DSP Lab 7

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Task 1 Warm Up Question

Solve the following Convolution Problem. Let $X_1(z) = 2 + 3z^{-1} + 4z^{-2}$ **and** $X_2(z) = 3 + 4z^{-1} + 5z^{-2} + 6z^{-3}$**. Determine** $X_3(z) = X_1(z)X_2(z)$

Answer:

$\Rightarrow (z + 3z^{-1} + 4z^{-1})(3 + 4z^{-1} + 5z^{-2} + 6z^{-3})$

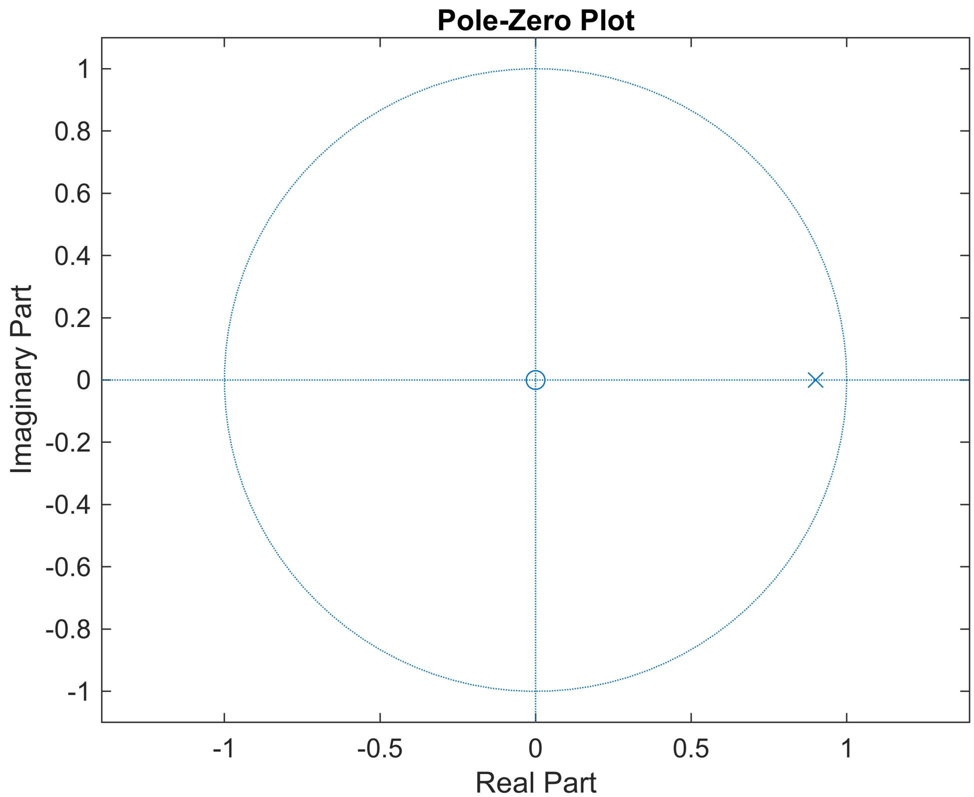
$\Rightarrow (6 + 8z^{-1} + 10z^{-2} + 12z^{-3}) + (9z^{-1} + 12z^{-2} + 15z^{-3} + 18z^{-4}) + (12z^{-2} + 16z^{-3} + 20z^{-4} + 24z^{-5})$

$\Rightarrow 6 + 17z^{-1} + 34z^{-2} + 43z^{-3} + 38z^{-4} + 24z^{-5}$

Task 2 Given a causal system determine H(z) and sketch its pole-zero plot

$H(z) = \dfrac{1}{1 - 0.9z^{-1}}; |z| > 0.9$

```
b = [1,0];
a = [1,-0.9];
zplane(b,a);
```



Questions

a) What does a and b and a represent

b represents the numerator which represents the zeros. **a** represents the denominator and represents the poles.

b) Is system casual, Explain the reason

System can be causal because all the poles are inside the **unit circle**

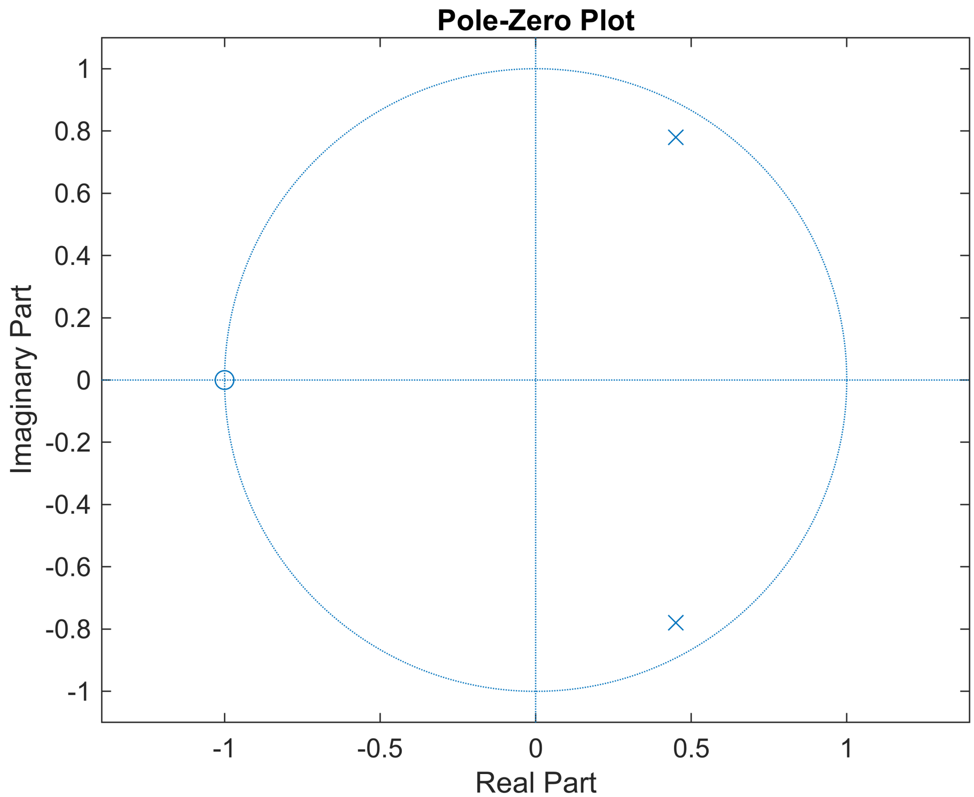
Task 2: Part(b)

$$\frac{Y(z)}{X(z)} = \frac{z+1}{z^2 - 0.9z + 0.81}$$

$$\Rightarrow \left(\frac{z+1}{z^2 - 0.9z + 0.81}\right) * \left(\frac{z^{-2}}{z^{-2}}\right)$$

$$\Rightarrow \frac{z^{-1} + z^{-2}}{1 - 0.9z^{-1} + 0.81z^{-2}}$$

```
Zeroz = [0, 1, 1];
Polez = [1,-0.9,0.81];
zplane(Zeroz,Polez);
```



Task 3 Using residue command solve the rational Z transform

Warm-up Question

To check our residue calculations, let us consider the rational function

$$X(z) = \frac{z}{3z^2 - 4z + 1}$$

```
b = [0,1];
a = [3,-4,1];
[R,p,C] = residuez (b,a);
str_Task3_a_Zeros = ['The Zeros of Task 3(a) are: ' num2str(R')];
disp(str_Task3_a_Zeros);
```

```
The Zeros of Task 3(a) are: 0.5        -0.5
```

```
str_Task3_a_Poles = ['The Poles of Task 3(a) are: ' num2str(p')];
disp(str_Task3_a_Poles);
```

```
The Poles of Task 3(a) are: 1     0.33333
```

Similarly, to convert back to the rational function form

```
[b,a] = residuez(R,p,C)
```

```
b = 1×2
   -0.0000    0.3333
```

```
a = 1×3
    1.0000   -1.3333    0.3333
```

This can be written as:

$$X(z) = \frac{0 + \frac{z^{-1}}{3}}{1 - \frac{4}{3}z^{-1} + \frac{z^{-2}}{3}} = \frac{z}{3z^2 - 4z + 1}$$

Question 1: Analyzing results obtained from residue Matlab Command

$$X(z) = \frac{z}{5z^3 + 0z^{-2} - 4z + 1}$$

```
Zeroz = [0,1];
Polez = [5,0,-4,1];
```

What is the value of R and P

```
[R,p,C] = residuez (Zeroz,Polez)
```

```
R = 3×1
   -0.0909
    0.1877
   -0.0968
p = 3×1
   -1.0000
    0.7236
    0.2764
C =

     []
```

Referring from above:

$$\frac{0.0909}{1 - z^{-1}} + \frac{0.1877}{1 - 0.7236z^{-1}} - \frac{0.0968}{1 - 0.2764z^{-1}}$$

What is C. What does it represent?

Direct terms of partial fraction are represented by C.

The C is [ ] : C represents constant which doesn't exist here.

Write down the equation using value of R (Zeros) and P (Poles)?

```
[Zeros,Polez] = residuez(R,p,C)
```

```
Zeros = 1×3
   -0.0000    0.2000    0.0000
Polez = 1×4
    1.0000    0.0000   -0.8000    0.2000
```

Converting back to the rational function form

$$X(z) = \frac{-0 + 0.2z^{-1} + 0z^{-2}}{1 + 0z^{-1} - 0.8z^{-2} + 0.2z^{-3}} = \frac{0.2z^{-1}}{1 - 0.8z^{-2} + 0.2z^{-3}}$$

Question 2:

Given a stable system with z-transform; $H(z) = \dfrac{1}{(1 - 0.25z^{-1})(1 - 0.5z^{-1})^2}$

find the inverse transform h[n]:

$$H(z) = \frac{1}{(1 - 0.9z^{-1})^2(1 + 0.9z^{-1})}$$

$$\frac{A}{1 + 0.9z^{-1}} + \frac{B}{1 - 0.9z^{-1}} + \frac{C}{(1 - 0.9z^{-1})^2}$$

$$= A(1 - 0.9z^{-1})^2 + B(1 - 0.9z^{-1})(1 + 0.9z^{-1}) + C(1 + 0.9z^{-1})$$

For A: $z^{-1} = \dfrac{-1}{0.9}$

$$4A = 1 \Rightarrow A = 0.25$$

For C: $z^{-1} = \dfrac{1}{0.9}$

$$1 = 2C \Rightarrow C = 0.5$$

Using Differentiation

$$0 = -0.9A(1 - 0.9z^{-1}) - 0.81B + 0.9C$$

Set $z^{-1} = \dfrac{1}{0.9}$

$$0 = -0.81B + 0.9(0.5)$$

$$B = \frac{5}{9}$$

$$\frac{0.25}{1 + 0.9z^{-1}} + \frac{5}{9(1 - 0.9z^{-1})} + \frac{0.5}{(1 - 0.9z^{-1})^2}$$

$$x[n] = -(-0.9)^n u(-n - 1)(0.25) + \frac{5}{9}0.9^n u[n]$$

```
syms z n
exp = (1/(1-0.25*(z^-1))*(1-0.5*(z^-1))^2);
disp(exp);
```

$$-\frac{\left(\frac{1}{2\,z}-1\right)^2}{\frac{1}{4\,z}-1}$$

D.1 Resolving Partial Fractions

```
% Use the poly command to obtain the polynomial coefficients from its roots
poly1 = poly([0.25 0.5 0.5])
```

```
poly1 = 1×4
    1.0000   -1.2500    0.5000   -0.0625
```

Equation:

We can infer that we got, $f(x) = x^3 - 1.25x^2 + 0.5x - 0.0625$

Thus, we got

$$H(z) = \frac{1}{-0.0625 + 0.5z^{-1} - 1.25z^{-2} + z^{-3}}$$

```
% For extra info, we will use the roots commond to obtain the roots of the
polnomial.
roots1 = roots([1 -1.25 0.5 -0.0625]); %This will give us back 0.5 0.5 0.25 which
is to be expected
```

Task-4 (D3)

Note: sir asked to use the following fraction for this part instead of the one written in the manual

$$H(z) = \frac{1 - 4z^{-1} + 8z^{-2}}{1 - 2.5z^{-1} + z^{-2}}:$$

To convert this irrational fraction into a rational fraction, long division operation was performed. And then using partial fraction expansion, the ROC was determined:

$$H(z) = \frac{1 - 4z^{-1} + 8z^{-2}}{1 - \frac{3}{2}z^{-1} + z^{-2}}$$

$$\begin{array}{r} 8 \\ z^{-2} - 2.5z^{-1} + 1 \overline{\big) \, 8z^{-2} - 4z^{-1} + 1} \\ -8z^{-2} + 20z^{-1} - 8 \\ \hline 16z^{-1} - 7 \end{array}$$

$$H(z) = 8 + \frac{16z^{-1} - 7}{z^{-2} - 2.5z^{-1} + 1}$$

$$H(z) = 8 + \frac{16z^{-1} - 7}{(z^{-1} - 2)(z^{-1} - \frac{1}{2})}$$

$$\cancel{8} + \frac{16z^{-1} - 7}{(z^{-1} - 2)(z^{-1} - \frac{1}{2})} = \cancel{8} + \frac{A}{(z^{-1} - 2)} + \frac{B}{(z^{-1} - 1/2)}$$

$$16z^{-1} - 7 = A(z^{-1} - 1/2) + B(z^{-1} - 2)$$

when $z^{-1} = 2$.

$$32 - 7 = A\left(\frac{3}{2}\right) \Rightarrow A = \frac{50}{3} = 16.667$$

when $z^{-1} = 1/2$

$$8 - 7 = B\left(\frac{-3}{2}\right) \Rightarrow B = \frac{-2}{3} = -0.667$$

$$H(z) = 8 + \frac{50/3}{z^{-1} - 2} - \frac{2/3}{z^{-1} - 1/2}$$

D.2

Verifying this part using matlab.

$$H(z) = \frac{1 + 0.4z^{-1} - 2.2z^{-2} + 0.8z^{-3}}{1 - 1.3z^{-1} + 0.4^{-2}}$$

```
% Define the numerator and denominator coefficients of the "transfer
% function"
B = [1, 0.4, -2.2, 0.8];
A = [1, -1.3, 0.4];

% Compute the partial fraction decomposition
[R, p, C] = residuez(B,A)
```

```
R = 2×1
   -1.0000
    1.0000
p = 2×1
    0.8000
    0.5000
C = 1×2
     1     2
```

Now we have the desired result: the sum of a polynomial in z^{-1} plus a proper rational function in z^{-1}

$$H(z) = \underbrace{1 + 2z^{-1}}_{\text{Polynomial in } z^{-1}} + \underbrace{\frac{-0.3z^{-1}}{1 - 1.3z^{-1} + 0.4z^{-2}}}_{\text{Proper rational function}}.$$

Splitting the second term by partial fractions gives

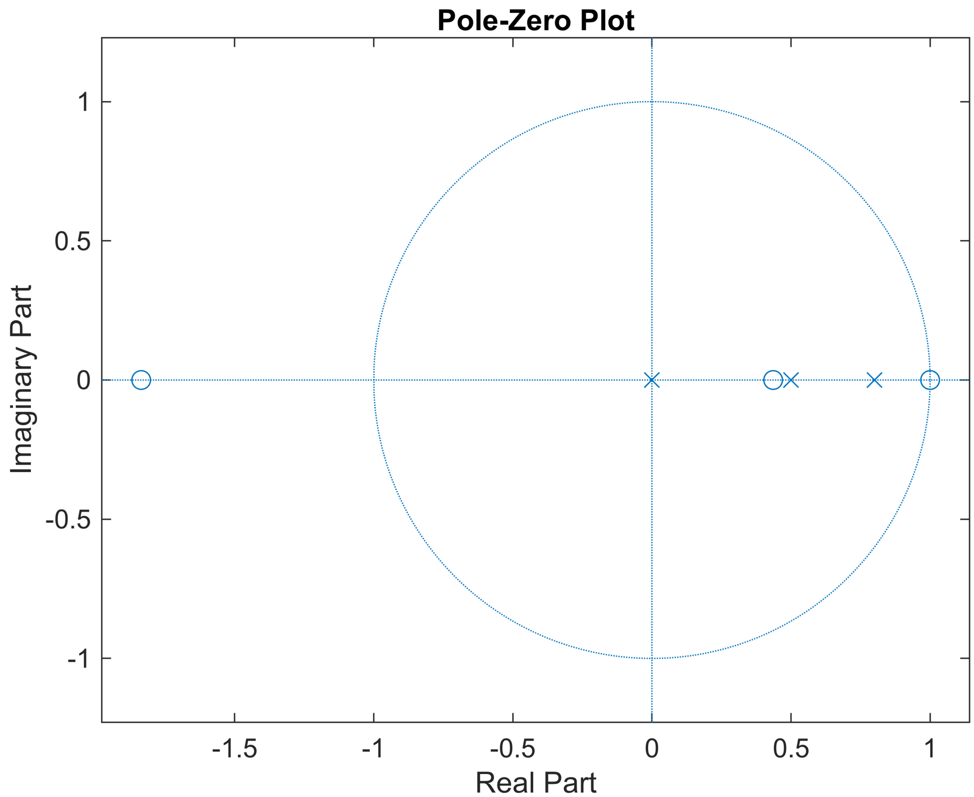
$$\begin{array}{llllllllll} H(z) & = & 1 & + & 2z^{-1} & + & \dfrac{0.5}{1-0.5z^{-1}} & - & \dfrac{0.8}{1-0.8z^{-1}} & \\ \text{ROC:} & & \text{all} & \cap & |z| > 0 & \cap & |z| > 0.5 & \cap & |z| > 0.8 & \Rightarrow |z| > 0.8. \end{array}$$

Analysis:

As can be observed, in **C,** we are getting direct polynomial terms aka 1 and 2, which don't contribute to poles in the partial fraction decomposition. So they are similar.

P have 0.5 and 0.8, and as can be referred from "D2", we are getting teh same poles that contribute to ROC!

```
%Plotting the poles and zeros
zplane(B,A);
```



Solving for D3, whch is (same question in which we did the long division)

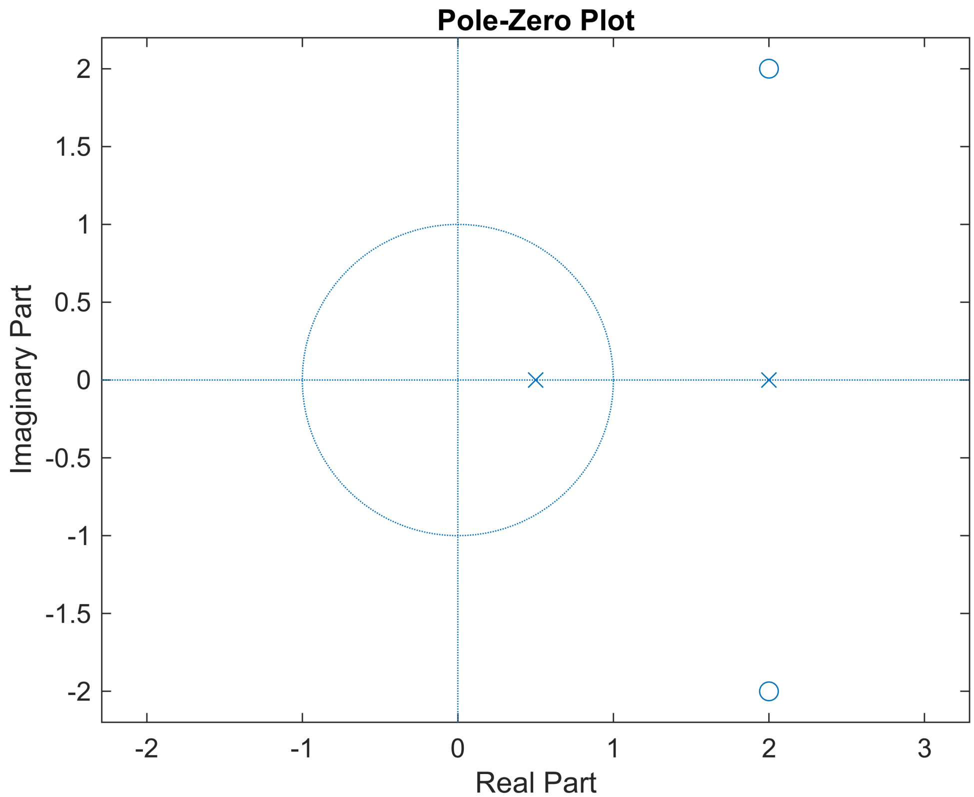
$$H(z) = \frac{1 - 4z^{-1} - 8z^{-2}}{1 - 2.5z^{-1} + z^{-2}}$$

```
% Define the numerator and denominator coefficients of the transfer function
B = [1, -4, 8];
A = [1, -2.5, 1];

% Compute the partial fraction decomposition
[R, p, C] = residuez(B,A)
```

```
R = 2×1
    1.3333
   -8.3333
p = 2×1
    2.0000
    0.5000
C = 8
```

```
zplane(B,A);
```


Pole-Zero Plot
Imaginary Part
Real Part
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2
-1
0
1
2
3