DIGITAL SIGNAL PROCESSING - HOMEWORK # 2

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Q.1:) a) $x_1[n] = a^n u[n-1]$

$$\Rightarrow a^n u[n] \xleftrightarrow{DTFT} \frac{1}{1 - ae^{-j\omega}}$$

$\rightarrow$ Replacing $n \rightarrow n-1$

$$\Rightarrow a^{n-1} u[n-1] \xleftrightarrow{DTFT} e^{-j\omega}\left(\frac{1}{1-ae^{-j\omega}}\right)$$

$$\Rightarrow (a)^{-1}(a)^n u[n-1] \xleftrightarrow{DTFT} e^{-j\omega}\left(\frac{1}{1-ae^{-j\omega}}\right)$$

$\rightarrow$ Multiplying Both sides by a

$$\Rightarrow a^n u[n-1] \xleftrightarrow{\text{DTFT}} \frac{ae^{-j\omega}}{1-ae^{-j\omega}} = X(e^{j\omega})$$

b) $x_2[n] = a^n u[-n-1] = a^n u[-(n+1)]$

$$\Rightarrow X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} a^n u[-n-1] e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{-1} a^n e^{-j\omega n}$$

$$\Rightarrow \sum_{n=-\infty}^{-1} (ae^{-j\omega})^n$$

taking $m=-n$

$$\Rightarrow \sum_{m=1}^{\infty} (ae^{-j\omega})^{-m}$$

$$\Rightarrow \sum_{m=1}^{\infty} \left(a^{-1} e^{jw}\right)^m$$

$$\Rightarrow \left(a^{-1} e^{jw}\right) \sum_{m=0}^{\infty} \left(a^{-1} e^{jw}\right)^m$$

$$\Rightarrow \frac{a^{-1} e^{jw}}{1 - a^{-1} e^{jw}}$$

$$\Rightarrow \frac{e^{jw} \cancel{\left(\frac{1}{a}\right)}}{\frac{a - e^{jw}}{\cancel{a}}}$$

$$\Rightarrow \frac{e^{jw}}{a - e^{jw}}$$

Q.2:) a) $H_2(e^{j\omega}) = 1 + 2\cos(\omega) + 3\cos(2\omega)$

$$= 1 + \cancel{2}\left(\frac{e^{j\omega} + e^{-j\omega}}{\cancel{2}}\right) + 3\left(\frac{e^{j2\omega} + e^{-j2\omega}}{2}\right)$$

$$= 1 + e^{j\omega} + e^{-j\omega} + \frac{3}{2}e^{j2\omega} + \frac{3}{2}e^{-j2\omega}$$

$* \; \delta[n] \xrightarrow{\text{DTFT}} 1$

$* \; \delta[n-n_0] \xrightarrow{\text{DTFT}} (1)e^{-j\omega n_0}$

Time Shift   Freq Shift

$\rightarrow \; 1 \xrightarrow{\text{DTFT}^{-1}} \delta[n]$

$\rightarrow \; e^{j\omega} \xrightarrow{\text{DTFT}^{-1}} \delta[n+1]$

$\rightarrow e^{-j\omega} \xrightarrow{DTFT^{-1}} \delta[n-1]$

$\rightarrow e^{j2\omega} \xrightarrow{DTFT^{-1}} \delta[n+2]$

$\rightarrow e^{-j2\omega} \xrightarrow{DTFT^{-1}} \delta[n-2]$

Using Linearity ...

$\Rightarrow h_2[n] = \delta[n] + \delta[n+1] + \delta[n-1] + \frac{3}{2}\delta[n+2] + \frac{3}{2}\delta[n-2]$

b) $H_3(e^{jw}) = j[3 + 4\cos(w) + 2\cos(2w)]\sin(w)$

$= j3\sin(w) + j4\cos(w)\sin(w) + j2\cos(2w)\sin(w)$

$* \sin(2w) = 2\sin(w)\cos(w)$

$= j3\sin(w) + j2\sin(2w) + j(\sin(3w) - \sin(w))$

$= j2\sin(w) + j2\sin(2w) + j\sin(3w)$

$= \cancel{j}\cancel{2}\left(\frac{e^{jw} - e^{-jw}}{\cancel{2}j}\right) + \cancel{j}\cancel{2}\left(\frac{e^{j2w} - e^{-j2w}}{\cancel{2}j}\right) + \cancel{j}\left(\frac{e^{j3w} - e^{-j3w}}{2\cancel{j}}\right)$

$= e^{jw} - e^{-jw} + e^{j2w} - e^{-j2w} + \frac{1}{2}e^{j3w} - \frac{1}{2}e^{-j3w}$

$h_3[n] = \delta[n+1] - \delta[n-1] + \delta[n+2] - \delta[n-2] + \frac{1}{2}\delta[n+3] - \frac{1}{2}\delta[n-3]$

Q.3:) a) i) $x_1[n] = (0.3)^n u[n+1]$

$$\Rightarrow X_1(Z) = \sum_{n=-\infty}^{\infty} (0.3)^n u[n+1] Z^{-n}$$

$$= \sum_{n=-1}^{\infty} (0.3)^n Z^{-n}$$

$$= \frac{Z}{0.3} \sum_{n=0}^{\infty} (0.3)^n Z^{-n}$$

$$= \frac{Z}{0.3} \sum_{n=0}^{\infty} (0.3Z^{-1})^n$$

$$= \frac{Z}{0.3} \left(\frac{1}{1-0.3Z^{-1}} \right)$$

$$= \frac{Z}{0.3} \left(\frac{Z}{Z-0.3} \right)$$

$$= \frac{Z^2}{0.3(Z-0.3)}$$

$$\boxed{\begin{array}{c} \underline{\text{ROC}} \\ |0.3z^{-1}| < 1 \\ \downarrow \\ |z| > |0.3| \end{array}}$$

ii) $x_2[n] = (0.7)^n u[n-1]$

$$\Rightarrow X_2(Z) = \sum_{n=-\infty}^{\infty} (0.7)^n u[n-1] Z^{-n}$$

$$= \sum_{n=1}^{\infty} (0.7)^n Z^{-n}$$

<u>Taking $m = n-1$</u>

$$= \sum_{m=0}^{\infty} (0.7)^{m+1} Z^{-(m+1)}$$

$$= \frac{0.7}{Z} \sum_{m=0}^{\infty} (0.7)^m Z^{-m}$$

$$= \frac{0.7}{Z} \sum_{m=0}^{\infty} (0.7Z^{-1})^m$$

$$= \frac{0.7}{Z} \left(\frac{1}{1-0.7z^{-1}} \right)$$

$$= \frac{0.7}{\cancel{Z}}\left(\frac{\cancel{Z}}{Z - 0.7}\right)$$

$$= \frac{0.7}{Z - 0.7}$$

$\boxed{\begin{array}{c} \underline{\text{ROC}} \\ |0.7Z^{-1}| < 1 \\ |z| > |0.7| \end{array}}$

iii) $x_3[n] = (0.4)^n u[n-5]$

$$\Rightarrow X_3(Z) = \sum_{n=-\infty}^{\infty} (0.4)^n u[n-5] Z^{-n}$$

$$= \sum_{n=5}^{\infty} (0.4)^n Z^{-n}$$

<u>Taking $m = n-5$</u>

$$= \sum_{m=0}^{\infty} (0.4)^{m+5} Z^{-(m+5)}$$

$$= \left(\frac{0.4}{z}\right)^5 \sum_{m=0}^{\infty} (0.4)^m z^{-m}$$

$$= \left(\frac{0.4}{z}\right)^5 \sum_{m=0}^{\infty} (0.4z^{-1})^m$$

$$= \left(\frac{0.4}{z}\right)^5 \left(\frac{1}{1-0.4z^{-1}}\right)$$

ROC

$|0.4z^{-1}| < 1$

$|z| > |0.4|$

$$= \left(\frac{0.4}{z}\right)^5 \left(\frac{z}{z-0.4}\right)$$

$$= \frac{(0.4)^5}{(z)^4(z-0.4)}$$

iv) $x_4[n] = (-0.4)^n u[-n-2]$

$$\Rightarrow X_4(Z) = \sum_{n=-\infty}^{\infty} (-0.4)^n u[-n-2] Z^{-n}$$

$$= \sum_{n=-\infty}^{n=-2} (0.4)^n Z^{-n}$$

<u>Taking $m = -n$</u>

$$= \sum_{m=2}^{\infty} (0.4)^{-m} Z^m$$

<u>Taking $L = m - 2$</u>

$$= \sum_{L=0}^{\infty} (0.4)^{-(L+2)} Z^{L+2}$$

$$= \left(\frac{Z}{0.4}\right)^2 \sum_{L=0}^{\infty} (0.4)^{-L} Z^L$$

$$= \left(\frac{Z}{0.4}\right)^2 \sum_{L=0}^{\infty} (0.4^{-1} Z)^L$$

$$= \left(\frac{Z}{0.4}\right)^2 \left(\frac{1}{1 - 0.4^{-1} Z}\right)$$

$$= \left(\frac{Z}{0.4}\right)^2 \left(\frac{0.4}{0.4 - Z}\right)$$

$$= \frac{Z^2}{0.4(0.4 - Z)}$$

$$\boxed{\begin{array}{c} \underline{\text{ROC}} \\ |0.4^{-1} z| < 1 \\ |z| < |0.4| \end{array}}$$

b) i) $y_1[n] = x_1[n] + x_2[n]$

Using Linearity Property

$\Rightarrow Y_1(Z) = X_1(Z) + X_2(Z)$

$$= \underbrace{\frac{Z^2}{0.3(Z-0.3)}}_{ROC:\ |Z| > |0.3|} + \underbrace{\frac{0.7}{Z-0.7}}_{ROC:\ |Z| > |0.7|}$$

$\Rightarrow$ ROC of $y_1[n]$: $|Z| > |0.7|$

ii) $y_2[n] = x_2[n] + x_3[n]$

<u>Using Linearity Property</u>

$$\Rightarrow Y_2(Z) = X_2(Z) + X_3(Z)$$

$$= \underbrace{\frac{0.7}{Z - 0.7}}_{\text{ROC: } |Z| > |0.7|} + \underbrace{\frac{(0.4)^5}{(Z)^4 (Z - 0.4)}}_{\text{ROC: } |Z| > |0.4|}$$

$\Rightarrow$ ROC of $y_2[n]$: $|Z| > |0.7|$

iii) $y_3[n] = x_3[n] + x_4[n]$

<u>Using Linearity Property</u>

$\Rightarrow Y_3(Z) = X_3(Z) + X_4(Z)$

$$= \underbrace{\frac{(0.4)^5}{(Z)^4(Z-0.4)}}_{ROC:\ |Z| > |0.4|} + \underbrace{\frac{Z^2}{0.4(0.4-Z)}}_{ROC:\ |Z| < |0.4|}$$

ROC for $y_3[n]$: No ROC

Q.4:) $x[n] = 2.1(0.4)^n u[n] + 0.3(-0.3)^n u[n]$

$$\Rightarrow X(Z) = \frac{2.1}{1-0.4z^{-1}} + \frac{0.3}{1+0.3z^{-1}}$$

$$Y(Z) = X(Z)H(Z)$$

$$\Rightarrow Y(Z) = \left[\frac{2.1}{(1-0.4z^{-1})} + \frac{0.3}{(1+0.3z^{-1})}\right]\left[\frac{1-3.3z^{-1}+0.36z^{-2}}{1+0.3z^{-1}-0.18z^{-2}}\right]$$

$$= \left[\frac{2.1(1+0.3z^{-1})+0.3(1-0.4z^{-1})}{(1-0.4z^{-1})(1+0.3z^{-1})}\right]\left[\frac{1-3.3z^{-1}+0.36z^{-2}}{\left(z^{-1}-\frac{10}{3}\right)\left(\frac{5}{3}+z^{-1}\right)}\right]$$

$$= \left[\frac{2.4+0.51z^{-1}}{(1-0.4z^{-1})(1+0.3z^{-1})}\right]\left[\frac{1-3.3z^{-1}+0.36z^{-2}}{\left(z^{-1}-\frac{10}{3}\right)\left(\frac{5}{3}+z^{-1}\right)}\right]$$

$$= \frac{2.4 - 7.92z^{-1} + 0.864z^{-2} + 0.51z^{-1} - 1.683z^{-2} + 0.1836z^{-3}}{(1-0.4z^{-1})(1+0.3z^{-1})(z^{-1}-\frac{10}{3})(\frac{5}{3}+z^{-1})}$$

$$= \frac{2.4 - 7.41z^{-1} - 0.819z^{-2} + 0.1836z^{-3}}{(1-0.4z^{-1})(1+0.3z^{-1})(z^{-1}-\frac{10}{3})(\frac{5}{3}+z^{-1})}$$

