Session 5:

Growing Causal Trees

- *Causal Forests and Generalized Random Forests*

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Agenda

1. Growing causal trees
    - Recap on Random Forest (Recap-on-Random-Forest)
    - Causal Forest
    - Generalized Random Forest (GRF)
2. Applying GRF
    - In-class research: heterogeneous treatment effects
    - A tutorial on using grf within Python (A-tutorial-on-using-grf-within-Python)

Recap on Random Forest

The forest full of trees

What is the difference between a Decision Tree and a Random Forest?

- Decision tree iteratively splits data into subsets (partitions) and calculates mean outcome in leaves (end of splits)
- Minimize on some criteria, often entropy or similar loss function
- Collection/ensemble of decision trees
    - Subset of data by bootstrap (sampling with replacement)
    - Subset of features

A special tree

So what distinguishes a Causal Tree from a Decision Tree?

- Causal tree estimates partition of data where treatment effects can be computed locally
- In order to have valid estimates we need **honesty** of trees by estimating partitions and treatment effects on different subsets of data
    - Analogy to train / test split

A tradeoff in structure of heterogeneity

Two approaches?

- Data driven heterogeneity
    - Based on causal trees etc.
- A priori sensible heterogeneity
    - e.g. gender, socioeconomic, ethnicity
    - we use regression model and have interaction with desired variable

When to choose which?

- Choose data driven heterogeneity for policy where you want to maximize impact given data (no theory)
- If we want to test whether certain subgroups are adversely affected

Limitations of Decision Trees

Random forests are nice but no asymptotic normality of prediction.

- Crucial for inference! (corresponds to MLR6 in Econometrics 1)
- Also holds for causal trees

Random forest for inference and treatment effecs

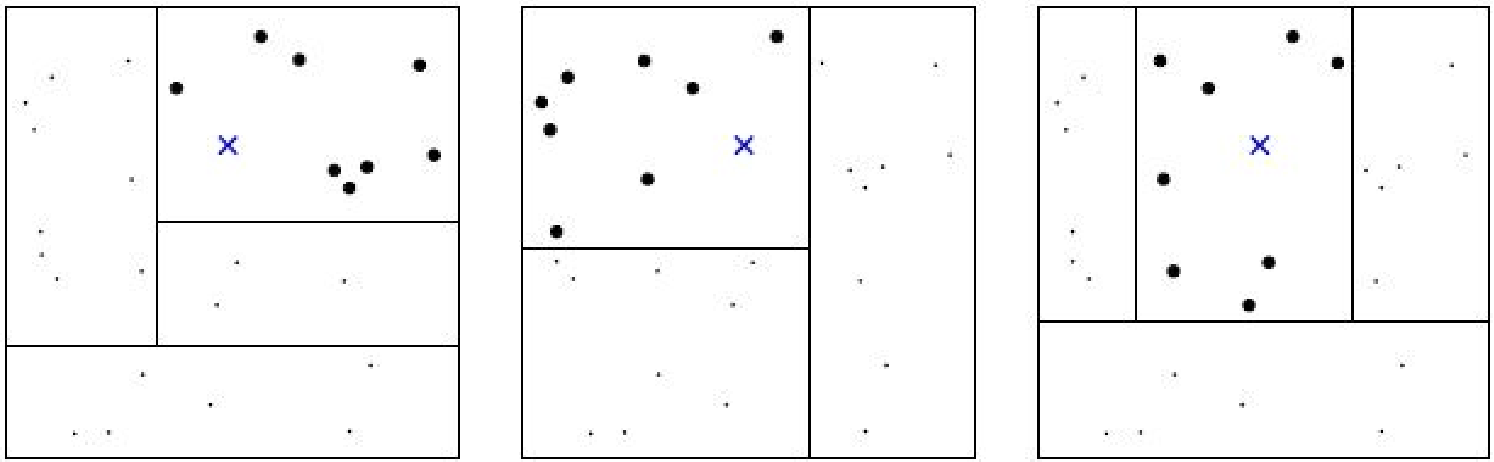
Causal Trees

The goal of causal trees is to establish unbiased, consistent estimates of heterogeneous treatment effects

- also known as conditional average treatment effects (**CATE**)
- the effect size is denoted $\hat{\tau}\left(x\right)$;
- standard tools for inference, e.g. using statistical tests locally

Causal Forest

What is the output from the decisions trees? Each tree produces a partitioning of the feature space X. Example of three trees:



Double Sample Trees

For Causal Trees

- first half ($\mathcal{J}$, $|\mathcal{J}| = \lceil s/2 \rceil$)
    - training Decision Tree
        - minimize adjusted MSE
        - require at least k observations for both treatment and control in all leaves of $\mathcal{I}$-sample
- other half ($\mathcal{I}$, $|\mathcal{I}| = \lfloor s/2 \rfloor$)
    - estimating treatment effects, $\hat{\tau}(x)$

Double Sample Trees (2)

For Regression Trees

- first half ($\mathcal{J}$, $|\mathcal{J}| = \lceil s/2 \rceil$)
    - training Decision Tree
        - minimize MSE / Gini etc.
        - require at least k observations in all leaves of $\mathcal{I}$-sample
- other half ($\mathcal{I}$, $|\mathcal{I}| = \lfloor s/2 \rfloor$)
    - estimating outcome, $\hat{\mu}(x)$

Quiz: How is this different from normal Decision Trees for regression problems?

- Unlike normal decision trees outcomes are estimated honestly.

Main results: econometric properties (1)

[Wager and Athey (2017) (https://doi.org/10.1080/01621459.2017.1319839)](https://doi.org/10.1080/01621459.2017.1319839) show

- We can estimate the variance of CATE
- $\hat{V}_{IJ}(x) = \frac{n-1}{n}\left(\frac{n}{n-s}\right)^2 \sum_{i=1}^{n} \mathrm{Cov}_* \left[\hat{\tau}_b^*(x), N_{ib}^*\right]^2$

Main results: econometric properties (2)

From Theorem 4.1 in [Wager and Athey (2017) (https://doi.org/10.1080/01621459.2017.1319839)](https://doi.org/10.1080/01621459.2017.1319839)

- The conditional average treatment estimates are unbiased and consistent
  - unbiased: no systematic error of measurement
  - consistency: with more data our estimate approaches true value
- Moreover, we can do inference:
  - The variance estimator $\hat{V}_{IJ}(x)$ is consistent.
  - Treatment effect estimates are asymptotic normal and unbiased
    - $(\hat{\tau}(x) - \tau(x))/\sqrt{\mathrm{Var}[\hat{\tau}(x)]} \Rightarrow \mathcal{N}(0,1)$

Caveat: only works for evaluating treatment effects in one point x! Do not perform multiple tests.

Useful forests

Two more procedures

1. Double Sampled Trees
    - using Regression trees for predicting outcome (=$\hat{\mu}(x)$)
2. Propensity Trees
    - using propensity trees for propensity score matching

What is the shared procedure?

- Each tree is estimated using repeated subsampling (**no replacement**)
    - Constrast to bootstrap aggregation in random forest (sample **with replacment**)
- Random subsample of features

More results

[Wager and Athey (2017) (https://doi.org/10.1080/01621459.2017.1319839)](https://doi.org/10.1080/01621459.2017.1319839) show that the same properties of Double Sample Trees using causal trees also hold analogously for regression trees.

- Random forests have the property of being asymptotic normal and can thus be used for inference
- Similar intuition as idea of nested CV where we could do inference

Simulation experiment

[Wager, and Athey (2017) (https://doi.org/10.1214/18-aos1709)](https://doi.org/10.1214/18-aos1709) compare causal forest to nearest neighbor methods

- random forest is kind of local nearest neighbor estimate
- based on work by Lin and Jeon (2006).

Simulation (1)

- simulation setup: no treatment effect, only confounding factors
- method: propensity trees
- comparison of estimated treatment effects
  - lower MSE and better coverage
  - coverage falls for increasing number of variables d

	Mean-squared error			Coverage		
d	CF	10-NN	100-NN	CF	10-NN	100-NN
2	0.02 (0)	0.21 (0)	0.09 (0)	0.95 (0)	0.93 (0)	0.62 (1)
5	0.02 (0)	0.24 (0)	0.12 (0)	0.94 (1)	0.92 (0)	0.52 (1)
10	0.02 (0)	0.28 (0)	0.12 (0)	0.94 (1)	0.91 (0)	0.51 (1)
15	0.02 (0)	0.31 (0)	0.13 (0)	0.91 (1)	0.90 (0)	0.48 (1)
20	0.02 (0)	0.32 (0)	0.13 (0)	0.88 (1)	0.89 (0)	0.49 (1)
30	0.02 (0)	0.33 (0)	0.13 (0)	0.85 (1)	0.89 (0)	0.48 (1)

Simulation (2)

- setup: heterogeneous treatment effect, **no** confounding factors
- comparison of estimated treatment effects
  - lower MSE and better coverage
  - coverage falls for increasing number of variables d

	Mean-squared error			Coverage		
d	CF	7-NN	50-NN	CF	7-NN	50-NN
2	0.04 (0)	0.29 (0)	0.04 (0)	0.97 (0)	0.93 (0)	0.94 (0)
3	0.03 (0)	0.29 (0)	0.05 (0)	0.96 (0)	0.93 (0)	0.92 (0)
4	0.03 (0)	0.30 (0)	0.08 (0)	0.94 (0)	0.93 (0)	0.86 (1)
5	0.03 (0)	0.31 (0)	0.11 (0)	0.93 (1)	0.92 (0)	0.77 (1)
6	0.02 (0)	0.34 (0)	0.15 (0)	0.93 (1)	0.91 (0)	0.68 (1)
8	0.03 (0)	0.38 (0)	0.21 (0)	0.90 (1)	0.90 (0)	0.57 (1)

Meta learners for heterogeneous treatment effects

Other procedures have been investigated

- Künzel et al. (2019) (https://doi.org/10.1073/pnas.1804597116) investigates more general class of prediction tools for partitioning data using
    - Lower EMSE in many cases relative to causal forest and BART (Bayesian tree based method)
- Nie and Wager (2017) (https://arxiv.org/pdf/1712.04912.pdf) investigates another class of methods called R-learners that leverages a smart representation of CATE.

Round-up causal forest

Summary of [Wager and Athey (2017) (https://doi.org/10.1080/01621459.2017.1319839)](https://doi.org/10.1080/01621459.2017.1319839)

- builds on Causal Trees method
- strong econometric properties
    - unbiased and consistent
    - asymptotic normality given x
        - causal and regression forest allows inference!
- problem:
    - must choose focus
        - unconfounding (propensity) or
        - estimate CATE
    - coverage was not good, especially for higher d!

Generalized Random Forest

A higher aim

Causal forests are pretty cool. Can we use our honest procedure more generally?

- Estimate any quantity $\theta(x)$ identified via local moment conditions, e.g.
    - simultaneously unconfound and find heterogeneity?
    - find heterogeneous treatment effects from IV estimation?

Different purpose

How does this look?

- The general moment conditions
  - $\mathbb{E}\left[\psi_{\theta(x),\nu(x)}\left(O_i\right)|X_i = x\right] = 0, \quad \forall x.$
- Where ψ estimating function, maps parameters and data into moment equations
  - Parameters
    - θ parameter we want estimate
    - ν is nuisance we want to "partial out"
  - Data
    - O_i main objects we are interested in modelling, e.g. Y_i, D_i
    - X_i covariates

Different purpose (2)

What is a moment condition?

- Similiar to solution to first order condition
- More general - can incorporate extra restrictions (e.g. unconfounding)

Different purpose (3)

