Flavor Mixing Parameters from Generation Hierarchy

An Empirical Observation

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Abstract

We report an empirical observation: the nine flavor mixing parameters of the Standard Model (three CKM angles, three PMNS angles, two CP-violating phases, and the Weinberg angle) can all be expressed as simple rational fractions constructed from powers of 3 and the integer 13. The generation hierarchy rule $p_i = 3^{i-1}$ for generation $i = 1, 2, 3$, combined with the cluster number 13 (the total count of gauge bosons plus one), reproduces all nine parameters with deviations below 2% from PDG 2024 central values. We identify cross-relations between the quark and lepton sectors, including a shared CP-phase numerator and a factor-of-6 link between θ_{13} mixing angles. The hypothesis is pre-registered and falsifiable: the JUNO and DUNE experiments will provide definitive tests within this decade. We emphasize that this is a phenomenological pattern, not a derivation from first principles, and the risk of coincidental fitting cannot be excluded.

Keywords: flavor mixing, CKM matrix, PMNS matrix, Weinberg angle, generation hierarchy, neutrino oscillations

1 Introduction

The Standard Model of particle physics contains a set of seemingly arbitrary parameters governing the mixing between quark and lepton mass eigenstates. The Cabibbo–Kobayashi–Maskawa (CKM) matrix describes quark mixing through three angles $(\theta_{12}, \theta_{23}, \theta_{13})$ and one CP-violating phase δ [1]. The Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix parameterizes neutrino mixing with an analogous set of three angles and one Dirac phase [2]. The weak mixing angle θ_W (Weinberg angle) governs electroweak symmetry breaking. Together, these nine parameters are measured to high precision but lack a theoretical explanation for their specific values.

Numerous attempts have been made to derive these parameters from symmetry principles, texture zeros, or discrete flavor symmetries (see [3] for a review). In this note,

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we take a different and deliberately modest approach: we report an *empirical observation* that all nine parameters can be expressed as simple rational fractions built from two integers, 3 and 13, via a generation hierarchy rule.

We make no claim that this observation constitutes a theory. The formulas were identified by inspection of the data and may represent coincidence rather than structure. However, the observation is *falsifiable*: upcoming neutrino experiments will either confirm or rule out the predicted values at high precision. This hypothesis has been pre-registered at the Open Science Framework [7] prior to these experimental results.

2 The Generation Hierarchy Rule

We define a generation weight for each fermion generation:

$$p_i = 3^{i-1}, \quad i = 1, 2, 3 \tag{1}$$

giving $p_1 = 1$, $p_2 = 3$, $p_3 = 9$. The base 3 corresponds to the number of fermion generations in the Standard Model.

We further identify a *cluster number*:

$$N_g = 13 = 1 + 12 \tag{2}$$

where 12 is the total number of gauge bosons in the Standard Model (8 gluons + W^+, W^-, Z^0, γ). A secondary number $N_g - 2 = 11$ also appears.

We emphasize that the identification of $13 = 1 + 12$ is a *motivation*, not a derivation. Whether the number 13 has deeper gauge-theoretic significance is an open question.

3 CKM Matrix Predictions

Using the standard parameterization of the CKM matrix [1], the three mixing angles and CP phase can be expressed as:

$$\sin\theta_{12} = \frac{p_2 \cdot 13 - p_1}{13^2} = \frac{38}{169} \approx 0.2249 \tag{3}$$

$$\sin\theta_{23} = \frac{13 - (p_3 - p_2)}{13^2} = \frac{7}{169} \approx 0.04142 \tag{4}$$

$$\sin\theta_{13} = \frac{p_3 - p_1}{13^3} = \frac{8}{2197} \approx 0.003641 \tag{5}$$

$$\sin\delta_{\mathrm{CKM}} = \frac{13 - p_2}{13 - 2} = \frac{10}{11} \approx 0.9091 \tag{6}$$

Table 1: CKM matrix predictions compared with PDG 2024 values.

Parameter	**Formula**	**Predicted**	**PDG 2024**	**Dev.**
$\sin\theta_{12}$	38/169	0.2249	0.2243 ± 0.0005	1.1σ
$\sin\theta_{23}$	7/169	0.04142	0.0422 ± 0.0008	1.0σ
$\sin\theta_{13}$	8/2197	0.003641	0.00369 ± 0.00011	0.4σ
δ_{CKM}	arcsin(10/11)	65.38°	$65.4° \pm 3.0°$	$< 0.1\sigma$

All four CKM parameters fall within 1.1σ of their measured values. We note that the hierarchy of CKM elements ($\theta_{12} \gg \theta_{23} \gg \theta_{13}$) is naturally reproduced by the increasing powers of 13 in the denominators: 13^2, 13^2, 13^3.

