Introduction to Robotics - L1

Huzaifah and Asghar

Question 1:

Let the position of the end-point for the three link planar manipulator shown in Figure 1 be (x, y). Find expressions for x and y in terms of joint coordinates θ_1, d_2, and θ_3, using elementary geometry and trigonometry.

Problem 1
CLO2-C4

10 points

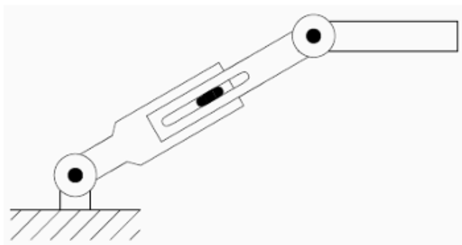
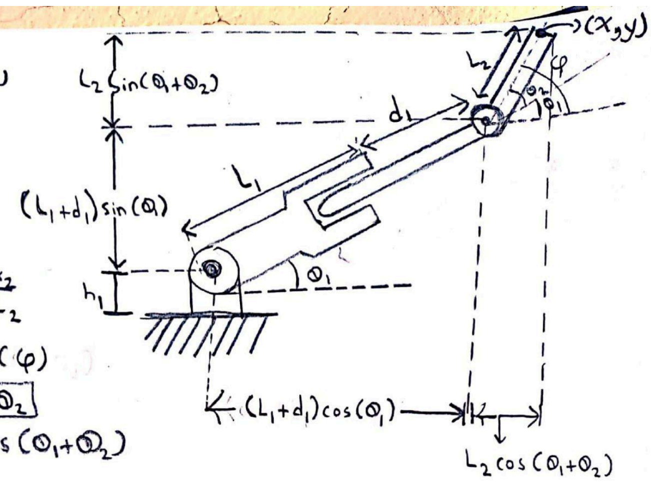


Figure 1: Three link planar robot

Intro to robotics (HW)

Q1



For x:

$\cos \theta_1 = \frac{x_1}{l_1+d_1}$

$\Rightarrow x_1 = (l_1+d_1)\cos\theta_1$

$\cos(\varphi) = \frac{x_2}{L_2}$

$x_2 = L_2 \cos(\varphi)$

$\boxed{\text{As } \varphi = \theta_1+\theta_2}$

$x_2 = L_2 \cos(\theta_1+\theta_2)$

$$\boxed{x = (l_1+d_1)\cos(\theta_1) + l_2 \cos(\theta_1+\theta_2)}$$

For y:

$\sin \theta_1 = \frac{y_1}{l_1+d_1}$

$\Rightarrow y_1 = (l_1+d_1)\sin(\theta_1)$

$\sin(\varphi) = \frac{y_2}{L_2}$

$$\boxed{y = h + (l_1+d_1)\sin(\theta_1) + L_2 \sin(\theta_1+\theta_2)}$$

$$(x, y) = \begin{bmatrix} (l_1+d_1)\cos(\theta_1) + l_2\cos(\theta_1+\theta_2), \\ h + (l_1+d_1)\sin(\theta_1) + L_2 \sin(\theta_1+\theta_2) \end{bmatrix}$$

D1 = D2
θ3 = θ2

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Question 2:

Figure 2 shows an RPRRR open chain in its zero (rest) configuration. Frames $\{0\}$ and $\{b\}$ have already been assigned in the figure. Assign the remaining frames according to standard DH convention, determine all link parameters between valid DH consecutive frames, write the homogeneous transformation matrices between all consecutive frames, and the final homogeneous transformation ${}^{0}T_{b}$ for this robot manipulator, illustrated in Figure 2, in terms

Problem 2
CLO2-C3

30 points

of the joint variables $\{\theta_1, \theta_2, \theta_3, \theta_4, \theta_5\}$[1]. You may make any reasonable assumptions. Also, determine the values of the joint variables in the present configuration as seen in Figure 2 and find the corresponding end-effector transformation, ${}^{0}T_{b}$.

