Discrete-Time Signals and Systems

EE 453 / CE 352

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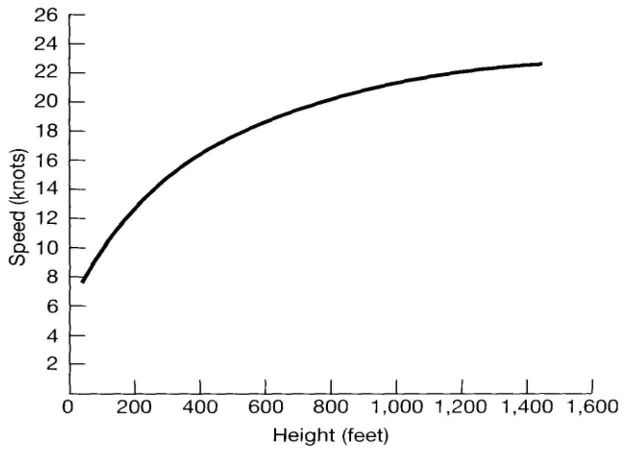
Agenda

- Digitals Signal
  - Properties
  - Types of Signals
- Linear Time-Invariant Systems
  - Impulse Response
  - Convolution
- Digital Systems
  - Representation, block diagrams, Direct Form I and II

Signal: Analog vs Digital

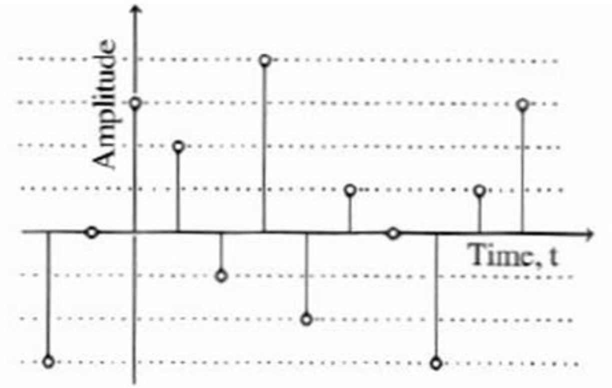
- Analog Signal
  - Continuous-Time

    AND
  - Continuous-Valued



- Digital Signal
  - Discrete-Time

    AND
  - Discrete Valued

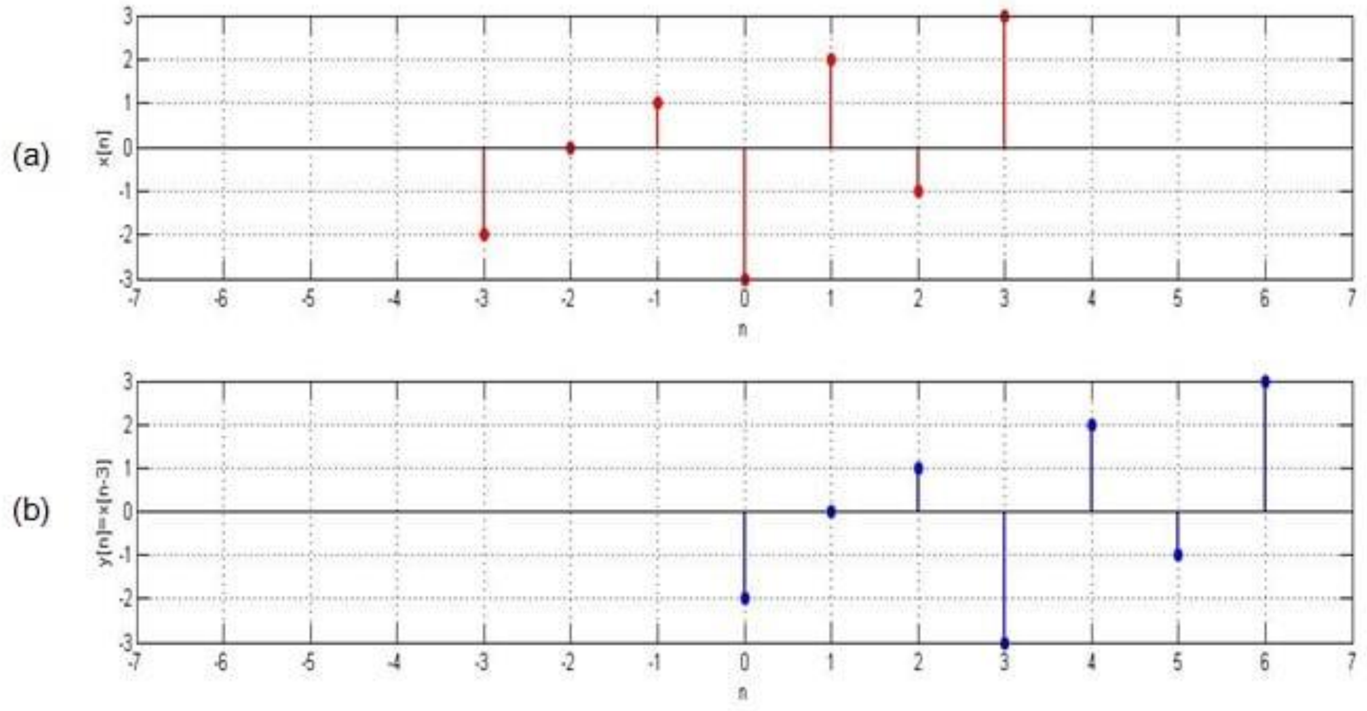


Elementary Signal Transformations

- Time Shifting
- Time Reversal
- Time Scaling
- Time Scaling with Shifting

Time Shifting (Translation)

- $x[n]$ is mapped to $y[n]$ as: $y[n] = x[n - n_0]$
  - Where n_0 is a real number.

