Kullback–Leibler divergence

$$\mathcal{KL}(q \parallel p) = \int q(x) \log \frac{q(x)}{p(x)} dx$$

1. $\mathcal{KL}(q \parallel p) \neq \mathcal{KL}(p \parallel q)$
2. $\mathcal{KL}(q \parallel q) = 0$
3. $\mathcal{KL}(q \parallel p) \geq 0$

Proof: $$-\mathcal{KL}(q \parallel p) = \mathbb{E}_q\left(-\log \frac{q}{p}\right) = \mathbb{E}_q\left(\log \frac{p}{q}\right)$$

$$\leq \log(\mathbb{E}_q \frac{p}{q}) = \log \int q(x) \frac{p(x)}{q(x)} dx = 0$$