- Q) Computers purchased by a public utility costs Rs 35,000 each. Past records indicate that they have a useful life of five years, after which, they will be disposed of with a scrap value of Rs 500. Determine the following by using decline balance method.

i) Depreciation charge during year two and three

ii) The book value of the computers at the end of fourth year.

Answer:

$$P = 35000$$

$$n = 5 \text{ years}$$

$$F = 500$$

$$D_2 \text{ & } D_3 = ?$$

$$B_4 = ?$$

$$R = 1 - \sqrt[n]{\frac{F}{P}} \\ = 1 - \sqrt[5]{\frac{500}{35000}} = 1 - (0.0143)^{1/5} = 0.572$$

$$D_t = R (1-R)^{t-1} \times P$$

$$D_2 = 0.572 (1-0.572)^{2-1} \times 35000 \\ = \text{Rs } 8568.56$$

$$D_3 = 0.572 (1-0.572)^{3-1} \times 35000 \\ = \text{Rs } 3667.34$$

$$B_4 = (1-R)^4 \times P \\ = (1-0.572)^4 \times 35000 \\ = \text{Rs } 1174.5$$

University Test (July 2015) Make-up

- Q) An asset has a first cost of Rs 2,50,000 and an expected life of ten years. The estimated salvage value at the end of ten years is Rs 50,000. Using the straight line method and declining balance method of depreciation determine the,

i. Undepreciated amount of capital remaining in the asset at the end of sixth year.

ii. The depreciation charges for fourth year and eighth year.

Answer:

$$P = 2,50,000$$

$$n = 10 \text{ years}$$

$$F = 50,000$$

(59)

$$B_6 = ?$$

$$D_4 = ?$$

$$D_8 = ?$$

$$R = 1 - \sqrt{\frac{B_t}{F}}$$

$$= 1 - \sqrt{\frac{50000}{250000}}$$

$$= 0.1487 \text{ for DBM.}$$

straight Line Method

$$B_t = P - t \left[\frac{P-F}{n} \right]$$

$$D_t = \frac{P-F}{n}$$

$$B_6 = 250000 - 6 \left[\frac{250000-50000}{10} \right]$$

$$= 130,000$$

$$D_8 = D_4 = \frac{P-F}{n} = \frac{250000-50000}{10}$$

$$= 20,000$$

Decline Balance Method

$$D_t = R(1-R)^{t-1} P$$

$$B_t = (1-R)^t P$$

$$B_6 = 250000(1-0.1487)^6$$

$$= 95155.7$$

$$D_4 = 0.1487(1-0.1487)^{4-1} \times 250000$$

$$= 22935$$

$$D_8 = 0.1487(1-0.1487)^8-1 \times 250000$$

$$= 12045.6$$

University Test (May 2015)

Q) An eight year old machine had a first cost of one lakh rupees and it can sell at Rs 5,000 now. If the machine's life is 10 years and salvage value is expected to be Rs 1,000; determine the sunk cost/capital gain if the machine is sold now using,

i) straight line method of depreciation

ii) Double declining balance method

Answer: (i) Straight Line Method

$$D_t = \frac{\text{Purchase Price} - \text{Salvage Value}}{\text{Years of useful life}}$$

$$= \frac{1,00,000 - 1,000}{10} = 9900.$$

$$\therefore B_8 = 1,00,000 - 8(9900)$$

$$= 20,800$$

$$\text{Market Value} = 5000$$

$$\text{Sunk Cost} = \text{Book Value} - \text{Market Value}$$

$$= 20,800 - 5000$$

$$= 15,800$$

$$B_8 = R(1-R)^t P$$

$$= 0.2(1-0.2)^8 \times 1,00,000$$

$$= 3333.44$$

$$\text{Sunk Cost} = \text{Book Value} - \text{Salvage Value}$$

$$= 3333 - 5000$$

$$= -1667$$

(ii) Double Declining Balance Method

$$\text{Annual rate of depreciation} = \frac{2}{n} = \frac{2}{10} = 20\%$$

University Test (May 2015) (60)

- Q) A company is planning to replace their three year old machine with a new machine now. The company uses a declining balance method of depreciation in their accounting procedure. Also, it is expected that the market value of the old machine will be equal to book value. The other details of the two machines are given below.

Particulars	Old Machine	New Machine
First Cost	1,50,000	1,90,000
Annual Operating Cost	23,000	18,000
Life of the machine	15 years	15 years
Salvage value at the end of life	10,000	15,000

Using an appropriate replacement analysis approach with an interest rate of 15%, determine whether the company should replace their old machine or not?

