EE/CE 453/352: Digital Signal Processing

Saad Baig

Homework 1 SOLUTION
Assigned: January 22nd, 2024
Due by: January 29th, 2024
Total marks: 50

Question 1 [8 pts]: Use the following input sequences to compute the requested sequences. For indices where the input sequences are not specified, consider values to be zero. Clearly specify the index range of nonzero values for each output.

$a[n] = \{3\ \ 1\ \ \text{-4}\ \ 5\ \ \text{-7}\ \ 2\ \ 0\ \ \text{-1}\ \ 6\}$ $\qquad -4 \leq n \leq 4$

$b[n] = \{2\ \ 4\ \ 3\ \ \text{-5}\ \ \text{-2}\ \ 1\ \ 6\ \ 1\ \ \text{-3}\ \ \text{-2}\ \ 2\}$ $\qquad -8 \leq n \leq 2$

$c[n] = \{\text{-1}\ \ 2\ \ \text{-3}\ \ 4\ \ \text{-1}\ \ 2\ \ \text{-3}\ \ 4\}$ $\qquad -6 \leq n \leq 1$

a) $y_1[n] = b[3-n] + c[n]$
b) $y_2[n] = 3a[n] * c[n-1]$ (convolution)

Solution:

n	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11
b[-n]					2	-2	-3	1	6	1	-2	-5	3	4	2			
b[3-n]								2	-2	-3	1	6	1	-2	-5	3	4	2
c[n]	-1	2	-3	4	-1	2	-3	4										
y_1[n]	-1	2	-3	4	-1	2	-3	6	-2	-3	1	6	1	-2	-5	3	4	2

n	-5	-4	-3	-2	-1	0	1	2	3	4
3a[n]		9	3	-12	15	-21	6	0	-3	18
3a[-n]		18	-3	0	6	-21	15	-12	3	9
c[n-1]	-1	2	-3	4	-1	2	-3	4		

$$y_2[-9] = -9$$
$$y_2[-8] = 18 - 3 = 15$$
$$y_2[-7] = -27 + 6 + 12 = -9$$
$$y_2[-6] = 36 - 9 - 24 - 15 = -12$$
$$y_2[-5] = -9 + 12 + 36 + 30 + 21 = 90$$
$$y_2[-4] = -27 + 18 - 3 - 48 - 45 - 42 - 6 = -126$$
$$y_2[-3] = -27 + 18 - 3 - 48 - 45 - 42 - 6 = 126$$
$$y_2[-2] = 36 - 9 - 24 - 15 - 84 - 18 + 3 = -111$$
$$y_2[-1] = 12 + 36 + 30 + +21 + 24 - 6 - 18 = 99$$
$$y_2[0] = 36 + 9 - 6 - 42 - 45 - 48 = -96$$
$$y_2[1] = -54 - 12 + 12 + 63 + 60 = 69$$
$$y_2[2] = 72 + 3 - 18 - 84 = -27$$
$$y_2[3] = -18 - 6 + 24 = 0$$
$$y_2[4] = 36 + 9 = 45$$
$$y_2[5] = -54 - 12 = -66$$
$$y_2[6] = 72$$

$$y_2[n] = \{-9, 15, -9, -12, 90, -126, 126, -111, 99, -96, 69, -27, 0, 45, -66, 72\}$$

Question 2 [10 pts]: Consider the following system properties discussed in class:

1) Memoryless
2) Causality
3) Linearity
4) Time Invariance
5) Stability

Determine which of these properties hold and which do not hold for each of the following systems (justify your answers):

a) $y[n] = x[2-n]$
b) $y[n] = nx[-n]$

Solution:

Property	System (a)	System (b)
Memory-less	System has memory: $y[n] = x[-n+2]$	System has memory: $y[n] = nx[-n]$
Causality	Non-causal because if $n=-1$: $y[-1] = x[1+2] = x[3]$ Which means there is a value before $n = 0$.	Non-causal because if $n=-1$: $y[-1] = -x[1]$ Which means there is a value before $n = 0$.
Linearity	System is linear: $\alpha y_1[n] + \beta y_2[n] = \alpha x_1[-n+2] + \beta x_2[-n+2]$	System is linear: $\alpha y_1[n] + \beta y_2[n] = \alpha n x_1[-n] + \beta n x_2[-n]$
Time-invariance	System is time-variant: $y[n-k] \neq x[-n+2-k]$ $\rightarrow y[n-k] = x[-(n-k)+2] = x[-n+2+k]$	System is time-variant: $y[n-k] \neq nx[-n-k]$ $\rightarrow y[n-k] = (n-k)x[-(n-k)] = (n-k)x[-n+k]$
Stability	System is stable because time reversal or shifting does not change the max and min value of signal: $\lvert x[-n+2] \rvert < B$	System is unstable: If $x[n] = u[n]$ then: $y[n] = nu[-n]$

Question 3 [4 pts]: Consider the following system properties discussed in class:

1) Causality
2) Stability

Determine which of these properties hold and which do not hold for each of the LTI systems whose impulse response is given below (justify your answers):

a) $h[n] = \delta[n] - \delta[n-1] + \delta[n+1]$
b) $h[n] = (0.2)^n u[n]$

Solution:

Property	System (a)	System (b)
Causality	Non-causal because of $\delta[n+1]$. $h[n] \neq 0, \quad n < 0$	System is causal (step function starts at 0).
Stability	System is stable (completely summable).	System is stable as $(0.2)^n$ converges. $\sum_{n=-\infty}^{\infty} h[n] = \frac{0.2}{1-0.2} = \frac{1}{4} < \infty$

Question 4 [12 pts]: A LTI system has an impulse response:

$$h[n] = \delta[n] + 0.5\delta[n-1] + 0.25\delta[n-2]$$

Compute and hand-sketch the output $y[n]$ for the following inputs:

a) $x[n] = \delta[n] + 2\delta[n-4] - 0.5\delta[n-6]$
b) $x[n] = \delta[n] - 0.5\delta[n-1]$

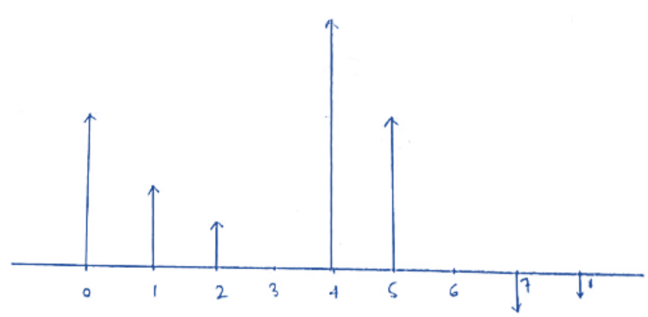
Solution:

a) $x[n] = \delta[n] + 2\delta[n-4] - 0.5\delta[n-6]$

$$y[n] = x[n] + 0.5x[n-1] + 0.25x[n-2]$$

$$y[n] = \{\delta[n] + 2\delta[n-4] - 0.5\delta[n-6]\} + 0.5\{\delta[n-1] + 2\delta[n-5] - 0.5\delta[n-7]\} + 0.25\{\delta[n-2] + 2\delta[n-6] - 0.5\delta[n-8]\}$$

$$y[n] = \delta[n] + 0.5\delta[n-1] + 0.25\delta[n-2] + 2\delta[n-4] + \delta[n-5] + 0.5\delta[n-6] - 0.25\delta[n-7] - 0.125\delta[n-8]$$