Using Partial Fractions

$$= \frac{A}{(1-0.4z^{-1})} + \frac{B}{(1+0.3z^{-1})} + \frac{C}{(z^{-1}-\frac{10}{3})} + \frac{D}{(\frac{5}{3}+z^{-1})}$$

$$= A(1+0.3z^{-1})(z^{-1}-\tfrac{10}{3})(\tfrac{5}{3}+z^{-1}) + B(1-0.4z^{-1})(z^{-1}-\tfrac{10}{3})(\tfrac{5}{3}+z^{-1})$$

$$+ C(1-0.4z^{-1})(1+0.3z^{-1})(\tfrac{5}{3}+z^{-1}) + D(1-0.4z^{-1})(1+0.3z^{-1})(z^{-1}-\tfrac{10}{3})$$

Taking $Z^{-1} = 10/4$

$$\Rightarrow 2.4 - 7.41\left(\frac{10}{4}\right) - 0.819\left(\frac{10}{4}\right)^2 + 0.1836\left(\frac{10}{4}\right)^3$$

$$= A\left(1+0.3\left(\frac{10}{4}\right)\right)\left(\frac{10}{4}-\frac{10}{3}\right)\left(\frac{5}{3}+\frac{10}{4}\right)$$

$$\Rightarrow +18.375 = +6.076A$$

$$\Rightarrow A = \frac{18.375}{6.076} = -3.024$$

Taking $Z = -10/3$

$$\Rightarrow 2.4 - 7.41\left(-\frac{10}{3}\right) - 0.819\left(-\frac{10}{3}\right)^2 + 0.1836\left(-\frac{10}{3}\right)^3$$

$$= B\left(1-0.4\left(-\frac{10}{3}\right)\right)\left(-\frac{10}{3}-\frac{10}{3}\right)\left(\frac{5}{3}-\frac{10}{3}\right)$$

$$\Rightarrow 11.2 = 25.925B$$

$$\Rightarrow B = \frac{11.2}{25.925} = 0.432$$

<u>Taking $Z^{-1} = \frac{10}{3}$</u>

$$\Rightarrow 2.4 - 7.41\left(\frac{10}{3}\right) - 0.819\left(\frac{10}{3}\right)^2 + 0.1836\left(\frac{10}{3}\right)^3$$

$$= C\left(1 - 0.4\left(\frac{10}{3}\right)\right)\left(1 + 0.3\left(\frac{10}{3}\right)\right)\left(\frac{5}{3} + \frac{10}{3}\right)$$

$$\Rightarrow \cancel{-}24.6 = \cancel{-}3.33C$$

$$\Rightarrow C = \frac{24.6}{3.33} = 7.38$$

Taking $z^{-1} = -10/6$

$$\Rightarrow 2.4 - 7.41\left(-\frac{10}{6}\right) - 0.819\left(-\frac{10}{6}\right)^2 + 0.1836\left(-\frac{10}{6}\right)^3$$

$$= D\left(1 - 0.4\left(-\frac{10}{6}\right)\right)\left(1 + 0.3\left(-\frac{10}{6}\right)\right)\left(-\frac{10}{6} - \frac{10}{3}\right)$$

$$\Rightarrow 11.625 = -4.166 D$$

$$\Rightarrow D = \frac{11.625}{-4.166} = -2.79$$

$$= -\frac{3.024}{(1-0.4z^{-1})} + \frac{0.432}{(1+0.3z^{-1})} + \frac{7.38}{\left(-\frac{10}{3}+z^{-1}\right)} - \frac{2.79}{\left(\frac{5}{3}+z^{-1}\right)}$$

$$= \frac{-3.024}{(1-0.4z^{-1})} + \frac{0.432}{(1+0.3z^{-1})} + \frac{7.38}{\left(\frac{-3}{10}\right)\left(-\frac{10}{3}+z^{-1}\right)} - \frac{2.79}{\left(\frac{3}{5}\right)\left(\frac{5}{3}+z^{-1}\right)}$$

$$= \frac{-3.024}{1-0.4z^{-1}} + \frac{0.432}{1+0.3z^{-1}} - \frac{2.214}{1-0.3z^{-1}} - \frac{1.674}{1+0.6z^{-1}}$$

$$\Rightarrow y[n] = -3.024(0.4)^n u[n] + 0.432(-0.3)^n u[n]$$

$$-2.214(0.3)^n u[n] - 1.674(-0.6)^n u[n]$$

Q.5:) $$H(Z) = \frac{1 - z^{-2}}{1-(1+\alpha)\cos(\omega_c)z^{-1}+\alpha z^{-2}}$$

$$\Rightarrow H(e^{j\omega}) = H(Z) \text{ for all } z = e^{j\omega}$$

$$\Rightarrow H(e^{j\omega}) = \frac{1 - e^{-j2\omega}}{1-(1+\alpha)\cos(\omega_c)e^{-j\omega}+\alpha e^{-j2\omega}}$$