4 PMNS Matrix Predictions

The PMNS mixing parameters [2] are conventionally quoted as $\sin^2\theta_{ij}$ rather than $\sin\theta_{ij}$. We find:

$$\sin^2\theta_{12} = \frac{p_1 + p_2}{13} = \frac{4}{13} \approx 0.3077 \tag{7}$$

$$\sin^2\theta_{23} = \frac{p_3 - p_2}{13 - 2} = \frac{6}{11} \approx 0.5455 \tag{8}$$

$$\sin^2\theta_{13} = \frac{(13 - p_3)(13 - p_1)}{13^3} = \frac{48}{2197} \approx 0.02185 \tag{9}$$

$$\sin\delta_{\mathrm{PMNS}} = -\frac{13 - p_2}{13} = -\frac{10}{13} \approx -0.7692 \tag{10}$$

The physical PMNS phase corresponds to $\delta_{\mathrm{PMNS}} = \pi - \arcsin(10/13) \approx -129.7°$.

Table 2: PMNS matrix predictions compared with PDG 2024 values (normal ordering).

Parameter	**Formula**	**Predicted**	**PDG 2024**	**Dev.**
$\sin^2\theta_{12}$	$4/13$	0.3077	0.307 ± 0.013	0.05σ
$\sin^2\theta_{23}$	$6/11$	0.5455	0.546 ± 0.021	0.03σ
$\sin^2\theta_{13}$	$48/2197$	0.02185	0.0220 ± 0.0007	0.2σ
δ_{PMNS}	$-\arcsin(10/13)$	$-129.7°$	$-130° \pm 40°$	$< 0.01\sigma$

The PMNS predictions are remarkably close to central values, though the current experimental uncertainties (particularly for δ_{PMNS}) are large.

5 Weinberg Angle

The weak mixing angle is:

$$\sin^2\theta_W = \frac{p_2}{13} = \frac{3}{13} \approx 0.2308 \tag{11}$$

The measured value at the M_Z scale in the $\overline{\mathrm{MS}}$ scheme is $\sin^2\theta_W = 0.23121 \pm 0.00004$ [1]. The prediction deviates by approximately 0.2%, corresponding to $\sim 11\sigma$.

This is the only parameter showing significant tension with data. A possible interpretation is that $3/13$ represents a high-energy ("bare") value, with the measured value receiving radiative corrections from renormalization group running. Within the Standard Model, $\sin^2\theta_W$ runs from $\sim 3/8 = 0.375$ at unification to ~ 0.231 at M_Z, so the question is whether there exists an energy scale at which $\sin^2\theta_W$ passes through $3/13$. We note that this would occur at a scale somewhat above M_Z, but a precise determination requires specifying the running scheme and is beyond the scope of this empirical note.

6 Cross-Relations

Two structural relationships connect the quark and lepton sectors.

6.1 The θ_{13} Connection

The reactor and V_{ub} mixing parameters satisfy:

$$\sin^2\theta_{13}^{\mathrm{PMNS}} = 6 \times \sin\theta_{13}^{\mathrm{CKM}} \tag{12}$$

since $48/13^3 = 6 \times 8/13^3$. The factor $6 = (p_3 - p_2)(13 - p_1)/[(p_3 - p_1) \cdot 13]$ arises naturally from the generation hierarchy.

6.2 The CP Phase Structure

Both CP-violating phases share the same numerator:

$$\sin\delta_{\mathrm{CKM}} = +\frac{10}{11} = +\frac{13 - p_2}{13 - 2} \tag{13}$$

$$\sin\delta_{\mathrm{PMNS}} = -\frac{10}{13} = -\frac{13 - p_2}{13} \tag{14}$$

The common numerator $10 = 13 - 3 = N_g - p_2$ links CP violation in both sectors. The sign difference (positive for quarks, negative for leptons) and the different denominators (11 vs. 13) distinguish the two sectors.

