Habib University

EE/CE 453/352: Digital Signal Processing - Spring 2024
Saad Baig

Time = 35 minutes	**Quiz 03 SOLUTION**	Max Points: 20

Instructions:

i. **Smart watches, laptops, and similar electronics are strictly NOT allowed.**
ii. **Answer sheets should contain all steps, working, explanations, and assumptions.**
iii. Attempt the quiz with black/blue ink.
iv. Print your name and HU ID on all sheets.
v. This is a closed-book examination but you are allowed a single-sided A4 sized cheat sheet.
vi. You are not allowed to ask/share your method or answer with your peers. The work submitted by you is solely your own work. Any violation of this will be the violation of HU Honor code and proper action will be taken as per university policy if found to be involved in such an activity.

CLO Assessment:

This quiz will assess students for the following course learning outcomes.

Course Learning Outcome		Learning Domain Level
CLO 2	Design various types of digital filters to meet given specifications.	Cog-5

Undertaking:

I hereby affirm that I have read the instructions. I am fully aware of the HU honor code and the repercussions of its violation, and hereby pledge that the work I am going to submit is clearly my own.

Signature: ______________________

Name: INSTRUCTOR SOLUTION

HU ID: ______________________

Question 1 [4 pts]: The bilinear transformation is used to design an ideal discrete-time low pass filter with cutoff frequency $\omega_c = 3\pi/5$ from an ideal continuous-time low pass filter with cutoff frequency $\Omega_c = 2\pi(300)$ rad/s. Give a choice for the parameter T that is consistent with this information.

Solution:

$$\Omega_c = \frac{2}{T}\tan\frac{\omega_c}{2} \quad \rightarrow \quad T = \frac{2}{2\pi(300)}\tan\frac{3\pi/5}{2} = 1.46 \text{ ms}$$

Question 2 [8 pts]: Consider a differentiator with the following system function:

$$H_a(s) = s$$

a) Determine $H(z)$ by applying the bilinear transformation to $H_a(s)$.
b) If $x[n]$ is the input and $y[n]$ is the output, write the difference equation satisfied by the input and output.

Solution:

Assume $T = 1\ s$:

$$H(z) = H_a(s)\Big|_{s=\frac{2}{T}\left(\frac{z-1}{z+1}\right)} \quad \rightarrow \quad H(z) = \frac{Y(z)}{X(z)} = \frac{2(z-1)}{z+1} = \frac{2(1-z^{-1})}{(1+z^{-1})}$$

$$Y(z)(1+z^{-1}) = X(z)\{2(1-z^{-1})\}$$

$$y[n] = 2x[n] - 2x[n-1] + y[n-1]$$

Question 3 [8 pts]: A FIR lowpass filter with cutoff frequency of $\omega = 0.3\pi$ has the following specifications:

$$0.95 < H(e^{j\omega}) < 1.05, \qquad 0 \le |\omega| \le 0.25\pi$$

$$-0.1 < H(e^{j\omega}) < 0.1, \qquad 0.35\pi \le |\omega| \le \pi$$

a) Determine the passband ripple and the minimum stopband attenuation in dB.
b) If the maximum passband error is -26 dB, determine the appropriate windowing function/s and approximate their minimum length.

Type of Window	Peak side-lobe amplitude (dB)	Peak approximation error (dB)	Approximate width of main lobe
Rectangular	-13	-21	$4\pi/(M+1)$
Bartlett	-25	-25	$8\pi/M$
Hanning	-31	-44	$8\pi/M$
Hamming	-41	-53	$8\pi/M$
Blackman	-57	-74	$12\pi/M$

Solution:

$$R_p = -20\log_{10}\frac{1-0.05}{1+0.05} = 0.869 \text{ dB}$$

$$A_s = -20\log_{10}\frac{0.1}{1+0.05} = 20.424 \text{ dB}$$

As the maximum passband error is -26 dB, we can only use the Hann, Hamming or Blackman windows. This is because the peak approximation error is the same as the maximum passband error.

Filter lengths using approximate width of main lobe:

$$M_{\text{Hann}} = M_{\text{Hamming}} = \frac{8\pi}{0.35 - 0.25} = 251$$

$$M_{\text{Blackman}} = \frac{12\pi}{0.35 - 0.25} = 377$$