Suppose we want to estimate treatment effects

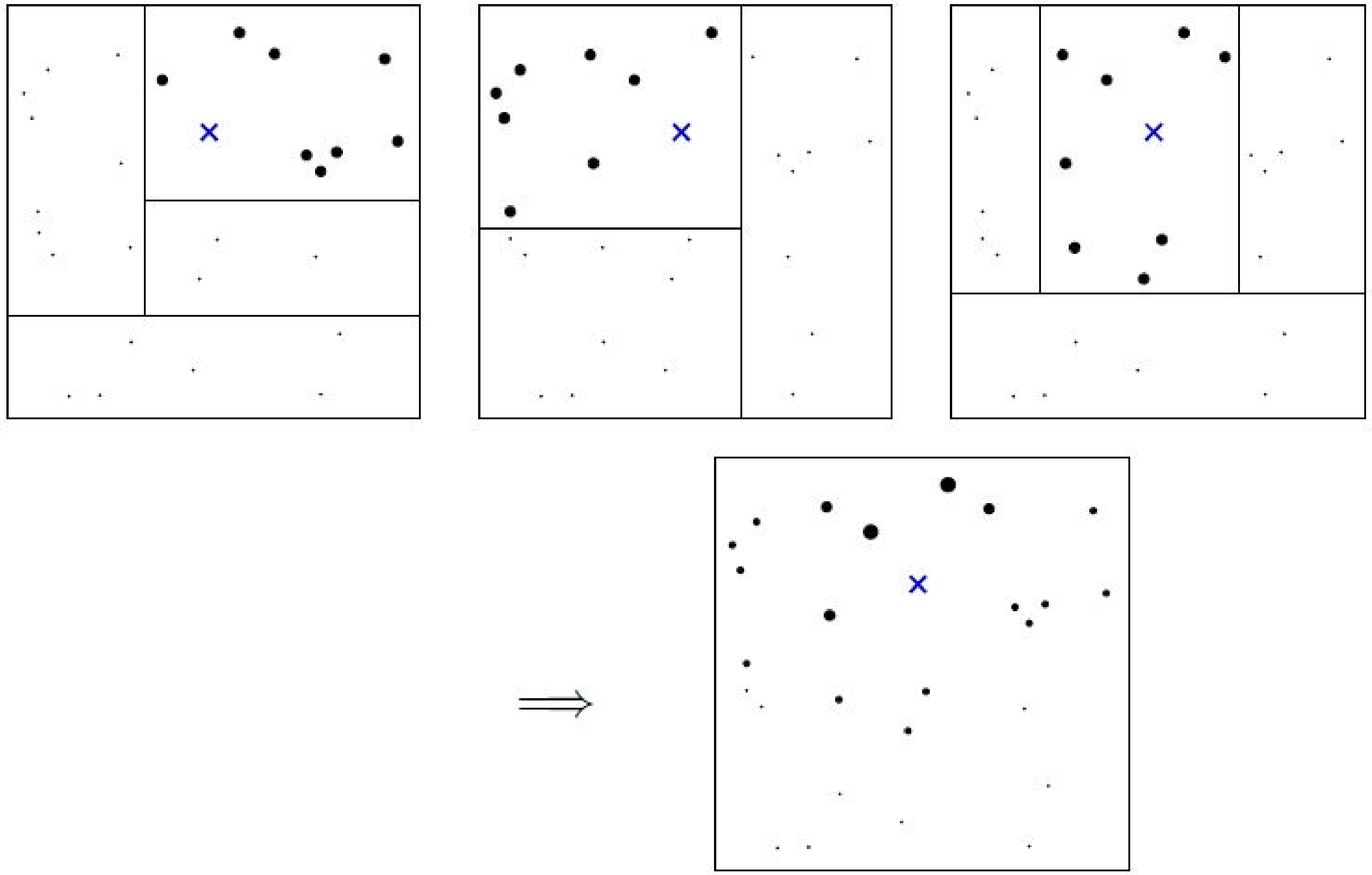
Functional form: $\psi_{\beta(x),c(x)} = Y_i - \beta(x)W_i - c(x)$ where

- β is treatment effect
- c is nuisance parameter

Using a kernel

Kernel methods can be used to unconfound and compute heterogeneous effects simulateneously

- Problem how to decide weights?



