

CS229 Problem Set 1

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1 Question 1: Linear Classifiers

1.1 Question 1(a)

Lemma 1.1.

$$g'(z) = g(z) (1 - g(z)) \quad (1.1)$$

Lemma 1.2. For every $x, z \in \mathbb{R}^n$,

$$\sum_i \sum_j z_i x_i z_j x_j = (x^T z)^2 \geq 0 \quad (1.2)$$

$$\nabla_{\theta} J(\theta) = -\frac{1}{n} \sum_{i=1}^n \left(y^{(i)} \frac{g'(\theta^T x^{(i)})}{g(\theta^T x^{(i)})} - (1 - y^{(i)}) \frac{g'(\theta^T x^{(i)})}{1 - g(\theta^T x^{(i)})} \right) x^{(i)} \quad (1.3)$$

$$= -\frac{1}{n} \sum_{i=1}^n \left(y^{(i)} (1 - g(\theta^T x^{(i)})) - (1 - y^{(i)}) g(\theta^T x^{(i)}) \right) x^{(i)} \quad (1.4)$$

$$= -\frac{1}{n} \sum_{i=1}^n \left(y^{(i)} - g(\theta^T x^{(i)}) \right) x^{(i)} \quad (1.5)$$

$$\implies \frac{\partial J(\theta)}{\partial \theta_j} = -\frac{1}{n} \sum_{i=1}^n \left(y^{(i)} - g(\theta^T x^{(i)}) \right) x_j^{(i)} \quad \forall j \in [d] \quad (1.6)$$

$$\implies \forall k \in [d], \frac{\partial^2 J(\theta)}{\partial \theta_j \partial \theta_k} = -\frac{1}{n} \sum_{i=1}^n \left(-g(\theta^T x^{(i)}) (1 - g(\theta^T x^{(i)})) x_k^{(i)} \right) x_j^{(i)} \quad (1.7)$$

$$= \frac{1}{n} \sum_{i=1}^n \left(g(\theta^T x^{(i)}) (1 - g(\theta^T x^{(i)})) x_j^{(i)} x_k^{(i)} \right) \quad (1.8)$$

Therefore, $H_J(\theta)$ can be constructed from the array of second order derivatives of $J(\theta)$ as

$$H_J(\theta)_{j,k} := \frac{1}{n} \sum_{i=1}^n \left(g(\theta^T x^{(i)}) (1 - g(\theta^T x^{(i)})) x_j^{(i)} x_k^{(i)} \right) \quad (1.9)$$

Notice that since $g(\theta^T x^{(i)}) \in (0, 1)$, therefore $g(\theta^T x^{(i)}) (1 - g(\theta^T x^{(i)})) > 0$ for every θ and $x^{(i)}$.³

Proof. Show that $H_J(\theta) \succeq 0$: let $z = (z_1, \dots, z_d) \in \mathbb{R}^d$, then

$$z^T H_J(\theta) \in \mathbb{R}^{1 \times d} \quad (1.10)$$

Then the β^{th} column of $z^T H_J(\theta)$ is

$$z^T H_J(\theta)_{\beta} = \frac{1}{n} \sum_{\alpha=1}^d \sum_{i=1}^n g(\theta^T x^{(i)}) (1 - g(\theta^T x^{(i)})) z_{\alpha} x_{\alpha}^{(i)} x_{\beta}^{(i)} \quad (1.11)$$

Therefore

$$z^T H_J(\theta) z = \frac{1}{n} \sum_{\beta=1}^d \sum_{\alpha=1}^d \sum_{i=1}^n g(\theta^T x^{(i)}) (1 - g(\theta^T x^{(i)})) z_{\alpha} x_{\alpha}^{(i)} x_{\beta}^{(i)} z_{\beta} \quad (1.12)$$

$$= \frac{1}{n} \sum_{i=1}^n g(\theta^T x^{(i)}) (1 - g(\theta^T x^{(i)})) \sum_{\beta=1}^d \sum_{\alpha=1}^d z_{\alpha} x_{\alpha}^{(i)} x_{\beta}^{(i)} z_{\beta} \quad (1.13)$$

$$= \frac{1}{n} \sum_{i=1}^n \underbrace{g(\theta^T x^{(i)}) (1 - g(\theta^T x^{(i)}))}_{>0 \quad \because g(\cdot) \in (0,1)} \underbrace{(z^T x)^2}_{\geq 0} \geq 0 \quad (1.14)$$

