Project 4: Degradation of Data Integrity

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Part 1:

Using a system of differential equations, we first develop a model for transferring data among processors. As a result of the fact that each processor is composed of a group of processors, we can add up the rate of input supplied by each processor, multiplied by the ratio of data each of these processors contains. To get a total output, we subtract and multiply the processor output.

Our next step is to find the eigenvalues of the matrix created by these DEs:

Part 1

$x_1(t) \quad \frac{1}{100} x_2(t) - \frac{6}{100} x_1(t)$

$x_2(t) \quad \frac{6}{100} x_1(t) + \frac{1}{100} (x_3(t)) - \frac{7}{100} x_2(t)$

$x_3(t) \quad \frac{5}{100} x_2(t) - \frac{5}{100} x_3(t)$

$$x_1(t) = -\frac{3}{50} x_1 + \frac{1}{50} x_2(t) + 0 x_3(t)$$

$$x_2(t) = \frac{3}{50} x_1 - \frac{7}{100} x_2(t) + \frac{1}{100} x_3(t)$$

$$x_3(t) = 0 x_1 + \frac{1}{20} x_2(t) - \frac{1}{20} x_3(t)$$

$$\begin{bmatrix} -\frac{3}{50} & \frac{1}{50} & 0 \\ \frac{3}{50} & -\frac{7}{100} & \frac{1}{100} \\ 0 & \frac{1}{20} & -\frac{1}{20} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\det \begin{bmatrix} -\frac{3}{50} - \lambda & \frac{1}{50} & 0 \\ \frac{3}{50} & -\frac{7}{100} - \lambda & -\frac{1}{100} \\ 0 & \frac{1}{20} & -\frac{1}{20} - \lambda \end{bmatrix} = 0$$

$$\lambda = -0.0214, -0.0531, -0.1054$$

The eigenvalues are: **λ = -0.0214, -0.0531, -0.1054**

Using an online calculator, we solved the third-degree polynomial.

Part 2:

Using a system of differential equations, we repeat steps in part 1, making another system of DEs. Our task was to solve for e^At, followed by solving the DE system using the <u>matrix method</u>.

Once e^At is solved, the solution can be found:

$$x_1' = -2/100\, x_1(t) + 3/100\, x_2(t)$$

$$x_2' = 2/100\, x_1(t) - 3/100\, x_2(t)$$

$$\begin{bmatrix} 1-2/100 & 3/100 \\ 2/100 & -3/100 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$e^{At} = \alpha_1 At + \alpha_0 I$$

$$= \begin{bmatrix} -2/100\,\alpha_1 t + \alpha_0 & 3/100\,\alpha_1 t \\ 2/100\,\alpha_1 t & -3/100\,\alpha_1 t + \alpha_0 \end{bmatrix}$$

$$\det(At - \lambda I) = 0$$

$$\begin{vmatrix} -2/100\, t - \lambda & 3/100\, t \\ 2/100\, t & -3/100\, t - \lambda \end{vmatrix} = 0$$

$$\lambda = 0,\ -1/20 \qquad e^{0} = \alpha_0$$

$$e^{-1/20\, t} = -1/20\, t\, \alpha_1 + 1$$

$$e^{-1/20\, t} - 1 = -1/20\, \alpha_1 \qquad 1 = \alpha_0$$

$$-20(e^{-1/20\, t} - 1) = \alpha_1$$

$$e^{At} = \begin{bmatrix} -2/100\,\alpha_1 t + \alpha_0 & 3/100\,\alpha_1 t \\ 2/100\,\alpha_1 t & -3/100\,\alpha_1 t + \alpha_0 \end{bmatrix}$$

$$x(t) = e^{At} C \qquad C_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

therefore

$$x(t) = \begin{bmatrix} e^{-1/20 t}, \\ -e^{-1/20 t} \cdot 1 \end{bmatrix}$$

Here is a graph of the solution using Python: