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EE 471 POWER SYSTEM ANALYSIS I

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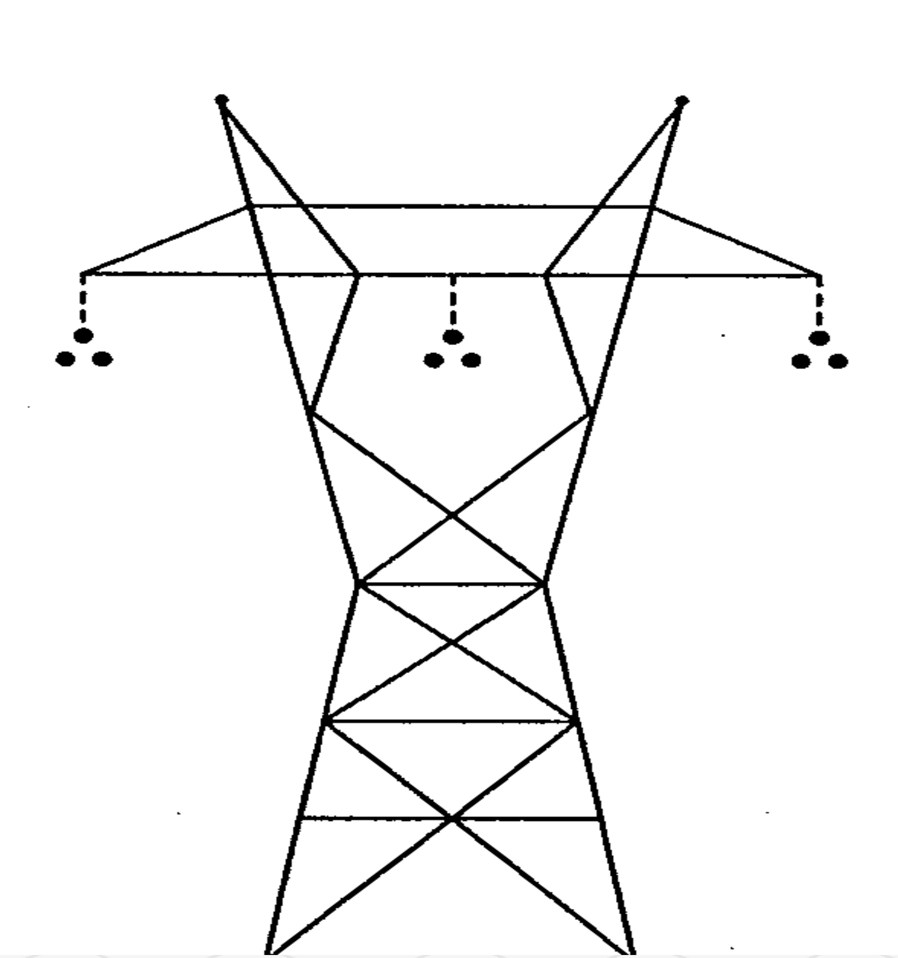
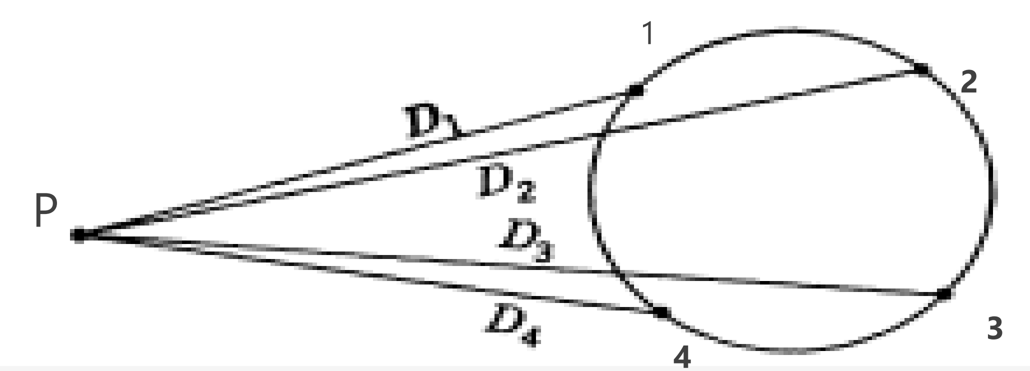
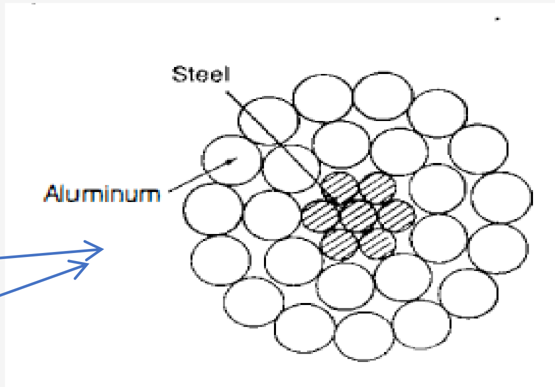
OUTLINES 1



INDUCTANCE AND CAPACITANCE CALCULATIONS AND GMD



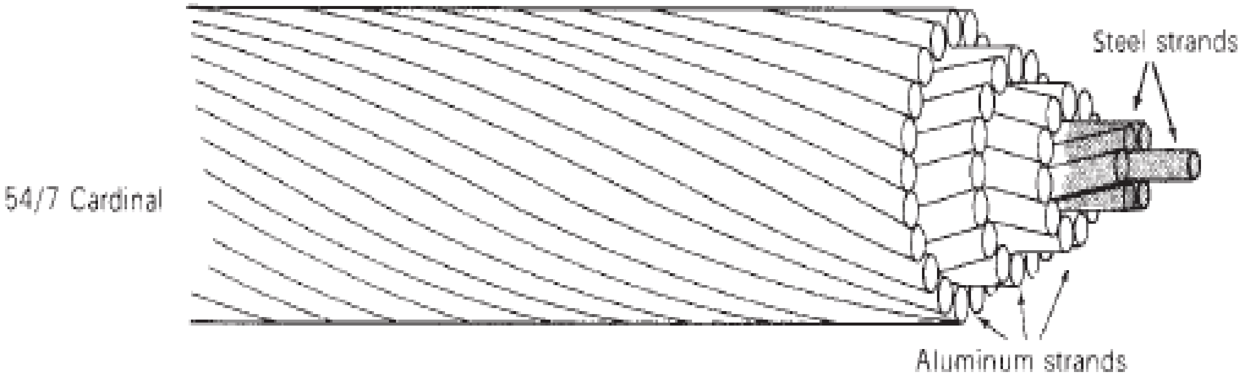
$$GMD = \sqrt[4]{D_1 D_2 D_3 D_4}$$



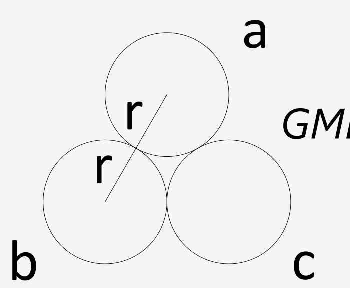
$$L = 2 \times 10^{-7} \ln \frac{D_m}{D_s} \; henrys/meter$$

$$C_{an} = \frac{2\pi\varepsilon}{\ln(D_{eq}/r)} \quad F/m$$

$$D_{eq} = \sqrt[3]{D_{ab} D_{bc} D_{ac}}$$



Self GMD (GMR)



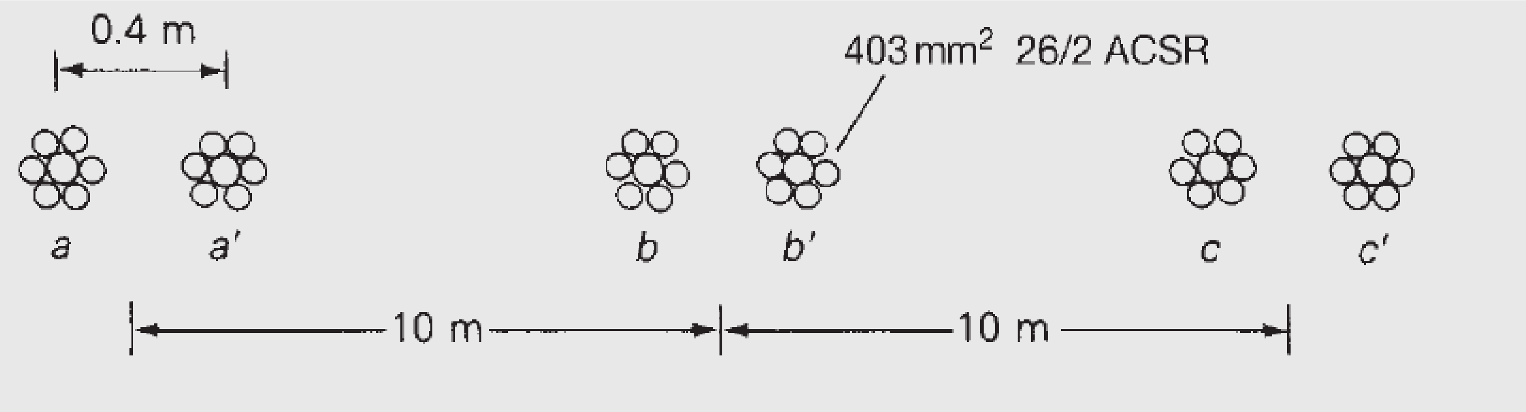
GMR(Self GMD)=?

$$GMR = (D_{aa}D_{ab}D_{ac}D_{bb}D_{ba}D_{bc}D_{cc}D_{ca}D_{cb})^{1/9}$$

$$GMR = (0.7788r \times 2r \times 2r \times 0.7788r \times 2r \times 2r \times 0.7788r \times 2r \times 2r)^{1/9}$$

$$GMR = (0.7788r \times 2r \times 2r)^{1/3} = 1.46r$$

EXAMPLE



3 Phase Line with bundle conductors. SGMR of stranded conductor is 0.0114 m .

Line is transposed. Calculate the line inductance.. (f=60 Hz, l =200 km))

$$D_{eq} = \sqrt[3]{(10)(10)(20)} = 12.6 \text{ m}$$

$$D_{SL} = \sqrt{(0.0114)(0.40)} = 0.0676 \text{ m}$$

$$L_a = 2 \times 10^{-7} \ln\left(\frac{12.6}{0.0676}\right)(1000)(200) = 0.209 \text{ H}$$

$$X_a = 2\pi f L_1 = (2\pi)(60)(0.209) = 78.8 \ \Omega$$

BUNDLED CONDUCTORS

Two-conductor bundle:

$$D_{SL} = \sqrt[4]{(D_S \times d)^2} = \sqrt{D_S d}$$

Three-conductor bundle:

$$D_{SL} = \sqrt[9]{(D_S \times d \times d)^3} = \sqrt[3]{D_S d^2}$$

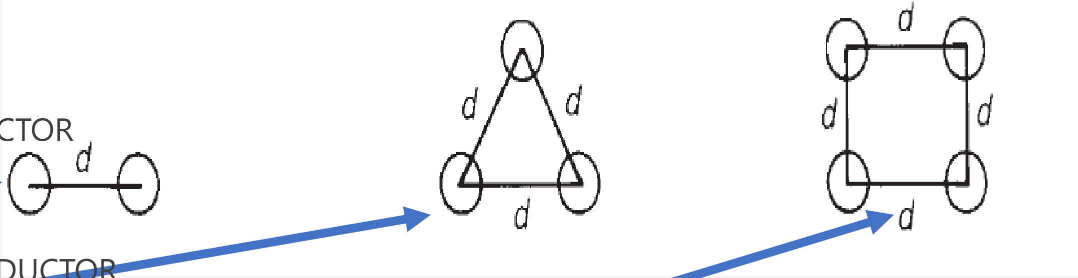
Four-conductor bundle:

$$D_{SL} = \sqrt[16]{(D_S \times d \times d \times d\sqrt{2})^4} = 1.091\sqrt[4]{D_S d^3}$$

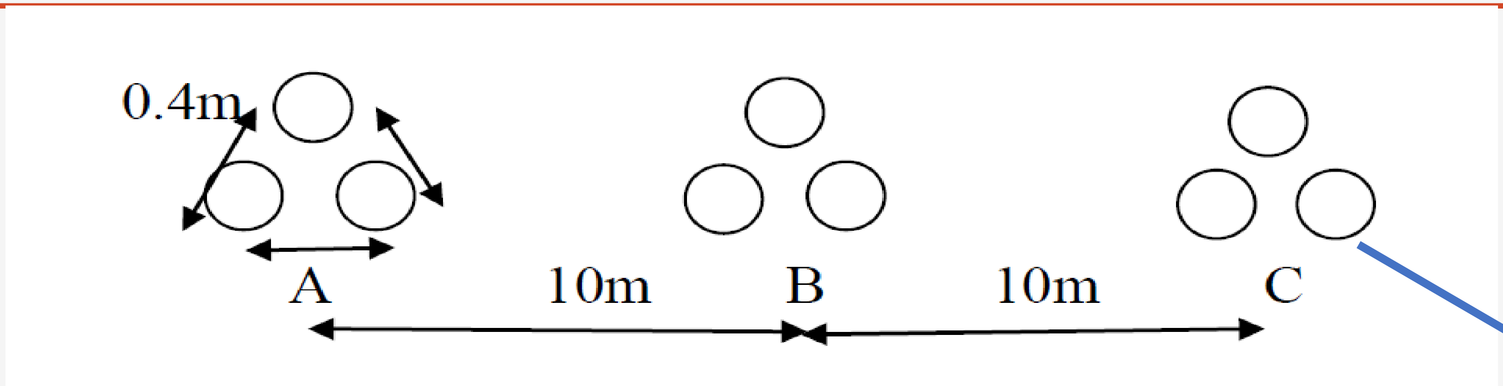
TWO CONDUCTOR

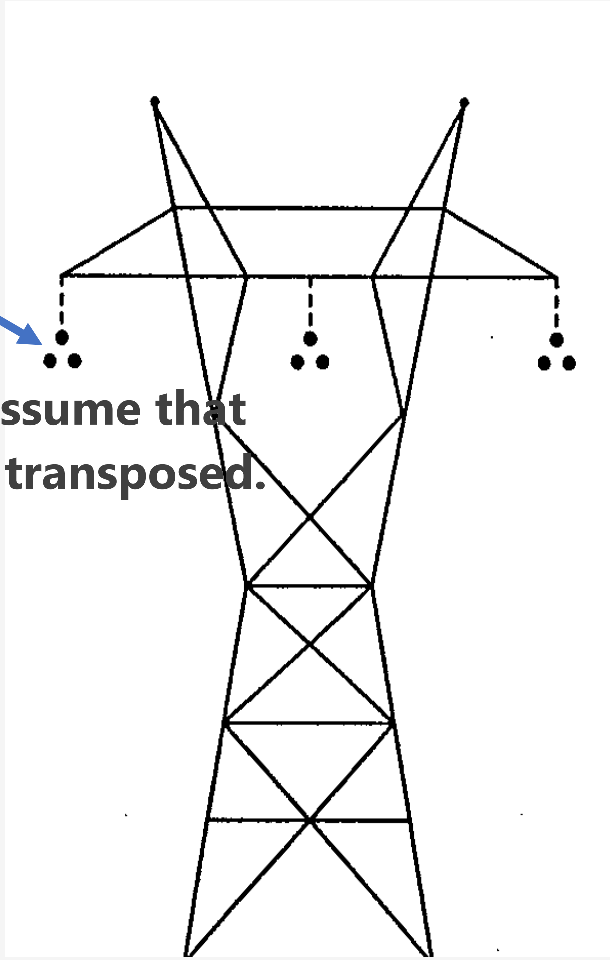
THREE CONDUCTOR

FOUR CONDUCTOR



TUTORIAL





Three phase, 50 Hz transmission line with three conductor bundles. Assume that sub conductor is solid round with r = 1.8 cm. The line is completely transposed.

a) Find the inductance per phse per unit length.

b) If the line is a 350 km line calculate the line reactance per phase.

SOLUTION

a)

$$D_S = \left(r^{1} \times 0.4 \times 0.4 \times r^{1} \times 0.4 \times 0.4 \times r^{1} \times 0.4 \times 0.4 \right)^{1/9}$$

$$r^{I} = 0.7788 \times 1.8 \times 10^{-2} \text{ m}$$

$$D_S = 0.13 \text{ m}$$

$$D_m = (D_{AB}\, D_{AC}\, D_{BC})^{1/3} \quad , \quad D_m = 12.6 \text{ m}$$

$$L = 2 \times 10^{-7} \ln \left(D_m / D_S \right) \text{ H/m per phase}$$

$$L = 9.133 \times 10^{-7} \text{ H/m per phase}$$

b)

$$X = j\,(2\Pi \times 50) \times 9.133 \times 10^{-7} \times 350000 = j\,100.4 \text{ ohm}$$

LINE MODELS

SHORT LINE

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \left[\begin{array}{c|c} 1 & Z \\ \hline 0 & 1 \end{array}\right] \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$A = D = 1$$
$$B = Z \quad \Omega$$
$$C = 0 \quad \mathrm{S}$$

MEDIUM LENGTH

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \left[\begin{array}{c|c} \left(1+\frac{YZ}{2}\right) & Z \\ \hline Y\left(1+\frac{YZ}{4}\right) & \left(1+\frac{YZ}{2}\right) \end{array}\right] \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$A = D = 1 + \frac{YZ}{2}$$
$$B = Z \quad \Omega$$
$$C = Y\left(1 + \frac{YZ}{4}\right)$$

LONG LINE

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$A = D = \cosh(\gamma l)$$
$$B = Z_c \sinh(\gamma l) \quad \Omega$$
$$C = \frac{1}{Z_c} \sinh(\gamma l) \quad \mathrm{S}$$

LONG LINE

$$V(x) = \left(\frac{e^{\gamma x} + e^{-\gamma x}}{2}\right) V_R + Z_c \left(\frac{e^{\gamma x} - e^{-\gamma x}}{2}\right) I_R$$

$$I(x) = \frac{1}{Z_c}\left(\frac{e^{\gamma x} - e^{-\gamma x}}{2}\right) V_R + \left(\frac{e^{\gamma x} + e^{-\gamma x}}{2}\right) I_R$$

$$\cosh(\gamma l) = \frac{e^{\gamma l} + e^{-\gamma l}}{2} = \frac{1}{2}\left(e^{\alpha l}\underline{/\beta l} + e^{-\alpha l}\underline{/-\beta l}\right)$$

$$\sinh(\gamma l) = \frac{e^{\gamma l} - e^{-\gamma l}}{2} = \frac{1}{2}\left(e^{\alpha l}\underline{/\beta l} - e^{-\alpha l}\underline{/-\beta l}\right)$$

$$V(x) = V_R \cosh \gamma x + I_R Z_C \sinh \gamma x \quad (4.22)$$

$$I(x) = I_R \cosh \gamma x + \frac{V_R}{Z_C} \sinh \gamma x$$

$$V = \frac{V_R + I_R Z_C}{2} e^{\alpha x} e^{j\beta x} + \frac{V_R - I_R Z_C}{2} e^{-\alpha x} e^{-j\beta x}$$

$$I = \frac{V_R/Z_C + I_R}{2} e^{\alpha x} e^{j\beta x} - \frac{V_R/Z_C - I_R}{2} e^{-\alpha x} e^{-j\beta x}$$

Incedent wave

Reflected wave

Probagation constant

$$\gamma = \alpha + j\beta \quad \text{m}^{-1}$$

The quantity γl is dimensionless. Also

$$e^{\gamma l} = e^{(\alpha l + j\beta l)} = e^{\alpha l} e^{j\beta l} = e^{\alpha l}\underline{/\beta l}$$

TUTORIAL PROBLEM

3 Phase, 50 Hz power transmission line has the following line parameters. Receiving end is unloaded with Vr = 132 KV.

- Determine the incident voltage wave at the receiving end.
- Determine the reflected voltage wave at the receiving end.
- Find the voltage at 120 km away from the receiving end.

$R = 0.2 \text{ ohm}, L = 1.3 \text{ mH and } C = 0.01 \ \mu\text{F}$ → **Per km.**

$$z = r + jx = (0.2 + j1.3 \times 314 \times 10^{-3}) = (0.2 + j0.408)$$
$$= 0.454 \angle 63.88°$$

$$\text{shunt admittance} = j\omega C = j314 \times 0.01 \times 10^{-6}$$
$$= 3.14 \times 10^{-6} \angle +90°$$

$$Z_c = \sqrt{\frac{z}{y}} = \sqrt{\frac{0.454}{0.314} \times 10^5 \angle 63.88 - 90°}$$
$$= 380 \angle -13.06°$$

$$\gamma = \sqrt{yz} = \sqrt{0.314 \times 0.454 \times 10^{-6}} \angle (90 + 63.88)°/2$$
$$= (0.2714 + j1.169) \times 10^{-3}$$
$$= (\alpha + j\beta)$$

$$\text{Vr} = \frac{132 \times 1000}{\sqrt{3}} = 76200 \text{ volts}$$

Incident voltage wave at the Receiving end $= \dfrac{V_r + I_r Z_c}{2}$ $\quad I_r = 0.$

$$\frac{V_r}{2} = \frac{76200}{2} = 38100 \text{ volts}$$

Reflected voltage wave At the receiving end. → $\dfrac{V_r - I_r Z_c}{2} = \dfrac{V_r}{2} = 38100 \text{ volts}$

SOLUTION

Incident wave at

120 km. = 38.1 $\exp(0.2714 \times 120 \times 10^{-3}) \exp(j1.169 \times 120 \times 10^{-3})$

$= 39.35 \angle 8.02°$ kV

Reflected voltage

Wave at 120 km= 38.1 $\exp(-\alpha x)\exp(-j\beta x)$ = 38.1 $\exp(-0.0325)\exp(-j0.140)$

$= 36.88 \angle -8.02°$ kV.

Voltage at 120 km away from the receiving end is the summation of the voltage waves.

TUTORIAL

$R = 0.2$ ohm, $L = 1.3$ mH and $C = 0.01\ \mu F$

Km başına

$$z = r + jx = (0.2 + j1.3 \times 314 \times 10^{-3}) = (0.2 + j0.408)$$
$$= 0.454 \angle 63.88°$$

$$\text{shunt admittance} \quad = j\omega C = j314 \times 0.01 \times 10^{-6}$$
$$= 3.14 \times 10^{-6} \angle{+90°}$$

$$Z_c = \sqrt{\frac{z}{y}} = \sqrt{\frac{0.454}{0.314} \times 10^5 \angle 63.88 - 90°}$$
$$= 380 \angle{-13.06°}$$

$$\gamma = \sqrt{yz} = \sqrt{0.314 \times 0.454 \times 10^{-6}} \quad \angle(90 + 63.88)°/2$$
$$= (0.2714 + j1.169) \times 10^{-3}$$
$$= (\alpha + j\beta)$$

For the same line assume that 40 MW is transferred to The reciving end with pf 0.8 lag. If the receiving end is at 132 KV find the sending end voltage.