Hence, $H_J(\theta) \succeq 0$ is shown by showing $z^T H_J(\theta) z$ for every $z \in \mathbb{R}^d$. ■

1.2 Question 1(c)

Proof. By Bayes' theorem,

$$p(y = 1|x; \phi, \mu_0, \mu_1, \Sigma) = \frac{p(x|y = 1; \phi, \mu_0, \mu_1, \Sigma)p(y = 1; \phi, \mu_0, \mu_1, \Sigma)}{p(x; \phi, \mu_0, \mu_1, \Sigma)} \quad (1.15)$$

Define

$$z := \frac{p(x|y = 1; \phi, \mu_0, \mu_1, \Sigma)p(y = 1; \phi, \mu_0, \mu_1, \Sigma)}{p(x; \phi, \mu_0, \mu_1, \Sigma)} \quad (1.16)$$

$$\Theta := \{\phi, \mu_0, \mu_1, \Sigma\} \quad (1.17)$$

Conditioned on particular x , y is either 0 or 1, therefore,

$$p(y = 0|x; \Theta) = 1 - z \quad (1.18)$$

$$\implies \frac{z}{1 - z} = \frac{p(y = 1|x; \Theta)}{p(y = 0|x; \Theta)} \quad (1.19)$$

$$= \frac{p(x|y = 1; \Theta)p(y = 1; \Theta)}{p(x|y = 0; \Theta)p(y = 0; \Theta)} \quad (1.20)$$

$$= \frac{\phi \exp\left(-\frac{1}{2}(x - \mu_1)^T \Sigma^{-1}(x - \mu_1)\right)}{1 - \phi \exp\left(-\frac{1}{2}(x - \mu_0)^T \Sigma^{-1}(x - \mu_0)\right)} \quad (1.21)$$

$$\implies \log \frac{z}{1 - z} = \log \frac{\phi}{1 - \phi} \quad (1.22)$$

$$+ \left(-\frac{1}{2}x^T \Sigma^{-1}x + \mu_1^T \Sigma^{-1}x - \frac{1}{2}\mu_1^T \Sigma^{-1}\mu_1 \right) \quad (1.23)$$

$$- \left(-\frac{1}{2}x^T \Sigma^{-1}x + \mu_0^T \Sigma^{-1}x - \frac{1}{2}\mu_0^T \Sigma^{-1}\mu_0 \right) \quad (1.24)$$

$$= \log \frac{\phi}{1 - \phi} + \left((\mu_1 - \mu_0)^T \Sigma^{-1}x + \frac{1}{2}\mu_0^T \Sigma^{-1}\mu_0 - \frac{1}{2}\mu_1^T \Sigma^{-1}\mu_1 \right) \quad (1.25)$$

$$\implies \frac{z}{1 - z} = \exp \left(\underbrace{\log \frac{\phi}{1 - \phi} + \left((\mu_1 - \mu_0)^T \Sigma^{-1}x + \frac{1}{2}\mu_0^T \Sigma^{-1}\mu_0 - \frac{1}{2}\mu_1^T \Sigma^{-1}\mu_1 \right)}_{=: \Delta} \right) \quad (1.26)$$

$$\implies z = \frac{\exp(\Delta)}{1 + \exp(\Delta)} = \frac{1}{1 + \exp(-\Delta)} \quad (1.27)$$

Therefore

$$\frac{p(x|y = 1; \phi, \mu_0, \mu_1, \Sigma)p(y = 1; \phi, \mu_0, \mu_1, \Sigma)}{p(x; \phi, \mu_0, \mu_1, \Sigma)} = \frac{1}{1 + \exp(-(\theta^T x + \theta_0))} \quad (1.28)$$

where

$$\theta = (\Sigma^{-1})^T(\mu_1 - \mu_0) \quad (1.29)$$

$$\theta_0 = \log \frac{\phi}{1 - \phi} + \frac{1}{2}\mu_0^T \Sigma^{-1}\mu_0 - \frac{1}{2}\mu_1^T \Sigma^{-1}\mu_1 \quad (1.30)$$



1.3 Question 1(d)

1.3.1 ϕ

Proof.

$$\frac{\partial}{\partial \phi} \ell(\phi, \cdot) = \frac{\partial}{\partial \phi} \sum_{i=1}^n \underbrace{\log p(x^{(i)} | y^{(i)}; \mu_0, \mu_1, \Sigma)}_{\perp \phi} + \log p(y^{(i)}; \phi) \quad (1.31)$$

$$= \frac{\partial}{\partial \phi} \sum_{i=1}^n \log p(y^{(i)}; \phi) \quad (1.32)$$

$$= \frac{\partial}{\partial \phi} \sum_{i=1}^n \log \phi^{y^{(i)}} (1 - \phi)^{1-y^{(i)}} \quad (1.33)$$

$$= \frac{\partial}{\partial \phi} \sum_{i=1}^n y^{(i)} \log \phi + (1 - y^{(i)}) \log(1 - \phi) \quad (1.34)$$

$$= \sum_{i=1}^n y^{(i)} \frac{1}{\phi} - (1 - y^{(i)}) \frac{1}{1 - \phi} \quad (1.35)$$

The first order condition of maximizing likelihood becomes

$$\sum_{i=1}^n y^{(i)} \frac{1}{\phi} - (1 - y^{(i)}) \frac{1}{1 - \phi} = 0 \quad (1.36)$$

$$\implies \sum_{i=1}^n \frac{y^{(i)}}{\phi} + \frac{y^{(i)}}{1 - \phi} - \frac{1}{1 - \phi} = 0 \quad (1.37)$$

$$\implies \sum_{i=1}^n y^{(i)} \frac{1 - \phi + \phi}{\phi(1 - \phi)} - \frac{1}{1 - \phi} = 0 \quad (1.38)$$

$$\implies \sum_{i=1}^n y^{(i)} \frac{1}{\phi(1 - \phi)} = n \frac{1}{1 - \phi} \quad (1.39)$$

$$\implies \phi = \frac{1}{n} \sum_{i=1}^n y^{(i)} \quad (1.40)$$

■

1.3.2 μ_0

Proof.

$$\frac{\partial}{\partial \mu_0} \ell(\mu_0, \cdot) = \frac{\partial}{\partial \mu_0} \sum_{i=1}^n \log p(x^{(i)} | y^{(i)}; \mu_0, \mu_1, \Sigma) + \underbrace{\log p(y^{(i)}; \phi)}_{\perp \mu_0} \quad (1.41)$$

$$= \frac{\partial}{\partial \mu_0} \sum_{i=1}^n \log p(x^{(i)} | y^{(i)}; \mu_0, \mu_1, \Sigma) \quad (1.42)$$

$$= \frac{\partial}{\partial \mu_0} \sum_{i=1}^n \left\{ \overbrace{y^{(i)} \log \left[\frac{1}{(2\pi)^{d/2} |\Sigma|^{1/2}} \exp \left(-\frac{1}{2} (x^{(i)} - \mu_1)^T \Sigma^{-1} (x^{(i)} - \mu_1) \right) \right]}^{\perp \mu_0} \right\} \quad (1.43)$$

$$+ (1 - y^{(i)}) \log \left[\frac{1}{(2\pi)^{d/2} |\Sigma|^{1/2}} \exp \left(-\frac{1}{2} (x^{(i)} - \mu_0)^T \Sigma^{-1} (x^{(i)} - \mu_0) \right) \right] \Big\} \quad (1.44)$$

$$= \frac{\partial}{\partial \mu_0} \sum_{i=1}^n (1 - y^{(i)}) \left(\underbrace{\log \frac{1}{(2\pi)^{d/2} |\Sigma|^{1/2}}}_{\perp \mu_0} - \frac{1}{2} (x^{(i)} - \mu_0)^T \Sigma^{-1} (x^{(i)} - \mu_0) \right) \quad (1.45)$$

$$= \frac{\partial}{\partial \mu_0} (-1) \sum_{i=1}^n (1 - y^{(i)}) \frac{1}{2} (x^{(i)} - \mu_0)^T \Sigma^{-1} (x^{(i)} - \mu_0) = 0 \quad (1.46)$$

$$\implies \sum_{i=1}^n (1 - y^{(i)}) \Sigma^{-1} (x^{(i)} - \mu_0) = 0 \quad (1.47)$$

$$(1.48)$$

$$\implies \sum_{i=1}^n \Sigma^{-1} (1 - y^{(i)}) x^{(i)} = \sum_{i=1}^n \Sigma^{-1} (1 - y^{(i)}) \mu_0 \quad (1.49)$$

$$\implies \sum_{i=1}^n (1 - y^{(i)}) x^{(i)} = \sum_{i=1}^n (1 - y^{(i)}) \mu_0 \quad (1.50)$$

$$\implies \mu_0 = \frac{\sum_{i=1}^n (1 - y^{(i)}) x^{(i)}}{\sum_{i=1}^n (1 - y^{(i)})} = \frac{\sum_{i=1}^n \mathbf{1}\{y^{(i)} = 0\} x^{(i)}}{\sum_{i=1}^n \mathbf{1}\{y^{(i)} = 0\}} \quad (1.51)$$

■

1.3.3 μ_1

Proof.

$$\frac{\partial}{\partial \mu_1} \ell(\mu_1, \cdot) = \frac{\partial}{\partial \mu_1} \sum_{i=1}^n \log p(x^{(i)} | y^{(i)}; \mu_0, \mu_1, \Sigma) + \underbrace{\log p(y^{(i)}; \phi)}_{\perp \mu_1} \quad (1.52)$$

$$= \frac{\partial}{\partial \mu_1} \sum_{i=1}^n \log p(x^{(i)} | y^{(i)}; \mu_0, \mu_1, \Sigma) \quad (1.53)$$

$$= \frac{\partial}{\partial \mu_1} \sum_{i=1}^n \left\{ y^{(i)} \log \left[\frac{1}{(2\pi)^{d/2} |\Sigma|^{1/2}} \exp \left(-\frac{1}{2} (x^{(i)} - \mu_1)^T \Sigma^{-1} (x^{(i)} - \mu_1) \right) \right] \right\} \quad (1.54)$$

$$+ \underbrace{(1 - y^{(i)}) \log \left[\frac{1}{(2\pi)^{d/2} |\Sigma|^{1/2}} \exp \left(-\frac{1}{2} (x^{(i)} - \mu_0)^T \Sigma^{-1} (x^{(i)} - \mu_0) \right) \right]}_{\perp \mu_1} \quad (1.55)$$

$$= \frac{\partial}{\partial \mu_1} \sum_{i=1}^n y^{(i)} \left(\underbrace{\log \frac{1}{(2\pi)^{d/2} |\Sigma|^{1/2}}}_{\perp \mu_1} - \frac{1}{2} (x^{(i)} - \mu_1)^T \Sigma^{-1} (x^{(i)} - \mu_1) \right) \quad (1.56)$$

$$= \frac{\partial}{\partial \mu_1} (-1) \sum_{i=1}^n y^{(i)} \frac{1}{2} (x^{(i)} - \mu_1)^T \Sigma^{-1} (x^{(i)} - \mu_1) = 0 \quad (1.57)$$

$$\implies \sum_{i=1}^n y^{(i)} \Sigma^{-1} (x^{(i)} - \mu_1) = 0 \quad (1.58)$$

$$\implies \sum_{i=1}^n \Sigma^{-1} y^{(i)} x^{(i)} = \sum_{i=1}^n \Sigma^{-1} y^{(i)} \mu_1 \quad (1.59)$$

$$\implies \sum_{i=1}^n y^{(i)} x^{(i)} = \sum_{i=1}^n y^{(i)} \mu_1 \quad (1.60)$$

$$\implies \mu_1 = \frac{\sum_{i=1}^n y^{(i)} x^{(i)}}{\sum_{i=1}^n y^{(i)}} = \frac{\sum_{i=1}^n \mathbb{1}\{y^{(i)} = 1\} x^{(i)}}{\sum_{i=1}^n \mathbb{1}\{y^{(i)} = 1\}} \quad (1.61)$$

■

1.3.4 Σ^{-1}

Proof. **TODO**

■

2 Question 2: Incomplete, Positive-Only Labels

2.1 Question 2(c)

Proof.

$$p(t^{(i)} = 1 | y^{(i)} = 1, x^{(i)}) = \frac{p(y^{(i)} = 1 | t^{(i)} = 1, x^{(i)})p(t^{(i)} = 1 | x^{(i)})}{p(y^{(i)} = 1 | x^{(i)})} \quad (2.1)$$

$$= \frac{p(y^{(i)} = 1 | t^{(i)} = 1, x^{(i)})p(t^{(i)} = 1 | x^{(i)})}{p(y^{(i)} = 1 | t^{(i)} = 1, x^{(i)})p(t^{(i)} = 1 | x^{(i)}) + p(y^{(i)} = 1 | t^{(i)} = 0, x^{(i)})p(t^{(i)} = 0 | x^{(i)})} \quad (2.2)$$

$$= \frac{\alpha p(t^{(i)} = 1 | x^{(i)})}{\alpha p(t^{(i)} = 1 | x^{(i)}) + 0 p(t^{(i)} = 0 | x^{(i)})} \quad (2.3)$$

$$= \frac{\alpha p(t^{(i)} = 1 | x^{(i)})}{\alpha p(t^{(i)} = 1 | x^{(i)})} = 1 \quad (2.4)$$

■

2.2 Question 2(d)

Proof.

$$p(t^{(i)} = 1|x^{(i)}) = p(t^{(i)}, y^{(i)} = 1|x^{(i)}) + p(t^{(i)} = 1, y^{(i)} = 0|x^{(i)}) \quad (2.5)$$

$$= p(t^{(i)} = 1|y^{(i)} = 1, x^{(i)})p(y^{(i)} = 1|x^{(i)}) \quad (2.6)$$

$$+ p(y^{(i)} = 0|t^{(i)} = 1, x^{(i)})p(t^{(i)} = 1|x^{(i)}) \quad (2.7)$$

$$= 1p(y^{(i)} = 1|x^{(i)}) + (1 - \alpha)p(t^{(i)} = 1|x^{(i)}) \quad (2.8)$$

$$\implies p(t^{(i)} = 1|x^{(i)}) = \frac{1}{\alpha}p(y^{(i)} = 1|x^{(i)}) \quad (2.9)$$

■

2.3 Question 2(e)

Proof.

$$h(x^{(i)}) = p(y^{(i)} = 1|x^{(i)}) \quad (2.10)$$

$$\implies \mathbb{E}[h(x^{(i)})|y^{(i)} = 1] = \mathbb{E}[p(y^{(i)} = 1|x^{(i)})|y^{(i)} = 1] \quad (2.11)$$

$$= \mathbb{E}\{p(y^{(i)} = 1|t^{(i)} = 1, x^{(i)})p(t^{(i)} = 1|x^{(i)}) \quad (2.12)$$

$$+ p(y^{(i)} = 1|t^{(i)} = 0, x^{(i)})p(t^{(i)} = 0|x^{(i)})|y^{(i)} = 1\} \quad (2.13)$$

$$= \mathbb{E}[\alpha p(t^{(i)} = 1|x^{(i)}) + 0|y^{(i)} = 1] \quad (2.14)$$

$$= \alpha \mathbb{E}[p(t^{(i)} = 1|x^{(i)})|y^{(i)} = 1] \quad (2.15)$$

From part (c), we proved that given $y^{(i)} = 1$, $t^{(i)} = 1$ with probability 1, conditioned on $x^{(i)}$. Hence,

$$\mathbb{E}[p(t^{(i)} = 1|x^{(i)})|y^{(i)} = 1] = 1 \quad (2.16)$$

$$\implies \mathbb{E}[h(x^{(i)})|y^{(i)} = 1] = \alpha \quad (2.17)$$

■

3 Question 3: Poisson Regression

3.1 Question 3(a)

Proof.

$$p(y; \lambda) = \frac{\exp(-\lambda)\lambda^y}{y!} \quad (3.1)$$

$$= \exp \log\left(\frac{\exp(-\lambda)\lambda^y}{y!}\right) \quad (3.2)$$

$$= \exp(-\lambda + y \log(\lambda) - \log(y!)) \quad (3.3)$$

$$= \frac{1}{y!} \exp(\log(\lambda)y - \lambda) \quad (3.4)$$

therefore, Poisson distribution belongs to the exponential family with

$$b(y) := \frac{1}{y!} \quad (3.5)$$

$$\eta(\lambda) := \log(\lambda) \quad (3.6)$$

$$T(y) := y \quad (3.7)$$

$$a(\eta) := \exp(\eta) = \lambda \quad (3.8)$$

■

3.2 Question 3(b)

Answer. By definition, the canonical response function maps η to the expectation $\mathbb{E}[T(y); \eta]$, which equals $\mathbb{E}[y; \eta] = \lambda$ here. Based on the fact that $\eta(\lambda) = \log(\lambda)$, the exponential function maps $\eta(\lambda)$ to $\mathbb{E}[y; \eta]$. Hence, the canonical response function here is the exponential function. ■

3.3 Question 3(c)

Derive.