The Generalized Random Forest

[Athey, Wager, Tibshirani (2019) (https://doi.org/10.1214/18-aos1709)](https://doi.org/10.1214/18-aos1709) show that kernel weights can be estimated using forest methods

- can be adapted for different purposes
  - quantile regression
  - heterogeneous treatment effects
  - instrumental variables

The Generalized Random Forest (2)

Athey, Wager, Tibshirani (2019) (https://doi.org/10.1214/18-aos1709) use a procedure as follows:

1. Use estimating equation, ψ to estimate tree splits iteratively on subsample.
2. View forests as a weights of similar neighbors
   - Amount of partitions where observations
$$\alpha_i(x) = \frac{1}{B}\sum_{b=1} \frac{1(X_i \in L_b(x))}{|L_b(x)|}$$
3. Re-estimate ψ using weights on entire sample.

Difference from Causal Forest - trees are used for constructing weights!

In-class research: heterogeneous treatment effects

The student as a researcher

We will try to make a collaborative effort in doing a research project.

The primary goal is to learn how to apply the methods. Our effort may turn into research.

The task

Work alone or in pairs.

1. Find a research paper that runs an experiment, either field or lab. Requirements for paper:
    - There are is at least one or more covariates for each treated unit (e.g. gender if individuals).
    - There is experimental data is available. You may look in the dataverse at Harvard or papers from experimental economics etc.
2. Make a function that loads and structure the data in Python. The output should contain:
    - Outcomes, y a vector with n-observations
    - Treatments, D a vector with n-observations with $0, 1$
    - Covariates, X a matrix with $n \times k$ dimensions
3. Try to replicate the results in terms of computing ATE or ATT.
4. Use `grf` to compute average treatment effects and heterogeneous treatment effects.

    - Hint: you can use the `grf` tutorial [here (https://grf-labs.github.io/grf/articles/grf.html)](https://grf-labs.github.io/grf/articles/grf.html).

A tutorial on using grf within Python

Leveraging the rpy2 package

Installing R in Anaconda

Note if the script below fails go to the bottom.

In [ ]:
```
!conda install -c r rpy2 -y
```

Import R in python

If the code below fails, check out the guidance on adding PATH variables [for Linux or Mac (https://stackoverflow.com/questions/51486081/install-and-use-rpy2-using-conda-so-that-it-uses-default-r-installation-in-us)](https://stackoverflow.com/questions/51486081/install-and-use-rpy2-using-conda-so-that-it-uses-default-r-installation-in-us) and [for Windows (https://anaconda.zendesk.com/hc/en-us/articles/360023857134-Setting-up-rpy2-on-Windows)](https://anaconda.zendesk.com/hc/en-us/articles/360023857134-Setting-up-rpy2-on-Windows)

In [4]:

```
import rpy2
import rpy2.robjects as robjects
```

Installing R from Python

In [1]:

```
# import rpy2's package module
import rpy2.robjects.packages as rpackages

# import R's utility package
utils = rpackages.importr('utils')

# select a mirror for R packages
utils.chooseCRANmirror(ind=1) # select the first mirror in the list

# R package names
packnames = ('ggplot2', 'hexbin', 'grf')

# R vector of strings
from rpy2.robjects.vectors import StrVector

# Selectively install what needs to be install.
# We are fancy, just because we can.
names_to_install = [x for x in packnames if not rpackages.isinstalled(x)]
if len(names_to_install) > 0:
    utils.install_packages(StrVector(names_to_install))
```

Importing R package from Python

In [31]:

```
from rpy2.robjects.packages import importr
grf = importr('grf')
```

Make synthetic data

In [117]:
```
import numpy as np
import rpy2.robjects.numpy2ri
rpy2.robjects.numpy2ri.activate()
import rpy2.robjects as ro

n, p = 2000, 10
X = np.random.randn(n,p)
P = 0.4 + 0.2 * (X[:, 0] > 0)
D = (np.random.rand(n) > P).astype('float')
Y = np.max([X[:, 0] * D, np.zeros(n)],0) + \
    X[:, 1] + np.max([X[:, 2] * W, np.zeros(n)],0)
```

Apply grf's causal forest

In [118]:
```
from rpy2.robjects.packages import importr
grf = importr('grf')

X_r = ro.r.matrix(X, nrow=n, ncol=p)
Y_r = ro.r.matrix(Y, nrow=n, ncol=1)
D_r = ro.r.matrix(D, nrow=n, ncol=1)

tau_forest = grf.causal_forest(X_r, Y_r, D_r)
```

