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EE 471 POWER SYSTEM ANALYSIS I

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LONG LINES II

LONG LINE EXPRESSIONS

$$V(x) = \left(\frac{e^{\gamma x} + e^{-\gamma x}}{2}\right) V_R + Z_c \left(\frac{e^{\gamma x} - e^{-\gamma x}}{2}\right) I_R$$

$$I(x) = \frac{1}{Z_c}\left(\frac{e^{\gamma x} - e^{-\gamma x}}{2}\right) V_R + \left(\frac{e^{\gamma x} + e^{-\gamma x}}{2}\right) I_R$$

$$\cosh(\gamma l) = \frac{e^{\gamma l} + e^{-\gamma l}}{2} = \frac{1}{2}\left(e^{\alpha l}\underline{/\beta l} + e^{-\alpha l}\underline{/-\beta l}\right)$$

$$\sinh(\gamma l) = \frac{e^{\gamma l} - e^{-\gamma l}}{2} = \frac{1}{2}\left(e^{\alpha l}\underline{/\beta l} - e^{-\alpha l}\underline{/-\beta l}\right)$$

(4.22)

$$V(x) = V_R \cosh \gamma x + I_R Z_C \sinh \gamma x$$

$$I(x) = I_R \cosh \gamma x + \frac{V_R}{Z_C} \sinh \gamma x$$

$$V = \frac{V_R + I_R Z_C}{2} e^{\alpha x} e^{j\beta x} + \frac{V_R - I_R Z_C}{2} e^{-\alpha x} e^{-j\beta x}$$

$$I = \frac{V_R/Z_C + I_R}{2} e^{\alpha x} e^{j\beta x} - \frac{V_R/Z_C - I_R}{2} e^{-\alpha x} e^{-j\beta x}$$

Incident wave

Reflected wave

Propagation constant

$$\gamma = \alpha + j\beta \quad \mathrm{m}^{-1}$$

The quantity γl is dimensionless. Also

$$e^{\gamma l} = e^{(\alpha l + j\beta l)} = e^{\alpha l} e^{j\beta l} = e^{\alpha l}\underline{/\beta l}$$

Long Line Sending-Receiving end Current-Voltage Relationships

l : length of the line

x=l ,

$$V_S = V_R \cosh \gamma l + I_R Z_C \sinh \gamma l$$

$$I_S = I_R \cosh \gamma l + \frac{V_R}{Z_C} \sinh \gamma l$$

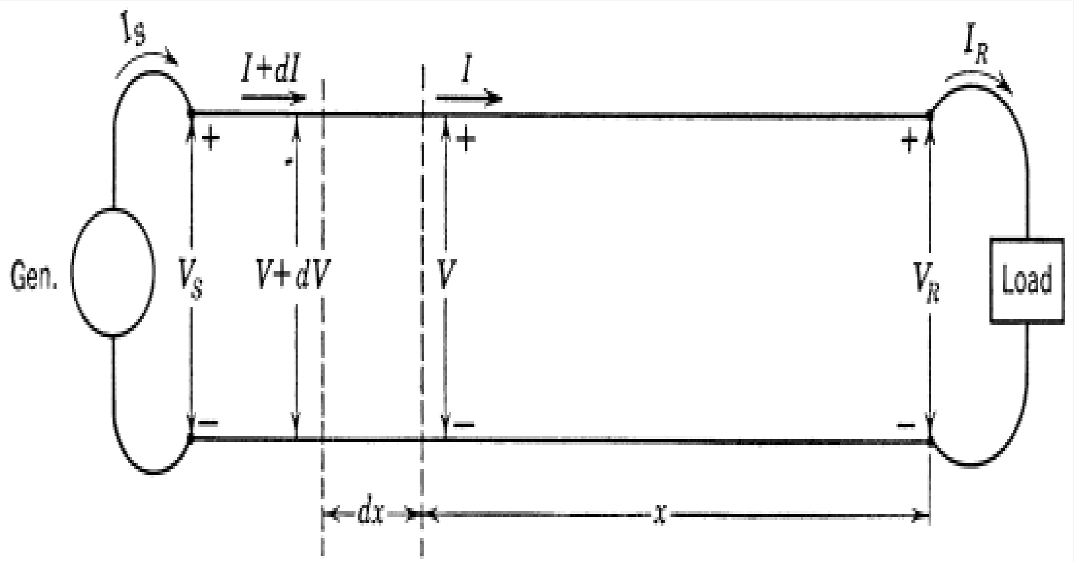
Therefore,

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$A = D = \cosh(\gamma l)$$

$$B = Z_c \sinh(\gamma l) \quad \Omega$$

$$C = \frac{1}{Z_c} \sinh(\gamma l) \quad S$$



x : distance measured from the receiving end

EXAMPLE

A three-phase 765-kV, 60-Hz, 300-km, completely transposed line has the following positive-sequence impedance and admittance:

$$z = 0.0165 + j0.3306 = 0.3310\underline{/87.14^\circ} \quad \Omega/\text{km}$$

$$y = j4.674 \times 10^{-6} \quad \text{S/km}$$

Assuming positive-sequence operation, calculate the exact *ABCD* parameters of the line. Compare the exact *B* parameter with that of the nominal π circuit.

Solution

$$Z_c = \sqrt{\frac{0.3310\angle 87.14^\circ}{4.674 \times 10^{-6}\angle 90^\circ}} = \sqrt{7.082 \times 10^4 \angle -2.86^\circ}$$

$$= 266.1\angle -1.43^\circ \quad \Omega$$

and

$$\gamma l = \sqrt{(0.3310\angle 87.14^\circ)(4.674 \times 10^{-6}\angle 90^\circ)} \times (300)$$

$$= \sqrt{1.547 \times 10^{-6}\angle 177.14^\circ} \times (300)$$

$$= 0.3731\angle 88.57^\circ = 0.00931 + j0.3730 \quad \text{per unit}$$

Solution

$$e^{\gamma l} = e^{0.00931} e^{+j0.3730} = 1.0094\underline{/0.3730} \quad \text{radians}$$

$$= 0.9400 + j0.3678$$

and

$$e^{-\gamma l} = e^{-0.00931} e^{-j0.3730} = 0.9907\underline{/-0.3730} \quad \text{radians}$$

$$= 0.9226 - j0.3610$$

$$\cosh(\gamma l) = \frac{(0.9400 + j0.3678) + (0.9226 - j0.3610)}{2}$$

$$= 0.9313 + j0.0034 = 0.9313\underline{/0.209^\circ}$$

$$\sinh(\gamma l) = \frac{(0.9400 + j0.3678) - (0.9226 - j0.3610)}{2}$$

$$= 0.0087 + j0.3644 = 0.3645\underline{/88.63^\circ}$$

Solution

$$A = D = \cosh(\gamma l) = 0.9313\angle 0.209^\circ \quad \text{per unit}$$

$$B = (266.1\angle -1.43^\circ)(0.3645\angle 88.63^\circ) = 97.0\angle 87.2^\circ \quad \Omega$$

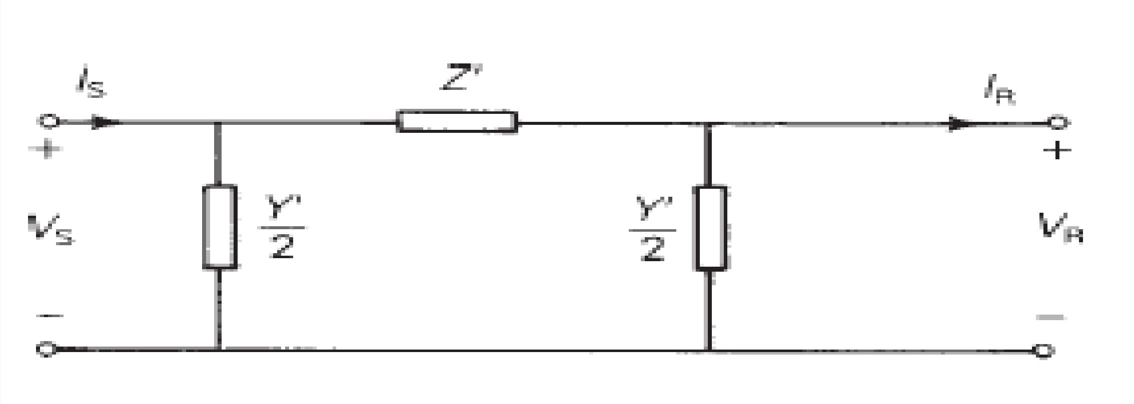
$$C = \frac{0.3645\angle 88.63^\circ}{266.1\angle -1.43^\circ} = 1.37 \times 10^{-3}\angle 90.06^\circ \quad \text{S}$$

