Practice Problem Set 01

CAI 2.0
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Problem 1. A machine learning model's accuracy follows the function $f(x) = -0.02x^2 + 0.8x + 70$, where x represents the number of training epochs (in hundreds).

- Plot this parabola for $x \in [0, 20]$
- Find the y-intercept and interpret its meaning
- For what values of x is the accuracy above 75%?

Problem 2. In a neural network, the activation function ReLU is defined as a piecewise function:

$$f(x) = \begin{cases} 0 & \text{for } x < 0 \\ x & \text{for } x \geq 0 \end{cases}$$

- Plot this function for $x \in [-5, 5]$
- Determine if this is an even or odd function
- Find the domain and range of the function

Problem 3. A drone delivery system operates in a 2D plane. The drone needs to deliver packages to three points: A(2,3), B(8,12), and C(-4,7).

- Plot these points on a coordinate system
- Find the distance between each pair of points
- Write the equation of the line connecting points A and B in point-slope form

Problem 4. Two AI-powered security cameras are mounted at points P(0,0) and Q(8,0). Their combined coverage forms a semi-circle.

- Write the equation of the semi-circle
- Determine if point R(3,4) lies within the coverage area
- Find the length of the curved boundary of the coverage area

Problem 5. A computer vision system uses rectangular coordinates to identify objects. Two objects are located at A(3,4) and B(-2,6).

- Find the midpoint of line segment AB
- Calculate the slope of line AB
- Write the equation of the perpendicular bisector of AB

Problem 6. The probability of correct classification in a binary classifier is given by:

$$p(x) = \frac{x^2}{x^2 + 1}$$

- Find the domain and range of this rational function
- Is this function even or odd?
- Plot the function for $x \in [-3, 3]$

Problem 7. A robot arm can move along the line $y = 2x + 5$.

- Plot this line and mark the y-intercept
- Find two points on this line where $x > 0$
- Write this line equation in two-point form using the points you found

Problem 8. An image processing algorithm maps pixel intensities using the function:

$$f(x) = \begin{cases} x^2 & \text{for } x < 1 \\ \sqrt{x} & \text{for } x \geq 1 \end{cases}$$

- Plot this piecewise function
- Find f(0.5), f(1), and f(4)
- Determine the range of the function

Problem 9. Three sensors form a triangle with coordinates A(0,0), B(6,0), and C(3,4).

- Verify if this triangle is right-angled using the Pythagorean theorem
- Find the area of the triangle
- Write equations for all three sides of the triangle

Problem 10. The accuracy of two different AI models are given by:

$$f(x) = 90 - \frac{100}{x+1}$$

$$g(x) = 75 + 15\cos(\frac{\pi x}{12})$$

where x is the training time in hours.

- Plot both functions on the same coordinate system
- Find the domain and range of each function
- Find g(6) and interpret its meaning

Problem 11. A data point P(x,y) needs to satisfy the constraint $x^2 + y^2 \leq 25$.

- Sketch the region that satisfies this constraint
- Determine if points (3,4) and (4,5) satisfy this constraint
- Write the equation of the boundary of this region

Problem 12. The path of a mobile robot is described by the equations:

$$x = 3\cos(t)$$

$$y = 3\sin(t)$$

- Identify the type of curve this represents
- Find the points where this path intersects the x-axis
- Find the points where this path intersects the y-axis

Problem 13. An object recognition system uses the exponential function $f(x) = 2^x$.

- Plot this function for $x \in [-2, 3]$
- Find f(0), f(1), and f(-1)
- Determine the y-intercept of this function

Problem 14. Two parallel lines in a computer vision system are given by:

$$y = 2x + 3$$

$$y = 2x - 1$$

- Plot both lines
- Find the vertical distance between these lines
- Write an equation for a line perpendicular to both lines that passes through (0,0)

Problem 15. A logarithmic function models data compression: $f(x) = log_2(x + 1)$

- Find the domain and range of this function
- Plot the function for $x \geq 0$
- Find f(1), f(3), and f(7)

Problem 16. Given vector components $\vec{a} = (3, 4)$ and $\vec{b} = (-1, 2)$:

- Plot these vectors on a coordinate system
- Find the magnitude of each vector
- Find the vector $\vec{c} = 2\vec{a} + \vec{b}$ and plot it

Problem 17. A semi-circle with radius 5 is centered at the origin, lying above the x-axis.

- Write the equation of this semi-circle
- Find the area of the region bounded by the semi-circle
- Determine if point (3,4) lies inside, outside, or on the semi-circle

Problem 18. The line $y = mx + b$ passes through points A(2,5) and B(4,9).

- Find the values of m and b
- Write the equation in point-slope form
- Find a point on this line where $x = 6$

Problem 19. A polynomial function $f(x) = x^3 - x$ models a system's behavior.

- Plot this function
- Determine if this is an even or odd function
- Find all points where $f(x) = x$

Problem 20. A square region is defined by vertices at (0,0), (4,0), (4,4), and (0,4).

- Find the equations of all four sides
- Calculate the length of each diagonal
- Determine if point (2,3) lies within the square