Basis Check

Friday, 7 March 2025 10:06 am

Are vectors $v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ basis for R^2?

$e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

1. Do they span $\mathbb{R}^2$?
2. Are there exactly $\dim(\mathbb{R}^2) = 2$ vectors?

$$\text{span}(V_1, V_2) = c_1 V_1 + c_2 V_2 = \begin{bmatrix} c_1 + c_2 \\ c_2 - c_2 \end{bmatrix} \begin{matrix} \to x \\ \to y \end{matrix}$$

Problem 1

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Check if the given vectors form a basis for R^3

$$u_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$u_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$u_3 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 10 \\ -2 \\ 5 \end{bmatrix}$$

$\text{span}(u_1, u_2, u_3)$

$= c_1 u_1 + c_2 u_2 + c_3 u_3$

$$= \begin{bmatrix} c_1 + c_2 + 0 \\ c_1 + 0 + c_3 \\ c_1 - c_2 - c_3 \end{bmatrix} \begin{array}{l} \rightarrow x = c_1 + c_2 \\ \rightarrow y = c_1 + c_3 \\ z = c_1 - c_2 - c_3 \end{array}$$

Example 1

Friday, 7 March 2025 10:10 am

Find basis and dimension for the given subspace

$$W = \{(x, y, z) \in \mathbb{R}^3 : x + y + z = 0\}.$$

$$x = -y - z$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -y-z \\ y \\ z \end{bmatrix} = y \overset{v_1}{\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}} + z \overset{v_2}{\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}}$$

Problem 2

Friday, 7 March 2025 10:13 am

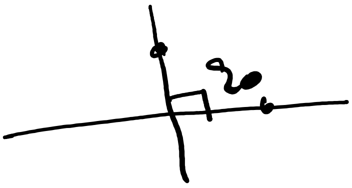
find a basis for the subspace $V = \text{span}\left(\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}2\\4\\6\end{bmatrix}, \begin{bmatrix}3\\1\\0\end{bmatrix}\right)$.

V_1 V_2

$V_1 \cdot V_2 = 0$

$3+2+0 = 5 \neq 0$

$\begin{bmatrix}1\\0\end{bmatrix} \begin{bmatrix}0\\1\end{bmatrix}$



$v_1 = \begin{bmatrix}1\\0\\0\end{bmatrix}, \; v_2 = \begin{bmatrix}0\\1\\0\end{bmatrix} \quad v_3 = \begin{bmatrix}0\\0\\1\end{bmatrix}$

Basis

orthogonal Basis

orthonormal "

$2x - y + 3z = 0$

(i) Find Basis

$y = 2x + 3z$

$$\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}x\\2x+3z\\z\end{bmatrix} = x\begin{bmatrix}1\\2\\0\end{bmatrix} + z\begin{bmatrix}0\\3\\1\end{bmatrix}$$

?

$\downarrow v_1$?

Basis : $\left(\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix} \right)$

(ii) Find orthogonal Basis : $2x - y + 3z = 0$

$$V_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \qquad V_2 = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$2x = y$

$$V_1 \cdot V_2 = a + 2b = 0 \Rightarrow a = -2b$$

$$b = 1 \Rightarrow a = -2 \quad c = 11$$

$$V_2 = \begin{bmatrix} -2 \\ 1 \\ 11 \end{bmatrix}$$

$$2x - y + 3z = 0$$
$$2(-2) - 1 + 33 \neq 0$$

$$\vec{n} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} \qquad V_2 = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \qquad V_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

$$\vec{n} \cdot V_2 = 0 \Rightarrow 2a - b + 3c = 0 \Rightarrow -4b - b + 3c = 0$$

$$V_1 \cdot V_2 = 0 \Rightarrow a + 2b = 0 \Rightarrow \boxed{\begin{matrix} c = \frac{5b}{3} \\ a = -2b \end{matrix}}$$

$$2x - y + 3z = 0$$
$$-12 - 3 + 15 = 0$$
$$0 = 0$$

$$V_2 = \begin{bmatrix} -6 \\ 3 \\ 5 \end{bmatrix}$$

let $b = 3$
$a = -6$
$c = 5$

$$3x - 2y + z = 0 \qquad \vec{n} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$

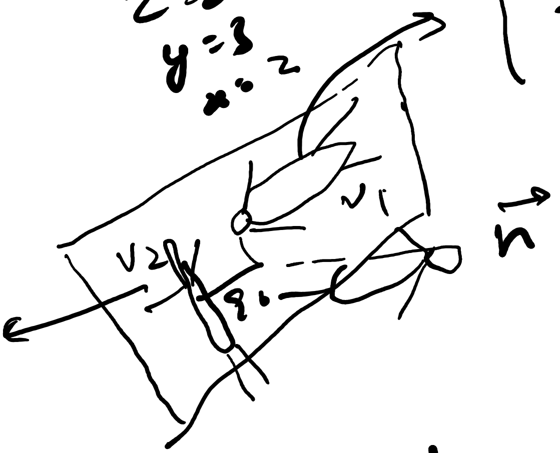
$$3x - 2y + z = 0$$

$$\vec{n} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$3x = 2y \qquad z = 0$$

$$x = \frac{2}{3}y \qquad y = 3, \; x = 2$$

$$\begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$$



$$V_2 = \begin{bmatrix} 3a \\ 2b \\ 13c \end{bmatrix}$$

$$V_2 \cdot \vec{n} = 3a - 2b + c = 0$$

$$V_2 = \begin{bmatrix} -3 \\ 2 \\ 13 \end{bmatrix} \qquad V_2 \cdot V_1 = 2a + 3b = 0$$

$$2a = -3b$$

$$3\left(-\tfrac{3}{2}b\right) - 2b + c = 0 \qquad \boxed{a = -\frac{3}{2}b} \qquad b = 2$$

$$-\frac{9}{2}b - 2b + c = 0 \qquad a = -3$$

$$c = \frac{9}{2}b + 2b = \frac{13b}{2} \qquad c = 13$$

(iii) Orthonormal Basis

$$V_1 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} \qquad V_2 = \begin{bmatrix} -3 \\ 2 \\ 13 \end{bmatrix}$$

$$\|V_1\| = \sqrt{4+9} = \sqrt{13} \qquad \|V_2\| = \sqrt{9+4+169} = \sqrt{182}$$

$$u_1 = \frac{V_1}{\|V_1\|} = \frac{1}{\sqrt{13}}\begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} \qquad u_2 = \frac{1}{\sqrt{182}}\begin{bmatrix} -3 \\ 2 \\ 13 \end{bmatrix}$$

Orthogonal Basis

Friday, 7 March 2025 10:14 am

Verify of the given vectors form an orthogonal basis for R^3

$$v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$$

Orthogonal Basis for a plane

Friday, 7 March 2025 10:16 am

Find an orthogonal basis for the plane P in $\mathbb{R}^3$ given by $2x + 3y + z = 0$.

1. Find normal vector n to plane
2. Find any vector in plane v_1
3. Find another vector v_2 that is orthogonal to both n and v_1

Orthonormal Vectors

Friday, 7 March 2025 10:19 am

Convert following orthogonal vectors to orthonormal vectors

$$v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$$

Orthonormal Basis

Friday, 7 March 2025 10:21 am

Find orthonormal basis for the plane

$$P : 2x + 3y + z = 0$$

Fourier Formula

Friday, 7 March 2025 10:23 am

[Fourier Formula] For any orthogonal collection of non-zero vectors $v_1, v_2, \ldots, v_k$ in $\mathbb{R}^n$ and vector v in their span,

$$v = \sum_{i=1}^{k} \left(\frac{v \cdot v_i}{v_i \cdot v_i} \right) v_i$$

Consider the orthogonal basis $\{v_1, v_2, v_3\}$ where:

$$v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$$

We want to express the vector $v = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$ in terms of this basis.