Problem: Find Integrals

Wednesday, 12 February 2025 4:09 pm

(a) $f(x) = x^5$ **(b)** $g(x) = \dfrac{1}{\sqrt{x}}$ **(c)** $h(x) = \sin 2x$

(d) $i(x) = \cos\dfrac{x}{2}$ **(e)** $j(x) = e^{-3x}$ ✓ **(f)** ~~$k(x) = 2^x$~~

a

$$\int x^5\,dx = \frac{x^6}{6}$$

b/

$$\int \frac{1}{\sqrt{x}}\,dx = \int x^{-1/2}\,dx$$

$$= \frac{x^{-1/2+1}}{-1/2+1} = \frac{x^{1/2}}{+1/2} = 2\sqrt{x}$$

c

$$\int \underbrace{\sin 2x}_{f(x)}\,\underbrace{dx}_{dx} = \frac{-\cos 2x}{2} = \boxed{-\frac{1}{2}\cos 2x}$$

d

$$\int \cos\frac{x}{2}\,dx = \frac{+\sin\frac{x}{2}}{1/2} = +2\sin\frac{x}{2}$$

e

$$\int e^{-3x}\,dx = \frac{e^{-3x}}{-3} = -\frac{1}{3}e^{-3x}$$

$$\Big\downarrow \frac{d}{dx}$$

$$-\frac{1}{\cancel{3}}\,e^{-3x}\cdot(-\cancel{3})$$

$$+e^{-3x}$$

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Evaluate

$$\int (x^2 - 2x + 5)\,dx.$$

$$= \frac{x^3}{3} - 2\frac{x^2}{2} + \frac{5x^1}{1} = \frac{1}{3}x^3 - x^2 + 5x$$

Problem

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$$\int_0^2 (2x + 1)\,dx = \left(2\frac{x^2}{2} + 1\frac{x^1}{1}\right)\Bigg|_0^2 = (x^2 + x)\Bigg|_0^2 = \overset{4+2}{(2^2+2)} - \overset{0+0}{(0^2+0)} = 6$$

$$\int_{-1}^0 (x - x^2)\,dx$$

$$\int_{-1}^1 x^3\,dx -$$

$$\int_0^1 (3x - x^3)\,dx = \left(\frac{3x^2}{2} - \frac{x^4}{4}\right)\Bigg|_0^1 = \left(\frac{3(1)^2}{2} - \frac{1^4}{4}\right) - \left(\frac{3(0)^2}{2} - \frac{0^4}{4}\right)$$

$$= \left(\frac{3}{2} - \frac{1}{4}\right) - (0 - 0) = \frac{5}{4}$$

Problem

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Find the average value of $f(x) = x$ on $[1, 3]$.

$$\text{Area} = \int_1^3 x\,dx = \left.\frac{x^2}{2}\right|_1^3 = \left(\frac{3^2}{2}\right) - \left(\frac{1^2}{2}\right)$$

$$= \frac{9}{2} - \frac{1}{2} = \frac{8}{2} = 4$$

$$\text{Avg. Value} = \frac{4}{3-1} = \frac{4}{2} = 2$$

<u>In general</u>

$$\text{Avg. Value} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

Problem

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$$\int_0^{\pi} \cos x \, dx = \sin x \Big|_0^{\pi}$$

$$= \sin \pi - \sin 0 = 0 - 0 = 0$$

Problem

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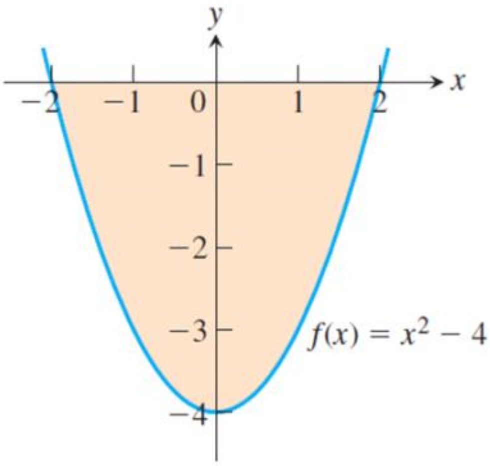

