Rode for question 3

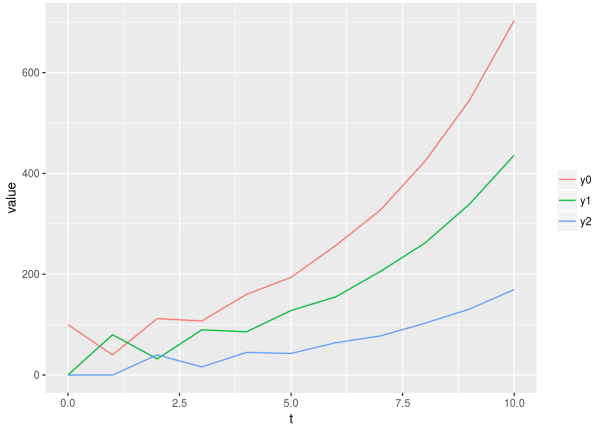
```
setwd('~/Desktop/Matematik og modeller/mod1/proj1')

library(magrittr)
library(tidyverse)
```

```
## ── Attaching packages ─────────────── tidyverse 1.2.1 ──
```

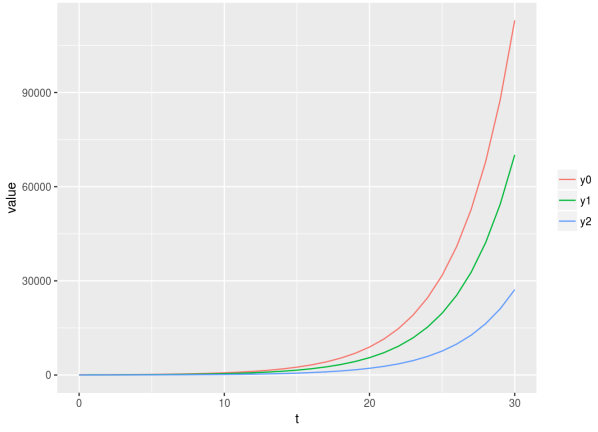
```
## ✔ ggplot2 2.2.1     ✔ purrr   0.2.4
## ✔ tibble  1.4.2     ✔ dplyr   0.7.4
## ✔ tidyr   0.8.0     ✔ stringr 1.3.0
## ✔ readr   1.1.1     ✔ forcats 0.3.0
```

```
## ── Conflicts ──────────────── tidyverse_conflicts() ──
## ✖ tidyr::extract()   masks magrittr::extract()
## ✖ dplyr::filter()    masks stats::filter()
## ✖ dplyr::lag()       masks stats::lag()
## ✖ purrr::set_names() masks magrittr::set_names()
```

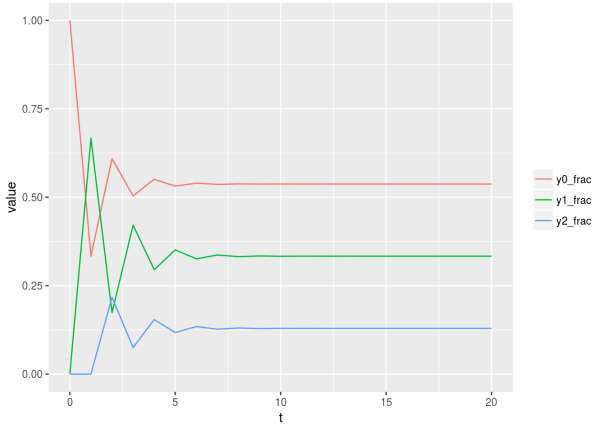
```
library(xtable)

sim_mod <- function(move, n, x1) {

  xs <- matrix(nrow = n+1, ncol = length(x1))
  xs[1,] <- x1
  for (t in 1:n) {
    xs[t+1,] <- move(xs[t,])
  }

  data_frame(t = 0:n, y0=xs[,1], y1=xs[,2], y2 = xs[,3])

}

M <- matrix(c(0.4,0.8,0,
              1.2,0,0.5,
              0.6,0,0),nrow=3)

move <- function(x) {
  M%*%x
}

n <- 50
x1 <- c(100,0,0)
sim <- sim_mod(move, n, x1)

sim[1:11,] %>%
  gather(key, value, y0, y1, y2) %>%
  ggplot(aes(t, value, color=key)) +
  geom_line() +
  theme(legend.title = element_blank())
```




```
sim[1:31,] %>%
  gather(key, value, y0, y1, y2) %>%
  ggplot(aes(t, value, color=key)) +
  geom_line() +
  theme(legend.title = element_blank())
```

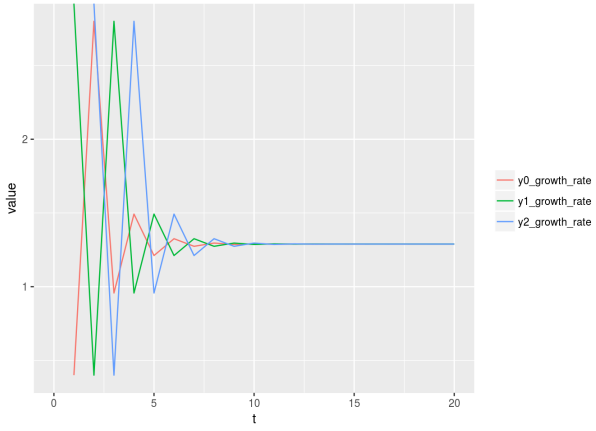


```
sim %<>%
  mutate(n = y0 + y1 + y2,
         y0_frac = y0/n,
         y1_frac = y1/n,
         y2_frac = y2/n,
         y0_growth_rate = y0/lag(y0),
         y1_growth_rate = y1/lag(y1),
         y2_growth_rate = y2/lag(y2))

sim[1:21,] %>%
  gather(key, value, y0_frac, y1_frac, y2_frac) %>%
  ggplot(aes(t, value, color=key)) +
  geom_line() +
  theme(legend.title = element_blank())
```

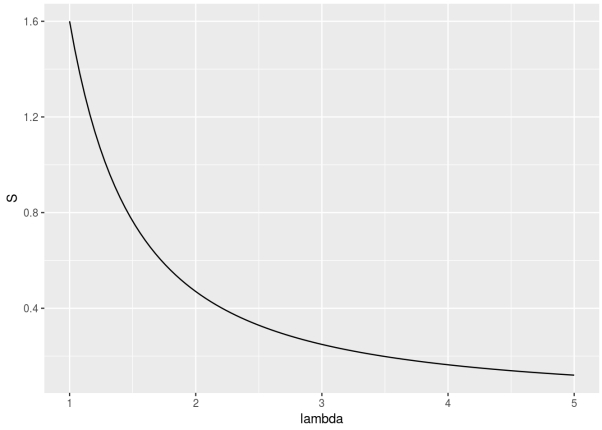



```
sim[1:21,] %>%
  gather(key, value, y0_growth_rate, y1_growth_rate, y2_growth_rate) %>%
  ggplot(aes(t, value, color=key)) +
  geom_line() +
  theme(legend.title = element_blank())
```

```
## Warning: Removed 4 rows containing missing values (geom_path).
```




```
#b

gen_S <- function(M) {
  bs <- M[1,]
  l1 <- sum(M[2,])
  l2 <- l1*sum(M[3,])
  function(lambda) {
    bs[1]/lambda +
      l1*bs[2]/(lambda**2) +
      l2*bs[3]/(lambda**3)
  }
}

S <- gen_S(M)
ggplot(data.frame(x=0), aes(x)) +
  stat_function(fun = S) +
  xlim(1,5) +
  xlab('lambda') + ylab('S')
```




```
uniroot(function(lambda) S(lambda) - 1,lower=1,upper=10)$root
```

```
## [1] 1.289115
```

```
#c
round(eigen(M)$values, 2)
```

```
## [1]  1.29 -0.55 -0.34
```

```
round(eigen(M)$vectors, 2)
```

```
##      [,1]  [,2]  [,3]
## [1,] 0.83 -0.45  0.23
## [2,] 0.52  0.66 -0.54
## [3,] 0.20 -0.60  0.81
```

```
print(xtable(round(eigen(M)$vectors, 2)), include.rownames=F, include.colnames=F, file='qs.tex')
Q <- eigen(M)$vectors
Q
```

```
##           [,1]       [,2]       [,3]
## [1,] 0.8324504 -0.4549322  0.2297340
## [2,] 0.5166023  0.6598009 -0.5445275
## [3,] 0.2003707 -0.5980798  0.8066672
```

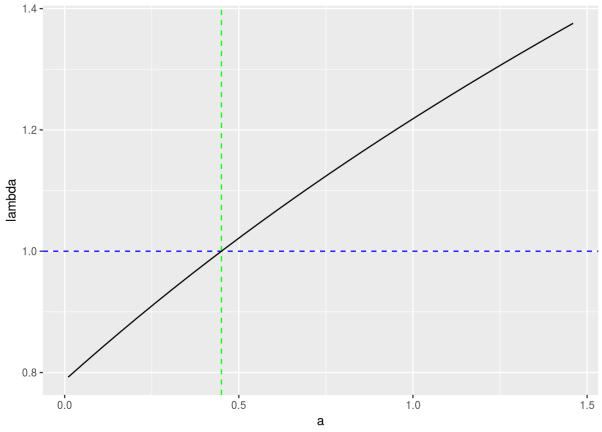
```
0.65*Q
```

```
##           [,1]       [,2]       [,3]
## [1,] 0.5410928 -0.2957059  0.1493271
## [2,] 0.3357915  0.4288706 -0.3539429
## [3,] 0.1302410 -0.3887519  0.5243337
```

```
round(solve(Q)%*%M%*%Q,2)
```

```
##      [,1]  [,2]  [,3]
## [1,] 1.29  0.00  0.00
## [2,] 0.00 -0.55  0.00
## [3,] 0.00  0.00 -0.34
```

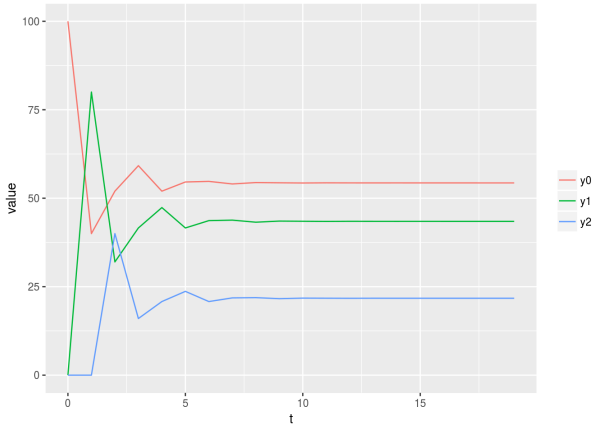
```
#e
make_M <- function(a) {
  matrix(c(0.4,0.8,0,
           a,0,0.5,
           0.6,0,0),nrow=3)
}

growth_rate <- function(a) {
  S <- gen_S(make_M(a))
  uniroot(function(lambda) S(lambda) - 1,lower=0.5,upper=2)$root
}

data_frame(a = seq(0.01, 1.5, 0.05)) %>%
  mutate(lambda = Vectorize(growth_rate)(a)) %>%
  ggplot(aes(a, lambda)) +
  geom_hline(yintercept = 1, color='blue', linetype='dashed') +
  geom_vline(xintercept = 0.45, color='green', linetype='dashed') +
  geom_line()
```



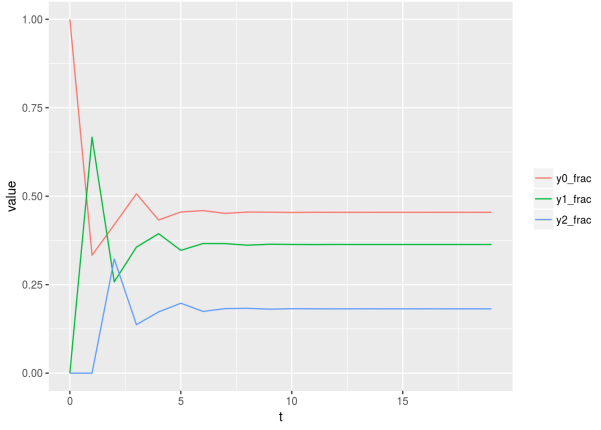

```
stable <- uniroot(function(a) growth_rate(a) - 1, lower=0.1,upper=1.5)$root
stable
```

```
## [1] 0.4499446
```

```
sim <- sim_mod(function(x) make_M(stable)%*%x, 30, c(100,0,0))

sim[1:20,] %>%
  gather(key, value, y0, y1, y2) %>%
  ggplot(aes(t, value, color=key)) +
  geom_line() +
  theme(legend.title = element_blank())
```

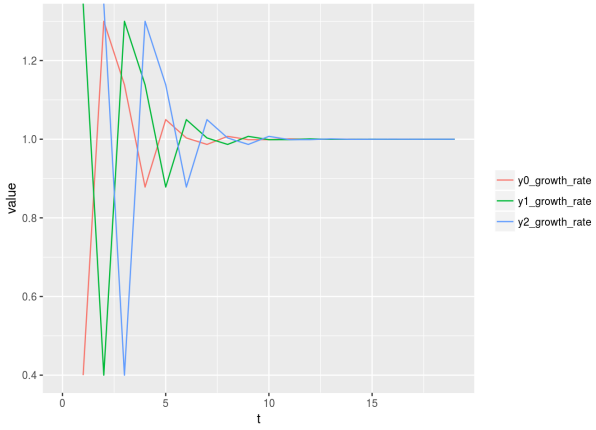



```
sim %<>%
  mutate(n = y0 + y1 + y2,
         y0_frac = y0/n,
         y1_frac = y1/n,
         y2_frac = y2/n,
         y0_growth_rate = y0/lag(y0),
         y1_growth_rate = y1/lag(y1),
         y2_growth_rate = y2/lag(y2))

sim[1:20,] %>%
  gather(key, value, y0_frac, y1_frac, y2_frac) %>%
  ggplot(aes(t, value, color=key)) +
  geom_line() +
  theme(legend.title = element_blank())
```

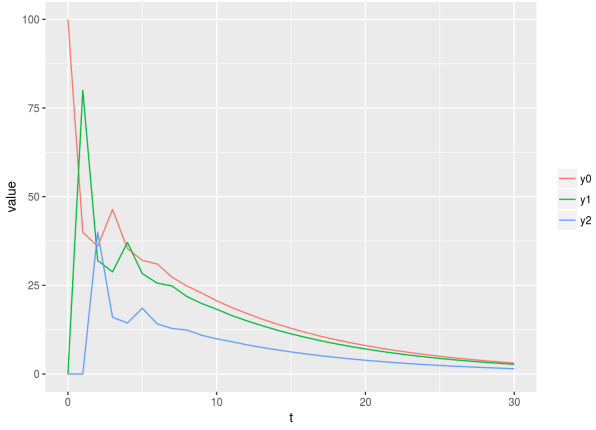



```
sim[1:20,] %>%
  gather(key, value, y0_growth_rate, y1_growth_rate, y2_growth_rate) %>%
  ggplot(aes(t, value, color=key)) +
  geom_line() +
  theme(legend.title = element_blank())
```

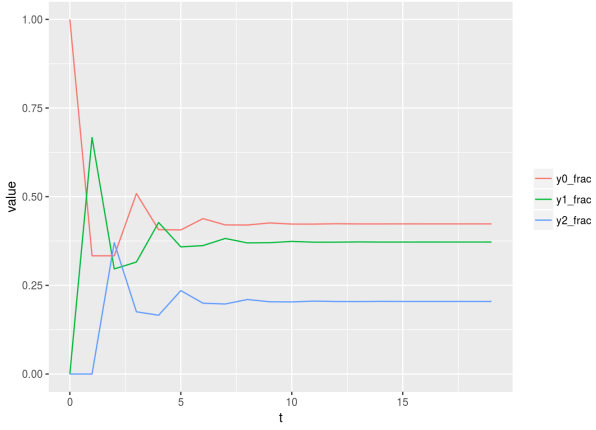
```
## Warning: Removed 4 rows containing missing values (geom_path).
```




```
sim <- sim_mod(function(x) make_M(stable-0.2)%*%x, 30, c(100,0,0))

sim %>%
  gather(key, value, y0, y1, y2) %>%
  ggplot(aes(t, value, color=key)) +
  geom_line() +
  theme(legend.title = element_blank())
```




```
sim %<>%
  mutate(n = y0 + y1 + y2,
         y0_frac = y0/n,
         y1_frac = y1/n,
         y2_frac = y2/n,
         y0_growth_rate = y0/lag(y0),
         y1_growth_rate = y1/lag(y1),
         y2_growth_rate = y2/lag(y2))

sim[1:20,] %>%
  gather(key, value, y0_frac, y1_frac, y2_frac) %>%
  ggplot(aes(t, value, color=key)) +
  geom_line() +
  theme(legend.title = element_blank())
```




```
sim[1:20,] %>%
  gather(key, value, y0_growth_rate, y1_growth_rate, y2_growth_rate) %>%
  ggplot(aes(t, value, color=key)) +
  geom_line() +
  theme(legend.title = element_blank())
```

```
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```

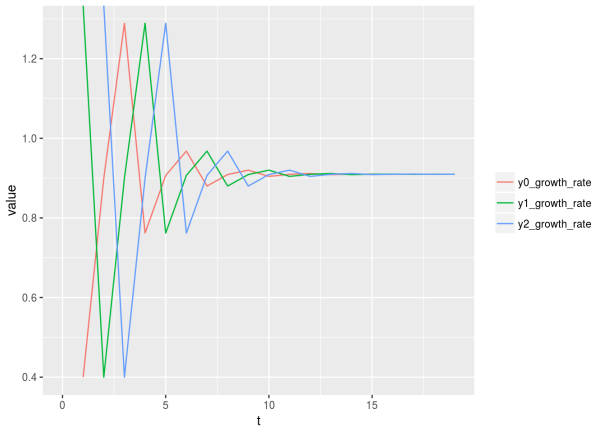
1.2
1.0
0.8
0.6
0.4
value
0
5
10
15
t
y0_growth_rate
y1_growth_rate
y2_growth_rate