<u>For $\omega = \omega_c$</u>

$$= \frac{1-e^{-j2\omega_c}}{1-(1+\alpha)\cos(\omega_c)e^{-j\omega_c}+\alpha e^{-j2\omega_c}}$$

$$= \frac{1-e^{-j2\omega_c}}{1-(1+\alpha)\left(\frac{e^{j\omega_c}+e^{-j\omega_c}}{2}\right)e^{-j\omega_c}+\alpha e^{-j2\omega_c}}$$

$$= \frac{1 - e^{-j2\omega_c}}{1-(1+\alpha)\left(\frac{1+e^{-j2\omega_c}}{2}\right)+\alpha e^{-j2\omega_c}}$$

$$= \frac{1 - e^{-j2\omega_c}}{1-\left(\frac{1+e^{-j2\omega_c}}{2}\right)-\alpha\left(\frac{1+e^{-j2\omega_c}}{2}\right)+\alpha e^{-j2\omega_c}}$$

$$= \frac{1 - e^{-j2\omega_c}}{\frac{1}{2}-\frac{e^{-j2\omega_c}}{2}-\frac{\alpha}{2}-\frac{\alpha e^{-j2\omega_c}}{2}+\alpha e^{-j2\omega_c}}$$

$$= \frac{1 - e^{-j2\omega_c}}{\frac{(1-\alpha)}{2}-\frac{(1+\alpha)}{2}e^{-j2\omega_c}+\alpha e^{-j2\omega_c}}$$

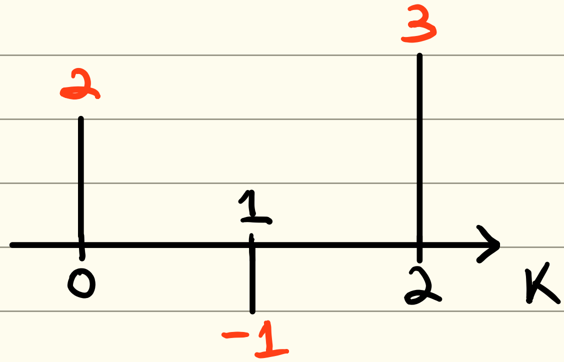
$$= \frac{1 - e^{-j2\omega_c}}{\frac{(1-\alpha)}{2} + e^{-j2\omega_c}\left(-\frac{1}{2} - \frac{\alpha}{2} + \alpha\right)}$$

$$= \frac{1 - e^{-j2\omega_c}}{\frac{(1-\alpha)}{2} - e^{-j2\omega_c}\left(\frac{1-\alpha}{2}\right)}$$

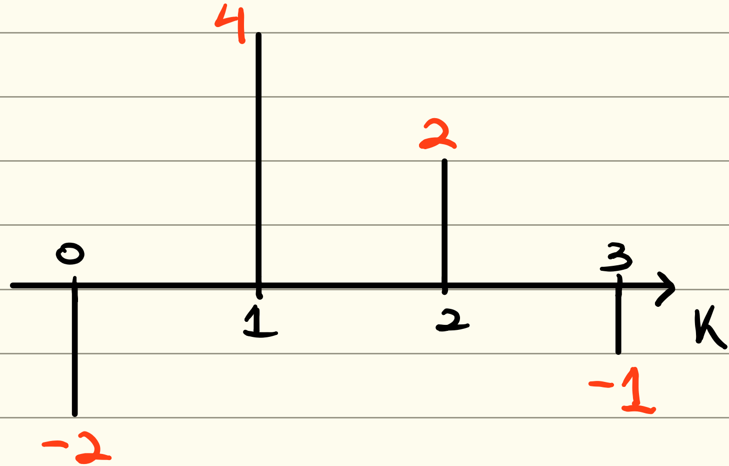
$$= \frac{\cancel{1 - e^{-j2\omega_c}}}{\frac{(1-\alpha)}{2}\cancel{\left(1 - e^{-j2\omega_c}\right)}}$$

