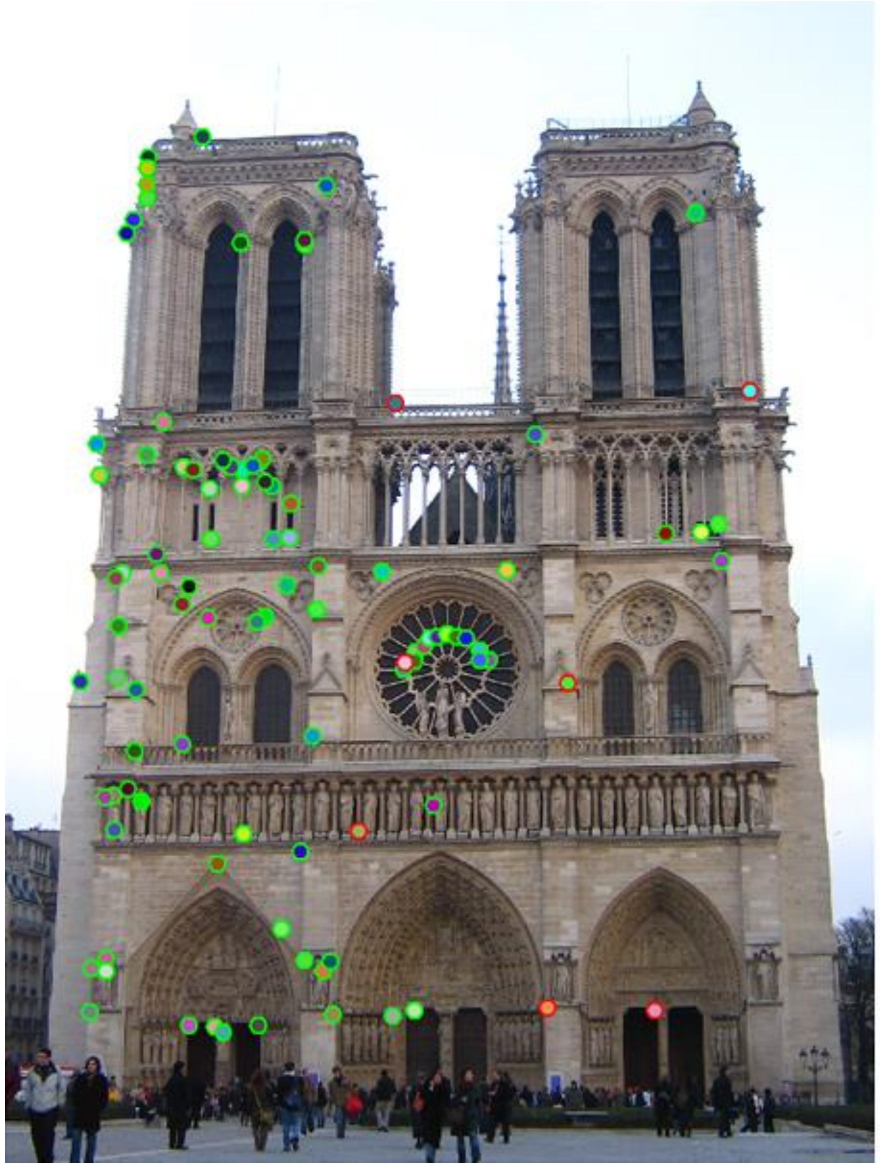
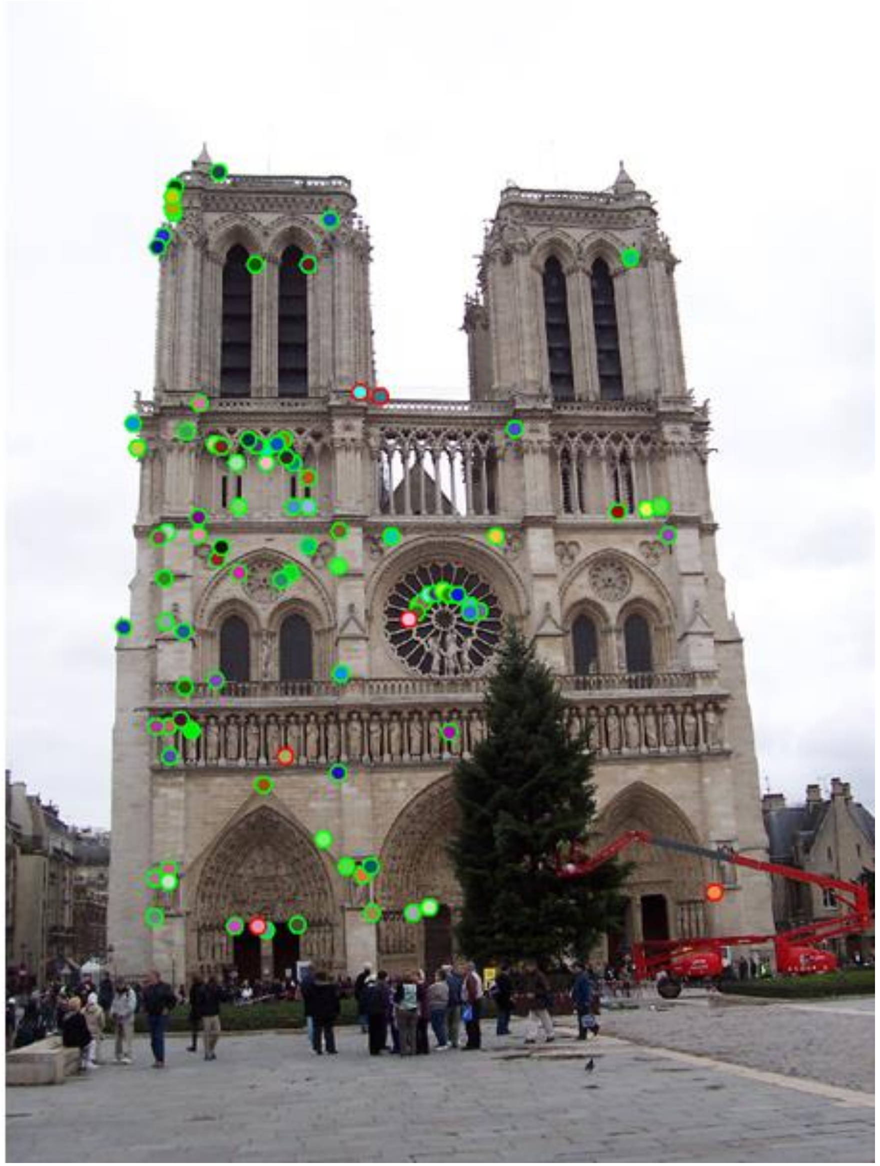
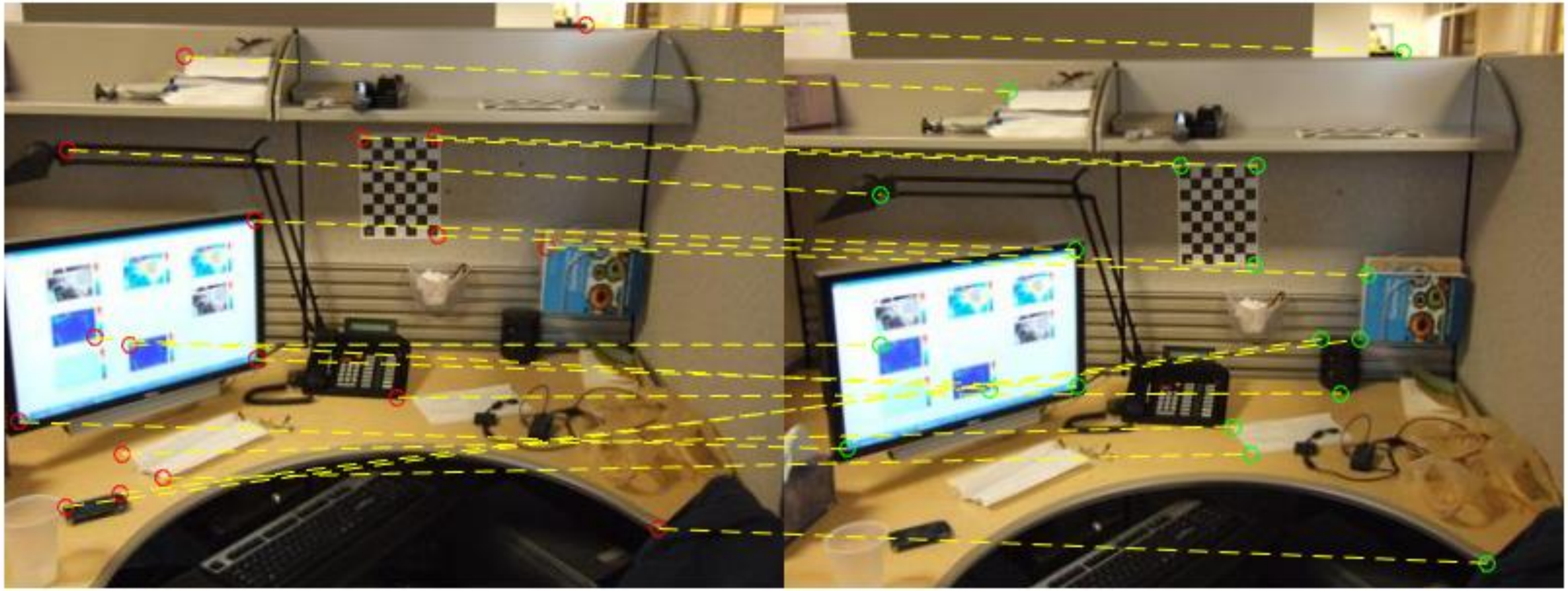
RANSAC
Matching Points

Given matches, what Next?

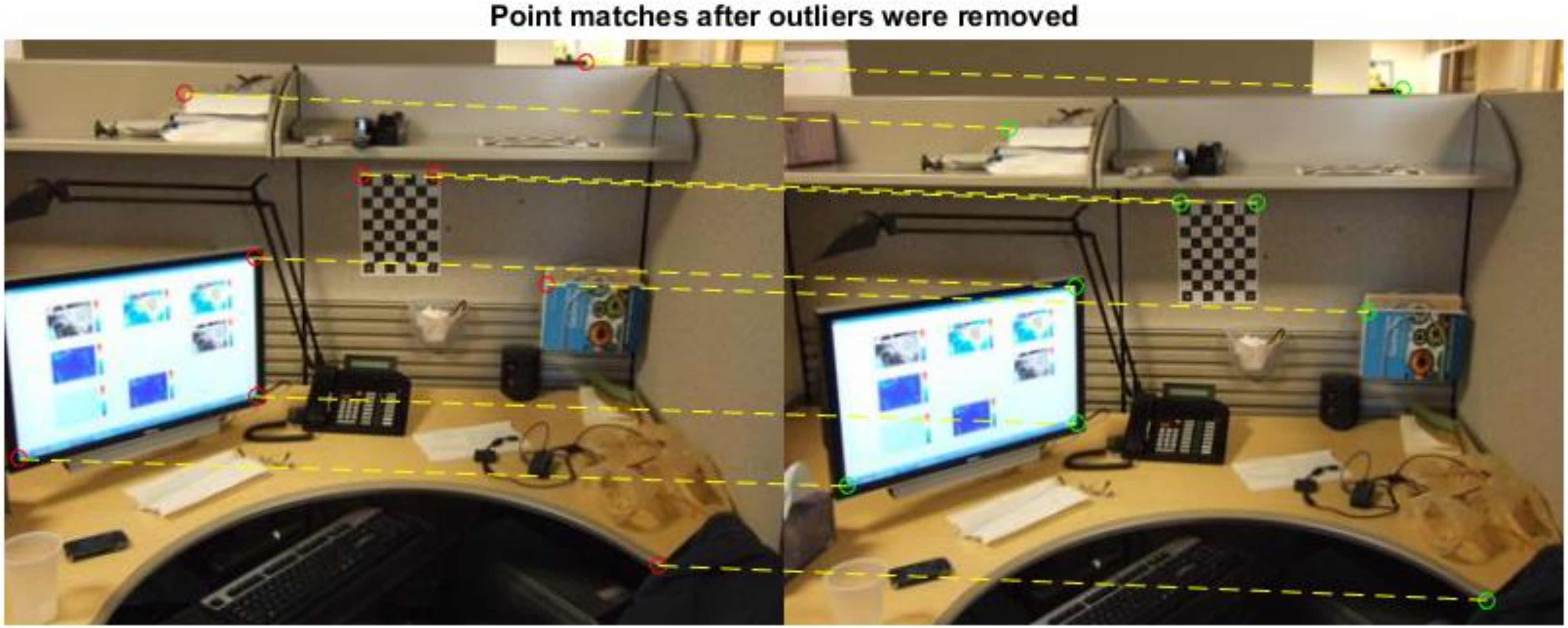




Stereo images with outliers



Stereo images without outliers



Fitting and Alignment

Fitting:

Find the parameters of a model that best fit the data.

Alignment:

Find the parameters of the transformation that best aligns matched points.

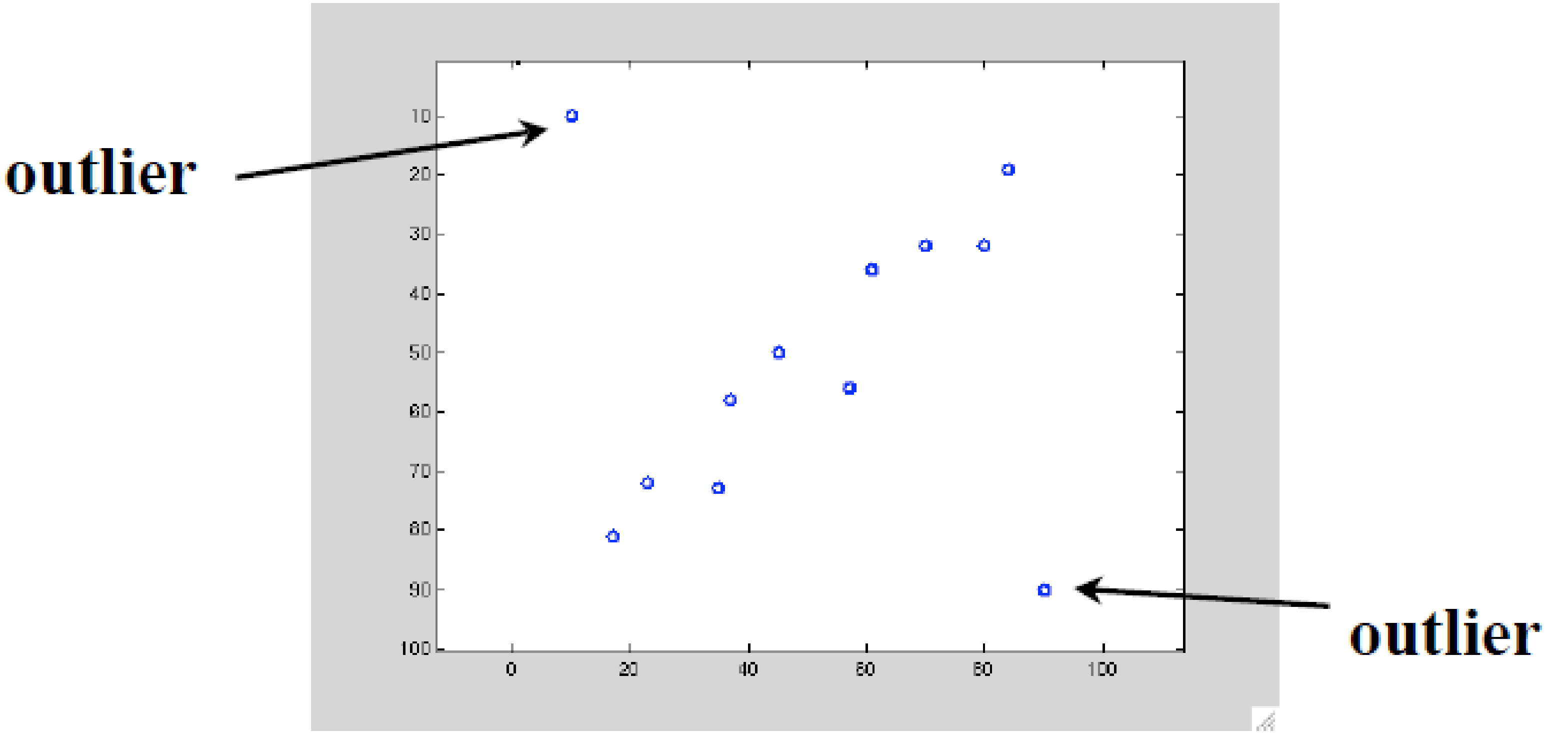
General Strategy:

- Least-Squares estimation from point correspondences

But there are problems with that approach….

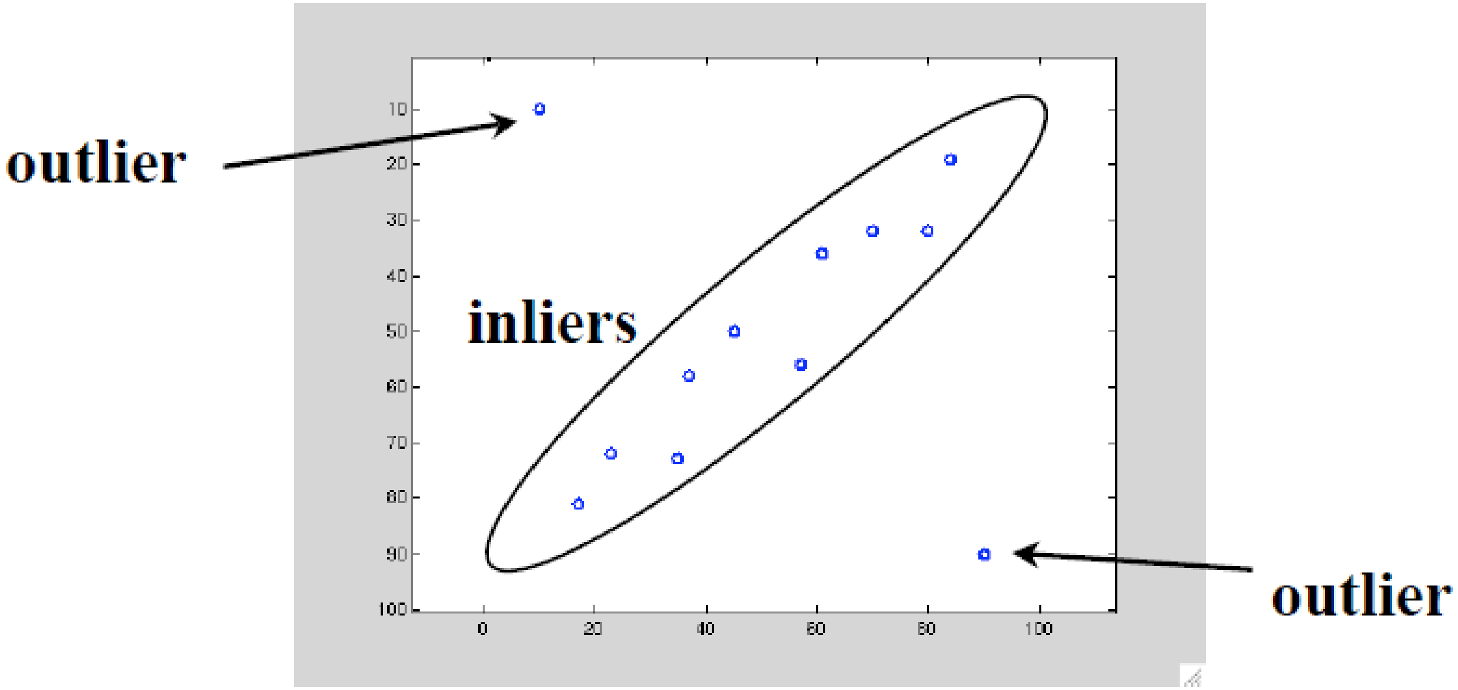
Problem : Outliers

- Loosely speaking, outliers are points that don't "fit" the model.



