

# Problem Statement:

• A laboratory test for detecting COVID-19 is **98%** efficient.

• It also has a false-positive rate of **1%**.

• **0.5%** of population is actually infected with COVID-19.

Suppose your Lab test results +ve, find the probability that you are actually infected with COVID-19.

You are <sup>actual</sup> Infected = **I**

Lab Result is +ve = **T**

$$P(T/I) = 0.98$$

$$P(I) = 0.005$$

$$P(T/I^c) = 0.01$$

$$P(I^c) = 1 - 0.005 = 0.995$$

$$= 1 - P(I)$$

$$P(I/T) = \frac{P(T/I)P(I)}{P(T)}$$

$$= \frac{(0.98)(0.005)}{(0.98)(0.005) + (0.01)(0.995)}$$

**Prior**

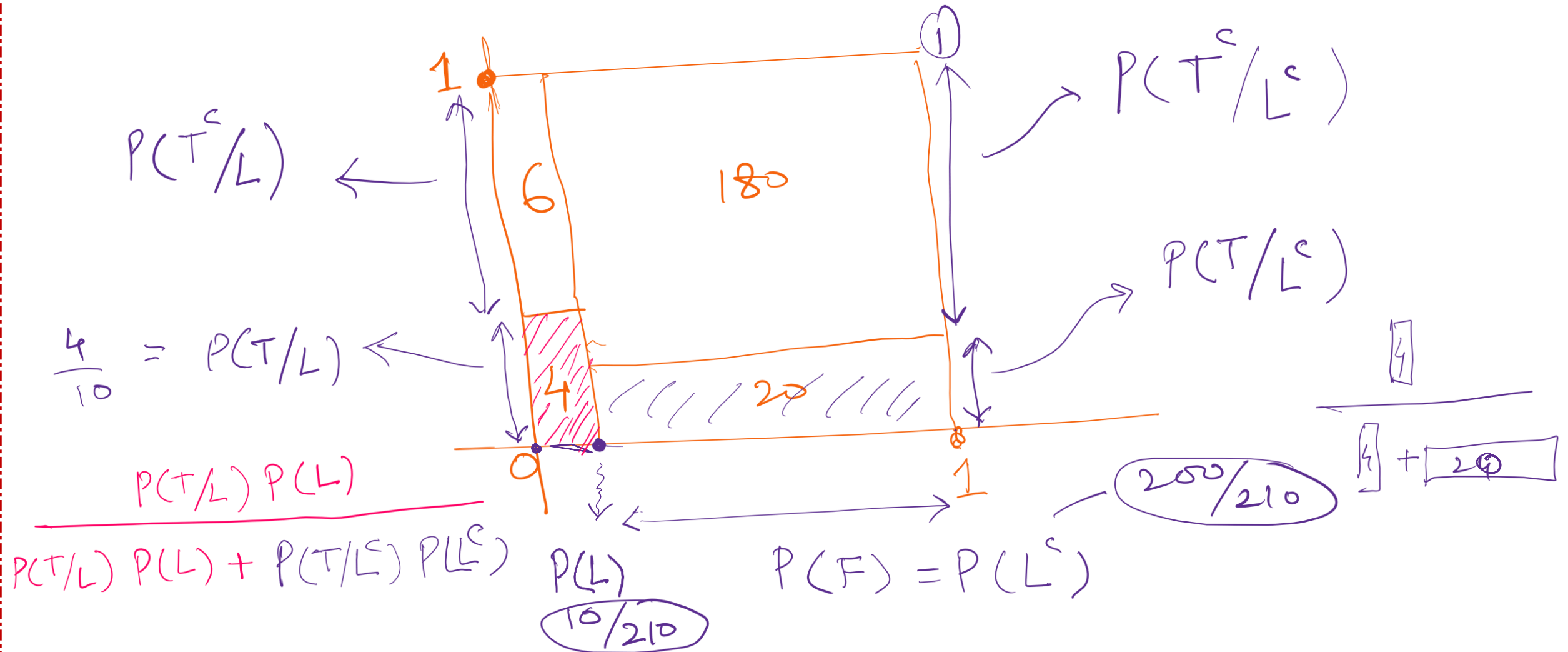
$$P(T) = P(T \cap I) + P(T \cap I^c)$$

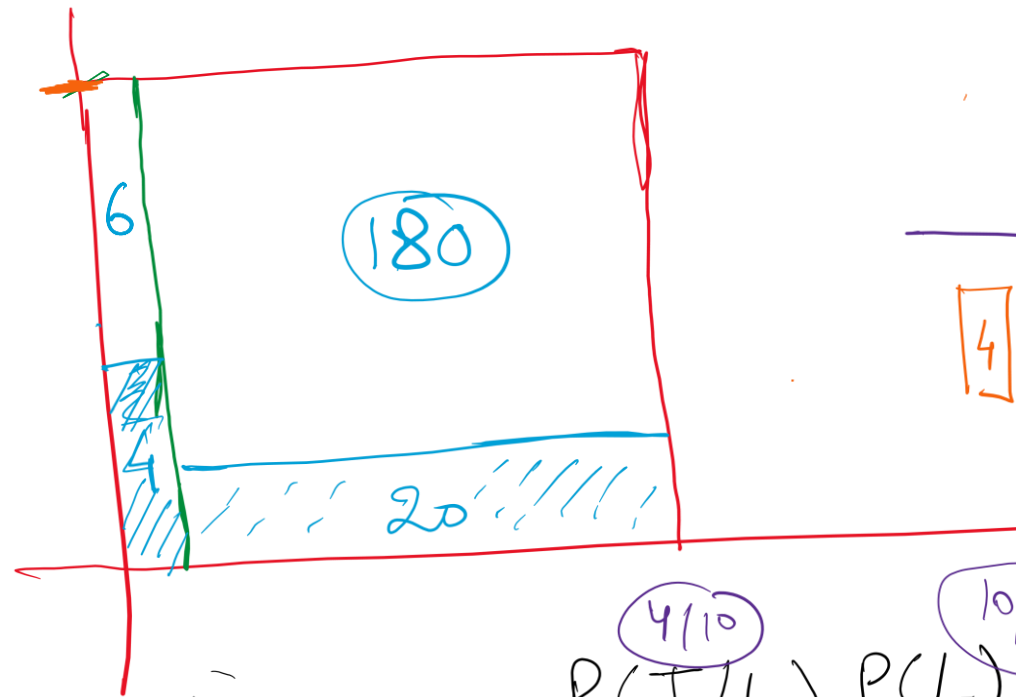
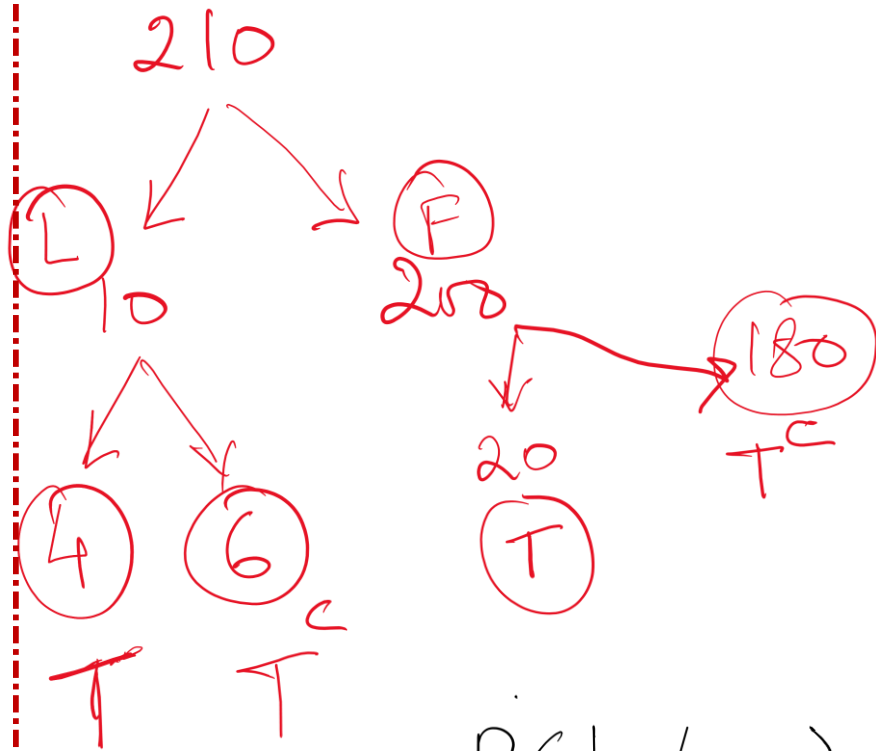
$$P(T) = P(T/I)P(I) + P(T/I^c)P(I^c)$$

$$= (0.98)(0.005) + (0.01)(0.995)$$

Librarian ~ Farmers ~ Tidy ~ Untidy

Example from Youtube Video





$$\frac{\boxed{4}}{\boxed{4} + \boxed{20}}$$

$$P(L/T) = \frac{4}{4+20} = \frac{P(T/L)P(L)}{P(T/L)P(L) + P(T/L^c)P(L^c)}$$

$\swarrow \quad \swarrow \quad \swarrow \quad \swarrow$   
 $4/10 \quad 10/210 \quad 20/200 \quad 200/210$

- 60% of all email in 2016 is spam.  $P(S) = 0.6$ ,  $P(S^c) = 0.4$
- 20% of spam has the word "Dear"  $P(D/S) = 0.2$
- 1% of non-spam (aka ham) has the word "Dear"  $\rightarrow P(D/S^c) = 0.01$

