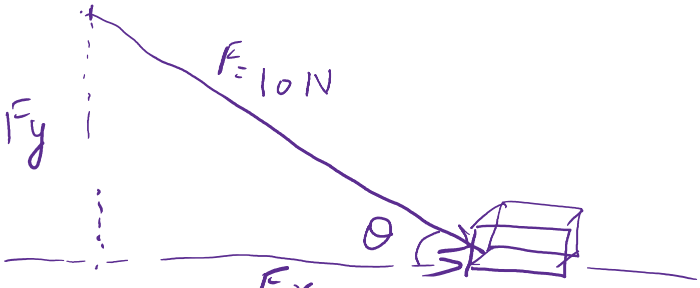
Dot Product

Saturday, 22 February 2025 9:35 am



$$\underline{F_x = F\cos\theta}$$

$$\boxed{W = \|F\| \, \|d\| \cos\theta}$$
$$= \vec{F} \cdot \vec{d}$$

$\theta = 45°$ $\quad d = 1\,m$

$F = 10N$ $\quad W = F_x \cdot d$

$$= F\cos\theta \cdot d$$
$$= \|F\| \cos\theta \cdot \|d\|$$

$$F \cdot d = \|F\| \, \|d\| \cos\theta$$

$$\cos\theta = \frac{F \cdot d}{\|F\| \, \|d\|} = \left(\frac{F}{\|F\|}\right) \cdot \left(\frac{d}{\|d\|}\right)$$

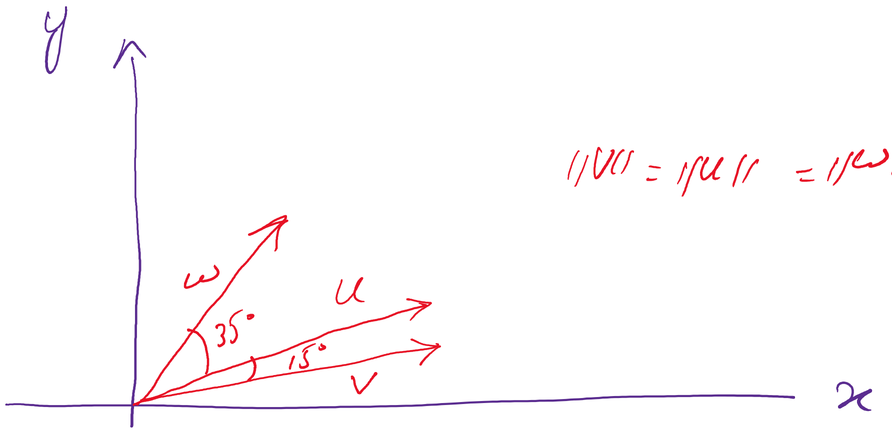
$$F = \begin{bmatrix} 7 \\ 3 \end{bmatrix} \begin{matrix} \rightarrow x \\ \rightarrow y \end{matrix} \qquad d = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \begin{matrix} \rightarrow x \\ \rightarrow y \end{matrix}$$

$$F \cdot d = 7 \times 3 + 3 + 2$$
$$= 21 + 6 = 27$$

$$V = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}, \quad \omega = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_m \end{bmatrix}$$

if $n == m$

$$V \cdot \omega = v_1\omega_1 + v_2\omega_2 + \cdots v_n\omega_m$$



$\|V\| = \|U\| = \|W\|$

$V \cdot U = \overbrace{\boxed{\|V\|\,\|U\|}}^{1} \cos\theta \quad 15° \rightarrow 0.96$

$V \cdot W = \underbrace{\boxed{\|V\|\,\|W\|}}_{1} \cos\theta \quad 50° \rightarrow 0.65$

$U \cdot W = \underbrace{\boxed{\|U\|\,\|W\|}}_{1} \cos\theta \quad 35° \rightarrow 0.81$

$\text{garden} = \begin{bmatrix} -0.25 \\ 0.165 \\ \vdots \\ \_ \end{bmatrix}_{4000}$ embedding vector

