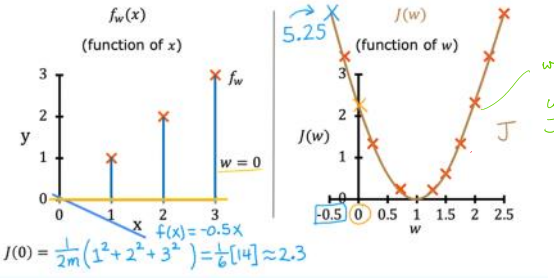
Gradient Descent

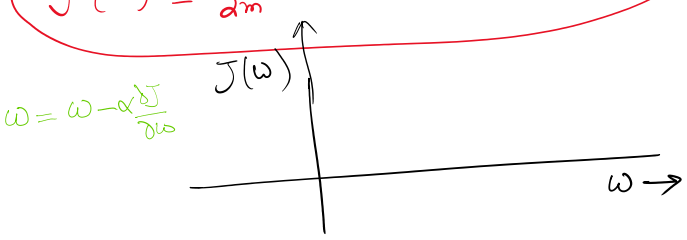
Thursday, March 13, 2025 9:32 PM

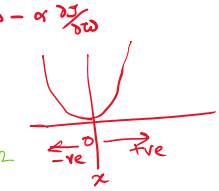
$$J(w,b) = \frac{1}{2m} \sum (wx^i + b - \hat{y}^i)^2 \qquad \hat{y} - y^i$$

$$J(w) = \frac{1}{2m} \sum (wx^i + b - y^i)^2$$

$w = w - \alpha \frac{\partial J}{\partial w}$







gradient descent

$J(w,b)$

not squared error cost

not linear regression

Gradient descent algorithm

Repeat until convergence

$$w = w - \alpha \frac{d}{dw} J(w,b)$$

$$b = b - \alpha \frac{d}{db} J(w,b)$$

Learning rate

Derivative

Simultaneously update w and b

Correct: Simultaneous update

$$tmp\_w = w - \alpha \frac{\partial}{\partial w} J(w,b)$$

$$tmp\_b = b - \alpha \frac{\partial}{\partial b} J(w, b)$$

$$w = tmp\_w$$

$$b = tmp\_b$$

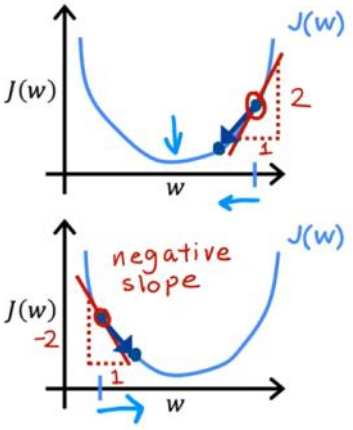
Incorrect

$$tmp\_w = w - \alpha \frac{\partial}{\partial w} J(w,b)$$

$$w = tmp\_w$$

$$tmp\_b = b - \alpha \frac{\partial}{\partial b} J(w, b)$$

$$b = tmp\_b$$



$$w = w - \alpha \frac{d}{dw} J(w)$$

$$w = w - \alpha \cdot (positive\ number)$$

$$\frac{d}{dw} J(w) < 0$$

$$w = w - \alpha \cdot (negative\ number)$$

$$\frac{\partial}{\partial w} J(w,b) = \frac{d}{dw} \frac{1}{2m} \sum_{i=1}^{m} \left(f_{w,b}(x^{(i)}) - y^{(i)} \right)^2 = \frac{d}{dw} \frac{1}{2m} \sum_{i=1}^{m} \left(wx^{(i)} + b - y^{(i)} \right)^2$$

$$= \frac{1}{2m} \sum_{i=1}^{m} \left(wx^{(i)} + b - y^{(i)} \right) 2x^{(i)} = \frac{1}{m} \sum_{i=1}^{m} \left(f_{w,b}(x^{(i)}) - y^{(i)} \right) x^{(i)}$$

$$\frac{\partial}{\partial b} J(w,b) = \frac{d}{db} \frac{1}{2m} \sum_{i=1}^{m} \left(f_{w,b}(x^{(i)}) - y^{(i)} \right)^2 = \frac{d}{db} \frac{1}{2m} \sum_{i=1}^{m} \left(wx^{(i)} + b - y^{(i)} \right)^2$$

$$= \frac{1}{2m} \sum_{i=1}^{m} \left(wx^{(i)} + b - y^{(i)} \right) 2 = \frac{1}{m} \sum_{i=1}^{m} \left(f_{w,b}(x^{(i)}) - y^{(i)} \right)$$

no $x^{(i)}$

$$w = w - \alpha \frac{d}{dw} J(w)$$

If α is too small...
Gradient descent may be slow.

If α is too large...

Gradient descent may:
- Overshoot, never reach minimum