SIFT: Detector and Descriptor

SIFT: David Lowe, UBC

D. Lowe. Distinctive image features from scale-invariant key points ., International Journal of Computer Vision 2004.

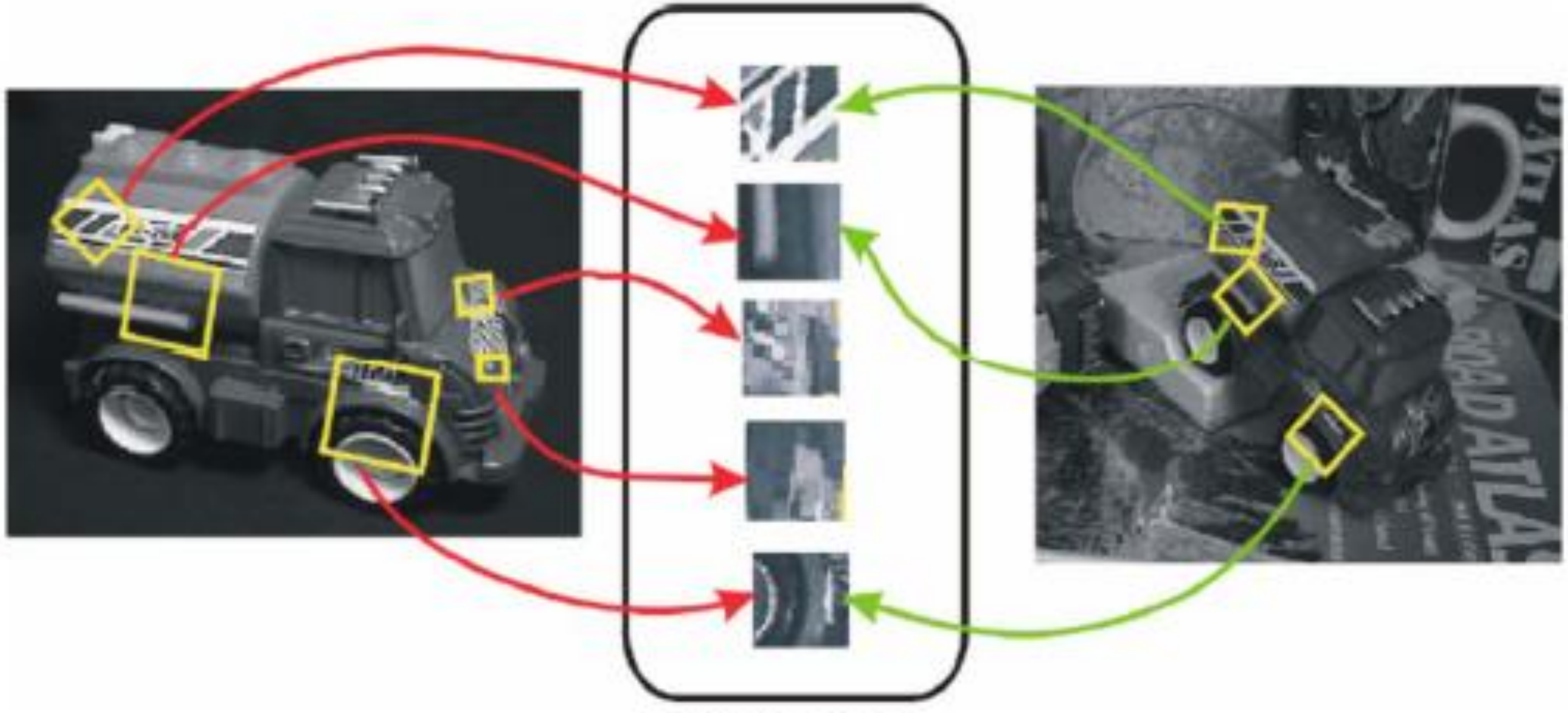
SIFT - Key Point Extraction

- Stands for Scale Invariant Feature Transform
- Patented by university of British Columbia
- Transforms image data into scale invariant coordinates

Goal

- Extract distinctive invariant features
  - Correctly matched against a large database of features from many images
- Invariance to image scale and rotation
- Robustness to
  - Affine (rotation, scale, shear) distortion,
  - Change in 3D viewpoint,
  - Addition of noise,
  - Change in illumination.

Invariant Local Features



SIFT Features

Steps for Extracting Key Points

Scale space peak selection

- Potential locations for finding features

Key point localization

- Accurately locating the feature key points

Orientation Assignment

- Assigning orientation to the key points

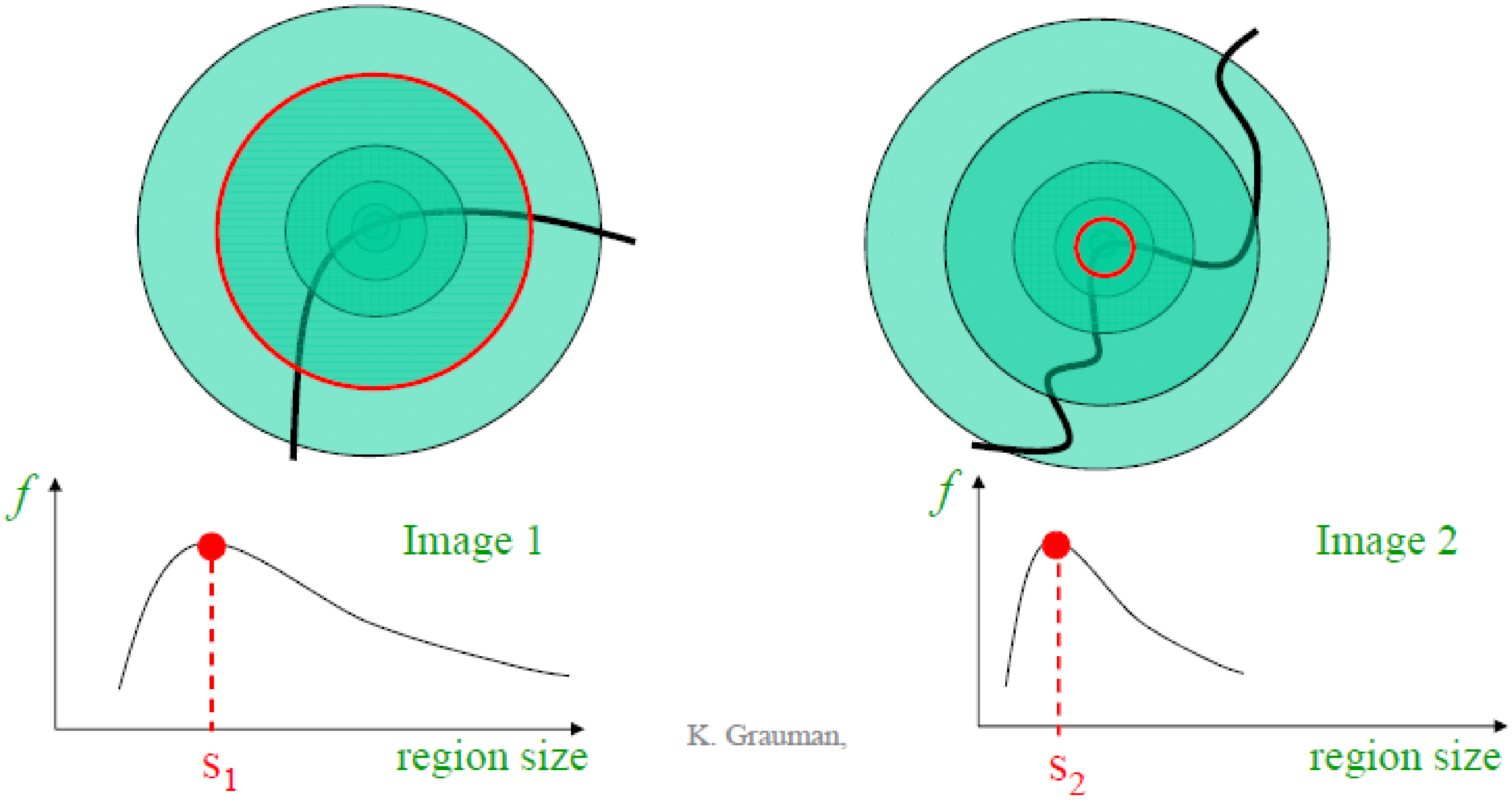
Key point descriptor

- Describing the key point as a vector of size 128 (SIFT Descriptor)

Automatic scale selection

Intuition:

- Find scale that gives local maxima of some function *f* in both position and scale.



K. Grauman,

Automatic Scale Selection

$$f(I_{i_1 \ldots i_m}(x, \sigma)) = f(I_{i_1 \ldots i_m}(x', \sigma'))$$

How to find corresponding patch sizes?

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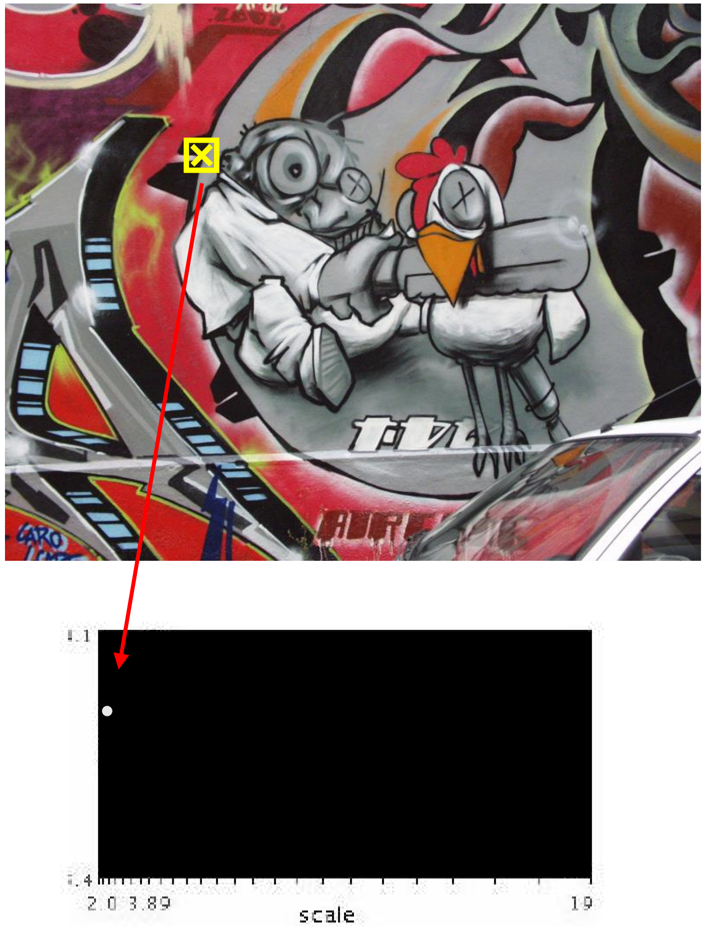
Automatic Scale Selection

- Function responses for increasing scale (scale signature)



$f(I_{i_1 \ldots i_m}(x, \sigma))$

$f(I_{i_1 \ldots i_m}(x', \sigma'))$

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Automatic Scale Selection

- Function responses for increasing scale (scale signature)

$f(I_{i_1 \ldots i_m}(x, \sigma))$ $\quad$ $f(I_{i_1 \ldots i_m}(x', \sigma'))$

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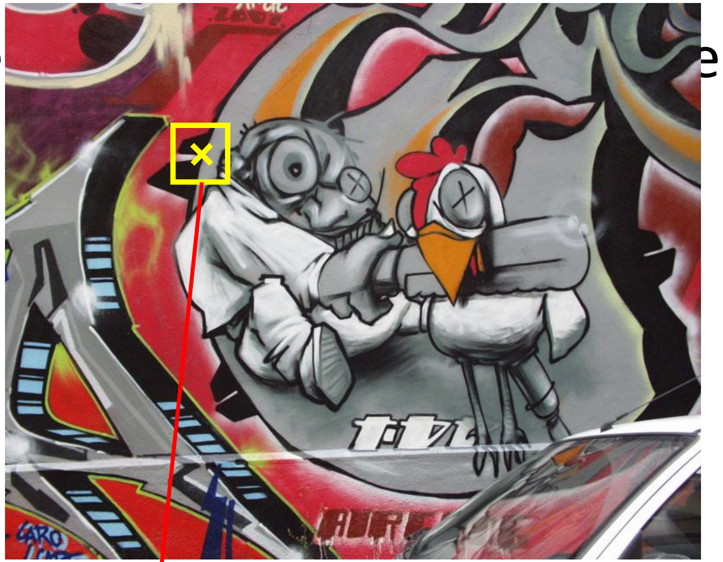
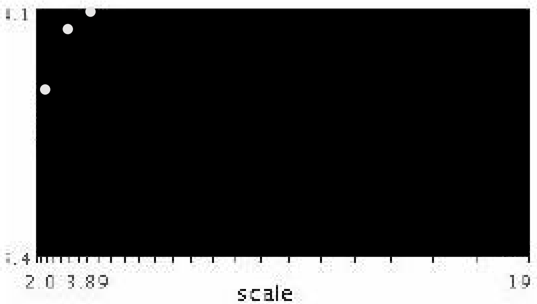
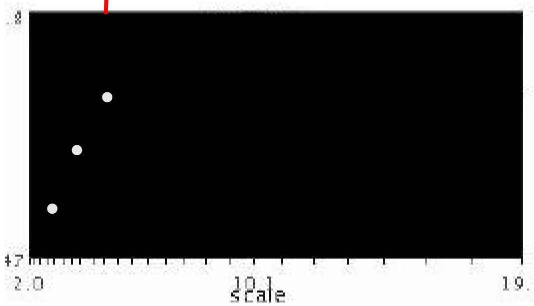
Automatic Scale Selection

- Function re e (scale





$f(I_{i_1 \ldots i_m}(x, \sigma))$



$f(I_{i_1 \ldots i_m}(x', \sigma'))$

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Automatic Scale Selection

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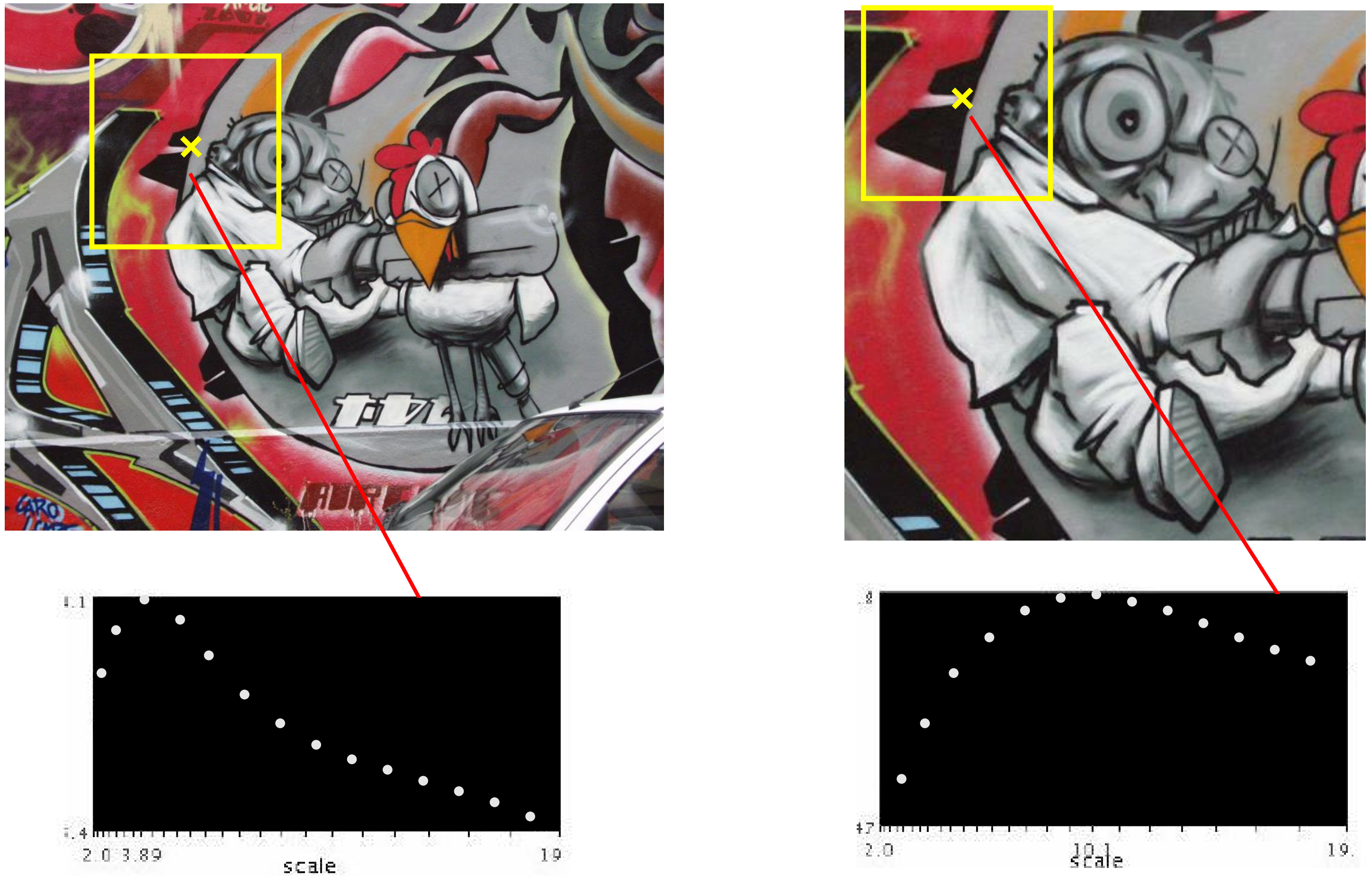

$f(I_{i_1 \dots i_m}(x, \sigma))$ $f(I_{i_1 \dots i_m}(x', \sigma'))$

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Automatic Scale Selection

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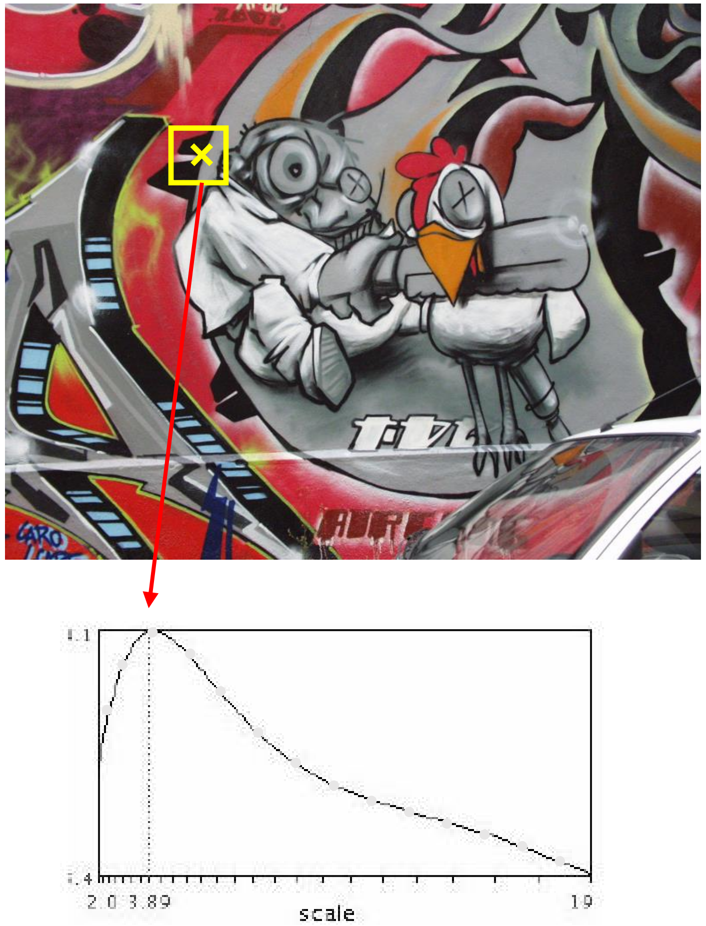
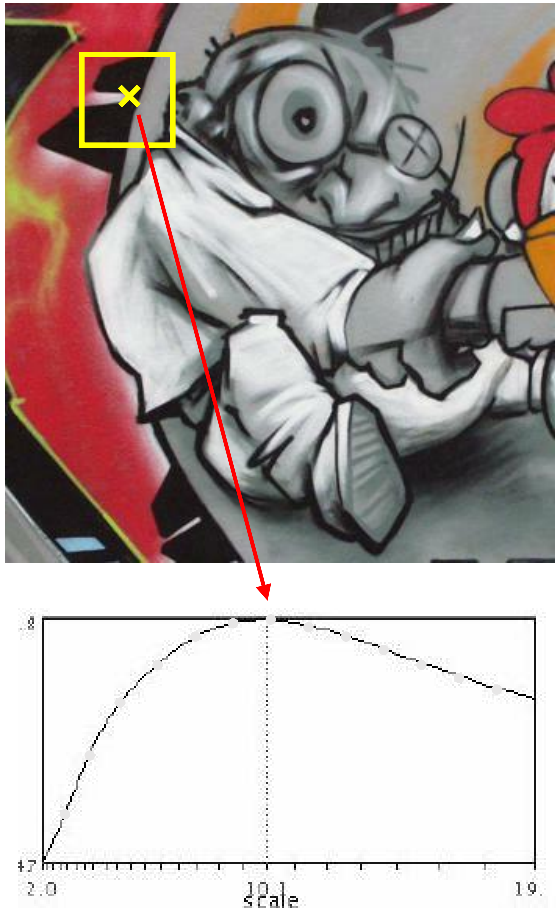


$f(I_{i_1 \ldots i_m}(x, \sigma))$



$f(I_{i_1 \ldots i_m}(x', \sigma'))$

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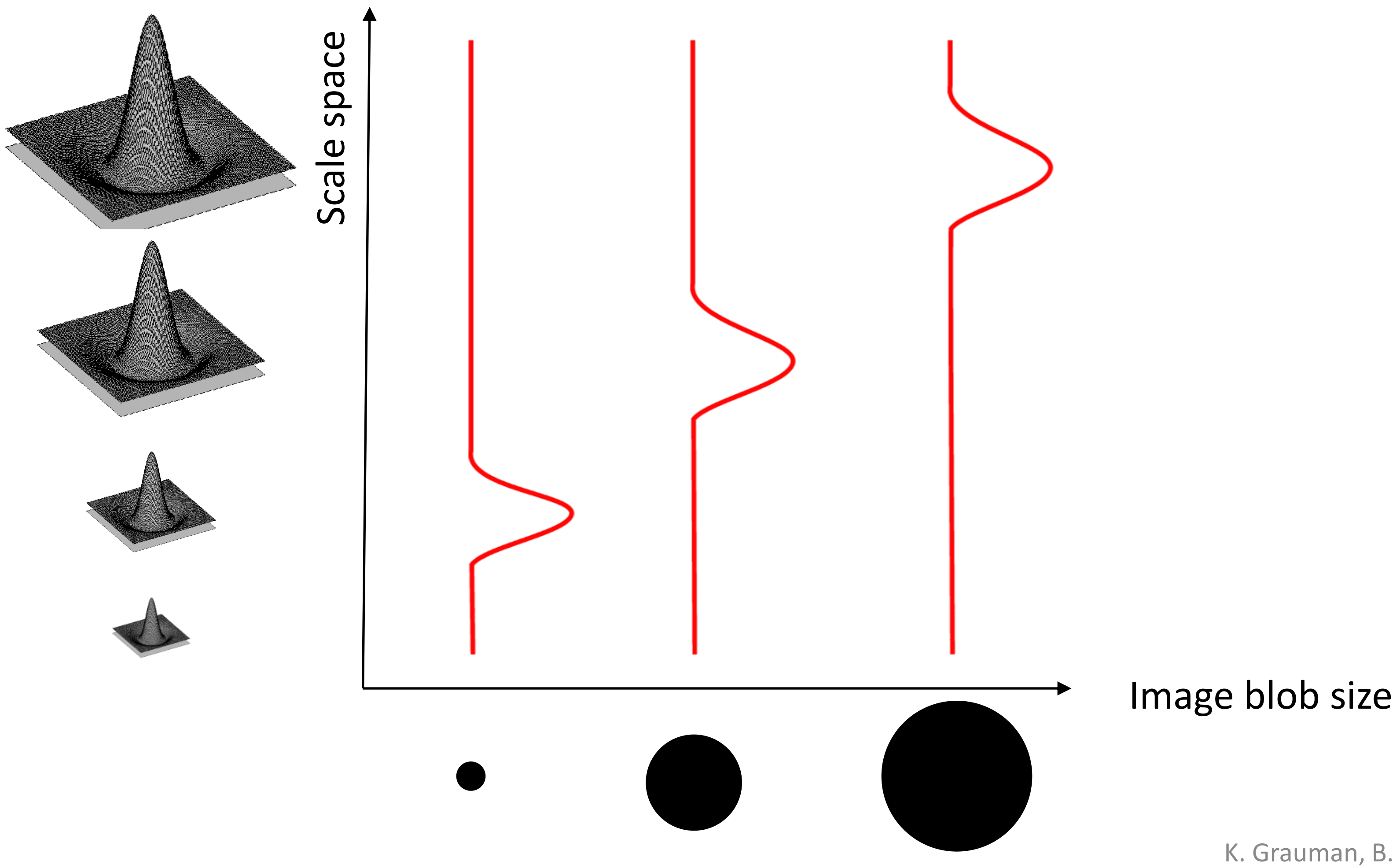
Blob detector