(a) the definite integral over the interval $[-2, 2]$, and

(b) the area between the graph and the x-axis over $[-2, 2]$.

a- $\int_{-2}^{2} (x^2 - 4)\,dx = \left(\frac{x^3}{3} - 4x\right)\Big|_{-2}^{2}$

$= \left(\frac{8}{3} - 8\right) - \left(-\frac{8}{3} + 8\right) = \frac{8}{3} - 8 + \frac{8}{3} - 8$

$= 2 \cdot \frac{8}{3} - 16 = \frac{16}{3} - 16 = \frac{16 - 48}{3} = -\frac{32}{3}$

b/ Area $= \frac{32}{3}$ (magnitude)

Problem

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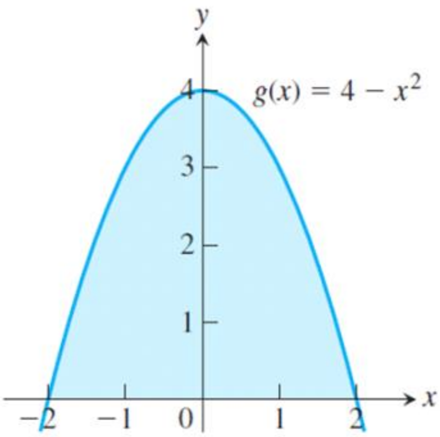

(a) the definite integral over the interval $[-2, 2]$, and

(b) the area between the graph and the x-axis over $[-2, 2]$.

a,

$$\int_{-2}^{2} (4-x^2)\,dx = -\boxed{\int_{-2}^{2} (x^2-4)\,dx} = -\left(-\frac{32}{3}\right)$$

$$= +\frac{32}{3}$$

Problem

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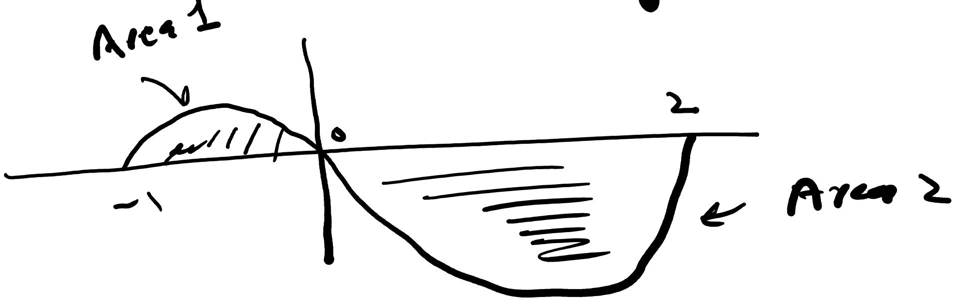
EXAMPLE 8 Find the area of the region between the x-axis and the graph of $f(x) = x^3 - x^2 - 2x,\ -1 \le x \le 2$.

$$\int_{-1}^{2} (x^3 - x^2 - 2x)\,dx = \left(\frac{x^4}{4} - \frac{x^3}{3} - \cancel{2}\frac{x^2}{\cancel{2}}\right)\Big|_{-1}^{2}$$

$$= \left(\cancel{4} - \frac{8}{3} - \cancel{4}\right) - \left(\frac{1}{4} + \frac{1}{3} - 1\right) = \left(-\frac{8}{3}\right) - \frac{1}{4} - \frac{1}{3} + 1$$

$$= \frac{-32 - 3 - 4 + 12}{12} = -\frac{27}{12}$$

not equal to area.



