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Q1

a) $x_1[n] = a^n u[n-1]$

Answer: $X_1(e^{j\omega}) = \sum_{n=-\infty}^{\infty} \{ a^n u[n-1] \} e^{-j\omega n} \Rightarrow \sum_{n=1}^{\infty} a^n e^{-j\omega n}$

$\Rightarrow \sum_{n=1}^{\infty} (a e^{-j\omega})^n = \frac{a e^{-j\omega}}{1 - a e^{-j\omega}} \Rightarrow \frac{\frac{a}{e^{j\omega}}}{1 - \frac{a}{e^{j\omega}}} \Rightarrow \frac{a}{e^{j\omega} - a}$

b) $x_2[n] = a^n u[-n-1]$

$X_2(e^{j\omega}) = \sum_{n=-\infty}^{\infty} a^n u(-n-1) e^{-j\omega n}$

$\Rightarrow \sum_{n=1}^{\infty} a^n u[-n] e^{-j\omega n}$

Let $m = -n$

$\Rightarrow \sum_{m=1}^{\infty} (a^{-1} e^{j\omega})^m = \frac{a^{-1} e^{j\omega}}{1 - a^{-1} e^{j\omega}} = \frac{e^{j\omega}/a}{1 - e^{j\omega}/a} = \frac{e^{j\omega}}{a - e^{j\omega}}$

Q2 a) $H_2(e^{jw}) = 1 + 2\cos(w) + 3\cos(2w)$

$\Rightarrow 1 + \cancel{2}\left(\dfrac{e^{jw} + e^{-jw}}{\cancel{2}}\right) + 3\left(\dfrac{e^{2jw} + e^{-2jw}}{2}\right)$ ①

$\Rightarrow 1 + e^{jw} + e^{-jw} + \frac{3}{2}e^{j2w} + \frac{3}{2}e^{-j2w}$

* $\delta[n] \xrightarrow{DTFT} 1$

$\underbrace{\delta[n-n_0]}_{\text{Time shift}} \longrightarrow \underbrace{(1)e^{-jwn_0}}_{\text{Frequency shift}}$

$1 \xrightarrow{DTFT^{-1}} \delta[n]$

$e^{jw} \longrightarrow \delta[n+1]$

① Using linearity

$\Rightarrow h_2[n] = \delta[n] + \delta[n+1] + \delta[n-1] + \frac{3}{2}\delta[n+2] + \frac{3}{2}\delta[n-2]$

b) $H_3(e^{jw}) = j[3 + 4\cos(w) + 2\cos(2w)]\sin(w)$

$\Rightarrow j3\sin(w) + j4\cos(w)\sin(w) + j2\cos(2w)\sin(w)$

$\cos x \sin y = \frac{1}{2}[\sin(x+y) + \sin(x-y)]$

$\boxed{\text{As, } \sin(2w) = 2\sin(w)\cos(w)}$ $\boxed{\text{Using Product-to-sum property}}$

$\Rightarrow j3\sin(w) + j[2][\sin(2w)] + j[\sin(3w) - \sin(w)]$

$\Rightarrow j2\sin(w) + j2\sin(2w) + j\sin(3w)$

$\Rightarrow \cancel{j}\cancel{2}\left[\dfrac{e^{jw} - e^{-jw}}{\cancel{2}\cancel{j}}\right] + \cancel{j}\cancel{2}\left[\dfrac{e^{j2w} - e^{-j2w}}{\cancel{2}\cancel{j}}\right] + \cancel{j}\left[\dfrac{e^{j3w} - e^{-j3w}}{2\cancel{j}}\right]$

$\Rightarrow e^{jw} - e^{-jw} + e^{j2w} - e^{-j2w} + \frac{1}{2}(e^{j3w}) - \frac{1}{2}(e^{-j3w})$

Calculating Inverse DTFT

$h_3[n] = \delta[n+1] - \delta[n-1] + \delta[n+2] - \delta[n-2] + \frac{1}{2}\delta[n+3] - \frac{1}{2}\delta[n-3]$

Question 3

1) a) $x_1[n] = (0.3)^n \mu[n+1]$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}$$

Answer: $X_1(z) = \sum_{n=\infty}^{\infty} x_1(n)\, z^{-n}$

$$X_1(z) = \sum_{n=-1}^{\infty} (0.3)^n z^{-n} = \sum_{n=-1}^{\infty} (0.3z^{-1})^n \overset{①}{\Rightarrow} \frac{z}{0.3} \sum_{n=0}^{\infty} (0.3z^{-1})^n$$

Geometric Series Formula:

$$S_n = a_1 \left(\frac{1-r^n}{1-r}\right)$$ with a_1 = First term, r = common ratio, n = number of terms

$ar^0 + ar^1 + \dots + ar^{n-1}$

$$\sum_{k=0}^{\infty} ar^k \Rightarrow \frac{a}{1-r} \quad \text{for } |r|<1$$

① Let $k = n+1$

$$\sum_{k=(n+1)-1}^{\infty} (0.3z^{-1})^{k-1} \Rightarrow \sum_{k=0}^{\infty} (0.3)^k \underset{\text{constant}}{(0.3)^{-1}} (z^{-1})^k \underset{\text{constant}}{(z^{-1})^{-1}} \Rightarrow \frac{z}{0.3} \sum_{k=0}^{\infty} (0.3z^{-1})^k$$

Applying Geometric Series.

$$\frac{z}{0.3} \sum_{k=0}^{\infty} \underset{a}{(1)} \underset{r}{(0.3z^{-1})^k} = \frac{z}{0.3}\left[\frac{1}{1-0.3z^{-1}}\right] \xRightarrow{\text{rationaliging}} \frac{z}{0.3}\left[\frac{1}{1-0.3z^{-1}}\right]\left[\frac{z}{z}\right] \Rightarrow \frac{z}{0.3}\left[\frac{z}{z-0.3}\right]$$

$$\Rightarrow \frac{z^2}{0.3\cdot(z-0.3)}$$

Finding ROC

$$\boxed{|0.3z^{-1}| < 1 \;\Rightarrow\; |z| > |0.3|}$$

it can not be greater than 1 due to the geometric property being $|r|<1$

2) $x_2[n] = (0.7)^n \mu[n-1]$

$$X_2[z] = \sum_{n=1}^{\infty} [0.7]^n z^{-n} \Rightarrow$$

Take $k = n-1$

$$\sum_{k=0}^{\infty} [0.7]^{k+1} z^{-k-1} \Rightarrow \frac{0.7}{z} \sum_{k=0}^{\infty} \underset{a}{(1)} \underset{r}{[0.7z^{-1}]^k}$$

$$\Rightarrow \frac{0.7}{z}\left[\frac{1}{1-0.7z^{-1}}\right] \xRightarrow{\text{rationaliging}} \left[\frac{0.7}{z}\right]\left[\frac{1}{1-0.7z^{-1}}\right]\left[\frac{z}{z}\right] \Rightarrow \frac{0.7\cancel{z}}{(\cancel{z})(z-0.7)} \Rightarrow \boxed{\frac{0.7}{z-0.7}}$$

Finding ROC

$$|0.7z^{-1}| < 1 \;\Rightarrow\; |z| > |0.7|$$

iii) $x_3[n] = (0.4)^n \mu[n-5]$

$$X_2(z) = \sum_{n=5}^{\infty} [0.4]^n z^{-n} \Rightarrow \text{Take } k=n-5 \quad \sum_{k=0}^{\infty} [0.4]^{k+5} z^{-(k+5)} = \sum_{k=0}^{\infty} [0.4]^5 [0.4]^k [z]^{-5} [z]^{-k}$$

$$\Rightarrow \left(\frac{0.4}{z}\right)^5 \sum_{k=0}^{\infty} \underbrace{(1)}_{a} \underbrace{[0.4z^{-1}]^k}_{r} \Rightarrow \left(\frac{0.4}{z}\right)^5 \left[\frac{1}{1-0.4z^{-1}}\right] \xRightarrow{\text{rationalizing}} \left(\frac{(0.4)^5 \cancel{z}}{\cancel{z}(z-0.4)}\right)$$

ROC

$$\boxed{|0.4z^{-1}| < 1 \quad z > 0.4}$$

v) $x_4[n] = (-0.4)^n \mu[-n-2]$

$$\Rightarrow X_4(z) = \sum_{n=-\infty}^{-2} (-0.4)^n z^{-n}$$

Taking $k = -n \Rightarrow \sum_{k=2}^{\infty} (0.4)^{-k} z^k$

Taking $L = k-2 \Rightarrow \sum_{L=0}^{\infty} (0.4)^{-L-2} z^{L+2} \Rightarrow \sum_{L=0}^{\infty} (0.4)^{-2}(0.4)^{-L}(z)^L(z)^2 \Rightarrow \left(\frac{z}{0.4}\right)^2 \sum_{L=0}^{\infty} (0.4^{-1}z)$

$$\Rightarrow \left(\frac{z}{0.4}\right)^2 \sum_{L=0}^{\infty} \underbrace{(1)}_{a} (\underbrace{0.4^{-1}z}_{r})^L \Rightarrow \left(\frac{z}{0.4}\right)^2 \left(\frac{1}{1-0.4^{-1}z}\right) \Rightarrow \frac{z^2}{0.4-z}$$