$\text{car} = \begin{bmatrix} 0.716 \\ \vdots \\ \\ \end{bmatrix}_{4000}$

$\text{garden} \cdot \text{car} \Rightarrow \cos\theta =$

Angle between 2-vectors

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$$\cos\theta = \frac{\overset{a \cdot b}{a_1 b_1 + a_2 b_2}}{\|\mathbf{a}\|\|\mathbf{b}\|}.$$

$= \hat{a} \cdot \hat{b}$

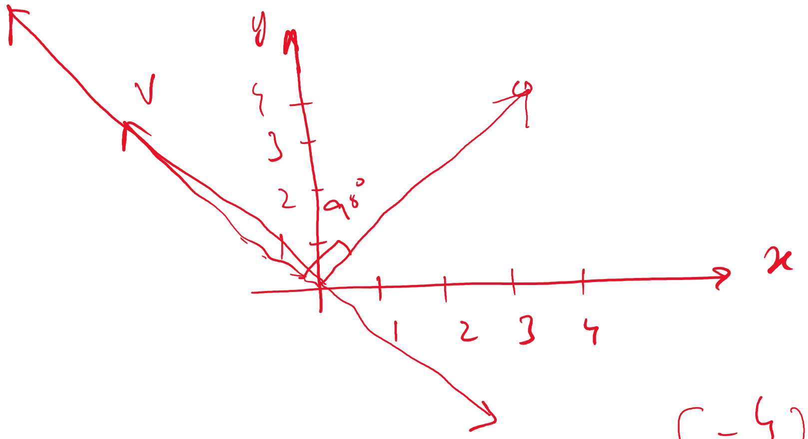
$a = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}, \quad b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

$\|a\| = \sqrt{a_1^2 + a_2^2}$

$\|b\| = \sqrt{b_1^2 + b_2^2}$

$u = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \quad v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \begin{matrix} -4/3, -4 \\ 1, 3 \end{matrix} \quad u \cdot v = 3v_1 + 4v_2 = 0$

$= \|u\| \|v\| \cos 90^\circ = 0$



$3v_1 + 4v_2 = 0$

$$\boxed{v_1 = -\frac{4}{3} v_2}$$

let $v_2 = 3$

$v_1 = -4$

$v = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$

$v = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}, \quad u = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

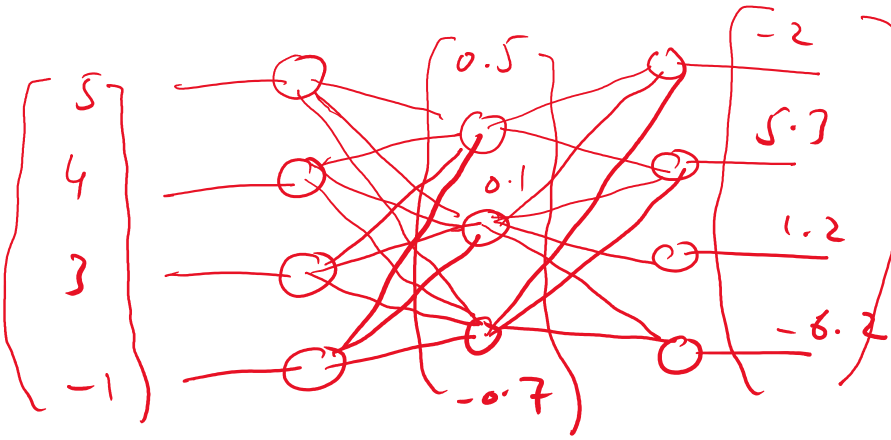
$v \cdot u = 2x - y + z = 0$

$\underbrace{2x}_{2} + \underbrace{z}_{2} = \underbrace{y}$

$\overset{2}{2x} - \overset{2}{y} = z$

$\begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$

$\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$


5
4
3
-1
0.5
0.1
-0.7
-2
5.3
1.2
-6.2

Angle between n-vectors

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$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \text{ and } \mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}.$$

The *dot product* of $\mathbf{x}$ and $\mathbf{y}$ is defined to be the scalar

$$\mathbf{x} \cdot \mathbf{y} = x_1 y_1 + x_2 y_2 + \cdots + x_n y_n = \sum_{i=1}^{n} x_i y_i.$$

