

Demaskammur 2 GHSR

a) DeMorgan's theorem for three variables:

$$(x + y + z)' = x'y'z' \quad \text{and} \quad (xyz)' = x' + y' + z'$$

xyz	$x+y+z$	$(x+y+z)'$	x'	y'	z'	$x'y'z'$
000	0	1	1	1	1	1
001	1	0	1	1	0	0
010	1	0	1	0	1	0
011	1	0	1	0	0	0
100	1	0	0	1	1	0
101	1	0	0	1	0	0
110	1	0	0	0	1	0
111	1	0	0	0	0	0

and

$$(xyz)' = x' + y' + z'$$

xyz	$x+y+z$	$(x+y+z)'$	x'	y'	z'	$x' + y' + z'$
000	0	1	1	1	1	1
001	1	0	1	1	0	0
010	1	0	1	0	1	0
011	1	0	1	0	0	0
100	1	0	0	1	1	0
101	1	0	0	1	0	0
110	1	0	0	0	1	0
111	1	0	0	0	0	0

b) The distributive law: $x + yz = (x+y)(x+z)$

xyz	$x+yz$	$(x+y)$	$(x+z)$	$(x+y)(x+z)$
000	0	0	0	0
001	0	0	1	0
010	0	1	0	0
011	1	1	1	1
100	1	1	1	1
101	1	1	1	1
110	1	1	1	1
111	1	1	1	1

c) $x(y+z) = xy + xz$

xyz	$x+y+z$	xy	xz	$xy + xz$
000	0	0	0	0
001	0	0	0	0
010	0	0	0	0
011	1	0	0	0
100	1	0	0	0
101	1	0	1	1
110	1	1	0	1
111	1	1	1	1

d) The associative law: $x + (y + z) = (x + y) + z$

x	y	z	x	(y+z)	x+(y+z)	(x+y)	(x+y)+z
0	0	0	0	0	0	0	0
0	0	1	0	1	1	0	1
0	1	0	0	1	1	1	1
0	1	1	0	1	1	1	1
1	0	0	1	0	1	1	1
1	0	1	1	1	1	1	1
1	1	0	1	1	1	1	1
1	1	1	1	1	1	1	1

e) The associative law and $x(yz) = (xy)z$

x	y	z	yz	x(yz)	xy	(xy)z
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	0	0
0	1	1	1	0	0	0
1	0	0	0	0	0	0
1	0	1	0	0	0	0
1	1	0	0	0	1	0
1	1	1	1	1	1	1

Reduce

2.4) a) $A'C' + ABC + AC'$ to three literals

A	C'	+	ABC	+	A	C'
2	0	1	0	7	1	1
0	0	0	0	6	1	0
				4	1	0

$\Sigma(0, 2, 4, 6, 7)$

ABC	00	01	11	10
0	1			1
1			1	1

$$F(ABC) = \bar{C} + AB$$

b) $(x'y + z)' + z + xy + wy$

$x + y\bar{z} + z + xy + wy$ to three literals

$$(x+y)(x+z) + z + xy + wy$$

x	y	z	w	(x+y)(x+z)	xy	wy	(x+y)(x+z) + z + xy + wy
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	1	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	0	1	0	0	1
0	1	0	1	1	0	0	1
0	1	1	0	0	0	0	0
0	1	1	1	0	0	1	0
1	0	0	0	1	0	0	1
1	0	0	1	1	0	0	1
1	0	1	0	1	0	0	1
1	0	1	1	1	0	0	1
1	1	0	0	1	1	0	1
1	1	0	1	1	1	0	1
1	1	1	0	1	1	0	1
1	1	1	1	1	1	1	1

Framhald af b) Fattadi að það er snúugt að einfalda án K-korts

$$= (\bar{x}y + z) + z + xy + wz$$

$$= (\bar{x}y + z) + z(1 + w) + xy$$

$$= (\bar{x}y + z) + z + xy$$

$$= (x + y)z + z + xy$$

$$= x + y + z + xy$$

$$= \underline{\underline{x + y + z}}$$

$$\begin{aligned} \bar{A}B + A &= B + A \\ A + AB &= A \end{aligned}$$

K-kort

AB \ CD	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	1	1	1	1
10	0	0	0	0

c) $\bar{A}B(\bar{D} + \bar{C}D) + B(A + \bar{A}CD) \rightarrow$ to one literal

$$\bar{A}B\bar{D} + \bar{A}BCD + AB + \bar{A}BCD$$

4 bita svæði

$$\begin{array}{r} \bar{A}B\bar{D} \\ 6 \ 0 \ 1 \ 1 \ 0 \\ 4 \ 0 \ 1 \ 0 \ 0 \end{array}$$

$$\bar{A}BCD$$

$$\begin{array}{r} ABC \\ 12 \ 1 \ 1 \ 0 \ 0 \\ 13 \ 1 \ 1 \ 0 \ 1 \\ 14 \ 1 \ 1 \ 1 \ 0 \\ 15 \ 1 \ 1 \ 1 \ 1 \end{array}$$

