CAI 2.0, Linear Algebra
Worksheet 4: Span, Subspaces, and Dimension

1. **Identifying Spans**

   Let $v_1 = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$, $v_2 = \begin{pmatrix} 2 \\ 4 \\ 0 \end{pmatrix}$, and $v_3 = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$.

   (a) Determine whether v_2 is in the span of v_1.

   (b) Determine whether v_3 is in the span of v_1 and v_2.

   (c) Find a geometric description of $\text{span}(v_1, v_2)$ in $\mathbb{R}^3$.

2. **Testing for Subspaces**

   For each of the following sets, determine whether it is a subspace of $\mathbb{R}^3$. If it is not a subspace, explain which condition(s) fail.

   (a) $S_1 = \{(x, y, z) \in \mathbb{R}^3 : x + 2y + 3z = 0\}$

   (b) $S_2 = \{(x, y, z) \in \mathbb{R}^3 : x + 2y + 3z = 5\}$

   (c) $S_3 = \{(x, y, z) \in \mathbb{R}^3 : xy = 0\}$

   (d) $S_4 = \{(x, y, z) \in \mathbb{R}^3 : x = 2y\}$

3. **Finding Span and Dimension**

   Consider the following sets of vectors.

   (a) $A = \{(1, 0, 1), (2, 1, 0), (3, 1, 1)\}$

   (b) $B = \{(1, 2, 3), (2, 4, 6), (0, 0, 0)\}$

   (c) $C = \{(1, 1, 0), (0, 1, 1), (1, 0, 1)\}$

   For each set:

   (a) Find the dimension of the span.

   (b) Identify a basis for the span.

4. **Vectors in a Subspace**

   Let $W = \{(x, y, z) \in \mathbb{R}^3 : 2x - y + 3z = 0\}$.

   (a) Show that W is a subspace of $\mathbb{R}^3$.

   (b) Find the dimension of W.

   (c) Find a basis for W.

   (d) Determine whether the vector $(3, 6, 0)$ is in W.

5. **Orthogonal Subspaces**

   Let $v = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$.

(a) Find a description of the set S of all vectors in $\mathbb{R}^3$ that are perpendicular to v.

(b) Prove that S is a subspace of $\mathbb{R}^3$.

(c) Find the dimension of S.

(d) Find a basis for S.

6. **Dimensions of Spans**

   For each of the following collections of vectors, find the dimension of their span and identify a basis for the span.

   (a) $\{(1, 2, 1, 0), (2, 3, 0, 1), (3, 5, 1, 1)\}$ in $\mathbb{R}^4$

   (b) $\{(1, 0, 1, 0), (0, 1, 0, 1), (1, 1, 1, 1), (2, 1, 2, 1)\}$ in $\mathbb{R}^4$

   (c) $\{(1, 2, 3), (4, 5, 6), (7, 8, 9)\}$ in $\mathbb{R}^3$