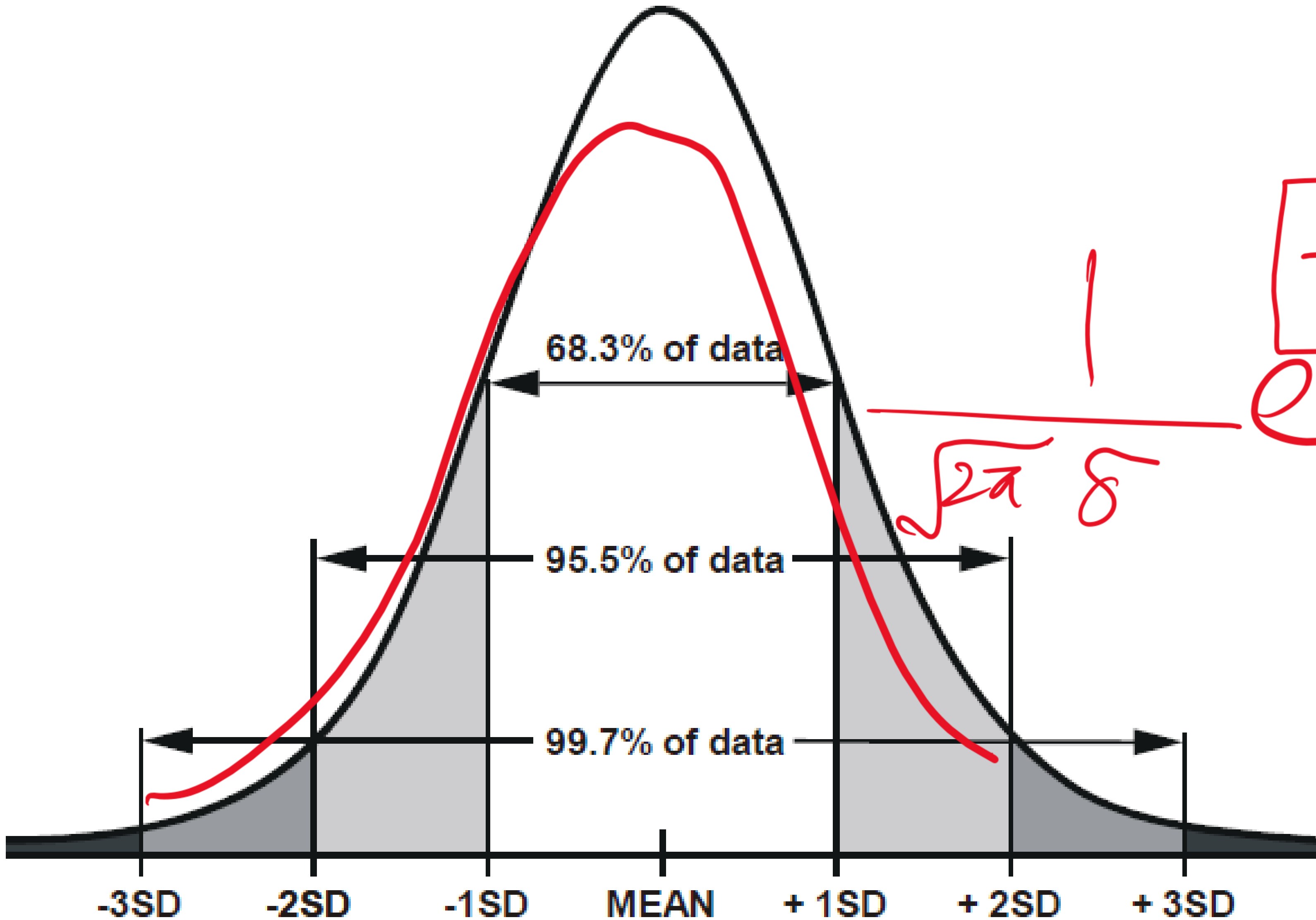
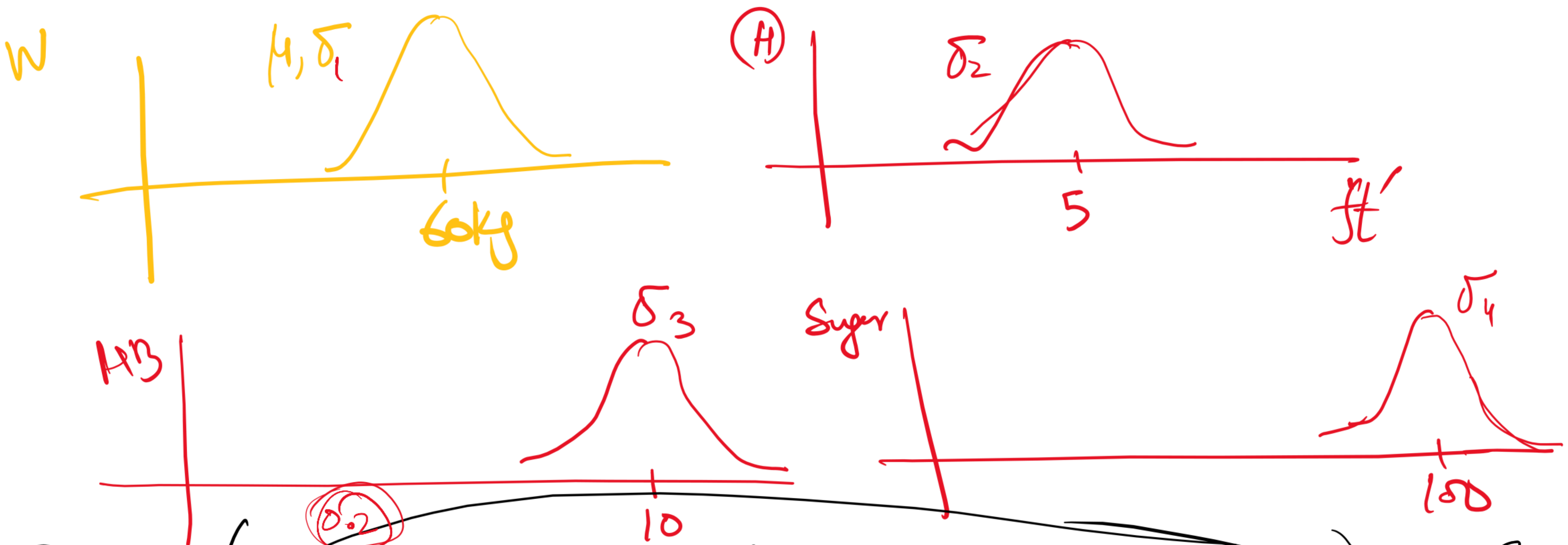
Figure 3.9
Areas under the normal curve that lie between 1, 2, and 3 standard deviations on each side of the mean



