CAI 2.0, Linear Algebra
Worksheet 2

Problem 1: Finding a perpendicular vector in $\mathbb{R}^2$

Let $\mathbf{u} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$. Find a nonzero vector $\mathbf{v}$ in $\mathbb{R}^2$ that is perpendicular to $\mathbf{u}$, i.e., a nonzero 2-vector $\mathbf{v}$ that makes an angle of $\pi/2$ radians (or $90°$) with $\mathbf{u}$.

Problem 2: Determining the angle between vectors in $\mathbb{R}^3$

Let $\mathbf{u} = \begin{bmatrix} 3 \\ 0 \\ 4 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$. Determine the angle between the vectors $\mathbf{u}$ and $\mathbf{v}$.

Problem 3: Vector operations in $\mathbb{R}^3$

Let $\mathbf{a} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, and $\mathbf{c} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$. For each of the following, calculate the number or indicate that it is not defined.

(a) $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c})$

(b) $(\mathbf{a} - \mathbf{b}) \cdot \mathbf{c}$

(c) $\|\mathbf{a} + \mathbf{c}\|$

(d) $(\mathbf{a} \cdot \mathbf{b}) + \mathbf{c}$

(e) $\| - \mathbf{a}\|$

Problem 4: Geometry with dot products

(a) Using that perpendicularity is governed by the dot products being equal to 0, find a nonzero vector in $\mathbb{R}^3$ that is perpendicular to $\mathbf{v} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$. Then find another that is not a scalar multiple of that one.

(b) Find an equation in x, y, z that characterizes when $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is perpendicular to $\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$. What does this collection of vectors look like?

Problem 5: Algebra with dot products

For $\mathbf{a} = \begin{bmatrix} 4 \\ -2 \\ 3 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 1 \\ 5 \\ -2 \end{bmatrix}$, and $\mathbf{c} = \begin{bmatrix} 6 \\ -4 \\ -1 \end{bmatrix}$, show that $\mathbf{a} \cdot (\mathbf{b} - \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} - \mathbf{a} \cdot \mathbf{c}$.

Problem 6: A correlation coefficient

Consider the collection of 5 data points: $(-2,5)$, $(-1,3)$, $(0,0)$, $(1,-2)$, $(2,-6)$.

(a) Plot the points to see if they look close to a line.

(b) Compute the correlation coefficient exactly. Plug that into a calculator to approximate it to three decimal digits to see if its nearness to ± 1 fits well with the visual quality of fit of the line to the data plot in (a).

Problem 7: Computing correlation coefficients

Below are four different sets of data with 5 data points. For each set, compute the corresponding 5-vectors $\mathbf{X}$ and $\mathbf{Y}$, and then compute the correlation coefficient r.

(a) $(-2,-4)$, $(-1,-2)$, $(0,0)$, $(1,2)$, $(2,4)$

(b) $(-2,-5)$, $(-1,3)$, $(0,1)$, $(1,-3)$, $(2,4)$

(c) $(-2,6)$, $(-1,2)$, $(0,-1)$, $(1,-2)$, $(2,-5)$

(d) $(-2,4)$, $(-1,-2)$, $(0,-1)$, $(1,3)$, $(2,-4)$