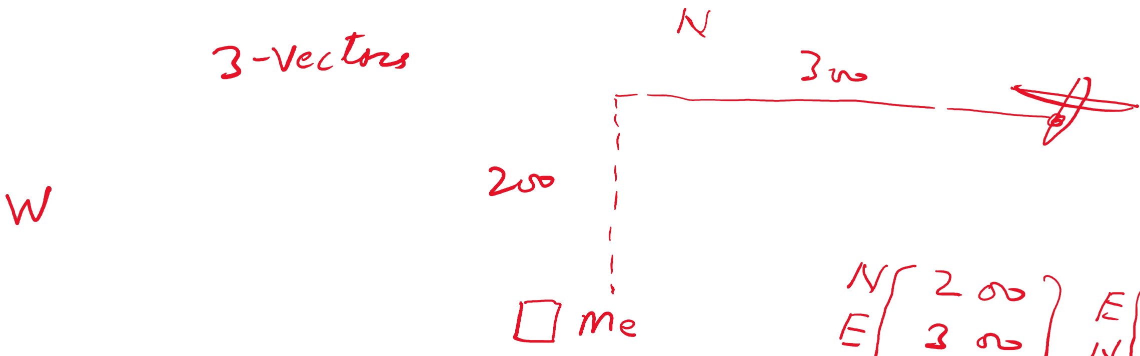
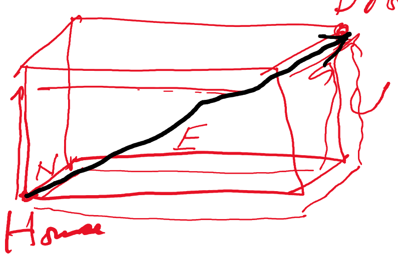
Vectors and Scalars

Thursday, 20 February 2025 2:20 pm

3-Vectors





Drone $(200, 300, 100)$

House $(0, 0)$

$$\begin{array}{c} N \\ E \\ U \end{array}\begin{pmatrix} 200 \\ 300 \\ 100 \end{pmatrix} \quad \begin{array}{c} E \\ N \\ U \end{array}\begin{pmatrix} 300 \\ 200 \\ 100 \end{pmatrix} \quad \begin{array}{c} U \\ E \\ N \end{array}\begin{pmatrix} 100 \\ 300 \\ 200 \end{pmatrix}$$

$$\boxed{\mathbf{a}} \qquad \vec{a} \qquad \overrightarrow{a} \qquad \hat{a}$$

$$a = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \qquad b = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$$

$$a = 2.5 \qquad b = -3.5$$

$$c = \pi$$

Vector Representation in 2D

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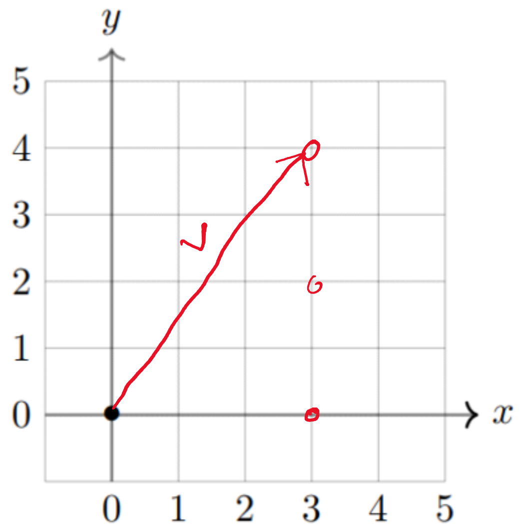
physics

CS

$[3, 4]$

Math

$\vec{v} + \vec{w}$



$$\mathbf{v} = \begin{array}{cl} \begin{bmatrix} 3 \\ 4 \end{bmatrix} & \begin{array}{l} \to x \\ \to y \end{array} \end{array}$$

Vector Representation in 3D

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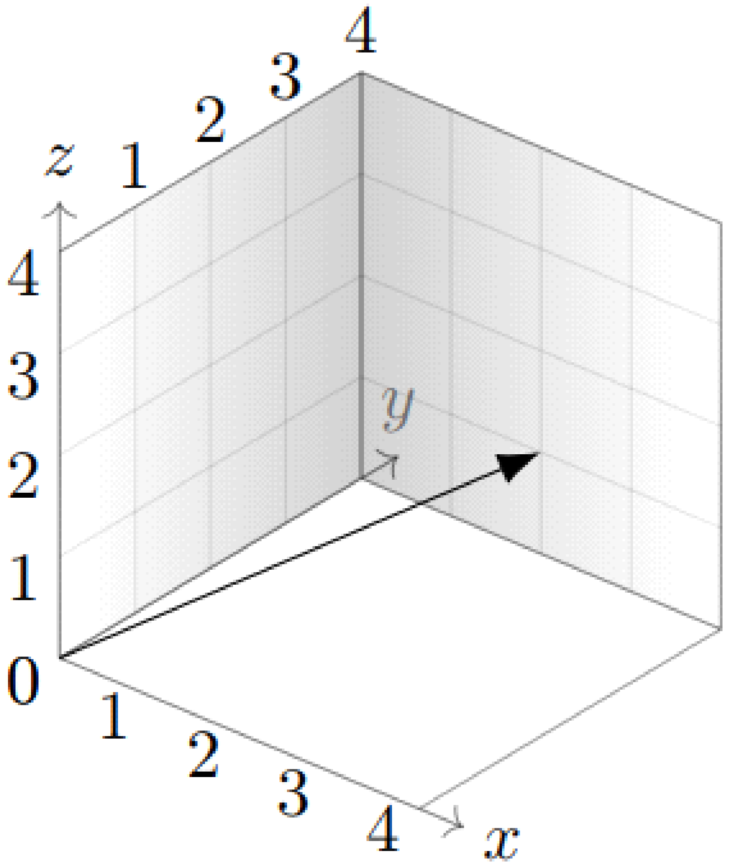

$$\mathbf{v} = \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}$$

n-Vcetor

Thursday, 20 February 2025 2:26 pm

$$V = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_n \end{bmatrix}$$

$$\begin{array}{c} x \\ y \end{array} \begin{bmatrix} 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \begin{array}{c} x \\ y \\ z \end{array}$$

Vector Addition 2D

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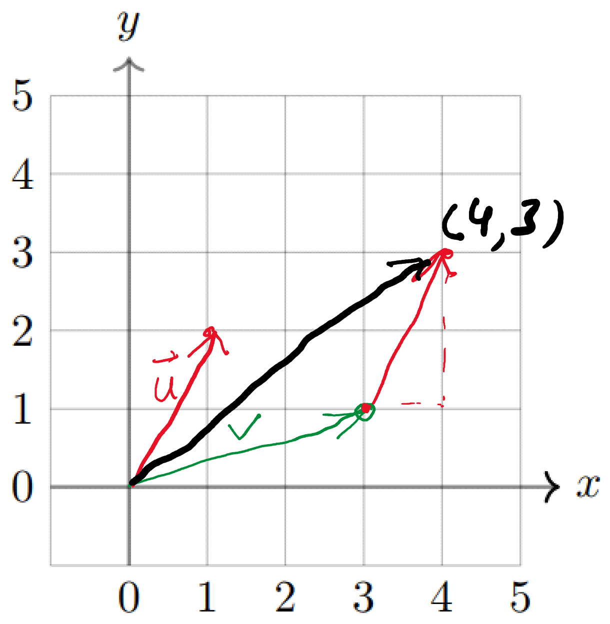


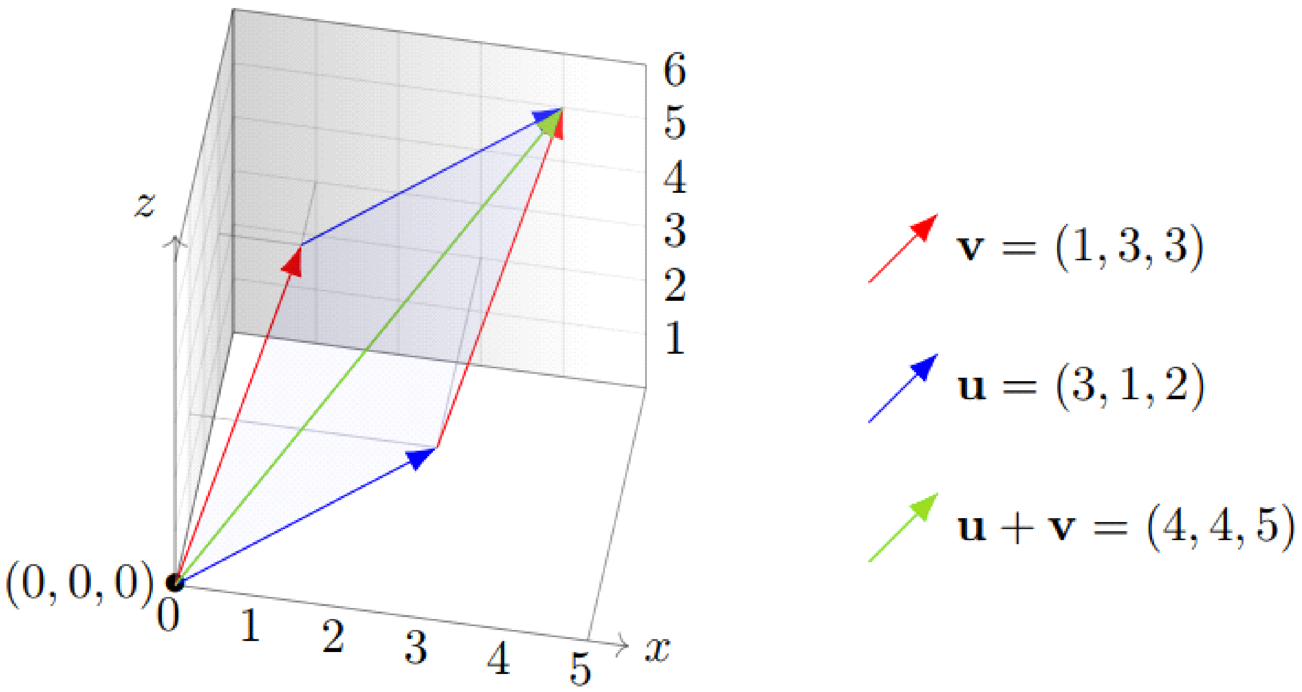
$$\mathbf{v} = (3, 1)$$

$$\mathbf{u} = (1, 2)$$

$$v + u = (3+1,\ 1+2)$$
$$= (4,\ 3)$$

Vector Addition in 3D

Thursday, 20 February 2025 2:31 pm



$$v = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}, \quad u = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix}$$

if $n = m$

$$v + u = \begin{bmatrix} v_1 + u_1 \\ v_2 + u_2 \\ \vdots \\ v_n + u_m \end{bmatrix}$$

n-Vector Addition

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$$\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} = \begin{bmatrix} v_1 + w_1 \\ v_2 + w_2 \\ \vdots \\ v_n + w_n \end{bmatrix}.$$

$$c \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} = \begin{bmatrix} cv_1 \\ cv_2 \\ \vdots \\ cv_n \end{bmatrix}.$$

$$a = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$-3a = \begin{bmatrix} -3 \\ -6 \end{bmatrix}$$

Linear Combination of Vectors

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(i) $\overset{v}{\begin{bmatrix}2\\1\\-8\end{bmatrix}} + \overset{u}{\begin{bmatrix}3\\2\\4\end{bmatrix}} \longrightarrow v+u$

(ii) $5 \cdot \overset{v}{\begin{bmatrix}-1\\8\end{bmatrix}} \longrightarrow 5v$

(iii) $\overset{v}{\begin{bmatrix}2\\1\\8\end{bmatrix}} - \overset{u}{\begin{bmatrix}3\\2\\4\end{bmatrix}} \longrightarrow v-u$