Bad Data => Outliers

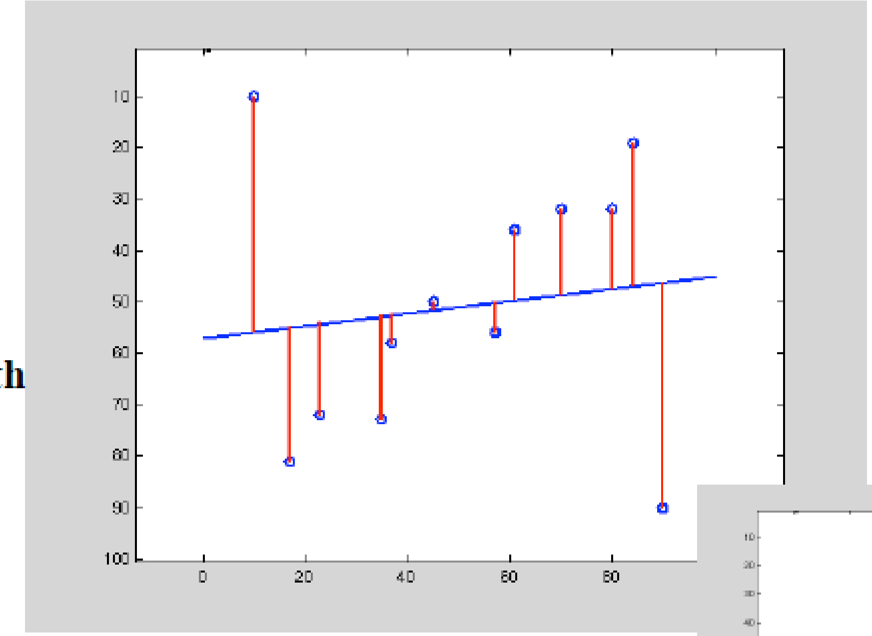
- Loosely speaking, outliers are points that don't "fit" the model. Points that do fit are called "inliers"



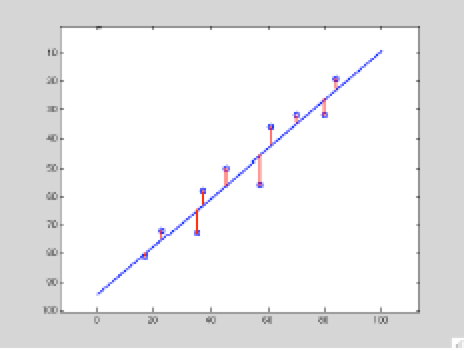
Problem with Outliers

- Least squares estimation is sensitive to outliers, so that a few outliers can greatly skew the result.

Least squares regression with outliers



compare



Solution: Estimation methods that are robust to outliers.

Robust Estimation

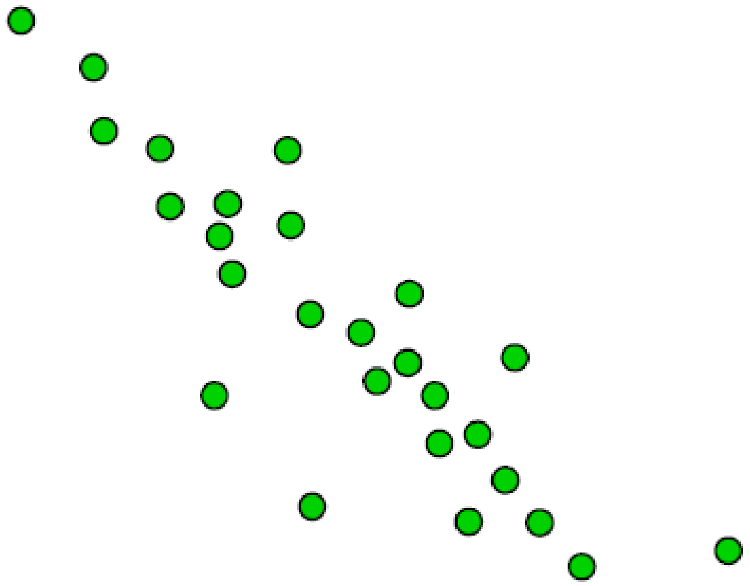
- View estimation as a two-stage process:
  - Classify data points as outliers or inliers
  - Fit model to inliers while ignoring outliers

M. A. Fischler and R. C. Bolles (June 1981). "Random Sample Consensus: A Paradigm for Model Fitting with Applications to Image Analysis and Automated Cartography". *Comm. of the ACM* 24: 381--395.

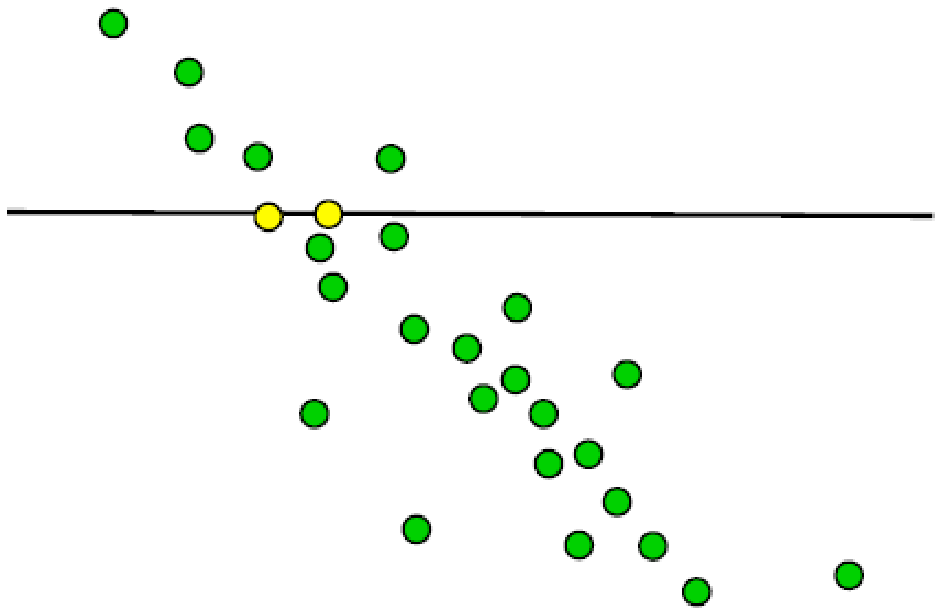
RANSAC

- RANdom SAmple Consensus
- Approach: we want to avoid the impact of outliers, so let's look for "inliers", and use only those.
- Intuition: if an outlier is chosen to compute the current fit, then the resul.ng line won't have much support from rest of the points.