$$\bar{A}BCD$$

$$\Sigma(4, 5, 6, 7, 12, 13, 14, 15)$$

AB \ CD	00	01	11	10
00	0 m0	0 m1	0 m3	0 m2
01	1 m4	1 m5	1 m7	1 m6
11	1 m8	1 m9	1 m11	1 m10
10	0 m12	0 m13	0 m15	0 m14

$$F(A, B, C, D) = \underline{\underline{B + A\bar{B} = 1}}$$

d) $(\bar{A} + C)(\bar{A} + \bar{C})(A + B + \bar{C}D)$

$$(\bar{A}\bar{A} + \bar{A}\bar{C} + C\bar{A} + C\bar{C})(A + B + \bar{C}D)$$

$$(\bar{A} + \bar{A}(C + \bar{C}) + 0)(A + B + \bar{C}D)$$

$$(\bar{A} + \bar{A} + 0)(A + B + \bar{C}D)$$

$$(\bar{A}A + \bar{A}B + \bar{A}\bar{C}D + \bar{A}A + \bar{A}B + \bar{A}\bar{C}D)$$

$$0 + \bar{A}B + \bar{A}\bar{C}D + 0 + \bar{A}B + \bar{A}\bar{C}D$$

$$\bar{A}D(\bar{C} + C) + \bar{A}B$$

$$\underline{\underline{\bar{A}D + \bar{A}B}}$$

$$x + \bar{x} = 1$$

$$x \cdot \bar{x} = 0$$

$$x + x = x$$

e) $AB\bar{C}D + \bar{A}BD + ABCD$

$AB\bar{C}D$	$\bar{A}BD$	$ABCD$
1 1 0 1	0 1 1 1	1 1 1 1
	0 1 0 1	

$F(A,B,C,D) = \underline{\underline{BD}}$

$AB \backslash CD$	00	01	11	10
00	0 m_0	0 m_1	0 m_3	0 m_2
01	0 m_4	1 m_5	1 m_7	0 m_6
11	0 m_8	1 m_9	1 m_{11}	0 m_{10}
10	0 m_{12}	0 m_{13}	0 m_{15}	0 m_{14}

2.12

$A = 10110001$

$B = 10101100$

a) $A \text{ AND } B = \underline{10100000}$

c) $A \text{ XOR } B = \underline{00011101}$

b) $A \text{ OR } B = \underline{10111101}$

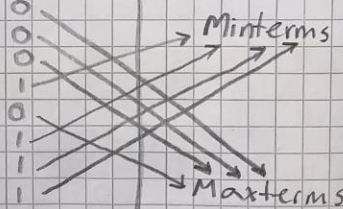
d) $\bar{A} = \underline{01001110}$

e) $\bar{B} = \underline{01010011}$

2.17

$F(b,c,d) = (b+cd)(c+bd)$

bcd	cd	bd	$(b+cd)$	$(c+bd)$	$(b+cd)(c+bd)$
0 0 0	0	0	0	0	0
1 0 0	0	0	0	0	0
2 0 1	0	0	0	1	0
3 0 1 1	1	0	1	1	1
4 1 0 0	0	0	1	0	0
5 1 0 1	0	1	1	1	1
6 1 1 0	0	0	1	1	1
7 1 1 1	1	1	1	1	1



Sum Of Minterms: $F(b,c,d) = \sum(3, 5, 6, 7)$

Product Of Maxterms: $F(b,c,d) = \prod(0, 1, 2, 4)$

2.27

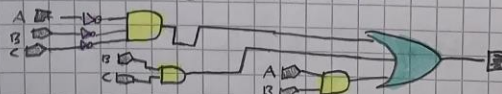
$f_2 = \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + ABC$

$f_1 = \bar{A}\bar{B}\bar{C} + \bar{A}BC + AB\bar{C} + ABC$

F1

ABC	00	01	11	10
0	1 m_0	0 m_1	1 m_3	0 m_2
1	0 m_4	0 m_5	1 m_7	1 m_6

$f_1(A,B,C) = \bar{A}\bar{B}\bar{C} + BC + AB$



F2

ABC	00	01	11	10
0	0	0	1	1
1	1	1	0	1

$f_2(A,B,C) = \bar{A}B + A\bar{C} + AB$

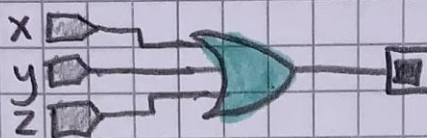


2.7

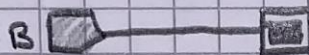
a: $\bar{C} + AB$



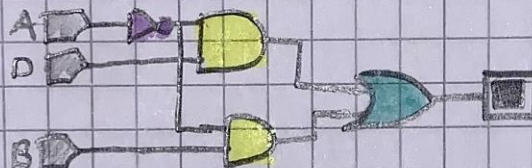
b: $x + y + z$



c: B



d: $\bar{A}D + \bar{A}B$



e: BD

