Question Bank:

Basic Probability

1. **What is the probability of rolling a sum of 8 on two six-sided dice?**
   - **Solution**:
   - The sample space for rolling two dice has 36 outcomes
   - Favorable outcomes for a sum of 8: (2,6),(3,5),(4,4),(5,3),(6,2)(2,6), (3,5), (4,4), (5,3), (6,2)(2,6),(3,5),(4,4),(5,3),(6,2) → 5 outcomes.
   - P=Total outcomes / Favorable outcomes = $\frac{5}{36}$

2. **What is the probability of drawing a king from a standard deck of 52 cards?**
   - **Solution**:
   - There are 4 kings in a deck.
   - P(King)= $\frac{4}{52} = \frac{1}{13}$

3. **A coin is flipped three times. What is the probability of getting exactly two heads?**
   - **Solution**: The sample space has $2^3 = 8$ outcomes.
   - Favorable outcomes: HHT, HTH, THH → 3 outcomes.
   - P = $\frac{3}{8}$

4. **If a lottery involves picking 6 numbers out of 49, what is the probability of picking exactly one correct number?**
   - **Solution**:
   - Total outcomes: $\binom{49}{6}$
   - Favorable outcomes: $\binom{1}{1}\binom{48}{5}$
   - P = $\binom{48}{5} / \binom{49}{6}$

5. **If the chance of rain on any day is 40%, what is the probability it does not rain on two consecutive days?**
   - **Solution**:

- Probability of no rain on a single day = 1 − 0.4 = 0.6
- Probability of no rain for two consecutive days:
- $(1-0.4)^2 = 0.6 \times 0.6 = 0.36$

OR, AND Probability

1. **If a six-sided die is rolled, what is the probability of getting a 3 or an even number?**
   - **Solution**:
   - $P(3 \cup \text{even}) = P(3) + P(\text{even}) - P(3 \cap \text{even})$
   - $P(3) = \frac{1}{6}$, $P(\text{even}) = \frac{3}{6}$, $P(3 \cap \text{even}) = 0$.
   - $P = \frac{1}{6} + \frac{3}{6} - 0 = \frac{4}{6} = \frac{2}{3}$

2. **What is the probability of drawing a spade or a face card from a deck?**
   - **Solution**:
   - $P(\text{Spade}) = \frac{13}{52}$, $P(\text{Face}) = \frac{12}{52}$, $P(\text{Spade} \cap \text{Face}) = \frac{3}{52}$
   - $P(\text{Spade} \cup \text{Face})$
   - $\frac{13}{52} + \frac{12}{52} - \frac{3}{52} = \frac{22}{52} = \frac{11}{26}$

3. **What is the probability of getting at least one head in three coin flips?**
   - **Solution**:
   - Complement rule: P(At least one head)=1−P(No heads)
   - P(No heads) = $\frac{1}{8}$
   - $P = 1 - \frac{1}{8} = \frac{7}{8}$

4. **If 10% of items are defective and 20% are oversized, with 5% being both, what is the probability an item is either defective or oversized?**
   - **Solution**:
   - $P(\text{Defective} \cup \text{Oversized}) = P(\text{Defective}) + P(\text{Oversized}) - P(\text{Both})$.
   - $P = 0.1 + 0.2 - 0.05 = 0.25$

5. **What is the probability of rolling a number greater than 4 and an even number on a single six-sided die?**
   - **Solution**:

- Favorable outcome: {6}.
- Total outcomes: 6.
- P = $\frac{1}{6}$

Conditional Probability

1. **If a coin is flipped twice, what is the probability the second flip is heads given the first is heads?**
   Solution:
   These are independent events. Probability remains:

   P(Heads on 2nd | Heads on 1st) = $\frac{1}{2}$

2. **What is the probability of drawing a red card given it is a face card?**
   Solution:
   There are 12 face cards: 6 red (hearts, diamonds) and 6 black (spades, clubs).

   P(Red | Face) = $\frac{6}{12} = \frac{1}{2}$

3. **Roll two dice. What is the probability their sum is 10 given the first die shows 6?**
   Solution:
   If the first die is 6, the second must be 4.
   Probability:

   P = $\frac{1}{6}$

4. **If a machine produces 80% good and 20% defective items, and a defective item is twice as likely to be selected for inspection, find P(Defective | Selected).**
   Solution:
   Using Bayes' theorem:

   P(Defective | Selected) = $\frac{0.2 * 2}{(0.8 * 1) + (0.2 * 2)} = \frac{0.4}{1.2} = \frac{1}{3}$

5. **A disease affects 1% of a population. A test has 95% sensitivity and 90% specificity. What is P(Has disease | Tests positive)?**
   Solution:
   Use Bayes' theorem:

   P(D|T) = $\frac{P(T|D)P(D)}{P(T)} = \frac{(0.95)(0.01)}{(0.95)(0.01) + (0.1)(0.99)}$

Bayes' Theorem

1. **A spam filter identifies 90% of spam emails correctly and 95% of non-spam emails correctly. If 20% of emails are spam, what is the probability an email is spam given the filter marks it as spam?**
   Solution:
   Use Bayes' theorem:

   P(Spam | Marked) = $\frac{\mathrm{P(Marked|Spam)P(Spam)}}{\mathrm{P(Marked)}}$

   Compute P(Marked) = P(Marked | Spam)P(Spam) + P(Marked | Not Spam)P(Not Spam)

   P(Marked) = (0.9)(0.2) + (0.05)(0.8) = 0.18+0.04 =0.22

   P(Spam | Marked) = $\frac{(0.9)(0.2)}{0.22} = \frac{0.18}{0.22} = 0.818$

2. **A disease has a prevalence of 0.1%. A test has 99% sensitivity and 98% specificity. If a person tests positive, what is the probability they have the disease?**
   Solution:
   Using Bayes' theorem:

   P(Disease | Positive) = $\frac{\mathrm{P(Positive \mid Disease)P(Disease)}}{\mathrm{P(Positive)}}$

   Compute P(Positive) = P(Positive | Disease)P(Disease) + P(Positive | No Disease)P(No Disease)

   P(Positive) = (0.99)(0.001) + (0.02)(0.999) = 0.00099 + 0.01998 = 0.02097

   P(Disease | Positive) = $\frac{(0.99)(0.001)}{0.0297}$ = 0.047

3. **Machine A produces 60% of items and Machine B 40%. If 10% of A's items are defective and 5% of B's are defective, what is the probability an item came from A given it is defective?**
   Solution:
   Using Bayes' theorem:

   P(A | Defective) = $\frac{\mathrm{P(Defective \mid A)P(A)}}{\mathrm{P(Defective)}}$

   Compute P(Defective) = P(Defective | A)P(A) + P(Defective | B)P(B)

   P(Defective) = (0.1)(0.6) + (0.05)(0.4) = 0.06 + 0.02 = 0.08

   P(A | Defective) = $\frac{(0.1)(0.6)}{0.08} = \frac{0.06}{0.08} = 0.75$

4. **In a population, 55% support candidate X. Supporters lie 10% of the time, while non-supporters lie 5%. Given someone says they support X, what is the probability they actually do?**
   Solution:
   Using Bayes' theorem:

   $P(S \mid C) = \frac{P(C \mid S)\, P(S)}{P(C)}$

   Compute P(C) = P(C | S)P(S) + P(C | NS)P(NS)

   P(C) = (0.9)(0.55) + (0.05)(0.45) = 0.495+0.0225 =0.5175

   $P(S|C) = \frac{(0.9)\,(0.55)}{0.5175} = \frac{0.495}{0.5175} \approx 0.956$

5. **An insurance company estimates 1% of drivers file claims. If 20% of claims are fraudulent and the company identifies fraud with 99% accuracy, what is the probability a flagged claim is fraudulent?**
   Solution:
   Using Bayes' theorem:

