DIGITAL IMAGE ENHACEMENT

CV

Image enhancement

"...process an image so that the result is more suitable than the original image for a *specific* application"

Make the image look "good"

What is "good"? How do we compare algorithms?

Good images

For human visual

The visual evaluation of image quality is a highly subjective process.

It is hard to standardize the definition of a good image.

For machine perception

The evaluation task is easier.

A good image is one which gives the best machine recognition results.

A certain amount of trial and error usually is required before a particular image enhancement approach is selected.

Image enhancement domains

- Two broad categories of image enhancement methods
  - **Spatial domain method**

$$g(x, y) = T[f(x, y)]$$

  - **Frequency domain method** – Operate in the Fourier transform domain

$$F(u, v) = FT\{f(x, y)\}$$
$$G(u, v) = T[F(u, v)]$$
$$g(x, y) = FT^{-1}[G(u, v)]$$

Example: Image Denoising

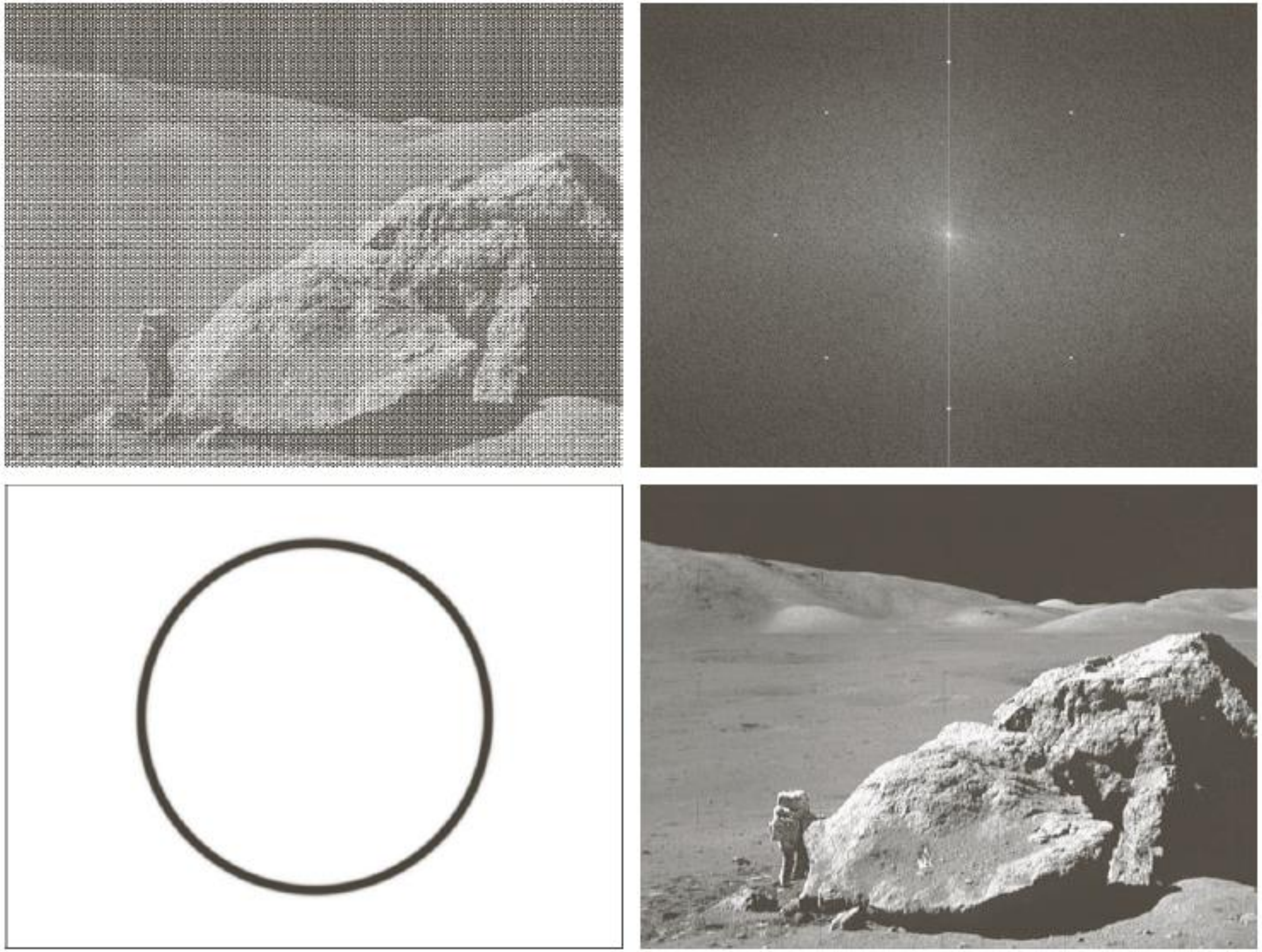
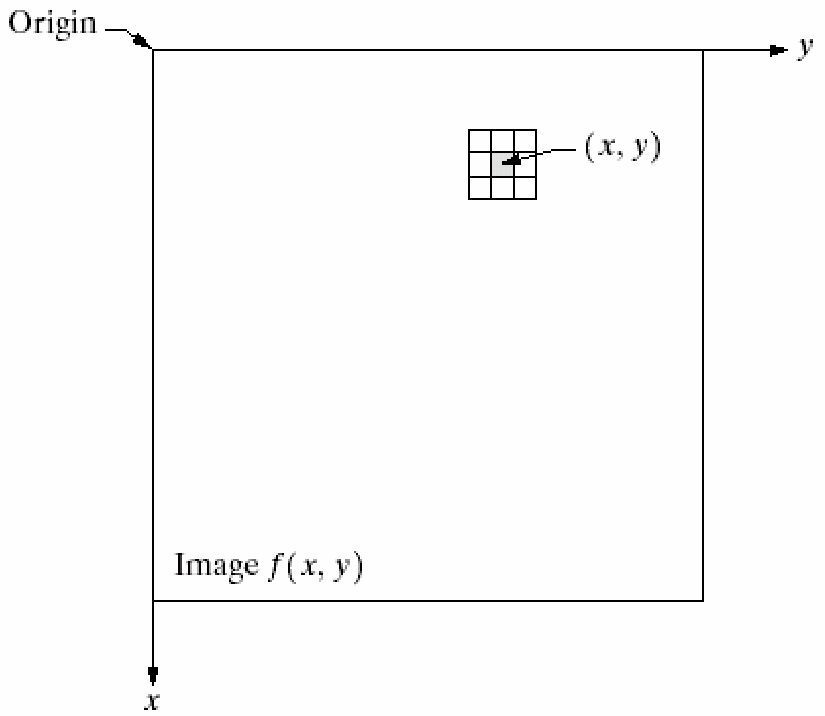


FIGURE 2.40
(a) Image corrupted by sinusoidal interference. (b) Magnitude of the Fourier transform showing the bursts of energy responsible for the interference. (c) Mask used to eliminate the energy bursts. (d) Result of computing the inverse of the modified Fourier transform. (Original image courtesy of NASA.)

Spatial domain image enhancement

- Point processing
  - Output pixel at (x,y) depends only on the input pixel at (x,y)
- Neighborhood processing

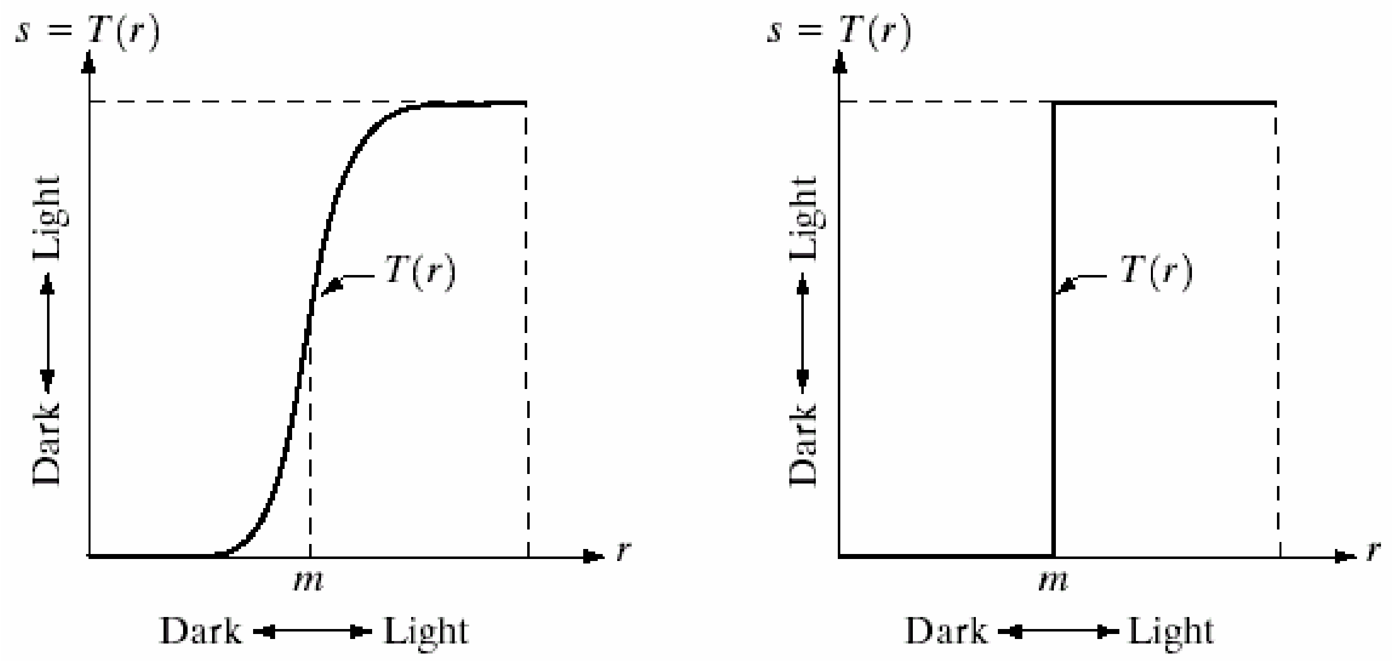
FIGURE 3.1 A 3×3 neighborhood about a point (x, y) in an image.



Gray level transformation

- Let r = pixel value at (x, y) in the input image (f)
- Let s = pixel value at (x, y) in the output image (g)
- Transformation given by

$$s = T(r)$$



a b

FIGURE 3.2 Gray-level transformation functions for contrast enhancement.

Gray level transformation

FIGURE 3.3 Some basic gray-level transformation functions used for image enhancement.