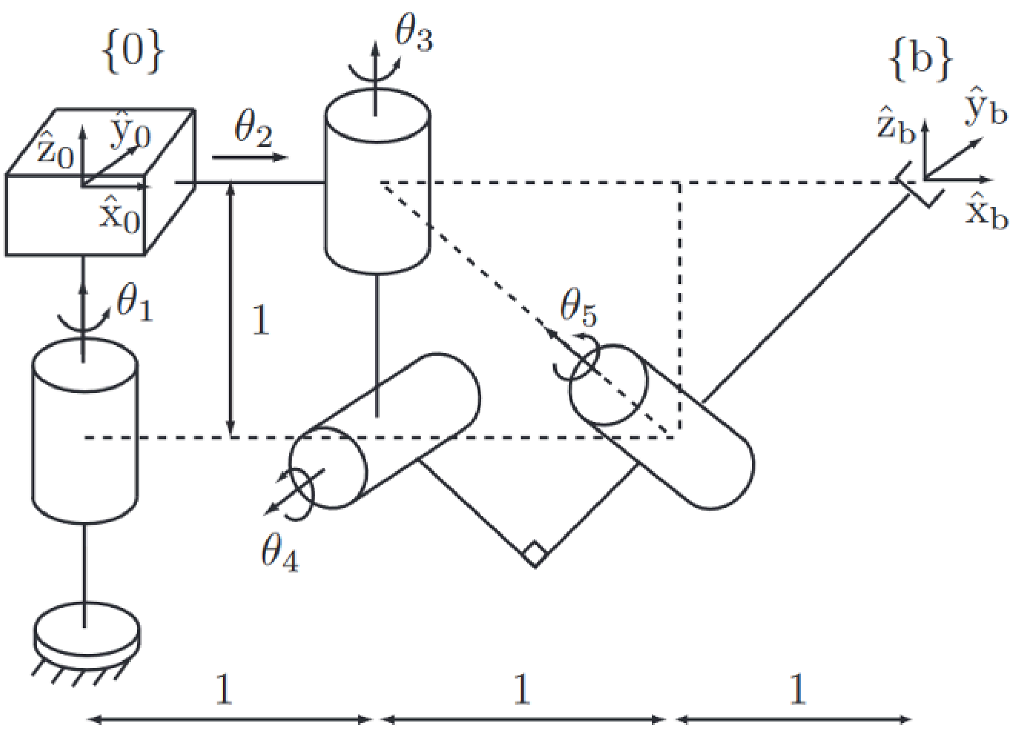

Figure 2: Manipulator for Problem 2

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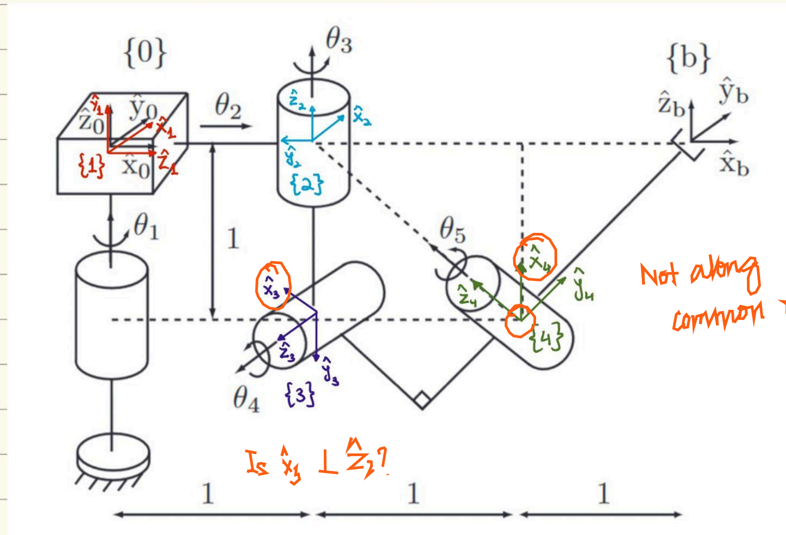
Q.2:)



-1.66

Not along common normal!

-0.135x3

DH Parameters:

Link	a_i	α_i	d_i	θ_i
1	0	90°	0	θ_1
2	0	-90°	θ_2	0
3	0	-90°	-1	θ_3
4	0	-90°	$\sqrt{2}$	θ_4
5	0	90°	$\sqrt{2}$	θ_5

-0.25

-0.5

-1

$$A_1 = {}^0T_1 = \begin{bmatrix} c\theta_1 & 0 & s\theta_1 & 0 \\ s\theta_1 & 0 & -c\theta_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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$$A_2 = {}^1T_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & \theta_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = {}^2T_3 = \begin{bmatrix} c\theta_3 & 0 & -s\theta_3 & 0 \\ s\theta_3 & 0 & c\theta_3 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = {}^3T_4 = \begin{bmatrix} c\theta_4 & 0 & -s\theta_4 & 0 \\ s\theta_4 & 0 & c\theta_4 & 0 \\ 0 & -1 & 0 & \sqrt{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5 = {}^4T_5 = \begin{bmatrix} c\theta_5 & 0 & s\theta_5 & 0 \\ s\theta_5 & 0 & -c\theta_5 & 0 \\ 0 & 1 & 0 & \sqrt{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ −2

<u>Current Values of $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5$:</u>

* $\theta_1 = 90°$ ✓
* $\theta_2 = 0$ x −0.8
* $\theta_3 = 90°$ ✓
* $\theta_4 = 270°$ —
* $\theta_5 = 90°$

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Finding the 0T5 Homogenous Transformation Matrix:

```
syms Q1 Q2 Q3 Q4 Q5

% Define the size of the matrices
matrixSize = 4;

% Initialize matrices separately
A1 = [cos(Q1) 0 sin(Q1) 0; sin(Q1) 0 -cos(Q1) 0; 0 1 0 0; 0 0 0 1];
A2 = [1 0 0 0; 0 0 1 0; 0 -1 0 Q2; 0 0 0 1];
A3 = [cos(Q3) 0 -sin(Q3) 0; sin(Q3) 0 cos(Q3) 0; 0 -1 0 1; 0 0 0 1];
A4 = [cos(Q4) 0 -sin(Q4) 0; sin(Q4) 0 cos(Q4) 0; 0 -1 0 sqrt(2); 0 0 0 1];
A5 = [cos(Q5) 0 sin(Q5) 0; sin(Q5) 0 -cos(Q5) 0; 0 1 0 sqrt(2); 0 0 0 1];

% Store matrices in a cell array
matrices = {A1, A2, A3, A4, A5};

% Perform matrix multiplication
resultMatrix = matrices{1};
for i = 2:5
    resultMatrix = resultMatrix * matrices{i};
end

% Display the matrices
disp('Input Matrices:');
```

```
Input Matrices:
```

```
for i = 1:5
    disp(['A ', num2str(i), ':']);
    disp(matrices{i});
end
```

A 1:

$$\begin{pmatrix} \cos(Q_1) & 0 & \sin(Q_1) & 0 \\ \sin(Q_1) & 0 & -\cos(Q_1) & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

A 2:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & Q_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

A 3:

$$\begin{pmatrix} \cos(Q_3) & 0 & -\sin(Q_3) & 0 \\ \sin(Q_3) & 0 & \cos(Q_3) & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
A 4:
```

$$\begin{pmatrix} \cos(Q_4) & 0 & -\sin(Q_4) & 0 \\ \sin(Q_4) & 0 & \cos(Q_4) & 0 \\ 0 & -1 & 0 & \sqrt{2} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
A 5:
```

$$\begin{pmatrix} \cos(Q_5) & 0 & \sin(Q_5) & 0 \\ \sin(Q_5) & 0 & -\cos(Q_5) & 0 \\ 0 & 1 & 0 & \sqrt{2} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
% Display the result matrix
disp('Result Matrix - 0T5:');
```

```
Result Matrix - 0T5:
```

```
disp(resultMatrix);
```

$$\begin{pmatrix} \sin(Q_5)\,\sigma_2 + \cos(Q_4)\cos(Q_5)\,\sigma_1 & -\sin(Q_4)\,\sigma_1 & \cos(Q_4)\sin(Q_5)\,\sigma_1 - \cos(Q_5)\,\sigma_2 & Q_2\sin(Q_1) - \sqrt{2}\,\sigma_2 - \sqrt{} \\ \cos(Q_4)\cos(Q_5)\,\sigma_2 - \sin(Q_5)\,\sigma_1 & -\sin(Q_4)\,\sigma_2 & \cos(Q_5)\,\sigma_1 + \cos(Q_4)\sin(Q_5)\,\sigma_2 & \sqrt{2}\,\sigma_1 - Q_2\cos(Q_1) - \sqrt{} \\ -\cos(Q_5)\sin(Q_4) & -\cos(Q_4) & -\sin(Q_4)\sin(Q_5) & 1 - \sqrt{2}\,\cos(Q \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where

$$\sigma_1 = \cos(Q_1)\cos(Q_3) - \sin(Q_1)\sin(Q_3)$$

$$\sigma_2 = \cos(Q_1)\sin(Q_3) + \cos(Q_3)\sin(Q_1)$$

Finding the 0T5 Homogenous Transformation Matrix for the Present Configuration:

```
Q1 = 90;
Q2 = 0;
Q3 = 90;
Q4 = 270;
Q5 = 90;

% Define the size of the matrices
matrixSize = 4;

% Initialize matrices separately
A1 = [cos(Q1) 0 sin(Q1) 0; sin(Q1) 0 -cos(Q1) 0; 0 1 0 0; 0 0 0 1];
A2 = [1 0 0 0; 0 0 1 0; 0 -1 0 Q2; 0 0 0 1];
A3 = [cos(Q3) 0 -sin(Q3) 0; sin(Q3) 0 cos(Q3) 0; 0 -1 0 1; 0 0 0 1];
```

```
A4 = [cos(Q4) 0 -sin(Q4) 0; sin(Q4) 0 cos(Q4) 0; 0 -1 0 sqrt(2); 0 0 0 1];
A5 = [cos(Q5) 0 sin(Q5) 0; sin(Q5) 0 -cos(Q5) 0; 0 1 0 sqrt(2); 0 0 0 1];

% Store matrices in a cell array
matrices = {A1, A2, A3, A4, A5};

% Perform matrix multiplication
resultMatrix = matrices{1};
for i = 2:5
    resultMatrix = resultMatrix * matrices{i};
end

% Display the matrices
disp('Input Matrices:');
```

```
Input Matrices:
```

```
for i = 1:5
    disp(['A ', num2str(i), ':']);
    disp(matrices{i});
end
```