The *angle* θ between two nonzero n-vectors $\mathbf{x}, \mathbf{y}$ is defined by the formula

$$\cos(\theta) = \frac{\mathbf{x} \cdot \mathbf{y}}{\|\mathbf{x}\| \|\mathbf{y}\|}$$

Properties of dot product

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For any n-vectors $\mathbf{v}$, $\mathbf{w}$, $\mathbf{w}_1$, and $\mathbf{w}_2$, the following hold:

(i) $\mathbf{v} \cdot \mathbf{w} = \mathbf{w} \cdot \mathbf{v}$,
(ii) $\mathbf{v} \cdot \mathbf{v} = \|\mathbf{v}\|^2$,
(iii) $\mathbf{v} \cdot (c\mathbf{w}) = c(\mathbf{v} \cdot \mathbf{w})$ for any scalar c, and $\mathbf{v} \cdot (\mathbf{w}_1 + \mathbf{w}_2) = \mathbf{v} \cdot \mathbf{w}_1 + \mathbf{v} \cdot \mathbf{w}_2$.
(iii′) Combining both rules in (iii), for any scalars c_1, c_2 we have

$$\mathbf{v} \cdot (c_1\mathbf{w}_1 + c_2\mathbf{w}_2) = c_1(\mathbf{v} \cdot \mathbf{w}_1) + c_2(\mathbf{v} \cdot \mathbf{w}_2).$$

$$v = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad w = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$$

Correlation coefficient

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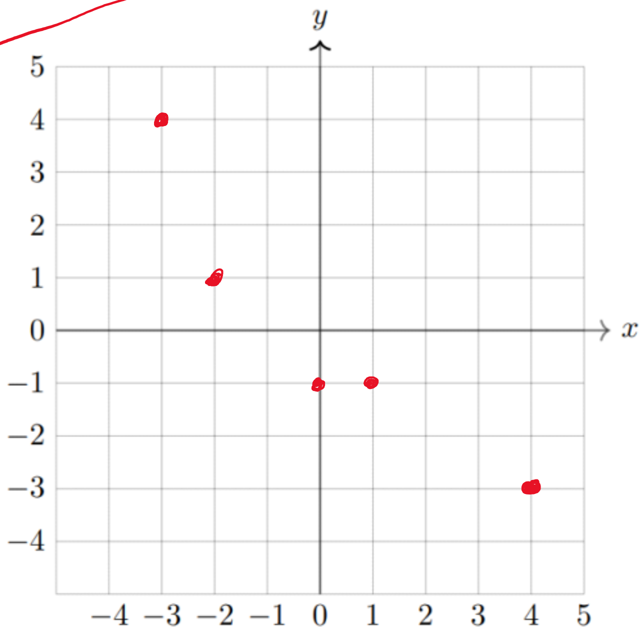
$$\boxed{r = -0.93}$$

$$(-3,4),\ (-2,1),\ (0,-1),\ (1,-1),\ (4,-3).$$



$$X = \begin{bmatrix} -3 \\ -2 \\ 0 \\ 1 \\ 4 \end{bmatrix}, \quad Y = \begin{bmatrix} 4 \\ 1 \\ -1 \\ -1 \\ -3 \end{bmatrix}$$

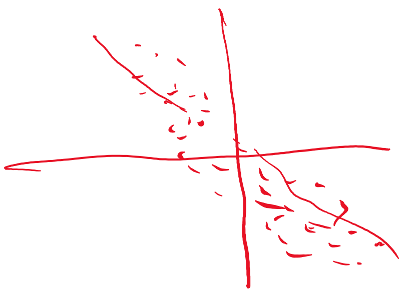
$$r = \cos\theta = \frac{X \cdot Y}{\|X\|\,\|Y\|} = \frac{-29}{\sqrt{30 \times 28}}$$