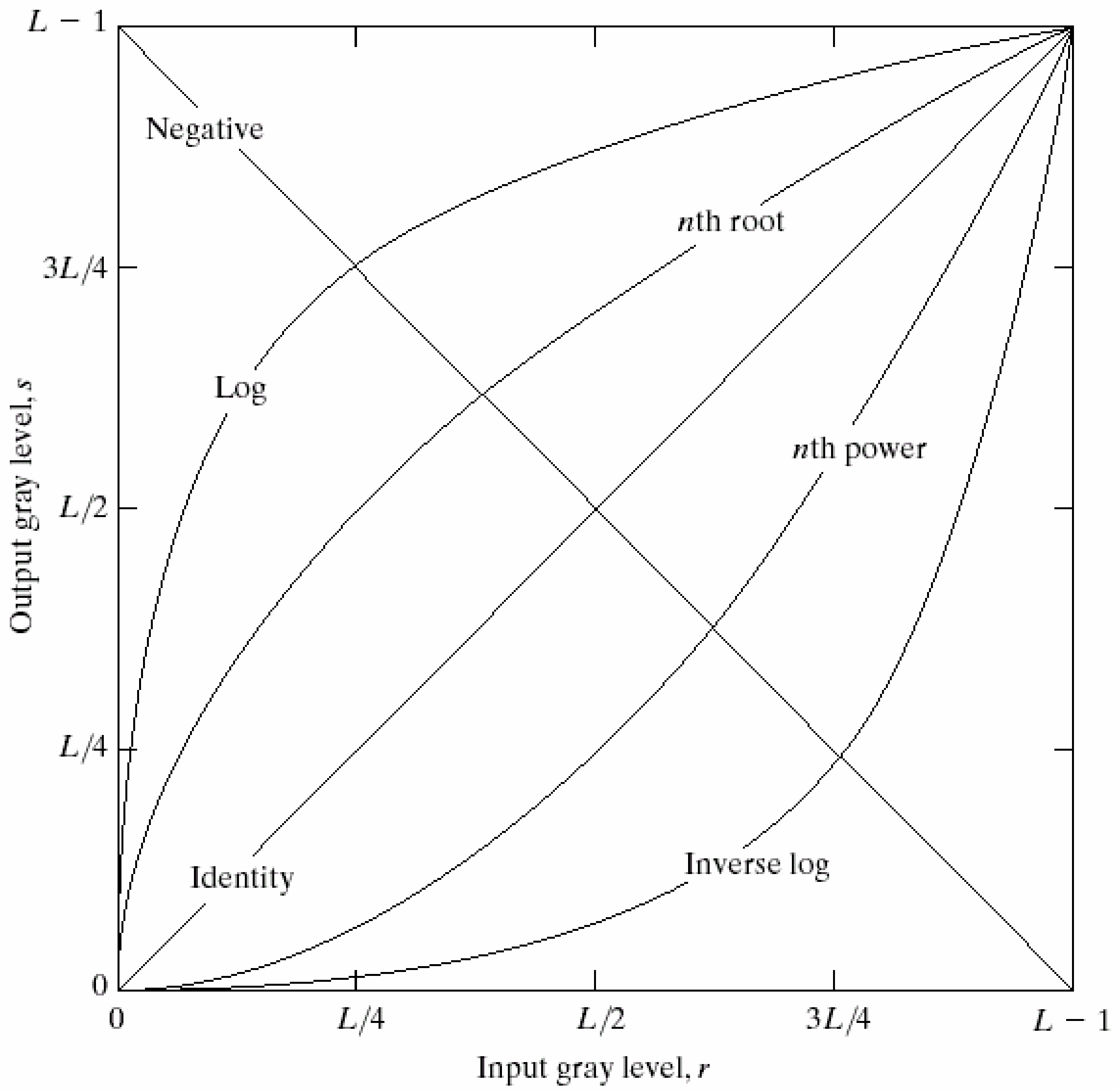
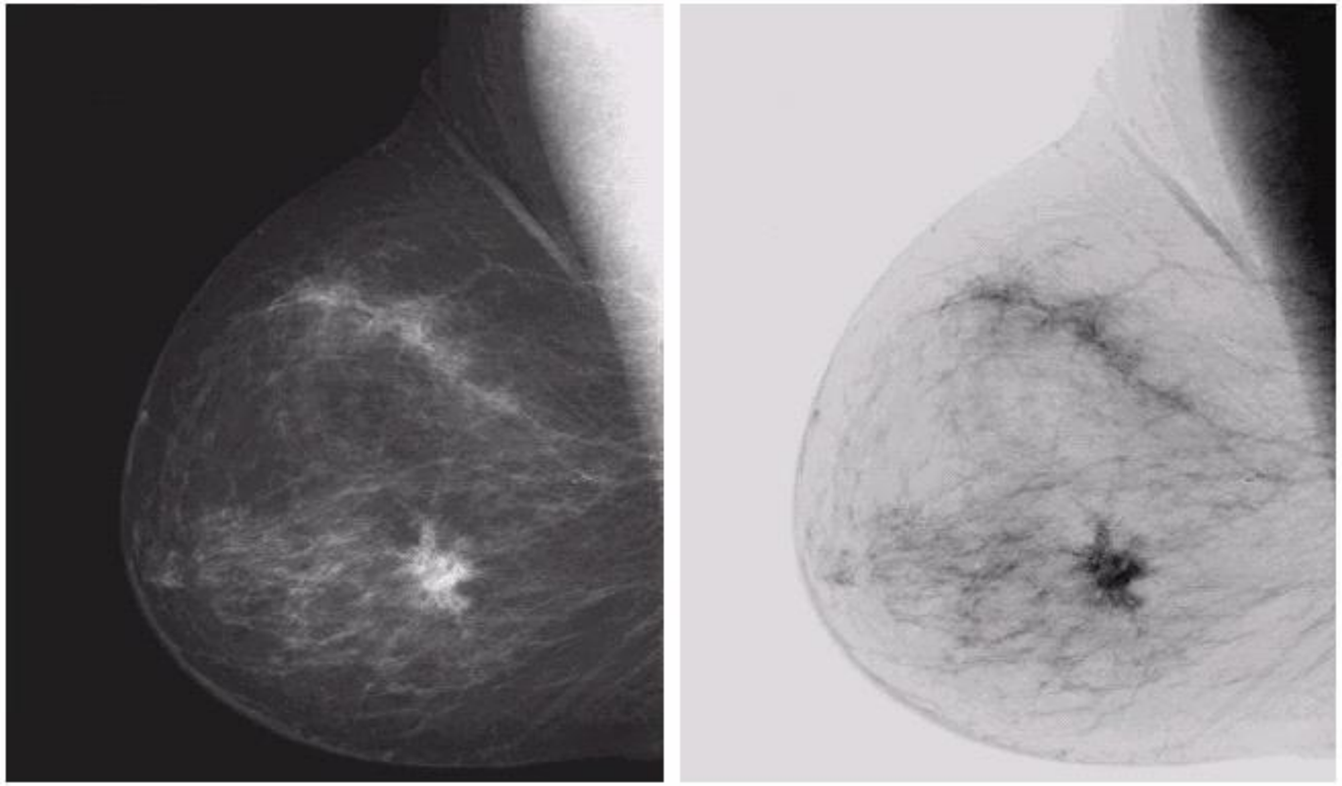


Image negatives



a b

FIGURE 3.4
(a) Original digital mammogram.
(b) Negative image obtained using the negative transformation in Eq. (3.2-1).
(Courtesy of G.E. Medical Systems.)

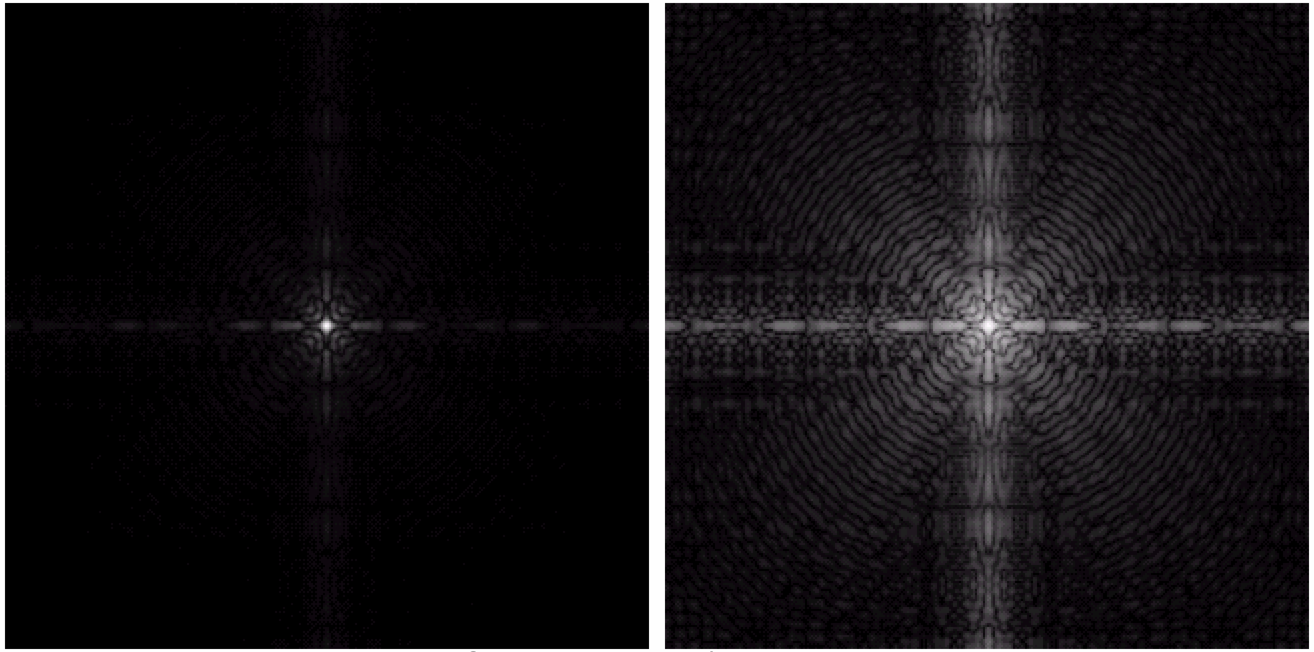
Log transforms

- Useful when we have to show images that have high dynamic range e.g. Fourier spectra of images