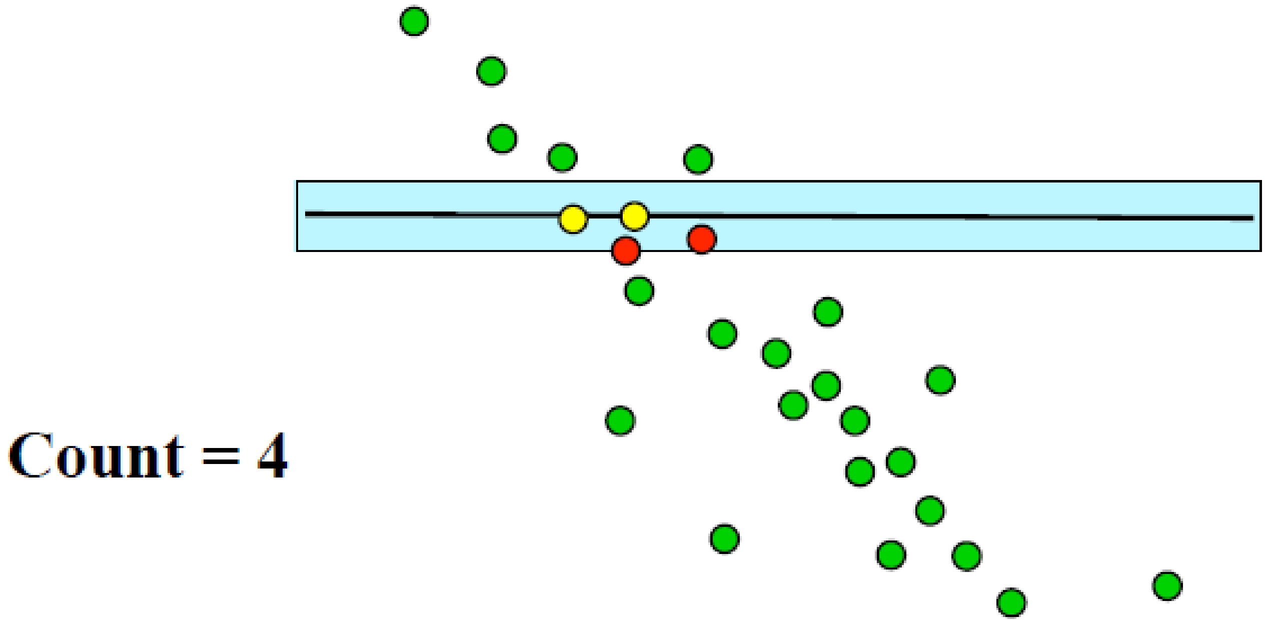
RANSAC Procedure



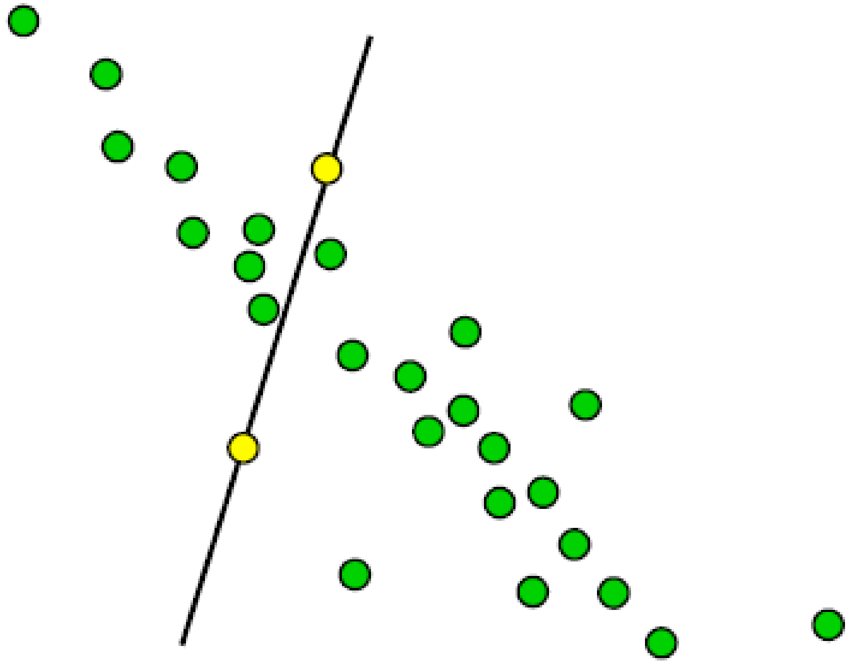
RANSAC Procedure



RANSAC Procedure



RANSAC Procedure



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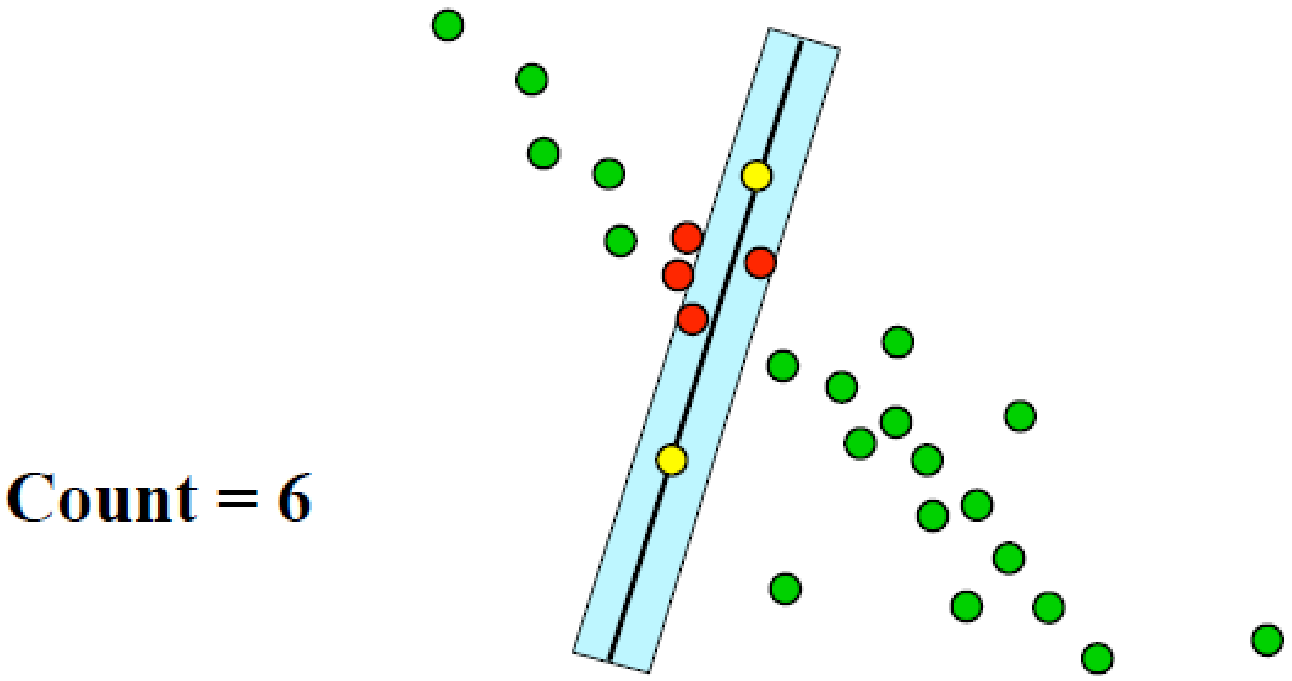

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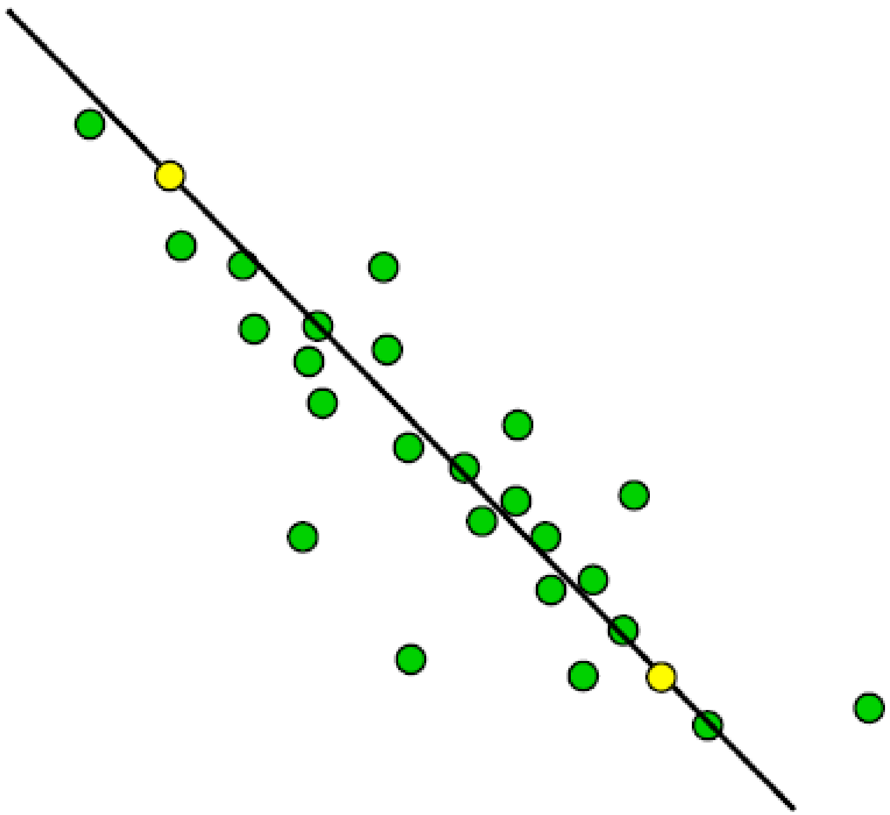
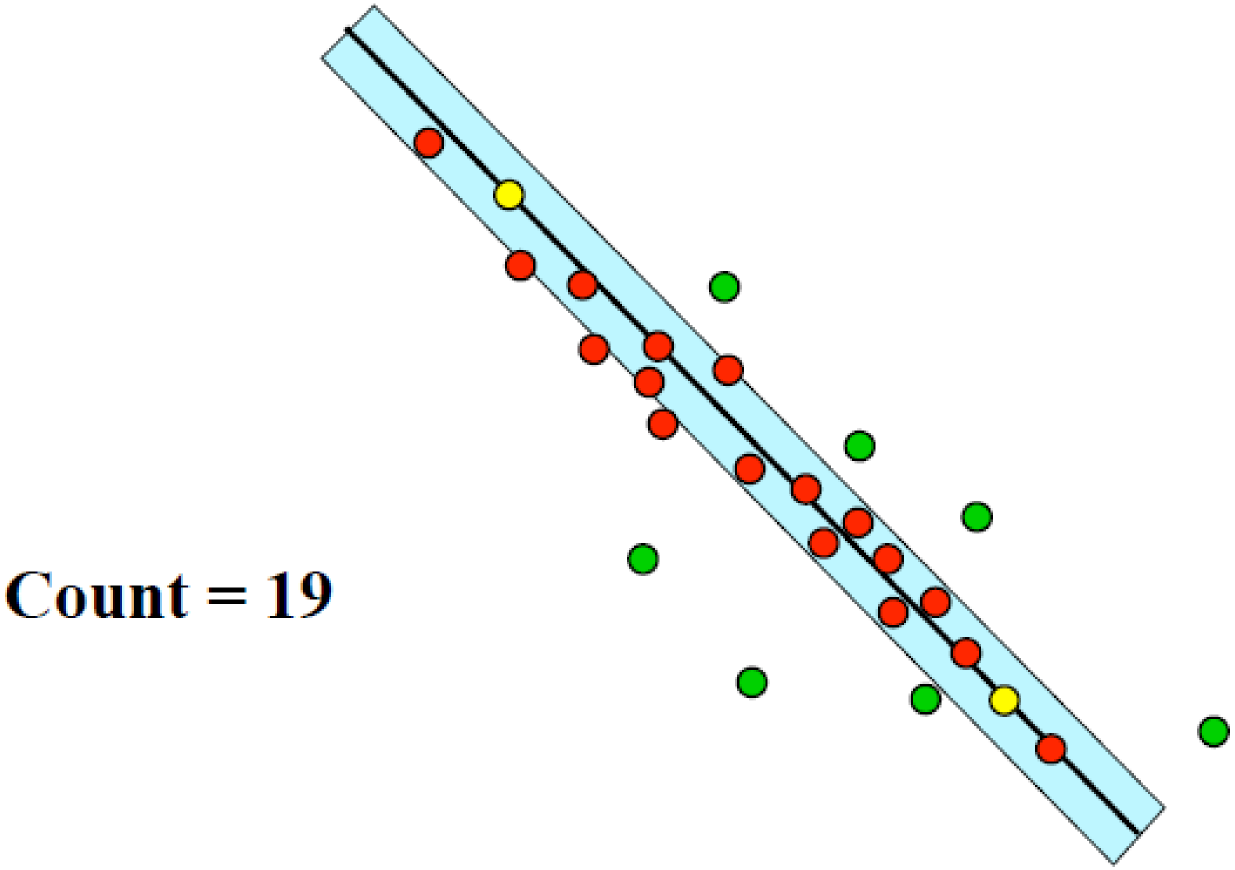
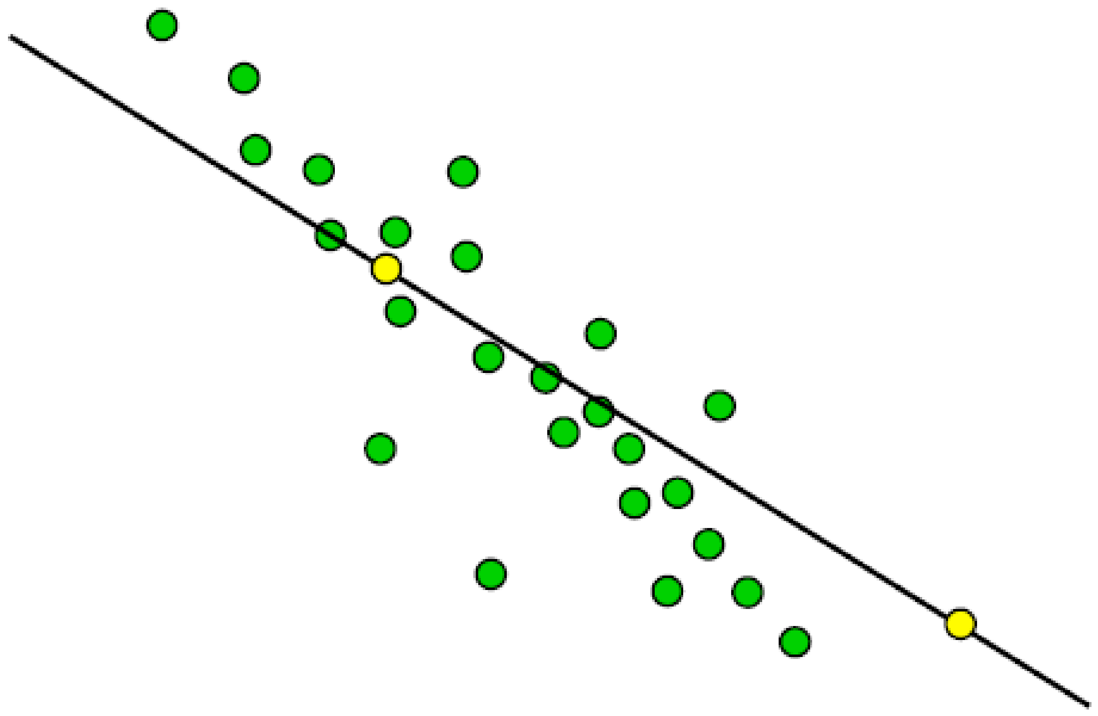