Getting predicted treatment effects for x range

In [119]:

```
from rpy2.robjects import pandas2ri
pandas2ri.activate()

X_range = np.zeros([201,p])
X_range[:,0] = np.linspace(-2,2,201)
X_range_r = ro.r.matrix(X_range, nrow=n, ncol=10)

tau_hat_r = ro.r.predict(tau_forest, X_range)
tau_hat = pandas2ri.ri2py(tau_hat_r)['predictions'].values
```

```
C:\Users\bvq720\AppData\Local\Continuum\anaconda3\lib\site-packages\rpy2\robjects\pan
das2ri.py:191: FutureWarning: from_items is deprecated. Please use DataFrame.from_dic
t(dict(items), ...) instead. DataFrame.from_dict(OrderedDict(items)) may be used to p
reserve the key order.
  res = PandasDataFrame.from_items(items)
```

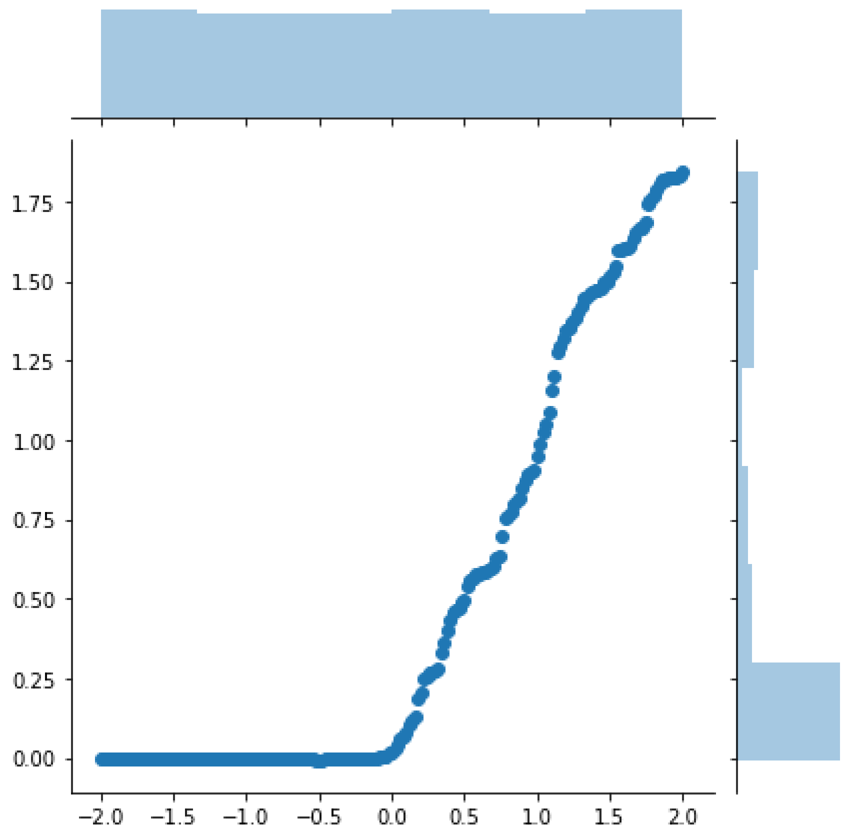
Plotting treatment effects

In [120]:

```
import seaborn as sns
%matplotlib inline
sns.jointplot(X_range[:,0], tau_hat)
```

Out[120]:

```
<seaborn.axisgrid.JointGrid at 0x2cb42438bc8>
```



Alternate way of installing GRF

- Step 1: download or look up the yaml file `sds_eml_2020.yml` from the GitHub repository (same folder as this file). This will be used to install a new conda environment from scratch.
- Step 2: go to the folder on your computer where you put the `sds_eml_2020.yml` file.
- Step 3: type in the lines below into the Anaconda Prompt / Terminal

```
conda env create -f sds_eml_2020.yml
conda activate sds_eml_2020
python -m ipykernel install --user --name python_grf --display-name "Python GRF"
```

- Step 4: restart your jupyter notebook. Select the new kernel called `Python GRF`.