Matrix Representation

Monday, 24 March 2025 10:07 am

$$\begin{array}{c|ccc} & \text{آلو} & \text{دال} & \text{چاول} \\ \hline \text{Ali} & & & \\ \text{Anas} & & & \end{array}$$

$$\begin{array}{c} \text{Ali} \\ \text{Anas} \end{array} \begin{bmatrix} 2 & 1.5 & 2 \\ 3 & 0.5 & 5 \end{bmatrix} \begin{bmatrix} x & 60 \\ y & 240 \\ z & 300 \end{bmatrix} = \begin{bmatrix} 1020 \\ 1800 \end{bmatrix}$$

آلو کا ریٹ = 60 Rs./kg = x

دال " " = 240 Rs./kg = y

چاول " " = 300 Rs./kg = z

$$\begin{cases} 2x + 1.5y + 2z = 1020 \\ 3x + 0.5y + 5z = 1800 \end{cases}$$

Linear System

$$\begin{array}{c} \\ \text{Ali} \\ \text{Anas} \end{array} \begin{array}{c} \begin{array}{ccc} \text{آلو} & \text{دال} & \text{چاول} \end{array} \\ \begin{bmatrix} 2 & 1.5 & 2 \\ 3 & 0.5 & 5 \end{bmatrix} \end{array} \begin{array}{c} \begin{array}{cc} \text{امتیاز} & \text{سوشی} \end{array} \\ \begin{bmatrix} x_1 & x_2 \\ y_1 & y_2 \\ z_1 & z_2 \end{bmatrix} \end{array}$$

3×3 $\quad$ 3×2

$$\begin{array}{c|cc} & \text{امتیاز} & \text{سوشی} \\ \hline \text{Ali} & - & - \\ \text{Anas} & - & - \end{array}_{2\times 2}$$

$$A_{10\times 21} \times B_{21\times 39} = C_{10\times 39}$$

$$A \in R^{m\times n}, \quad B \in R^{n\times p} \Rightarrow A\times B \in R^{m\times p}$$

$B \in R^{n \times r}$

Linear System

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A function $f : \mathbb{R}^n \to \mathbb{R}^m$ is linear if it satisfies two key properties:

1. **Additivity**: $f(x + y) = f(x) + f(y)$ for all vectors $x, y \in \mathbb{R}^n$
2. **Homogeneity**: $f(cx) = cf(x)$ for all $x \in \mathbb{R}^n$ and all scalars c

Together, these properties give us **superposition**: $f(ax + by) = af(x) + bf(y)$.

Example 1

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Let's examine the function $f : \mathbb{R}^2 \to \mathbb{R}^2$ defined by:

$$f(x_1, x_2) = (3x_1 - 2x_2, x_1 + 4x_2)$$

Is this linear? Let's verify both properties:

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad f(x) = \begin{bmatrix} 3x_1 - 2x_2 \\ x_1 + 4x_2 \end{bmatrix} \to \begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \quad f(y) = \begin{bmatrix} 3y_1 - 2y_2 \\ y_1 + 4y_2 \end{bmatrix}$$

$$x + y = \begin{bmatrix} x_1 + y_1 \\ x_2 + y_2 \end{bmatrix} \quad f(x+y) = \begin{bmatrix} 3(x_1 + y_1) - 2(x_2 + y_2) \\ x_1 + y_1 + 4(x_2 + y_2) \end{bmatrix}$$

$$= \begin{bmatrix} 3x_1 - 2x_2 \\ x_1 + 4x_2 \end{bmatrix} + \begin{bmatrix} 3y_1 - 2y_2 \\ y_1 + 4y_2 \end{bmatrix}$$

$$= f(x) + f(y)$$

$$cx = \begin{bmatrix} cx_1 \\ cx_2 \end{bmatrix} \quad f(cx) = \begin{bmatrix} 3(cx_1) - 2(cx_2) \\ cx_1 + 4(cx_2) \end{bmatrix}$$

$$= c \begin{bmatrix} 3x_1 - 2x_2 \\ x_1 + 4x_2 \end{bmatrix} = cf(x)$$

Problem 1

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Let's examine another function $g : \mathbb{R}^2 \to \mathbb{R}$ defined by:

$$g(x_1, x_2) = x_1^2 + x_2$$

$$\begin{bmatrix} x_1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Problem 2

