YAKIN DOĞU ÜNİVERSİTESİ
UZAKTAN EĞİTİM VE BİLİŞİM TEKNOLOJİLERİ MERKEZİ

EE 471 POWER SYSTEM ANALYSIS I

Prof. Dr. Sezai Dinçer

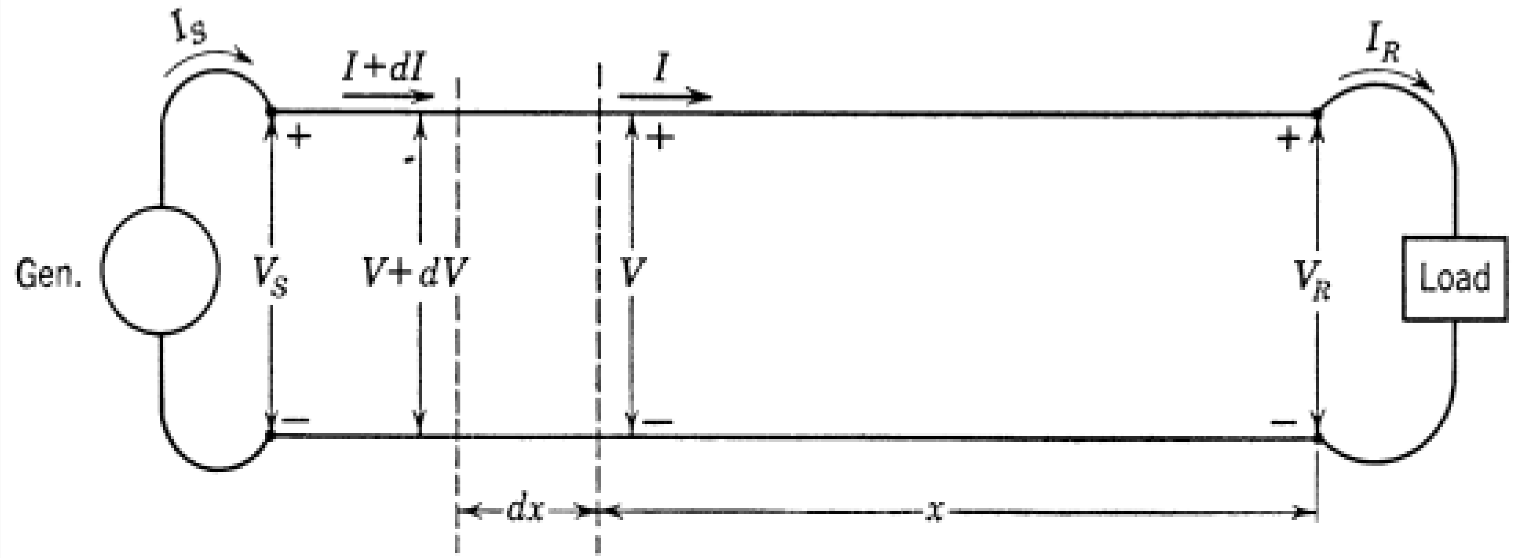
LONG LINE SOLUTION

The Long Transmission Line

-The parameters of the line are not lumped but are distributed uniformly throughout the length of the line.

- General solution with high accuracy is the long line solution for the power line which considers the fact that the parameters of the line are not lumped but are distributed uniformly throughout the length of the line.

Long Line Model



Schematic diagram of a transmission line showing one phase and the neutral return.

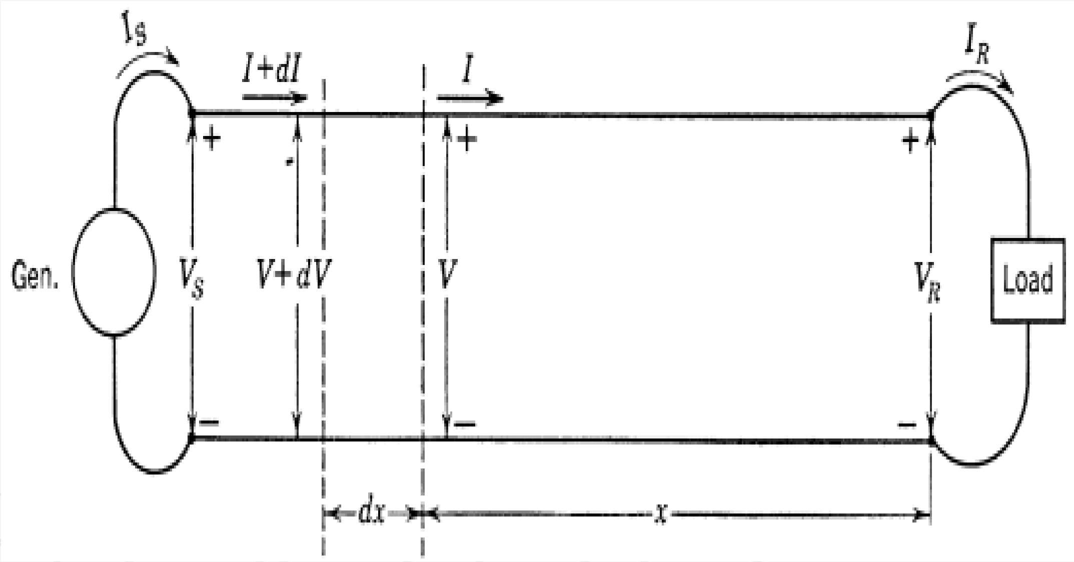
Long Line

x : distance measured from the receiving end

dx : length of line element

zdx: series impedance of line element

ydx: shunt admittance of the line element



Long Line

Voltage drop across the line element

$$dV = Izdx \longrightarrow \frac{dV}{dx} = Iz$$

Consider second derivative

$$\frac{d^2V}{dx^2} = z\frac{dI}{dx}$$

Current through the shunt admittance,

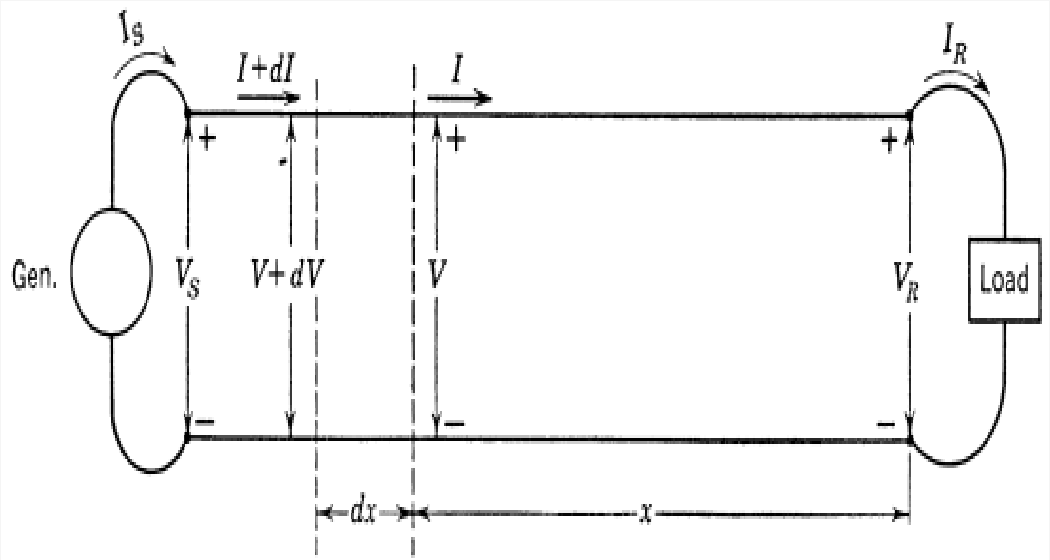
$$dI = Vydx \longrightarrow \frac{dI}{dx} = Vy$$

Consider the second derivative

$$\frac{d^2I}{dx^2} = y\frac{dV}{dx}$$

$$\frac{d^2V}{dx^2} = z\frac{dI}{dx}$$

Therefore, $\frac{d^2V}{dx^2} = yzV$ and $\frac{d^2I}{dx^2} = yzI$



Long Line

Assume that the solution is given as

$$V = A_1 e^{\sqrt{yz}x} + A_2 e^{-\sqrt{yz}x}$$

Taking the second derivative of V with respect to x

$$\frac{d^2V}{dx^2} = yz\left(A_1 e^{\sqrt{yz}x} + A_2 e^{-\sqrt{yz}x}\right)$$

$$\frac{d^2V}{dx^2} = yzV$$

which is yz times the assumed solution for V.

For the current

$$\frac{dV}{dx} = Iz$$

since

$$V = A_1 e^{\sqrt{yz}x} + A_2 e^{-\sqrt{yz}x}$$

$$I = \frac{1}{\sqrt{z/y}} A_1 e^{\sqrt{yz}x} - \frac{1}{\sqrt{z/y}} A_2 e^{-\sqrt{yz}x}$$

Long Line

The constants A_1 and A_2 may be evaluated by using the conditions at the receiving end of the line, namely, when $x = 0, V = V_R$ and I=I_R

$$V_R = A_1 + A_2$$

$$I_R = \frac{1}{\sqrt{z/y}}(A_1 - A_2)$$ and $$Z_C = \sqrt{z/y}$$

Therefore ,

$$A_{1=}\frac{V_R + I_R Z_C}{2}$$

$$A_{2=}\frac{V_R - I_R Z_C}{2}$$

Long Line

$$V(x) = \frac{V_R + I_R Z_C}{2} e^{\gamma x} + \frac{V_R - I_R Z_C}{2} e^{-\gamma x}$$

$$I(x) = \frac{V_R/Z_C + I_R}{2} e^{\gamma x} + \frac{V_R/Z_C - I_R}{2} e^{-\gamma x}$$

$Z_c = \sqrt{z/y}$ is called the *characteristic impedance* of the line, $\gamma = \sqrt{yz}$ *is the propagation constant*.

The equations give the rms values of V and I and their phase angles at any specified point along the line in terms of the distance x from the receving end.

Long Line

γ ve Z_C are complex numbers.

$$\gamma = \alpha + j\beta$$

where , α attenuation constant and β is the phase constant

$$V = \frac{V_R + I_R Z_C}{2} e^{\alpha x} e^{j\beta x} + \frac{V_R - I_R Z_C}{2} e^{-\alpha x} e^{-j\beta x}$$

$$I = \frac{V_R/Z_C + I_R}{2} e^{\alpha x} e^{j\beta x} - \frac{V_R/Z_C - I_R}{2} e^{-\alpha x} e^{-j\beta x}$$

Long Line

The properties of $e^{\alpha x}$ and $e^{j\beta x}$ help to explain the variation of the voltage and current at any instant with distance along the line. The term $e^{\alpha x}$changes in magnitude as x changes, but $e^{j\beta x}$, which is identical to $\cos\beta x + j\sin\beta x$, always has a magnitude of one and causes a shift in phase of β radians per unit length of the line.

The first term in Eq, $[(V_R + I_R Z_C)/2]e^{\alpha x}e^{j\beta x}$, increases in magnitude and advances in phase as distance from the receiving end increases. Conversely, as progress along the line from the sending end toward the receiving end is considered, the term diminishes in magnitude and is retarded in phase. This is the characteristic of a traveling wave (incident wave

The second term in Eq, $[(V_R - I_R Z_C)/2]e^{-\alpha x}e^{-j\beta x}$, diminishes in magnitude and is retarded in phase from the receiving end toward the sending end. It is called the reflected voltage. At any point along the line, the voltage is the sum of the component incident and reflected voltages at that point.