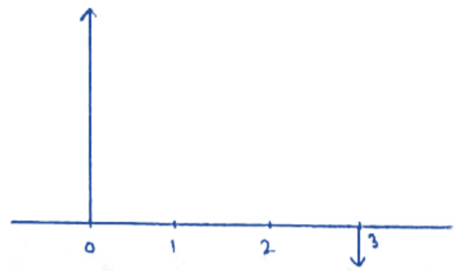


b) $x[n] = \delta[n] - 0.5\delta[n-1]$

$$y[n] = x[n] + 0.5x[n-1] + 0.25x[n-2]$$

$$y[n] = \delta[n] - 0.5\delta[n-1] + 0.5\{\delta[n-1] - 0.5\delta[n-2]\} + 0.25\{\delta[n-2] - 0.5\delta[n-3]\}$$

$$y[n] = \delta[n] - 0.125\delta[n-3]$$



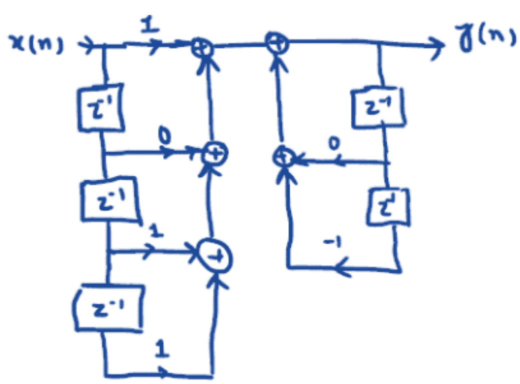
Question 5 [16 pts]: Consider the systems represented by the following Linear, Constant Coefficient Difference Equations (LCCDEs):

1) $y[n] + y[n-2] = x[n] + x[n-2] + x[n-3]$
2) $y[n] - 2x[n-2] = x[n-1] + 2x[n]$
3) $y[n] + 2y[n-2] = x[n] + 3x[n-2] + y[n-3]$

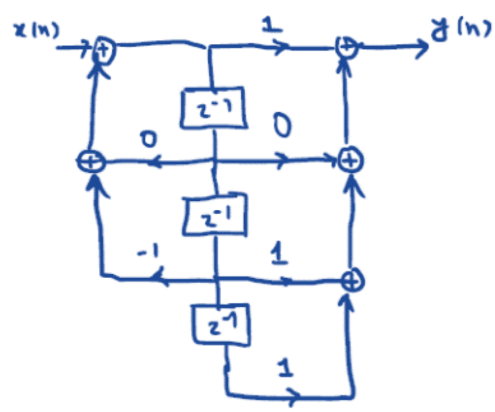
a) Identify which of these systems are FIR and IIR systems.
b) For IIR systems, draw the block diagrams for Direct Form I and Direct Form II realizations.
c) In part (b), make an attempt to minimize the number of adder blocks needed. (Hint: notice the coefficients of LCCDE).

Solution:

a) Systems 1 and 3 are IIR, and system 2 is FIR
b) For system 1:

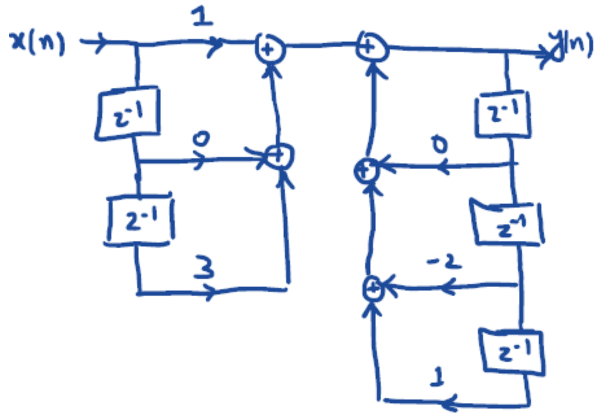


DIRECT FORM I

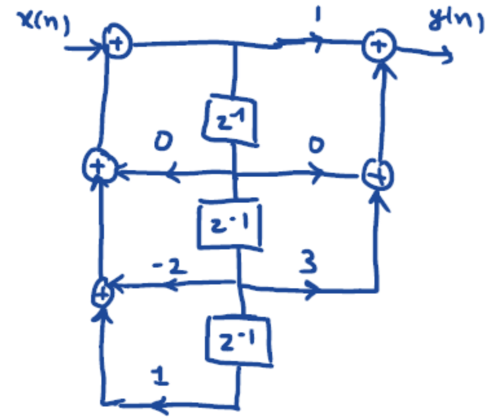


DIRECT FORM II

For system 3:

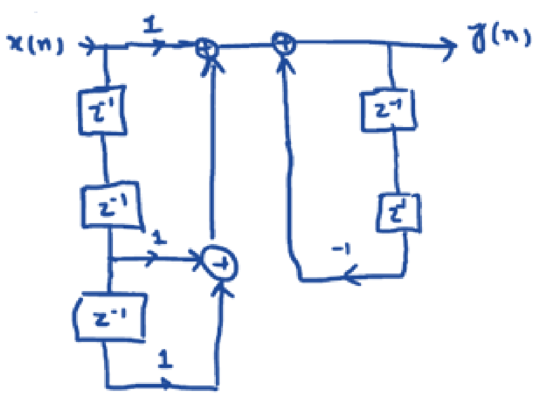


DIRECT FORM I

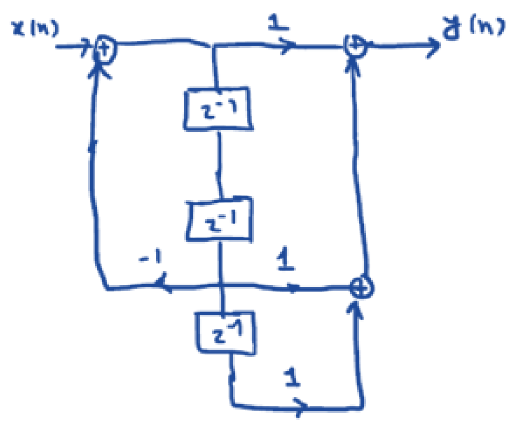


DIRECT FORM II

c) For system 1:

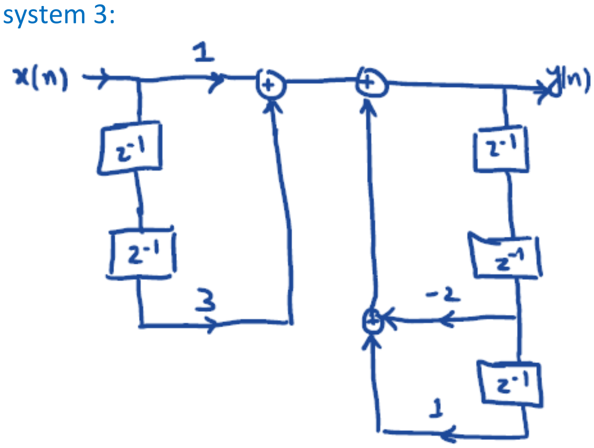


DIRECT FORM I

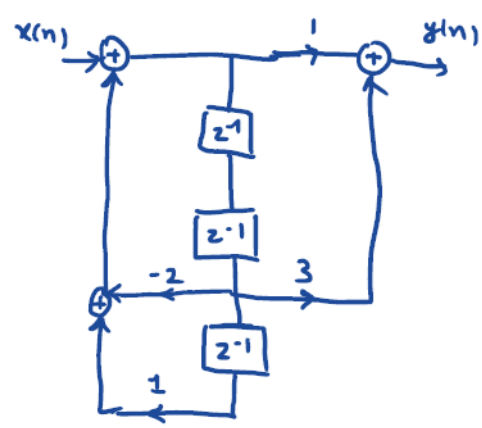


DIRECT FORM II

For system 3:



DIRECT FORM I



DIRECT FORM II