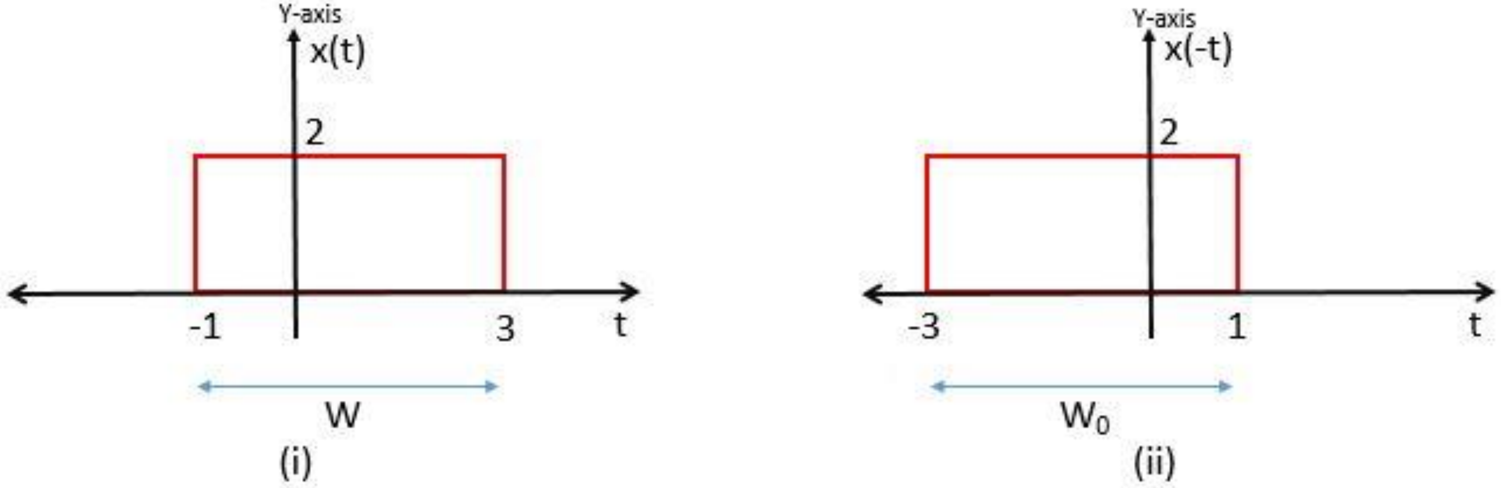


Time Reversal (Reflection)

- Maps the input signal $x[n]$ to the output signal $y[n]$ as given by

$$y[n] = x[-n]$$

  – Geometrically, the output signal $y[n]$ is a reflection of the input signal $x[n]$ about the (vertical) line $n = 0$.

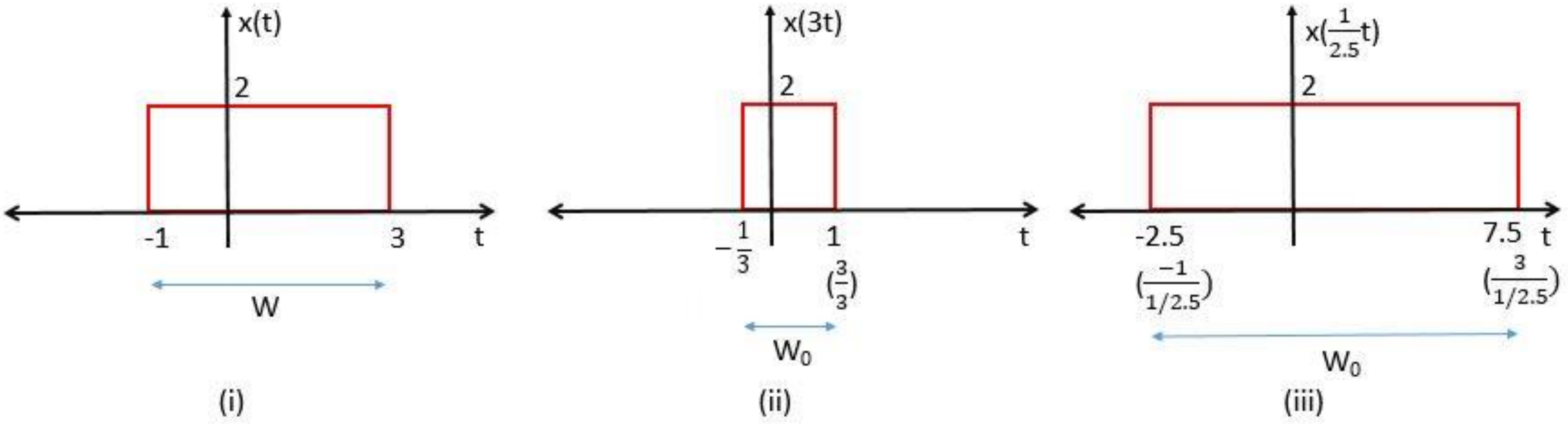


Time Scaling (Compression/Expansion)

- Maps the input signal $x[n]$ to the output signal $y[n]$ as given by

$$y[n] = x[\alpha n]$$

  - where α is a non-zero real number.



