

Kullback–Leibler divergence

$$\mathcal{KL}(q \parallel p) = \int q(x) \log \frac{q(x)}{p(x)} dx$$

1. $\mathcal{KL}(q \parallel p) \neq \mathcal{KL}(p \parallel q)$
2. $\mathcal{KL}(q \parallel \textcolor{red}{q}) = 0$
3. $\mathcal{KL}(q \parallel p) \geq 0$

Proof:
$$-\mathcal{KL}(q \parallel p) = \mathbb{E}_q \left(-\log \frac{q}{p} \right) = \mathbb{E}_q \left(\log \frac{p}{q} \right)$$
$$\leq \log(\mathbb{E}_q \frac{p}{q})$$