Introduction to Robotics - L1

Huzaifah and Asghar

Latest version of this document:

Robotics_HW1_Asghar_Huzaifah

QUESTION 1:

Q1 a) Space containing all possible configurations of robot is called configuration space. it is "n" dimensional space where n is the degrees of freedom.

~~For~~

We have "Two-link planar 2R robot". It has 2 Revolute joints where each joint gives one one degree of rotation as well as one degree of freedom.

Thus, the C-space would be 2-dimensional and would be constrained by set of all valid joint angle combination (θ_1, θ_2).

$$C\text{-space} = \{(\theta_1, \theta_2) \mid \theta_{1(min)} \leq \theta_1 \leq \theta_{1(max)}, \theta_{2(min)} \leq \theta_2 \leq \theta_{2(max)}\}$$

Limits ?

Question 1) a)

```
%Asghar and Huzaifah
%These are the range of joint angles
t1_r = 0:2:360; % 2-degree step-incrementation Theta_1 range
t2_r = 0:2:360;

%Meshgrid for theta1 and theta2
[t1, t2] = meshgrid(deg2rad(t1_r), deg2rad(t2_r));

% x and y values
x = 2*cos(t1) + cos(t1 + t2);
y = 2*sin(t1) + sin(t1 + t2);

% Ploting the C-Space
figure;
scatter(x(:), y(:), 15, 'MarkerFaceColor',
'green','MarkerEdgeColor','green') % Scatter plot with filled circles
title('Robot Configuration Space','Asghar and Huzaifah');
xlabel('x-axis'); ylabel('y-axis');
% equal scaling along both the x and y axes in a plot.
axis equal;
grid on;

hold on;
plot([zeros(size(t1,1), 1), 2*cos(t1)], [zeros(size(t1,1), 1), 2*sin(t1)],
'g-');
plot([2*cos(t1), x], [2*sin(t1), y], 'green');
hold off;
```

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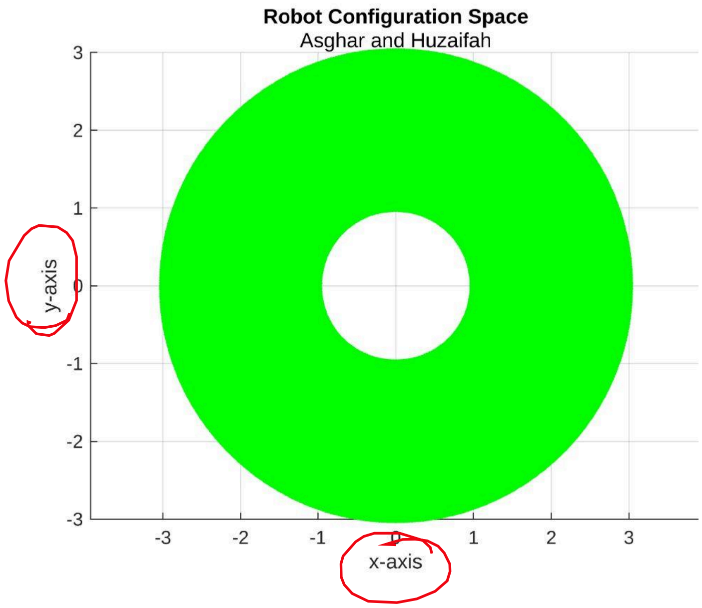

b) Workspace: The specification of the configuration, that the end-effector of the robot can reach.

As we know tip coordinates of x and y;

Workspace $= \{(x,y) \mid x = 2\cos\theta 1 + \cos(\theta 1 + \theta 2),\ y = 2\sin\theta_1 + \sin(\theta_1 + \theta_2),\ \theta_1, \theta_2 \in$

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Question 1) b)

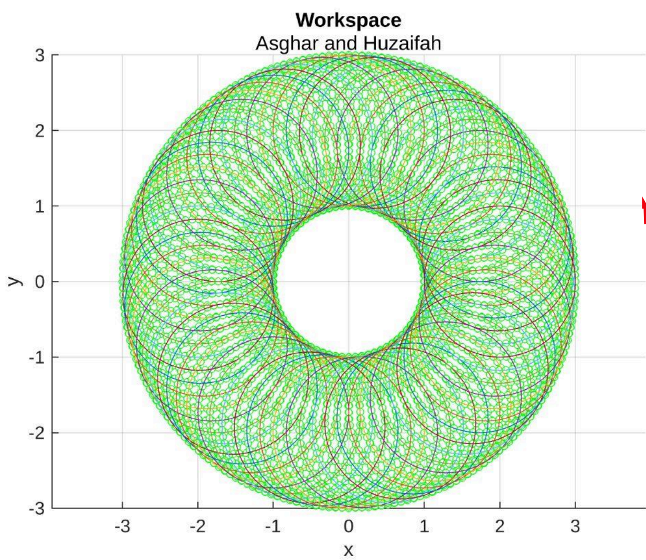
```
t1_r = 0:5:360;
t2_r = 0:5:360;

%Meshgrid for t1 and t2
[t1, t2] = meshgrid(deg2rad(t1_r), deg2rad(t2_r));

x = 2*cos(t1) + cos(t1 + t2);y = 2*sin(t1) + sin(t1 + t2);

figure;
scatter(x(:), y(:), 15, 'MarkerEdgeColor', 'green', 'MarkerFaceColor',
'white', 'MarkerFaceAlpha',0.5);
title('Workspace' , 'Asghar and Huzaifah');
xlabel('x');ylabel('y');axis equal;
grid on;

hold on;
plot([2*cos(t1), x], [2*sin(t1), y]);
plot([zeros(size(t1,1), 1), 2*cos(t1)], [zeros(size(t1,1), 1), 2*sin(t1)]);
hold off;
```



c) The suitable task space for the robot will be 2D plane.
As the task space itself should represent the environment in which the robot will operate and the task at hand it needs to accomplish.
Drawing on paper involves 2D-plane movements.

subset of plane would suffice.

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d) we will restrict the movements of the robot such that it does not collide with any vertical ~~ra~~barriers. We will restrict joint angles θ_1 and θ_2 ~~as~~ & like this,

$x=1$;

$2\cos\theta_1 + \cos(\theta_1+\theta_2) < 1$

$x=-1$;

$2\cos\theta_1 + \cos(\theta_1+\theta_2) > -1$

These constraints define free C-space of robot in presence of vertical barriers

Question 1) d)

```
t1_r = 0:5:360;
t2_r = 0:5:360;

%Meshgrid for t1 and t2
[t1, t2] = meshgrid(deg2rad(t1_r), deg2rad(t2_r));

x = 2*cos(t1) + cos(t1 + t2);y = 2*sin(t1) + sin(t1 + t2);

figure;
scatter(x(:), y(:), 15, 'MarkerEdgeColor', 'green', 'MarkerFaceColor',
'white', 'MarkerFaceAlpha',0.5);
title('Workspace' , 'Asghar and Huzaifah');
xlabel('x-axis');ylabel('y-axis');axis equal;
grid on;

hold on;

%Plotting the vertical barriers!
plot([1, 1], [-4, 4], 'k--'); % Line at x = 1
plot([-1, -1], [-4, 4], 'k--'); % Line at x = -1
```

free C- space = ?

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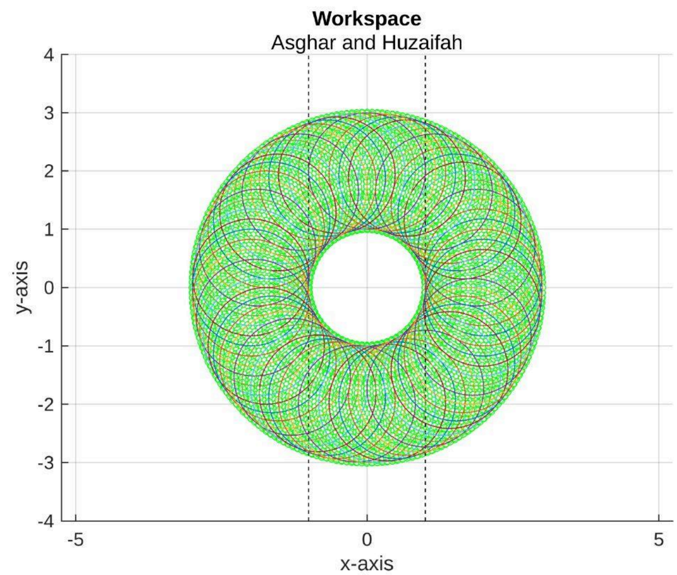
```
plot([2*cos(t1), x], [2*sin(t1), y]);
plot([zeros(size(t1,1), 1), 2*cos(t1)], [zeros(size(t1,1), 1), 2*sin(t1)]);
hold off;
```



Dated:

e) ~~Et~~ Claim is fallacious due to various reasons.
increasing the length of the last link in a planar open chain will not necessarily increase the workspace.
It certainly may extend the reach of the end-effector along certain direction but it will side-by-side introduce limitations and constraints in other directions. This is due to potential collisions imposed by environment. It could also lead to mechanical instability and make robot harder to control. It can introduce self-collision issues. It may also ~~intr~~ introduce singularities in robot so it depends.

Workspace is ~~introduced~~ influenced by range of motion of joints, configuration of the robot and lengths of all other links overall!

QUESTION 2:

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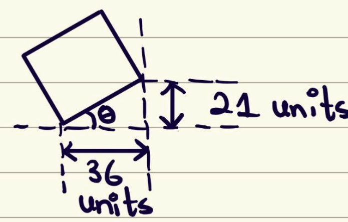
Q.2:) a) We can see from the figure that Red box is rotated about z-axis, and then translated in x any y axes to form the blue box

* Red Box $\longrightarrow \{0\}$
* Blue Box $\longrightarrow \{1\}$

$${}^{0}R_{1} = R_{z,\theta}$$



$$\Rightarrow \tan\theta = \frac{21}{36}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{21}{36}\right)$$

$$\Rightarrow \theta = 30.256^{\circ}$$

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$$
{}^{0}R_{1} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} \cos(30.256) & -\sin(30.256) & 0 \\ \sin(30.256) & \cos(30.256) & 0 \\ 0 & 0 & 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} 0.8638 & -0.5039 & 0 \\ 0.5039 & 0.8638 & 0 \\ 0 & 0 & 1 \end{bmatrix}
$$

* ${}^{0}R_{1}$ captures the rotation of the box. We also need to know translation, to be able to form homogenous transformation matrix.

* Considering the origin point of red box at (0,0), we see that it has moved to (2,1.5) in the blue box. So;

$$
{}^{0}P_{1} = \begin{bmatrix} 2 \\ 1.5 \\ 0 \end{bmatrix}
$$
, hence

$$
{}^{0}T_{1} = \begin{bmatrix} {}^{0}R_{1} & {}^{0}P_{1} \\ 0 & 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} 0.8638 & -0.5039 & 0 & 2 \\ 0.5039 & 0.8638 & 0 & 1.5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

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c) The frame attached to the blue frame in figure 3, defines the same pose which can be modelled by the homogenous transformation calculated in part (a)

$$\begin{bmatrix} 0.8638 & -0.5039 & 0 & 2 \\ 0.5039 & 0.8638 & 0 & 1.5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

d) Both points P_1 and P_2 in figure 3, can be expressed in both Red ($\{0\}$) and blue ($\{1\}$) box frames.

<u>For P_1</u>

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$${}^{0}P_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \longrightarrow P_1 \text{ in Red Box Frame}$$

$${}^{1}P_1 = \left({}^{1}T_0\right) {}^{0}P_1 \qquad P_1 \text{ in Blue Box Frame}$$

We first need to find ${}^{1}T_0$ to get ${}^{1}P_1$

$$\Rightarrow {}^{1}T_0 = \left({}^{0}T_1\right)^{-1}$$

$$= \left(\begin{bmatrix} 0.8638 & -0.5039 & 0 & 2 \\ 0.5039 & 0.8638 & 0 & 1.5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\right)^{-1}$$

Directly evaluating using matrix calculator;

$${}^{1}T_0 = \begin{bmatrix} 0.8638 & 0.5039 & 0 & -2.483 \\ -0.5039 & 0.8638 & 0 & -0.287 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{1}P_1 = \left({}^{1}T_0\right) {}^{0}P_1$$

$$= \begin{bmatrix} 0.8638 & 0.5039 & 0 & -2.483 \\ -0.5039 & 0.8638 & 0 & -0.287 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1.619 \\ -0.791 \\ 0 \\ 1 \end{bmatrix}$$

For P_2

$$^1P_2 = (^1T_0)\,^0P_2$$

$\downarrow$ P2 in Blue Box Frame

So we need $^0P_2$ for finding $^1P_2$.

$$^0P_2 = \begin{bmatrix} 2.865 \\ 2 \\ 0 \end{bmatrix} \rightarrow$$ P2 in Red Box Frame (Directly from Figure)

Now using this value of $^0P_2$, to find $^1P_2$.

$$^1P_2 = \begin{bmatrix} 0.8638 & 0.5039 & 0 & -2.483 \\ -0.5039 & 0.8638 & 0 & -0.287 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2.865 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

e) $^BP_1 = {}^1P_1$, if we know $^BP_2$ ($^1P_2$) and $^0T_1$ (^RT_B), we can find $^BP_1$ as;

$$\Rightarrow {}^BP_1 = (^RT_B)^{-1}(^RP_1) \quad \text{as } {}^RP_1 = {}^BP_2$$

$$\Rightarrow {}^BP_1 = (^RT_B)^{-1}(^BP_2)$$

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f) $^{B}P_2 = {}^{1}P_2$, if we know $^{R}P_2$ ($^{0}P_2$) and $^{0}T_1$ ($^{R}T_B$), we can find $^{B}P_2$ as;

$$\Rightarrow {}^{B}P_2 = \left({}^{R}T_B\right)^{-1}\left({}^{R}P_2\right)$$

as $^{B}T_R = \left({}^{R}T_B\right)^{-1}$

EE/CE/CS 366/366/380: Homework 1

Question 2 Part B

```
% Generating points for square
% You don't need to edit this
delta = 0.01;                        % Distance between consecutive points
l = 1;                               % Length of square
N = l/delta*4;                       % Number of points
pts = zeros(4,N);                    % 1st row has x coordinates and 2nd row
has y coordinates, 3rd row z coordinates
pts(1,1:N/4) = 0:delta:l-delta;
pts(1,N/4+1:N/2) = l;
pts(2,N/4+1:N/2) = 0:delta:l-delta;
pts(1,N/2+1:3*N/4) = l:-delta:delta;
pts(2,N/2+1:3*N/4) = l;
pts(1,3*N/4+1:N) = 0;
pts(2,3*N/4+1:N) = l:-delta:delta;
pts(3,:) = 0;                        % Z-coordinate is zero for all
pts(4,:) = 1;                        % Converting coordinates to homogenous
coordinates


% Draw original square
% You don't need to edit this
clf;
plot(pts(1,:),pts(2,:),'red','LineWidth',2);
% Set the axes limits
xlim([0 3.3])
ylim([0 3.2])
zlim([0 2]);
% Grid Options
grid on;
grid minor;
axis square;

% Displacing the square
% Homogeneous coordinates of the new points should be placed in the
% variable pts2. Points are column in pts2 and in the same order as in pts,
% i.e. 1st column of pts2 is the new location of point in the 1st column of
% pts, with first row being x-coordinate, second the y-coordinate, third
% z-coordinate, and the fourth row entry is 1.

% Write your code here
% The transformation matrix 0T1 or RTB is expressed below
RTB = [0.86377 -0.50387 0 2; 0.50387 0.86377 0 1.5; 0 0 1 0; 0 0 0 1]
```

```
RTB = 4x4
    0.8638   -0.5039         0    2.0000
    0.5039    0.8638         0    1.5000
```

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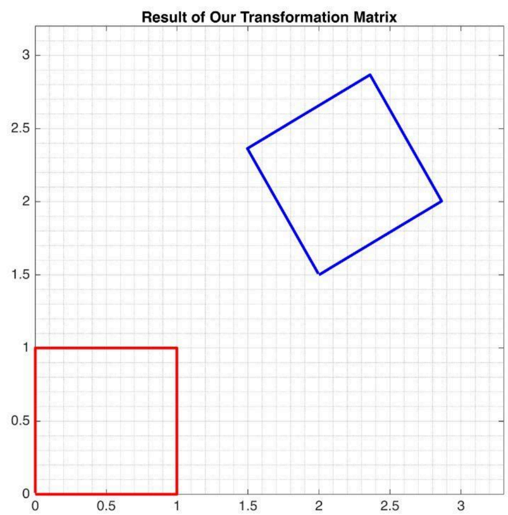
```
         0         0    1.0000         0
         0         0         0    1.0000
```

```
ptsBlue = RTB * pts; % This will create the coordinates for Blue Box.

hold on;
plot(ptsBlue(1,:),ptsBlue(2,:),'blue','LineWidth',2);
title("Result of Our Transformation Matrix")
```



QUESTION 3:

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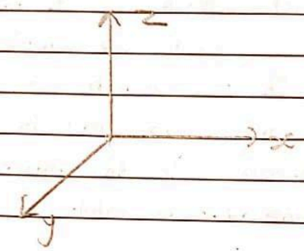
Q3 Fixed frame = {0}
Moving frame = {1} (initially aligned with {0})

a) Frame {0} ⇒



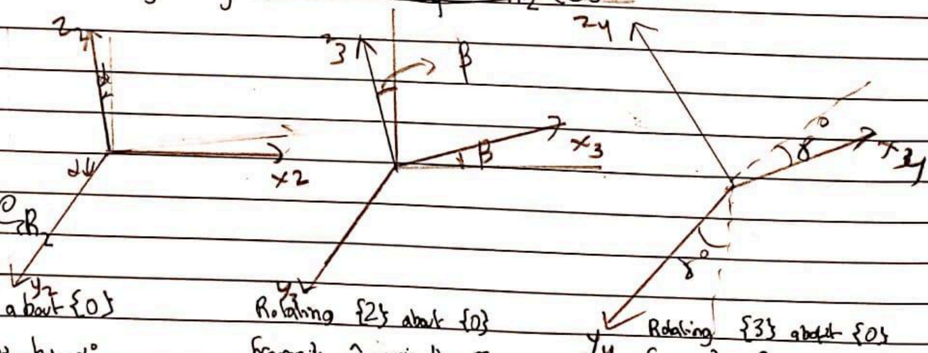
Rotational Matrices =

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \quad R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$^0R_2 = R_x(\theta) \quad ^0R_3 = R_y(\theta) \quad ^0R_4 = R_z(\theta)$

$^0R_4 \, ^0R_3 \, ^0R_2$



Rotating {1} about {0} frame's x-axis by α° so we get {2}

Rotating {2} about {0} frame's $\hat{y}$ axis by β° so we get {3}

Rotating {3} about {0} frame's $\hat{z}$ axis by γ°. we get {4} frame

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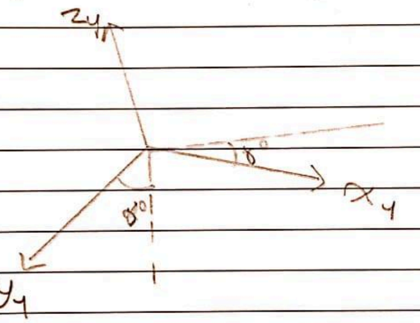
Using Left ~~Mat~~ Multiplication.

$$ {}^0R_4 = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \cos\alpha & \cos\alpha \end{bmatrix} $$

b) After replacing the third statement, ~~the~~ following changes would take place.



Final Orientation will be:

$$ {}^0R_4 = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} $$

c) ~~Rotate~~

Rotate {1} about the {1} frame y-axis by β; call this frame {2}

Rotate {2} about the {2} frame x-axis by α; call this frame {3}

Rotate {3} about the {3} frame z-axis by γ; call this frame {4}

QUESTION 4:

Q.4:) a) If we attach frame {1} at origin of frame {0}, this initial orientation {1}' with respect to {0} will be;

-1 for right handed.

$$ {}^{0}\tilde{T}_{1'} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -\cos(30) & -\sin(30) & 0 & 0 \\ -\sin(30) & \cos(30) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

* {1}' is rotated about $\hat{y}_1$ axis with angular velocity $\frac{V_1}{R_1}$,

V_1 is angular velocity.

Hence

$$ {}^{0}T_{1'} = {}^{0}\tilde{T}_{1'} \begin{bmatrix} \cos\left(\frac{V_1}{R_1}t\right) & 0 & \sin\left(\frac{V_1}{R_1}t\right) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\left(\frac{V_1}{R_1}t\right) & 0 & \cos\left(\frac{V_1}{R_1}t\right) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ = \begin{bmatrix} \sin\left(\frac{V_1}{R_1}t\right) & 0 & -\cos\left(\frac{V_1}{R_1}t\right) & 0 \\ -\frac{\sqrt{3}}{2}\cos\left(\frac{V_1}{R_1}t\right) & -1/2 & -\frac{\sqrt{3}}{2}\sin\left(\frac{V_1}{R_1}t\right) & 0 \\ -\frac{1}{2}\cos\left(\frac{V_1}{R_1}t\right) & \sqrt{3}/2 & -\frac{1}{2}\sin\left(\frac{V_1}{R_1}t\right) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Position of {1} in {1}' is $\begin{bmatrix} -R1 \\ 0 \\ 0 \end{bmatrix}^T$

Hence,

how?

$$
{}^{0}T_{1} = \begin{bmatrix} \sin\left(\frac{v_1}{R_1}t\right) & 0 & -\cos\left(\frac{v_1}{R_1}t\right) & -R_1\sin\left(\frac{v_1}{R_1}t\right) \\ -\frac{\sqrt{3}}{2}\cos\left(\frac{v_1}{R_1}t\right) & -\frac{1}{2} & -\frac{\sqrt{3}}{2}\sin\left(\frac{v_1}{R_1}t\right) & \frac{\sqrt{3}}{2}R_1\cos\left(\frac{v_1}{R_1}t\right) \\ -\frac{1}{2}\cos\left(\frac{v_1}{R_1}t\right) & \frac{\sqrt{3}}{2} & -\frac{1}{2}\sin\left(\frac{v_1}{R_1}t\right) & \frac{R_1}{2}\cos\left(\frac{v_1}{R_1}t\right) \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Similarly for $\{2\}$,

$$
{}^{0}\tilde{T}_{2'} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$\{2\}'$ is rotated about $\hat{y}_2$ axis with angular velocity, $\frac{v_2}{R_2}$

$$
{}^{0}T_{2'} = {}^{0}\tilde{T}_{2'} \begin{bmatrix} \cos\left(\frac{v_2}{R_2}t\right) & 0 & \sin\left(\frac{v_2}{R_2}t\right) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\left(\frac{v_2}{R_2}t\right) & 0 & \cos\left(\frac{v_2}{R_2}t\right) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Therefore;

$$
{}^{0}T_{2} = \begin{bmatrix} \sin\left(\frac{v_2}{R_2}t\right) & 0 & -\cos\left(\frac{v_2}{R_2}t\right) & -R_2\sin\left(\frac{v_2}{R_2}t\right) \\ -\cos\left(\frac{v_2}{R_2}t\right) & 0 & -\sin\left(\frac{v_2}{R_2}t\right) & R_2\cos\left(\frac{v_2}{R_2}t\right) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

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Question 4 Part b

$$ {}^{1}T_{2} = \left({}^{0}T_{1}\right)^{-1}\left({}^{0}T_{2}\right) $$

$$ \Rightarrow {}^{1}T_{2} = \left(\begin{bmatrix} \sin\left(\frac{V_1}{R_1}t\right) & 0 & -\cos\left(\frac{V_1}{R_1}t\right) & -R_1\sin\left(\frac{V_1}{R_1}t\right) \\ -\frac{\sqrt{3}}{2}\cos\left(\frac{V_1}{R_1}t\right) & -\frac{1}{2} & -\frac{\sqrt{3}}{2}\sin\left(\frac{V_1}{R_1}t\right) & \frac{\sqrt{3}}{2}R_1\cos\left(\frac{V_1}{R_1}t\right) \\ -\frac{1}{2}\cos\left(\frac{V_1}{R_1}t\right) & \frac{\sqrt{3}}{2} & -\frac{1}{2}\sin\left(\frac{V_1}{R_1}t\right) & \frac{R_1}{2}\cos\left(\frac{V_1}{R_1}t\right) \\ 0 & 0 & 0 & 1 \end{bmatrix}\right)^{-1} \left(\begin{bmatrix} \sin\left(\frac{V_2}{R_2}t\right) & 0 & -\cos\left(\frac{V_2}{R_2}t\right) & -R_2\sin\left(\frac{V_2}{R_2}t\right) \\ -\cos\left(\frac{V_2}{R_2}t\right) & 0 & -\sin\left(\frac{V_2}{R_2}t\right) & R_2\cos\left(\frac{V_2}{R_2}t\right) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\right) $$

Now we solve this below:

```
syms x; % Represents (V1/R1)*t
syms y; % Represents (V2/R2)*t
syms d; % Represents R1
syms h; % Represents R2

% Define a 4x4 matrix A
A = [sin(x), 0, -cos(x), -d*sin(x);
     -(sqrt(3)/2)*cos(x), -1/2, -(sqrt(3)/2)*sin(x), (sqrt(3)/2)*d*cos(x);
     (-1/2)*cos(x), sqrt(3)/2, -(1/2)*sin(x), (1/2)*d*cos(x);
     0, 0, 0, 1];

% Define another 4x4 matrix B (with symbolic elements)
B = [sin(y), 0, -cos(y), -h*sin(y);
    -cos(y), 0, -sin(y), h*cos(y);
    0, 1, 0, 0;
    0, 0, 0, 1];

% Check if the matrices A and B are invertible
if det(A) ~= 0
    % If the matrices are invertible, find their symbolic inverses
    A_inv = inv(A);

    % Perform matrix multiplication
    result_matrix = A_inv * B;

    % Display the original matrices, their symbolic inverses, and the result
    disp('Original Matrix A:');
    disp(A);

    disp('Symbolic Inverse of Matrix A:');
    disp(A_inv);

    disp('Matrix B:');
```

```
    disp(B);

    disp('Result of A_inv * B:');
    disp(result_matrix);
else
    disp('Matrix A is singular and not invertible.');
end
```

```
Original Matrix A:
```

$$\begin{pmatrix} \sin(x) & 0 & -\cos(x) & -d\sin(x) \\ -\frac{\sqrt{3}\ \cos(x)}{2} & -\frac{1}{2} & -\frac{\sqrt{3}\ \sin(x)}{2} & \frac{\sqrt{3}\ d\cos(x)}{2} \\ -\frac{\cos(x)}{2} & \frac{\sqrt{3}}{2} & -\frac{\sin(x)}{2} & \frac{d\cos(x)}{2} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
Symbolic Inverse of Matrix A:
```

$$\begin{pmatrix} \frac{\sin(x)}{\sigma_1} & -\frac{\sqrt{3}\ \cos(x)}{2\,\sigma_1} & -\frac{\cos(x)}{2\,\sigma_1} & d \\ 0 & -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{\cos(x)}{\sigma_1} & -\frac{\sqrt{3}\ \sin(x)}{2\,\sigma_1} & -\frac{\sin(x)}{2\,\sigma_1} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where

$$\sigma_1 = \cos(x)^2 + \sin(x)^2$$

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Matrix B:
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$$\begin{pmatrix} \sin(y) & 0 & -\cos(y) & -h\sin(y) \\ -\cos(y) & 0 & -\sin(y) & h\cos(y) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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Result of A_inv * B:
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$$\begin{pmatrix} \frac{\sin(x)\sin(y)}{\sigma_1} + \frac{\sqrt{3}\ \cos(x)\cos(y)}{2\,\sigma_1} & -\frac{\cos(x)}{2\,\sigma_1} & \frac{\sqrt{3}\ \cos(x)\sin(y)}{2\,\sigma_1} - \frac{\cos(y)\sin(x)}{\sigma_1} & d - \frac{h\sin(x)\sin(y)}{\sigma_1} - \frac{\sqrt{3}\ \cdots}{\cdots} \\ \frac{\cos(y)}{2} & \frac{\sqrt{3}}{2} & \frac{\sin(y)}{2} & -\frac{h\cos(y)}{2} \\ \frac{\sqrt{3}\ \cos(y)\sin(x)}{2\,\sigma_1} - \frac{\cos(x)\sin(y)}{\sigma_1} & -\frac{\sin(x)}{2\,\sigma_1} & \frac{\cos(x)\cos(y)}{\sigma_1} + \frac{\sqrt{3}\ \sin(x)\sin(y)}{2\,\sigma_1} & \frac{h\cos(x)\sin(y)}{\sigma_1} - \frac{\sqrt{3}\ h\cdots}{\cdots} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where

$$\sigma_1 = \cos(x)^2 + \sin(x)^2$$

QUESTION 6:

Introduction to Robotics - L1

Huzaifah and Asghar

a) We both have given this homework around 13 hours. Although both of us have contributed to answer all four questions, Asghar has contributed more in Q1 and Q3, while Huzaifah have contributed more to doing Q2 and Q4.

b) We would advise them to make sure that they have revised the lecture material and book properly before attempting the questions. We would also advise them to spare ample amount of time for this homework as it isn’t something you leave for the final day.

c) **Huzaifah Tariq Ahmed:** I have successfully grasped the concept of manipulating rotation matrices and homogeneous transformations to describe frames and perform rigid body motions. Understanding these concepts has been crucial in my ability to model physical situations involving multiple rigid objects through the composition of homogeneous transforms. However, I find myself struggling at times to convert between implicit and explicit representations of orientation. This aspect requires more practice and exploration on my part. I plan to address this gap by revisiting relevant exercises and seeking additional resources to strengthen my understanding.

Syed Asghar Abbas Zaidi: My comprehension of manipulating rotation matrices and homogeneous transformations for describing frames and executing rigid body motions has grown considerably. This knowledge has proven valuable in modeling scenarios with multiple rigid objects through the composition of homogeneous transforms. While I can handle explicit representations of orientation well, a challenge I've identified is applying these concepts to real-world scenarios. I aim to enhance my skills by working on practical projects or case studies that involve implementing rotation matrices and homogeneous transformations in a broader context.