Angle between 2 planes

Tuesday, 4 March 2025 10:34 am

For planes with normal vectors $\vec{n}_1$ and $\vec{n}_2$, the angle θ between them is:

$$\cos\theta = \frac{|\vec{n}_1 \cdot \vec{n}_2|}{||\vec{n}_1|| \cdot ||\vec{n}_2||}$$

Example 1

Tuesday, 4 March 2025 10:42 am

Find the angle between the planes $2x + 3y - z = 5$ and $4x + y + 3z = 2$.

$\vec{n_1} = (2, 3, -1)$, $\vec{n_2} = (4, 1, 3)$

$\vec{n_1} \cdot \vec{n_2} = 8 + 3 - 3 = 8$

$\|\vec{n_1}\| = \sqrt{4 + 9 + 1} = \sqrt{14}$

$\|\vec{n_2}\| = \sqrt{16 + 1 + 9} = \sqrt{26}$

$\cos\theta = \dfrac{8}{\sqrt{14 \times 26}}$

$= 0.419$

$\theta = 65.8°$

Problem 1

Tuesday, 4 March 2025 10:44 am

Find the angle between the planes:

$3x - 4y + 2z = 7$

and

$6x + 8y - 4z = 3$

Problem 2

Tuesday, 4 March 2025 10:46 am

Find the angle between the planes:

$x + 2y + 2z = 4$

and

$2x + 4y + 4z = 10$

Line of Intersection of 2 planes

Tuesday, 4 March 2025 10:47 am

Find the line of intersection of the planes

$2x + y - z = 4$ —— (1)

And

$x - y + z = 2.$ —— (2)

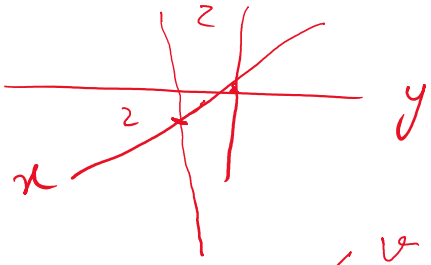
1. Solve the two plane equations simultaneously to find a parametric form
2. Express the line in the form $P + tv$, where P is a point on the line and v is the direction vector

(1) + (2) $\Rightarrow$

$$\begin{array}{r} 2x + y - z = 4 \\ x - y + z = 2 \\ \hline 3x \qquad\quad = 6 \end{array} \Rightarrow x = 2$$

(2) $\Rightarrow$ $x - y + z = x \Rightarrow y = z$

Let $y = t \Rightarrow z = t$ $\quad x = 2$



$P + tv_1 + t'v_2$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ t \\ t \end{bmatrix} = t\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

(v: $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$, P: $\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$)

$= P + tv$

Problem 2

Tuesday, 4 March 2025 10:52 am

Find the parametric equations of the line of intersection of the planes:

$x + y + z = 2$

and

$2x - y + z = 3$

$x = t$

$y = ?$ in for of x

$z = ?$ x

$3x + 2z = 5$

$2z = 5 - 3x$

$z = \frac{5}{2} - \frac{3}{2}x = \frac{5}{2} - \frac{3}{2}t$

$x + y + \frac{5}{2} - \frac{3}{2}x = 2 \Rightarrow -\frac{1}{2}x + y = -\frac{1}{2}$

$y = \frac{1}{2}x - \frac{1}{2} = \frac{1}{2}t - \frac{1}{2}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} t \\ \frac{1}{2}t - \frac{1}{2} \\ \frac{5}{2} - \frac{3}{2}t \end{bmatrix} = \begin{bmatrix} 0 \\ -1/2 \\ 5/2 \end{bmatrix} + t\begin{bmatrix} 1 \\ 1/2 \\ -3/2 \end{bmatrix}$$

$P + tv$

Problem 3

Tuesday, 4 March 2025 11:03 am

Find the parametric equations of the line of intersection of the planes:
$x - y + 2z = 4$
and
$3x + 2y - z = 1$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6/7 \\ 0 \\ 11/7 \end{bmatrix} + t \begin{bmatrix} -3/7 \\ 1 \\ 5/7 \end{bmatrix}$$

P

$x = 0$

$y = 0$ —— ?

$z = 0$

R_4 | R_3 | R_2 | R_1

Distance from a point to plane

Tuesday, 4 March 2025 11:11 am

The distance from a point (x_0, y_0, z_0) to a plane $ax + by + cz + d = 0$ is:

$$\text{Distance} = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

$2x + y - 7z = 5$

$2x + y - 7z - 5 = 0$

$|2(0) + (0) - 7(1) - 5| = |-12| = 12$

$P = (0, 0, 1)$

$$\frac{12}{\sqrt{4 + 1 + 49}} = \frac{12}{\sqrt{54}} = 1.6$$

Problem 4

Tuesday, 4 March 2025 11:12 am

Find the distance from the point $(3, -1, 2)$ to the plane $4x + 3y - z + 5 = 0$.