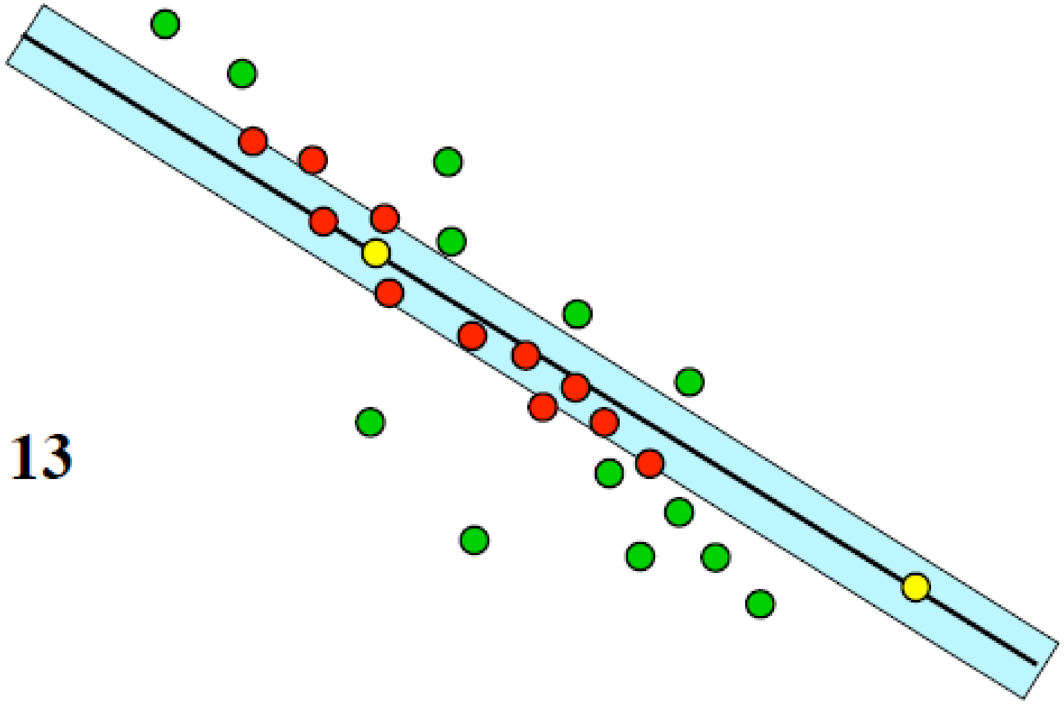
RANSAC Procedure



RANSAC Procedure

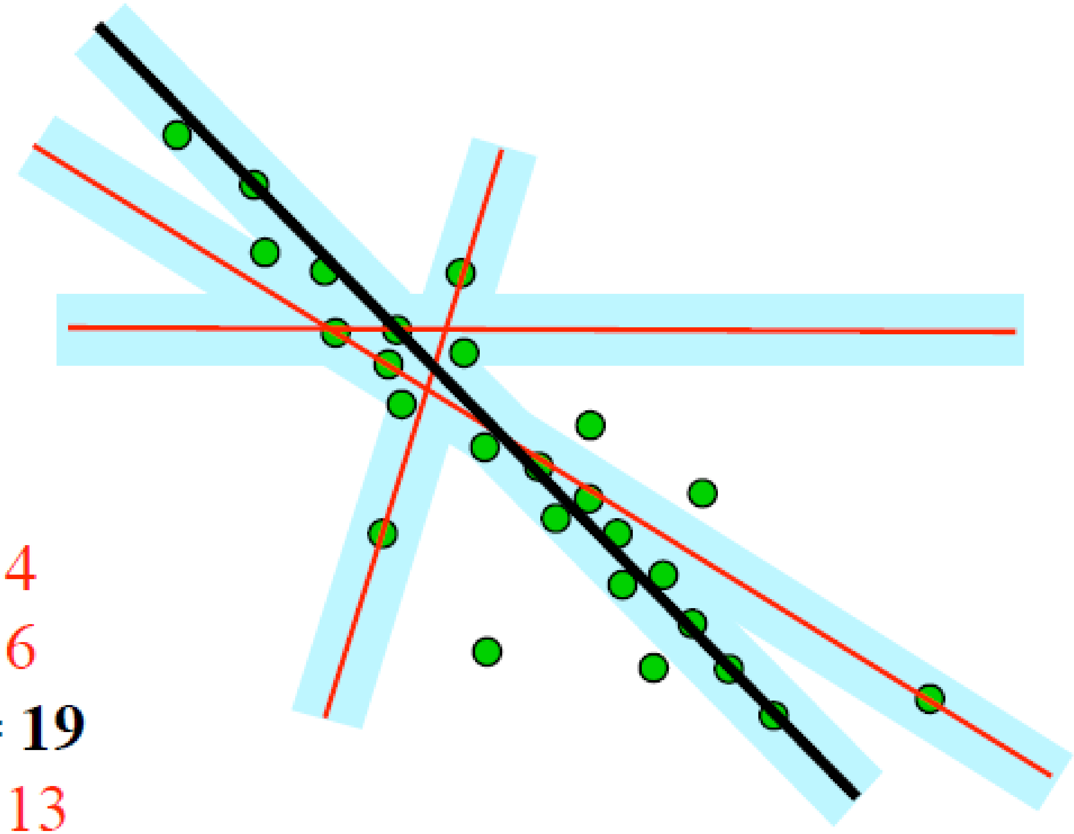


RANSAC Procedure



Count = 13

RANSAC Procedure

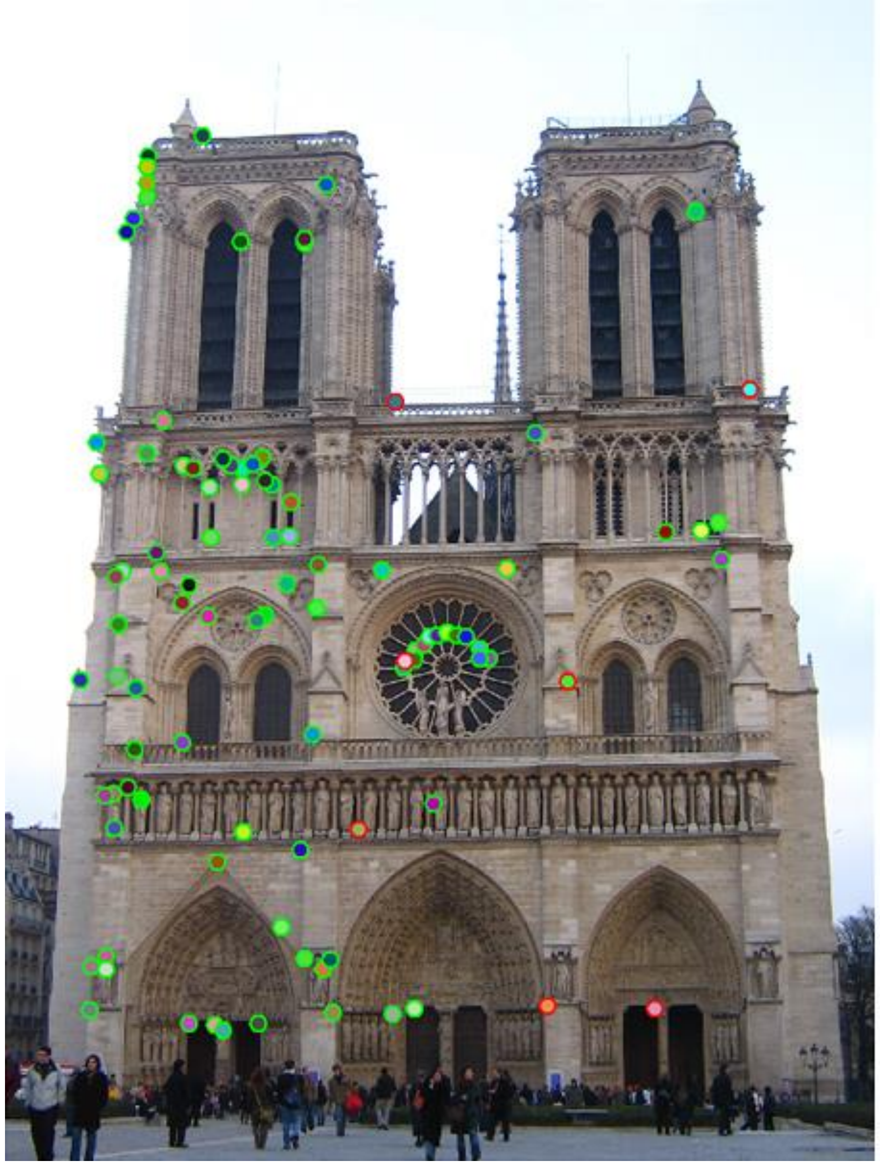


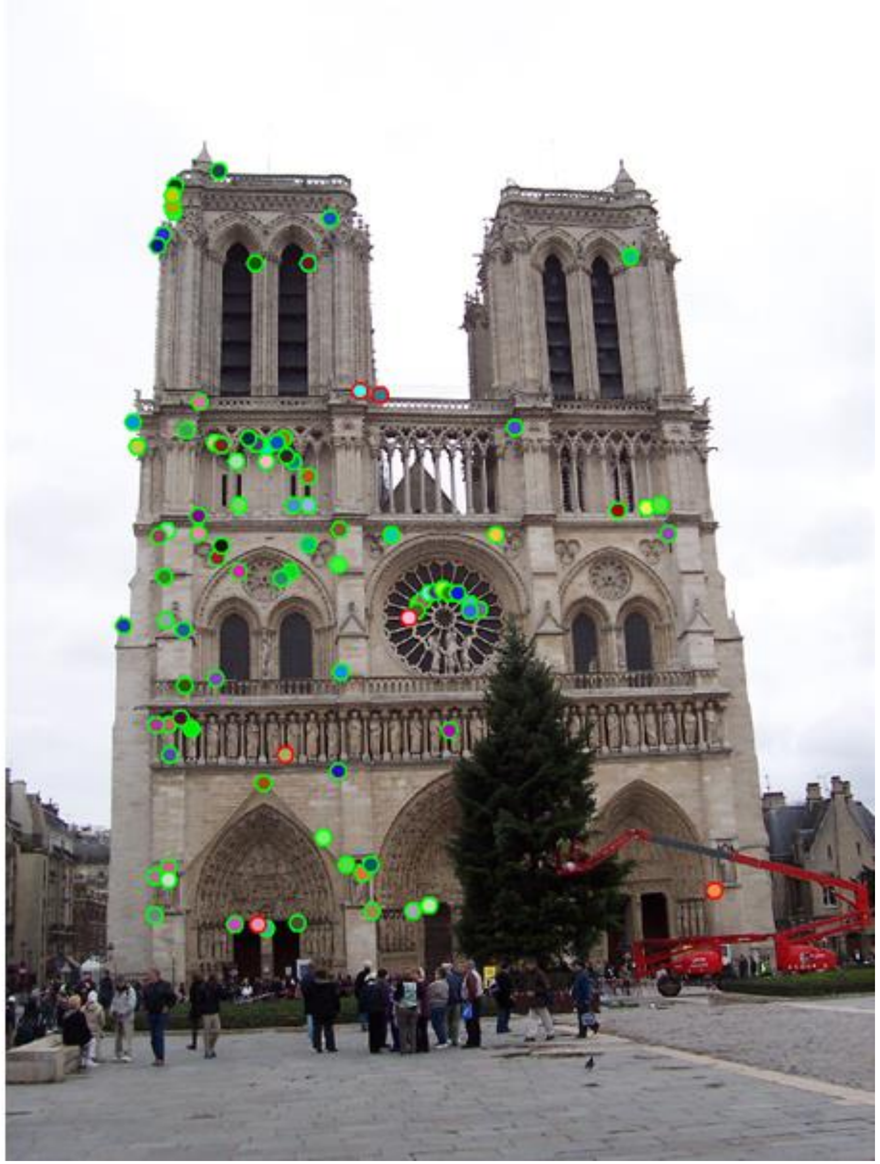
Count = 4
Count = 6
Count = 19
Count = 13

RANSAC - *Loop*

1. Randomly select a *seed group* of points on which to base transformation estimate (e.g., a group of matches)
2. Find *inliers* to this transformation
3. If the number of inliers is sufficiently large, re-compute least-squares estimate of transformation on all of the inliers
4. Keep the transformation with the largest number of inliers

Given matches, what Next?

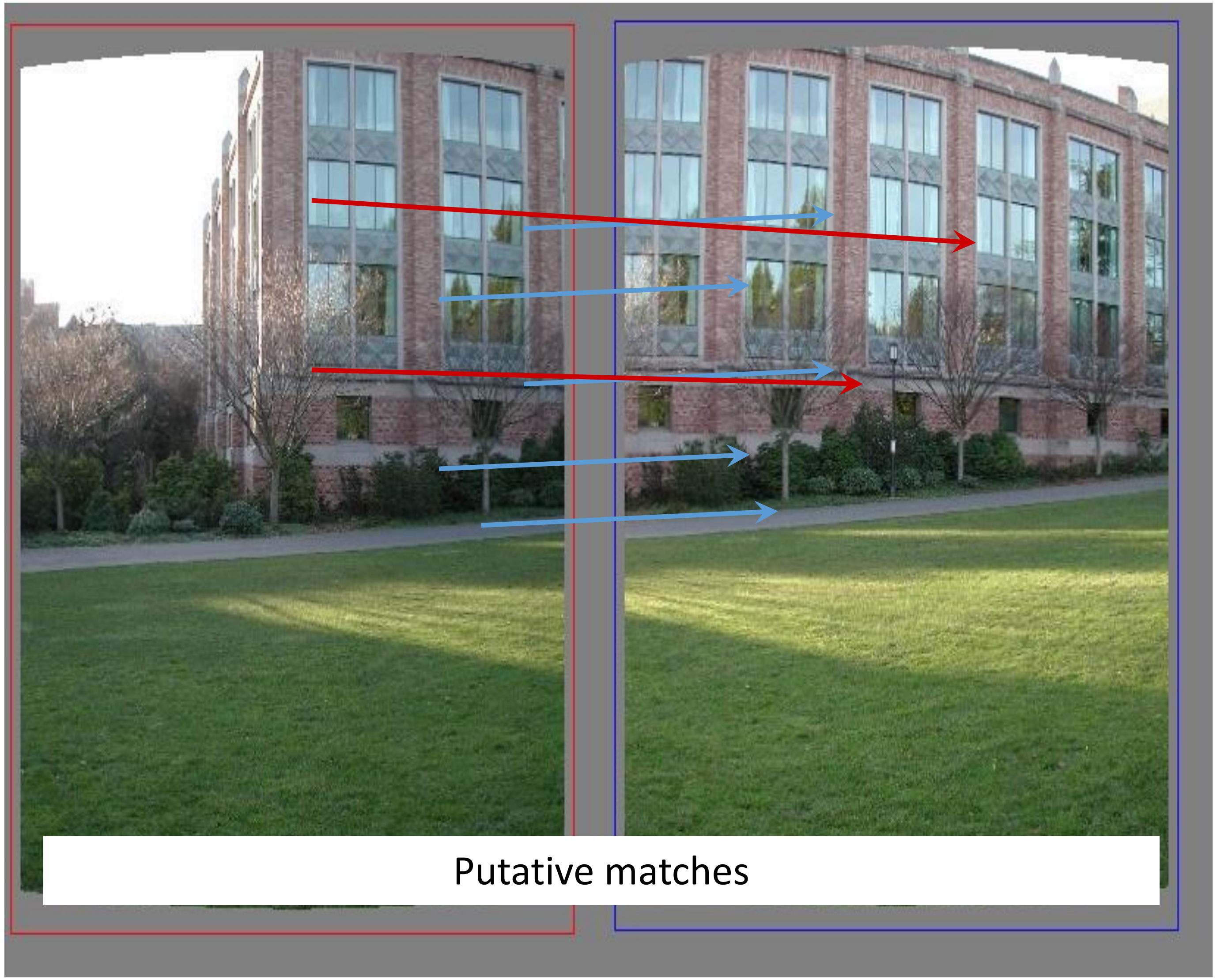




Algorithm 15.4: RANSAC: fitting lines using random sample consensus

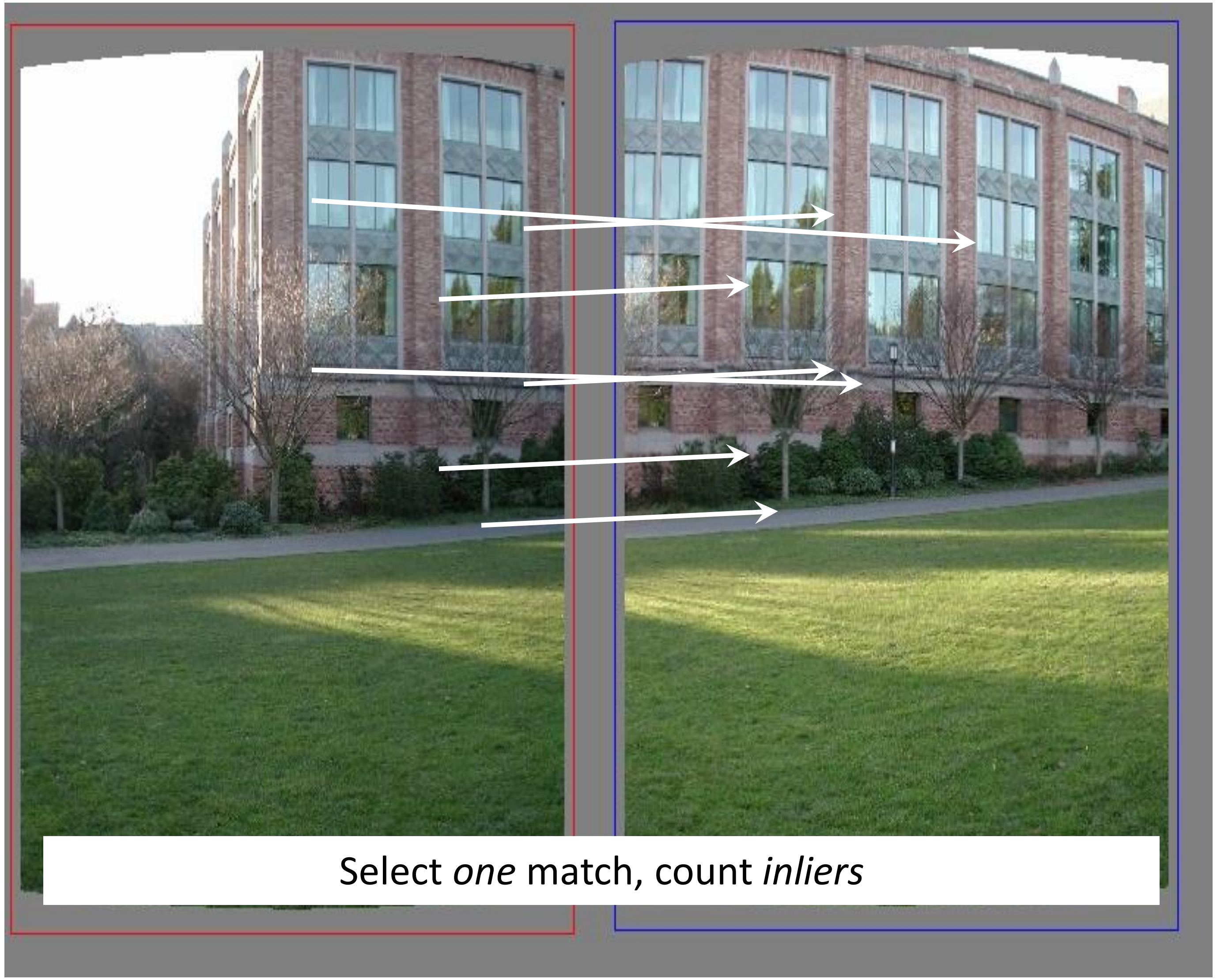
```
Determine:
    n — the smallest number of points required
    k — the number of iterations required
    t — the threshold used to identify a point that fits well
    d — the number of nearby points required
      to assert a model fits well
Until k iterations have occurred
    Draw a sample of n points from the data
      uniformly and at random
    Fit to that set of n points
    For each data point outside the sample
      Test the distance from the point to the line
        against t; if the distance from the point to the line
        is less than t, the point is close
    end
    If there are d or more points close to the line
      then there is a good fit. Refit the line using all
      these points.
end
Use the best fit from this collection, using the
  fitting error as a criterion
```

