Linear Algebra
Worksheet 4: Projections

Problem 1: Projection onto a Line
For each of the following vectors, find the projection onto the given line:

(a) Find the projection of $\vec{x} = \begin{bmatrix} 4 \\ 2 \\ -3 \end{bmatrix}$ onto the line spanned by $\vec{w} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$.

(b) Find the projection of $\vec{y} = \begin{bmatrix} -2 \\ 5 \\ 3 \end{bmatrix}$ onto the line spanned by $\vec{u} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$.

(c) Find the distance from the point $\vec{z} = \begin{bmatrix} 7 \\ 4 \\ 1 \end{bmatrix}$ to the line through the origin in the direction of $\vec{v} = \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix}$.

Problem 2: Finding Orthogonal Bases
For each of the following pairs of vectors:

(a) $\vec{a} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$

(b) $\vec{c} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$ and $\vec{d} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

(i) Determine if the vectors are already orthogonal.

(ii) If not, construct an orthogonal basis for the plane they span.

Problem 3: Projection onto a Plane
For each of the following vectors and planes:

(a) Find the projection of $\vec{v} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$ onto the plane P with equation $x + y + z = 0$.

(b) Find the projection of $\vec{w} = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$ onto the plane Q spanned by $\vec{u}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\vec{u}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$.

(c) Find the distance from the point $\vec{p} = \begin{bmatrix} 5 \\ -2 \\ 3 \end{bmatrix}$ to the plane R with equation $2x - y + 3z = 4$.