Practice Questions

Topic: **Normal Distribution**

1	For some computers, the time period between charges of the battery is normally distributed with a mean of 50 hours and a standard deviation of 15 hours. Rohan has one of these computers and needs to know the probability that the time period will be between 50 and 70 hours. Solution: Let x be the random variable that represents the time period. Given Mean, $\mu = 50$ and standard deviation, $\sigma = 15$ To find: Pprobability that x is between 50 and 70 or $P(50 < x < 70)$ By using the transformation equation, we know; $z = (X - \mu) / \sigma$ For $x = 50$, $z = (50 - 50) / 15 = 0$ For $x = 70$, $z = (70 - 50) / 15 = 1.33$ $P(50 < x < 70) = P(0 < z < 1.33)$ = [area to the left of $z = 1.33$] – [area to the left of $z = 0$] From the table we get the value, such as; $P(0 < z < 1.33) = 0.9082 - 0.5 = 0.4082$ The probability that Rohan's computer has a time period between 50 and 70 hours is equal to 0.4082.
2	The speeds of cars are measured using a radar unit, on a motorway. The speeds are normally distributed with a mean of 90 km/hr and a standard deviation of 10 km/hr. What is the probability that a car selected at chance is moving at more than 100 km/hr? Solution: Let the speed of cars is represented by a random variable 'x'. Now, given mean, $\mu = 90$ and standard deviation, $\sigma = 10$. To find: Probability that x is higher than 100 or $P(x > 100)$ By using the transformation equation, we know; $z = (X - \mu) / \sigma$ Hence, For $x = 100$, $z = (100 - 90) / 10 = 1$ $P(x > 90) = P(z > 1)$ = [total area] – [area to the left of $z = 1$] $P(z > 1) = 1 - 0.8413 = 0.1587$

3

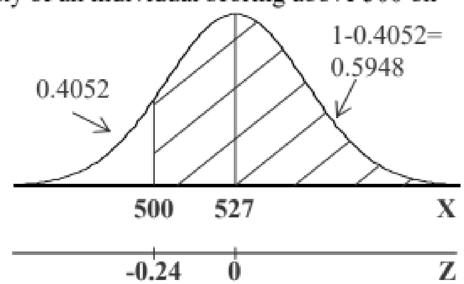
Most graduate schools of business require applicants for admission to take the Graduate Management Admission Council's GMAT examination. Scores on the GMAT are roughly normally distributed with a mean of 527 and a standard deviation of 112. What is the probability of an individual scoring above 500 on the GMAT?

Normal Distribution $$Z = \frac{500-527}{112} = -0.24107$$

$\mu = 527$

$\sigma = 112$

$\Pr\{X > 500\} = \Pr\{Z > -0.24\} = 1 - 0.4052 = \boxed{0.5948}$



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How high must an individual score on the GMAT in order to score in the highest 5%?

Normal Distribution

$\mu = 527$

$\sigma = 112$

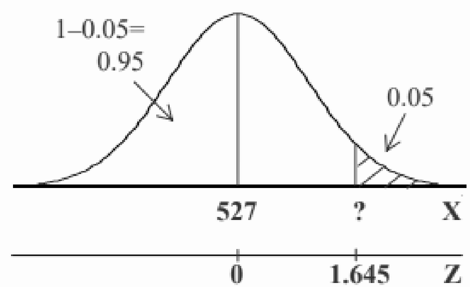
$P(X > ?) = 0.05 \Rightarrow P(Z > ?) = 0.05$

$P(Z < ?) = 1 - 0.05 = 0.95 \Rightarrow Z = 1.645$

$X = 527 + 1.645(112)$

$X = 527 + 184.24$

$X = \boxed{711.24}$



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The length of human pregnancies from conception to birth approximates a normal distribution with a mean of 266 days and a standard deviation of 16 days. What proportion of all pregnancies will last between 240 and 270 days (roughly between 8 and 9 months)?

Normal Distribution $$Z = \frac{240-266}{16} = -1.625$$

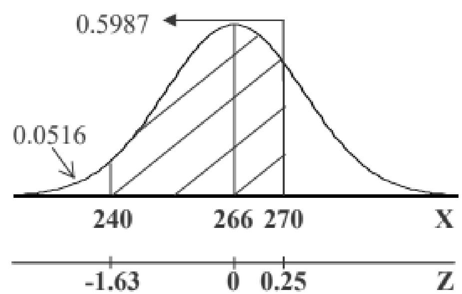
$\mu = 266$ $$Z = \frac{270-266}{16} = 0.25$$

$\sigma = 16$

$P(240 < X < 270) = P(-1.63 < Z < 0.25)$

$P(-1.63 < Z < 0.25) = P(Z < 0.25) - P(Z < -1.63)$

$P(-1.63 < Z < 0.25) = 0.5987 - 0.0516 = \boxed{0.5471}$



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What length of time marks the shortest 70% of all pregnancies?

