Counting Theory

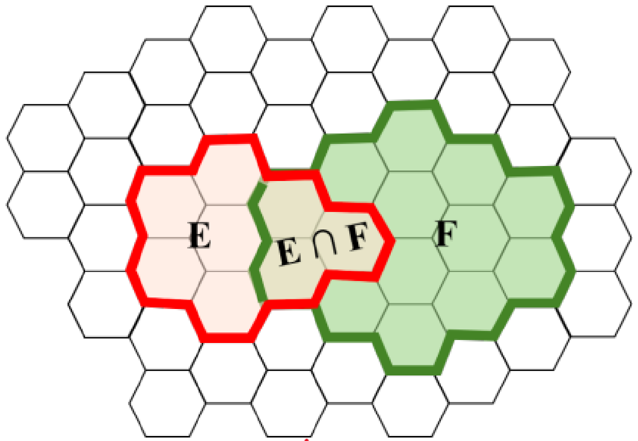
Core Probability

Random Variables

Probabilistic Models

Uncertainty Theory

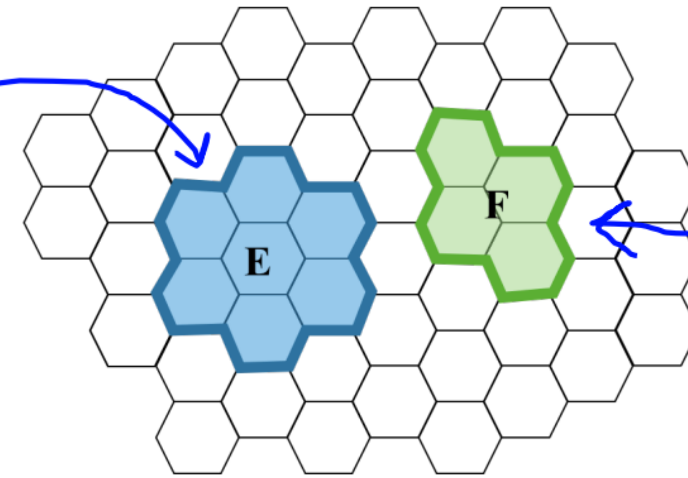
Machine Learning



$P(E) = \frac{8}{50}$

$P(E|F) = \frac{3}{14}$

7/50

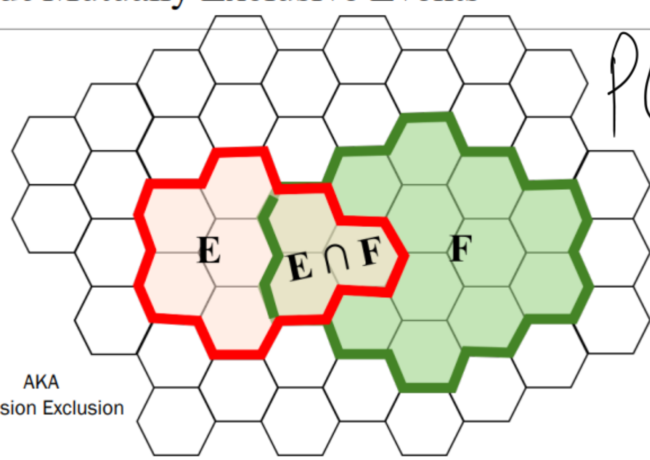


4/50

$P(E \cap F) = P(E|F)P(F)$

$$P(E \cup F) = P(E) + P(F)$$

OR ***without*** Mutually Exclusive Events

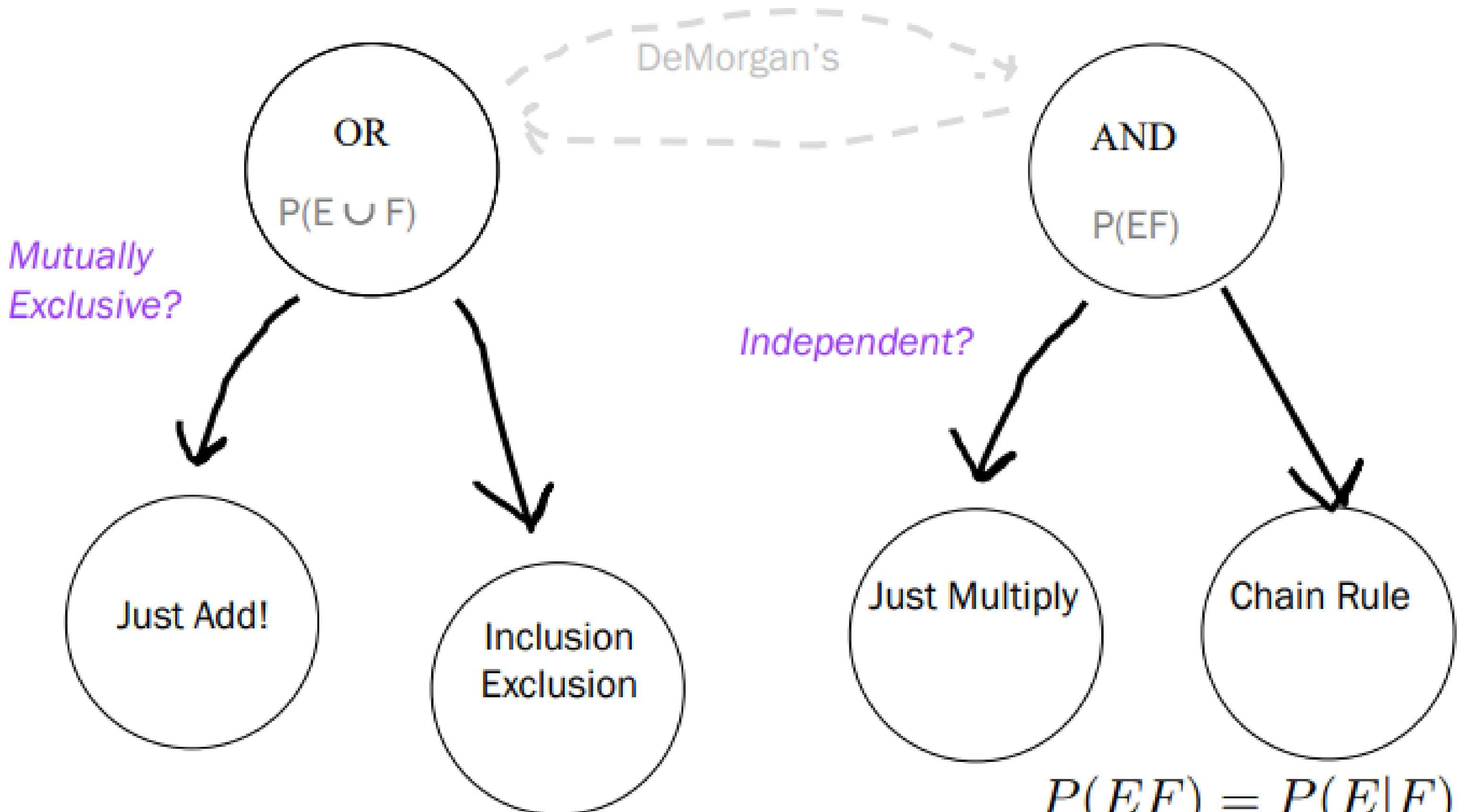


$P(E \cap F) = \frac{3}{50}$

$= P(E \cup F) =$

AKA
Inclusion Exclusion

$$P(E \cup F) = P(E) + P(F) - P(EF)$$



$$P(EF) = P(E|F)P(F)$$

$$P(E \cup F) = P(E) + P(F) - P(EF)$$

1000 intel

1

- n chips manufactured, 1 of which is defective.
- k chips randomly selected from n for testing.
  - What is P{defective chip is in k selected chips}?

hp = 50

$$P = \frac{|E|}{|S|}$$

Chip Defect Detection

1000

- n chips manufactured, 1 of which is defective.
- 50 → k chips randomly selected from n for testing.
  - What is P{defective chip is in k selected chips}?
- $|S| = \binom{n}{k}$
- $|E| = \binom{1}{1}\binom{n-1}{k-1}$
- P(defective chip is in k selected chips)

$$= \frac{\binom{1}{1}\binom{n-1}{k-1}}{\binom{n}{k}} = \frac{\frac{(n-1)!}{(k-1)!(n-k)!}}{\frac{n!}{k!(n-k)!}} = \frac{k}{n}$$

$$= \frac{\binom{1}{1}\binom{999}{\cancel{50}\,49}}{\binom{1000}{50}}$$

Say the population of Stanford is 17,000 people

- You are friends with 80 people?
- Walk into a room, see 62 random people.
- What is the probability that you see someone you know?
- Assume you are equally likely to see each person at Stanford

$$P = \frac{|E|}{|S|}$$

?

?

$$\frac{\binom{17000-80}{62}}{\binom{17000}{62}}$$

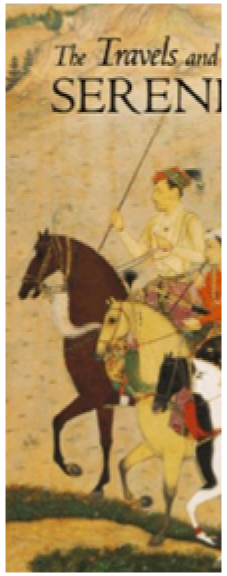
- Say the population of Stanford is 17,000 people
  - You are friends with 80 people?
  - Walk into a room, see 62 random people.
  - What is the probability that you see someone you know?
  - Assume you are equally likely to see each person at Stanford

$$|S| = \binom{17{,}000}{62}$$

$$|E^C| = \binom{17{,}000 - 80}{62}$$

$P(\text{see someone you know})$
$= P(\text{see 1 or more friends})$
$= 1 - P(\text{don't see anyone you know})$

$$P(E) = 1 - P(E^C) = 1 - \frac{|E^C|}{|S|} \approx 0.1914$$



Independence

$A \quad B$

$P(A \cap B) = P(A|B)\, P(B)$

Two events A and B are called **independent** if:

$P(A \cap B) = P(A)\, P(B)$

$$P(A) = P(A|B)$$

Knowing that event B happened, doesn't change our belief that A will happen.

$P(A \cap B \cap C \cap D \cap E \ldots$

Otherwise, they are called **dependent** events

M F

250 750

C T C T

100 150 300 450

M & C

M —— C

C —— M

$P(M|C)$ $\quad$ $P(M)$

$$\frac{100}{400} \qquad \frac{250}{1000}$$

$$\frac{1}{4} = \frac{1}{4}$$

$P(C|M)$ $\quad$ $P(C)$

$$\frac{100}{250} = \frac{400}{1000} = 2/5$$