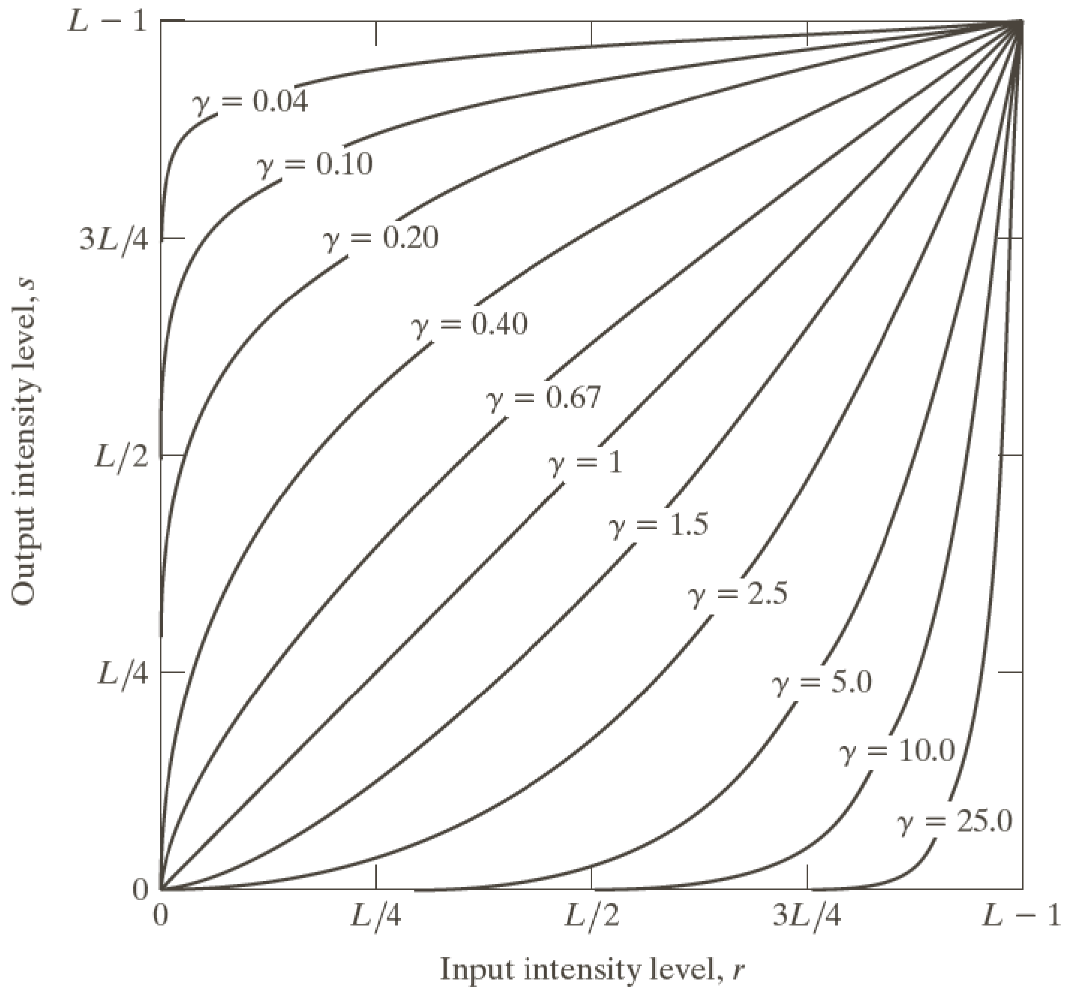
$$s = c \log(1+r)$$

a b

FIGURE 3.5
(a) Fourier spectrum.
(b) Result of applying the log transformation given in Eq. (3.2-2) with $c = 1$.



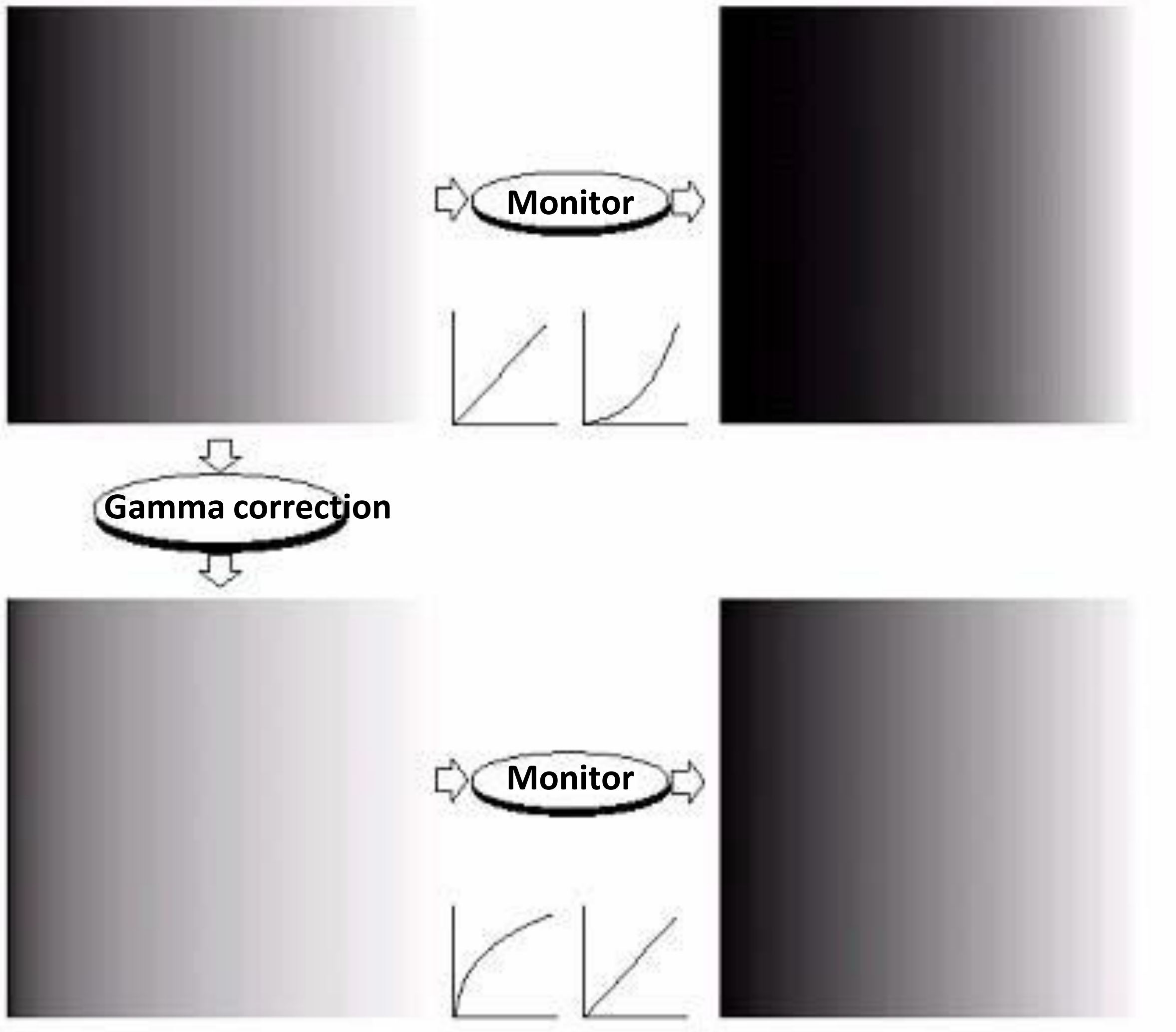
Power-Law (Gamma) Transformations



$$s = cr^{\gamma}$$

FIGURE 3.6 Plots of the equation $s = cr^{\gamma}$ for various values of γ ($c = 1$ in all cases). All curves were scaled to fit in the range shown.

Example: Gamma Transformations



$$s = r^{1/2.5}$$

a	b
c	d

FIGURE 3.7
(a) Intensity ramp image. (b) Image as viewed on a simulated monitor with a gamma of 2.5. (c) Gamma-corrected image. (d) Corrected image as viewed on the same monitor. Compare (d) and (a).

Example: Gamma Transformations

a	b
c	d

FIGURE 3.9
(a) Aerial image. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 3.0, 4.0,$ and 5.0, respectively. (Original image for this example courtesy of NASA.)

Piecewise-Linear Transformations

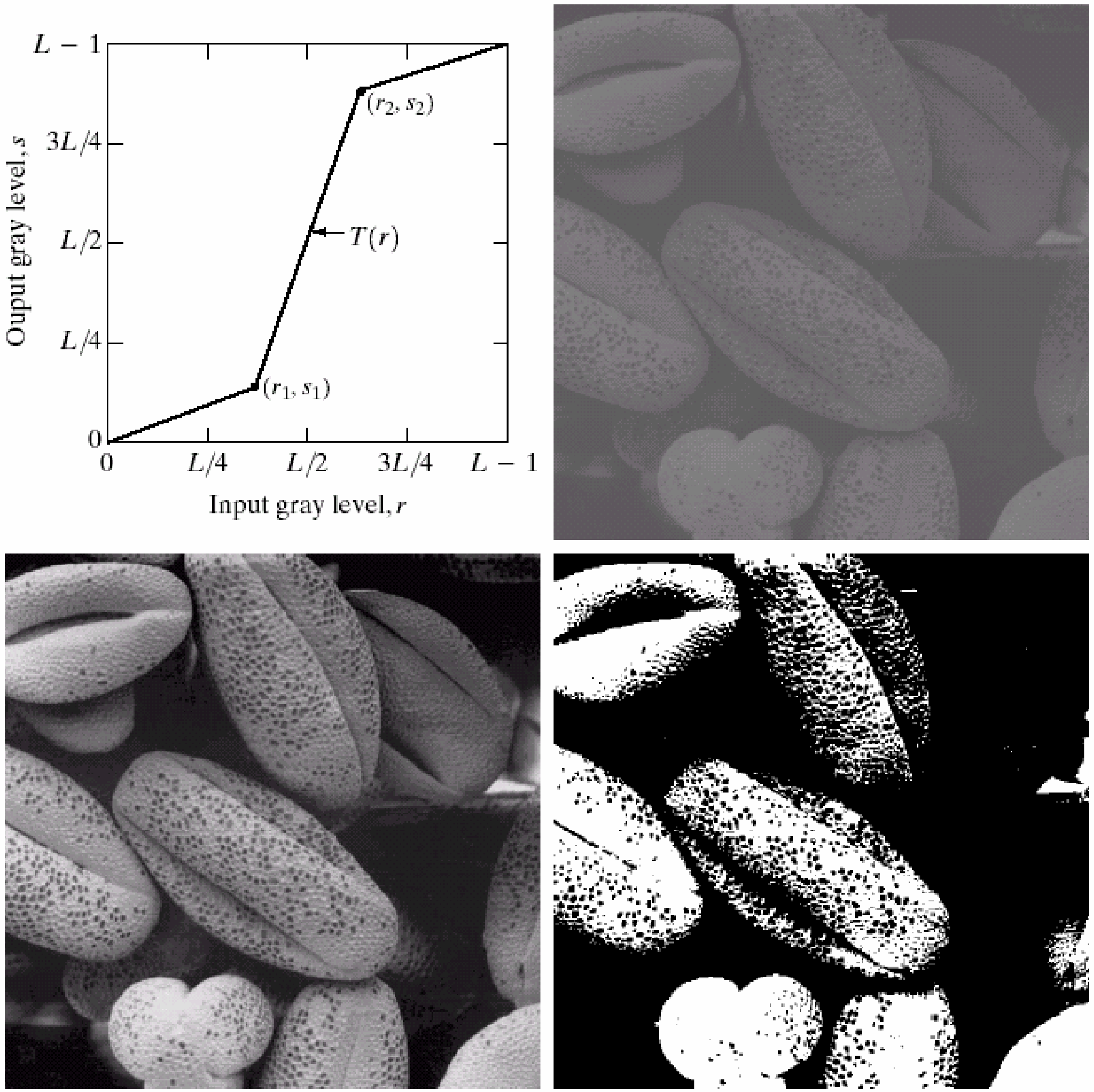
- Contrast Stretching

  — Expands the range of intensity levels in an image so that it spans the full intensity range of the recording medium or display device.

