Introduction to Digital Filters

EE 453 / CE 352
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Big Picture Till Now

$$\{a_k, b_k\} \xrightarrow{\textbf{Analysis}} H(\omega)$$

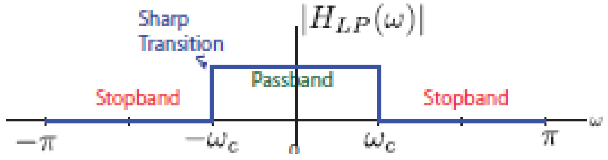
$$\{a_k, b_k\} \xleftarrow{\textbf{Design}} H(\omega)$$

DT LTI Systems
(Described by LCCDEs)

$$\sum_{k=0}^{N} a_k y(n-k) = \sum_{k=0}^{M} b_k x(n-k) \qquad a_0 \triangleq 1$$

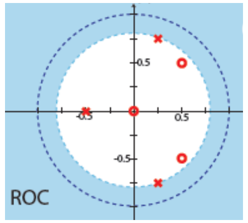
DT LTI Systems as Frequency Selective Filters
(Frequency Response)

$$h(n) \xleftrightarrow{\text{DTFT}} H(\omega)$$



Rational z-Transforms
(Pole-Zero Plots)

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{\left[1 + \sum_{k=1}^{N} a_k z^{-k}\right]}$$



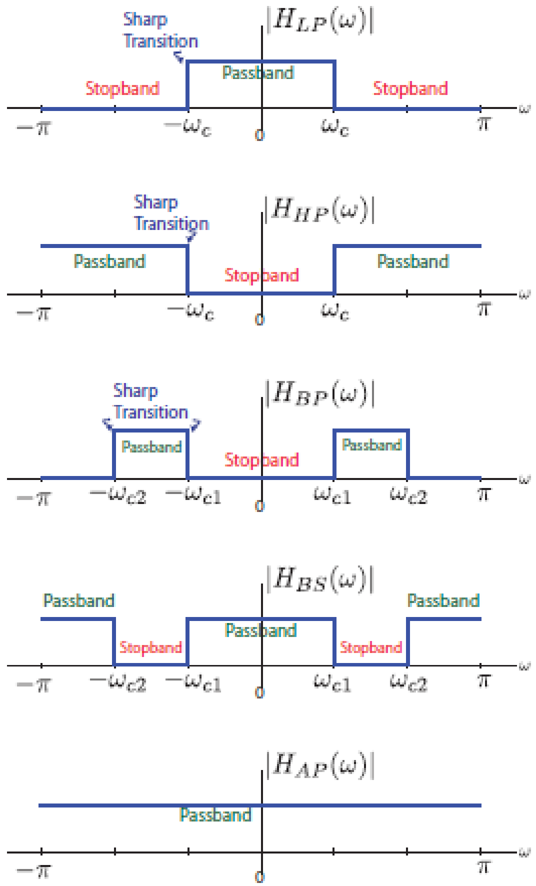
Big Picture Till Now

$$y[n] = x[n] * h[n] \overset{\mathcal{F}}{\leftrightarrow} Y(e^{j\omega}) = X(e^{j\omega})\, H(e^{j\omega})$$

- An LTI system can only change the amplitude and phase of a sinusoidal signal. It cannot change the frequency.
- Fourier Transform (frequency components) of the output of an LTI system is the Fourier Transform (frequency components) of the input, multiplied by the frequency response of the system
- This creates the possibility to change the relative amplitude of the frequency components in a signal.
- This process is referred to as filtering.

Big Picture Till Now

- A filter is a device that discriminates, according to some attribute of the input, what passes through it.
- For LTI systems, given $Y(\omega) = X(\omega)H(\omega)$, $H(\omega)$ acts as a kind of weighting function or spectral shaping function of the different frequency components of the signal.



