# Supporting information for "On the integration of decision trees with mixture cure model"

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### A1. R code for data generation in Section 4

```
mydata = function (n, alpha, beta1, beta2, beta3, beta4, beta5, beta6, beta7, beta8, beta9,
beta10, cens, setting) {
  z1 = rbern(n, prob = 0.5) \# z1 \text{ to } z4 \text{ are the four binary covariates generated}
  z2 = rbern(n, prob = 0.3) \# independently from Bernoulli distribution.
  z3 = rbern(n, prob = 0.5)
  z4 = rbern(n, prob = 0.7)
  z_5 = r_0(n, mean = 0, sd = 1) \# z_5  to z_10 are the six continuous covariates generated
  z6 = rnorm(n, mean=0, sd=1) \# independently from standard normal distributions.
  z7 = \mathbf{rnorm}(n, \mathbf{mean} = 0, \mathbf{sd} = 1)
  z8 = \mathbf{rnorm}(n, \mathbf{mean} = 0, \mathbf{sd} = 1)
  z9 = \mathbf{rnorm}(n, \mathbf{mean} = 0, \mathbf{sd} = 1)
  z10 = rnorm(n, mean=0, sd=1)
  piz = rep(NA, n) \# this is the uncured probability <math>pi(z)
  if(setting == 1){ # non-linear classification boundary
     piz = (\exp(0.3 + (10*z1*z1*z2) - (5*z2*z2)) / (1 + \exp(0.3 + (10*z1*z1*z2) - (5*z2*z2))))
  }
  if(setting == 2) { # complex classification boundary with high dimension
     piz = \exp(-\exp(-0.8*z1*z2+1.1*z2*z4+0.5*z3+0.2*z7*z7-1.3*sin(z5*z6)+
     1.9 \times \cos(z7 \times z8) - 1.5 \times \exp(z5 \times z6 \times z7) - 1.6 \times z7 \times z8 \times z9 \times z10 + 0.8 \times (z6 \times z7) \times (z8 \times z9 \times z8 \times z9)
     +1.8*\cos(z5*z6*z7*z8*z9)+1.2*(abs(z6*z7*z8*z9*z10)^0.5)-2.4)
  }
     \mathbf{C} = \mathbf{runif}(\mathbf{n}, 0, \mathbf{cens}) \# censoring time
  U = \mathbf{runif}(n, 0, 1)
  Y = \mathbf{rep}(NA, n) \# observed \ lifetime
  \mathbf{D} = \mathbf{rep}(NA, n) \# censoring indicator
  J = \mathbf{rep}(NA, n) \# cured indicator (J=0 implies cured)
```

```
Sp = \mathbf{rep}(NA, n) \# overall (population) survival function
    S1 = \mathbf{rep}(NA, n) \# survival function of susceptible group
    for (i in 1:n) {
          if (U[i]<= 1-piz[i]) {
              Y[i] = C[i]
              \mathbf{D}[i] = 0
               J[i] = 0
               S1[i] = \exp(-((Y[i]/\exp(-((beta1*z1[i])+(beta2*z2[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3*z3[i])+(beta3
               (beta4*z4[i])+(beta5*z5[i])+(beta6*z6[i])+(beta7*z7[i])+(beta8*z8[i])+
               (beta9*z9[i])+(beta10*z10[i]))/alpha))^alpha))
               Sp[i] = (1-piz[i]) + (piz[i]*exp(-((Y[i]/exp(-((beta1*z1[i])+(beta2*z2[i])+
               (beta3*z3[i])+ (beta4*z4[i])+(beta5*z5[i])+(beta6*z6[i])+(beta7*z7[i])+
               (beta8*z8[i])+(beta9*z9[i])+(beta10*z10[i]))/alpha))^alpha)))
          else{
                           rweibull(1, shape=alpha, scale=exp(-((beta1*z1[i])+(beta2*z2[i])+
              T =
               (beta3*z3[i])+(beta4*z4[i])+(beta5*z5[i])+(beta6*z6[i])+(beta7*z7[i])+
               (beta8*z8[i])+(beta9*z9[i])+(beta10*z10[i]))/alpha) )
              Y[i] = \min(T, C[i])
               J[i] = 1
               S1[i] = \exp(-((Y[i]/\exp(-((beta1*z1[i])+(beta2*z2[i])+(beta3*z3[i])+
               (beta4*z4[i])+(beta5*z5[i])+(beta6*z6[i])+(beta7*z7[i])+(beta8*z8[i])+
               (beta9*z9[i])+(beta10*z10[i]))/alpha))^alpha))
               Sp[i] = (1-piz[i]) + (piz[i]*exp(-((Y[i]/exp(-((beta1*z1[i])+(beta2*z2[i])
               +(beta3*z3[i])+(beta4*z4[i])+(beta5*z5[i])+(beta6*z6[i])+(beta7*z7[i])+
               (beta8*z8[i])+(beta9*z9[i])+(beta10*z10[i]))/alpha))^alpha)))
               if (Y[i]==C[i]) {
                   \mathbf{D}[i] = 0
               }
               else {
                  \mathbf{D}[i] = 1
               }
    }
    return (data.frame(Y,D,z1,z2,z3,z4,z5,z6,z7,z8,z9,z10,J,uncure=piz,S1,Sp=Sp))
} # function to generate data under 3 different classification boundaries
```