RANSAC example: Translation



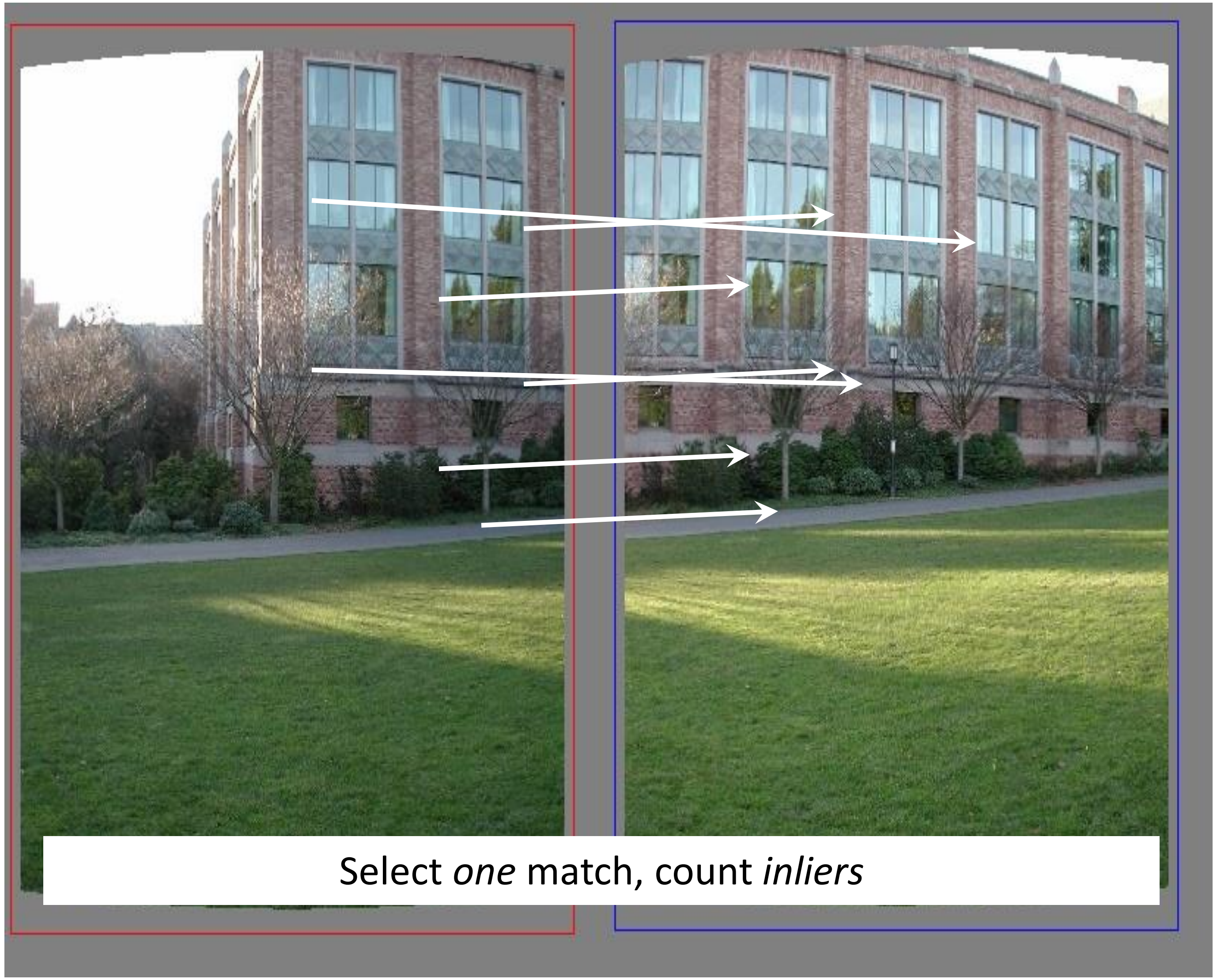
Putative matches

RANSAC example: Translation



Select *one* match, count *inliers*

RANSAC example: Translation



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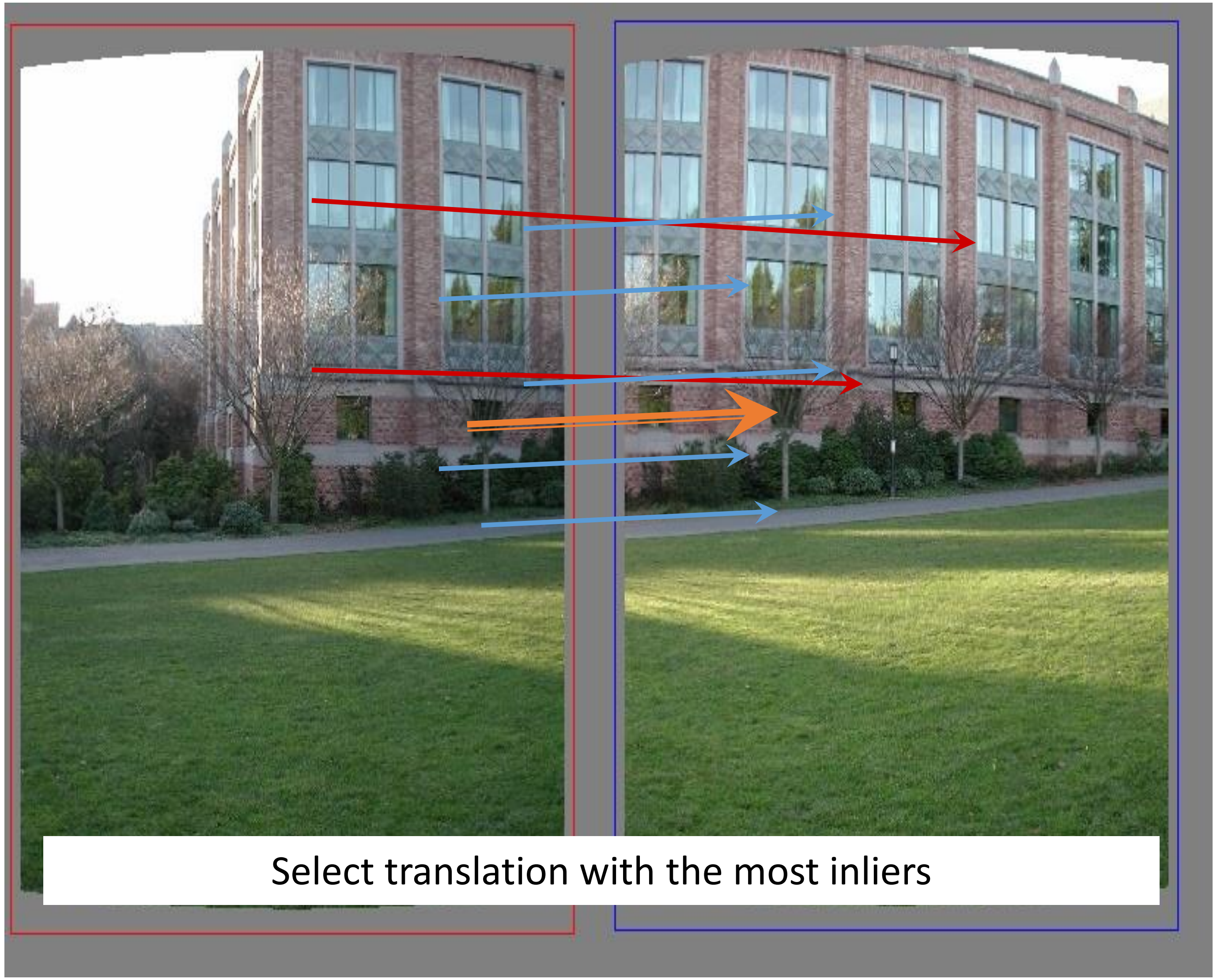

Robust feature-based alignment

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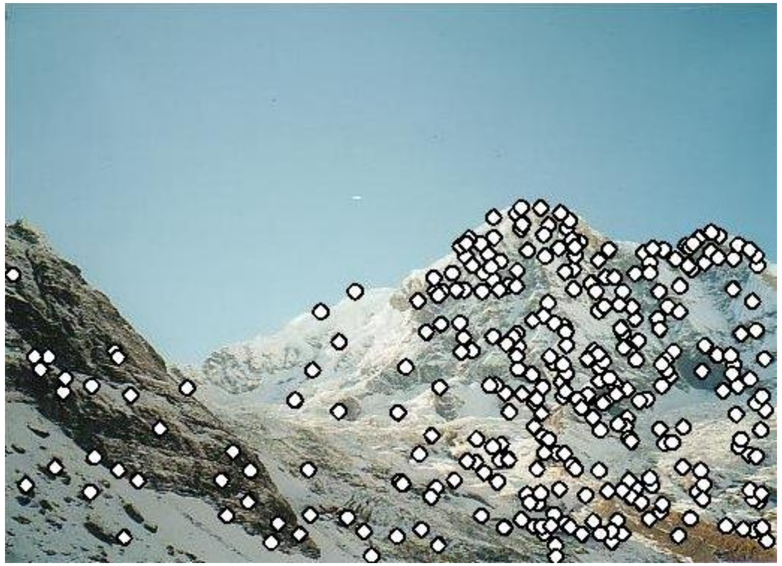
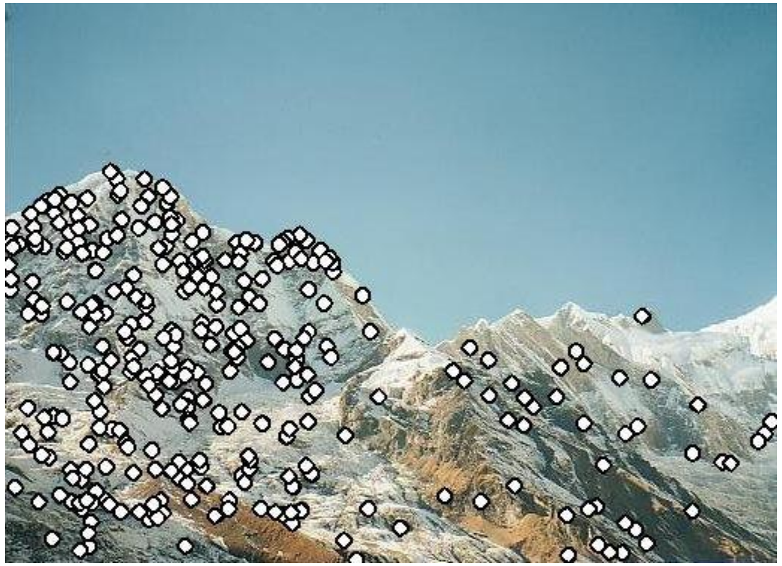


