Multirate Signal Processing

EE 453 / CE 352

Saad Baig

Multirate Signal Processing

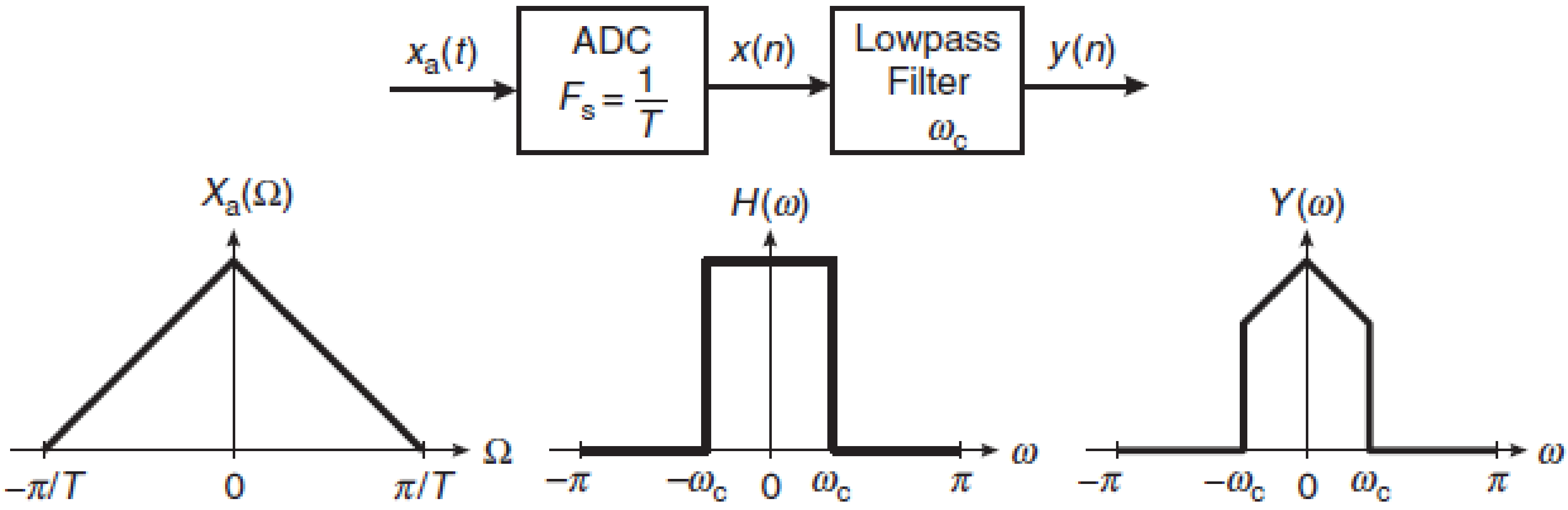
- Multirate signal processing is the study of processing signals at multiple rates.
- It has important applications in many fields:
  - In wireless communications, efficient processing of signals at multiple rates is critical.
  - In image processing, multirate signal processing can be used to perform efficient image compression, image filtering, and image resizing.
  - In audio processing, it can be used to implement high-quality digital audio effects, such as time-stretching and pitch-shifting.
  - In control systems, multirate signal processing can be used to design digital control systems with reduced sampling rates.

Multirate Signal Processing

- Downsampling and upsampling operations
- Polyphase decomposition
- Design of digital filters for multirate signal processing

Sampling Rate Conversion

- The process of converting a signal from a given rate to a different rate is called sampling rate conversion.



Sampling Rate Conversion

- Two methods:

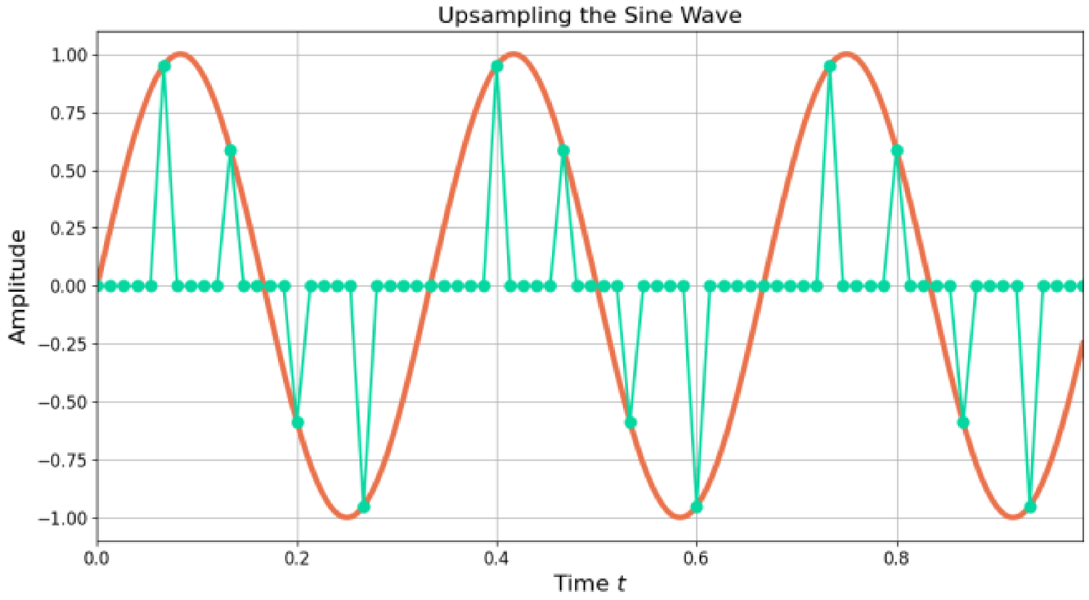
1. Convert to analog and then resample at a desired rate.
    - New rate is independent of previous rate.
    - D/A and A/D conversion can cause distortion and errors due to quantization.
2. Perform conversion in the digital domain.

Upsampling

- Add $L - 1$ zeros in between samples to increase the rate.

$$x[n] \longrightarrow \boxed{\uparrow L} \longrightarrow x_L[n] = x[n]_{\uparrow L}$$

F_T $\qquad\qquad\qquad\qquad$ LF_T



- Upsampling is a shift varying operation.

Upsampling

- Upsampling adds high frequency content to your signal, due to discontinuities.
- To remove discontinuities, we would need to convolve with a low pass filter, which is known as an interpolating filter.

$$x[n] \longrightarrow \boxed{\uparrow L} \xrightarrow{x_u[n]} \boxed{H(\omega)} \longrightarrow y[n]$$

$$\omega_c = \frac{\pi}{L}$$

$$x_u[n] = \begin{cases} x[n/L], & n = kL \\ 0 & \text{else} \end{cases} \qquad k = 0, \pm 1, \pm 2 \ldots$$

Upsampling

$$x_u[n] = \begin{cases} x[n/L], & n = kL \\ 0 & \text{else} \end{cases}$$

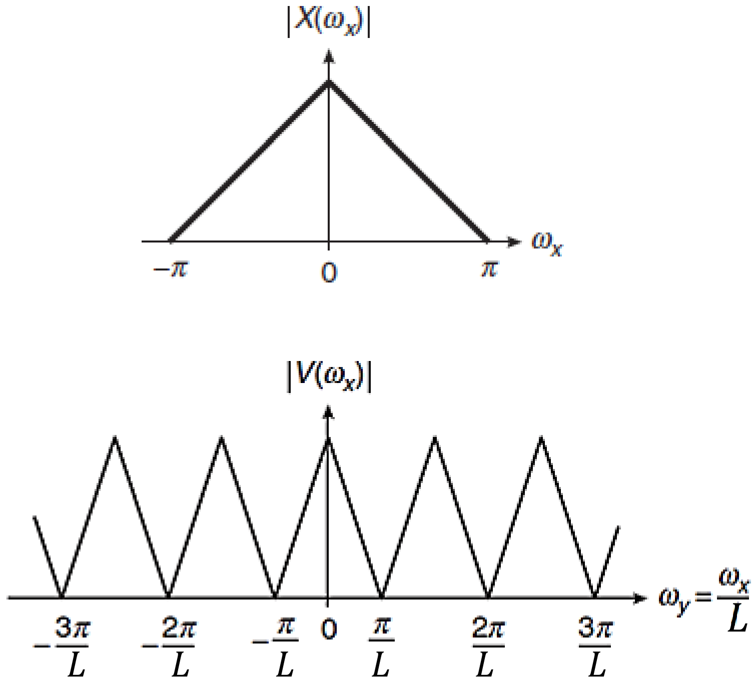
$$X_u(z) = \sum_{n=-\infty}^{\infty} x_u[n] z^{-n} = \sum_{n=-\infty}^{\infty} x[n/L] z^{-n}$$

$$X_u(z) = \sum_{k=-\infty}^{\infty} x[kL/L] z^{-kL} = \sum_{k=-\infty}^{\infty} x[k] (z^L)^{-k}$$

$$X_u(z) = X(z^L)$$

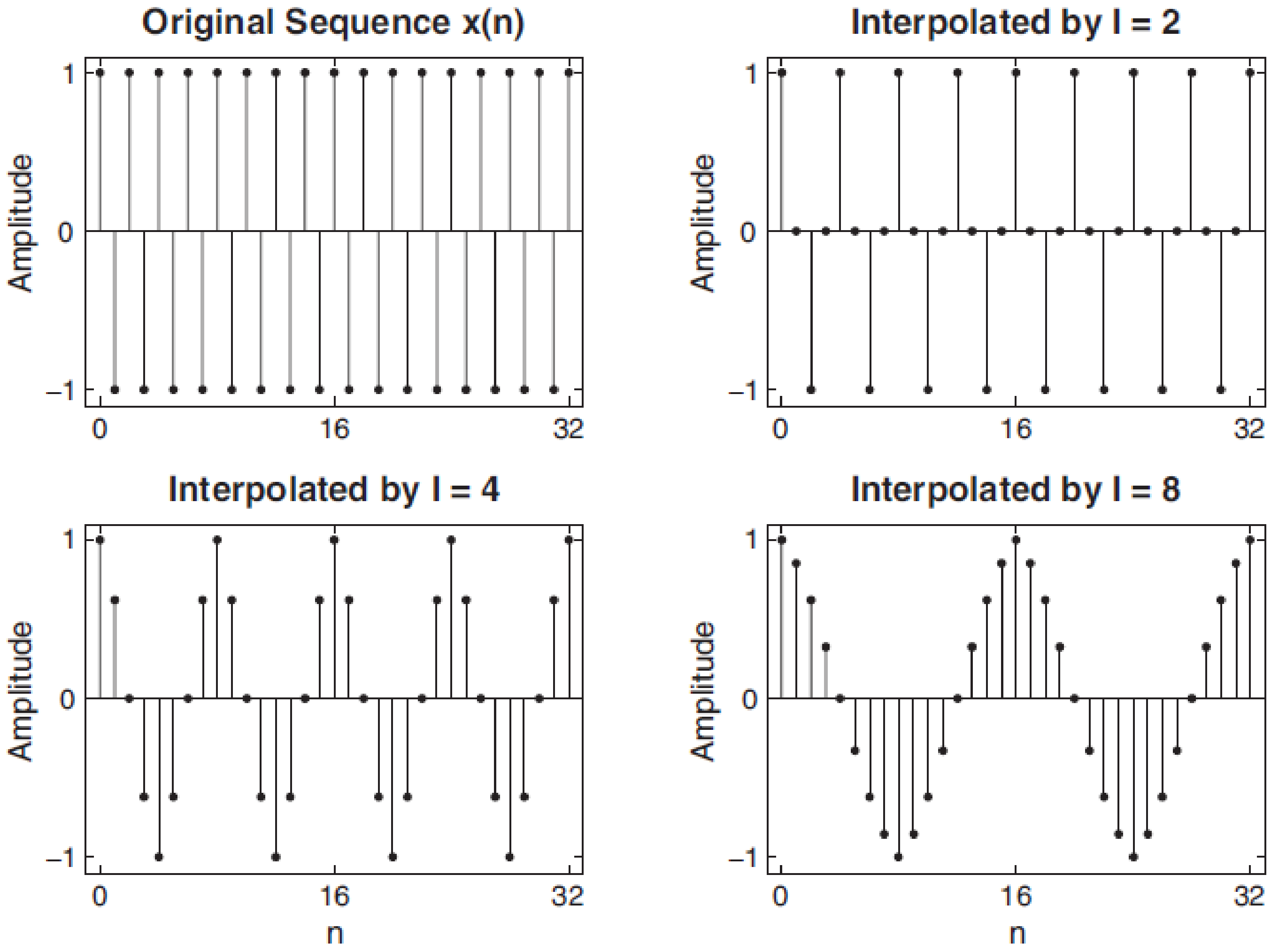
$$X_u\left(e^{j\omega}\right) = X\left(e^{j\omega L}\right)$$