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Determine whether the following function is linear: $f(x, y, z) = (xy, y + z, x - z)$

$$\begin{bmatrix} xy \\ y+z \\ x-z \end{bmatrix} = \begin{bmatrix} 0 & x & 0 \\ 0 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{bmatrix} y & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$f(x) \approx f(a) + \nabla f(a)\,(x-a)$$

Matrix representation of Linear Functions

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$$f(x) = Ax$$

$$f(x_1, x_2) = (3x_1 - 2x_2, x_1 + 4x_2)$$

$$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad f(x) = \begin{bmatrix} 3x_1 - 2x_2 \\ x_1 + 4x_2 \end{bmatrix} = \left[\begin{pmatrix} 3 \\ 1 \end{pmatrix} \begin{pmatrix} -2 \\ 4 \end{pmatrix}\right], \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \begin{matrix} 0 & 1 \\ & 0 \end{matrix}$$

$$\begin{array}{cc} e_1 & e_2 \\ \end{array}$$
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = e_1 \Rightarrow f(x) = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$X = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = e_2 \Rightarrow f(x) = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$$

$Ae_1 \to$ [illegible]

$Ae_2 -$ [illegible]

Problem 3

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$$f(x_1, x_2, x_3) = (2x_1 - x_2 + 3x_3, x_1 + x_2 - x_3)$$

$$\underbrace{\begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & -1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{X}$$

$$e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$Ae_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Problem 4

Monday, 24 March 2025 10:17 am

Find the matrix representation of the linear function $f(x, y, z) = (2x - y + z, 3x + 2z, -x + 4y - 2z)$

$A = \begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$ $|A| = (-1)(2) - (3)(-5)$

$= -2 + 15 = 13$

$2x - 5y = a$

$3x - y = b$

$A = \begin{bmatrix} 2 & 1 & 5 \\ -1 & 2 & 5 \\ -2 & 1 & -7 \end{bmatrix}$ $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

$|A| = +2 \begin{vmatrix} 2 & 5 \\ 1 & -7 \end{vmatrix} - 1 \begin{vmatrix} -1 & 5 \\ -2 & -7 \end{vmatrix} + 5 \begin{vmatrix} -1 & 2 \\ -2 & 1 \end{vmatrix}$

$= 2(-14-5) - 1(7+10) + 5(-1+4)$

$= -38 - 17 + 15 = ?$

$x =$ ———

$y =$

$z =$

الخوارزمي

↓

لوغاريتم

$\begin{array}{r} 37 \\ 15 \times \\ \hline 185 \\ 37\otimes \end{array}$ 3

$10^3 \quad 10^2 \quad 10 \quad 10$

Solving Linear Systems

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Solve the following system of equations using Cramer's rule:

$$3x + 2y = 7$$
$$5x - 4y = 3$$

$$\underbrace{\begin{bmatrix} 3 & 2 \\ 5 & -4 \end{bmatrix}}_{A}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

$Ax = b \Rightarrow x = A^{-1}b$

$|A| = -12 - 10$
$= -22$

τ:

$$2x + 3y = 8$$
$$4x + 6y = 7$$

$\tau = 8$ $\quad 2\tau = 7$

$2(2x + 3y) = 7$

$A_1 = \begin{bmatrix} 7 & 2 \\ 3 & -4 \end{bmatrix}$ $\quad A_2 = \begin{bmatrix} 3 & 7 \\ 5 & 3 \end{bmatrix}$

$x = \dfrac{|A_1|}{|A|}$

$y = \dfrac{|A_2|}{|A|}$

$|A_1| = -28 - 6$
$= -34$

$|A_2| = 9 - 35 = -26$

$x = \dfrac{-34}{-22} = \dfrac{17}{11}$ $\quad y = \dfrac{-26}{-22} = \dfrac{13}{11}$

Problem 5

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Find the solution to the system:

$$
\begin{aligned}
4x - 3y + z &= 10 \\
2x + y - 2z &= 5 \\
3x - 2y + 4z &= 12
\end{aligned}
$$