```
A 1:
   -0.4481         0    0.8940         0
    0.8940         0    0.4481         0
         0    1.0000         0         0
         0         0         0    1.0000
A 2:
     1     0     0     0
     0     0     1     0
     0    -1     0     0
     0     0     0     1
A 3:
   -0.4481         0   -0.8940         0
    0.8940         0   -0.4481         0
         0   -1.0000         0    1.0000
         0         0         0    1.0000
A 4:
    0.9844         0    0.1760         0
   -0.1760         0    0.9844         0
         0   -1.0000         0    1.4142
         0         0         0    1.0000
A 5:
   -0.4481         0    0.8940         0
    0.8940         0    0.4481         0
         0    1.0000         0    1.4142
         0         0         0    1.0000
```

```
% Display the result matrix
disp('Result Matrix - 0T5:');
```

```
Result Matrix - 0T5:
```

```
disp(resultMatrix);
```

```
   -0.4523   -0.1054   -0.8856    0.9840
    0.8884   -0.1410   -0.4369   -1.0458
```

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```
   -0.0789   -0.9844    0.1574   -0.3921
         0         0         0    1.0000
```

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Question 3

Problem 3
CLO2-C4

45 points

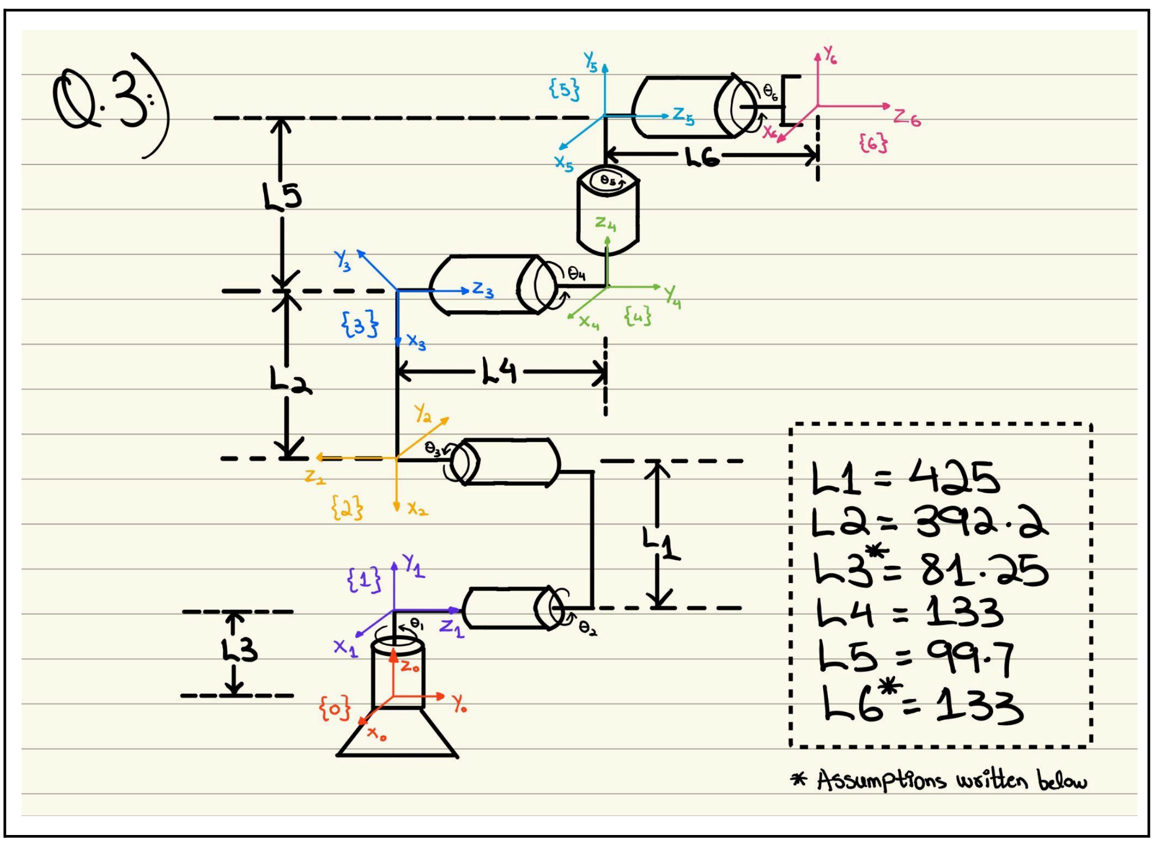
RoboCup is an annual robotics competition, with its finals being held in summer every year, to promote robotics and AI research. While the title comes from a soccer competition modeled after FIFA, Robocup has grown to include a variety of leagues and competitions. RoboCup Autonomous Robot Manipulation (ARM) Challenge, sponsored by Mathworks, has been a recent addition to the competition. This year the challenge will utilize Universal Robotics' UR5e arm, and your task is to determine its forward kinematics. You are to submit:

- schematic of kinematic chain using our class's convention for drawing revolute and prismatic joints schematically;

[1] θ_2 is translation due to prismatic joint.

 2

- a proper assignment of DH frames on the bottom right figure in the specifications sheet;
- a table of link parameters (You may indicate and assume reasonable values for missing information);
- an implementation of a workspace approximation algorithm that calculates an approximate end-effector position workspace of this robot. You should loop through all combinations of joint angles within its joint range, up to a resolution of 10°, and for each configuration calculates the end-effector position using forward kinematics. Output and plot these points. What kind of shape do the points approximate? How densely or sparsely is this shape sampled?



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DH Parameters:

Link	a_i	α_i	d_i	θ_i
1	0	$-90°$	L3	θ_1
2	-L1	$180°$	0	θ_2
3	-L2	$180°$	0	θ_3
4	0	$90°$	L4	θ_4
5	0	$-90°$	L5	θ_5
6	0	$0°$	L6	θ_6

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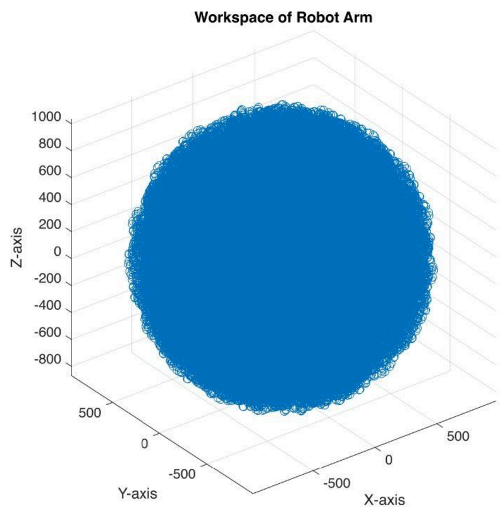
Computing Robot Workspace

```
% Number of joints
num_joints = 6;

% Resolution in degrees
resolution = 72;

% Generate all possible angles
possible_angles = -360:resolution:360;

% Create all combinations using ndgrid
angles_combinations = cell(1, num_joints);
[angles_combinations{:}] = ndgrid(possible_angles);

% Reshape to get all combinations
all_combinations = cell2mat(cellfun(@(x) x(:), angles_combinations,
'UniformOutput', false));

% Initialize transformation matrix and workspace coordinates
transformation_matrices = cell(size(all_combinations, 1), 1);
workspace_coordinates = zeros(size(all_combinations, 1), 3);

% Iterate through all combinations
for i = 1:size(all_combinations, 1)
    % Extract joint angles for the current combination
    current_angles = all_combinations(i, :);

    % Calculate the transformation matrix for the current combination
    transformation_matrix = calculateTransformationMatrix(current_angles);

    % Save transformation matrix
    transformation_matrices{i} = transformation_matrix;

    % Extract position from the transformation matrix
    position = transformation_matrix(1:3, 4)';

    % Save workspace coordinates
    workspace_coordinates(i, :) = position;
end
% Plot workspace coordinates
figure;
plot3(workspace_coordinates(:, 1), workspace_coordinates(:, 2),
workspace_coordinates(:, 3), 'o');
title('Workspace of Robot Arm');
xlabel('X-axis');
ylabel('Y-axis');
zlabel('Z-axis');
grid on;
axis equal;
```



Function for Computing Transformation Matrices (Used Above)

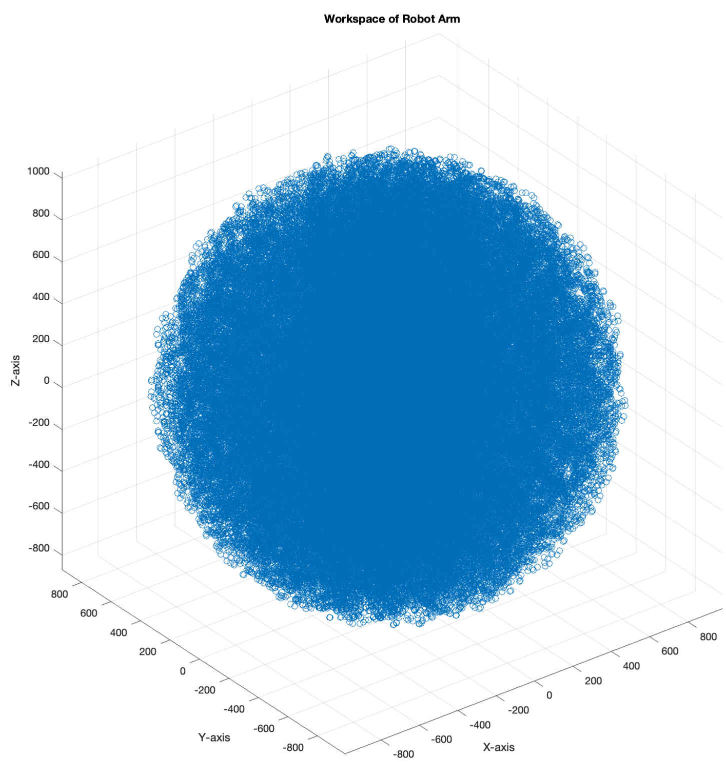
```
function T = calculateTransformationMatrix(angles)
    L1 = 425;
    L2 = 392.2;
    L3 = 81.25;
    L4 = 133;
    L5 = 99.7;
    L6 = 133;

    % DH parameters
    a = [0, L1, L2, 0, 0, 0];
    alpha = [-pi/2, pi, pi, pi/2, -pi/2, 0];
    d = [L3, 0, 0, L4, L5, L6];

    % Initialize the transformation matrix
    T = eye(4);

    % Loop through each joint and update the transformation matrix
    for i = 1:6
        Q = angles(i);
        A = [cos(Q) -sin(Q) * cos(alpha(i)) sin(Q) * sin(alpha(i)) a(i) *
cos(Q);
             sin(Q) cos(Q) * cos(alpha(i)) -cos(Q) * sin(alpha(i)) a(i) *
sin(Q);
```

```
                0 sin(alpha(i)) cos(alpha(i)) d(i);
                0 0 0 1];

           % Multiply the transformation matrix
           T = T * A;
       end
   end
```

Explanation:
We used the resolution of 72 degrees, because even using 36 degrees was proving to be too computationally extensive leading to our matlab getting crashed repeatedly.

When plotting these points, we observed that they formed a **spherical shape**. Interestingly, we noticed a **higher density of points towards the centre**. This suggests that the **robot exhibits greater precision in movements closer to the centre**, as indicated by the larger concentration of plotted points in that region.

I will now attach the plots that substantiate our conclusion. The first plot, depicted below, provides an external perspective, clearly illustrating the spherical formation of the workspace.



Zooming-into the sphere or workspace itself!

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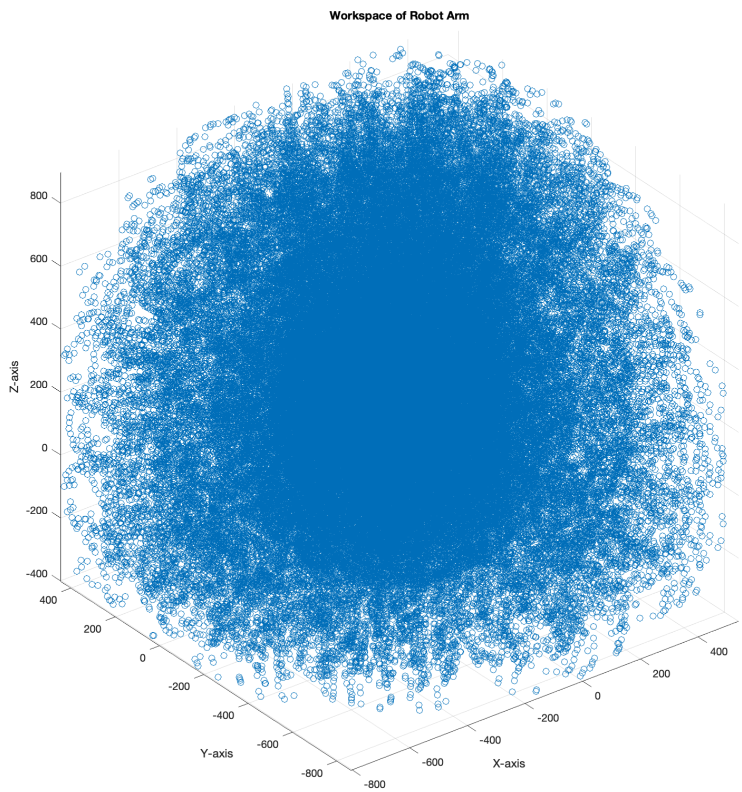
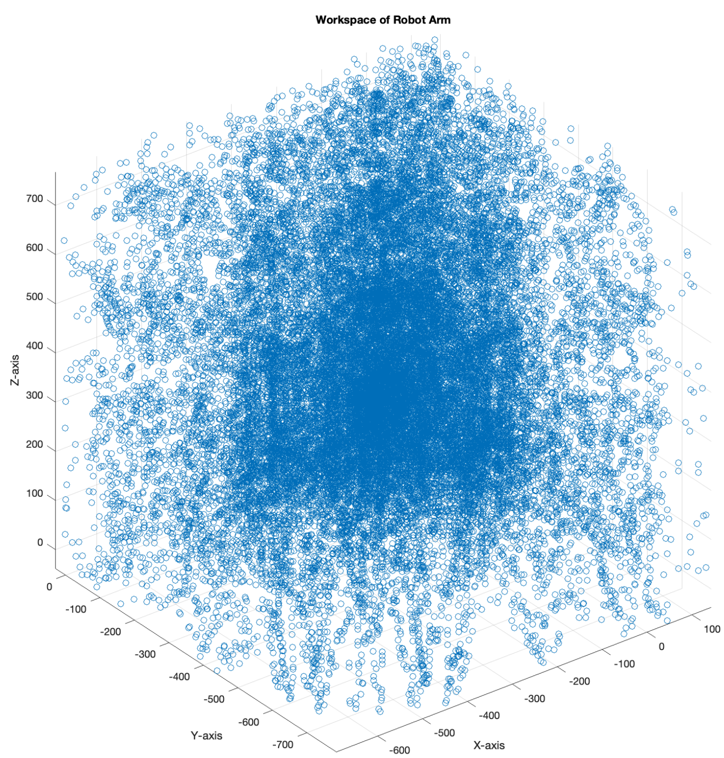


Zooming it Further!