At $\omega = \pi$, $X_u\left(e^{j\pi}\right) = X\left(e^{j\pi L}\right)$.



Upsampling

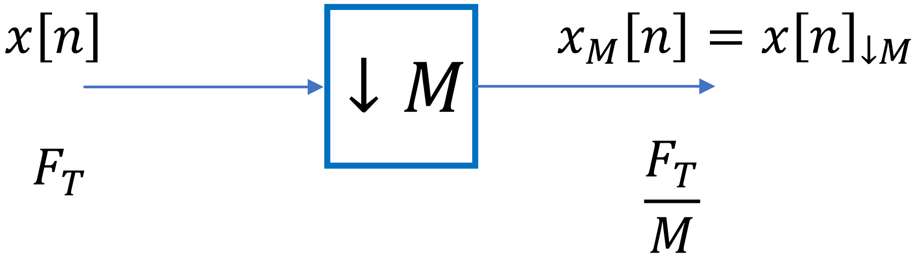
Let $x(n) = \cos(\pi n)$



Downsampling

- Keep only the M^{th} sample.
- Also varies with shifting.
- Decreases the rate, and also known as decimation.

$$x_d[n] = x[nD]$$



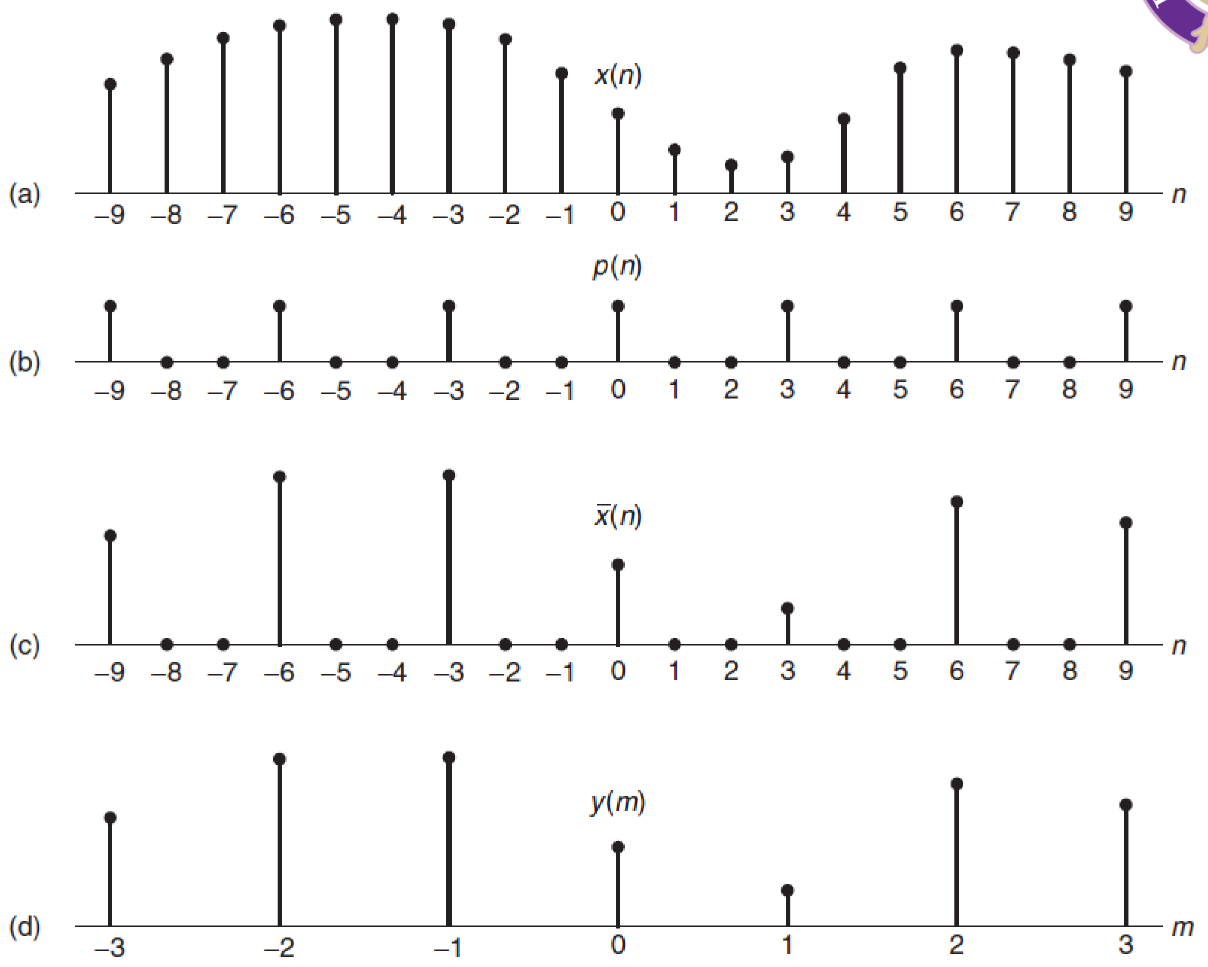


FIGURE 9.4 *Operation of downsampling: (a) original signal $x(n)$, (b) periodic impulse train $p(n)$ with period $D = 3$, (c) multiplication of $x(n)$ with $p(n)$, and (d) downsampled signal $y(m)$*

Downsampling

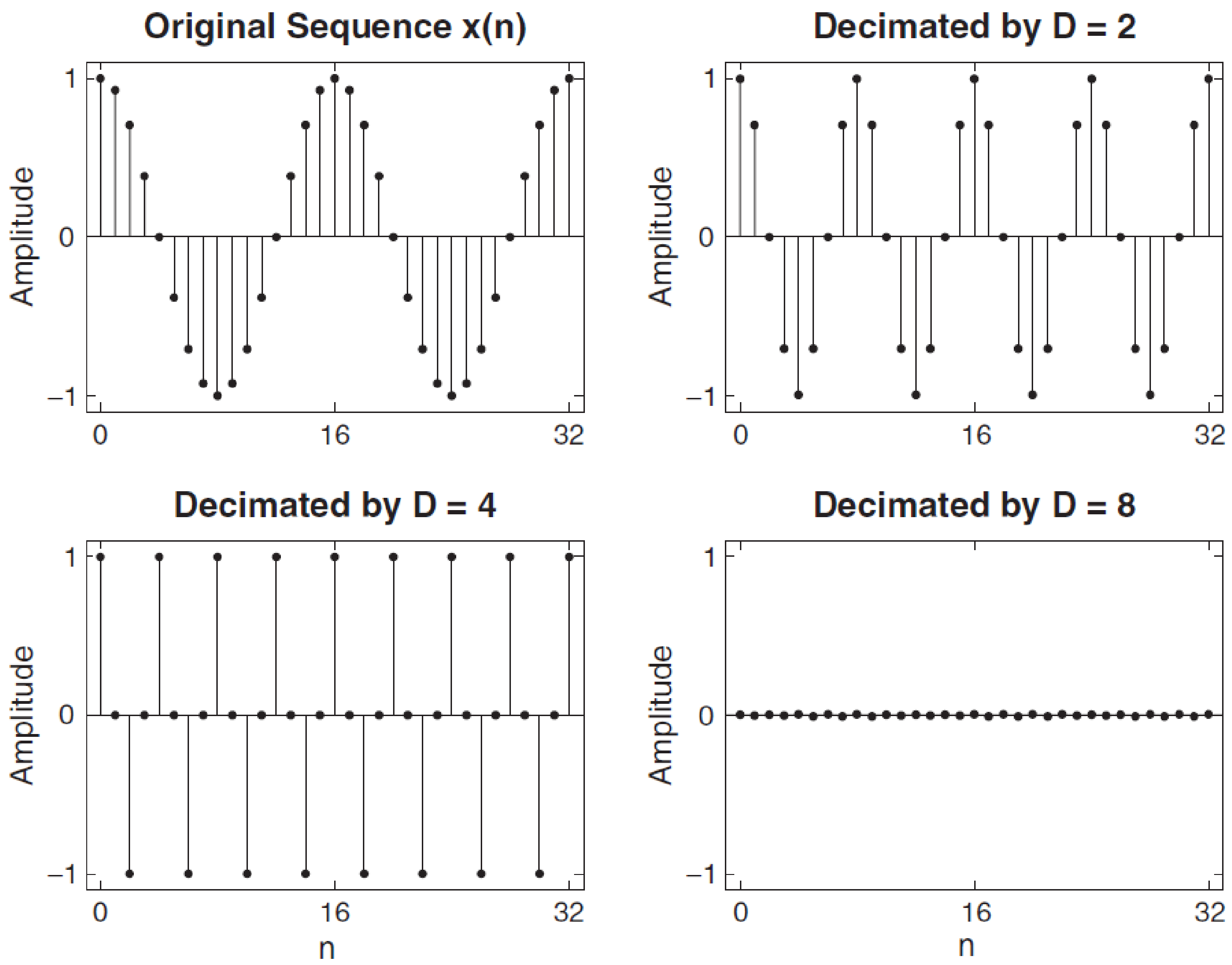
- Can cause aliasing to occur. To reduce aliasing, we can pass the signal through a LPF to reduce the bandwidth to $F_{T_{\max}} = F_T/2M$.

$$x[n] \longrightarrow \boxed{H(\omega)} \longrightarrow \boxed{\uparrow L} \longrightarrow y[n]$$

$$\omega_c = \frac{\pi}{M}$$

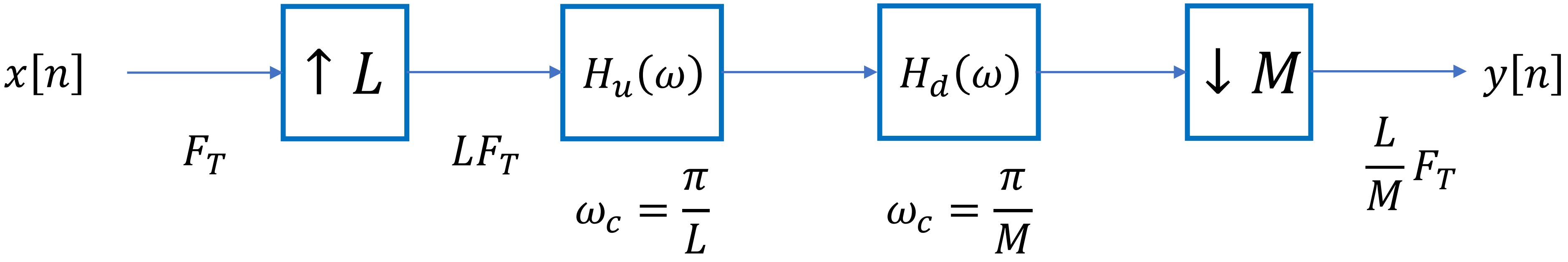
Downsampling

Let $x(n) = \cos(0.125\pi n)$

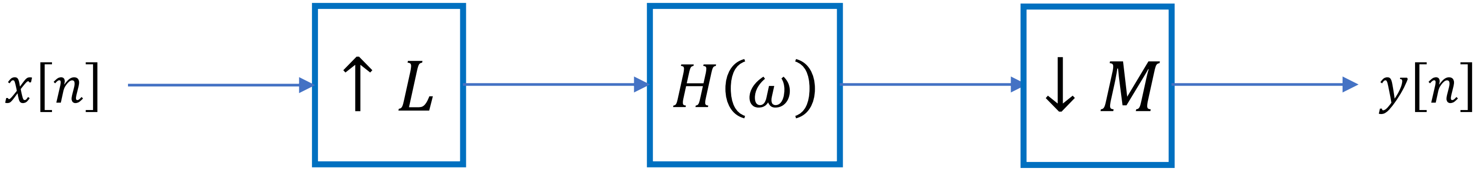


Rate Conversion by a Rational Factor

- Combine both upsampling and downsampling systems.



- Since both filters are performing at the same rate (LF_T), they can be combined together into a single filter.



Rate Conversion by a Rational Factor

$$x[n] \longrightarrow \boxed{\uparrow L} \longrightarrow \boxed{H(z)} \longrightarrow \boxed{\downarrow M} \longrightarrow y[n]$$

$$H(\omega) = \begin{cases} L, & 0 \leq |\omega_v| \leq \min(\pi/M\,, \pi/L) \\ 0 & \text{else} \end{cases}$$

$$\omega_v = \frac{2\pi F}{L F_T} = \frac{\omega}{L}$$

Rate Conversion by a Rational Factor

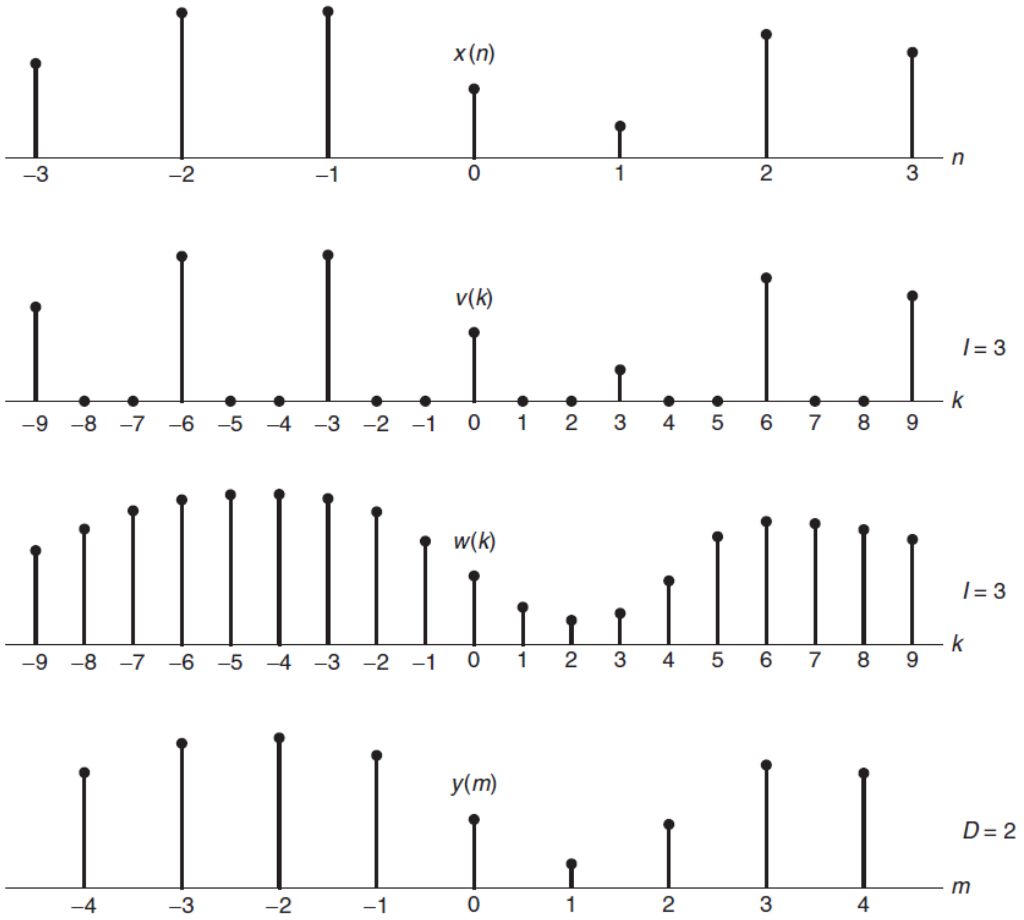


FIGURE 9.16 *Examples of signals $x(n)$, $v(k)$, $w(k)$, and $y(m)$ in the sampling rate converter of Figure 9.15 for $I = 3$ and $D = 2$*

Rate Conversion by a Rational Factor

Let $x(n) = \cos(0.125\pi n)$