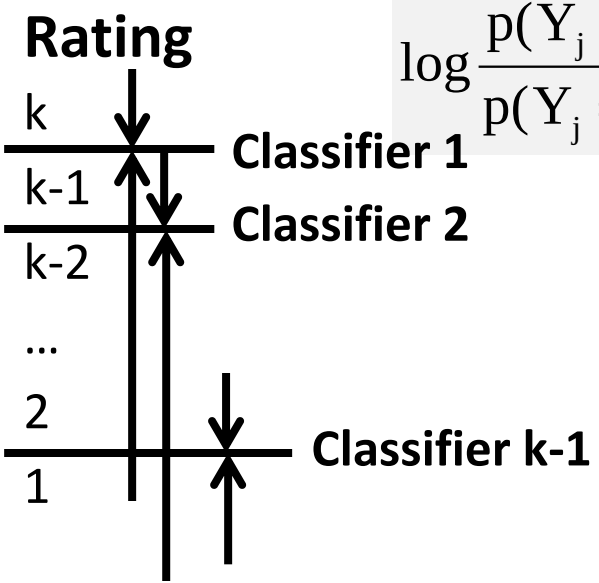
Logistic Regression for Multi-Level Ratings

$$Y_j = \begin{cases} 1 & \text{rating is } j \text{ or above} \\ 0 & \text{rating is lower than } j \end{cases}$$

Predictors: $X = (x_1, x_2, \ldots, x_M),\ x_i \in \Re$

Rating: $r \in \{1, 2, \ldots, k\}$



$$\log \frac{p(Y_j = 1 \mid X)}{p(Y_j = 0 \mid X)} = \log \frac{p(r \geq j \mid X)}{1 - p(r \geq j \mid X)} = \alpha_j + \sum_{i=1}^{M} x_i \beta_{ji} \qquad \beta_{ji} \in \Re$$

$$p(r \geq j \mid X) = \frac{e^{\alpha_j + \sum_{i=1}^{M} x_i \beta_{ji}}}{e^{\alpha_j + \sum_{i=1}^{M} x_i \beta_{ji}} + 1}$$