Habib University

EE/CE 453/352: Digital Signal Processing - Spring 2024
Saad Baig

Time = 120 minutes	**Midterm Solution**	Max Points: 40

Instructions:

i. **Smart watches, laptops, and similar electronics are strictly NOT allowed.**
ii. **Answer sheets should contain all steps, working, explanations, and assumptions.**
iii. Attempt the quiz with black/blue ink.
iv. Print your name and HU ID on all sheets.
v. This is a closed-book examination but you are allowed a single-sided A4 sized cheat sheet.
vi. You are not allowed to ask/share your method or answer with your peers. The work submitted by you is solely your own work. Any violation of this will be the violation of HU Honor code and proper action will be taken as per university policy if found to be involved in such an activity.

CLO Assessment:

This quiz will assess students for the following course learning outcomes.

Course Learning Outcome		Learning Domain Level	Questions
CLO 1	Analyze discrete-time signals and systems in time domain.	Cog-4	1, 2, 3
CLO 2	Analyze discrete-time signals and systems in transform domain using z-Transform, DTFT, and DFT.	Cog-4	4, 5

Undertaking:

I hereby affirm that I have read the instructions. I am fully aware of the HU honor code and the repercussions of its violation, and hereby pledge that the work I am going to submit is clearly my own.

Signature: ______________________

Name: INSTRUCTOR SOLUTION **HU ID:** ______________

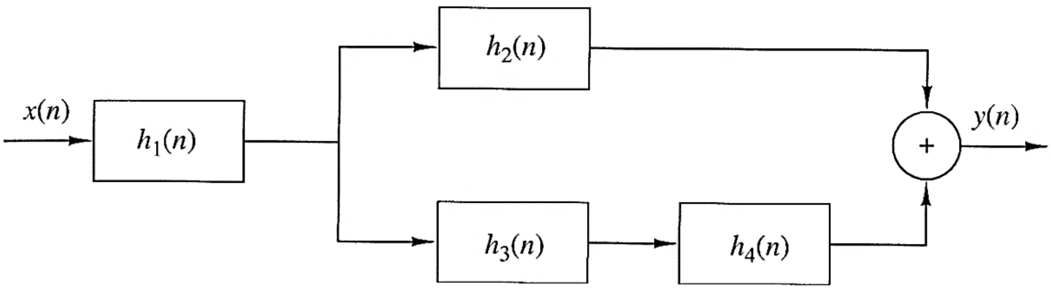
Question 1 [10 pts]: For the systems defined below, state with proof whether or not they are dynamic linear, causal, stable and/or time invariant.

a) $y[n] = 5x[3^n]$
b) $y[n] = x[n] + by[n-1]$

Solution:

Property	Part (a)	Part (b)
Dynamic	Yes, $y[1]$ depends on future value $x[3]$	Yes, $y[1]$ depends on past value $y[0]$
Linear	Yes	Yes
Causal	No, $y[1]$ depends on future value $x[3]$	Yes, $y[1] = x[1] + by[0]$
Stable	Yes, BIBO stable	Yes, BIBO stable
Time Invariant	Yes, $y[n-k] = 5x[3^{n-k}]$	Yes, $y[n-k] = x[n-k] + by[n-1-k]$

Question 2 [4 pts]: Consider the interconnection of LTI systems as shown in the figure below:



Express the overall impulse response $h(n)$ in terms of $h_1(n)$, $h_2(n)$, $h_3(n)$ and $h_4(n)$.

Solution:

$$h(n) = h_1(n) * \{h_2(n) + [h_3(n) * h_4(n)]\}$$

Question 3 [6 pts]: Two signals $s(n)$ and $v(n)$ are related through the following differential equation:

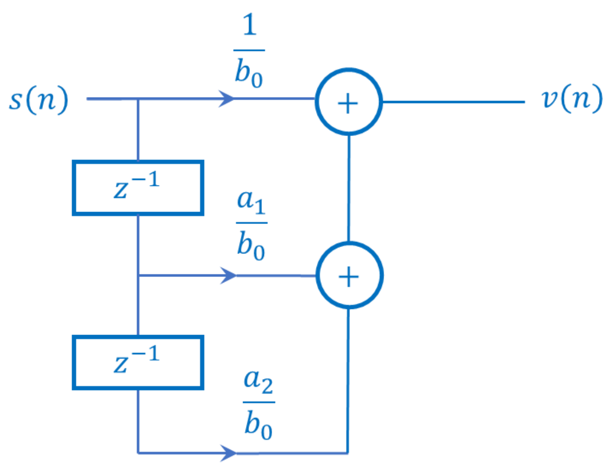
$$s(n) + a_1 s(n-1) + a_2 s(n-2) = b_0 v(n)$$