- Laplacian (2nd derivative) of Gaussian (LoG)



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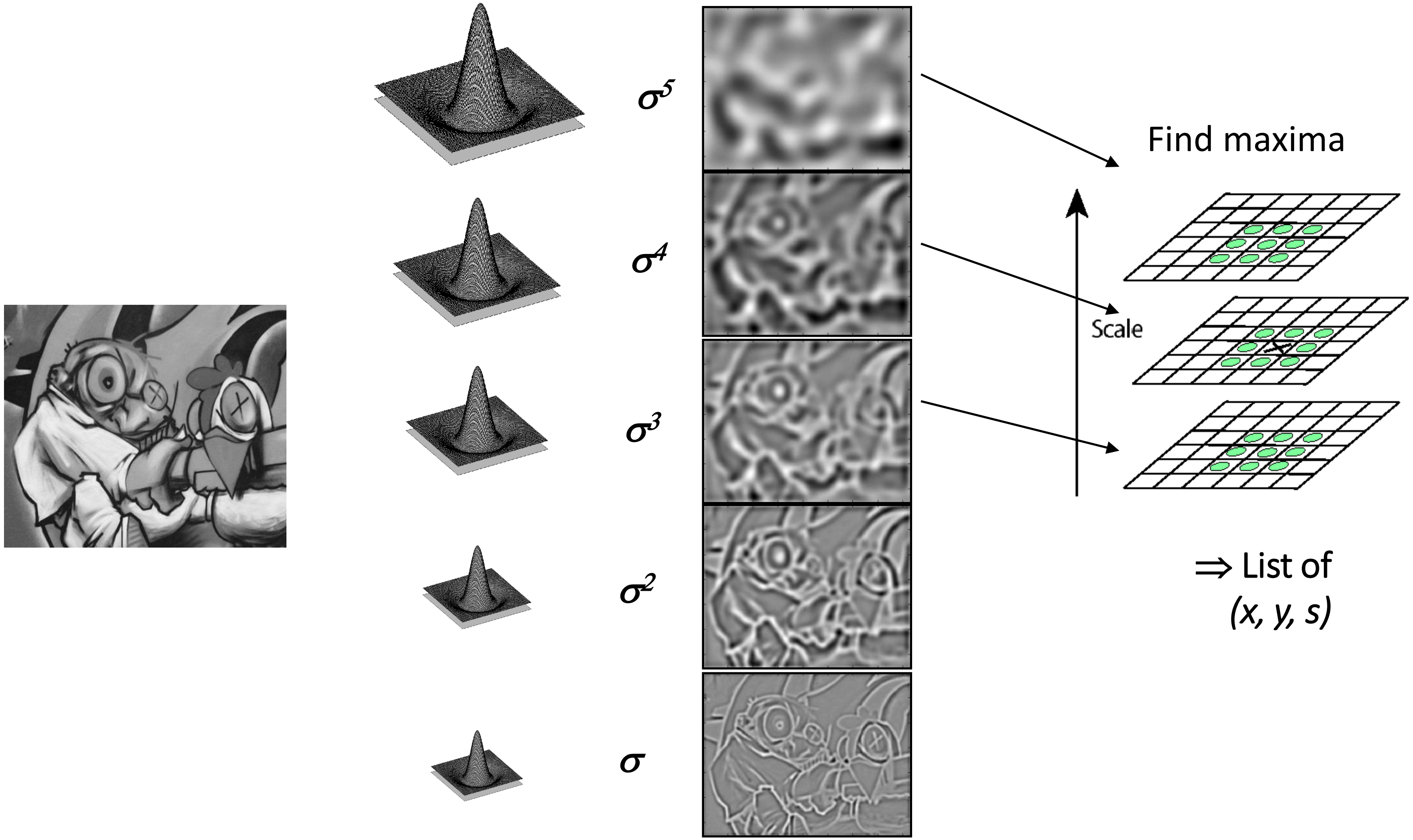
Find local maxima in scale space of Laplacian of Gaussian (LoG)



$\Rightarrow$ List of
(x, y, s)

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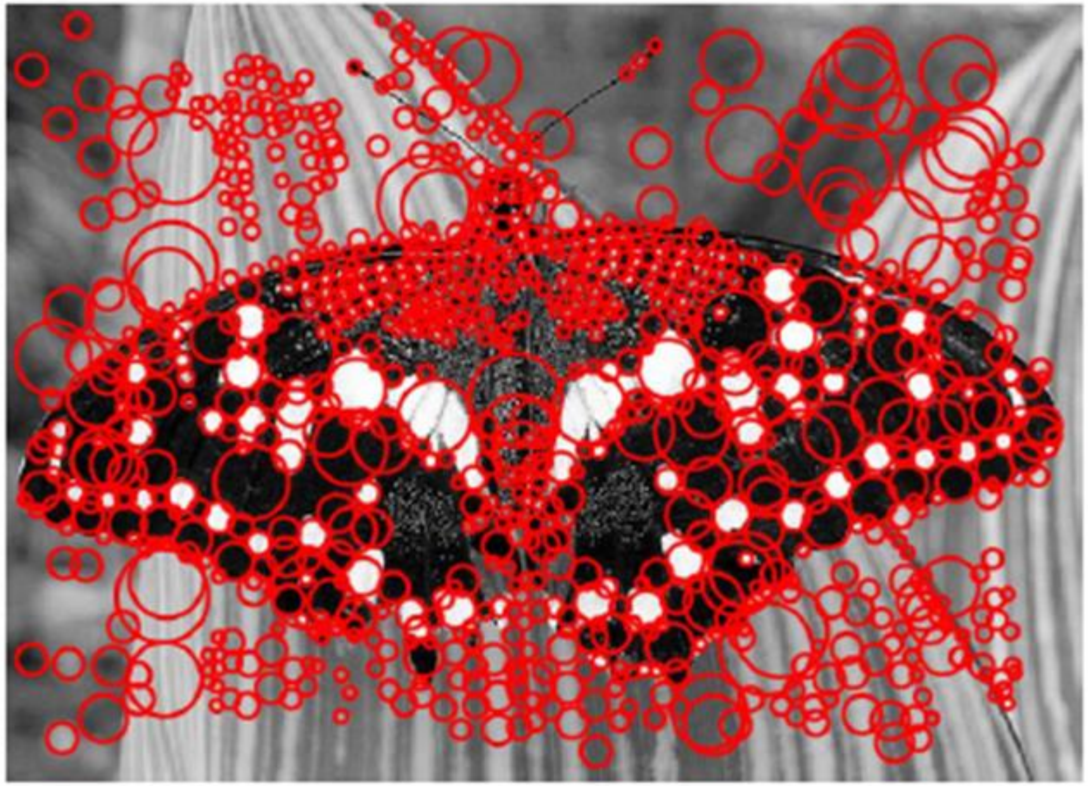
Scale-space blob detector: Example

Scale-space blob detector: Example

sigma = 11.9912



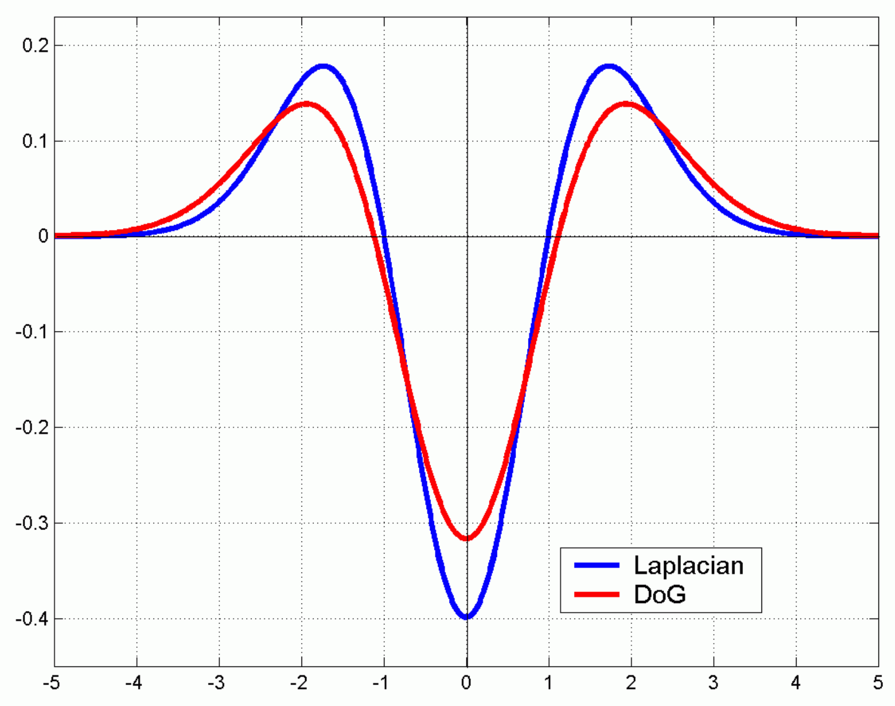
Approximation of LoG by Difference of Gaussians

$$\frac{\partial G}{\partial \sigma} = \sigma \Delta^2 G$$ Heat Equation

$$\sigma \Delta^2 G = \frac{\partial G}{\partial \sigma} = \frac{G(x, y, k\sigma) - G(x, y, \sigma)}{k\sigma - \sigma}$$

$$G(x, y, k\sigma) - G(x, y, \sigma) \approx (k-1)\sigma^2 \Delta^2 G$$

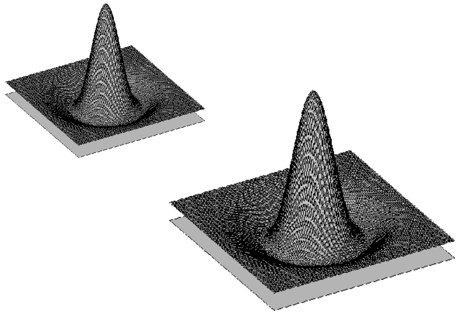
$$\text{Typical values} : \sigma = 1.6;\ k = \sqrt{2}$$



Difference-of-Gaussian (DoG)