(iv) $3\begin{bmatrix}1\\2\\3\end{bmatrix} + 6\begin{bmatrix}1\\-1\\2\end{bmatrix} \longrightarrow 3v+6u$

(v) $2\begin{bmatrix}-1\\0\\1\\2\end{bmatrix} + 3\begin{bmatrix}2\\1\\-3\\4\end{bmatrix} - 5\begin{bmatrix}0\\2\\-2\\1\end{bmatrix} \longrightarrow 2v+3u-5w$

$\alpha_1, \alpha_2, \alpha_3 \in R$

$\overbrace{\alpha_1 v + \alpha_2 u + \alpha_3 w + \dots}$

Linear Combination

$\boxed{t^3} v + \boxed{\log t} u + \boxed{\sin t} w$

Nonlinear Combination

$$S1 = \begin{bmatrix}80\\70\\90\end{bmatrix} \qquad S_2 = \begin{bmatrix}60\\70\\50\end{bmatrix}$$

$$\frac{1}{2}(S_1 + S_2) = \underbrace{\frac{1}{2}S_1 + \frac{1}{2}S_2}$$

$$\alpha = \frac{1}{2}$$

$$\frac{1}{4}\alpha^2 v + \overset{1/2}{\alpha} u$$

$$x = \sin t$$
$$y = \cos t$$

$\alpha_1 = \underbrace{\sin \alpha_2 =}$ $\qquad \alpha_2 = 2$

$\cancel{8} v = \begin{bmatrix}2\\1\end{bmatrix} + \cancel{6} u = \begin{bmatrix}-1\\2\end{bmatrix}$

$v = (1), \qquad = (2)$

$2v + 2u$?

cc $\longleftarrow \frac{1}{2}v + \frac{1}{2}u$?

$\frac{1}{3}v + 3u \rightarrow$

cc $\longleftarrow \frac{1}{3}v + \frac{2}{3}u$

cc $\longleftarrow \frac{1}{5}v + \frac{4}{5}u$?

$-v - u \rightarrow$

$-7v - 5u$

? $\frac{1}{7}v - \frac{6}{7}u$

$\alpha_1 v + \alpha_2 u$

Convex Combination of Vectors

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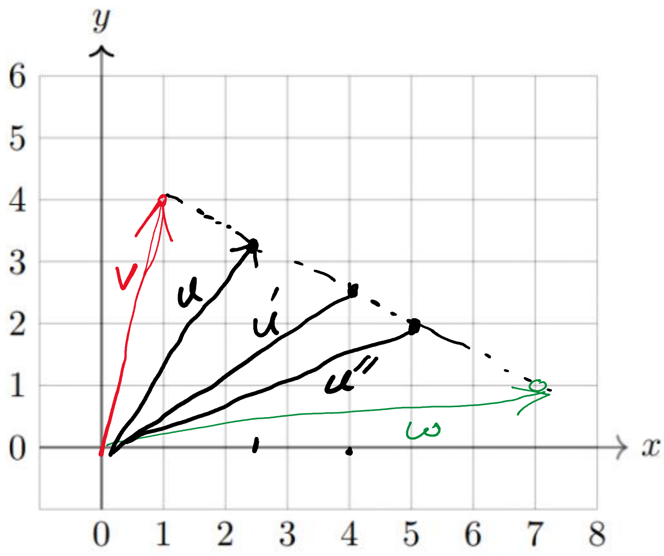

$u = (\frac{5}{2}, \frac{13}{4})$

$(\frac{3}{4}, 3) + (\frac{7}{4}, \frac{1}{4})$

$\mathbf{v} = (1, 4)$

$\mathbf{w} = (7, 1)$

$\mathbf{u} = (3/4)\mathbf{v} + (1/4)\mathbf{w} = \frac{3}{4}(1, 4) + \frac{1}{4}(7, 1)$