## A2. R code for DT-based EM algorithm in Section 4

```
library(e1071)
library(survival)
library(rpart)
smsurv <-function(Time, Status, X, beta, w, model){</pre>
```

```
death_point <- sort(unique(subset(Time, Status==1)))
if(model='ph') coxexp \leftarrow exp((beta)\%*\%t(X[,-1]))
lambda <- numeric()
event <- numeric()
for (i in 1: length (death_point)) {
event[i] <- sum(Status*as.numeric(Time=death_point[i]))
if (mode='ph') temp <- sum(as.numeric(Time>=death_point[i])*w*drop(coxexp))
if (model='aft') temp <- sum(as.numeric(Time>=death_point[i])*w)
temp1 <- event[i]
lambda [i] <- temp1/temp
HHazard <- numeric()
for (i in 1:length (Time)) {
HHazard[i] <- sum(as.numeric(Time[i]>=death_point)*lambda)
if (Time[i]>max(death_point)) HHazard[i] <- Inf
if (Time[i] < min(death_point)) HHazard[i] <- 0
survival \leftarrow exp(-HHazard)
list (survival=survival)
em. dt. RC = function (Time, Status, X, Z, uncureprob, beta, s0, emmax, eps) {
w <- Status
n <- length (Status)
s <- smsurv (Time, Status, X, beta, w, model="ph") $survival
convergence - 1000; i <-1
while (convergence > eps & i < emmax){
\#print(i)
survival < -drop(s^(exp((beta)% * \%t(X[,-1]))))
\# E step
w \leftarrow Status + (1 - Status) * (uncureprob * survival) / ((1 - uncureprob) + uncureprob * survival)
\# M step
multipleuncureprob=matrix (1:5*n, nrow=n, ncol=5)
for (j \text{ in } 1:n) \{ \text{multipleuncureprob}[j,] = \text{rbinom}(5, \text{ size} = 1, \text{ prob} = w[j]) \}
uncureprob1 = \mathbf{c}(1,1)
uncureprob2 = c(1,1)
uncureprob3 = c(1,1)
uncureprob4 = c(1,1)
uncureprob5 = \mathbf{c}(1,1)
for (j \text{ in } 1:n)\{\text{uncureprob1}[j] = \text{multipleuncureprob}[j,1]\}
```

```
for (j \text{ in } 1:n) \{ uncureprob 2 [j] = multipleuncureprob [j,2] \}
for (j \text{ in } 1:n)\{\text{uncureprob3}[j] = \text{multipleuncureprob}[j,3]\}
for (j in 1:n) {uncureprob4[j] = multipleuncureprob[j,4]}
for (j in 1:n) {uncureprob5[j] = multipleuncureprob[j,5]}
for (j \text{ in } 1:n) \{ uncureprob1[j] = uncureprob1[j] \}
for (j \text{ in } 1:n) \{ uncureprob 2 [j] = uncureprob 2 [j] \}
for (j \text{ in } 1:n) \{ uncureprob 3 [j] = uncureprob 3 [j] \}
for (j \text{ in } 1:n) \{ uncureprob 4 [j] = uncureprob 4 [j] \}
for (j \text{ in } 1:n)\{\text{uncureprob5}[j] = \text{uncureprob5}[j]\}
uncureprob1 = as.factor(uncureprob1)
uncureprob2 = as.factor(uncureprob2)
uncureprob3 = as.factor(uncureprob3)
uncureprob4 = as.factor(uncureprob4)
uncureprob5 = as.factor(uncureprob5)
update\_cureb = c(1,1)
obj3 \leftarrow tune.rpart(uncureprob1~Z[,-1], data = data, minsplit = seq(41,51,5),
cp = seq(0.002, 0.012, 0.005), xval = 3
bc—obj3$best.parameters[1]
bg - obj3 best. parameters [2]
mod1 \leftarrow rpart(formula = uncureprob1~Z[,-1], data = data, method = "class",
control = rpart.control(minsplit = bc[[1]], cp = bg[[1]])
proba1 <- predict (mod1, newdata = data, type = "prob")
update_cureb1 < -c(1,1)
for (z \text{ in } 1:n) \{ \text{update\_cureb1} [z] \leftarrow \text{proba1} [z, \text{colnames} (\text{proba1}) == 1] \}
uncureprob1 <- as . numeric (as . character (uncureprob1))
mod2 \leftarrow rpart(formula = uncureprob2~Z[,-1], data = data, method = "class",
control = rpart.control(minsplit = bc[[1]], cp = bg[[1]])
proba2 <- predict (mod2, newdata = data, type = "prob")
update_cureb2 < -c(1,1)
for (z \text{ in } 1:n) \{ \text{update\_cureb2} [z] \leftarrow \text{proba2} [z, \text{colnames} (\text{proba2}) == 1] \}
uncureprob2 (as. character (uncureprob2))
mod3 \leftarrow rpart(formula = uncureprob3~Z[,-1], data = data, method = "class",
control = rpart.control(minsplit = bc[[1]], cp = bg[[1]]))
proba3 <- predict (mod3, newdata = data, type = "prob")
update_cureb3 < -c(1,1)
for (z \text{ in } 1:n) \{ \text{update\_cureb3} [z] \leftarrow \text{proba3} [z, \text{colnames} (\text{proba3}) == 1] \}
uncureprob3<-as.numeric(as.character(uncureprob3))
mod4 \leftarrow rpart (formula = uncureprob4~Z[, -1], data = data, method = "class",
```

```
control = rpart.control(minsplit = bc[[1]], cp = bg[[1]]))
proba4 <- predict (mod4, newdata = data, type = "prob")
update_cureb4 < -c(1,1)
for (z \text{ in } 1:n) \{ \text{update\_cureb4} [z] \leftarrow \text{proba4} [z, \text{colnames} (\text{proba4}) == 1] \}
uncureprob4 (-as.numeric(as.character(uncureprob4))
mod5 \leftarrow rpart(formula = uncureprob5~Z[,-1], data = data, method = "class",
control = rpart.control(minsplit = bc[[1]], cp = bg[[1]])
proba5 <- predict (mod5, newdata = data, type = "prob")
update_cureb5 < -c(1,1)
for (z \text{ in } 1:n) \{ \text{update\_cureb5} [z] \leftarrow \text{proba5} [z, \text{colnames} (\text{proba5}) == 1] \}
uncureprob5 <- as.numeric(as.character(uncureprob5))
for (z in 1:n){update_cureb[z]<-(update_cureb1[z]+update_cureb2[z]
+\mathbf{update}_{-}\mathbf{cureb3}[z]+\mathbf{update}_{-}\mathbf{cureb4}[z]+\mathbf{update}_{-}\mathbf{cureb5}[z])/5
update_beta \leftarrow coxph(Surv(Time, Status)^X[,-1] + offset(log(w)), subset=w!=0,
method="breslow")$coef
update_s <--smsurv(Time, Status, X, beta, w, model="ph") $survival
convergence - sum (c (mean (update_cureb) - mean (uncureprob), update_beta - beta
mean(update_s)-mean(s)^2
uncureprob <- update_cureb
beta <- update_beta
s<-update_s
i \leftarrow i+1
\#print(i)
S1 = \mathbf{drop}(s^{(\mathbf{s}(\mathbf{exp}(\mathbf{beta})))} \times \mathbf{t}(\mathbf{X}[,-1])))) \# survival function of susceptible group)
Sp = (1-uncureprob) + (uncureprob *S1)
em. dt <- list (latencyfit=beta, Uncureprob=uncureprob, S0=s, S1=S1, Sp=Sp,
tau=convergence, Mod1=mod1, Mod2=mod2, Mod3=mod3, Mod4=mod4, Mod5=mod5)
}
```