Normal Distribution

$\mu = 266$

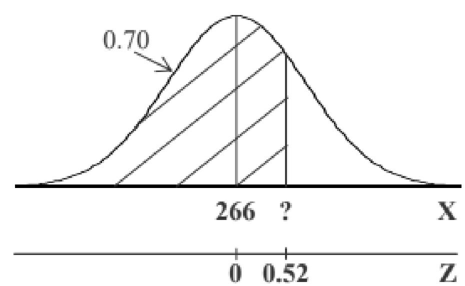
$\sigma = 16$

$P(X < ?) = 0.70 \Rightarrow P(Z < ?) = 0.70 \Rightarrow Z = 0.52$

$X = 266 + 0.52(16)$

$X = 266 + 8.32$

$X = \boxed{274.32}$



7 The average number of acres burned by forest and range fires in a large New Mexico county is 4,300 acres per year, with a standard deviation of 750 acres. The distribution of the number of acres burned is normal. What is the probability that between 2,500 and 4,200 acres will be burned in any given year?

Normal Distribution $Z = \dfrac{2500 - 4300}{750} = -2.40$

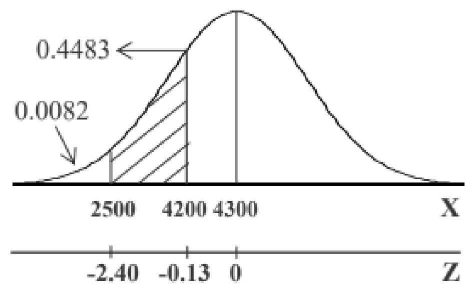
$\mu = 4300$ $Z = \dfrac{4200 - 4300}{750} = -0.13333$

$\sigma = 750$

P(2500 < X < 4200) = P(-2.40 < Z < -0.13)

P(-2.40 < Z < -0.13) = P(Z < -0.13) - P(Z < -2.40)

P(-2.40 < Z < -0.13) = 0.4483 - 0.0082 = $\boxed{\mathbf{0.4401}}$



8 What number of burnt acres corresponds to the 38th percentile?

Normal Distribution

$\mu = 4300$

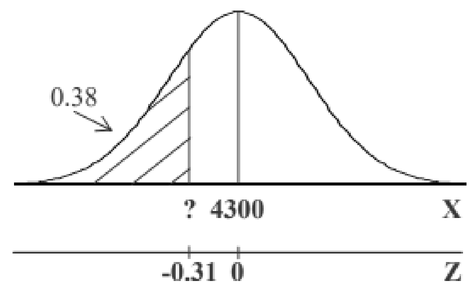
$\sigma = 750$

P(X < ?) = 0.38 ⇒ P(Z < ?) = 0.38 ⇒ Z = -0.31

X = 4300 + (-0.31)(750)

X = 4300 – 232.5

X = $\boxed{\mathbf{4067.5}}$



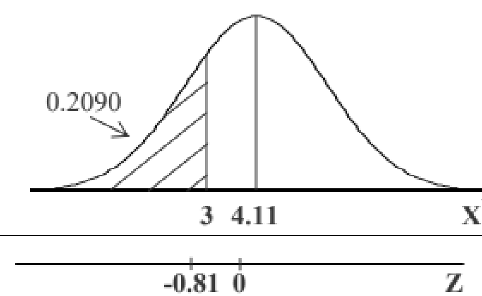
9 The Edwards's Theater chain has studied its movie customers to determine how much money they spend on concessions. The study revealed that the spending distribution is approximately normally distributed with a mean of $4.11 and a standard deviation of $1.37. What percentage of customers will spend less than $3.00 on concessions?

Normal Distribution $Z = \dfrac{3.00 - 4.11}{1.37} = -0.81021$

$\mu = 4.11$

$\sigma = 1.37$

P(X < 3.00) = P(Z < -0.81) = 0.2090 ⇒ $\boxed{\mathbf{20.9\%}}$



10 What spending amount corresponds to the top 87th percentile?

Normal Distribution

$\mu = 4.11$

$\sigma = 1.37$

P(X > ?) = 0.87 ⇒ P(Z > ?) = 0.87

P(Z > ?) = 0.87 ⇒ P(Z < ?) = 1 - 0.87 = 0.13 ⇒ Z = -1.13

X = 4.11 + (-1.13)(1.37)

X = 4.11 – 1.5481

X = 2.5619

X = $\boxed{\mathbf{\$2.56}}$