Let's Design an Ideal Filter

An ideal lowpass filter is given by:

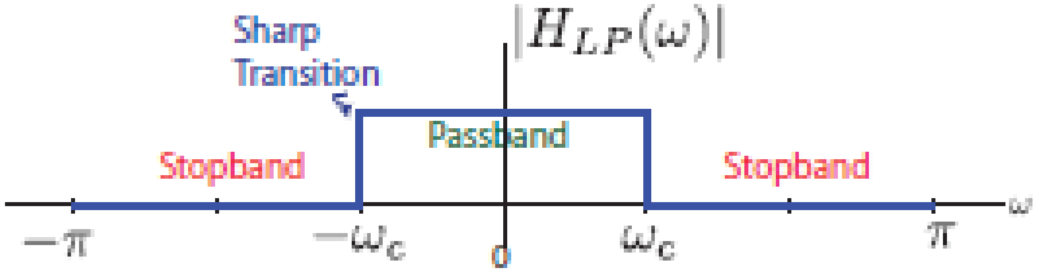
$$H(\omega) = \begin{cases} 1 & |\omega| \leq \omega_c \\ 0 & \omega_c < |\omega| \leq \pi \end{cases}$$

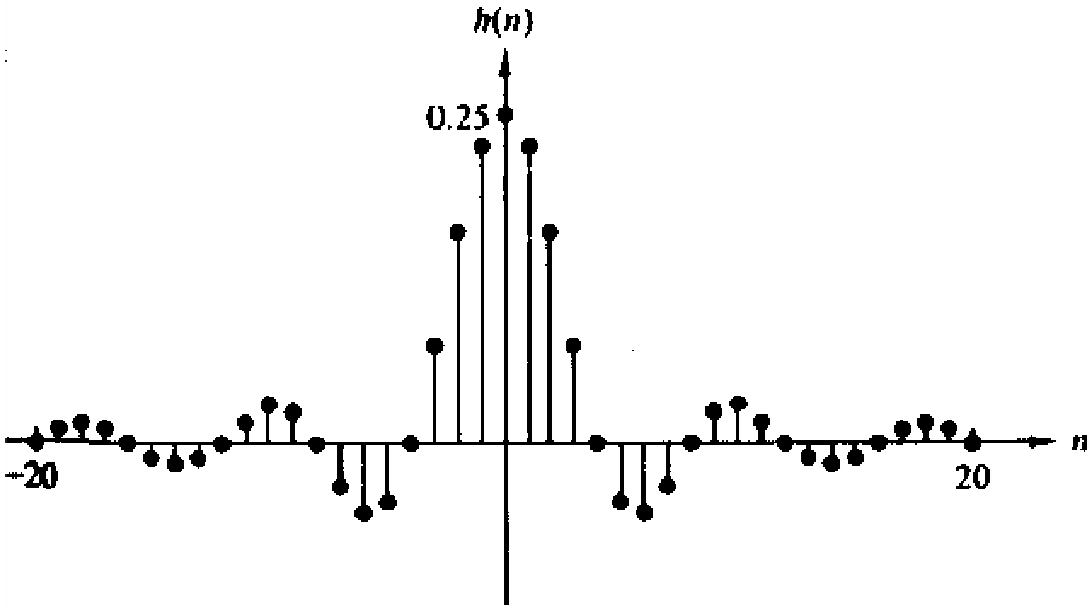
The impulse response is given by:

$$h(n) = \begin{cases} \frac{\omega_c}{\pi} & n = 0 \\ \frac{\omega_c}{\pi}\frac{\sin(\omega_c n)}{\omega_c n} & n \neq 0 \end{cases}$$

$$h(n) \neq 0 \text{ for } n < 0.$$

- Ideal filters are non-causal and hence cannot be realized in practice.





Practical Filters (Described by LCCDE's)

- Ideal filter characteristics of sharp transitions and flat gains may not be absolutely necessary for most practical applications.

- Relaxing these conditions allows us to realize causal finite parameter filters that can approximate ideal filters.

DT LTI Systems
(Described by LCCDEs)

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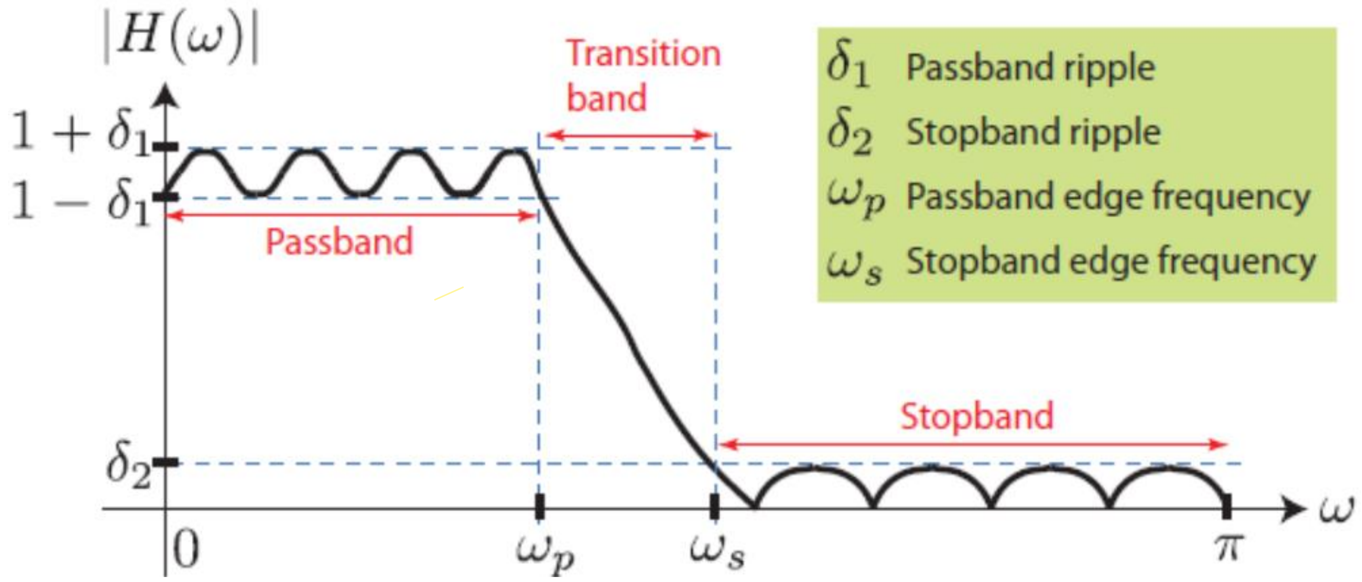
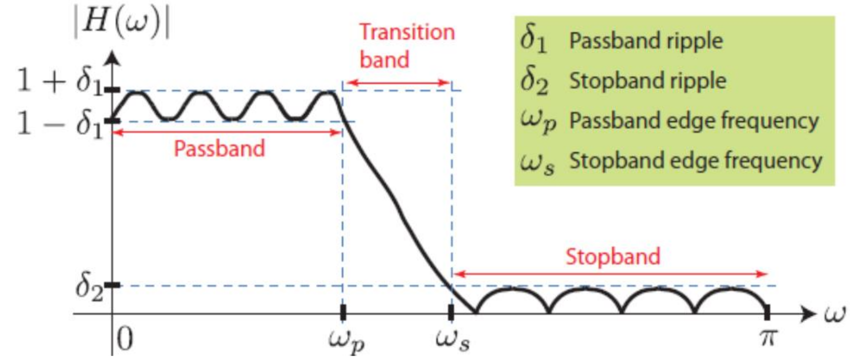


Three Stages of Filter Design

1. **Specifications** (determined by applications)
2. **Approximation** (come up with a filter description or coefficients to approximate the given specifications)
3. **Implementation** (realize a difference equation as hardware/software)



DT LTI Systems
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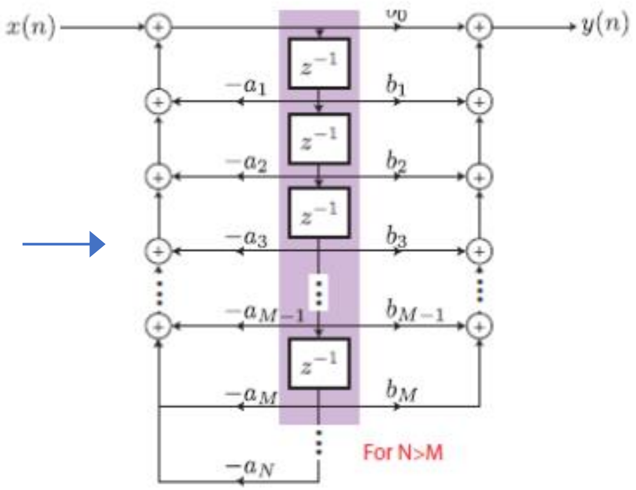
Direct Form II



Filter Specifications: Magnitude

- R_p is the passband ripple in dB:

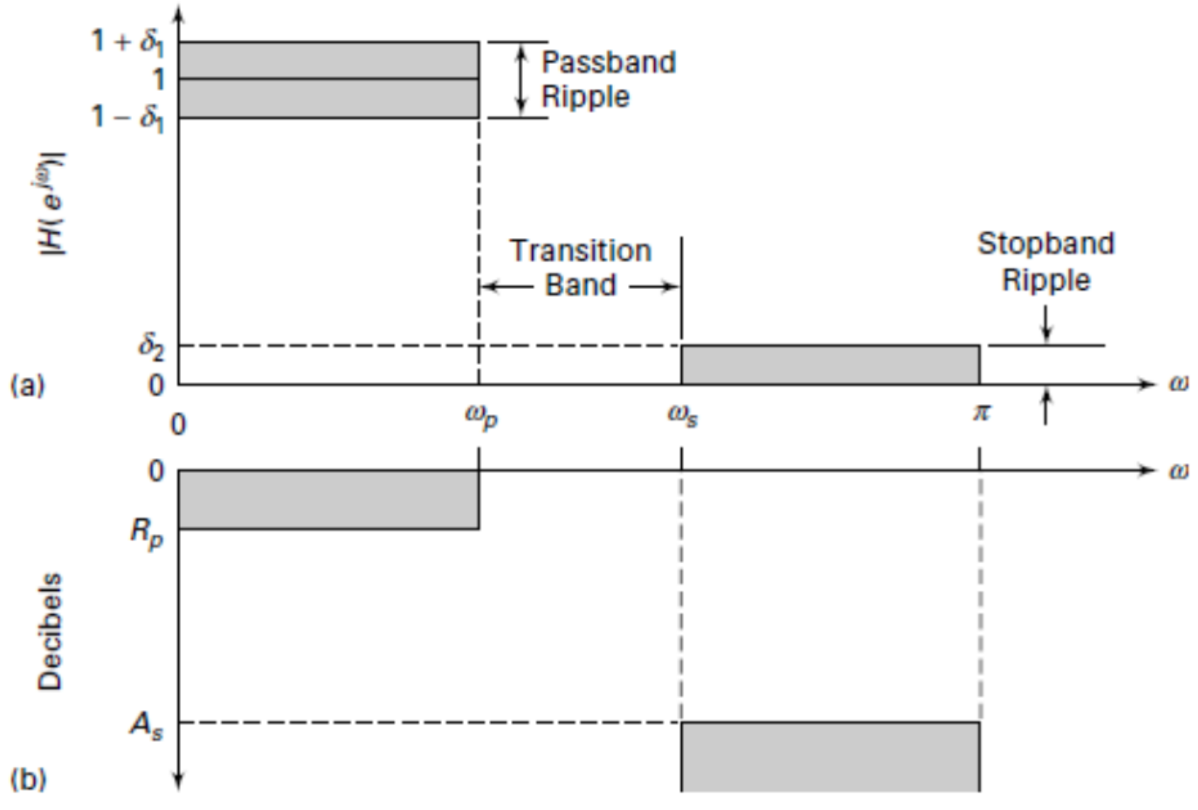
$$R_p = -20\log_{10}\frac{1-\delta_1}{1+\delta_1} > 0(\approx 0)$$

- A_s is the stopband attenuation in dB:

$$A_s = -20\log_{10}\frac{\delta_2}{1+\delta_1} > 0(\gg 1)$$

- Decibel scale:

$$\text{dB scale} = -20\log_{10}\frac{\left|H\left(e^{j\omega}\right)\right|}{|H(e^{j\omega})|_{\max}} \geq 0$$



filter specifications: (a) absolute (b) relative

Magnitude Specification: Example

- ❑ Given the passband tolerance $\delta_1 = 0.01$ and the stopband tolerance $\delta_2 = 0.001$, determine the passband ripple and the stopband attenuation.

- **Solution:**

$$R_p = -20\log_{10}\frac{1-\delta_1}{1+\delta_1} = -20\log_{10}\frac{1-0.01}{1+0.01}$$

$$R_p = 0.1737\ \mathrm{dB}$$

$$A_s = 50 = -20\log_{10}\frac{\delta_2}{1+\delta_1} = -20\log_{10}\frac{0.001}{1+0.01}$$

$$\delta_2 = 60\ \mathrm{dB}$$

Magnitude Specification: Example

- ❑ In a certain filter's specifications, the passband ripple is 0.25 dB and the stopband attenuation is 50 dB. Determine δ_1 and δ_2.

- **Solution:**

$$R_p = 0.25 = -20\log_{10}\frac{1-\delta_1}{1+\delta_1}$$

$$\delta_1 = 0.0144$$

$$A_s = 50 = -20\log_{10}\frac{\delta_2}{1+\delta_1} = -20\log_{10}\frac{\delta_2}{1+0.0144}$$

$$\delta_2 = 0.0032$$