SOLUTION

$$I_r = \frac{40 \times 1000}{\sqrt{3} \times 132 \times 0.8}$$

$$= 218.7 \text{ amps.}$$

$$I_r = 218.7 \angle -36.8^\circ$$

$$e^{\alpha x} e^{j\beta x} = 1.033 \angle 8.02^\circ$$

$$e^{-\alpha x} e^{-j\beta x} = 0.968 \angle -8.02^\circ$$

For 120 kms.

$$V_s^+ = \frac{V_r + I_r Z_c}{2} e^{\alpha x} e^{j\beta x}$$

$$= \frac{76200 + 380 \times 218.7 \angle -13.06 \angle -36.8}{2} \times 1.033 \angle 8.02^\circ$$

$$= 74.63 \angle -18^\circ$$

$$V_s^- = \frac{V_r - I_r Z_c}{2} e^{-\alpha x} e^{-j\beta x}$$

$$= \frac{76200 - 380 \times 218.7 \angle -49.86}{2} \times 0.968 \angle -8.02^\circ$$

$$= 32.619 \angle 62.37 \text{ kV}$$

$$V_s = V_s^+ + V_s^- = 74.63 \angle -18^\circ + 32.619 \angle 62.37^\circ$$

$$= 86077 + j5751 = 86.26 \angle 3.82^\circ$$

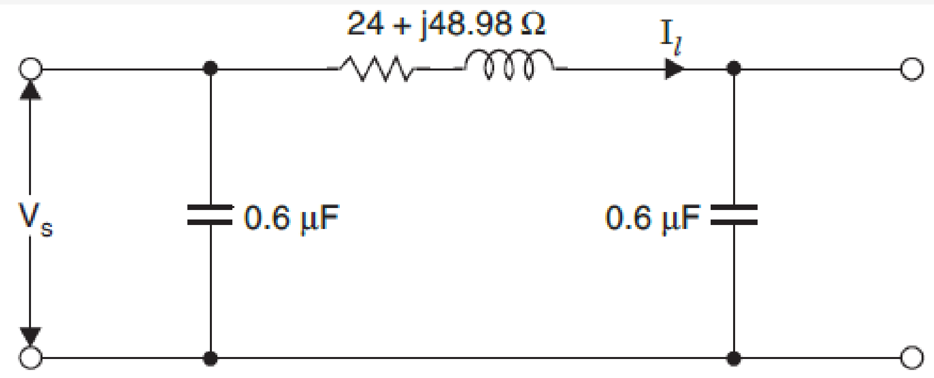
TUTORIAL

Solve the same problem with nominal -π.

Resistance $= 0.2 \times 120 = 24$ ohms

Reactance $= 1.3 \times 10^{-3} \times 120 \times 314 = 48.98\ \Omega$

Capacitance $= 0.01 \times 10^{-6} \times 120 = 1.2\ \mu F$



$$I_r = 218.7(0.8 - j0.6) = 174.96 - j131.22$$

$$I_{c_1} = j314 \times 0.6 \times 10^{-6} \times 76200 = j14.356 \text{ amp}$$

$$I_l = I_{c_1} + I_r = 174.96 - j116.86 = 210.39 \angle 33.73°$$

$$V_s = 76200 + (174.96 - j116.86)(24 + j48.38)$$
$$= 76200 + 4199 + j8596 - j2804 + 5723$$
$$= 86122 + j5765$$
$$= 86314 \angle 3.82° \text{ volts}$$

TUTORIAL

3 phase , 50 Hz power line. Distances between phases are in order 3.7 , 6.475 and 7.4 meters. The line is completely transposed . Conductor SGMR = 0.978 cm.

R= 0.1557 ohm/km is defined per phase. 50 MVA is transferred at 0.8 pf lag with the receiving end voltage of 132 KV. Find Vs , Is and A B C D line parameters.

SOLUTION

$$R = 0.1557 \times 160 = 24.9\ \Omega$$

$$\text{GMD} = \sqrt[3]{3.7 \times 6.475 \times 7.4}$$

$$= 5.6 \text{ metre.}$$

$$\text{Inductance} = 2 \times 10^{-7} \left(\ln \frac{560}{0.978} \right) \times 160 \times 1000$$

$$= 0.2032 \text{ H}$$

$$X_L = 63.8$$

$$C = \frac{2\pi \in_0}{\ln \frac{560}{0.978}} \times 160 \times 1000$$

$$= \frac{1}{36\pi} \times 10^{-9} \left(\frac{2\pi}{\ln \frac{560}{0.978}} \right) \times 160 \times 1000$$

$$= \frac{10^{-9}}{18} \times \frac{160000}{\ln \left(\frac{560}{0.978} \right)}$$

$$= 1399 \times 10^{-9} \text{ F}$$

$$= 1.399\ \mu\text{F}$$

$$Z = \sqrt{0.1557^2 + 0.39875^2} = 0.428\underline{/68.67^\circ}$$

Solution

$$j\omega C = j \times 314 \times \frac{1.399}{160} \times 10^{-6}$$
$$= j\,2.745 \times 10^{-6}$$
$$Z_c = \sqrt{\frac{0.428}{2.745} \times 10^{+6} \angle(68.67 - 90)}$$
$$= 394.8\angle -10.66^\circ$$
$$\gamma = \sqrt{0.428 \times 2.745 \times 10^{-6} \angle (90 + 68.67)}$$
$$= 1.084 \times 10^{-3}\angle 79.335^\circ$$
$$= (0.2 + j1.0653) \times 10^{-3}$$
$$\gamma l = (0.2 + j1.0653) \times 10^{-3} \times 160$$
$$= 0.0320 + j0.17$$
$$A = \cosh \gamma l = \cosh (0.032 + j0.17)$$
$$= \cosh 0.032 \cos 0.17 + \sinh 0.032 \sin 0.17$$
$$= 0.9855 + j0.032 \times 0.16926$$
$$= 0.9855 + j0.005417$$
$$= 0.9855\angle 0.32^\circ$$
$$B = Z_c \sinh \gamma l$$
$$\sinh \gamma l = \sinh 0.032 \cos 0.17 + j \cosh 0.032 \sin 0.17$$
$$= 0.032 \times 0.9835 + j0.9855 \times 0.17$$
$$= 0.0315 + j0.167535$$
$$= 0.17047\angle 79.35^\circ$$
$$B = 394.8 \times 0.17047\angle 79.35 - 10.66$$
$$= 67.3\angle 68.69$$
$$C = \frac{\sinh \gamma l}{Z_c} = \frac{0.17047}{394.8}$$
$$= 4.3 \times 10^{-4}\angle 90^\circ$$

Solution

$$I_r = \frac{50{,}000}{\sqrt{3} \times 132} = 218.68 \text{ A}$$

$$V_s = 0.9855\underline{/0.32°} \times 76.208 + 67.3\underline{/68.69°} \times 218.68\underline{/-36.87°} \times 10^{-3}$$

$$= 75.103\underline{/0.32°} + 14.717\underline{/31.82°}$$

$$= 75.103 + 12.505 + j7.76$$

$$= 87.608 + j7.76$$

$$= 87.95\underline{/5.06°}$$

V_s line to line = 152.34 kV. **Ans.**

$$I_s = 4.3 \times 10^{-4}\underline{/90°} \times 76.208 \times 10^3 + 0.9855\underline{/0.32°} \times 218.68\underline{/-36.87°}$$

$$= 32.76\underline{/90°} + 215.51\underline{/-36.55°}$$

$$= j\,32.76 + 173.12 - j\,128.34$$

$$= 173.12 - j\,95.58$$

$= 197.75\underline{/-28.9°}$. **Ans.**