$$= \frac{1}{\frac{(1-\alpha)}{2}} = \frac{2}{(1-\alpha)} \quad \text{(Shown)}$$

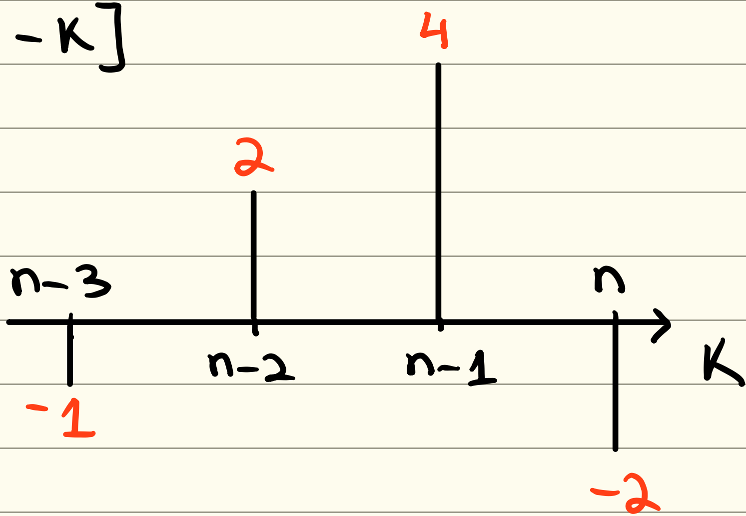
Q.6:) a) * g[K]



* h[K]



* $h[n-k]$



* $\underline{n < 0}$

$y_L[n] = 0$

* $\underline{n = 0}$

$y_L[n] = -4$

* $\underline{n = 1}$

$$y_L[n] = (-2 \times -1) + (4 \times 2)$$
$$= 2 + 8$$
$$= 10$$

* $\underline{n = 2}$

$$y_L[n] = (-2 \times 3) + (4 \times -1) + (2 \times 2)$$
$$= -6 - \cancel{4} + \cancel{4}$$
$$= -6$$

* $\underline{n - 1 = 2 \rightarrow n = 3}$

$$y_L[n] = (4 \times 3) + (2 \times -1) + (-1 \times 2)$$

$= 12 - 2 - 2 = 8$

* $\underline{n - 2 = 2 \longrightarrow n = 4}$

$y_L[n] = (2 \times 3) + (-1 \times -1)$
$= 6 + 1$
$= 7$

* $\underline{n - 3 = 2 \longrightarrow n = 5}$

$y_L[n] = (-1 \times 3) = -3$

* $\underline{n - 3 > 2 \longrightarrow n > 5}$

$y_L[n] = 0$

$$y_L[n] = \{\underset{\uparrow}{-4}, 10, -6, 8, 7, -3\} \;;\; 0 \leqslant n \leqslant 5$$

b) $g[n] = \{\underset{\uparrow}{2}, -1, 3, 0\} \rightarrow$ (After Zero Padding)

$$h[n] = \{\underset{\uparrow}{-2}, 4, 2, -1\}$$

<u>Using Matrix Method</u>

$$= \begin{bmatrix} 2 & 0 & 3 & -1 \\ -1 & 2 & 0 & 3 \\ 3 & -1 & 2 & 0 \\ 0 & 3 & -1 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 4 \\ 2 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} (2\times-2)+(0\times4)+(3\times2)+(-1\times-1) \\ (-1\times-2)+(2\times4)+(0\times2)+(3\times-1) \\ (3\times-2)+(-1\times4)+(2\times2)+(0\times-1) \\ (0\times-2)+(3\times4)+(-1\times2)+(2\times-1) \end{bmatrix}$$

$$= \begin{bmatrix} -4+0+6+1 \\ 2+8+0-3 \\ -6-4+4+0 \\ 0+12-2-2 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \\ -6 \\ 8 \end{bmatrix}$$

$$\Rightarrow y_c[n] = \{ \underset{\uparrow}{3}, 7, -6, 8 \}$$

c) $g[n] = \{2, -1, 3, 0\}$

$$\Rightarrow G(K) = \sum_{n=0}^{N-1} g[n] e^{-j 2\pi K \frac{n}{N}}$$

$$= \sum_{n=0}^{3} g[n] e^{-j \frac{\pi}{2} K n} \quad , K = 0, 1, 2, 3$$

$$= 2e^{0} - e^{-j\frac{\pi}{2}K} + 3e^{-j\pi K} + 0$$

$$= 2 - e^{-j\frac{\pi}{2}K} + 3e^{-j\pi K}$$

$$\Rightarrow G(0) = 2 - e^{0} + 3e^{0} = 2 - \underline{1} + 3 = 4$$

$$\Rightarrow G(1) = 2 - e^{-j\frac{\pi}{2}} + 3e^{-j\pi}$$

$$= 2 + j - 3$$

$$= j - 1$$

$$\Rightarrow H(2) = 2 - e^{-j\pi} + 3e^{-j2\pi}$$
$$= 2 + 1 + 3$$
$$= 6$$

$$\Rightarrow H(3) = 2 - e^{-j\frac{3\pi}{2}} + 3e^{-j3\pi}$$
$$= 2 - j - 3$$
$$= -j - 1$$