You get an email with the word "Dear" in it.

What is the probability that the email is spam?

$$\rightarrow P(S|D) = ?$$

$$\begin{aligned}
 P(S|D) &= \frac{P(D/S) P(S)}{P(D/S) P(S) + P(D/S^c) P(S^c)} = \frac{(0.2)(0.6)}{(0.2)(0.6) + (0.01)(0.4)} = \frac{0.12}{0.12 + 0.04} \\
 &= \frac{0.12}{0.124} = 0.96 = \boxed{96\%}
 \end{aligned}$$

# Bayes Theorem



probability a hypothesis is true given the evidence

probability a hypothesis is true (before any evidence is present)

probability of seeing the evidence if the hypothesis is true

probability of observing the evidence

$$P(H/E) = \frac{P(H) P(E/H)}{P(E)}$$

Explanation, Statement, Formula,  
Proof, Solved Examples

## LIKELIHOOD

The probability of "B" being True, given "A" is True

## PRIOR

The probability "A" being True. This is the knowledge.

$$P(A|B) = \frac{P(B|A).P(A)}{P(B)}$$

## POSTERIOR

The probability of "A" being True, given "B" is True

## MARGINALIZATION

The probability "B" being True.

## Likelihood

The probability that the evidence is true, given that your hypothesis is true

## Prior

The probability of your hypothesis before observing the evidence

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

## Posterior

The probability that your hypothesis is true, given the observed evidence

## Marginal

The probability of the new evidence under all possible hypotheses