# A3. Plots in Section 5

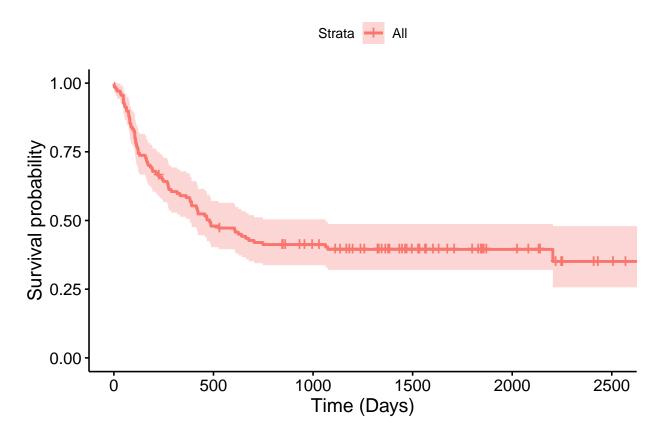


Figure A3.1: Kaplan-Meier survival curve for the leukemia data

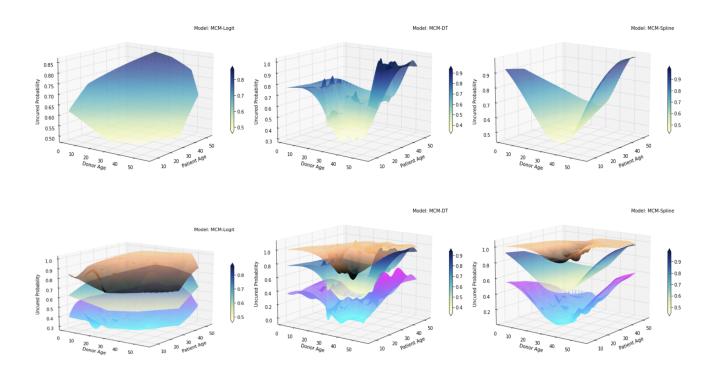


Figure A3.2: 3-dimensional surface plane of uncured probabilities, along with 95% confidence bounds, as a function of patient's age and donor's age