Planes in R^3

Monday, 3 March 2025 10:04 am

$R^3 \rightarrow$ 3-dimension

$R^2 \rightarrow$ 2-dim

$R^n \rightarrow$ n-dim

$(x, y) = (\sin t, \cos t)$

0,0

$0 \xrightarrow{t} 10$

0.1

Ways to describe Planes

Monday, 3 March 2025 10:05 am

1. **Equational form**: A set of points (x, y, z) satisfying an equation $ax + by + cz = d$ where at least one of a, b, c is nonzero
2. **Parametric form**: A set of points expressed as $P + te + t'e'$ for varying scalars t, t' (where P is a point in the plane and e, e' are displacement vectors in the plane)
3. **Point and normal vector form**: A plane determined by a point P in the plane and a normal vector $\vec{n}$ perpendicular to the plane
4. **Three points**: A plane determined by three non-collinear points

Example 1

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Consider the plane

$$x + 2y + 3z = 4$$

a. Find any 4 points A, B, C, D on plane
b. Find vectors AB, AC, AD
c. The normal vector to plane is n= (1,2,3). Find $AB.n, AC.n, AD.n$

a) $x = 1, \quad y = 2 \quad z = -1/3$

$$\overbrace{1 + \underbrace{2(2)}_{4}}^{5} + \overbrace{3(?)}^{-1} = 4 \qquad -\frac{1}{3}$$

$A = (1, 2, -\frac{1}{3})$

$B = (2, 1, 0)$

$C = (1, 0, 1)$

$D = (0, 2, 0)$

b) $\vec{AB} = B - A = (2-1, 1-2, 0+\frac{1}{3}) = (1, -1, \frac{1}{3})$

Example 2

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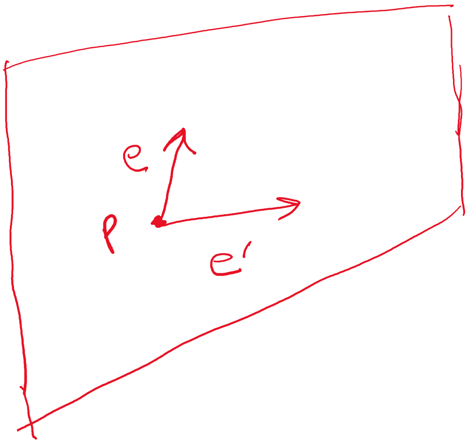
Find parametric form of plane with

$P = (1,0,1)$ and displacement vectors $e = (2,-1,0)$ and $e' = (0,3,-2)$

$$(x,y,z) = P + \underset{s}{\overset{v_1}{\cancel{t}\,\cancel{e}}} + \underset{t}{\overset{v_2}{\cancel{t'}\,\cancel{e'}}}$$

$$= \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + t\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} + t'\begin{bmatrix} 0 \\ 3 \\ -2 \end{bmatrix} = \begin{bmatrix} 1+2t \\ -t+3t' \\ 1-2t' \end{bmatrix} \qquad \begin{matrix} t \\ t' \end{matrix}$$



Let's check if e & e' are collinear.

$$\cos\theta = \frac{e \cdot e'}{\|e\| \cdot \|e'\|} = \frac{-3}{\sqrt{5}\cdot\sqrt{13}} \neq 1$$

Example 3

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Find the equation of the plane with

$P = (2,3,1)$ and $n = (4, -2, 5)$

$G = (x, y, z)$

a. Find PG

b. Find $PG . n = 0$

$\vec{PG} = G - P = (x-2, y-3, z-1)$

$\vec{PG} \cdot \vec{n} = 0$

$4(x-2) - 2(y-3) + 5(z-1) = 0$

$\boxed{4x - 2y + 5z = 7}$

Example 4

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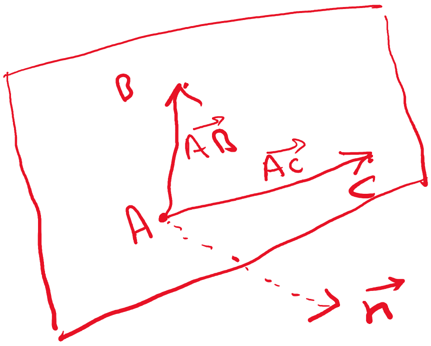
Find the equation of plane containing points
$A = (1,2,3), \quad B = (2,3,1), \quad C = (3,1,2)$

$ax + by + cz = d$

a. Find AB, AC
b. Find $n = AB \times AC$
c. Find $PG.n = 0$



$\vec{AB} = (1, 1, -2)$

$\vec{AC} = (2, -1, -1)$

$\vec{n} = \vec{AB} \times \vec{AC}$

$= \begin{vmatrix} i & j & k \\ 1 & 1 & -2 \\ 2 & -1 & -1 \end{vmatrix} = i(-1-2) - j(-1+4) + k(-1-2)$

$= -3i - 3j - 3k = (-3, -3, -3)$

$P = A = (1, 2, 3)$ $\quad G = (x, y, z)$

$\vec{PG} = (x-1, y-2, z-3)$

$\vec{PG} \cdot \vec{n} = -3(x-1) - 3(y-2) - 3(z-3) = 0$

$-3x + 3 - 3y + 6 - 3z + 9 = 0$

$-3x - 3y - 3z = -18$ $\quad \vec{n} = (-3, -3, -3)$

$\boxed{x + y + z = 6}$ $\quad \vec{n} = (1, 1, 1)$

Problem 1

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Find the equation of the plane containing the points

$$A = (0,2,3)$$
$$B = (1,0,-1)$$
$$C = (4,1,2)$$

Problem 2

Monday, 3 March 2025 10:47 am

Find the equation of the plane containing the points

$$A = (3, -2, 1)$$

$$B = (2, 0, 5)$$

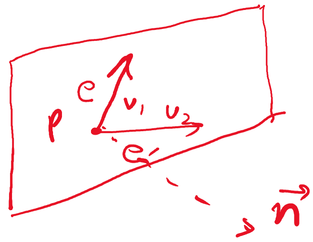
$$C = (-1, 4, 0)$$

Equational to Parametric Form

Monday, 3 March 2025 10:23 am

Convert the plane $2x - 3y + 4z = 12$ from equational to parametric form.

a. Find a point P on the plane
b. Find the normal vector n
c. Find any 2 vectors, v_1 and v_2, perpendicular to n
d. Parametric form = $P + sv_1 + tv_2$



a/ $P = (0, 0, 3)$

b/ $\vec{n} = (2, -3, 4)$

c) $V_1 = (0, 4, 3)$

$V_2 = (3, 2, 0)$

$(x, y, z) = P + sV_1 + tV_2$

$$= \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} + s\begin{bmatrix} 0 \\ 4 \\ 3 \end{bmatrix} + t\begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3t \\ 4s + 2t \\ 3 + 3s \end{bmatrix}$$

Problem 3

Monday, 3 March 2025 10:28 am

Convert the plane $x + 2y - 5z = 7$ from equational to parametric form.

Problem 4

Monday, 3 March 2025 10:31 am

Convert the plane $3x - 4y + 2z = 10$ from equational to parametric form.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4t \\ 2s+3t \\ 5+4s \end{bmatrix} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix}}_{P} + s\underbrace{\begin{bmatrix} 0 \\ 2 \\ 4 \end{bmatrix}}_{v_1} + t\underbrace{\begin{bmatrix} 4 \\ 3 \\ 0 \end{bmatrix}}_{v_2}$$

$$\vec{n} = \vec{v_1} \times \vec{v_2} = \begin{vmatrix} i & j & k \\ 0 & 2 & 4 \\ 4 & 3 & 0 \end{vmatrix} = i(-12) - j(-16) + k(-8)$$

$$= (-12, 16, -8)$$

$$\vec{PG} = (x-0, y-0, z-5) = (x, y, z-5)$$

$$\vec{PG} \cdot \vec{n} = 0 \Rightarrow -12x + 16y - 8(z-5) = 0$$

$$\frac{-12x}{-4} + \frac{16y}{-4} - \frac{8z}{-4} = \frac{-40}{-4}$$

$$3x - 4y + 2z = 10$$

Parametric to Equational Form

Monday, 3 March 2025 10:32 am

Convert the parametric form $(1, 2, 3) + s(1, 0, 2) + t(0, 1, -1)$ to equational form.

a. Identify displacement vectors v_1 and v_2
b. Find normal vector $n = v_1 \times v_2$
c. Form equation from P, G and n

Problem 5

Monday, 3 March 2025 10:35 am

Convert the parametric form $(2,1,4) + s(3,-1,2) + t(1,2,-3)$ to equational form.

Problem 6

Monday, 3 March 2025 10:37 am

Convert the parametric form $(0,3,-2)+s(2,1,0)+t(-1,4,1)$ to equational form.

Determining Points on different Sides

Monday, 3 March 2025 10:39 am

For the plane $2x - y + 3z = 5$, determine if the points $A = (1, 1, 1)$ and $B = (3, 0, 0)$ lie on the same side or opposite sides of the plane.

$A \to 2(1) - 1 + 3(1) = 2 - 1 + 3 = 4 < 5$

$B \to 2(3) - 0 + 3(0) = 6 > 5.$