$$X \cdot Y = -12 - 2 + 0 - 1 - 12 = -27$$

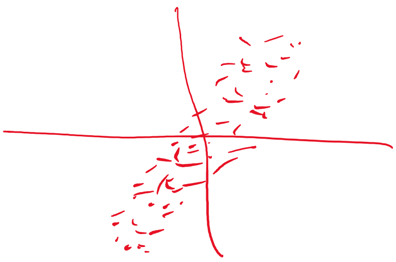
$$\|X\| = \sqrt{9 + 4 + 0 + 1 + 16} = \sqrt{30}$$

$$\|Y\| = \sqrt{16 + 1 + 1 + 1 + 9} = \sqrt{28}$$

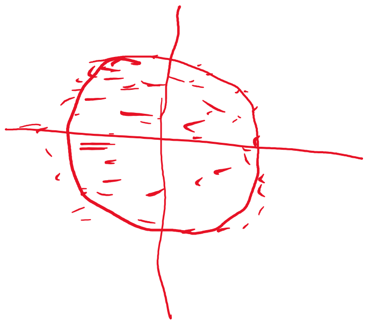
$r_1 = -\ ?$



$r_2 = +\ ?$



$r_3 \approx 0$



Problem: Find correlation coefficients

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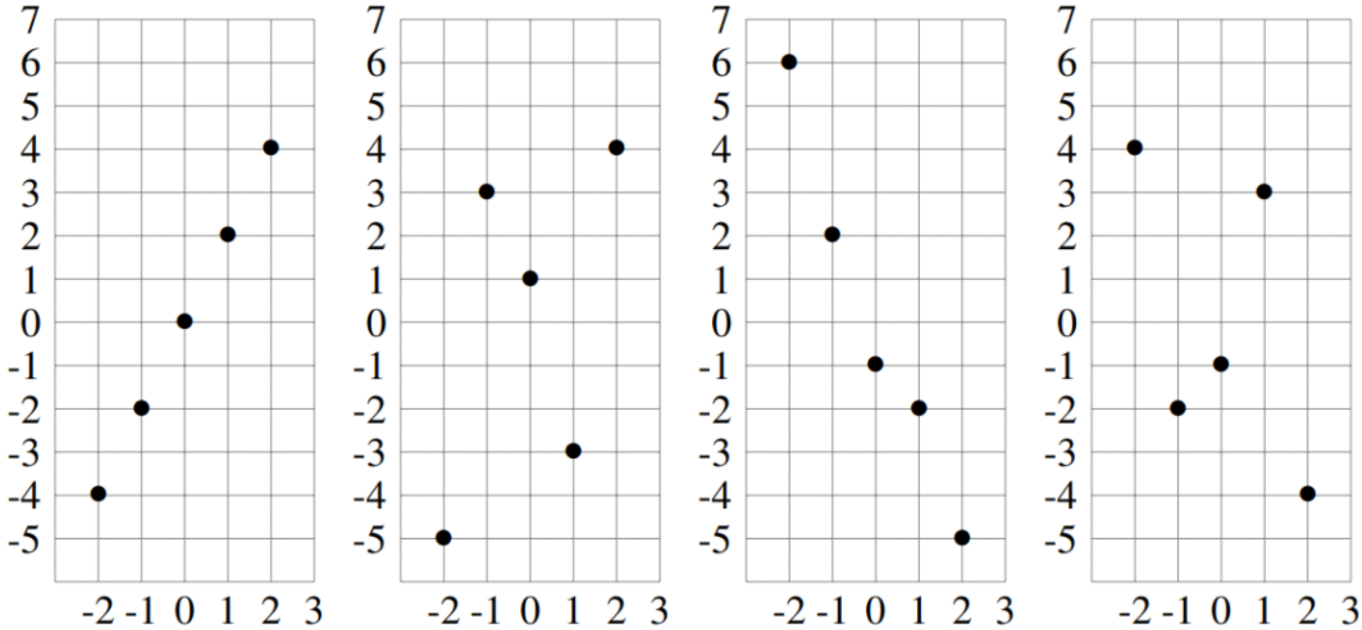
$r_1 = 1$ $r_2 = 0.68$ $r_3 = -0.98$ $r_4 = -0.5$



$$a = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \qquad \bar{a} = \frac{2+3+4}{3} = 3$$

$$a - \bar{a} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \qquad \bar{a} = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$$