Answer:

Old Machine

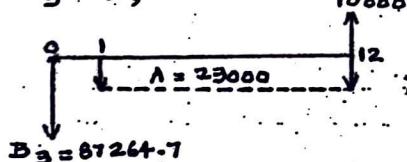
Market Value for a 3 year old = Book Value (B_3) according DDBM

$$1-R = \sqrt[15]{\frac{B_b}{P}} = \sqrt[15]{\frac{10000}{150000}}$$

$$R = 1 - 0.835$$

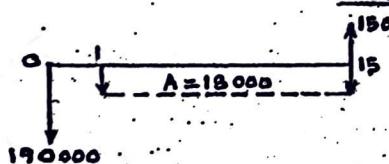
$$B_b = (1-R)^t \times P \Rightarrow B_3 = 0.1652 = R$$

$$B_3 = 87,264.7$$



$$\begin{aligned} AW &= 87264.7 \left(\frac{\Delta}{P}, i, 15 \right) + 23000 - 10000 \left(\frac{\Delta}{F}, i, 15 \right) \\ &= 87264.7 (0.1652) + 23000 - 10000 (0.0345) \\ &= 38,755.3 \end{aligned}$$

New Machine



$$\begin{aligned} AW &= 190000 \left(\frac{\Delta}{P}, i, 15 \right) + 18000 - 15000 \left(\frac{\Delta}{F}, i, 15 \right) \\ &= 190000 (0.1652) + 18000 - 15000 (0.0345) \\ &= 50,175. \end{aligned}$$

$AW_{\text{new machine}} > AW_{\text{old machine}}$ \therefore Retain old machine

University Test (May 2015) Make-up

- Q) A truck of model A has a first cost of Rs 15,00,000 and an expected life of ten years. The estimated salvage value at the end of ten years is Rs 75,000. Tabulate the depreciation schedule for this truck using,

i) straight Line Method.

(61)

ii) Declining Balance Method

iii) Double Declining Balance Method

Answer:

$$D_t = \frac{P - F}{n} = \frac{1500000 - 75000}{10}$$

$$D_t = 1,42,500$$

(i) straight Line Method

	D _t	B _t
0	-	15,00,000
1	142500	13,57,500
2	142500	12,15,000
3	"	10,72,500
4	"	9,30,000
5	"	7,87,500
6	"	6,45,000
7	"	5,02,500
8	"	3,60,000
9	"	2,17,500
10	"	75,000

(ii) Declining Balance Method

$$R = 1 - \sqrt{\frac{B_t}{P}}$$

$$= 1 - \sqrt{\frac{75000}{1500000}}$$

$$= 0.2589$$

	D _t	B _t
0	-	15,00,000
1	3,88,350	11,11,650
2	2,87,806.2	8,23,843.8
3	2,13,293.2	6,10,550.6
4	1,38,071.6	4,32,479.04
5	1,17,146.8	3,35,332.2
6	86,817.5	2,48,514.7
7	64,340.5	1,84,174.3
8	47,682.7	1,36,491.6
9	35,337.7	1,01,154
10	26,188.8	74,965.2

iii) Double Declining Balance Method (62)

$$R = \frac{2}{n} = \frac{2}{10} = 0.2$$

	D _t	B _t
0	-	15,00,000
1	3,00,000	12,00,000
2	2.4	9.6
3	1.92	7.68
4	1.536	6.144
5	1.2288	4.9152
6	0.98504	3.93216
7	0.786432	3.145728
8	0.629146	2.51658
9	0.50332	2.013264
10	0.4027	1,61,061.13

University Test (November 2014)

- (Q) Freeport Mining Company has purchased a computer-controlled gold ore grading unit for \$80,000. The unit has an anticipated life of 10 years and a salvage value of \$10,000. Use the straight-line and double declining balance methods to compare the schedule of depreciation and book values for each year.

Answer:

Year, t	Straight Line Method		Double Declining Method	
	D _t	B _t	D _t	B _t
0	-	80,000	-	80,000
1	7000	73,000	$80000 \times 0.2 = 16000$	64,000
2	7000	66,000	$64000 \times 0.2 = 12800$	51,200
3	7000	59,000	$51200 \times 0.2 = 10240$	40,960
4	7000	52,000	$40960 \times 0.2 = 8192$	32,768
5	7000	45,000	$32768 \times 0.2 = 6553.6$	26,214.4
6	7000	38,000	$26214.4 \times 0.2 = 5242.8$	20,971.5
7	7000	31,000	$20971.5 \times 0.2 = 4194.3$	16,771.19
8	7000	24,000	$16771.19 \times 0.2 = 3355.4$	13,421.9
9	7000	17,000	$13421.9 \times 0.2 = 2684.3$	10,737.6
10	7000	10,000	$10737.6 \times 0.2 = 2147.5$	8,590.08

$$D_t = \frac{P-F}{n} = \frac{80,000-10,000}{10} = 7,000$$

(For Straight Line Method)

$$D_t = \frac{2}{n} = \frac{2}{10} = 0.2$$

(for Double Declining Method)

University Test (July 2014) Make-up

(63)

- Q) An asset was purchased for Rs 2,50,000. It has an expected life of 10 years and a salvage value of Rs 50,000 at the end of 10th year. What will be the undepreciated amount of capital remaining in the asset at the end of 6th year? If the asset is being depreciated according to the declining balance method. Also calculate the depreciation charge for the 8th year.