- Intensity-level Slicing

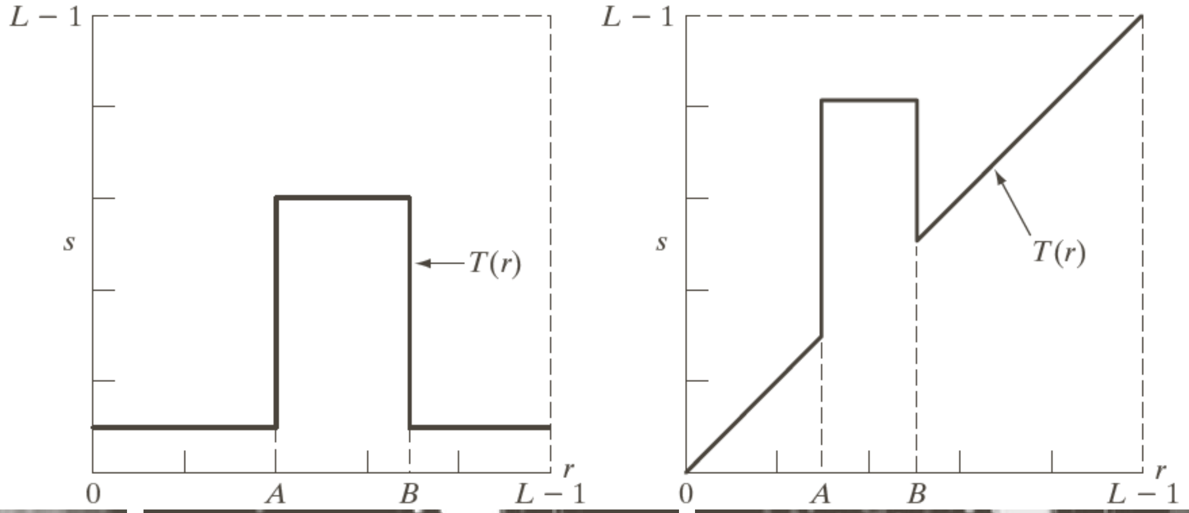
  — Highlighting a specific range of intensities in an image often is of interest.

Contrast Stretching



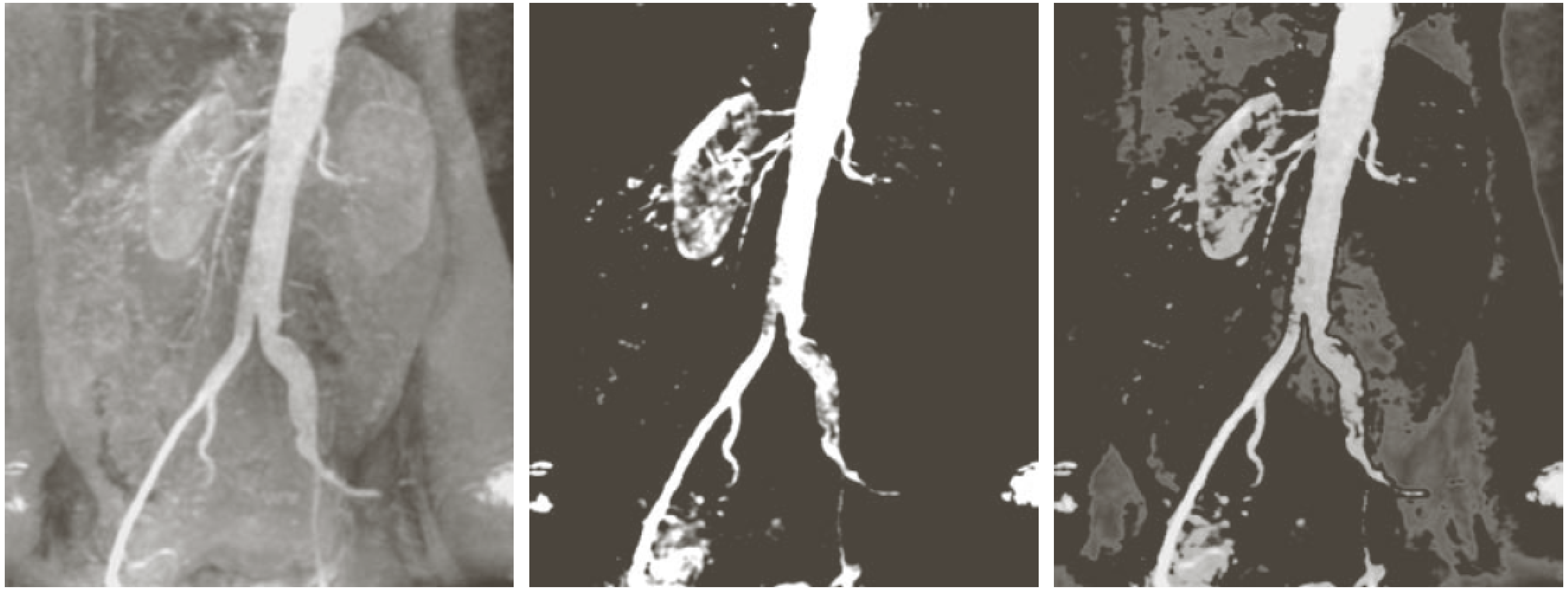
a b
c d

FIGURE 3.10 Contrast stretching. (a) Form of transformation function. (b) A low-contrast image. (c) Result of contrast stretching. (d) Result of thresholding. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)



a b

FIGURE 3.11 (a) This transformation highlights intensity range [A, B] and reduces all other intensities to a lower level. (b) This transformation highlights range [A, B] and preserves all other intensity levels.

