Collapsed Gibbs for LDA

Model

$$p(\theta_d) = \mathrm{Dir}(\beta) \qquad p(\phi_t) = \mathrm{Dir}(\alpha)$$

$$p(z_{dn}|\theta_d) = \Theta_{dz_{dn}} \qquad p(w_{dn}|z_{dn}, \Phi) = \Phi_{z_{dn}w_{dn}}$$

$$p(Z \mid W) \sim \{Gibbs\ Sampling\}$$

$$\begin{aligned} p(\Phi|W) &= \int p(\Phi|W, Z)p(Z|W)dZ \\ &= \mathbb{E}_{p(Z|W)}p(\Phi|W, Z) \\ &\approx p(\Phi|W, \widehat{Z}) \end{aligned}$$