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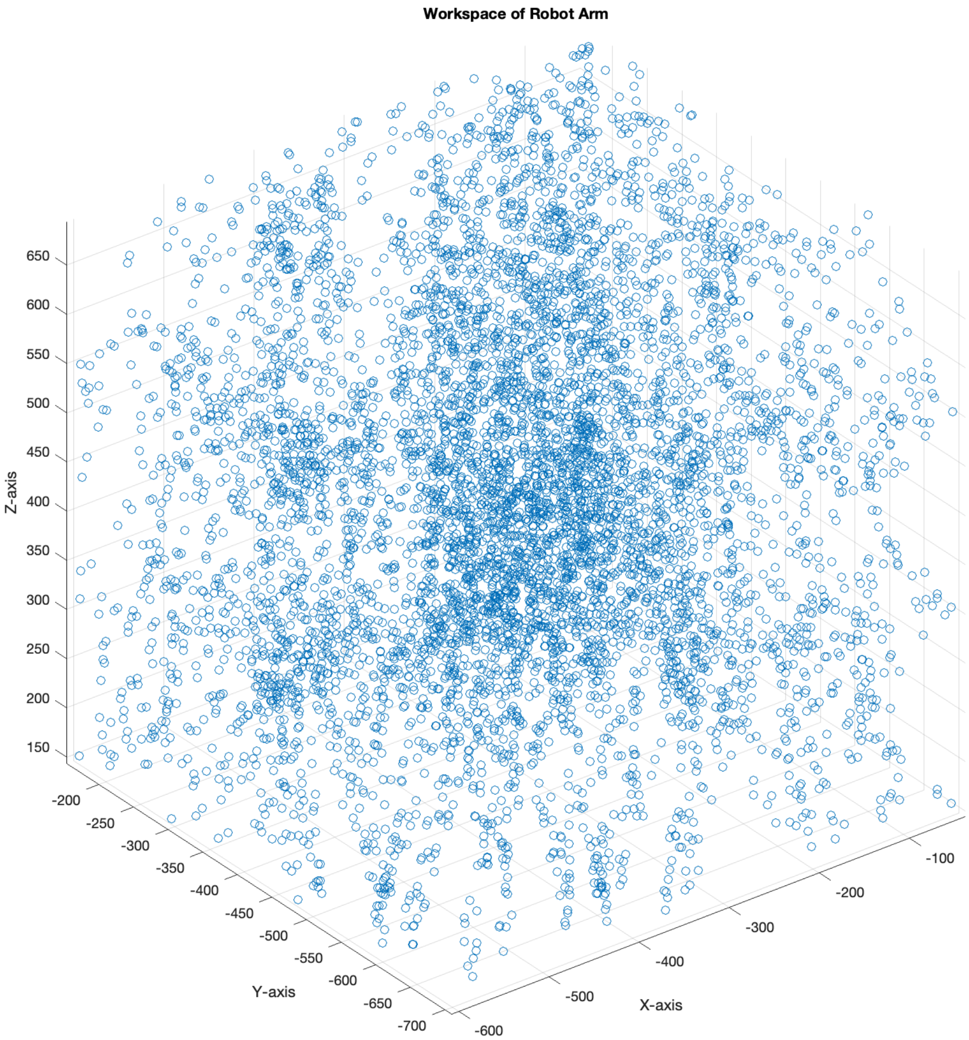
Zooming it even Furtherer!



***Assumptions:**

L3: The length from the base of the zeroth frame till the end of the joint itself is 162.5. Thus, wanting half of the base and half of the revolute joint, we approximated 81.25.

L6: There weren't any direct measurements given. If the diagram is scaled properly, we took the length of 133 cm represented in the diagrams, then compared it to what is supposed to be L6 , and interestingly, it looked like it was of the same length. Thus we are assuming that this link is of 133 cms as well.

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Question 4:

Answer the following questions individually:

Problem 4
CLO2-C2

15 points

(a) How many hours did each of you spend on this homework and specifically state your contribution in this homework assignment? Answer as accurately as you can, as this will be used to structure next year's class.

(b) Do you have any specific advice for students attempting this homework next year?

(c) Each group member is to provide a self-reflection in the form of a note or a concept map. This requires you to reflect on your learning in relation to each of the outcomes stated at the beginning of this document.

Some questions that may help in this regard are: Have I achieved this outcome? What do I currently understand about content related to this outcome? How does it help me understand or build any robot? Do I have unanswered questions? What went wrong? How can I enable myself to achieve this outcome? What could I do to know more or enhance my skills in this context?

(a) How many hours did each of you spend on this homework and specifically state your contribution in this homework assignment? Answer as accurately as you can, as this will be used to structure next year's class.

Asghar:
I spent 10 hours on this homework, with 7 hours being together with Huzaifah solving Question 3 together on an online discord call.