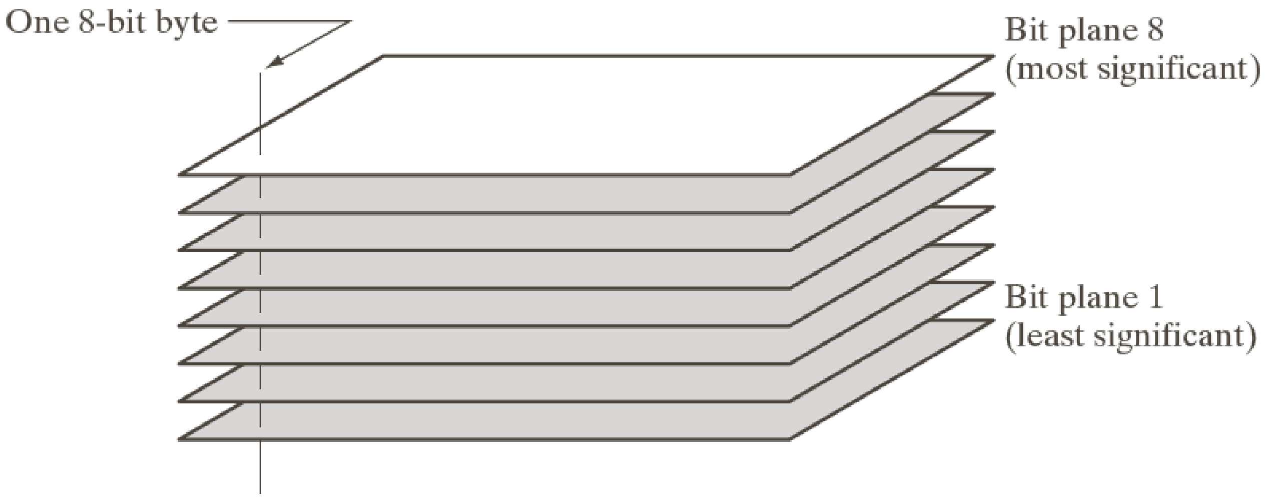


a b c

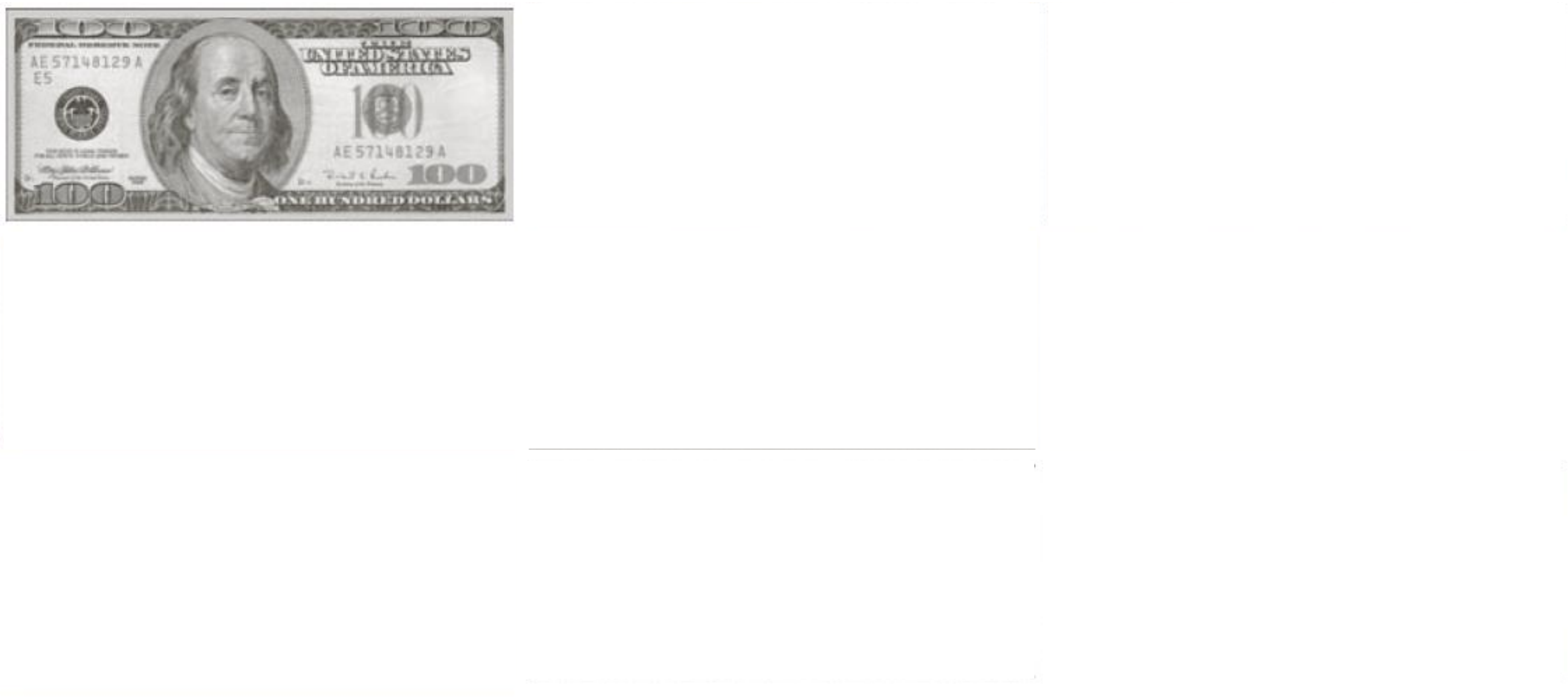
FIGURE 3.12 (a) Aortic angiogram. (b) Result of using a slicing transformation of the type illustrated in Fig. 3.11(a), with the range of intensities of interest selected in the upper end of the gray scale. (c) Result of using the transformation in Fig. 3.11(b), with the selected area set to black, so that grays in the area of the blood vessels and kidneys were preserved. (Original image courtesy of Dr. Thomas R. Gest, University of Michigan Medical School.)

Bit plane slicing

- Highlighting the contribution made to total image appearance by specific bits
- Suppose each pixel is represented by 8 bits
- Higher-order bits contain the majority of the visually significant data
- Useful for analyzing the relative importance played by each bit of the image



Bit-plane Slicing



a	b	c
d	e	f
g	h	i

FIGURE 3.14 (a) An 8-bit gray-scale image of size 500 × 1192 pixels. (b) through (i) Bit planes 1 through 8, with bit plane 1 corresponding to the least significant bit. Each bit plane is a binary image.

Bit-plane Slicing

a b c

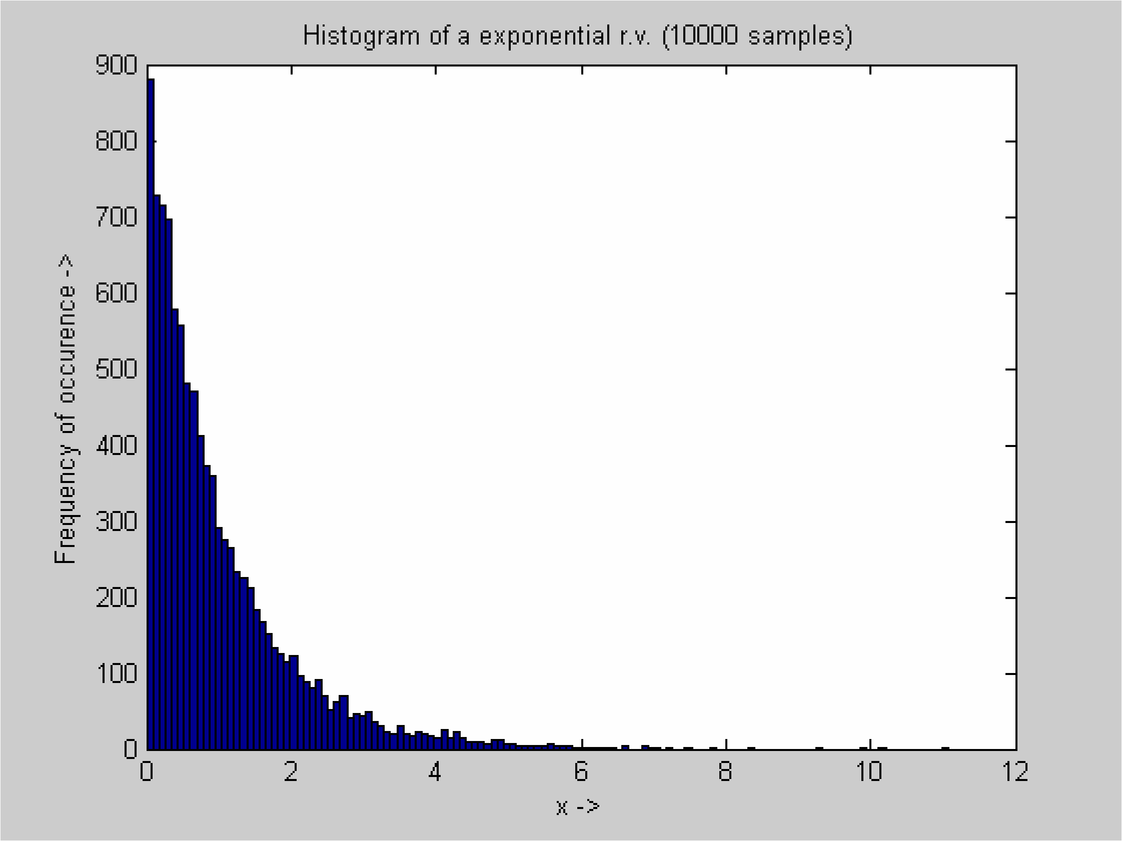
FIGURE 3.15 Images reconstructed using (a) bit planes 8 and 7; (b) bit planes 8, 7, and 6; and (c) bit planes 8, 7, 6, and 5. Compare (c) with Fig. 3.14(a).

Histogram Processing

- Histogram Equalization
- Histogram Matching
- Local Histogram Processing
- Using Histogram Statistics for Image Enhancement