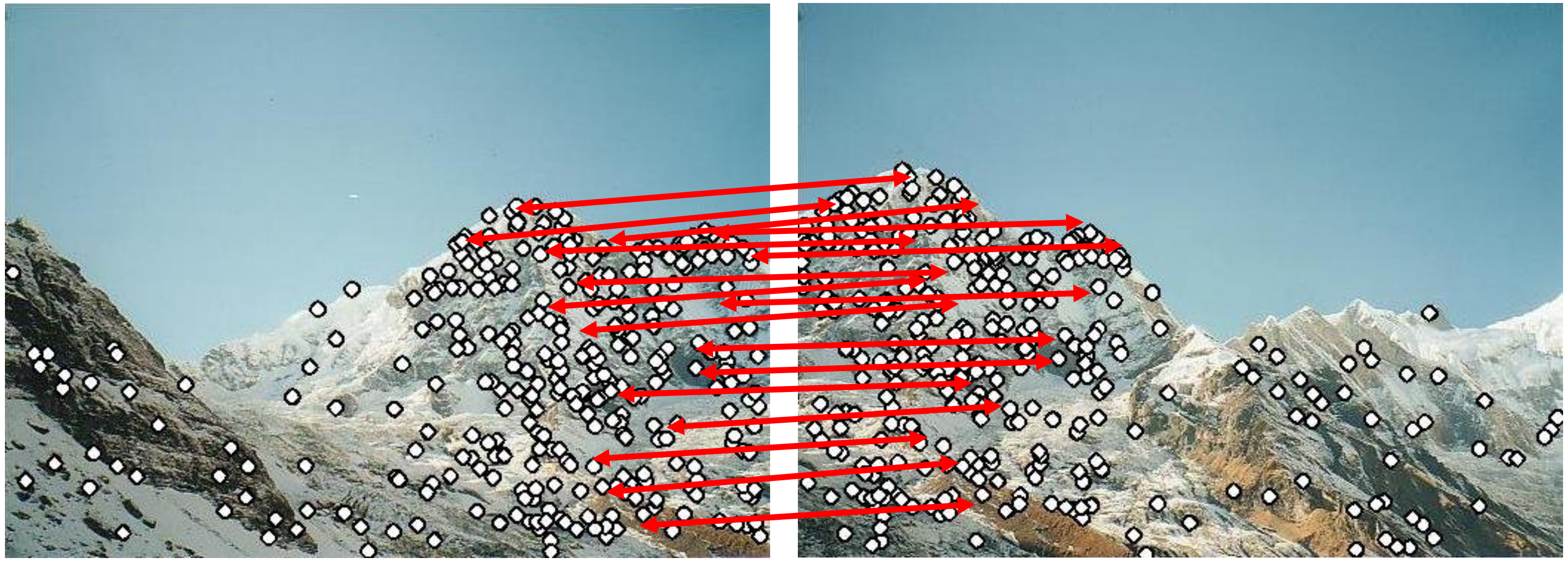


- Extract features

Robust feature-based alignment

- Extract features
- Compute *putative matches*

Robust feature-based alignment



- Extract features
- Compute *putative matches*
- Loop:
  - *Hypothesize* transformation *T*
  - *Verify* transformation (search for other matches consistent with *T*)

Robust feature-based alignment

- Extract features
- Compute *putative matches*
- Loop:
  - *Hypothesize* transformation *T*
  - *Verify* transformation (search for other matches consistent with *T*)

How Many Samples to Choose?

e = probability that a point is an outlier
s = number of points in a sample
N = number of samples (we want to compute this)
p = desired probability that we get a good sample

Solve the following for N:

$$1 - \left(1 - (1 - e)^{s}\right)^{N} = p$$

Where in the world did that come from? ….

How Many Samples to Choose?

e = probability that a point is an outlier
s = number of points in a sample
N = number of samples (we want to compute this)
p = desired probability that we get a good sample

$$1 - \left(1 - \left(\underbrace{\mathbf{1 - e}}_{\text{Probability that choosing one point yields an inlier}}\right)^s\right)^N = p$$

How Many Samples to Choose?

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$$1 - (1 - \underbrace{(1 - e)}_{\text{Probability of choosing s inliers in a row (sample only contains inliers)}}{}^{s})^{N} = p$$

How Many Samples to Choose?

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$$1 - (\underbrace{1 - (1 - e)^s}_{\text{Probability that one or more points in the sample were outliers (sample is contaminated).}})^N = p$$

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How Many Samples to Choose?

e = probability that a point is an outlier
s = number of points in a sample
N = number of samples (we want to compute this)
p = desired probability that we get a good sample

$$\underbrace{1 - \left(1 - (1 - e)^{s}\right)^{N}}_{\text{Probability that at least one sample was not contaminated (at least one sample of s points is composed of only inliers).}} = p$$

How many samples?

Choose N so that, with probability p, at least one random sample is free from outliers. e.g. p=0.99

$$\left(1-(1-e)^s\right)^N = 1-p$$

$$N = \frac{\log(1-p)}{\log\left(1-(1-e)^s\right)}$$

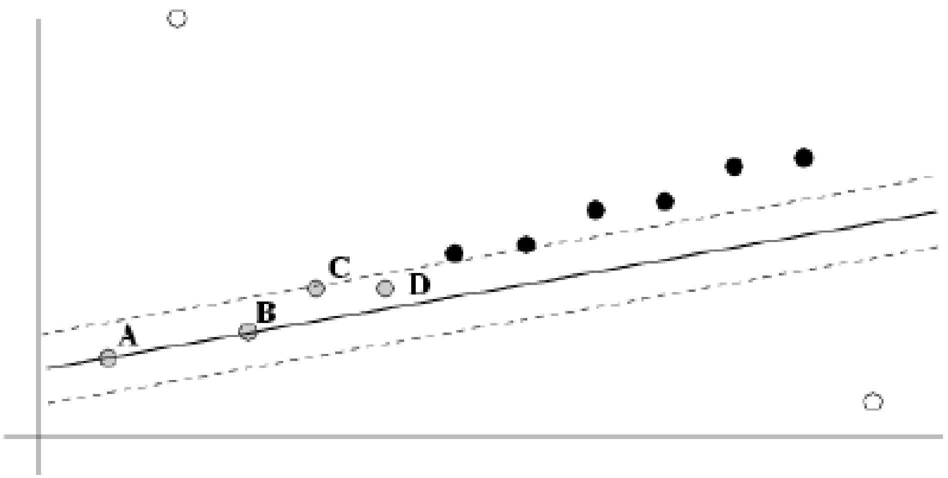
	proportion of outliers e						
s	5%	10%	20%	25%	30%	40%	50%
2	2	3	5	6	7	11	17
3	3	4	7	9	11	19	35
4	3	5	9	13	17	34	72
5	4	6	12	17	26	57	146
6	4	7	16	24	37	97	293
7	4	8	20	33	54	163	588
8	5	9	26	44	78	272	1177

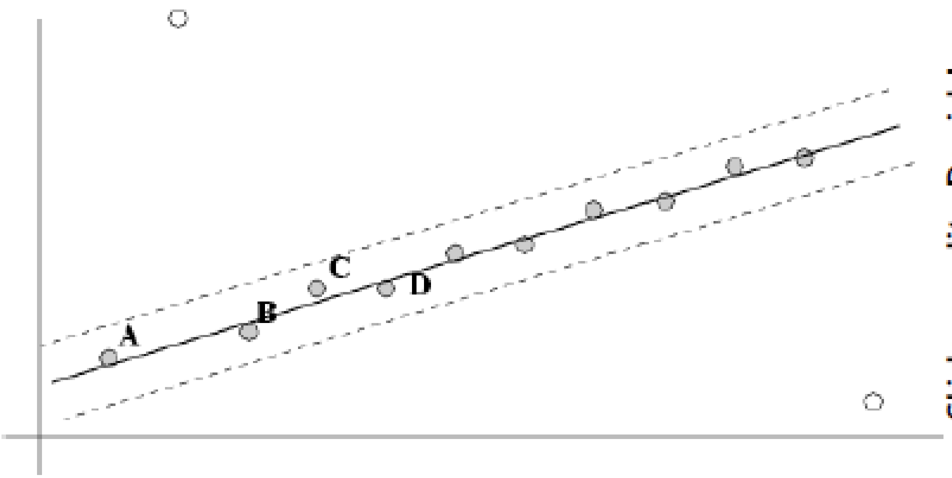
How to choose the parameters?

- Number of sample points **S**
  - Minimum number needed to fit a model
- Outlier ratio **e** (e = #outliers/#datapoints)
- Number of trials **T**
  - Choose T so that, with probability p, at least one random sample is free from outliers
- Distance threshold $delta$
  - Choose $delta$ so that a good point with noise is likely (prob = 0.95) within threshold

After RANSAC

- RANSAC divides data into inliers and outliers and yields estimate computed from minimal set of inliers.
- Improve this initial estimate with estimation over all inliers (e.g. with standard least-squares minimization).
- But this may change inliers, so alternate fitting with re-classification as inlier/outlier.





Slide credit: David Lowe

RANSAC : In Summary

Pros

- General method suited for a wide range of model fitting problems
- Easy to implement and easy to calculate its failure rate

Cons

- Computational time grows quickly with fraction of outliers and number of parameters

Common applications

- Computing a homography (e.g., image stitching)
- Estimating fundamental matrix (relating two views)