$\mathbf{u}' = (1/2)\mathbf{v} + (1/2)\mathbf{w} = (4, \frac{5}{2})$

$\mathbf{u}'' = (1/3)\mathbf{v} + (2/3)\mathbf{w} = (5, 2)$

$\alpha_1 \qquad \alpha_2$

40% 30% 30%

$$Q_1 = \begin{bmatrix} 70 \\ 80 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} 90 \\ 50 \end{bmatrix}, \quad Q_3 = \begin{bmatrix} 70 \\ 70 \end{bmatrix}$$

$$0.4Q_1 + 0.3Q_2 + 0.3Q_3 = \begin{bmatrix} 76 \\ 68 \end{bmatrix}$$

$$\begin{bmatrix} 28 \\ 32 \end{bmatrix} + \begin{bmatrix} 27 \\ 15 \end{bmatrix} + \begin{bmatrix} 21 \\ 21 \end{bmatrix}$$

Convex Combination of n-Vectors

Thursday, 20 February 2025 2:41 pm

For any n-vectors $\mathbf{v}_1, \ldots, \mathbf{v}_k$, a *convex combination* of them means a linear combination

$$t_1\mathbf{v}_1 + \cdots + t_k\mathbf{v}_k$$

for which all $t_j \geq 0$ and the sum of the coefficients is equal to 1; that is, $t_1 + \cdots + t_k = 1$

Example: Draw Convex Combinations

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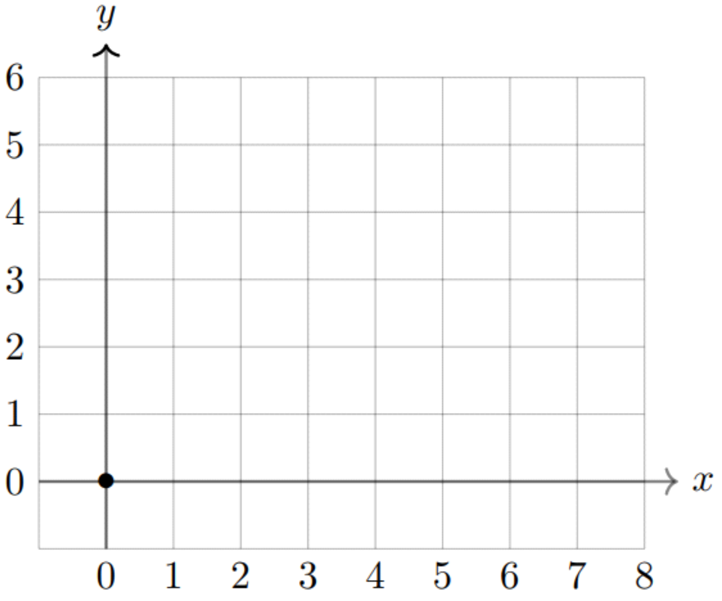

$$\mathbf{v}_1 = (2, 2)$$

$$\mathbf{v}_2 = (7, 1)$$

$$\mathbf{v}_3 = (1, 5)$$

$$\mathbf{w} = (1/3)\mathbf{v}_1 + (1/3)\mathbf{v}_2 + (1/3)\mathbf{v}_3$$

$$\mathbf{u} = (1/10)\mathbf{v}_1 + (4/5)\mathbf{v}_2 + (1/10)\mathbf{v}_3$$

Example: Draw Convex Combinations

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$a = [1\ 1]^T$
$b = [2\ 4]^T$
$c = [4\ 1]^T$
$d = [5\ 4]^T$