Histograms

- – a practical way of estimating the pdf of a random variable



Histogram Processing

Histogram $h(r_k) = n_k$

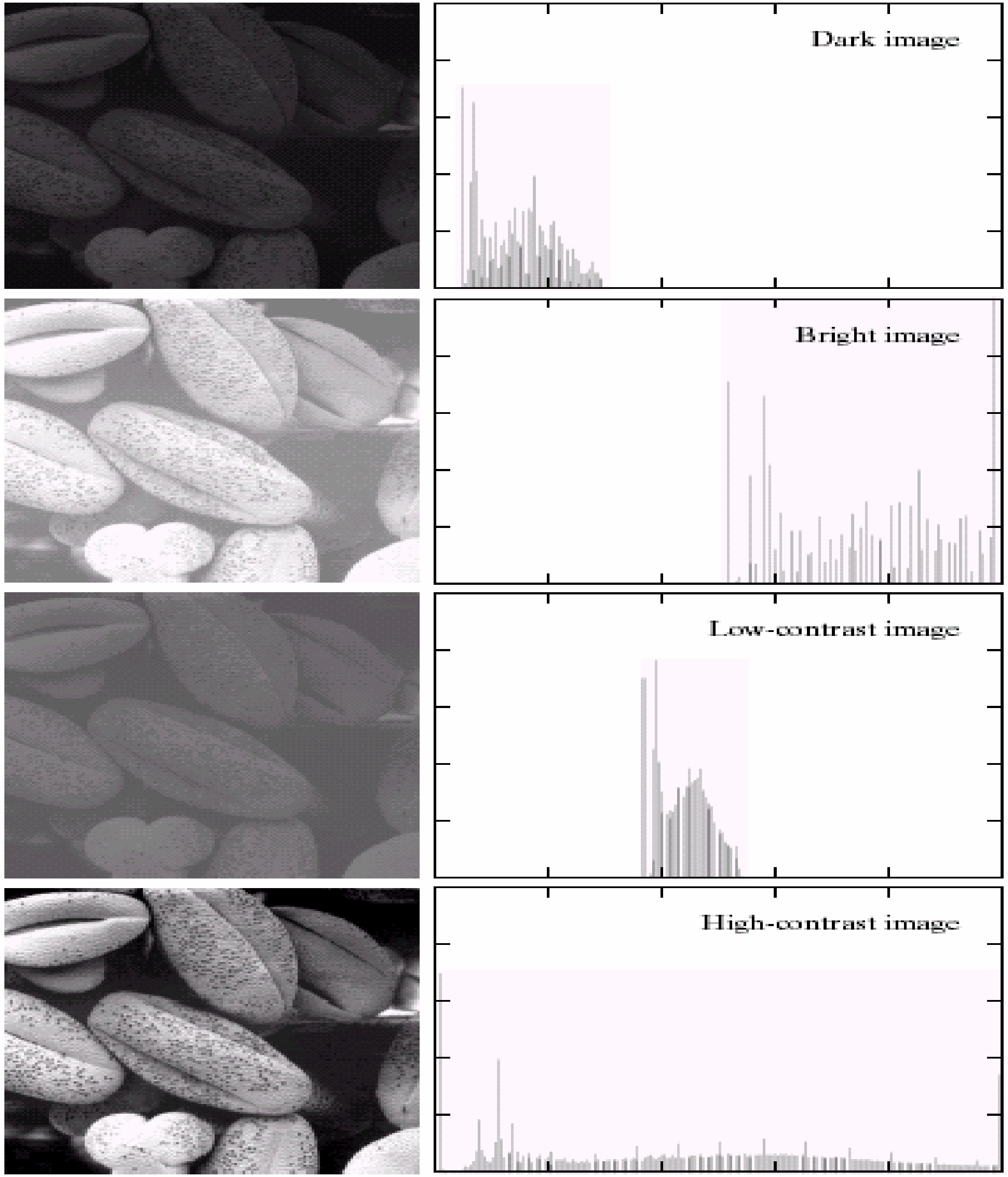
r_k is the k^{th} intensity value

n_k is the number of pixels in the image with intensity r_k

Normalized histogram $p(r_k) = \dfrac{n_k}{MN}$

n_k: the number of pixels in the image of

size $\mathrm{M} \times \mathrm{N}$ with intensity r_k

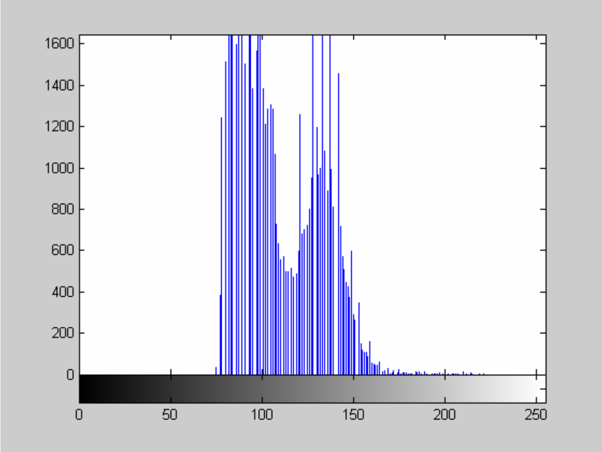


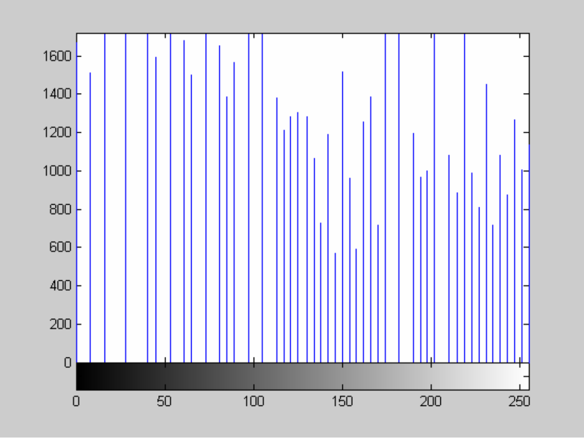
Dark image
Bright image
Low-contrast image
High-contrast image

Image enhancement using histogram equalization

Original

Enhanced





Histogram Equalization

The intensity levels in an image may be viewed as random variables in the interval [0, L-1].

Let $p_r(r)$ and $p_s(s)$ denote the probability density function (PDF) of random variables r and s.

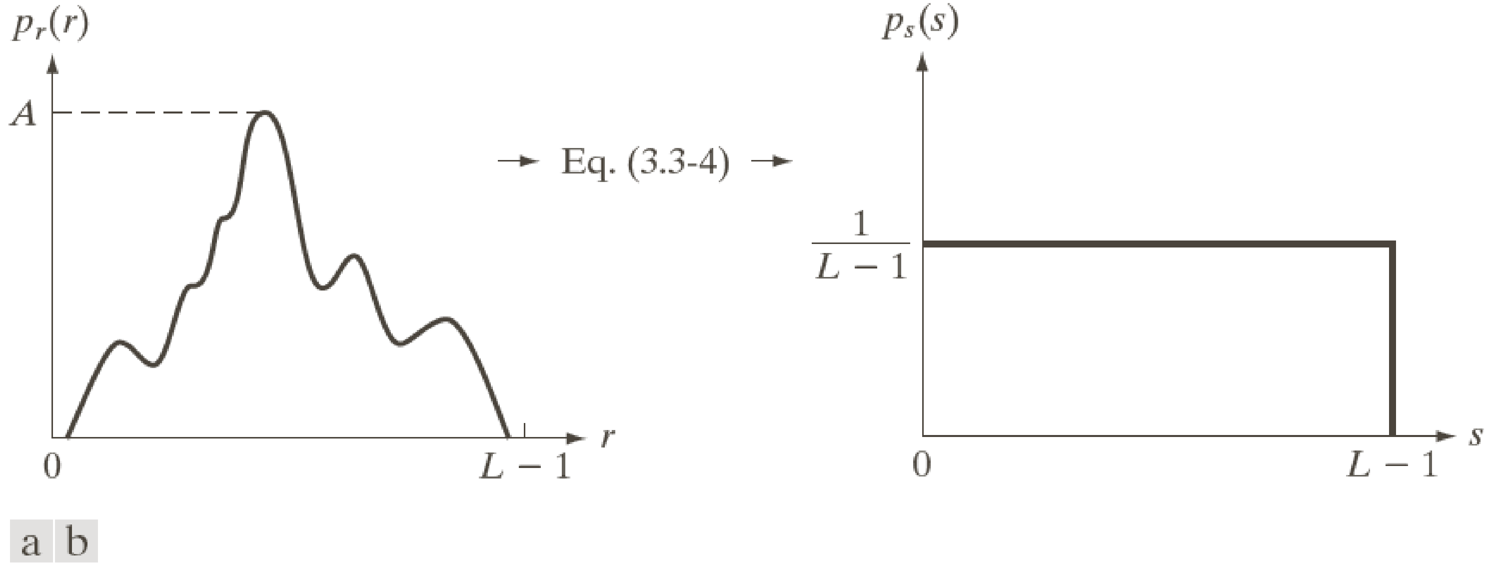


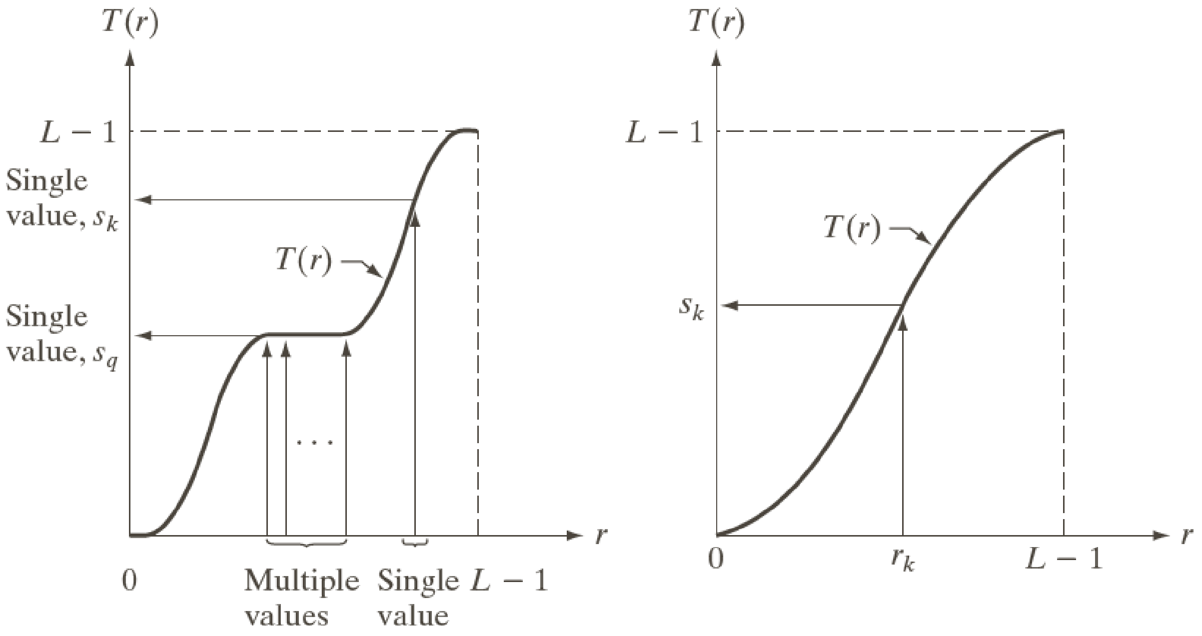
FIGURE 3.18 (a) An arbitrary PDF. (b) Result of applying the transformation in Eq. (3.3-4) to all intensity levels, r. The resulting intensities, s, have a uniform PDF, independently of the form of the PDF of the r's.

Histogram Equalization

$$s = T(r) \qquad 0 \le r \le L-1$$

$a.$ T(r) is a strictly monotonically increasing function in the interval $0 \le r \le L\text{-}1$;

$b.$ $0 \le T(r) \le L\text{-}1 \quad \text{for} \quad 0 \le r \le L\text{-}1.$



a b

FIGURE 3.17 (a) Monotonically increasing function, showing how multiple values can map to a single value. (b) Strictly monotonically increasing function. This is a one-to-one mapping, both ways.

Histogram Equalization

$$s = T(r) \qquad 0 \le r \le L-1$$

a. T(r) is a strictly monotonically increasing function
in the interval $0 \le r \le L$ -1;

b. $0 \le T(r) \le L$ -1 for $0 \le r \le L$ -1.

$T(r)$ is continuous and differentiable.

$$\boxed{p_s(s)ds = p_r(r)dr}$$

Histogram Equalization

$$s = T(r) = (L-1)\int_0^r p_r(w)\,dw$$

$$\frac{ds}{dr} = \frac{dT(r)}{dr} = (L-1)\frac{d}{dr}\left[\int_0^r p_r(w)\,dw\right]$$

$$= (L-1)\,p_r(r)$$

$$p_s(s) = \frac{p_r(r)dr}{ds} = p_r(r) \Big/ \left(\frac{ds}{dr}\right) = p_r(r) \Big/ \left((L-1)\,p_r(r)\right) = \frac{1}{L-1}$$

Histogram Equalization

Continuous case:

$$s = T(r) = (L-1)\int_0^r p_r(w)\,dw$$

Discrete values:

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j)$$

$$= (L-1)\sum_{j=0}^{k} \frac{n_j}{MN} = \frac{L-1}{MN}\sum_{j=0}^{k} n_j \qquad \text{k=0,1,..., L-1}$$

Example: Histogram Equalization

Suppose that a 3-bit image (L=8) of size 64 × 64 pixels (MN = 4096) has the intensity distribution shown in following table.
Get the histogram equalization transformation function and give the $p_s(s_k)$ for each s_k.