Huzaifah:
I spent 12 hours on this homework, with 7 hours being together with Asghar solving Question 3 together on an online discord call.

(b) Do you have any specific advice for students attempting this homework next year?

Asghar:
Although this is basic advice, I would highly recommend students to brush up on their fundamentals before attempting this homework. It might seem enticing to understand many of the concepts as you go along with the homework, but it will prove to be greatly helpful if you do it with some kind of basic understanding beforehand.

I also believe that having a firm grasp in the Robotics Labs beforehand will greatly help ease this homework. As you will be able to visualise all the configurations as well as map the linkages in your head far more easily. This helped me alot substantially in question 3 when we had to assign frames e.t.c to a real robot hand. And as we had already done so previously (although not completely, our labs aren't all complete sadly), it only made me realise that if those basics were strong, this homework could have taken a lot less time to do.

Huzaifah:
I would like to emphasise conceptual understanding. Better understanding the underlying concepts rather than memorising the formulas as they are, helps you to

adapt to different problem scenarios more effectively. Even if the subject at hand feels like something you memorise, it is in homeworks and lab work you realise how important having strong fundamentals really are.

This is more of a personal note to me, but I would also recommend practising on a timely basis, this would help me greatly in the upcoming labs and future assignments. As can also be observed from our homework, I would stress upon the importance of paying attention to details. When assigning DH frames and calculating transformation matrices, even smaller errors can lead to significant discrepancies in the eventual final results! ~~Cough Question 3 Cough~~

(c) Each group member is to provide a self-reflection in the form of a note or a concept map. This requires you to reflect on your learning in relation to each of the outcomes stated at the beginning of this document.

Some questions that may help in this regard are: Have I achieved this outcome? What do I currently understand about content related to this outcome? How does it help me understand or build any robot? Do I have unanswered questions? What went wrong? How can I enable myself to achieve this outcome? What could I do to know more or enhance my skills in this context?

Asghar:
To be honest, at first, I thought it was kinda tedious to assign frames to the robot. However with time, it became one of my favourite activities. It's really fun when you are able to model the entire possible configuration and movement of a robot with a mathematical expression.
Forward Kinematics so far has definitely been my favourite topic in robotics so far in the course. It encompasses the determination of the position and orientation of a robot manipulator's end-effector based on its joint variables. This involves mathematical representations using transformation matrices and Denavit-Hartenberg (DH) parameters, where-in each link's position and orientation relative to its adjacent link are defined.
Understanding forward kinematics entails

- **grasping the geometric interpretation of joint movements** on the **end-effector's position and orientation**.
- Visualising the kinematic chain, and comprehending coordinate frames and frames of reference.

It also inevitably helps us recognize the interplay between forward and inverse kinematics. While forward kinematics establishes the end-effector's position given joint variables, inverse kinematics calculates the joint variables needed to achieve a desired end-effector position.

Forward Kinematics practical usage is in feedback loop mechanism of a robot where using inverse kinematics it tries to find out the joint angles given a certain position of the end-effector, however it does so when sweeping, when doing so, it repeatedly does forward kinematics where given certain joint angles, it tries finding the predicted "end-effector" position. If the predicted position is not as expected, which is expected in itself ironically given the physical nature of the robot, it re-adjusts its' motor with certain parameters. This is the very reason, forward kinematics is indispensable in the feedback loop mechanism of a robot, particularly in trajectory planning and motion control algorithms.

Thus, Forward-Kinematics and Inverse-Kinematics **together** make a very important pair for a robot's movement.

And as previously mentioned, despite the confidence gained from doing this homework, visualising it and having everything come together in your head to form a visualisation during the lab is pretty difficult. I plan to assign more "larger chunks" of my time to do labs.
Mostly what happens is that I try to do labs within these 1 hour slots of mine which are spent "catching up" before I produce anything useful?

Huzaifah:

Through completing this homework assignment, I believe I have made significant progress towards achieving the learning outcomes outlined in the course syllabus. Specifically, I have developed a deeper understanding of forward kinematics and its application in determining the position of robot manipulators.

I believe that my current understanding of forward kinematics and DH parameters is solid. I feel confident in my ability to apply these concepts to solve problems related to robot manipulators. However, I still believe that there is room to grow, considering how much difficulty I am facing in my lab right now.

Understanding forward kinematics is absolutely crucial for designing and controlling robot manipulators effectively. It allows us to predict and control the motion of robot end-effectors, enabling tasks such as pick-and-place operations, assembly, and manipulation in various industries.

I did encounter challenges during the homework assignment, particularly in accurately assigning DH frames and determining link parameters **ESPECIALLY IN QUESTION 3**. However, taking my sweet time, not hurrying, revising the basic standards of DH conventions with Asghar, we slowly revised the diagram multiple times until we were satisfied that we were able to come to the correct one.

APPENDIX:

Latest version of this document, as well as **livescript code of Q2 and Q4** can be found in this drive folder:

Homework 2

THANK YOU!