CAI 2.0, Linear Algebra
Worksheet 3: Planes in $\mathbb{R}^3$

Problem 1: Different forms of a plane

Let $A = (1, 2, 3)$, $B = (2, 0, 1)$, and $C = (3, 1, 0)$.

(a) Use displacement vectors to show these points are not collinear.

(b) Find the equation of the plane P containing these three points.

(c) Give a parametric form for the plane P.

(d) Find the normal vector to the plane P.

Problem 2: Finding points on different sides of a plane

For the plane given by the equation $3x - 2y + 4z = 12$, determine whether the following points lie on the same side, opposite sides, or on the plane.

(a) $P = (1, 0, 2)$ and $Q = (3, 3, 3)$

(b) $R = (2, 2, 2)$ and $S = (4, 0, 0)$

(c) $T = (0, -2, 3)$ and the origin $(0, 0, 0)$

Problem 3: Converting between forms

(a) Convert the plane with equation $2x + 3y - z = 6$ to parametric form.

(b) Convert the plane given parametrically as $(1, 2, 0) + s(1, 0, 1) + t(0, 1, 2)$ to equation form.

Problem 4: Angle between planes

(a) Find the angle between the planes $x + y + z = 1$ and $x - y + z = 5$.

(b) Determine if the planes $2x - 4y + 6z = 3$ and $x - 2y + 3z = 5$ are parallel, perpendicular, or neither.

Problem 5: Intersection of planes

Find the line of intersection between the planes $P_1 : 2x - y + 3z = 4$ and $P_2 : x + 2y - z = 5$.

(a) Express the line in parametric form $P + tV$.

(b) Find a point on the line with $y = 0$.

Problem 6: Distance from a point to a plane

(a) Find the distance from the point $(3, 1, 4)$ to the plane $2x + 4y - 4z = 12$.

(b) Find the point on the plane $2x + 4y - 4z = 12$ that is closest to $(3, 1, 4)$.

Problem 7: Multiple planes

Three planes are given by the equations:

$$\begin{aligned} P_1 &: x + y + z = 1 \\ P_2 &: 2x - y + z = 0 \\ P_3 &: 3x + 2y - z = 2 \end{aligned}$$

(a) Do these three planes have a common point of intersection? If so, find it.

(b) If two of these planes are parallel, identify them.

(c) Find the angle between P_1 and P_2.