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

Example: Histogram Equalization

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

$$s_0 = T(r_0) = 7\sum_{j=0}^{0} p_r(r_j) = 7 \times 0.19 = 1.33 \quad \rightarrow 1$$

$$s_1 = T(r_1) = 7\sum_{j=0}^{1} p_r(r_j) = 7 \times (0.19 + 0.25) = 3.08 \quad \rightarrow 3$$

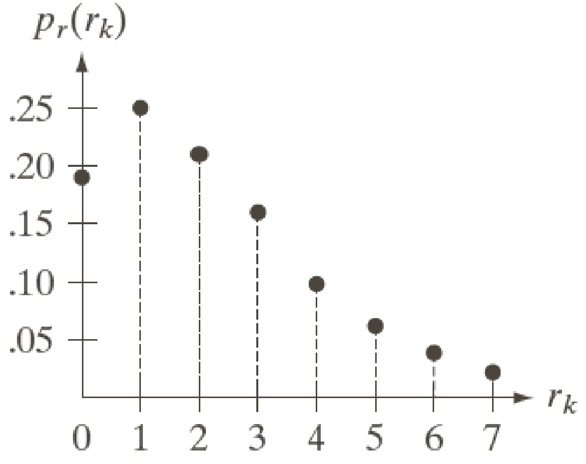
$$s_2 = 4.55 \quad \rightarrow 5 \qquad s_3 = 5.67 \quad \rightarrow 6$$

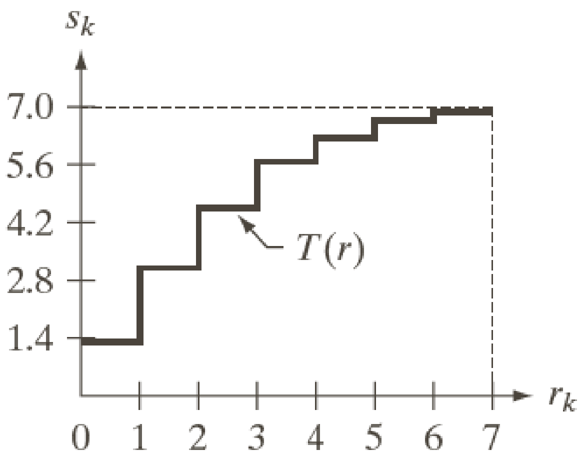
$$s_4 = 6.23 \quad \rightarrow 6 \qquad s_5 = 6.65 \quad \rightarrow 7$$

$$s_6 = 6.86 \quad \rightarrow 7 \qquad s_7 = 7.00 \quad \rightarrow 7$$

Example: Histogram Equalization

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

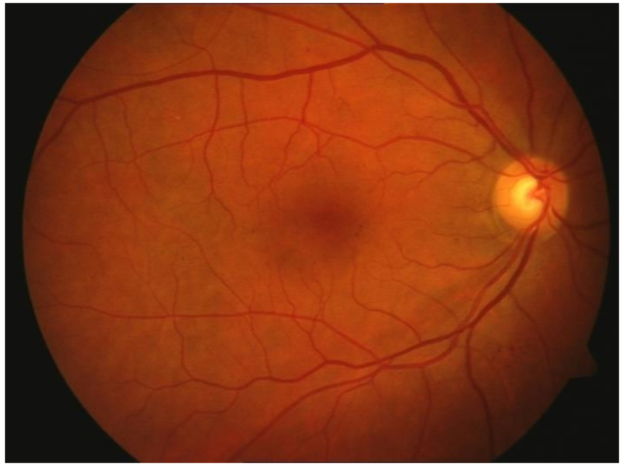




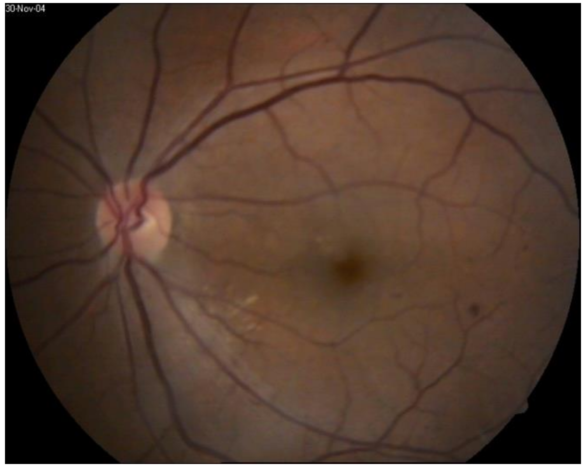
a b c

FIGURE 3.19 Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

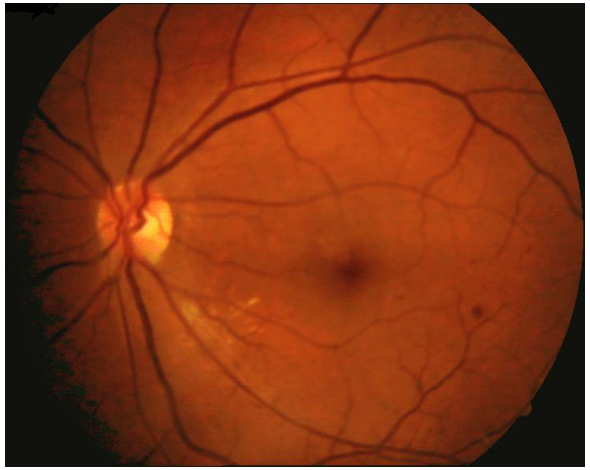
Histogram Matching



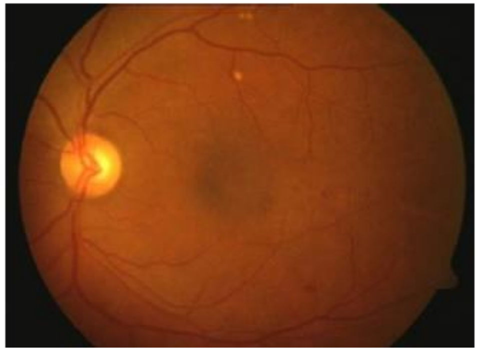
Standard image



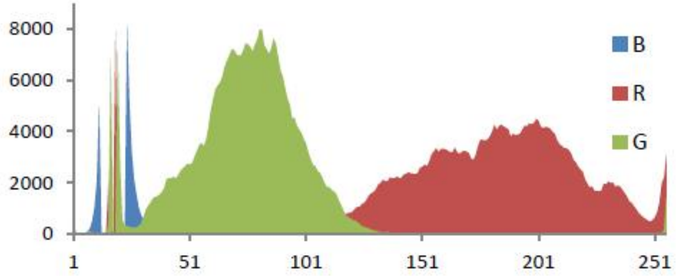
Input image

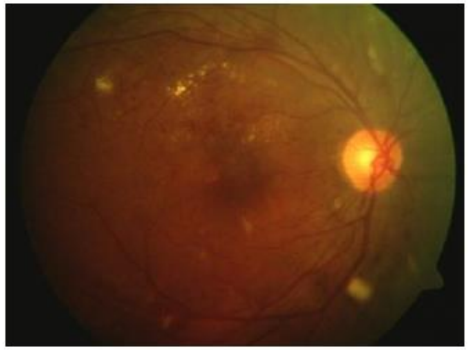


Colour normalized image

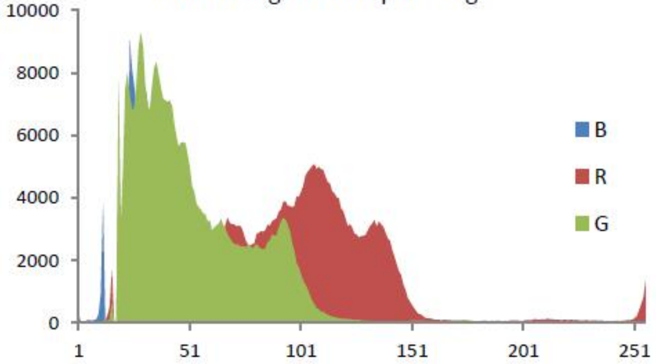


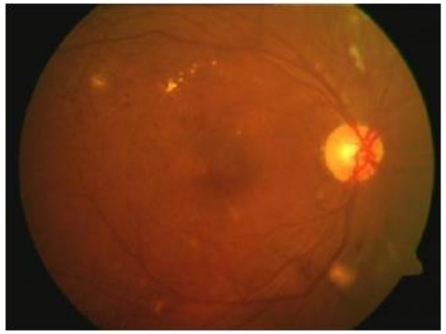
RGB Histogram of reference image



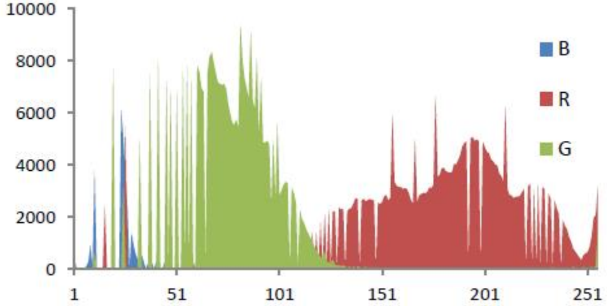


RGB Histogram of input image





RGB Histogram ofcolour normalized image



Histogram Matching

Histogram matching (histogram specification)

— generate a processed image that has a specified histogram

Let $p_r(r)$ and $p_z(z)$ denote the continous probability density functions of the variables r and z. $p_z(z)$ is the specified probability density function.

Let s be the random variable with the probability

$$s = T(r) = (L-1)\int_0^r p_r(w)dw$$

Define a random variable z with the probability

$$G(z) = (L-1)\int_0^z p_z(t)dt = s$$

Histogram Matching

$$s = T(r) = (L-1)\int_0^r p_r(w)dw$$

$$G(z) = (L-1)\int_0^z p_z(t)dt = s$$

$$z = G^{-1}(s) = G^{-1}\left[T(r)\right]$$

Histogram Matching: Discrete Cases

- Obtain $p_r(r_j)$ from the input image and then obtain the values of s_k, round the value to the integer range [0, L-1].

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) = \frac{(L-1)}{MN}\sum_{j=0}^{k} n_j$$

- Use the specified PDF and obtain the transformation function $G(z_q)$, round the value to the integer range [0, L-1].