Answer:

$$R = 1 - \sqrt[10]{\frac{P-F}{P}} = 1 - \sqrt[10]{\frac{50000}{250000}} = 14.87\%$$

Undepreciated amount at the end of sixth year

$$B_6 = (1-R)^6 P = (1-0.1487)^6 * 250000 = 95156$$

Depreciation charge in 8th year

$$\begin{aligned} D_8 &= R(1-R)^{8-1} P \\ &= 0.1487 (1-0.1487)^{8-1} * 250000 \\ &= 12045.62 \end{aligned}$$

University Test (May 2014)

- Q) An asset was purchased 10 years ago for Rs 4800. It is being depreciated according to straight line method for an estimated life of 20 years and a salvage value by Rs 800. What is the difference in its current book value and the book value that would have resulted if double declining balance method is adopted.

Answer:

Initial Cost, P = 4800; Life, n = 20 years; Salvage Value, F = 800; t = by years

Straight Line Method

$$\begin{aligned} B_{10} &= P - \frac{t(P-F)}{n} \\ &= 4800 - 10 \left(\frac{4800-800}{20} \right) \\ &= 2800 \end{aligned}$$

Double Declining Balance Method

$$\begin{aligned} R &= \frac{2}{20} = \frac{1}{10} = 0.1 \\ B_{10} &= (1-R)^{10} * P \\ &= (1-0.1)^{10} * 4800 \\ &= 1673 \end{aligned}$$

$$\text{Difference} = 2800 - 1673 = 1126.34$$

Sessional Test (January 2015) Make-up

- Q) An asset has a first cost of Rs 48,000 with an estimated life of 20 years. What is the total accumulated depreciation charge during the first five years of the assets life, if it is depreciated according to Double Declining Balance method?

Answer:

(64)

First cost = Rs 48,000 ; Estimated Life = 20 years

Total accumulated depreciation amount for first five years

$$R = \frac{2}{n} = \frac{2}{20} = 0.1$$

$$D_5 = R(1 - R)^{t-1} p$$

$$= 0.1(1 - 0.1)^{5-1} \times 48000$$

$$= 3149.28$$

Objective Type Questions (From Sessional Tests)

- Q) An interest rate of 2% per quarter is same as, (Feb. 2015, 1st Test)

- a) 8% per year b) 6% per year, compounded quarterly c) 4% per semi-annual compounded quarterly d) 4.04% per semi-annual

Options: i) only c ii) only c and d iii) only a iv) Only b v) Only b, c and d

- Q) Annual equivalent of an income stream of Rs 1000, to be received at the end of each of the next three years at an interest rate of 12% is,

- a) Less than Rs 1000 per year b) Greater than Rs 1000 per year c) Rs 1000 per year d) Rs 1000 ($\frac{1}{P}, 12\%, 3$)

(c) Answer (April 2014 2nd sessional)

- Q) When the interest rate is 10% per year, all of the following are equivalent to \$5,000 now except: (Feb. 2014, 1st Test)

- a) \$4,545 one year ago b) \$5,500 one year hence c) \$4,021 two years ago d) \$6,050 two years hence

(c) Answer

- Q) State whether the below statements are true/false:

(i) All other things being equal, I'd rather have \$10,000 in 10 years than to receive \$10,000 today. (Feb. 2014, 1st Test)

(ii) The present value of a dollar on hand today is 0.

(d) Answer

- (a) i. True ii. False (b) i. True ii. True (c) i. False ii. True
(d) i. False ii. False

- Q) Which of the following banks offer you a better interest deal for your deposit. Justify your answer. (September 2014, 1st Test)

Bank A : 8.5% compounded Quarterly Bank B: 8.65% Compounded Annually

Answer: Bank A : 8.5% compounded quarterly

$$I_{eff} = \left(1 + \frac{0.085}{4}\right)^4 - 1 = 8.69\% \text{ per annum}$$

Bank B : 8.65% compounded annually

Bank A is the answer due to more returns

Understand the basics from the main book & then work through this supplement

- Mr POTTY
(A teacher)