ROC

$$\boxed{|0.4^{-1}z| < 1, \quad |z| < |0.4|}$$

b) i) $y_1[n] = x_1[n] + x_2[n]$

Answer: Using the property of linearity $\Rightarrow y_1(z) = X_1(z) + X_2(z)$

$$\Rightarrow \boxed{\frac{z^2}{0.3(z-0.3)}} + \boxed{\frac{0.7}{(z-0.7)}}$$

ROC: $|z| > |0.3|$ ROC: $|z| > |0.7|$

$$\Rightarrow \boxed{\text{ROC of } y_1[n]: |z| > |0.7|}$$

ii) $\underbrace{\frac{0.7}{z-0.7}}_{ROC: |z|>|0.7|} + \underbrace{\frac{(0.4)^5}{(z)^4(z-0.4)}}_{ROC: |z|>|0.4|}$ → ROC of this entire thing: $|z|>|0.7|$

iii) $\underbrace{\frac{(0.4)^5}{(z)^4(z-0.4)}}_{ROC: |z|>|0.4|} + \underbrace{\frac{z^2}{(0.4)(0.4-z)}}_{ROC: |z|<|0.4|}$ There is no Region of Convergence for this

Question 4 $x[n] = 2.1(0.4)^n \mu[n] + 0.3(-0.3)^n \mu[n]$

$X(z)$ = Using linearity = $\sum_{n=0}^{\infty} (2.1)(0.4)^n z^{-1} + \sum_{n=0}^{\infty} (0.3)(-0.3)^n z^{-1}$

$\Rightarrow \frac{2.1}{1-0.4z^{-1}} + \frac{0.3}{1+0.3z^{-1}}$

$\boxed{Y(z) = X(z)H(z)}$

$\Rightarrow Y(z) = \left[\frac{2.1}{1-0.4z^{-1}} + \frac{0.3}{1+0.3z^{-1}}\right]\left[\frac{1-3.3z^{-1}+0.36z^{-2}}{1+0.3z^{-1}-0.18z^{-2}}\right]$

$\Rightarrow \left[\frac{2.1(1+0.3z^{-1})+0.3(1-0.4z^{-1})}{(1-0.4z^{-1})(1+0.3z^{-1})}\right]\left[\frac{1-3.3z^{-1}+0.36z^{-2}}{1+0.3z^{-1}-0.18z^{-2}}\right]$

$\Rightarrow \left[\frac{2.4+0.51z^{-1}}{(1-0.4z^{-1})(1+0.3z^{-1})}\right]\left[\frac{1-3.3z^{-1}+0.36z^{-2}}{(z^{-1}-\frac{10}{3})(\frac{5}{3}+z^{-1})}\right]$

$\Rightarrow \frac{2.4-7.92z^{-1}+0.864z^{-2}+0.51z^{-1}+0.1836z^{-3}-1.683z^{-2}}{(1-0.4z^{-1})(1+0.3z^{-1})(z^{-1}-10/3)(5/3+z^{-1})}$

$\Rightarrow \frac{2.4-7.41z^{-1}-0.819z^{-2}+0.1836z^{-3}}{(1-0.4z^{-1})(1+0.3z^{-1})(z^{-1}-10/3)(5/3+z^{-1})}$

x[Using Partial Fractions Calculator on the internet:

$$\Rightarrow \frac{A}{(1-0.4z^{-1})} + \frac{B}{(1+0.3z^{-1})} + \frac{C}{(z^{-1}-10/3)} + \frac{D}{(5/3+z^{-1})}$$

$$\Rightarrow \frac{-3.024}{(1-0.4z^{-1})} + \frac{0.432}{(1+0.3z^{-1})} + \frac{7.38}{(-10/3+z^{-1})} - \frac{2.79}{(5/3+z^{-1})}$$

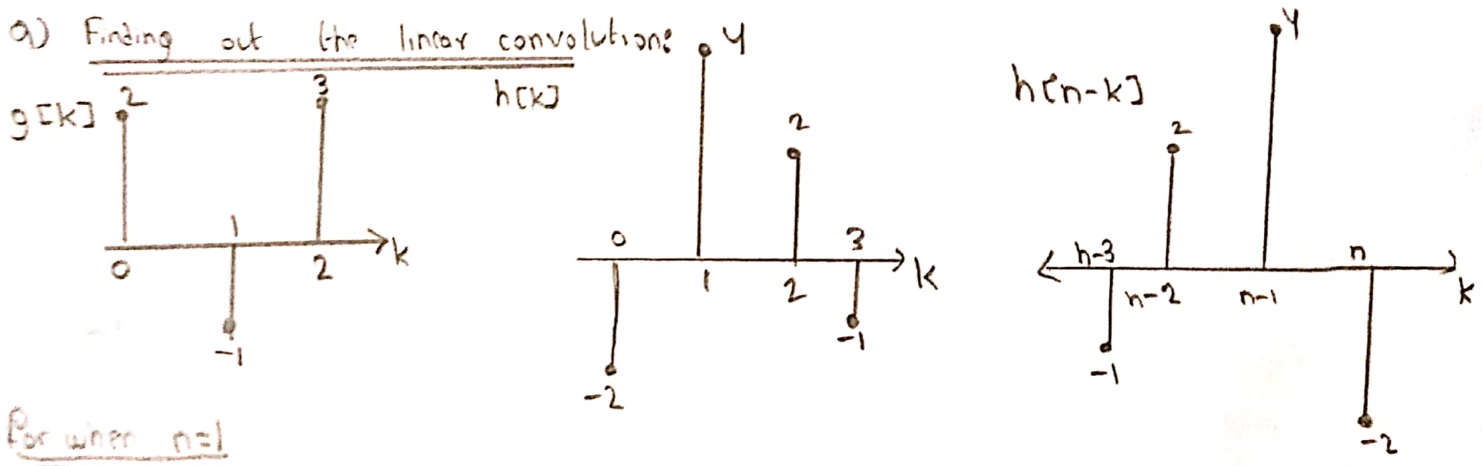
$$\Rightarrow \frac{-3.024}{(1-0.4z^{-1})} + \frac{0.432}{(1+0.3z^{-1})} + \frac{7.38}{(1-0.3z^{-1})} - \frac{1.674}{1+0.6z^{-1}}$$

$$\Rightarrow y[n] = -3.024(0.4)^n\mu[n] + 0.432(-0.3)^n\mu[n] - 2.214(0.3)^n\mu[n] - 1.67(-0.6)^n\mu[n]$$

Question 6

$$g[n] = \{2, -1, 3\}, \quad 0 \leq n \leq 2$$
$$h[n] = \{-2, 4, 2, -1\}, \quad 0 \leq n \leq 3$$

a) Finding out the linear convolution:



For when n=1

$y_L[1] = (-2)(-1) + (4)(2) = 2+8 = 10$

$y_L[2] = (-2)(3) + (4)(-1) + (2)(2) = -6$

$y_L[3] = (4)(3) + (2)(-1) + (-1)(2) = 8$

$y_L[4] = (2)(3) + (-1)(-1) = 7$

$y_L[5] = (-1)(3) = -3$

when $n > 5$, $y_L[n] = 0$

Thus,

$$\boxed{y_L[n] = \{\underset{\uparrow}{-4}, 10, -6, 8, 7, -3\}; \quad 0 \leq n \leq 5}$$

Question 6:

b) $g[n] = \{2, -1, 3, \boxed{0}\}$ we need to zero pad cause the length of both arrays should be same for circular convolution

$h[n] = \{-2, 4, 2, -1\}$

$$\begin{bmatrix} 2 & 0 & 3 & -1 \\ -1 & 2 & 0 & 3 \\ 3 & -1 & 2 & 0 \\ 0 & 3 & -1 & 2 \end{bmatrix}\begin{bmatrix} -2 \\ 4 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} (2)(-2)+(0)(4)+(3)(2)+(-1)(-1) \\ (-1)(-2)+(2)(4)+(0)(2)+(3)(-1) \\ (3)(-2)+(-1)(4)+(2)(2)+(0)(-1) \\ (0)(-2)+(3)(4)+(-1)(2)+(2)(-1) \end{bmatrix} = \begin{bmatrix} -4+0+6+1 \\ 2+8+0-3 \\ -6-4+4\ 0 \\ 0\ 12-2\ -2 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \\ -6 \\ 8 \end{bmatrix}$$

$$y_c[n] = \{\underset{\uparrow}{3}, 7, -6, 8\}$$

c) $g[n] = \{2, -1, 3, 0\}$

$$\Rightarrow G(k) = \sum_{n=0}^{N-1} g[n] e^{-j2\pi k \frac{n}{N}} = \sum_{n=0}^{3} g[n] e^{-j\frac{\pi}{2}kn}$$

$$\Rightarrow 2e^0 - e^{-j\frac{\pi}{2}k} + 3e^{-j\pi k} + 0$$

$$\Rightarrow 2 - e^{-j\frac{\pi}{2}k} + 3e^{-j\pi k}$$

$$\Rightarrow G(0) = 2 - e^0 + 3e^0 = 2-1+3 = 4$$
$$\Rightarrow G(1) = 2 - e^{-j\pi/2} + 3e^{-j\pi} = 2+j-3 = j-1$$
$$\Rightarrow G(2) = 2 - e^{-j\pi} + 3e^{-j2\pi} \Rightarrow 2+1+3 \Rightarrow 6$$
$$\Rightarrow G(3) = 2 - e^{-j\frac{3\pi}{2}} + 3e^{-j3\pi} \Rightarrow 2-j-3 \Rightarrow -j-1$$

$$G(k) = \{4, j-1, 6, -j-1\}$$

$$h[n] = \{-2, 4, 2, -1\}$$

$$H(k) = \sum_{n=0}^{N-1} h[n] e^{-j2\pi k \frac{n}{N}} \Rightarrow \sum_{n=0}^{3} h[n] e^{-j\frac{\pi}{2}kn}, \quad k = 0, 1, 2, 3$$

$$= -2e^0 + 4e^{-j\frac{\pi}{2}k} + 2e^{-j\pi k} - e^{-j\frac{3\pi}{2}k}$$
$$= -2 + 4e^{-j\frac{\pi}{2}k} + 2e^{-j\pi k} - e^{-j\frac{3\pi}{2}k}$$

$$H(0) = -2 + 4e^0 + 2e^0 - e^0 = -2+4+2-1 = 3$$
$$H(1) = -2 + 4e^{-j\pi/2} + 2e^{-j\pi} - e^{-j\frac{3\pi}{2}} = -2-4j-2-j = -4-5j$$
$$H(2) = -2 + 4e^{-j\pi} + 2e^{-j2\pi} - e^{-j3\pi} = -2-4+2+1 = -3$$
$$H(3) = -2 + 4e^{-j\frac{3\pi}{2}} + 2e^{-j3\pi} - e^{j\frac{9\pi}{2}} = -2+4j-2+j = -4+5j$$

$$\Rightarrow H(k) = \{3, -4-5j, -3, -4+5j\}$$

$Y(k) = G(k)\,H(k)$

$Y(0) = (4)(3) = 12$

$Y(1) = (j-1)(-4-5j) = -4j + 5 + 4 + 5j = 9 + j$

$Y(2) = (6)(-3) = -18$

$Y(3) = (-j-1)(-4+5j) = +4j - 5 + 4 - 5j = 9 - j$

<u>IDFT</u>

$$y[n] = \frac{1}{N}\sum_{k=0}^{N-1} Y(k)\, e^{j2\pi kn/N}$$

$$= \frac{1}{4}\sum_{k=0}^{3} Y(k)\, e^{j\frac{\pi}{2}kn}$$

$$= \frac{1}{4}\left[12 + (9+j)e^{j\frac{\pi}{2}n} - 18e^{j\pi n} + (9-j)e^{j\frac{3\pi}{2}n}\right]$$

$y[0] = \frac{1}{4}[12 + 9 + j - 18 + 9 - j] = 3$

$y[1] = \frac{1}{4}[12 + (9+j)j + 18 + (9-j)(-j)] = 7$

$y[2] = \frac{1}{4}[12 + (9+j)(-1) - 18 + (9-j)(-1)] = -6$

$y[3] = \frac{1}{4}[12 + (9+j)(-j) - 18(-1) + (9-j)(j)] = 8$

$y_c[n] = \{\underset{\uparrow}{3}, 7, -6, 8\}$

d) $g[n] = \{\underset{\uparrow}{2}, -1, 3, 0, 0, 0\}$

$h[n] = \{-2, 4, 2, -1, 0, 0\}$

$$\begin{bmatrix} 2 & 0 & 0 & 0 & 3 & -1 \\ -1 & 2 & 0 & 0 & 0 & 3 \\ 3 & -1 & 2 & 0 & 0 & 0 \\ 0 & 3 & -1 & 2 & 0 & 0 \\ 0 & 0 & 3 & -1 & 2 & 0 \\ 0 & 0 & 0 & 3 & -1 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 4 \\ 2 \\ -1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -4 \\ 10 \\ -6 \\ 8 \\ 7 \\ -3 \end{bmatrix}$$

$y_c[n]$

$\{-4, 10, -6, 8, 7, -3\}$

SAME AS IN (a)

Q7 $x[n] = \sin\left(\frac{n\pi}{2}\right)$

$N = 4,\ L = 4$

$N = L$

$\Rightarrow W_N = e^{-j\frac{2\pi}{N}} = e^{-j\frac{2\pi}{4}} = e^{-j\frac{\pi}{2}}$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix} = \begin{bmatrix} 0+1+0-1 \\ 0-j+0-j \\ 0-1+0+1 \\ 0+j+0+j \end{bmatrix}$$

$$\hookrightarrow \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 \\ -2j \\ 0 \\ 2j \end{bmatrix} \qquad X(k) = \{0, -2j, 0, 2j\}$$

Q8 $X(k) = \{1, 0, 1, 0\}$

$N = 4 \quad L = 4$

DFT in Matrix Method

$\Rightarrow X(k) = W_N\,(x[n])$

Inverse DFT in Matrix

$\Rightarrow x[n] = \frac{1}{N} W_N^{*} X(k)$

$W_N^{*} = W_N^{-kn} = \left(e^{-j\frac{2\pi}{N}}\right)^{-kn} = \left(e^{j\frac{\pi}{2}}\right)^{kn} = j^{kn}$

$$W_N^{*} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$

$$\Rightarrow x[n] = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}\begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix}$$

$$\begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} (1)(1)+(1)(0)+(1)(1)+(1)(0) \\ (1)(1)+(j)(0)+(-1)(1)+(-j)(0) \\ (1)(1)+(-1)(0)+(1)(1)+(1)(0) \\ (1)(1)+(-j)(0)+(-1)(1)+(j)(0) \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 2 \\ 0 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0 \\ 0.5 \\ 0 \end{bmatrix}$$

$$x[n] = \{0.5, 0, 0.5, 0\}$$

Q5) $$H(z) = \frac{1 - z^{-2}}{1-(1-\alpha)\cos(w_c)z^{-1} + az^{-2}}$$

$$H(e^{jw}) = \frac{1 - e^{-j2w}}{1-(1-\alpha)\cos(w_c)e^{-jw} + \alpha e^{-j2w}}$$

For $w = w_c$

$$\Rightarrow \frac{1 - e^{-j2w_c}}{1-(1-\alpha)\cos(w_c)\, e^{-jw_c} + \alpha e^{-j2w_c}}$$

$$\Rightarrow \frac{1 - e^{-j2w_c}}{1-(1+\alpha)\left(\frac{e^{jw_c}+e^{-jw_c}}{2}\right)e^{-jw_c} + \alpha e^{-j2w_c}}$$

$$\Rightarrow \frac{1-e^{-j2w_c}}{1-(1+\alpha)\left(\frac{1+e^{-j2w_c}}{2}\right)+\alpha e^{-j2w_c}}$$

$$\Rightarrow \frac{1-e^{-j2w_c}}{1-\left(\frac{1+e^{-j2w_c}}{2}\right)-\alpha\left(\frac{1+e^{-j2w_c}}{2}\right)+\alpha e^{-j2w_c}}$$

$$\Rightarrow \frac{1-e^{-j2w_c}}{\frac{1}{2}-\frac{e^{-j2w_c}}{2}-\frac{\alpha}{2}-\frac{\alpha e^{-j2w_c}}{2}+\alpha e^{-j2w_c}}$$

$$\Rightarrow \frac{1-e^{-j2w_c}}{\frac{(1-\alpha)}{2}-\frac{(1+\alpha)}{2}e^{-j2w_c}+\alpha e^{-j2w_c}}$$

$$\Rightarrow \frac{1-e^{-j2w_c}}{\frac{(1-\alpha)}{2}+e^{-j2w_c}\left(-\frac{1}{2}-\frac{\alpha}{2}+\alpha\right)}$$

$$\Rightarrow \frac{\cancel{1-e^{-j2w_c}}}{\frac{(1-\alpha)}{2}\cancel{(1-e^{-j2w_c})}} = \frac{2}{(1-\alpha)}$$