$$B_{\text{nominal }\pi} = Z = (0.3310\angle 87.14^\circ)(300) = 99.3\angle 87.14^\circ \quad \Omega$$

which is 2% larger than the exact value.

EQUIVALENT π CIRCUIT OF A LONG LINE

The nominal-π circuit do not represent a transmission line exactly because it does not account for the parameters of the line being uniformly distributed. The discrepancy between the nominal π and the actual line becomes larger as the length of line increases. It is possible, however, to find the equivalent circuit of a long transmission line and to represent the line accurately, in so far as measurements at the ends of the line are concerned, by a network of lumped parameters.



The circuit shown in Figure is called an equivalent π circuit. It is identical in structure to the nominal π circuit except that the series arm of our equivalent-π circuit Z' and the shunt arms $Y'/2$ to distinguish them from the arms of the nominal-π circuit.

Equivalent Circuit of a Long Line

$$V_S = \left(\frac{Z'Y'}{2} + 1\right) V_R + Z' I_R$$

$$V_S = V_R \cosh\gamma l + I_R Z_C \sinh\gamma l$$

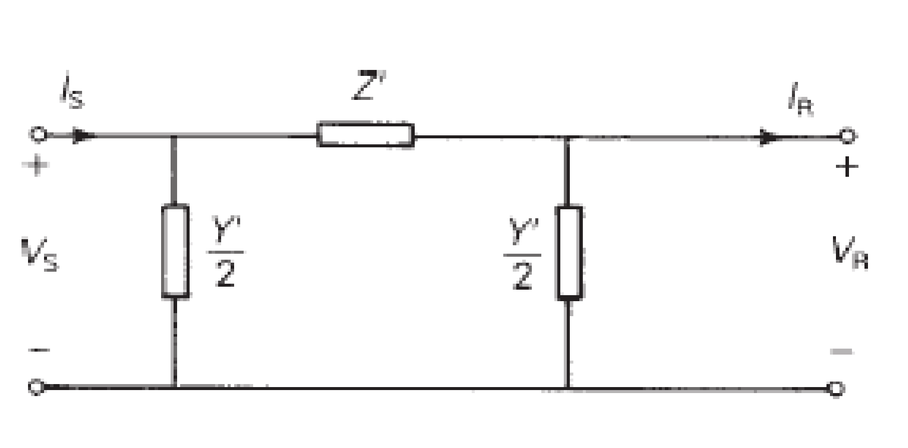
THEN,

$$Z' = Z_C \sinh\gamma l$$

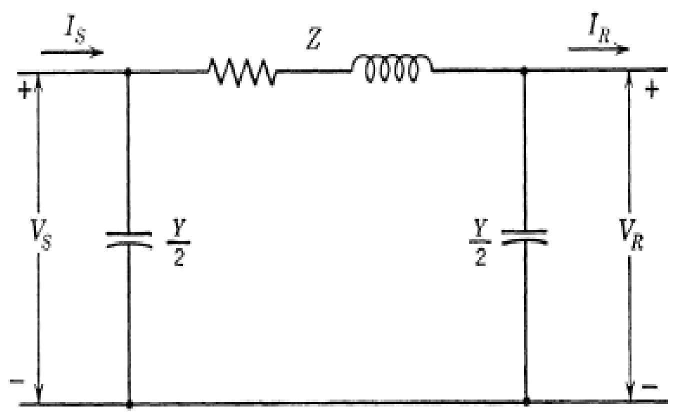
$$Z' = \sqrt{\frac{z}{y}} \sinh\gamma l = zl \frac{\sinh\gamma l}{\sqrt{zy}\, l}$$

$$Z' = Z \frac{\sinh\gamma l}{\gamma l}$$

$Z = zl$ total series impedance



Equivalent - π Model



Nominal π Model

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} \left(1 + \frac{YZ}{2}\right) & Z \\ Y\left(1 + \frac{YZ}{4}\right) & \left(1 + \frac{YZ}{2}\right) \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

Equivalent Circuit of a Long Line

(sinh γl)/γl term is a correction factor to convert series impedance for the nominal π circuit to and series impedance for the equivalent π circuit.

In order to find the shunt admittance

$$\mathrm{A} = \frac{Z'Y'}{2} + 1 = \cosh \gamma l$$

use $Z_C \sinh \gamma l$ instead of Z' then

$$\frac{Y' Z_C \sinh \gamma l}{2} + 1 = \cosh \gamma l$$

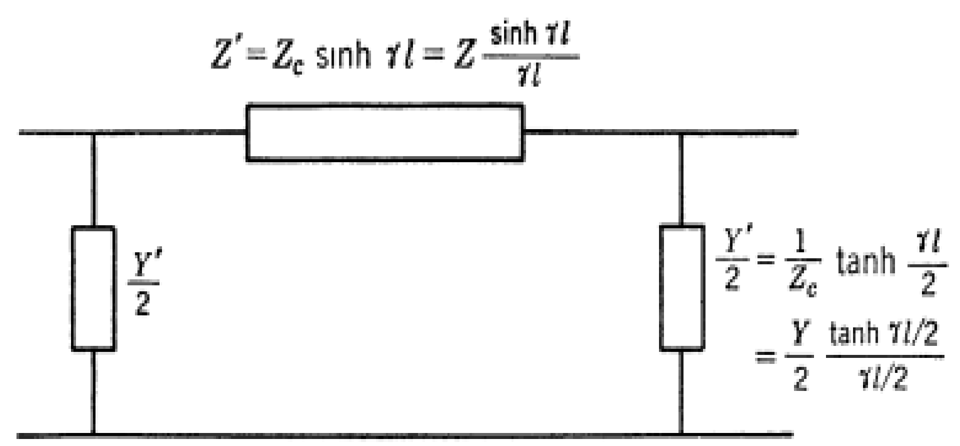
$$\frac{Y'}{2} = \frac{1}{Z_C} \frac{\cosh \gamma l - 1}{\sinh \gamma l}$$

o $$\tanh \frac{\gamma l}{2} = \frac{\cosh \gamma l - 1}{\sinh \gamma l}$$

Equivalent–π Circuit

$$\frac{Y'}{2}=\frac{\cosh(\gamma l)-1}{Z_c\sinh(\gamma l)}=\frac{\tanh(\gamma l/2)}{Z_c}=\frac{\tanh(\gamma l/2)}{\sqrt{\dfrac{z}{y}}}$$

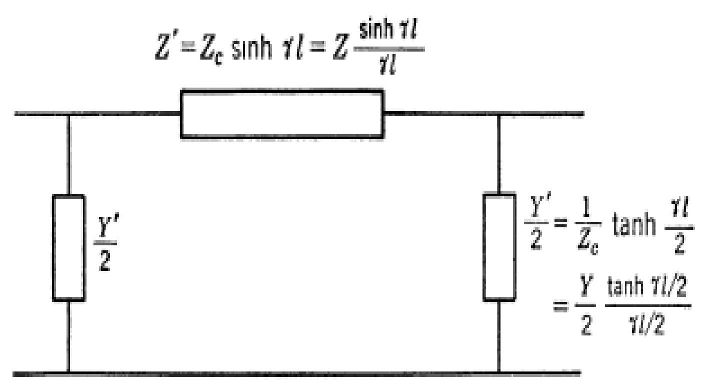
$$\frac{Y'}{2}=\frac{yl}{2}\left[\frac{\tanh(\gamma l/2)}{\sqrt{\dfrac{z}{y}}\dfrac{yl}{2}}\right]=\frac{yl}{2}\left[\frac{\tanh(\gamma l/2)}{\sqrt{zy}\,l/2}\right]$$



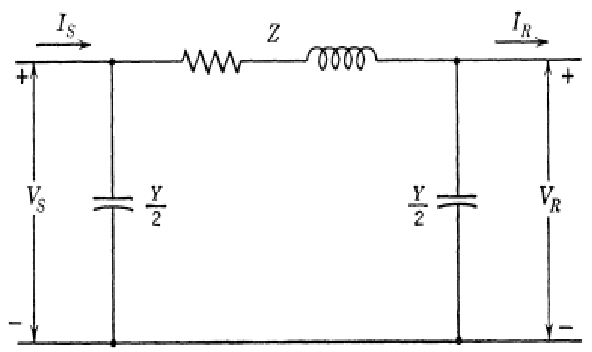
Correction Factors

$$F_1 = \frac{\sinh(\gamma l)}{\gamma l}$$

$$F_2 = \frac{\tanh(\gamma l/2)}{\gamma l/2}$$



Equivalent - π Model



Nominal – π Model

EXAMPLE

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$$z = 0.0165 + j0.3306 = 0.3310\angle 87.14^\circ \quad \Omega/\text{km}$$

$$y = j4.674 \times 10^{-6} \quad \text{S/km}$$

Compare the equivalent and nominal π circuits for the line

SOLUTION For the nominal π circuit,

$$Z = zl = (0.3310\angle 87.14^\circ)(300) = 99.3\angle 87.14^\circ \quad \Omega$$

$$\frac{Y}{2} = \frac{yl}{2} = \left(\frac{j4.674 \times 10^{-6}}{2}\right)(300) = 7.011 \times 10^{-4}\angle 90^\circ \quad \text{S}$$

$$Z_c = \sqrt{\frac{0.3310\angle 87.14^\circ}{4.674 \times 10^{-6}\angle 90^\circ}} = \sqrt{7.082 \times 10^4\angle -2.86^\circ}$$

$$= 266.1\angle -1.43^\circ \quad \Omega$$

and

$$\gamma l = \sqrt{(0.3310\angle 87.14^\circ)(4.674 \times 10^{-6}\angle 90^\circ)} \times (300)$$

$$= \sqrt{1.547 \times 10^{-6}\angle 177.14^\circ} \times (300)$$

$$= 0.3731\angle 88.57^\circ = 0.00931 + j0.3730 \quad \text{per unit}$$

$$e^{\gamma l} = e^{0.00931} e^{+j0.3730} = 1.0094\angle 0.3730 \quad \text{radians}$$

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Correction Factors

$$F_1 = \frac{\sinh(\gamma l)}{\gamma l}$$

$$F_1 = \frac{0.3645\angle 88.63^\circ}{0.3731\angle 88.57^\circ} = 0.9769\angle 0.06^\circ \quad \text{per unit}$$

$$F_2 = \frac{\tanh(\gamma l/2)}{\gamma l/2} = \frac{\cosh(\gamma l) - 1}{(\gamma l/2)\,\sinh(\gamma l)}$$

$$= \frac{0.9313 + j0.0034 - 1}{\left(\frac{0.3731}{2}\angle 88.57^\circ\right)(0.3645\angle 88.63^\circ)}$$

$$= \frac{-0.0687 + j0.0034}{0.06800\angle 177.20^\circ}$$

$$= \frac{0.06878\angle 177.17^\circ}{0.06800\angle 177.20^\circ} = 1.012\angle -0.03^\circ \quad \text{per unit}$$

EXAMPLE

$$Z' = (99.3\angle 87.14^\circ)(0.9769\angle 0.06^\circ) = 97.0\angle 87.2^\circ \quad \Omega$$

$$\frac{Y'}{2} = (7.011 \times 10^{-4}\angle 90^\circ)(1.012\angle -0.03^\circ) = 7.095 \times 10^{-4}\angle 89.97^\circ \quad \text{S}$$

$$= 3.7 \times 10^{-7} + j7.095 \times 10^{-4} \quad \text{S}$$

Comparing these nominal and equivalent π circuit values, Z' is about 2% smaller than Z, and $Y'/2$ is about 1% larger than $Y/2$. Although the circuit values are approximately the same for this line, the equivalent π circuit should be used for accurate calculations involving long lines. Note the small shunt conductance, $G' = 3.7 \times 10^{-7}$ S, introduced in the equivalent π circuit.

TUTORIAL

3 Phase 60 Hz transmission line with l = 230 mil es. 125 MW is transferred to the receiving end at unity poer factor. Receiving end voltage is 215 KV (line-line)

Line parameters are

z = $0.8431\angle 79.04^\circ \ \Omega/\text{mi}$

y = $5.105 \times 10^{-6}\angle 90^\circ \ \text{S/mi}$

Find the sending end voltage and current.

$$\gamma l = \sqrt{yz}\, l = 230\sqrt{0.8431 \times 5.105 \times 10^{-6}}\ \angle \frac{79.04^\circ + 90^\circ}{2}$$

$$= 0.4772\angle 84.52^\circ = 0.0456 + j0.4750$$

$$Z_c = \sqrt{\frac{z}{y}} = \sqrt{\frac{0.8431}{5.105 \times 10^{-6}}}\ \angle \frac{79.04^\circ - 90^\circ}{2} = 406.4\angle -5.48^\circ \ \Omega$$

$$V_R = \frac{215{,}000}{\sqrt{3}} = 124{,}130\angle 0^\circ \ \text{V to neutral}$$

$$I_R = \frac{125{,}000{,}000}{\sqrt{3} \times 215{,}000} = 335.7\angle 0^\circ \ \text{A}$$

TUTORIAL

$$\cosh(\alpha + j\beta) = \frac{\varepsilon^{\alpha}\varepsilon^{j\beta} + \varepsilon^{-\alpha}\varepsilon^{-j\beta}}{2} = \frac{1}{2}\left(\varepsilon^{\alpha}\angle\beta + \varepsilon^{-\alpha}\angle-\beta\right)$$

$$\sinh(\alpha + j\beta) = \frac{\varepsilon^{\alpha}\varepsilon^{j\beta} - \varepsilon^{-\alpha}\varepsilon^{-j\beta}}{2} = \frac{1}{2}\left(\varepsilon^{\alpha}\angle\beta - \varepsilon^{-\alpha}\angle-\beta\right)$$

$$\cosh \gamma l = \frac{1}{2}\varepsilon^{0.0456}\angle 27.22^\circ + \frac{1}{2}\varepsilon^{-0.0456}\angle -27.22^\circ$$

$$= 0.4654 + j0.2394 + 0.4248 - j0.2185$$

$$= 0.8902 + j0.0209 = 0.8904\angle 1.34^\circ$$

$$\sinh \gamma l = 0.4654 + j0.2394 - 0.4248 + j0.2185$$

$$= 0.0406 + j0.4579 = 0.4597\angle 84.93^\circ$$

$$0.4750 \text{ rad} = 27.22^\circ$$

ANGLE= 180x0.475/3.14

= 27.22 degrees

TUTORIAL

$$V_S = V_R \cosh \gamma l + I_R Z_c \sinh \gamma l$$

$$I_S = I_R \cosh \gamma l + \frac{V_R}{Z_c} \sinh \gamma l$$

$$V_S = 124{,}130 \times 0.8904\angle 1.34^\circ + 335.7 \times 406.4\angle -5.48^\circ \times 0.4597\angle 84.93^\circ$$

$$= 110{,}495 + j2{,}585 + 11{,}483 + j61{,}656$$

$$= 137{,}860\angle 27.77^\circ \text{ V}$$

$$I_S = 335.7 \times 0.8904\angle 1.34^\circ + \frac{124{,}130}{406.4\angle -5.48^\circ} \times 0.4597\angle 84.93^\circ$$

$$= 298.83 + j6.99 - 1.00 + j140.41$$

$$= 332.31\angle 26.33^\circ \text{ A}$$

$$V_S = \sqrt{3} \times 137.86 = 238.8 \text{ kV}$$