Find the dimension of Null space of the matrix; $\begin{pmatrix} 1 & -1 & 2 \\ -1 & 1 & -2 \end{pmatrix}$.

Solution:

Let $A = \begin{pmatrix} 1 & -1 & 2 \\ -1 & 1 & -2 \end{pmatrix}$ and the solution to $A\overrightarrow{u} = \overrightarrow{0}$ will be $\overrightarrow{u} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$.

Now the augmented matrix of the associated system of equation is given by the following row operations;

$$\begin{pmatrix} 1 & -1 & 2 \\ -1 & 1 & -2 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & -1 & 2 \\ 0 & 0 & 0 \end{pmatrix} \qquad \because \text{by } R_2^{/} \longrightarrow R_2 - R_1$$

This reduced form will give the solution;

$x - y + 2z = 0$

$\Longrightarrow x = y - 2z$

And the solution in the vector form as;

$$\overrightarrow{u} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} y - 2z \\ y \\ z \end{pmatrix} = \begin{pmatrix} y - 2z \\ y + 0z \\ 0y + z \end{pmatrix} = \begin{pmatrix} y \\ y \\ 0y \end{pmatrix} + \begin{pmatrix} -2z \\ 0z \\ z \end{pmatrix}$$

$$\Longrightarrow \overrightarrow{u} = y \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + z \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}$$

$\Longrightarrow$ Null space of matrix A is the set of all linear combinations of the vectors $\overrightarrow{v} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ and $\overrightarrow{w} = \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}$.

And since $\overrightarrow{v}$ and $\overrightarrow{w}$ are not multiple of each other

$\Longrightarrow \overrightarrow{v}$ and $\overrightarrow{w}$ are linearly independent.

$\Longrightarrow \overrightarrow{v}$ and $\overrightarrow{w}$ form the basis of *Null(A)*

$\therefore Dim\,[Null(A)] = 2.$