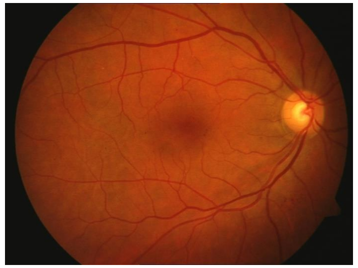
$$G(z_q) = (L-1)\sum_{i=0}^{q} p_z(z_i) = s_k$$

- Mapping from s_k to z_q

$$z_q = G^{-1}(s_k)$$

Example: Histogram Matching

Suppose that a 3-bit image (L=8) of size 64 × 64 pixels (MN = 4096) has the intensity distribution shown in the following table (on the left). Get the histogram transformation function and make the output image with the specified histogram, listed in the table on the right.



r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

z_q	Specified $p_z(z_q)$
$z_0 = 0$	0.00
$z_1 = 1$	0.00
$z_2 = 2$	0.00
$z_3 = 3$	0.15
$z_4 = 4$	0.20
$z_5 = 5$	0.30
$z_6 = 6$	0.20
$z_7 = 7$	0.15

Example: Histogram Matching

Obtain the scaled histogram-equalized values,

$$s_0 = 1, s_1 = 3, s_2 = 5, s_3 = 6, s_4 = 7,$$
$$s_5 = 7, s_6 = 7, s_7 = 7.$$

Compute all the values of the transformation function G,

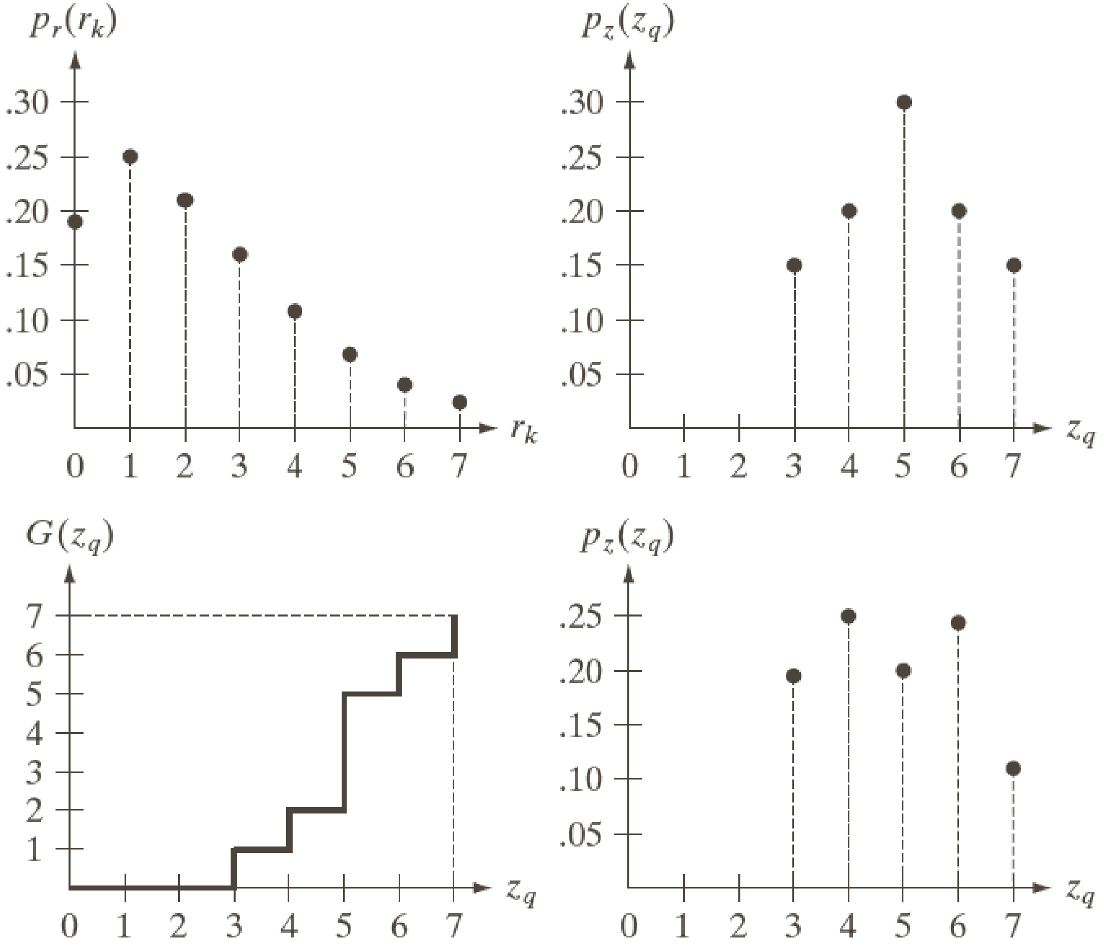
$$G(z_0) = 7\sum_{j=0}^{0} p_z(z_j) = 0.00 \rightarrow 0$$

$$G(z_1) = 0.00 \rightarrow 0 \qquad G(z_2) = 0.00 \rightarrow 0$$
$$G(z_3) = 1.05 \rightarrow 1 \qquad G(z_4) = 2.45 \rightarrow 2$$
$$G(z_5) = 4.55 \rightarrow 5 \qquad G(z_6) = 5.95 \rightarrow 6$$
$$G(z_7) = 7.00 \rightarrow 7$$

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

z_q	Specified $p_z(z_q)$
$z_0 = 0$	0.00
$z_1 = 1$	0.00
$z_2 = 2$	0.00
$z_3 = 3$	0.15
$z_4 = 4$	0.20
$z_5 = 5$	0.30
$z_6 = 6$	0.20
$z_7 = 7$	0.15

Example: Histogram Matching



Example: Histogram Matching

Obtain the scaled histogram-equalized values,

$$s_0 = 1, s_1 = 3, s_2 = 5, s_3 = 6, s_4 = 7,$$
$$s_5 = 7, s_6 = 7, s_7 = 7.$$

Compute all the values of the transformation function G,

$$G(z_0) = 7\sum_{j=0}^{0} p_z(z_j) = 0.00 \rightarrow 0$$

$$G(z_1) = 0.00 \rightarrow 0 \qquad G(z_2) = 0.00 \rightarrow 0$$
$$G(z_3) = 1.05 \rightarrow 1 \quad \mathbf{s_0} \qquad G(z_4) = 2.45 \rightarrow 2 \quad \mathbf{s_1}$$
$$G(z_5) = 4.55 \rightarrow 5 \quad \mathbf{s_2} \qquad G(z_6) = 5.95 \rightarrow 6 \quad \mathbf{s_3}$$
$$G(z_7) = 7.00 \rightarrow 7 \quad \mathbf{s_4} \quad \mathbf{s_5} \quad \mathbf{s_6} \quad \mathbf{s_7}$$

Example: Histogram Matching

$$s_0 = 1, s_1 = 3, s_2 = 5, s_3 = 6, s_4 = 7,$$
$$s_5 = 7, s_6 = 7, s_7 = 7.$$

r_k

0
1
2
3
4
5
6
7

s_k	$\rightarrow$	z_q
1	$\rightarrow$	3
3	$\rightarrow$	4
5	$\rightarrow$	5
6	$\rightarrow$	6
7	$\rightarrow$	7

$r_k \rightarrow z_q$

$0 \rightarrow 3$

$1 \rightarrow 4$

$2 \rightarrow 5$

$3 \rightarrow 6$

$4 \rightarrow 7$

$5 \rightarrow 7$

$6 \rightarrow 7$

$7 \rightarrow 7$

Local Histogram Processing

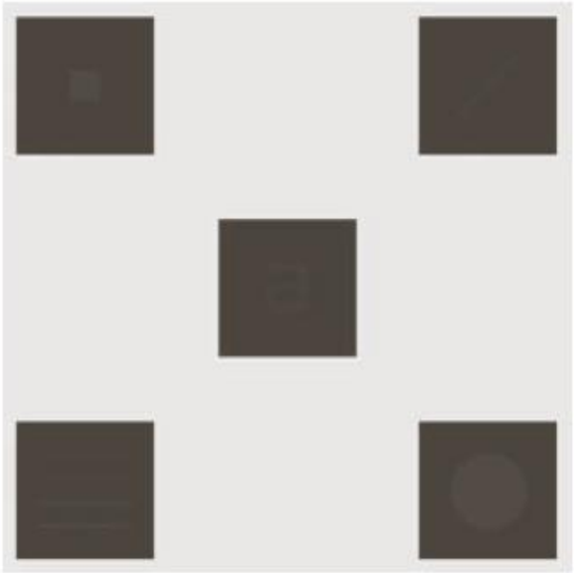
Define a neighborhood and move its center from pixel to pixel

At each location, the histogram of the points in the neighborhood is computed. Either histogram equalization or histogram specification transformation function is obtained

Map the intensity of the pixel centered in the neighborhood

Move to the next location and repeat the procedure

Local Histogram Processing: Example



a b c

FIGURE 3.26 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization applied to (a), using a neighborhood of size 3 × 3.

Using Histogram Statistics for Image Enhancement

Average Intensity

$$m = \sum_{i=0}^{L-1} r_i p(r_i) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y)$$

Variance

$$\sigma^2 = u_2(r) = \sum_{i=0}^{L-1} (r_i - m)^2 p(r_i) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [f(x, y) - m]^2$$

Using Histogram Statistics for Image Enhancement

Local average intensity

$$m_{s_{xy}} = \sum_{i=0}^{L-1} r_i \, p_{s_{xy}}(r_i)$$

s_{xy} denotes a neighborhood

Local variance

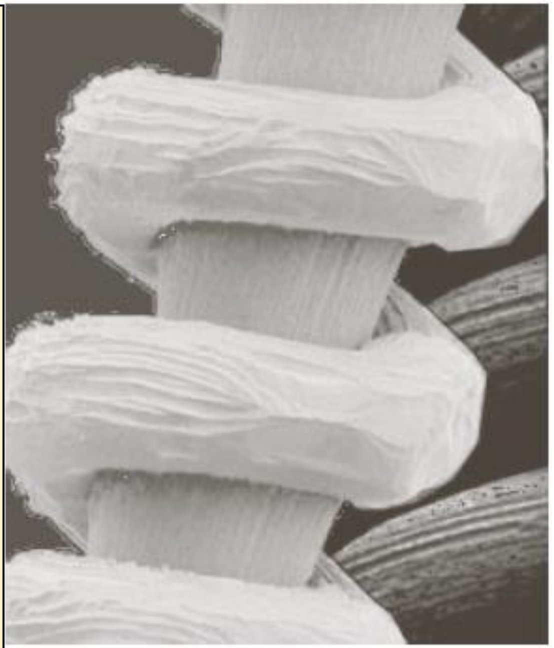
$$\sigma_{s_{xy}}^2 = \sum_{i=0}^{L-1} (r_i - m_{s_{xy}})^2 \, p_{s_{xy}}(r_i)$$

Using Histogram Statistics for Image Enhancement: Example

$$g(x,y)=\begin{cases} E\cdot f(x,y), & \text{if } m_{s_{xy}} \le k_0 m_G \text{ and } k_1\sigma_G \le \sigma_{s_{xy}} \le k_2\sigma_G \\ f(x,y), & \text{otherwise} \end{cases}$$

m_G : global mean; σ_G : global standard deviation

$k_0 = 0.4;\quad k_1 = 0.02;\; k_2 = 0.4;\; E = 4$



a b c

FIGURE 3.27 (a) SEM image of a tungsten filament magnified approximately 130×. (b) Result of global histogram equalization. (c) Image enhanced using local histogram statistics. (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)