Approximate LoG with DoG

1. Blur image with σ Gaussian kernel
2. Blur image with *k*σ Gaussian kernel
3. Subtract 2. from 1.



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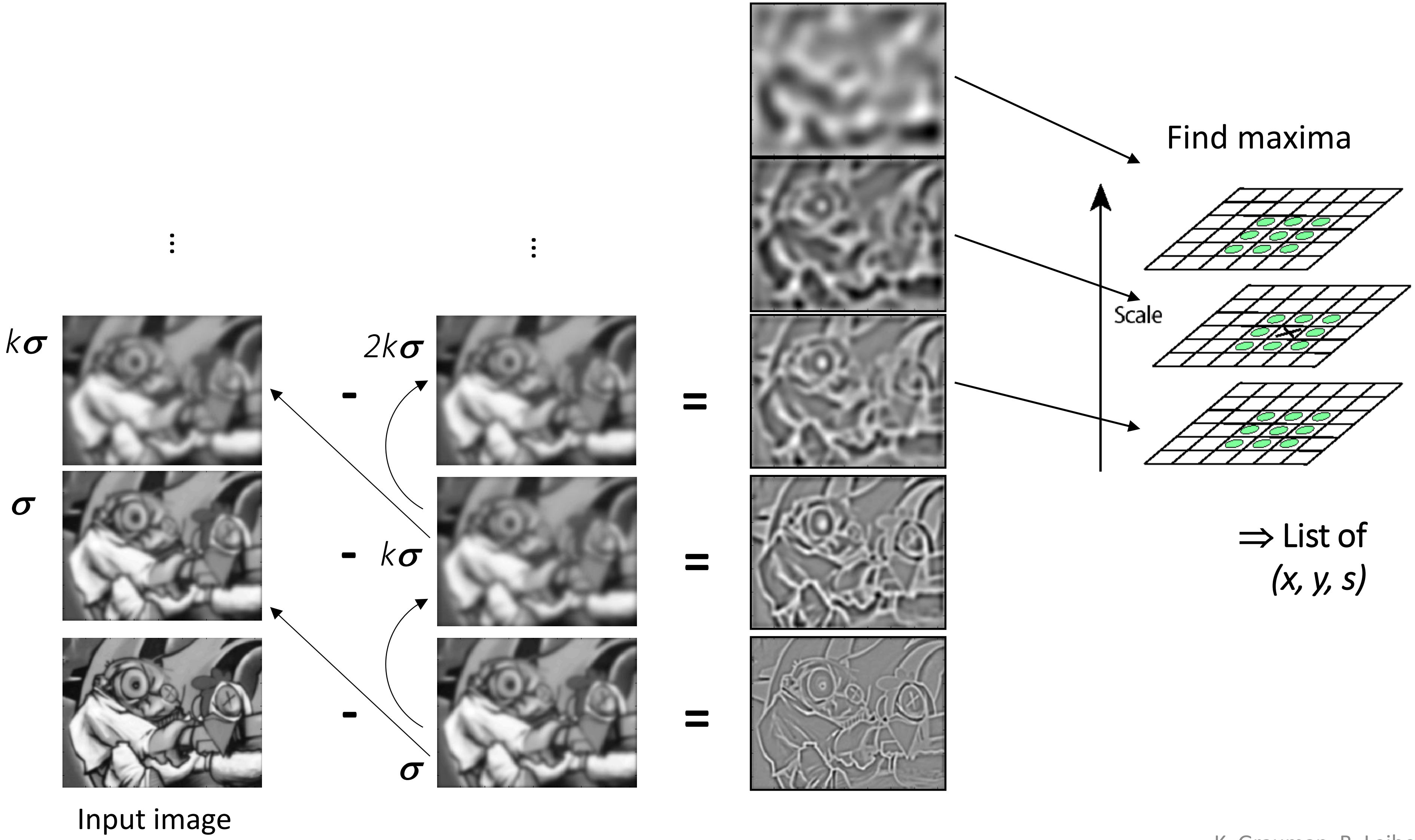
Find local maxima in scale space of Difference of Gaussian

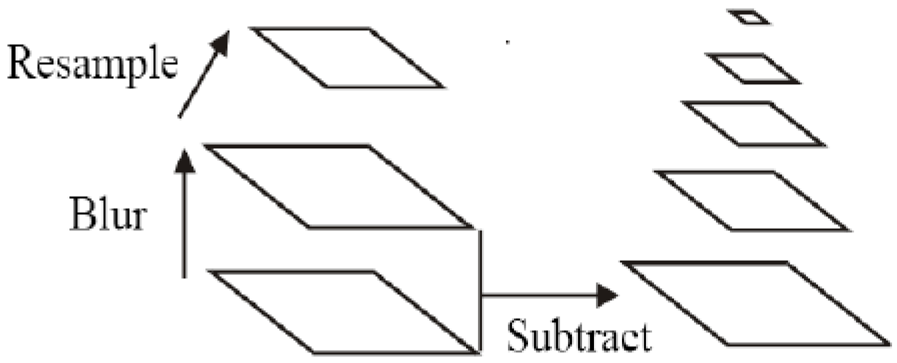


$\Rightarrow$ List of *(x, y, s)*

Building a Scale Space

- All scales must be examined to identify scale invariant features

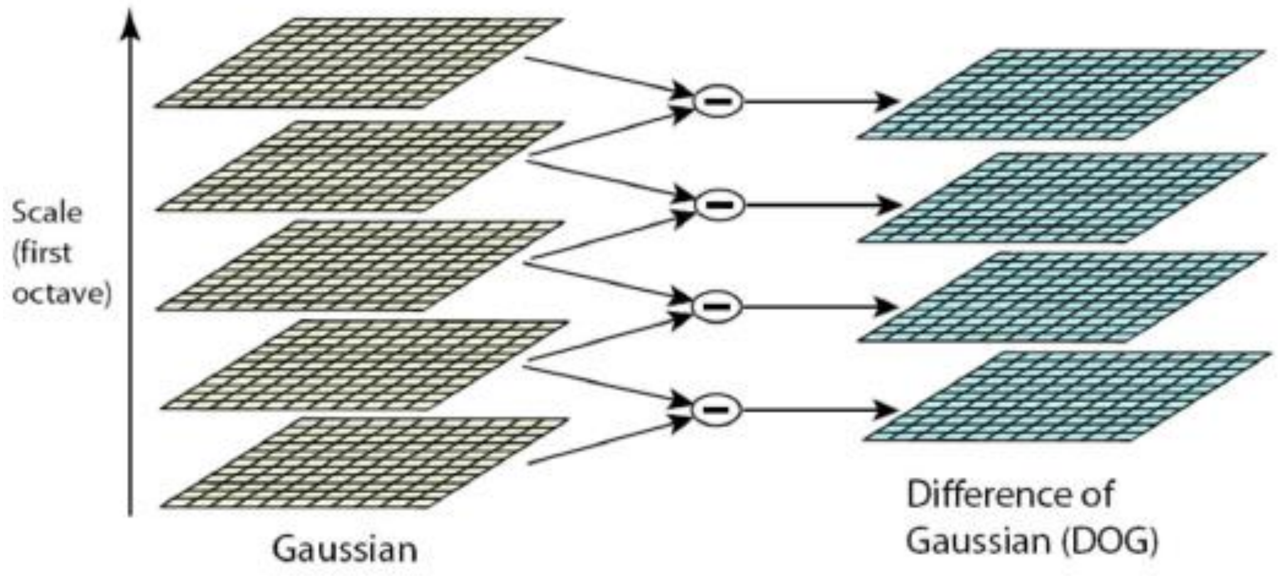
- An efficient function is to compute the Laplacian Pyramid (Difference of Gaussian)



Building a Scale Space

…

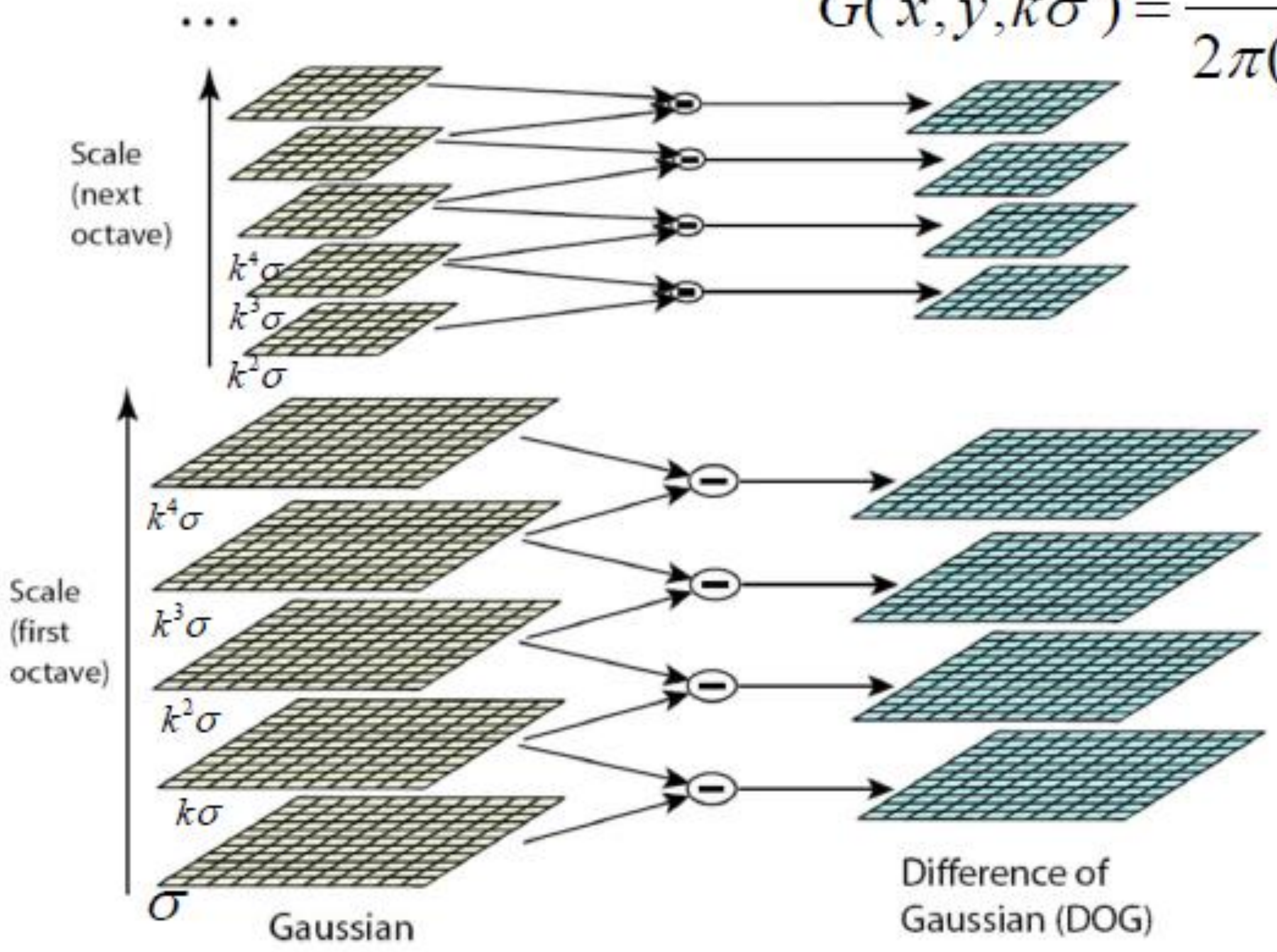
$$G(x,y,k\sigma) = \frac{1}{2\pi(k\sigma)^2} e^{-(x^2+y^2)/2k^2}$$



$$k = \sqrt{2}$$

Building a Scale Space

$$G(x, y, k\sigma) = \frac{1}{2\pi(k\sigma)^2} e^{-(x^2+y^2)/2k^2\sigma^2}$$



$$k = \sqrt{2}$$

Building a Scale Space

scale →

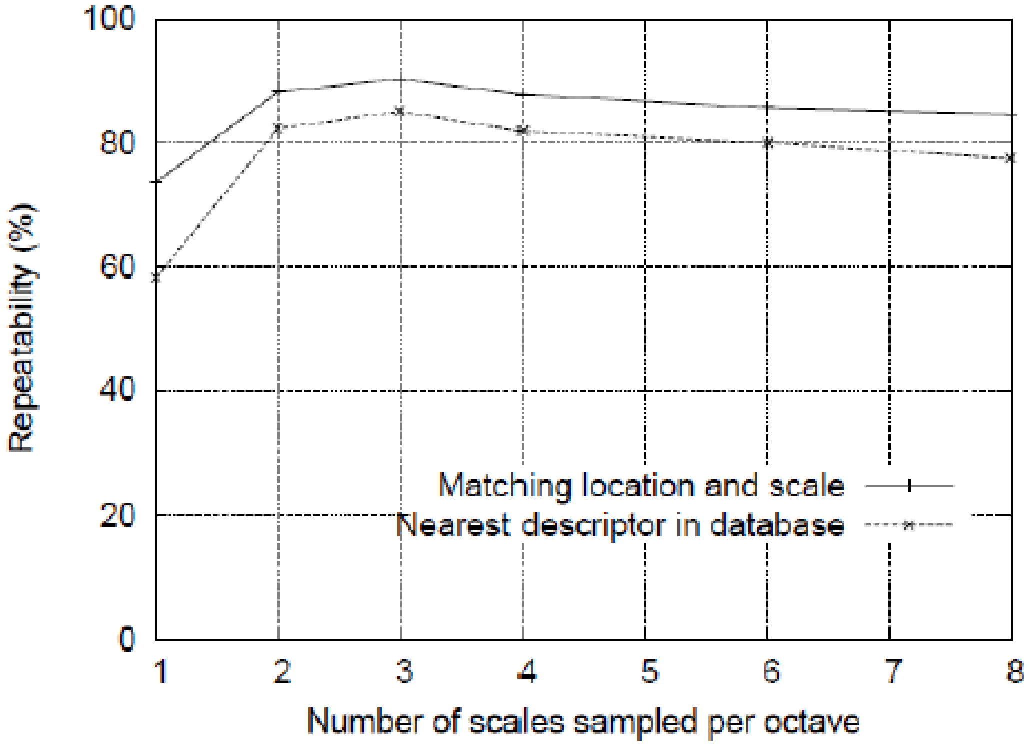
octave

0.707107	1.000000	1.414214	2.000000	2.828427
1.414214	2.000000	2.828427	4.000000	5.656854
2.828427	4.000000	5.656854	8.000000	11.313708
5.656854	8.000000	11.313708	16.000000	22.627417

$$\sigma = .707187.6;\ k = \sqrt{2}$$

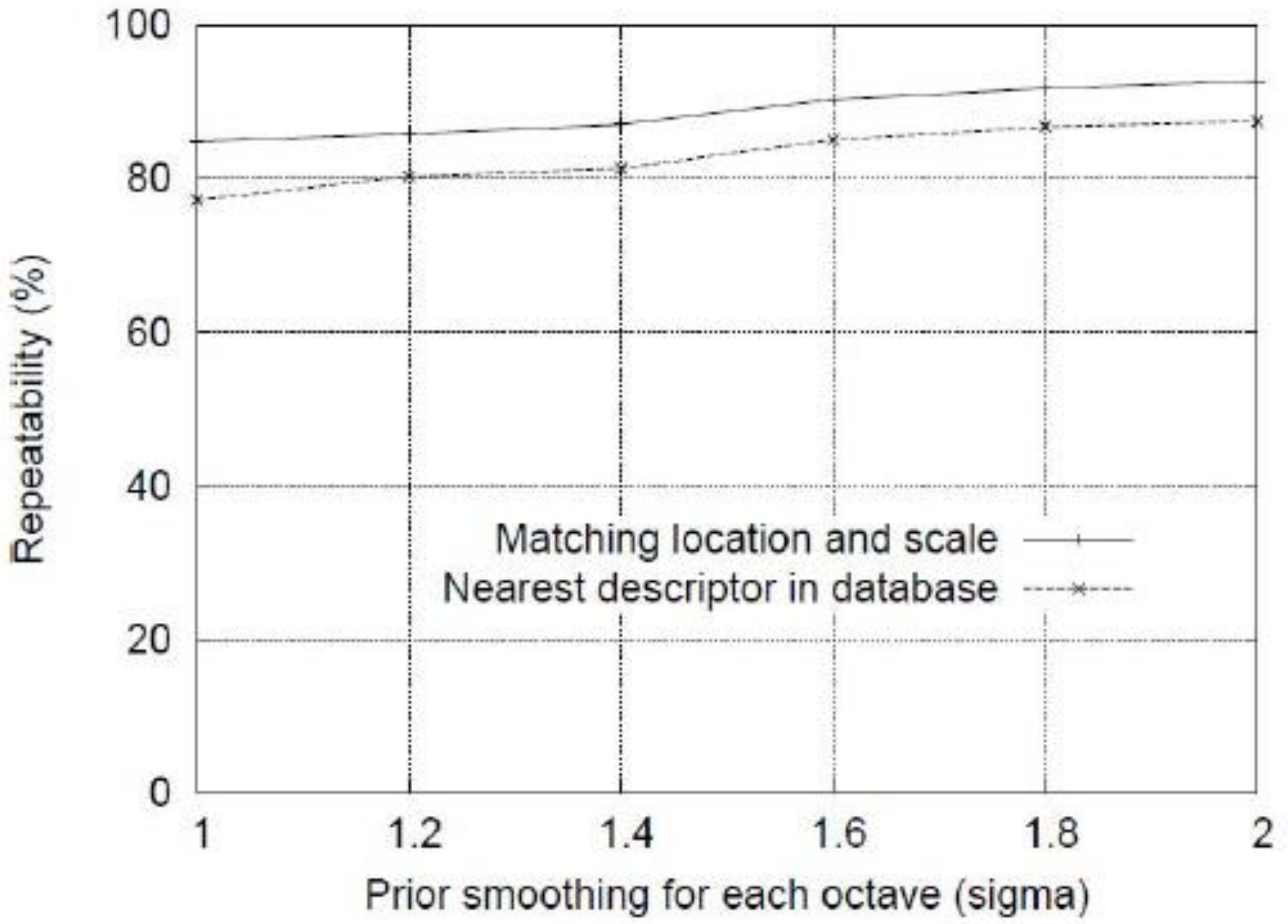
How many scales per octave?

- A collection of 32 real images drawn from a diverse range, including
  - outdoor scenes, human faces, aerial photographs, and industrial
- Each image was then subject to a range of transformations:
  - rotation, scaling, affine stretch, change in brightness and addition of image noise.



3 Scales

Initial value of sigma



1.6

Steps for Extracting Key Points

Scale space peak selection

- Potential locations for finding features

Key point localization

- Accurately locating the feature key points

Orientation Assignment

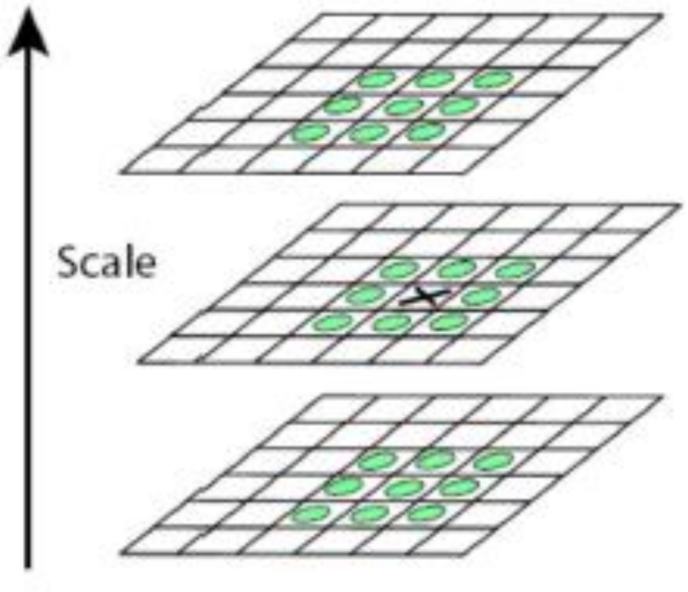
- Assigning orientation to the key points

Key point descriptor

- Describing the key point as a vector of size 128 (SIFT Descriptor)

Scale Space Peak Detection

- Compare a pixel (**X**) with 26 pixels in current and adjacent scales (Green Circles)
- Select a pixel (**X**) if larger/smaller than all 26 pixels
- Large number of extrema, computationally expensive
  - Detect the most stable subset of scales



Key Point Localization

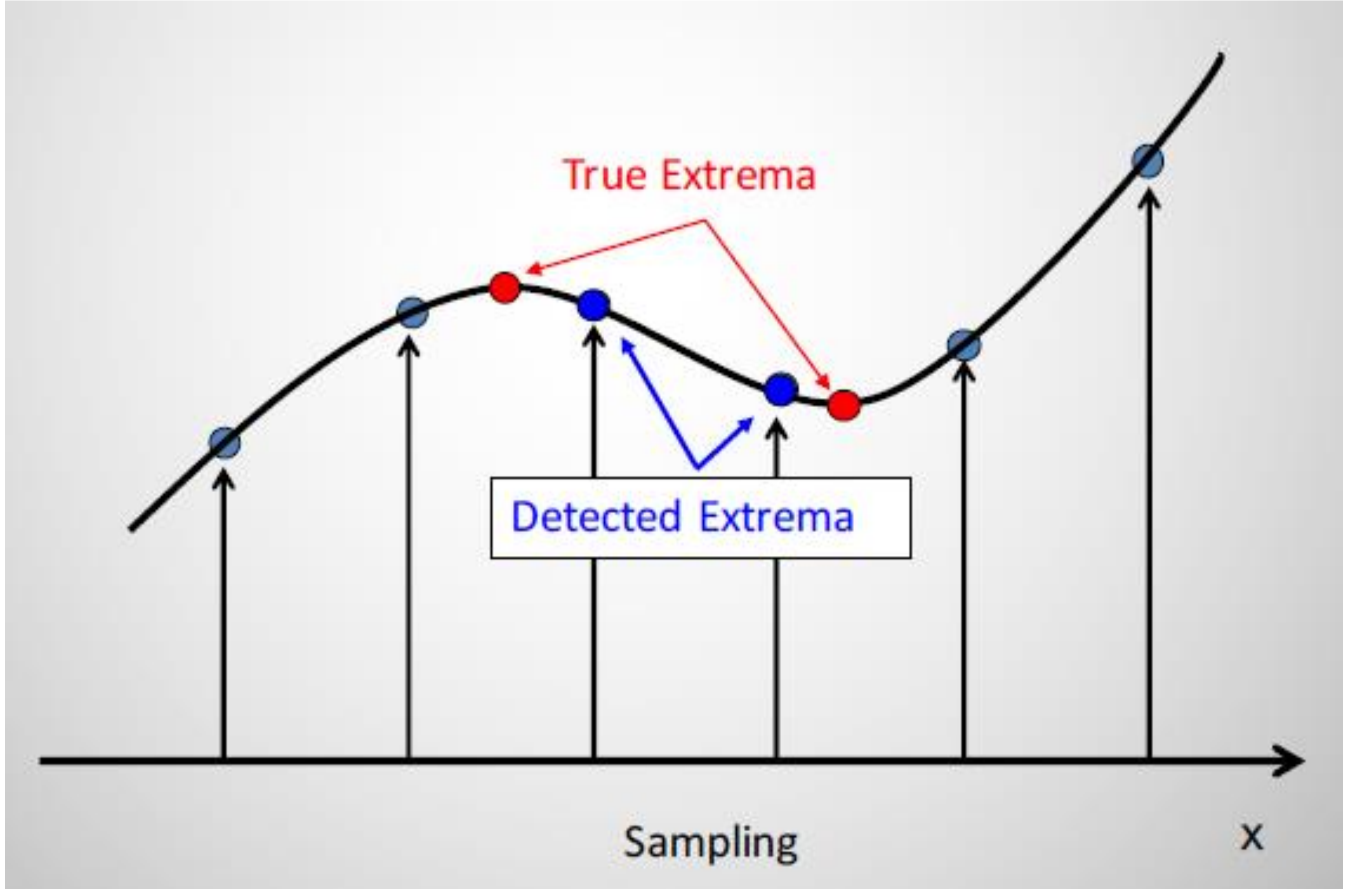
- Candidates are chosen from extrema detection
- There are lot of points, some of them are not good enough
- The locations of keypoints may be not accurate

original image

extrema locations

Inaccurate Keypoint Localization

- Inaccurate Keypoint localization due to poor contrast



Inaccurate Keypoint Localization

- The Solution:
  - Taylor expansion:

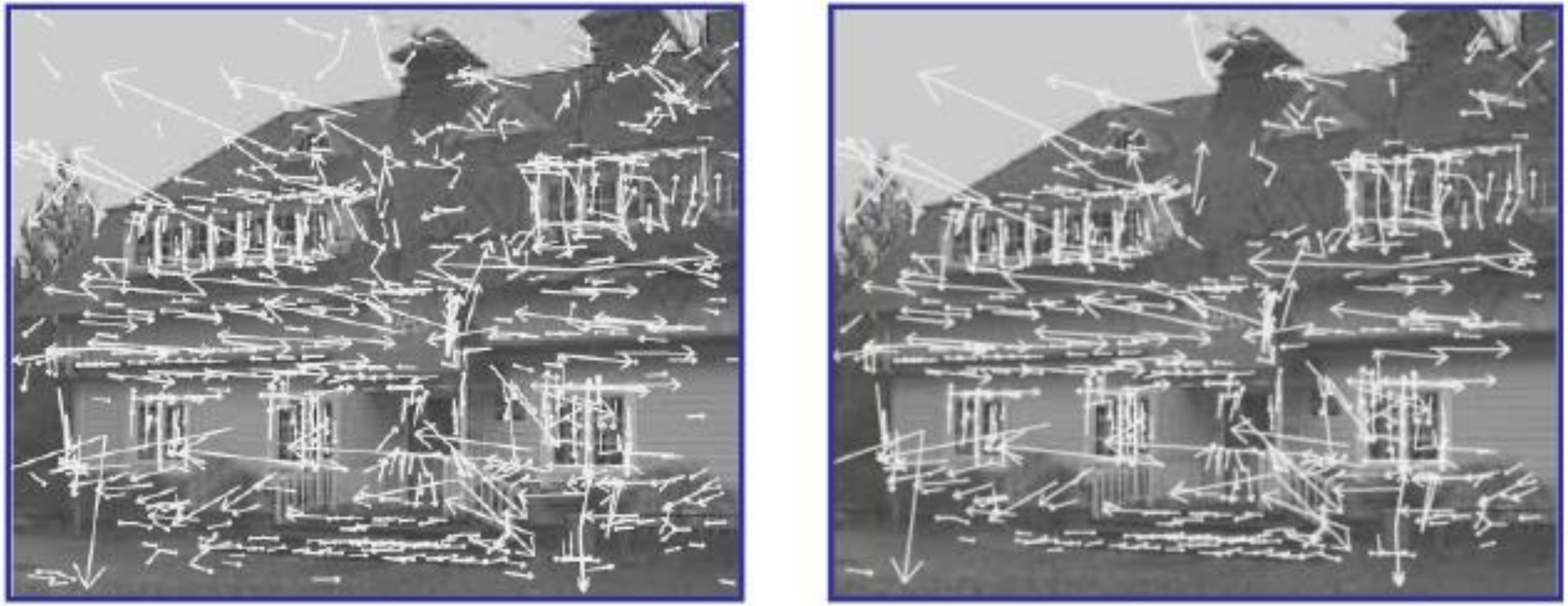
$$D(\vec{x}) = D + \frac{\partial D^T}{\partial \vec{x}}\vec{x} + \frac{1}{2}\vec{x}^T \frac{\partial^2 D^T}{\partial \vec{x}^2}\vec{x}$$

  - Minimize to find accurate extrema:

$$\hat{x} = -\frac{\partial^2 D}{\partial \vec{x}^2}^{-1} \frac{\partial D}{\partial \vec{x}}$$

  - If offset from sampling point is larger than 0.5 - Keypoint should be in a different sampling point.

Initial Outlier Rejection



from 832 key points to 729 key points, th=0.03.

Further Outlier Rejection

- DOG has strong response along edge

- Assume DOG as a surface
  - Compute principal curvatures (PC)
  - Poorly defined peak will have a large principal curvature across the edge.

Further Outlier Rejection

- Compute Hessian of D (principal curvature)

$$\mathbf{H} = \begin{bmatrix} D_{xx} & D_{xy} \\ D_{xy} & D_{yy} \end{bmatrix}$$

$$Tr(H) = D_{xx} + D_{yy} = \lambda_1 + \lambda_2$$

$$Det(H) = D_{xx} D_{yy} - D_{xy}^2 = \lambda_1 \lambda_2$$

- Remove outliers by evaluating

$$\frac{Tr(H)^2}{Det(H)} = \frac{(r+1)^2}{r}$$

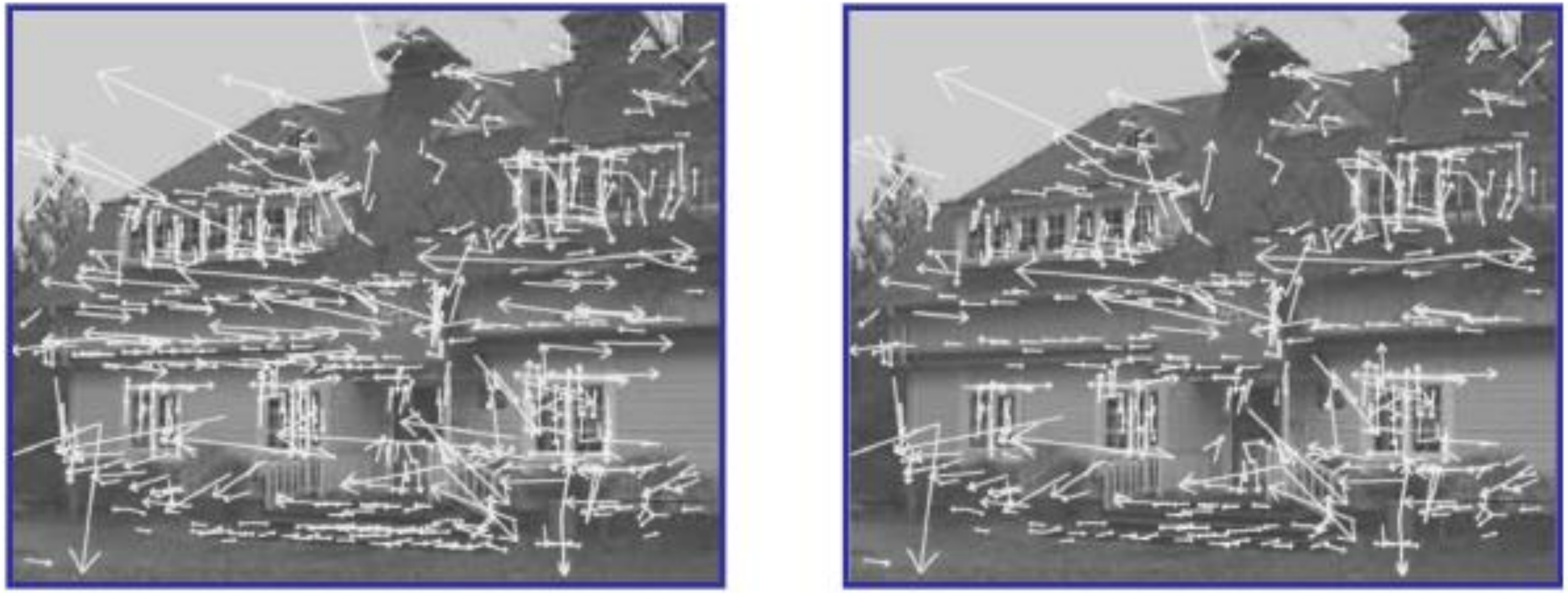
$$r = \frac{\lambda_1}{\lambda_2}$$

$$\frac{Tr(H)^2}{Det(H)} = \frac{(\lambda_1 + \lambda_2)^2}{\lambda_1 \lambda_2} = \frac{(r\lambda_2 + \lambda_2)^2}{r\lambda_2^2} = \frac{(r+1)^2}{r}$$

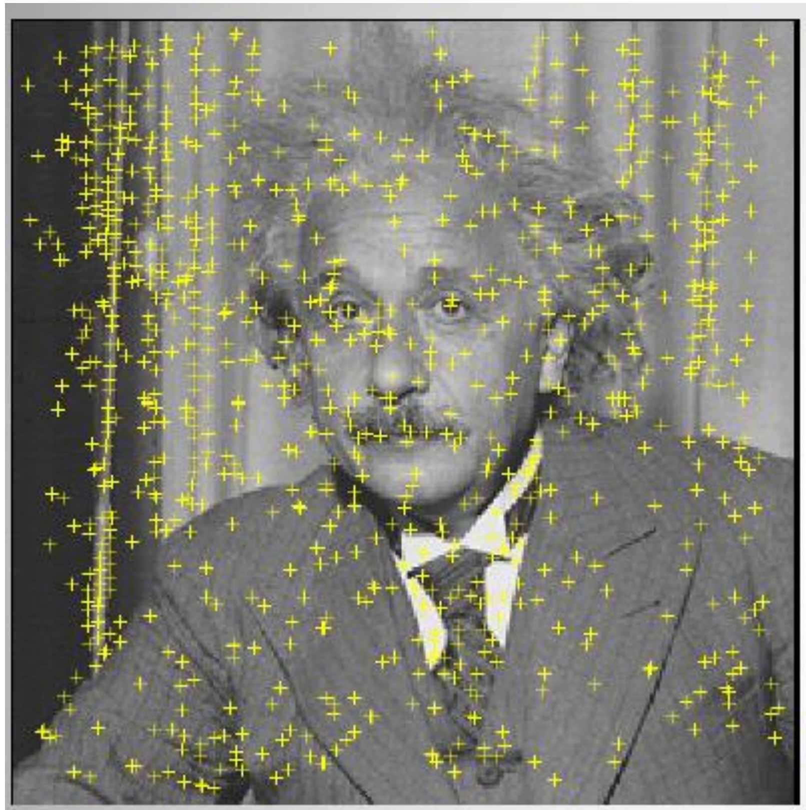
Eliminate key points if

$$\frac{Tr(H)^2}{Det(H)} \prec \frac{(r+1)^2}{r} \qquad r > 10$$

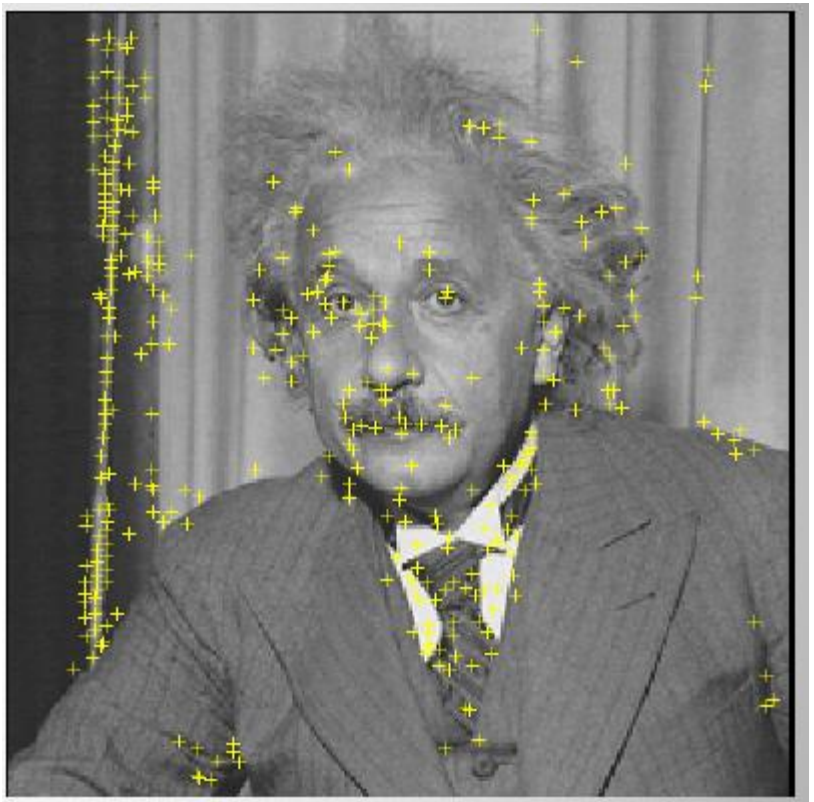
Further Outlier Rejection



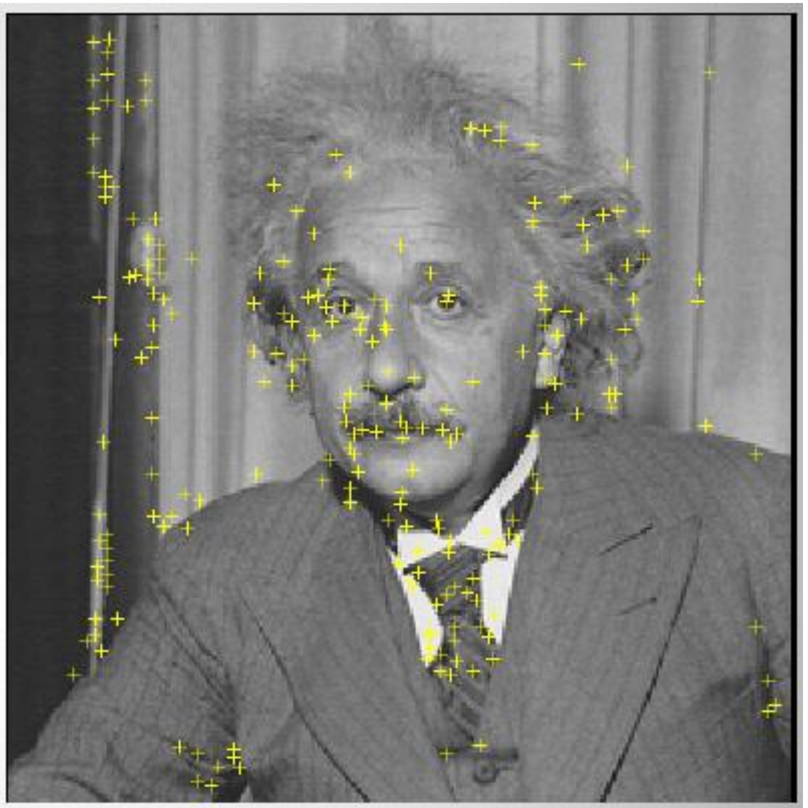
from 729 key points to 536 key points.