Time Scaling with Shifting

- Maps the input signal $x[n]$ to the output signal $y[n]$ as given by

$$y[n] = x[\alpha n + \beta]$$

  - where α and β are real numbers and α is non-zero.
  - A systematic approach to doing this is to first delay or advance $x[n]$ in accordance with the value of β, and then perform time scaling on the resulting signal in accordance with the value of α.
    - I.e., replace n with $n + \beta$, then replace n with αn to get $\alpha n + \beta$.

Other Elementary Signal Transformations

- Amplitude Shifting
- Amplitude Scaling
- Amplitude Scaling with Shifting
- Signal Addition
- Signal Multiplication

Amplitude Shifting

- Maps the input signal $x[n]$ to the output signal $y[n]$ as given by:

$$y[n] = x[n] - n_0$$

  – where n_0 is a real number.

Amplitude Scaling

- Maps the input signal $x[n]$ to the output signal $y[n]$ as given by:

$$y[n] = \alpha x[n]$$

  - where α is a non-zero real number.

Amplitude Scaling with Shifting

- Maps the input signal $x[n]$ to the output signal $y[n]$ as given by

$$y[n] = \alpha x[n] + \beta$$

  - where α and β are real numbers and α is non-zero.

Signal Addition

- The sum of two signals and is a signal y(n) whose value at any instant is equal to the sum of the values of these two signals at that instant, as given by:

$$y[n] = x_1[n] + x_2[n], \qquad -\infty < n < \infty$$

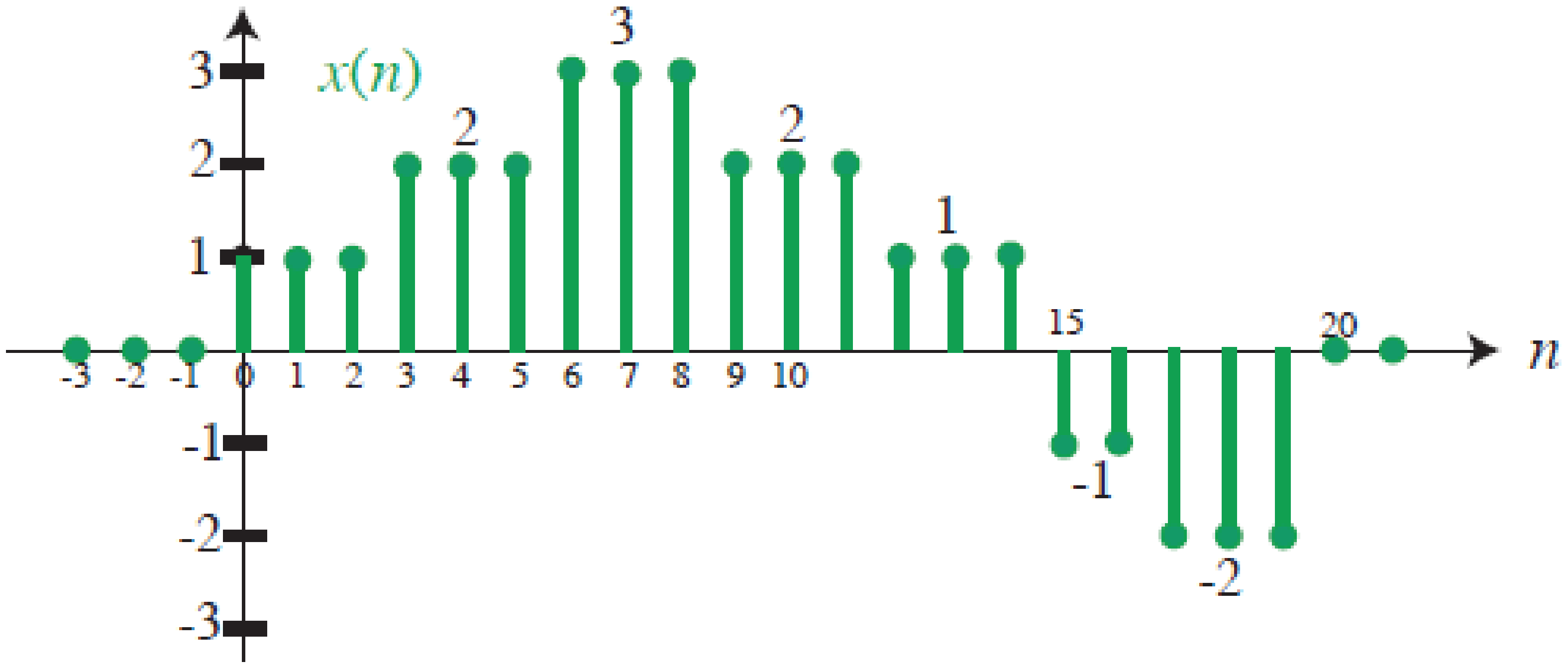
Signal Multiplication

- The product of two signals $x_1[n]$ and $x_2[n]$ is a signal $y[n]$ whose value at any instant is equal to the product of the values of these two signals at that instant, that is