   $P(F \mid Flag) = \frac{P(Flag \mid F)\, P(F)}{P(Flag)}$

   Compute P(Flag) = P(Flag | F)P(F) + P(Flag | NF)P(NF)

   P(Flag) = (0.99)(0.002) + (0.01)(0.998) = 0.00198+0.00998 = 0.01196

   $P(F \mid Flag) = \frac{(0.99)\,(0.002)}{0.0196} \approx 0.165$

Chain Rule

1. **What is the probability of drawing two aces consecutively from a deck?**
   Solution:
   The probability of drawing the first ace is $\frac{4}{52}$. After one ace is removed, the probability of drawing another is $\frac{3}{51}$.
   Using the chain rule:

   $P(A1 \cap A2) = P(A1)\, P(A2 \mid A1) = \frac{4}{52} \times \frac{3}{51} = \frac{1}{221}$

2. **If P(Rain)=0.3, P(Cold | Rain) = 0.7, and P(Snow | Rain and Cold) = 0.5, find P(Rain ∩ Cold ∩ Snow).**

Solution:
Using the chain rule:

P=P(Rain)P(Cold | Rain)P(Snow | Rain and Cold) = $0.3 \times 0.7 \times 0.5 = 0.10$

Independence

1. **Are "rolling an even number" and "rolling a number >4" independent?**
   Solution:
   Check if P(A ∩ B) = P(A)P(B)
   P(A) = $\frac{3}{6}$, P(B) = $\frac{2}{6}$, P(A∩B) = $\frac{1}{6}$.
   P(A)P(B) = $\frac{3}{6} \times \frac{2}{6} = \frac{1}{6}$. They are independent.
2. **Are the events "drawing a heart" and "drawing a queen" independent in a single draw from a deck?**
   Solution:
   Check independence: P(A ∩ B) = P(A)P(B).

   P(Heart) = $\frac{13}{52}$, P(Queen) = $\frac{4}{52}$

   P(Heart ∩ Queen) = $\frac{1}{52}$ (Queen of Hearts).

   P(Heart)P(Queen) = $\frac{13}{52} \text{x} \frac{4}{52} = \frac{52}{2704} = \frac{1}{52}$
   Since P(A∩B)=P(A)P(B), the events are independent.

3. **If P(Rain) = 0.4, P(Lightning)=0.2, and P(Rain ∩ Lightning)=0.1, are rain and lightning independent?**
   Solution:

   P(Rain)P(Lightning) = $0.4 \times 0.2 = 0.08$

   P(Rain ∩ Lightning) = 0.1
   Since P(A∩B) ≠ P(A)P(B), the events are not independent.

4. **A factory has two machines. The probability Machine A produces a defective part is 5%, and the probability Machine B produces a defective part is 5%. Are the events "defective from Machine A" and "defective from Machine B" independent?**
   Solution:
   Since the defects from Machine A and Machine B do not influence each other, the events are independent by definition.
   P(A∩B) = P(A)P(B) = $0.05 \times 0.05 = 0.0025$.

5. **Are the events "getting heads on the first flip" and "getting heads on the second flip" independent?**
   Solution:
   The outcomes of each coin flip are independent.
   a. P(Heads on first) = P(Heads on second) = $\frac{1}{2}$
   b. P(Both heads) = P(Heads on first)P(Heads on second) = $\frac{1}{2}$ x $\frac{1}{2}$ $=$ $\frac{1}{4}$
      Hence, the events are independent.

Total Law of Probability

1. **A weather forecast predicts 60% chance of rain in the morning and 40% in the afternoon. If the overall chance of rain is 50%, what is the probability it rains in the afternoon given it rains?**

Solution:
Define events:

- A: It rains in the afternoon.
- B: It rains in the morning.
- C: It rains.