$(i)\ \frac{1}{4}a + \frac{1}{4}b + \frac{1}{4}c + \frac{1}{4}d$

$(ii)\ \frac{1}{2}a + \frac{1}{6}b + \frac{1}{6}c + \frac{1}{6}d$

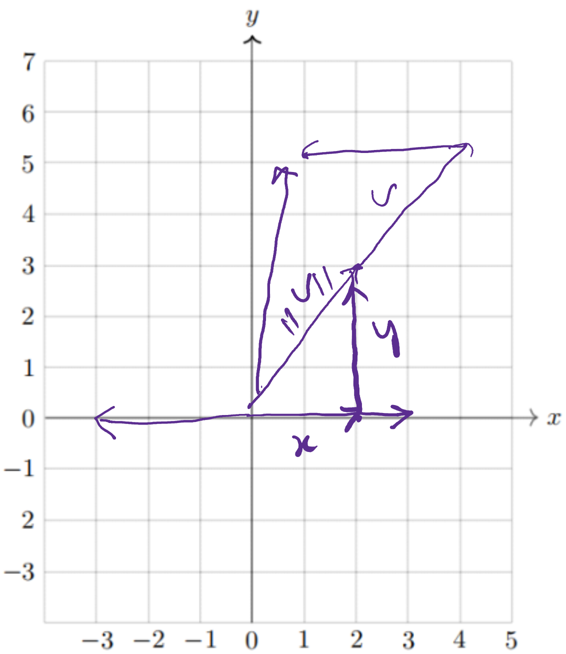
Generalized Convex Combination

Thursday, 20 February 2025 2:50 pm

$$(1-t)\mathbf{v} + t\mathbf{w} = \mathbf{v} + t(\mathbf{w} - \mathbf{v}) \text{ with } 0 \le t \le 1$$

Interpretation of 2D Velocity Vectors

Thursday, 20 February 2025 2:53 pm



$\mathbf{v} = (2, 3)$

$\mathbf{w} = (3, 0)$

$2\mathbf{v}$

$\mathbf{v} - \mathbf{w}$

$-\mathbf{w}$

$\mathbf{v}$

$v = 120$ km/h

$w = 80$ km/h

$$\|v\| = \sqrt{x^2 + y^2}$$

Length of a Vector

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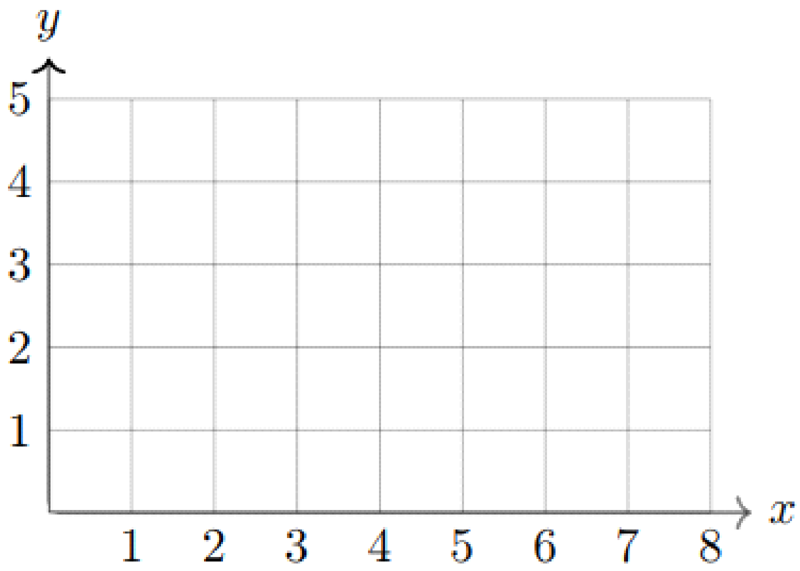
$$\begin{bmatrix} 0.3 \\ -0.7 \\ 2.4 \end{bmatrix}$$

Distance Between 2 Vectors

Thursday, 20 February 2025 3:00 pm

. For $\mathbf{v} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$, the distance between them is the length of the difference



Unit Vector

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$$\mathbf{v} = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$$

$$\|v\| = \sqrt{(-2)^2 + 1^2 + 2^2} = \sqrt{4+1+4}$$
$$= \sqrt{9} = 3$$

$$\hat{v} = \frac{v}{\|v\|} = \frac{1}{3}\begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -2/3 \\ 1/3 \\ 2/3 \end{bmatrix}$$

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$a = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 3 \end{bmatrix}, \quad c = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

$u = \frac{1}{3}a + \frac{1}{3}b + \frac{1}{3}c$

$= \begin{bmatrix} 1/3 \\ 1 \end{bmatrix}$