$$\gamma = \alpha + j\beta$$

,

$$V(x) = \left(\frac{e^{\gamma x} + e^{-\gamma x}}{2}\right)V_R + Z_c\left(\frac{e^{\gamma x} - e^{-\gamma x}}{2}\right)I_R$$

$$I(x) = \frac{1}{Z_c}\left(\frac{e^{\gamma x} - e^{-\gamma x}}{2}\right)V_R + \left(\frac{e^{\gamma x} + e^{-\gamma x}}{2}\right)I_R$$

$$\cosh(\gamma l) = \frac{e^{\gamma l} + e^{-\gamma l}}{2} = \frac{1}{2}\left(e^{\alpha l}\underline{/\beta l} + e^{-\alpha l}\underline{/-\beta l}\right)$$

$$\sinh(\gamma l) = \frac{e^{\gamma l} - e^{-\gamma l}}{2} = \frac{1}{2}\left(e^{\alpha l}\underline{/\beta l} - e^{-\alpha l}\underline{/-\beta l}\right)$$

(4.22)

$$V(x) = V_R \cosh\gamma x + I_R Z_C \sinh\gamma x$$

$$I(x) = I_R \cosh\gamma x + \frac{V_R}{Z_C}\sinh\gamma x$$

Long Line Current – Voltage Relationships

for $x=l$

$$V_S = V_R \cosh\gamma l + I_R Z_C \sinh\gamma l$$

$$I_S = I_R \cosh\gamma l + \frac{V_R}{Z_C}\sinh\gamma l$$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

ve

$$A = D = \cosh(\gamma l)$$

$$B = Z_c \sinh(\gamma l) \quad \Omega$$

$$C = \frac{1}{Z_c}\sinh(\gamma l) \quad \mathrm{S}$$

Evaluation of the functions

$$\cosh(\gamma l) = \frac{e^{\gamma l} + e^{-\gamma l}}{2} = \frac{1}{2}(e^{\alpha l} \underline{/\beta l} + e^{-\alpha l} \underline{/-\beta l})$$

$$\sinh(\gamma l) = \frac{e^{\gamma l} - e^{-\gamma l}}{2} = \frac{1}{2}(e^{\alpha l} \underline{/\beta l} - e^{-\alpha l} \underline{/-\beta l})$$

EXAMPLE

50 Hz 3 phase line $R = 0.11\ \Omega/km$, $X = 0.38\ \Omega/km$, $Y = 2.5 \times 10^{-6}\ ℧/km$, $l = 400\ km$ at the receiving and 100 MW is drawn with unity $p.f.$ and $V_R = 154\ kV\ (line - to - line)$, find V_S

SOLUTION:

$$Z = 0.11 + j0.38 = 0.395\angle 73.85^0\ \Omega/km$$

$$Y = 2.5 \times 10^{-6}\ ℧/km$$

$$\gamma l = \sqrt{ZY}l = \sqrt{(0.395\angle 73.85^0)(2.5 \times 10^{-6}\angle 90^0)}.400$$

$$\gamma l = 0.397\angle 81.92^0 = 0.0558 + j0.393 = \alpha l + j\beta l$$

ohm

$$Z_C = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{0.395\angle 73.85^0}{2.5 \times 10^{-6}\angle 90^0}} = 397.5\angle -8.025^0$$

$$|V_R| = \frac{154 \times 10^3}{\sqrt{3}} = 88912\ V$$

Solution

$$\gamma l = 0.397\angle 81.92^0 = 0.0558 + j0.393 = \alpha l + j\beta l$$

$$\cosh\gamma l = \frac{1}{2}\left(e^{\alpha l\angle\beta l} + e^{-\alpha l\angle-\beta l}\right)$$

$$\cosh\gamma l = \frac{1}{2}\left(e^{0.0558\angle 22.57} + e^{-0.0558\angle-22.57}\right) = 0.924 + j0.021 = 0.9242\angle 1.8^0$$

$$\sinh\gamma l = \frac{1}{2}\left(e^{\alpha l\angle\beta l} - e^{-\alpha l\angle-\beta l}\right)$$

$$\sinh\gamma l = \frac{1}{2}\left(e^{0.0558\angle 22.57} - e^{-0.0558\angle-22.57}\right) = 0.052 + j0.384 = 0.387\angle 82.29^0$$