$$\Rightarrow H(K) = \{4, j-1, 6, -j-1\}$$

$$h[n] = \{-2, 4, 2, -1\}$$

$$\Rightarrow H(K) = \sum_{n=0}^{N-1} h[n] e^{-j2\pi K \frac{n}{N}}$$

$$= \sum_{n=0}^{3} h[n] e^{-j\frac{\pi}{2}kn} \quad , K = 0, 1, 2, 3$$

$$= -2e^{0} + 4e^{-j\frac{\pi}{2}k} + 2e^{-j\pi k} - e^{-j\frac{3\pi}{2}k}$$

$$= -2 + 4e^{-j\frac{\pi}{2}k} + 2e^{-j\pi k} - e^{-j\frac{3\pi}{2}k}$$

$$\Rightarrow H(0) = -2 + 4e^{0} + 2e^{0} - e^{0}$$

$$= -\cancel{2} + 4 + \cancel{2} - 1$$

$$= 3$$

$$\Rightarrow H(1) = -2 + 4e^{-j\frac{\pi}{2}} + 2e^{-j\pi} - e^{-j\frac{3\pi}{2}}$$

$$= -2 - 4j - 2 - j$$

$$= -4 - 5j$$

$$\Rightarrow H(2) = -2 + 4e^{-j\pi} + 2e^{-j2\pi} - e^{-j3\pi}$$

$$= -\cancel{2} - 4 + \cancel{2} + 1$$
$$= -3$$

$$\Rightarrow H(3) = -2 + 4e^{-j\frac{3\pi}{2}} + 2e^{-j3\pi} - e^{j\frac{9\pi}{2}}$$
$$= -2 + 4j - 2 + j$$
$$= -4 + 5j$$

$$\Rightarrow H(K) = \{3, -4-5j, -3, -4+5j\}$$

$$Y(K) = G(K) \cdot H(K)$$

$$\Rightarrow Y(0) = 4 \times 3 = 12$$

$$\Rightarrow Y(1) = (j-1)(-4-5j)$$
$$= j(-4) + j(-5j) - (-4) - (-5j)$$

$$= -4j + 5 + 4 + 5j$$
$$= 9 + j$$

$$\Rightarrow Y(2) = 6 \times -3$$
$$= -18$$

$$\Rightarrow Y(3) = (-j-1)(-4+5j)$$
$$= -j(-4) - j(5j) - (-4) - (5j)$$
$$= +4j + 5 + 4 - 5j$$
$$= 9 - j$$

<u>Taking Inverse DFT of $Y(K)$</u>

$$\Rightarrow y[n] = \frac{1}{N} \sum_{k=0}^{N-1} Y(k)\, e^{j 2\pi k \frac{n}{N}}$$

$$= \frac{1}{4} \sum_{k=0}^{3} Y(k)\, e^{j \frac{\pi}{2} k n}$$

$$= \frac{1}{4} \left[12 + (9+j) e^{j\frac{\pi}{2} n} - 18\, e^{j \pi n} + (9-j) e^{j \frac{3\pi}{2} n} \right]$$

$$\Rightarrow y[0] = \frac{1}{4} \left[12 + \cancel{9} + \cancel{j} - \cancel{18} + \cancel{9} - \cancel{j} \right]$$

$$= \frac{12}{4} = 3$$

$$\Rightarrow y[1] = \frac{1}{4} \left[12 + (9+j) j + 18 + (9-j)(-j) \right]$$

$$= \frac{1}{4} \left[12 + \cancel{9j} - 1 + 18 - \cancel{9j} - 1 \right]$$

$$= \frac{1}{4}(28) = 7$$

$$\Rightarrow y[2] = \frac{1}{4}\left[12 + (9+j)(-1) - 18 + (9-j)(-1)\right]$$

$$= \frac{1}{4}\left[12 - 9 - \cancel{j} - 18 - 9 + \cancel{j}\right]$$

$$= \frac{1}{4}(-24) = -6$$

$$\Rightarrow y[3] = \frac{1}{4}\left[12 + (9+j)(-j) - 18(-1) + (9-j)(j)\right]$$

