

Let $P(A) = 0.2$, $P(A \cup B) = \frac{1}{2}$ and $P(B) = p$. For what val of p , A and B are indpt

Ans = $3/8$

② Given A, B , are indpt events with $P(C) = 0.2$, $P(A \cup B) = 0.3$
Then $P((A \cup B) \cap C) =$

Ans : $3/50$

③ Ajay is known to truth in 5 cases out of 6. He states that a white ball is drawn from a bag which contains 8 B + 1 W balls. What is the prob that white Ball is drawn

Ans = ~~$5/13$~~ $5/13$

④ A diagnostic test has 99% accuracy & 60% of all ppl have covid-19. What is the prob that a randomly chosen person will be tested +ve?

Ans

0.598

⑤ The prob that in a randomly selected permutation of the word 'AROMA', all 3 vowels are together is _____

Ans

$3/10$

⑥ A d.r.v has pdf $f(x) = \begin{cases} \frac{x}{18} & x=2, 4, 6 \\ \frac{(x-2)}{18} & x=8 \\ 0 & \text{else} \end{cases}$. $F(5) =$ _____

Ans = $1/3$

⑦ The length of time in minutes that a consumer goes in a post off is a r.v T w/ pdf $f(t) = \begin{cases} c(9-t) & 0 \leq t \leq 9 \\ 0 & \text{else} \end{cases}$

$c = ?$

Ans

$2/81$

⑧ $E(X) = 3$, $E(X^2) = 13$. Then lower bound for $P(|X-3| < 1)$ is $\frac{3}{4}$

⑨ $X \sim U(0, 5)$. $P(X > 2) = \underline{\hspace{2cm}}$
Ans
 $\frac{3}{5}$

⑩ Mean & var of Binomial dist are $16/3$, $16/9$. Then $n = 8$

⑪ If 5% of the items produced by a machine are defective. Then prob that there are 3 def items in a sample of 100 items using Poisson distⁿ
Ans
 0.1404

⑫ If $f(x, y) = x + y$, $0 < x < 1$, $0 < y < 1$, Then $E(X) = \underline{\hspace{2cm}}$
Ans
 $7/12$

⑬ $X \sim N(\mu, 1)$. Then $P(X > \mu) = \underline{\hspace{2cm}}$
Ans
 0.5

⑭ $X \sim N(75, 25^2)$. $P(X < 80) = \underline{\hspace{2cm}}$
Ans = 0.8413

⑮ $f(x, y) = 4x(1-y)$, $0 < x < 1$, $0 < y < 1$. Find $E(XY)$
Ans = $2/9$

⑯ If X has pdf $f(x) = \frac{1}{54} x^2 e^{-x/3}$, $x > 0$, $E(X) = \underline{\hspace{2cm}}$
Ans = 9

3) 1) 2 fair dice are rolled, what is the prob that the sum is 7
Ans = $6/36$

2) X has pdf $f(x=x) = \frac{c}{2^k}$ $k=0,1,2,\dots$ The $c =$ —
Ans = $1/2$

3) X has pdf $f(x) = \begin{cases} 4x-4x^3 & 0 < x < 1 \\ 0 & \text{else} \end{cases}$. Then mode —
Ans = $1/\sqrt{3}$

20) A student takes MCQ containing 2 probs. The first one has 3 possible ans & the second one has 5. The student selects one ans @ random for both probs. Let X be the no of right ans. