Habib University

EE/CE 453/352: Digital Signal Processing - Spring 2024
Saad Baig

Time = 35 minutes | **Quiz 02** | Max Points: 20

Instructions:

i. **Smart watches, laptops, and similar electronics are strictly NOT allowed.**
ii. **Answer sheets should contain all steps, working, explanations, and assumptions.**
iii. Attempt the quiz with black/blue ink.
iv. Print your name and HU ID on all sheets.
v. This is a closed-book examination but you are allowed a single-sided A4 sized cheat sheet.
vi. You are not allowed to ask/share your method or answer with your peers. The work submitted by you is solely your own work. Any violation of this will be the violation of HU Honor code and proper action will be taken as per university policy if found to be involved in such an activity.

CLO Assessment:

This quiz will assess students for the following course learning outcomes.

Course Learning Outcome		Learning Domain Level
CLO 2	Analyze discrete-time signals and systems in transform domain using z-Transform, DTFT, and DFT.	Cog-4

Undertaking:

I hereby affirm that I have read the instructions. I am fully aware of the HU honor code and the repercussions of its violation, and hereby pledge that the work I am going to submit is clearly my own.

Signature: ____________________

Name: INSTRUCTOR SOLUTION **HU ID:** ____________________

Question 1 [8 pts]: With the help of Table 4.1 below, evaluate the inverse z-transform of the following z-transform:

$$Y(z) = \frac{z(z-1)}{(z+1)\left(z+\frac{1}{3}\right)}, \qquad \frac{1}{3} < |z| < 1$$

Solution:

$$\frac{Y(z)}{z} = \frac{(z-1)}{(z+1)\left(z+\frac{1}{3}\right)} = \frac{A}{z+1} + \frac{B}{z+\frac{1}{3}}$$

Using partial fractions method, we find $A = 3$ and $B = -2$:

$$\frac{Y(z)}{z} = \frac{3}{z+1} - \frac{2}{z+\frac{1}{3}}, \qquad Y(z) = \frac{3z}{z+1} - \frac{2z}{z+\frac{1}{3}} = \frac{3}{1+z^{-1}} - \frac{2}{1+\frac{1}{3}z^{-1}}$$

We see that $1/3 < |z|$ is a causal implementation and $|z| < 1$ is an anti-causal implementation. Using relevant pairs from the table:

$$y[n] = -3(-1)^n u[-n-1] - 2\left(-\frac{1}{3}\right)^n u[n]$$

Question 2 [8 pts]: Calculate the 4-point DFT of the following sequence and draw its magnitude plot:

$$x[n] = \{1, 1, -2, -2\}$$

Solution:

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -2 \\ -2 \end{bmatrix} = \begin{bmatrix} -2 \\ 3-3j \\ 0 \\ 3+3j \end{bmatrix}$$

$$|X(k)| = \{2, 4.2, 0, 4.2\}$$

Question 3 [4 pts]: If a sequence $x(n)$ is periodic with periodicity of N samples, then N-point DFT of the sequence is also periodic with periodicity of N samples. Mathematically:

$$\text{If } x(n+N) = x(n), \qquad \text{then } X(k+N) = X(k)$$

Prove the above periodicity property using the DFT function.

Solution:

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi k \frac{n}{N}}$$

$$X(k+N) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi(k+N)\frac{n}{N}} = \sum_{n=0}^{N-1} x(n) e^{-j2\pi k \frac{n}{N}} . e^{-j2\pi N \frac{n}{N}} = \sum_{n=0}^{N-1} x(n) e^{-j2\pi k \frac{n}{N}} . e^{-j2\pi n}$$

Here $e^{-j2\pi n} = 1$ for all integer values of n.

$$X(k+N) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi k \frac{n}{N}} . 1 = X(k)$$