$$= \frac{1}{4}\left[12 - \cancel{9j} + 1 + 18 + \cancel{9j} + 1\right] = \frac{32}{4} = 8$$

$$\Rightarrow y_c[n] = \{\underset{\uparrow}{3}, 7, -6, 8\}$$

d) $g[n] = \{ \underset{\uparrow}{2}, -1, 3, 0, 0, 0 \}$

$h[n] = \{-2, 4, 2, -1, 0, 0\}$

<u>Using Matrix Method</u>

$$= \begin{bmatrix} 2 & 0 & 0 & 0 & 3 & -1 \\ -1 & 2 & 0 & 0 & 0 & 3 \\ 3 & -1 & 2 & 0 & 0 & 0 \\ 0 & 3 & -1 & 2 & 0 & 0 \\ 0 & 0 & 3 & -1 & 2 & 0 \\ 0 & 0 & 0 & 3 & -1 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 4 \\ 2 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} (2\times -2)+0+0+0+0+0 \\ (-1\times -2)+(2\times 4)+0+0+0+0 \\ (3\times -2)+(-1\times 4)+(2\times 2)+0+0+0 \\ 0+(3\times 4)+(-1\times 2)+(2\times -1)+0+0 \\ 0+0+(3\times 2)+(-1\times -1)+0+0 \\ 0+0+0+(3\times -1)+0+0 \end{bmatrix}$$

$$= \begin{bmatrix} -4 \\ 2+8 \\ -6-4+4 \\ 12-2-2 \\ 6+1 \\ -3 \end{bmatrix} = \begin{bmatrix} -4 \\ 10 \\ -6 \\ 8 \\ 7 \\ -3 \end{bmatrix}$$

$$y_c[n] = \{\underset{\uparrow}{-4}, 10, -6, 8, 7, -3\}$$

This is exactly the same as what we got in part (a).

Q.7:) $X[n] = \sin\left(\frac{n\pi}{2}\right)$; $n = 0, 1, 2, 3 \ldots$

$N = 4,\ L = 4 \longrightarrow N = L$ (No Zero Padding Required)

$\Rightarrow W_N = e^{-j\frac{2\pi}{N}} = e^{-j\frac{2\pi}{4}} = e^{-j\frac{\pi}{2}}$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} (1\times 0)+(1\times 1)+(1\times 0)+(1\times -1) \\ (1\times 0)+(-j\times 1)+(-1\times 0)+(j\times -1) \\ (1\times 0)+(-1\times 1)+(1\times 0)+(-1\times -1) \\ (1\times 0)+(j\times 1)+(-1\times 0)+(-j\times -1) \end{bmatrix}$$

$$= \begin{bmatrix} 0+1+0-1 \\ 0-j+0-j \\ 0-1+0+1 \\ 0+j+0+j \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -2j \\ 0 \\ 2j \end{bmatrix} \quad \rightarrow X(K) = \{0, -2j, 0, 2j\}$$

Q.8:) $X(K) = \{1, 0, 1, 0\}$, $N=4$, $L=4$

DFT in Matrix Method

$\Rightarrow X(K) = W_N \cdot x[n]$

Inverse DFT in Matrix Method

$\Rightarrow x[n] = \frac{1}{N} W_N^{*} \cdot X(K)$

$$W_N^{*} = W_N^{-Kn} = \left(e^{-j\frac{2\pi}{N}}\right)^{-Kn} = \left(e^{j\frac{\pi}{2}}\right)^{Kn} = j^{Kn}$$

$$\Rightarrow W_N^{*} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$

$$\Rightarrow \quad x[n] = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix}$$

$$\begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} (1\times 1)+(1\times 0)+(1\times 1)+(1\times 0) \\ (1\times 1)+(j\times 0)+(-1\times 1)+(-j\times 0) \\ (1\times 1)+(-1\times 0)+(1\times 1)+(-1\times 0) \\ (1\times 1)+(-j\times 0)+(-1\times 1)+(j\times 0) \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 1+0+1+0 \\ 1+0-1+0 \\ 1+0+1+0 \\ 1+0-1+0 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 2 \\ 0 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 0 \\ 1/2 \\ 0 \end{bmatrix}$$

$$\Rightarrow x[n] = \{1/2, 0, 1/2, 0\}$$