Local extremas



Remove low contrast features



Remove low edges

Orientation Assignment

- Use scale of point to choose correct image:

$$L(x,y) = G(x,y,\sigma) * I(x,y)$$

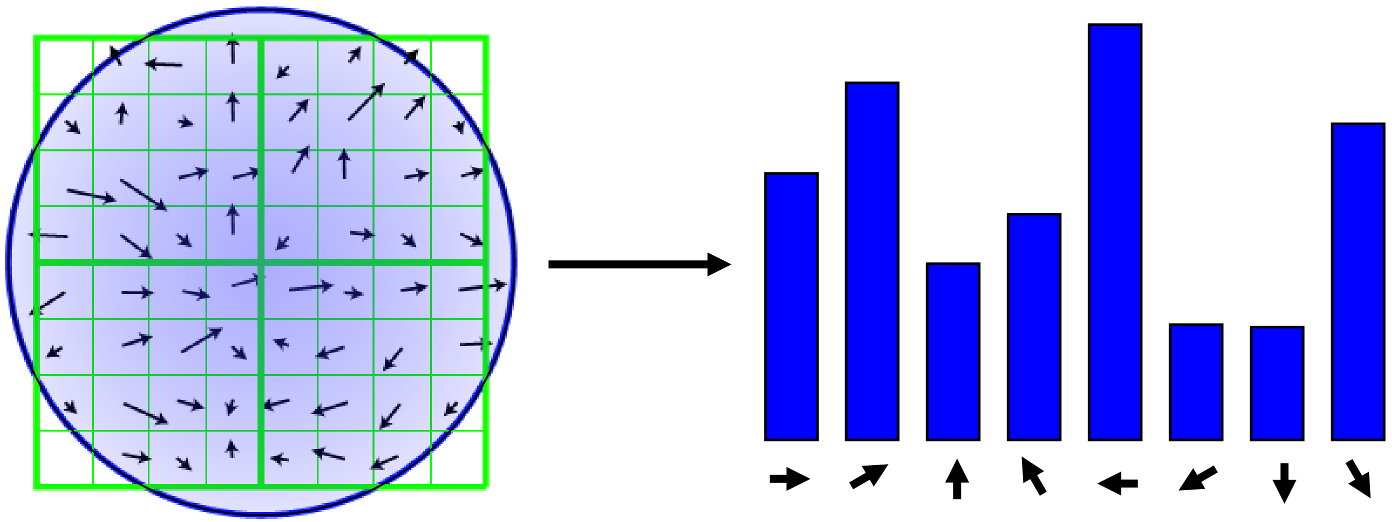
- Compute gradient magnitude and orientation using finite differences:

$$m(x,y) = \sqrt{\left(L(x+1,y) - L(x-1,y)\right)^2 + \left(L(x,y+1) - L(x,y-1)\right)^2}$$

$$\theta(x,y) = \tan^{-1}\left(\frac{\left(L(x,y+1) - L(x,y-1)\right)}{\left(L(x+1,y) - L(x-1,y)\right)}\right)$$

Orientation Assignment

- Create gradient histogram (36 bins)
  - Weighted by magnitude and Gaussian window (σ =1.5 times that of the scale of a keypoint)

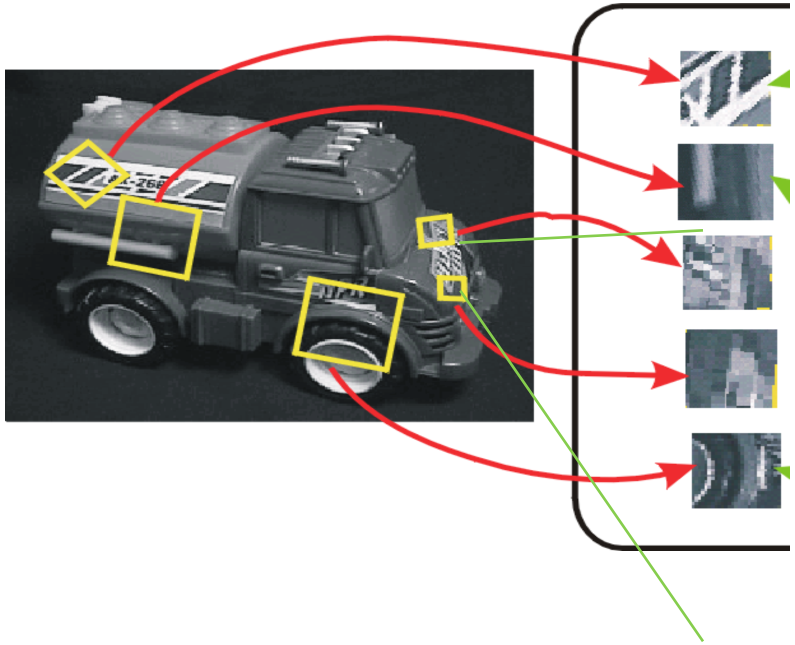


Orientation Assignment

- Any peak within 80% of the highest peak is used to create a keypoint with that orientation

SIFT descriptor formation

- Compute on local 16 x 16 window around detection.



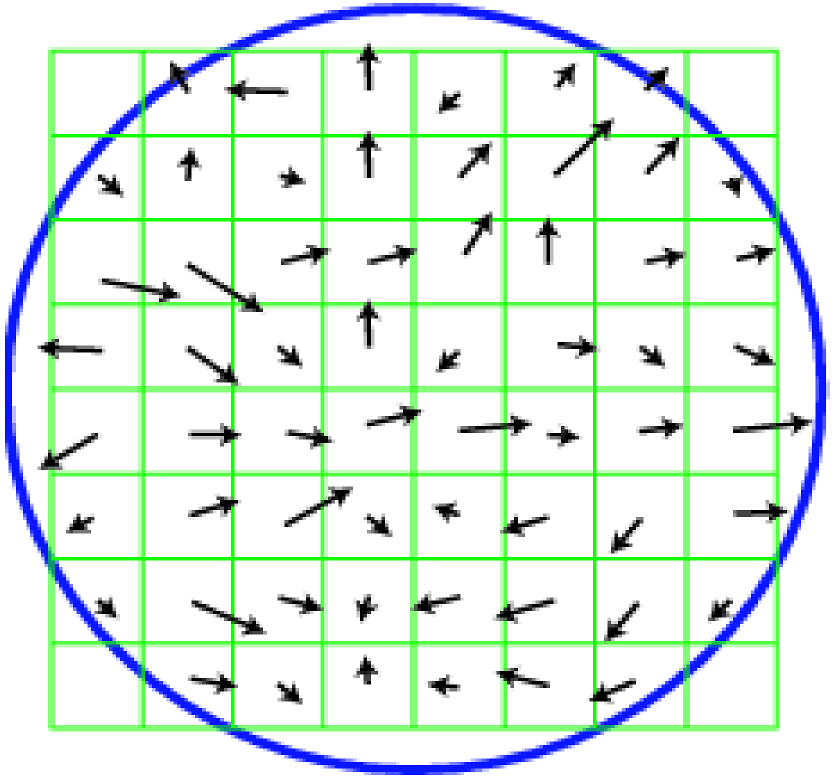
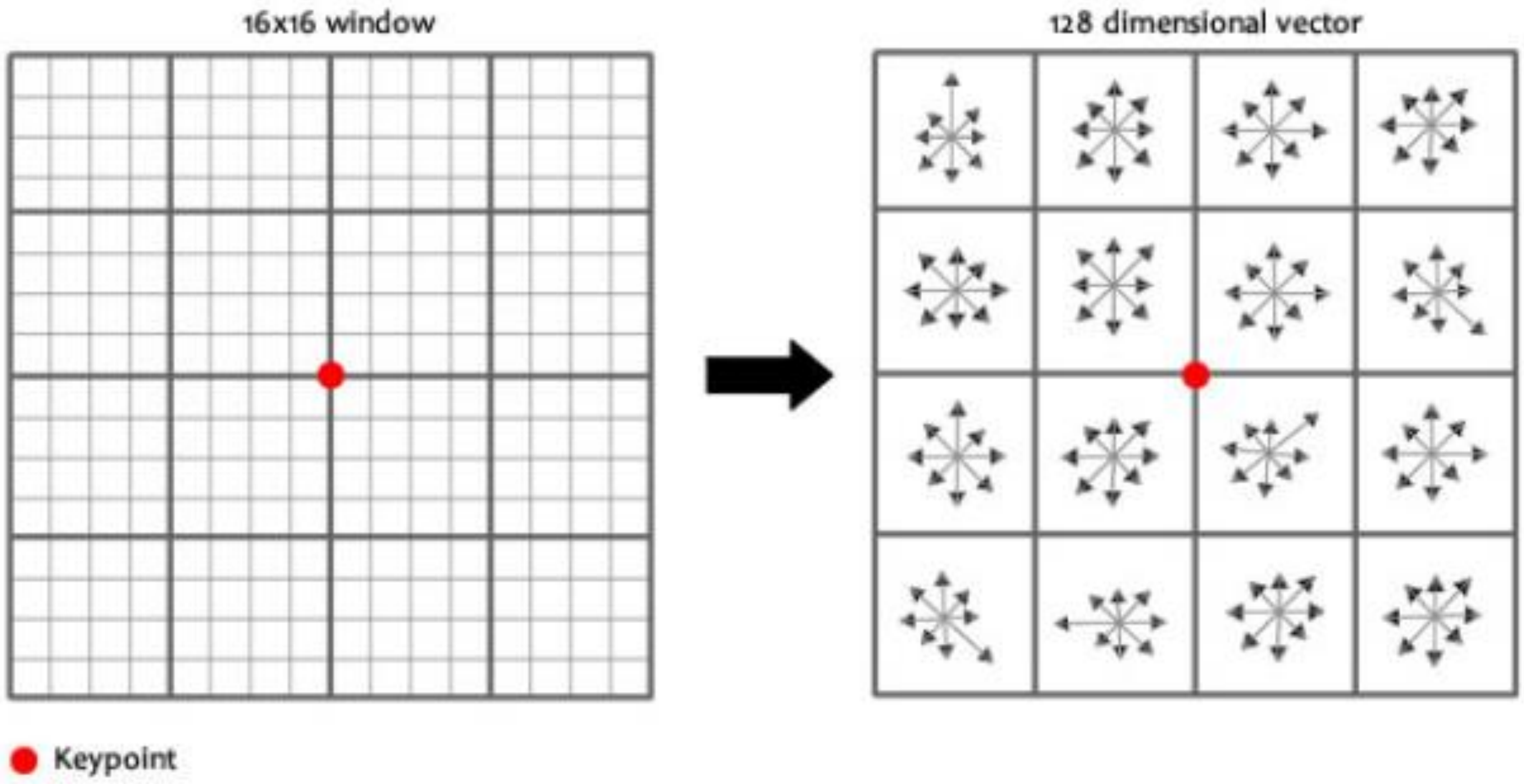