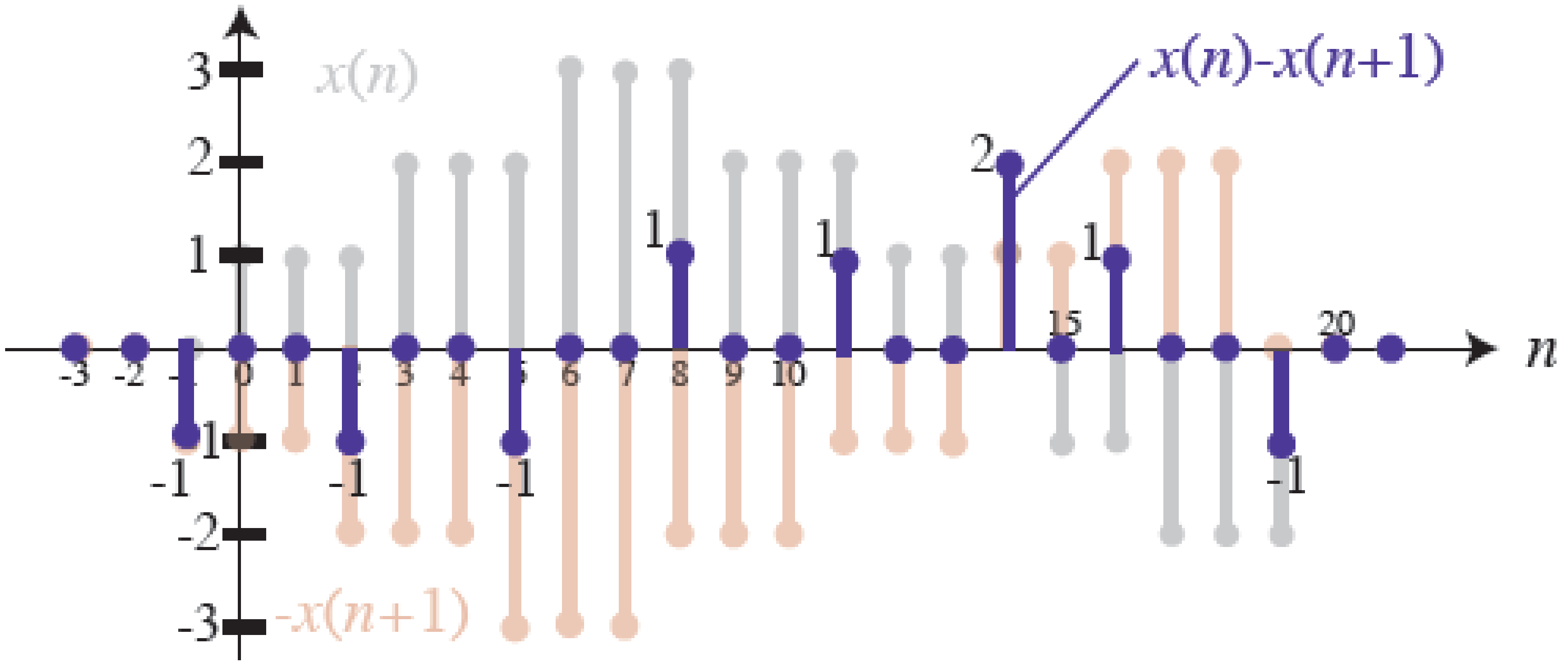
$$y[n] = x_1[n]x_2[n], \qquad -\infty < n < \infty$$

Signal Transformation: Example

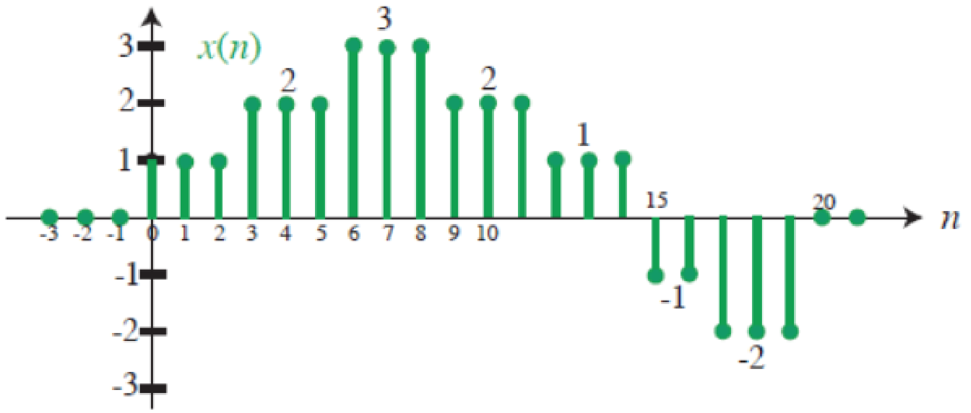
Find $x(n) - x(n+1)$.



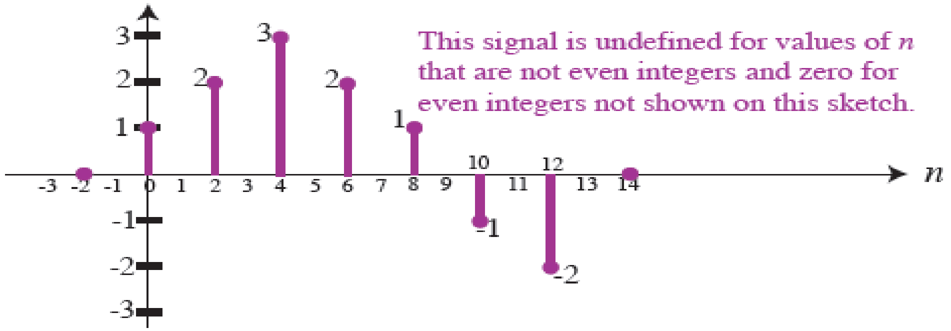
Signal Transformation: Example



Signal Transformation: Example



Graph of $x(\frac{3}{2}n+1)$.



Find $x(\frac{3}{2}n+1)$.

n	$\frac{3n}{2}+1$	$x(\frac{3n}{2}+1)$
< -1	$< -\frac{1}{2}$	0 if $\frac{3n}{2}+1$ is an integer; undefined otherwise
-1	$-\frac{1}{2}$	undefined
0	1	$x(1) = 1$
1	$\frac{5}{2}$	undefined
2	4	$x(4) = 2$
3	$\frac{11}{2}$	undefined
4	7	$x(7) = 3$
5	$\frac{17}{2}$	undefined
6	10	$x(10) = 2$
7	$\frac{23}{2}$	undefined
8	13	$x(13) = 1$
9	$\frac{29}{2}$	undefined
10	16	$x(16) = -1$
11	$\frac{35}{2}$	undefined
12	19	$x(19) = -2$
> 12	> 19	0 if $\frac{3n}{2}+1$ is an integer; undefined otherwise

Signal Energy

- The energy of a discrete-time signal $x[n]$ is defined as:

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

  - Note that for some signals the above sum might not converge and thus those signal have infinite energy.
  - Practical signals have finite energy. They have finite duration and finite amplitude.

Signal Power

- The average power of a discrete-time signal x[n] is defined as:

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

  - Note that for some signals the above sum might not converge and thus those signals have infinite power.
  - A signal whose amplitude is constant over infinite duration is a power signal.

Signal Characterization through "Energy" and "Power"

$$E \equiv \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

- A signal with finite energy is said to be an energy signal.
- A signal with non-zero finite power is said to be a power signal.
- It's possible for a signal to be neither an energy nor a power signal (if either of them is infinity).

Energy Signal and Power Signal: Interesting Observations

$$E \equiv \sum_{n=-\infty}^{\infty} |x(n)|^2 \qquad\qquad P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

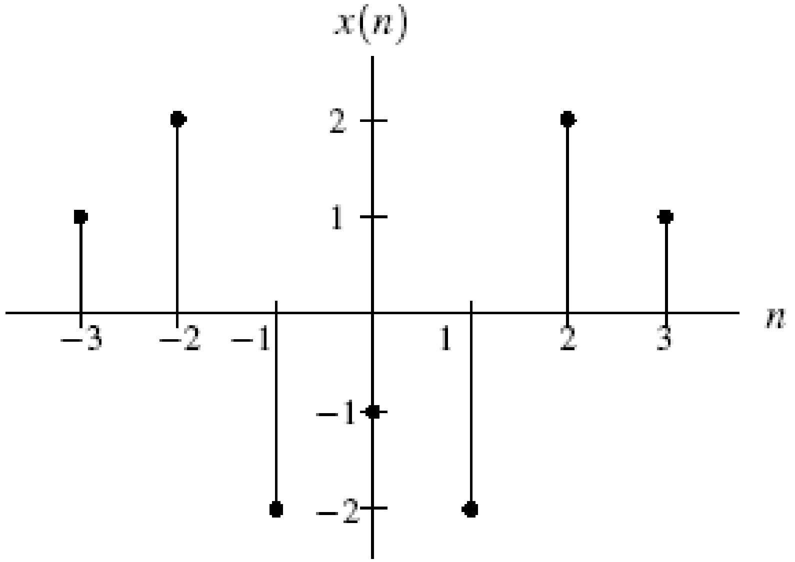
- An energy signal has zero average power (e.g. nonperiodic signal).
- If E is infinite, the average power P may be finite or infinite.
- A power signal has infinite energy (e.g. periodic signals).

Signal Properties: Even Signal

- A signal $x[n]$ is said to be even (or symmetric) if it satisfies:

$$x[n] = x[-n] \qquad \text{for all n}$$

  - Geometrically, the graph of an even signal is symmetric about the origin.

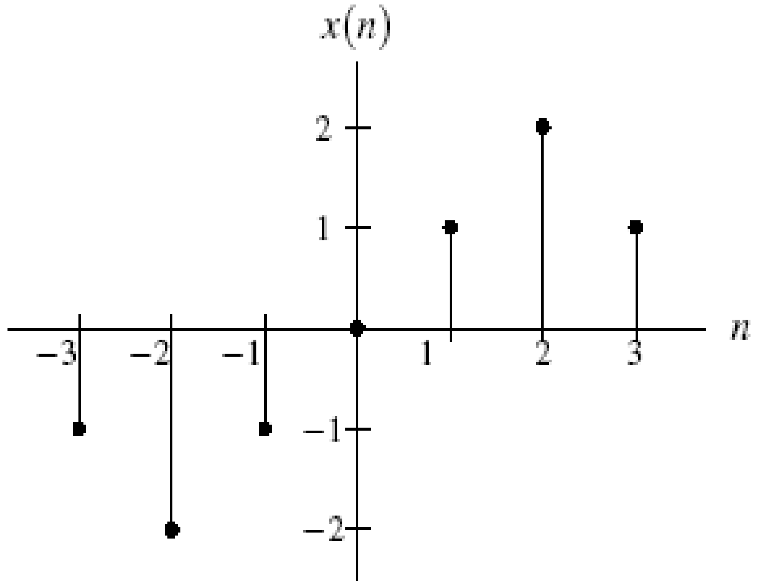


Signal Properties: Odd Signal

- A signal $x[n]$ is said to be odd (or antisymmetric) if it satisfies:

$$x[n] = -x[-n] \qquad \text{for all n}$$

  - Geometrically, the graph of an odd signal is antisymmetric about the origin.



Decomposition of a Signal

- If a signal $x[n]$ can be represented as

$$x[n] = x_1[n] + x_2[n]$$

we say that $x(n)$ can be decomposed into signals $x_1(n)$ and $x_2(n)$.

Even-Odd Decomposition of a Signal

- Every signal $x(n)$ has a unique representation of the form:

$$x(n) = x_e(n) + x_o(n)$$

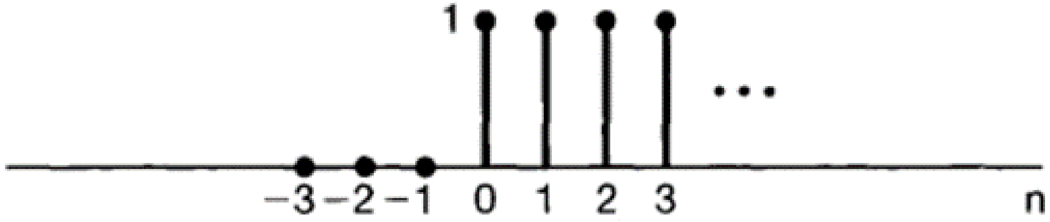
where the functions $x_e(n\,)$ and $x_o(n)$ are even and odd respectively, given by:

$$x_e(n\,) = \frac{1}{2}[x(n) + x(-n)] \qquad x_o(n\,) = \frac{1}{2}[x(n) - x(-n)]$$

- The functions $x_e(n)$ and $x_o(n)$ are called the even part and odd part of $x(n)$, respectively.

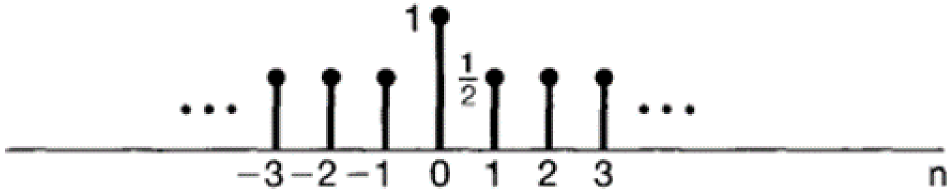
Even-Odd Decomposition of a Signal

$$x[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$



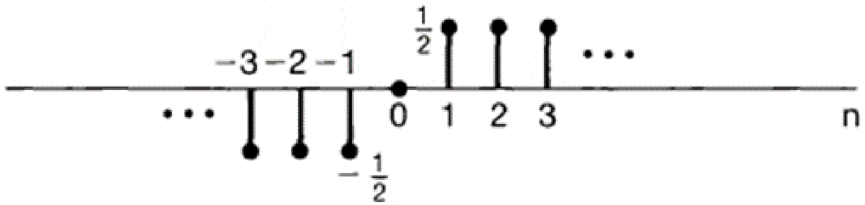
$$x_e(n) = \frac{1}{2}[x(n) + x(-n)]$$

$$\mathcal{E}v\{x[n]\} = \begin{cases} \frac{1}{2}, & n < 0 \\ 1, & n = 0 \\ \frac{1}{2}, & n > 0 \end{cases}$$



$$x_o(n) = \frac{1}{2}[x(n) - x(-n)]$$

$$\mathcal{O}d\{x[n]\} = \begin{cases} -\frac{1}{2}, & n < 0 \\ 0, & n = 0 \\ \frac{1}{2}, & n > 0 \end{cases}$$

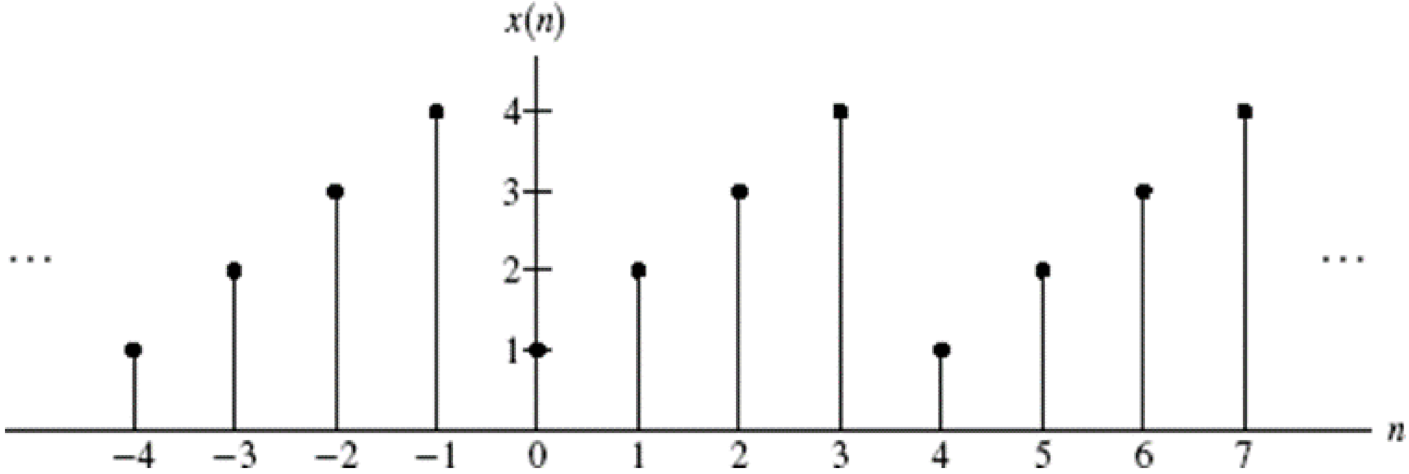


Signal Properties: Periodic Signal

- A signal $x[n]$ is said to be periodic with period N if it satisfies:

$$x[n] = x[n+N], \qquad \text{for all } n$$

- The smallest value of N for which the above equation holds is call the fundamental period of $x[n]$.



Signal Properties: Periodic Signal

- A periodic signal is unchanged by a time shift of T.
- If $x(t)$ is periodic then it must satisfy:

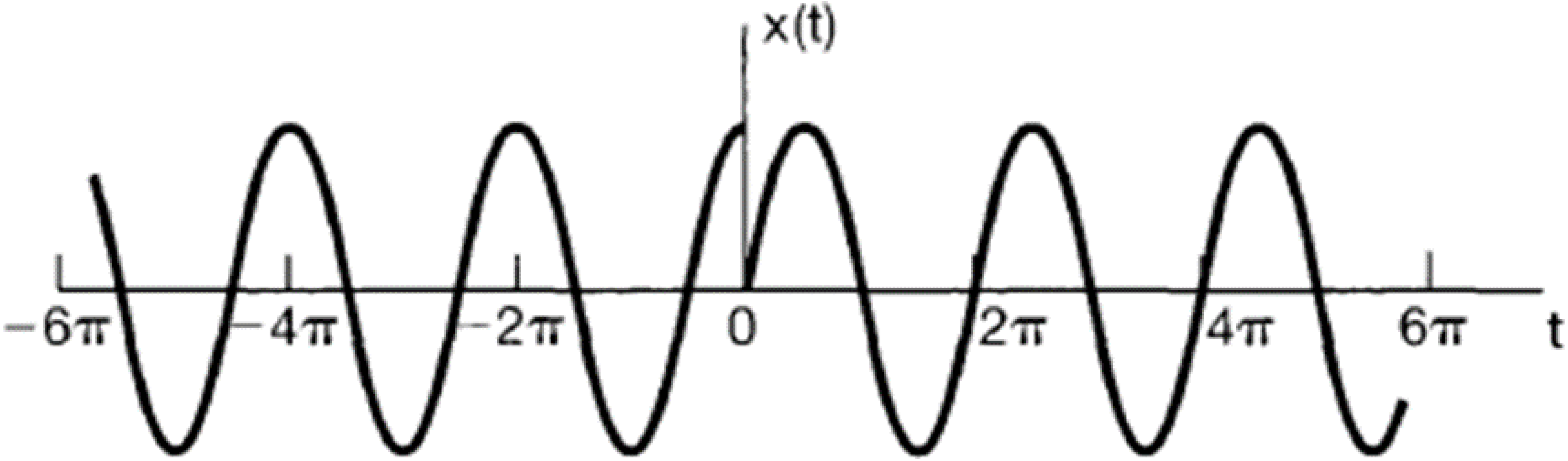
$$x(t) = x(t + mT) \qquad \text{for all } t \text{ and integers } m$$

- Fundamental period (T_0) of $x(t)$ is the smallest positive value of T for which the following equation holds:

$$x(t) = x(t + T) \qquad \text{for all } t$$

Signal Properties: Aperiodic Signal

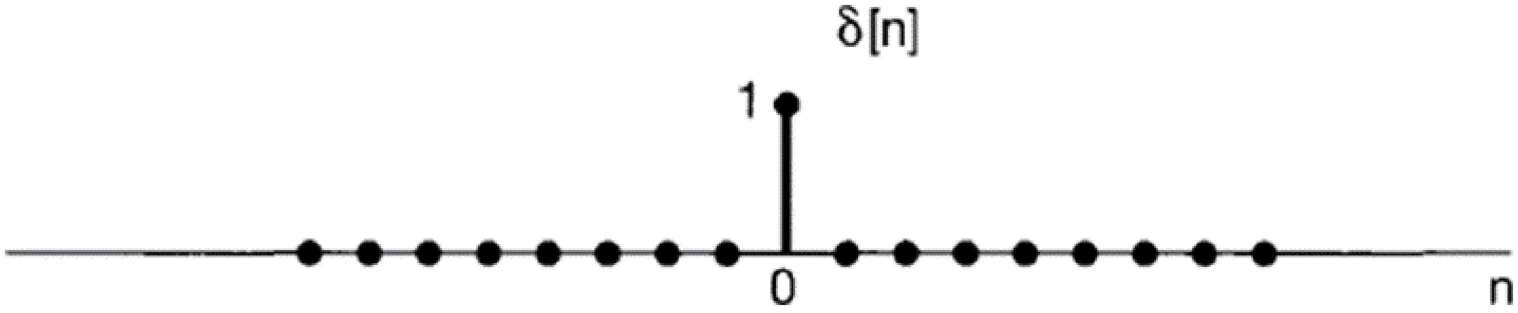
- A signal that is not periodic is said to be Aperiodic (or Nonperiodic) signal.



DT Unit Impulse Signal (Unit Sample Sequence)

- The discrete-time unit impulse signal is defined as:

$$\delta[n] = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}$$



DT Unit Step Signal

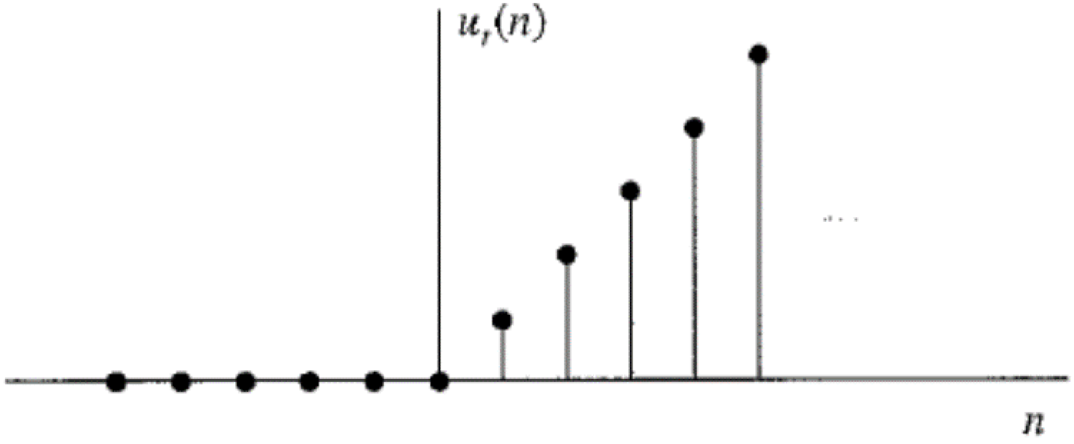
- The discrete-time unit step signal is defined as:

$$u[n] = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}$$

DT Unit Ramp Signal

- The discrete-time unit ramp signal is defined as:

$$u_r[n] = \begin{cases} 0, & n < 0 \\ n, & n \geq 0 \end{cases}$$



Relationship between DT Unit Impulse and Unit Step Signals

- The discrete-time unit impulse is the first difference of the discrete-time unit step.

$$\delta[n] = u[n] - u[n-1]$$

- The discrete-time unit step is the running sum of the discrete-time unit impulse.

$$u[n] = \sum_{m=0}^{n} \delta[n-m]$$

Sampling Property of the DT Unit Impulse

- The DT unit impulse signal can be used to sample the value of a signal at n = 0.

$$x[n]\delta[n] = x[0]\delta[n]$$

- More generally, a DT unit impulse signal $\delta[n-n_0]$ can be used to sample the value of a signal at $n = n_0$:

$$x[n]\delta[n-n_0] = x[n_0]\delta[n-n_0]$$