Design the block diagram realization of:

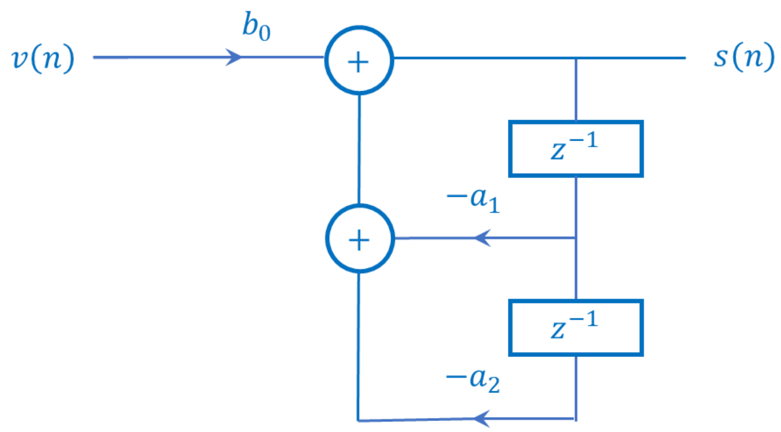
a) The system that generates $v(n)$ when excited by $s(n)$.
b) The system that generates $s(n)$ when excited by $v(n)$.

Solution:

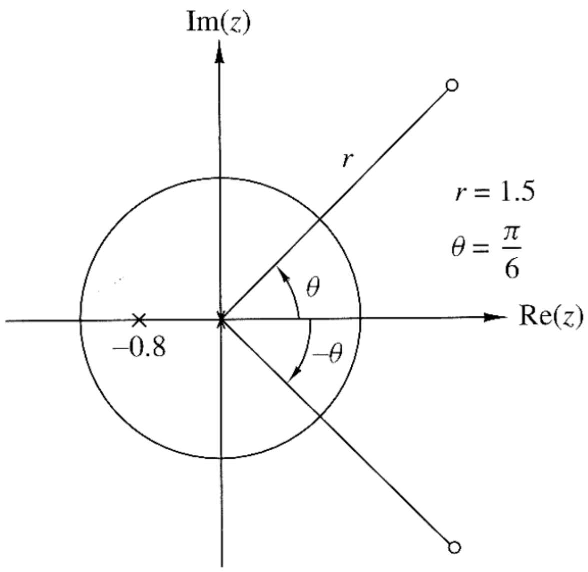
a) $v(n) = \frac{1}{b_0} s(n) + \frac{a_1}{b_0} s(n-1) + \frac{a_2}{b_0} s(n-2)$



b) $s(n) = b_0 v(n) - a_1 s(n-1) - a_2 s(n-2)$



Question 4 [12 pts]: Consider the causal system defined by the pole-zero pattern shown below (there is a pole at $z = 0$ also).



a) [4 pts] Determine the system function $H(z)$.
b) [4 pts] What is the region of convergence?
c) [4 pts] Is the system stable or unstable? State reasons.

Solution:

a) To determine the locations of the poles and zeroes:

$$p_1 = -0.8, \qquad p_2 = 0$$

$$r_1 = 1.5e^{j\frac{\pi}{6}} = \frac{3\sqrt{3}}{4} + j\frac{3}{4}$$

$$r_2 = 1.5e^{-j\frac{\pi}{6}} = \frac{3\sqrt{3}}{4} - j\frac{3}{4}$$

$$H(z) = \frac{\left(z - \frac{3\sqrt{3}}{4} - j\frac{3}{4}\right)\left(z - \frac{3\sqrt{3}}{4} + j\frac{3}{4}\right)}{z(z+0.8)}$$

b) As this is a causal system, the ROC is defined by $|z| > 0.8$
c) The system is stable because the ROC contains the unit circle.

Question 5 [8 pts]: Use the 4-point DFT and IDFT to determine the sequence $x_3(n) = x_1(n) \textcircled{4} x_2(n)$, where $x_1(n) = \{1,2,3,1\}$ and $x_2(n) = \{4,3,2,2\}$.

Solution:

$$X_1(k) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 7 \\ -2-j \\ 1 \\ -2+j \end{bmatrix}$$

$$X_2(k) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 11 \\ 2-j \\ 1 \\ 2+j \end{bmatrix}$$

$$X_3(k) = X_1(k) X_2(k) = \{77, -5, 1, -5\}$$

$$x_3(n) = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 77 \\ -5 \\ 1 \\ -5 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 68 \\ 76 \\ 88 \\ 76 \end{bmatrix} = \{17, 19, 22, 19\}$$