We know:

- P(B)= 0.6, P(A)=0.4 , and P(C)=0.5.

Using the Total Law of Probability:

P(C) = P(C ∣ B)P(B) + P(C ∣ A)P(A)

Assuming P(C ∣ B) = 1 and P(C ∣ A) =1 because rain in either period contributes to overall rain:

P(C) = (1)(0.6) + (1)(0.4) = 1.0

To find P(A|C)P(A|C), use Bayes' Theorem:

P(A|C) = $\frac{\mathrm{P(C|A)\ P(A)}}{\mathrm{P(C)}}$

Substitute:

P(A|C) = $\frac{(1)(0.4)}{0.5}$ = 0.8

Answer: The probability it rains in the afternoon given it rains is 0.8.

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2. Machine A produces 70% of parts, and Machine B produces 30%. Defect rates are 5% for A and 10% for B. What is the probability a randomly chosen part is defective?

Solution:
Define events:

- D: The part is defective.
- A: The part is from Machine A.
- B: The part is from Machine B.

Using the Total Law of Probability:

$P(D) = P(D \mid A)P(A) + P(D \mid B)P(B)$

Substitute values:

$P(D) = (0.05)(0.7) + (0.1)(0.3)$

$P(D) = 0.035 + 0.03 = 0.065$

Answer: The probability a randomly chosen part is defective is 0.065 (6.5%).

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3. A test for a disease has 95% sensitivity and 98% specificity. The disease prevalence is 1%. What is the probability of testing positive?

Solution:
Define events:

- T: The test result is positive.
- D: The person has the disease.
- D^c: The person does not have the disease.

Using the Total Law of Probability:

$P(T) = P(T \mid D)P(D) + P(T \mid D^c)P(D^c)$

Substitute values:

$P(T \mid D) =0.95$, $P(D)=0.01$, $P(T \mid D^c) = 1-0.98 = 0.02$, $P(D^c) = 1-0.01 = 0.99$

P(T) = (0.95)(0.01) + (0.02)(0.99)
P(T) = 0.0095 + 0.0198 = 0.029

Answer: The probability of testing positive is 0.0293 (2.93%).

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4. In a town, 60% support a policy, and 40% oppose it. Among supporters, 70% vote. Among opponents, 50% vote. What is the probability a randomly chosen person votes?

Solution:
Define events:

- V: The person votes.
- S: The person supports the policy.
- O: The person opposes the policy.

Using the Total Law of Probability:

P(V) = P(V | S)P(S) + P(V | O)P(O)

Substitute values:

P(V | S) = 0.7, P(S) = 0.6, P(V | O)=0.5, P(O)=0.4.

P(V) = (0.7)(0.6) + (0.5)(0.4)
P(V) = 0.42 + 0.2 = 0.62

Answer: The probability a randomly chosen person votes is 0.62 (62%).

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5. In a card game, you win if you draw a king or a queen. The deck is either shuffled (90% chance) or not shuffled (10% chance). In a shuffled deck, drawing a king or queen is $\frac{8}{52}$. In an unshuffled deck, drawing a king or queen is $\frac{4}{52}$. What is the probability of winning?

Solution:
Define events:

- W: Winning (drawing a king or queen).
- S: Deck is shuffled.
- U: Deck is not shuffled.

Using the Total Law of Probability:

$P(W) = P(W \mid S)P(S) + P(W \mid U)P(U)$

Substitute values:

- $P(W \mid S) = \frac{8}{52}$, $P(S) = 0.9$, $P(W|U) = \frac{4}{52}$, $P(U) = 0.1$.

$P(W) = \frac{8}{52}(0.9) + \frac{4}{52}(0.1)$
$P(W)= \frac{7.2}{52} + \frac{0.4}{52} = \frac{7.6}{52}$
$P(W) \approx 0.146$

Answer: The probability of winning is approximately 0.146 (14.6%).