$$\text{Area 1} = \int_{-1}^{0} (x^3 - x^2 - 2x)\,dx = \frac{5}{12} \quad (+\text{ive})$$

$$\text{Area 2} = \int_{0}^{2} (x^3 - x^2 - 2x)\,dx = -\frac{8}{3}$$ (-ive but area is +ive)

$$\frac{5}{12} + \frac{8}{3} = \frac{5 + 32}{12} = \frac{37}{12}$$

Problem: Find shaded area

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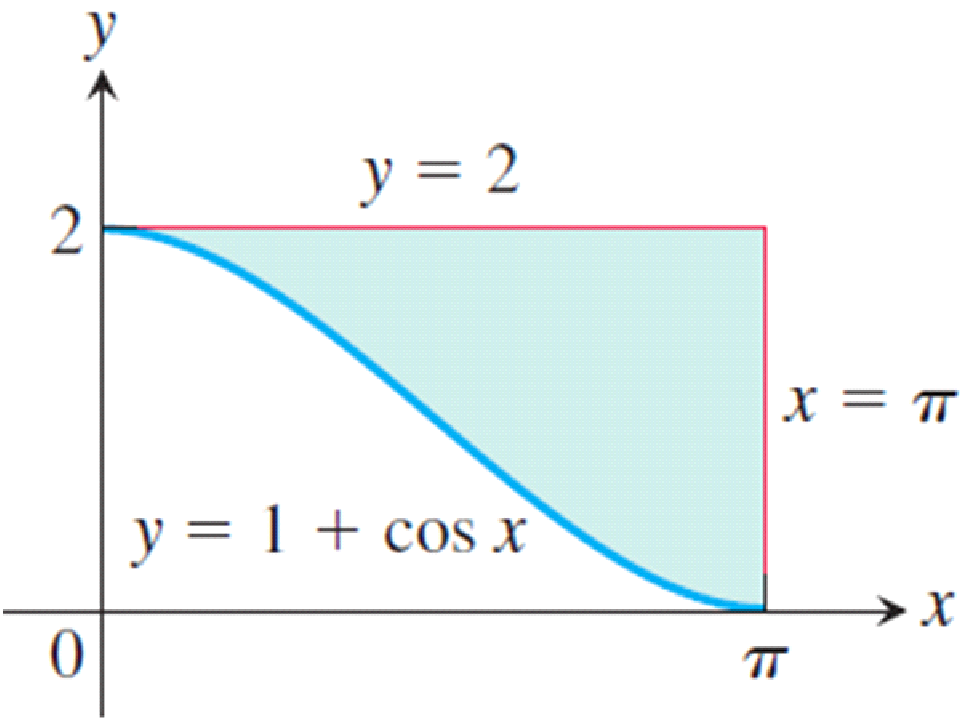

Problem: Find shaded area

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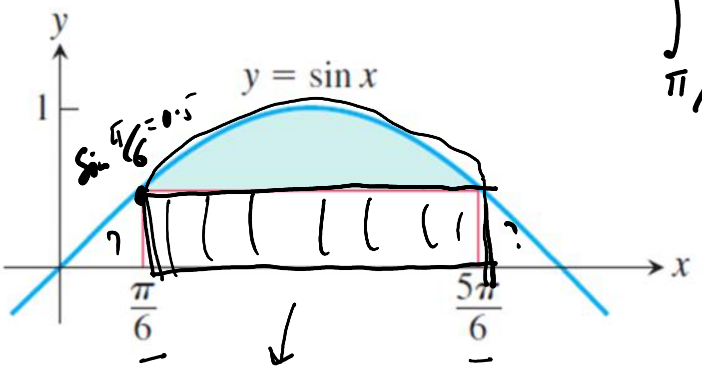

$$\int_{\pi/6}^{5\pi/6} \sin x \, dx = -\cos x \Big|_{\pi/6}^{5\pi/6}$$

$$= -\left(\cos\overset{150^\circ}{\frac{5\pi}{6}} - \cos\overset{30^\circ}{\frac{\pi}{6}}\right)$$

$$= -(-0.866 - 0.866)$$

$$= 1.73$$

$$\text{Rectangle} = \left(\frac{5\pi}{6} - \frac{\pi}{6}\right)(0.5)$$

$$= \frac{2\pi}{6} = 1.04$$

$$\text{Shaded Area} = 1.73 - 1.04$$
$$= 0.69$$