$$\frac{\partial}{\partial \theta_j} \log(p(y^{(i)} | x^{(i)}; \theta)) = \frac{\partial}{\partial \theta_j} \left(\log(b(y)) + \eta^T y^{(i)} - a(\eta) \right) \quad (3.9)$$

$$= \frac{\partial}{\partial \theta_j} \left(\theta^T x^{(i)} y^{(i)} - \exp(\theta^T x^{(i)}) \right) \quad (3.10)$$

$$= x_j^{(i)} y^{(i)} - \exp(\theta^T x^{(i)}) x_j^{(i)} \quad (3.11)$$

$$= \left(y^{(i)} - \exp(\theta^T x^{(i)}) \right) x_j^{(i)} \quad (3.12)$$

The stochastic gradient ascent update rule for parameter θ_j is

$$\theta_j \leftarrow \theta_j + \alpha \left(y^{(i)} - \exp(\theta^T x^{(i)}) \right) x_j^{(i)} \quad (3.13)$$

where $(x^{(i)}, y^{(i)})$ is the randomly selected sample, and $\alpha > 0$ denotes the learning rate. ■

4 Question 4: Convexity of Generalized Linear Models

4.1 Question 4(a)

Proof. The mean of y is simply

$$\mathbb{E}[Y; \eta] = \int_{\mathbb{R}} yp(y; \eta) dy \quad (4.1)$$

$$= \int_{\mathbb{R}} yb(y) \exp(\eta y - a(\eta)) dy \quad (4.2)$$

By definition of probability measure, it must be the case that

$$\int_{\mathbb{R}} p(y; \eta) dy = 1 \quad (4.3)$$

for every valid η . Therefore,

$$\int_{\mathbb{R}} b(y) \exp(\eta y - a(\eta)) dy = 1 \quad (4.4)$$

$$\implies \int_{\mathbb{R}} b(y) \exp(\eta y) \frac{1}{\exp(a(\eta))} dy = 1 \quad (4.5)$$

$$\implies \int_{\mathbb{R}} b(y) \exp(\eta y) dy = \exp(a(\eta)) \quad (4.6)$$

$$\implies \frac{\partial \exp(a(\eta))}{\partial \eta} = \frac{\partial}{\partial \eta} \int_{\mathbb{R}} b(y) \exp(\eta y) dy \quad (4.7)$$

$$\implies a'(\eta) \exp(a(\eta)) = \int_{\mathbb{R}} b(y) \frac{\partial \exp(\eta y)}{\partial \eta} dy \quad (4.8)$$

$$\implies a'(\eta) = \int_{\mathbb{R}} yb(y) \exp(\eta y) \frac{1}{\exp(a(\eta))} dy \quad (4.9)$$

$$\implies a'(\eta) = \int_{\mathbb{R}} yb(y) \exp(\eta y - a(\eta)) dy \quad (4.10)$$

$$= \mathbb{E}[Y; \eta] \quad (4.11)$$

■

4.2 Question 4(b)

Proof. From part (a),

$$a'(\eta) = \int_{\mathbb{R}} yb(y) \exp(\eta y - a(\eta)) \, dy \quad (4.12)$$

$$\implies \frac{\partial^2 a(\eta)}{\partial \eta^2} = \frac{\partial}{\partial \eta} \int_{\mathbb{R}} yb(y) \exp(\eta y - a(\eta)) \, dy \quad (4.13)$$

$$= \int_{\mathbb{R}} yb(y) \exp(\eta y - a(\eta)) (y - a'(\eta)) \, dy \quad (4.14)$$

$$= \int_{\mathbb{R}} y^2 b(y) \exp(\eta y - a(\eta)) \, dy - a'(\eta) \int_{\mathbb{R}} yb(y) \exp(\eta y - a(\eta)) \, dy \quad (4.15)$$

$$= \mathbb{E}[Y^2; \eta] - a'(\eta) \mathbb{E}[Y; \eta] \quad (4.16)$$

$$= \mathbb{E}[Y^2; \eta] - \mathbb{E}[Y; \eta]^2 \quad (4.17)$$

$$= \mathbb{V}[Y; \eta] \quad (4.18)$$

■

4.3 Question 4(c)

Proof.

■

5 Question 5: Linear Regression

5.1 Question 5(a)

$$J(\theta) := \frac{1}{2} \sum_{i=1}^n \left(y^{(i)} - \theta^T \phi(x^{(i)}) \right)^2 \quad (5.1)$$

$$\theta \leftarrow \theta + \alpha \sum_{i=1}^n \left(y^{(i)} - \theta^T \hat{x}^{(i)} \right) \hat{x}^{(i)} \quad (5.2)$$

where α denotes the learning rate.