$$\frac{1}{\sqrt{2\pi}\,\delta} e^{\left[\frac{-(x-\mu)^2}{2\delta^2}\right]}$$



$$P(59\text{-}61,\ 5.1'\text{-}5.3'',\ 11\text{-}12,\ 85\text{-}90) = ?$$

$\sigma\ .2$ $\sigma\ .1$ $\sigma\ .15$ $\sigma\ .01$

Bivariate Gaussian , Multivariate Gaussian

Univariate Gaussian Distribution

$$p(x;\mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2\sigma^2}(x-\mu)^2\right).$$

$(x-\mu)(x-\mu))$

Multivariate Gaussian Distribution

$$p(x;\mu,\Sigma) = \frac{1}{(2\pi)^{n/2}|\Sigma|^{1/2}} \exp\left(-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)\right).$$

Criteria	Covariance	Correlation
Definition	Measures the degree of joint variability between two variables.	Standardized measure of the linear relationship between two variables.
Formula	$\mathrm{Cov}(X, Y) = \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{n}$	$\mathrm{Corr}(X, Y) = \frac{\mathrm{Cov}(X,Y)}{\sigma_X \cdot \sigma_Y}$
Interpretation	Positive: Variables move in the same direction. Negative: Variables move in opposite directions. Zero: No linear relationship.	$\rho = 1$: Perfect positive correlation. $\rho = -1$: Perfect negative correlation. $\rho = 0$: No linear correlation.

covariance

$$Cov[X, Y] = E[(X - E[X])(Y - E[Y])] = E[XY] - E[X]E[Y].$$

For the **bivariate normal distribution**, where we have two jointly correlated normal random variables X_1 and X_2, the probability density function (PDF) is given by:

$$f(X_1, X_2) = \frac{1}{2\pi|\mathbf{\Sigma}|^{1/2}} \exp\left(-\frac{1}{2}\begin{bmatrix} X_1 - \mu_1 \\ X_2 - \mu_2 \end{bmatrix}^T \mathbf{\Sigma}^{-1} \begin{bmatrix} X_1 - \mu_1 \\ X_2 - \mu_2 \end{bmatrix}\right)$$

- The mean vector is:

$$\boldsymbol{\mu} = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}$$

- The covariance matrix is:

$$\mathbf{\Sigma} = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{bmatrix}$$

- The determinant of the covariance matrix is:

$$|\mathbf{\Sigma}| = \sigma_1^2\sigma_2^2 - \sigma_{12}^2$$

- The inverse of the covariance matrix is:

$$\mathbf{\Sigma}^{-1} = \frac{1}{|\mathbf{\Sigma}|}\begin{bmatrix} \sigma_2^2 & -\sigma_{12} \\ -\sigma_{12} & \sigma_1^2 \end{bmatrix}$$

Expanding the quadratic form in the exponent:

$$\begin{bmatrix} X_1 - \mu_1 \\ X_2 - \mu_2 \end{bmatrix}^T \mathbf{\Sigma}^{-1} \begin{bmatrix} X_1 - \mu_1 \\ X_2 - \mu_2 \end{bmatrix} = \frac{(X_1 - \mu_1)^2 \sigma_2^2 - 2\sigma_{12}(X_1 - \mu_1)(X_2 - \mu_2) + (X_2 - \mu_2)^2 \sigma_1^2}{\sigma_1^2 \sigma_2^2 - \sigma_{12}^2}$$

Thus, the final PDF for the **bivariate normal distribution** is:

$$f(X_1, X_2) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \exp\left(-\frac{1}{2(1-\rho^2)} \left[\frac{(X_1 - \mu_1)^2}{\sigma_1^2} + \frac{(X_2 - \mu_2)^2}{\sigma_2^2} - \frac{2\rho(X_1 - \mu_1)(X_2 - \mu_2)}{\sigma_1 \sigma_2} \right] \right)$$

where:

$$\rho = \frac{\sigma_{12}}{\sigma_1 \sigma_2}$$