7 Degree of Freedom Analysis

An honest assessment of the predictive content requires counting degrees of freedom. The model has:

Inputs: Two integers, 3 (generation base) and 13 (cluster number). Additionally, the *form* of each formula (which combinations of p_i, 13, and 11 appear in numerator and denominator) constitutes implicit degrees of freedom.

Outputs: Nine parameters (four CKM, four PMNS, one Weinberg).

If each formula form is considered a free choice, the model has effectively 9 implicit structural choices constrained to hit 9 targets, which would make the agreement unremarkable. The non-trivial content lies in:

1. All formulas use only $\{p_1, p_2, p_3, 13, 11\}$ with simple arithmetic.
2. The cross-relations (Section 6) are not independent fits but structural consequences.
3. The hierarchy 13^2 vs. 13^3 in denominators naturally reproduces the CKM hierarchy.
4. The Weinberg angle prediction was not fitted to data (it disagrees at 11σ).

We leave it to the reader to judge whether the pattern is sufficiently constrained to be non-trivial.

8 Experimental Tests

The hypothesis makes specific predictions testable by upcoming experiments:

8.1 JUNO (2027)

The Jiangmen Underground Neutrino Observatory [4] will measure $\sin^2\theta_{12}$ with precision $\sim \pm 0.003$. Our prediction:

$$\sin^2\theta_{12} = \frac{4}{13} = 0.30769\ldots \tag{15}$$

Falsified if: JUNO measures $\sin^2\theta_{12}$ outside $[0.298,\ 0.317]$ at 3σ.

8.2 DUNE (2029+)

The Deep Underground Neutrino Experiment [5] will measure δ_{PMNS} with precision $\sim \pm 10^\circ$. Our prediction:

$$\delta_{\mathrm{PMNS}} = -129.7^\circ \quad (\sin\delta = -10/13) \tag{16}$$

Falsified if: DUNE measures δ_{PMNS} outside $[-160^\circ,\ -100^\circ]$ at 3σ.

8.3 Hyper-Kamiokande

Hyper-Kamiokande [6] will provide complementary measurements of $\sin^2\theta_{23}$ with precision $\sim \pm 0.006$. Our prediction:

$$\sin^2\theta_{23} = \frac{6}{11} = 0.54545\ldots \tag{17}$$

Falsified if: Hyper-K measures $\sin^2\theta_{23}$ outside $[0.527,\ 0.564]$ at 3σ.

These predictions have been pre-registered at the Open Science Framework [7] to ensure that they are genuine *a priori* predictions for these specific experiments, even though the formulas were constructed using existing data.

9 Discussion

We have presented an empirical pattern, not a theory. Several important caveats must be stated clearly:

Post-hoc construction. The formulas were found by inspecting the experimental values and identifying rational approximations using the numbers $\{1, 3, 9, 11, 13\}$. This is fundamentally different from a first-principles derivation. The "predictions" for parameters already measured are retrodictions.

Risk of overfitting. With five building blocks $(p_1, p_2, p_3, 13, 11)$ and freedom to choose addition, subtraction, multiplication, division, and power operations, the space of achievable fractions is large. A systematic study of how many random numbers could be similarly approximated would strengthen or weaken the case, but is beyond our current scope.

Weinberg angle tension. The 11σ discrepancy for $\sin^2\theta_W$ is either evidence against the pattern or evidence that $3/13$ is a high-energy value. We cannot distinguish these possibilities without a theoretical framework.

No mechanism. We offer no explanation for *why* these patterns should hold. Without a mechanism, the observation remains a curiosity rather than physics.

Despite these caveats, we note that the pattern has a genuine falsifiable component: the PMNS predictions for $\sin^2\theta_{12}$, $\sin^2\theta_{23}$, and δ_{PMNS} will be tested at precisions that can definitively confirm or exclude the predicted values within this decade.

10 Conclusion

We have reported the observation that all nine Standard Model flavor mixing parameters can be expressed as simple rational fractions of the form $f(1,3,9)/g(13,11)$, with deviations below 2% from PDG 2024 values in eight of nine cases. Cross-relations between the quark and lepton sectors emerge naturally from the structure.

The observation is pre-registered and falsifiable. If confirmed by JUNO, DUNE, and Hyper-Kamiokande, it would suggest that the flavor sector has a simpler arithmetic structure than currently appreciated, and would call for a theoretical explanation. If falsified, it should be discarded as a numerical coincidence.

All code and data are publicly available [8].

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