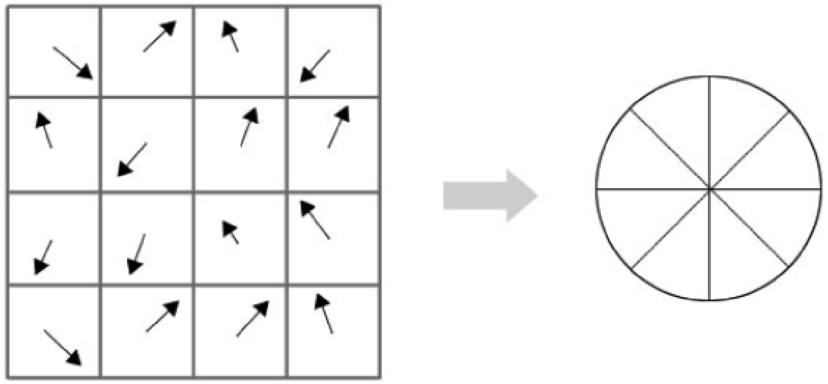
Image gradients

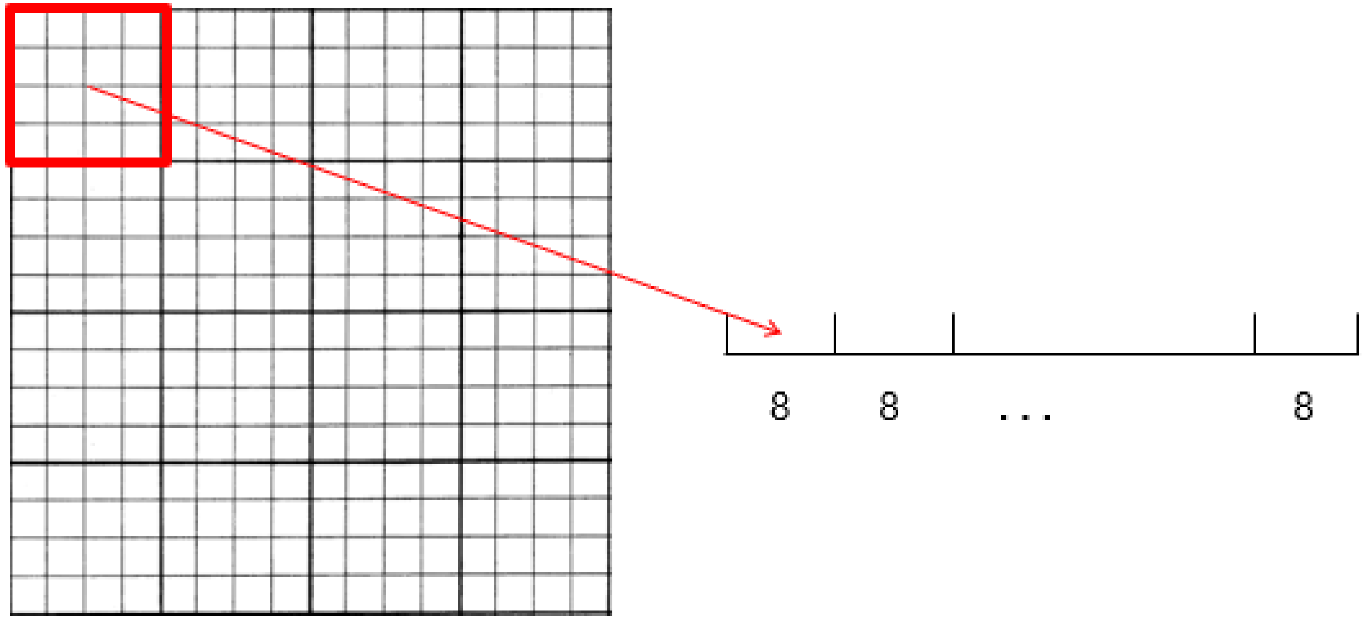
Actually 16x16, only showing 8x8

James Hays

- Compute Divide the 16x16 window into a 4x4 grid of cells
- an orientation histogram for each cell
- 16 cells * 8 orientations = 128 dimensional descriptor



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8
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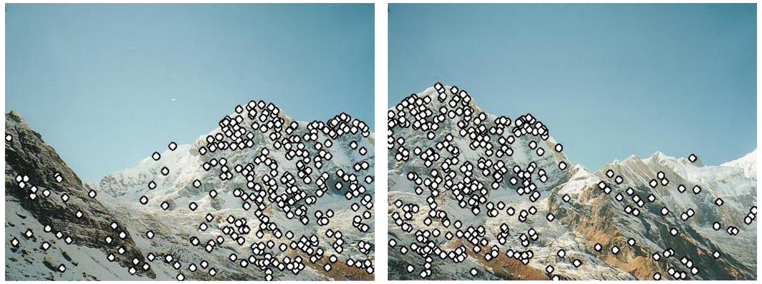
8

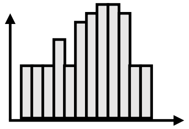
Local features: main components

1) **Detection:**
Find a set of distinctive key points.

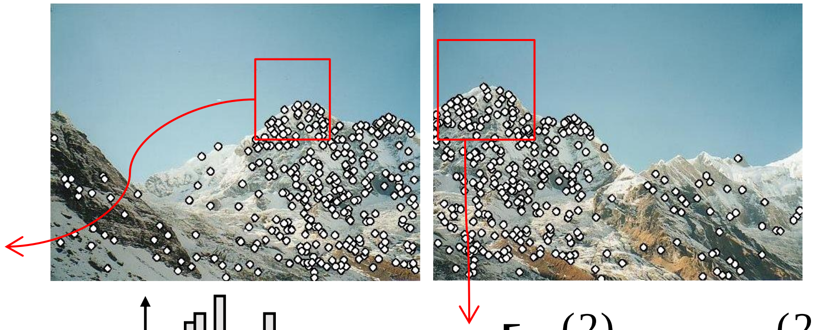


2) **Description:**
Extract feature descriptor around each interest point as vector.

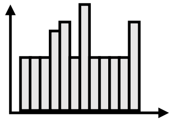
x_1



$$\mathbf{x}_1 = [x_1^{(1)}, \ldots, x_d^{(1)}]$$



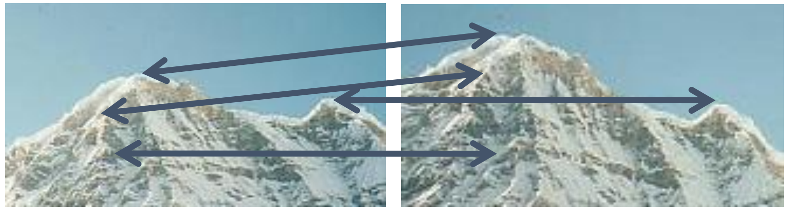
x_2



$$\mathbf{x}_2 = [x_1^{(2)}, \ldots, x_d^{(2)}]$$

3) **Matching:**
Compute distance between feature vectors to find correspondence.

$$d(\mathrm{x}_1, \mathrm{x}_2) < T$$

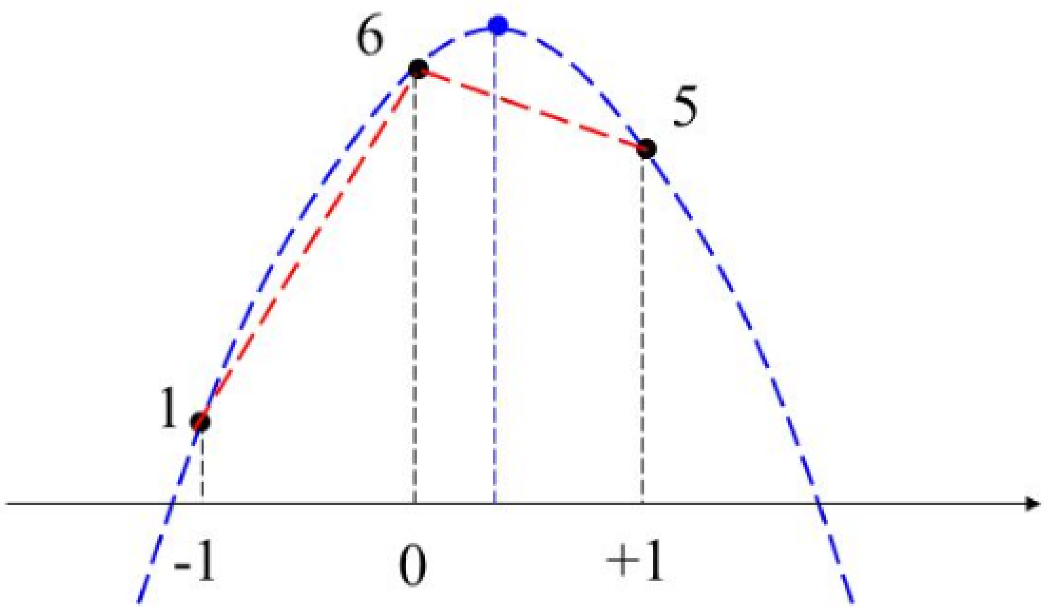


Object Recognition

- **For training images:**
  - Extracting keypoints by SIFT.
  - Creating descriptors database.
- **For query images:**
  - Extracting keypoints by SIFT.
  - For each descriptor - finding nearest neighbor in DB.
  - Finding cluster of at-least 3 keypoints.
  - Performing detailed geometric fit check for each cluster.

2. Accurate keypoint localization

- Reject points with low contrast (flat) and poorly localized along an edge (edge)
- Fit a 3D quadratic function for sub-pixel maxima



2. Accurate keypoint localization

- Taylor series of several variables

$$T(x_1,\cdots,x_d)=\sum_{n_1=0}^{\infty}\cdots\sum_{n_d=0}^{\infty}\frac{\partial^{n_1}}{\partial x_1^{n_1}}\cdots\frac{\partial^{n_d}}{\partial x_d^{n_d}}\frac{f(a_1,\cdots,a_d)}{n_1!\cdots n_d!}(x_1-a_1)^{n_1}\cdots(x_d-a_d)^{n_d}$$

- Two variables

$$f(x,y)\approx f(0,0)+\left(\frac{\partial f}{\partial x}x+\frac{\partial f}{\partial y}y\right)+\frac{1}{2}\left(\frac{\partial^2 f}{\partial x\partial x}x^2+2\frac{\partial^2 f}{\partial x\partial y}xy+\frac{\partial^2 f}{\partial y\partial y}y^2\right)$$

$$f\left(\begin{bmatrix}x\\y\end{bmatrix}\right)\approx f\left(\begin{bmatrix}0\\0\end{bmatrix}\right)+\begin{bmatrix}\frac{\partial f}{\partial x} & \frac{\partial f}{\partial y}\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}+\frac{1}{2}\begin{bmatrix}x & y\end{bmatrix}\begin{bmatrix}\frac{\partial^2 f}{\partial x\partial x} & \frac{\partial^2 f}{\partial x\partial y}\\ \frac{\partial^2 f}{\partial x\partial y} & \frac{\partial^2 f}{\partial y\partial y}\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}$$

$$f(\mathbf{x})\approx f(\mathbf{0})+\frac{\partial f}{\partial \mathbf{x}}^T\mathbf{x}+\frac{1}{2}\mathbf{x}^T\frac{\partial^2 f}{\partial \mathbf{x}^2}\mathbf{x}$$

Accurate keypoint localization

- Taylor expansion in a matrix form, $\mathbf{x}$ is a vector, f maps $\mathbf{x}$ to a scalar

$$f(\mathbf{x}) = f + \frac{\partial f}{\partial \mathbf{x}}^T \mathbf{x} + \frac{1}{2}\mathbf{x}^T \frac{\partial^2 f}{\partial \mathbf{x}^2}\mathbf{x}$$

Hessian matrix (often symmetric)

gradient $\begin{pmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_1} \\ \vdots \\ \frac{\partial f}{\partial x_n} \end{pmatrix}$

$$\begin{pmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_1 \partial x_n} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} & \cdots & \frac{\partial^2 f}{\partial x_2 \partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 f}{\partial x_n \partial x_1} & \frac{\partial^2 f}{\partial x_n \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_n^2} \end{pmatrix}$$

2D illustration

$$f(\mathbf{x}) = f + \frac{\partial f}{\partial \mathbf{x}}^T \mathbf{x} + \frac{1}{2}\mathbf{x}^T \frac{\partial^2 f}{\partial \mathbf{x}^2}\mathbf{x}$$

$f_{-1,1}$	$f_{0,1}$	$f_{1,1}$
$f_{-1,0}$	$f_{0,0}$	$f_{1,0}$
$f_{-1,-1}$	$f_{0,-1}$	$f_{1,-1}$

$$\begin{aligned}
\frac{\partial f}{\partial x} &= (f_{1,0} - f_{-1,0})/2 \\
\frac{\partial f}{\partial y} &= (f_{0,1} - f_{0,-1})/2 \\
\frac{\partial^2 f}{\partial x^2} &= f_{1,0} - 2f_{0,0} + f_{-1,0} \\
\frac{\partial^2 f}{\partial y^2} &= f_{0,1} - 2f_{0,0} + f_{0,-1} \\
\frac{\partial^2 f}{\partial x \partial y} &= (f_{-1,-1} - f_{-1,1} - f_{1,-1} + f_{1,1})/4
\end{aligned}$$