$$V_S = V_R\cosh\gamma l + I_R Z_C\sinh\gamma l$$

$$V_S = (88912)(0.9242\angle 1.8^0) + (375\angle 0^0)(397.5\angle -8.025^0)(0.387\angle 82.29^0)$$

$$V_S = 113421\angle 30.29^0$$

$$\left|V_{S(l-l)}\right| = \sqrt{3}\times 113421 = 196.45\ kV$$

Solution

$$I_R = \frac{100 \times 10^6}{\sqrt{3} \times 154 \times 10^3} = 375\angle 0^0\ A$$

$$A_1 = \frac{V_R + I_R Z_C}{2} = 118712\angle -5.05^0$$

$$A_2 = \frac{V_R - I_R Z_C}{2} = 31149\angle 160.3^0$$

$$V_S^+ = A_1 e^{\alpha l} e^{j\beta l}\ (incident\ wave)$$

$$V_S^+ = (118712\angle -5.05^0) e^{0.0558} e^{j0.393}$$

$$V_S^+ = 125476\angle 17.52^0\ V$$

$$V_S^- = A_2 e^{-\alpha l} e^{-j\beta l}\ (reflected\ wave)$$

$$V_S^- = (31149\angle 160.3^0) e^{-0.0558} e^{-j0.393}$$

$$V_S^- = 29436\angle 137.79^0\ V$$

$$V_S = V_S^+ + V_S^-$$

$$V_S = 113521\angle 30.46^0\ V \longrightarrow \left|V_{S(l-l)}\right| = \sqrt{3} \times 113521 = 196624\ V$$

EXAMPLE

For a three phase 765 kV , 60 Hz , line with l= 300 km, line parameters are,

$$z = 0.0165 + j0.3306 = 0.3310\underline{/87.14^\circ} \quad \Omega/\mathrm{km}$$

$$y = j4.674 \times 10^{-6} \quad \mathrm{S/km}$$

Find A B C D line constants

Solution :

$$Z_c = \sqrt{\frac{0.3310\underline{/87.14^\circ}}{4.674 \times 10^{-6}\underline{/90^\circ}}} = \sqrt{7.082 \times 10^4\underline{/-2.86^\circ}}$$

$$= 266.1\underline{/-1.43^\circ} \quad \Omega$$

$$\gamma l = \sqrt{(0.3310\underline{/87.14^\circ})(4.674 \times 10^{-6}\underline{/90^\circ})} \times (300)$$

$$= \sqrt{1.547 \times 10^{-6}\underline{/177.14^\circ}} \times (300)$$

$$= 0.3731\underline{/88.57^\circ} = 0.00931 + j0.3730$$

$$e^{\gamma l} = e^{0.00931} e^{+j0.3730} = 1.0094\underline{/0.3730} \quad \text{radians}$$

$$= 0.9400 + j0.3678$$

Example

$$e^{-\gamma l} = e^{-0.00931} e^{-j0.3730} = 0.9907\underline{/-0.3730} \quad \text{radians}$$
$$= 0.9226 - j0.3610$$

$$\cosh(\gamma l) = \frac{(0.9400 + j0.3678) + (0.9226 - j0.3610)}{2}$$
$$= 0.9313 + j0.0034 = 0.9313\underline{/0.209^\circ}$$
$$\sinh(\gamma l) = \frac{(0.9400 + j0.3678) - (0.9226 - j0.3610)}{2}$$
$$= 0.0087 + j0.3644 = 0.3645\underline{/88.63^\circ}$$

$$A = D = \cosh(\gamma l) = 0.9313\underline{/0.209^\circ}$$

$$B = (266.1\underline{/-1.43^\circ})(0.3645\underline{/88.63^\circ}) = 97.0\underline{/87.2^\circ} \quad \Omega$$
$$C = \frac{0.3645\underline{/88.63^\circ}}{266.1\underline{/-1.43^\circ}} = 1.37 \times 10^{-3}\underline{/90.06^\circ} \quad \text{S}$$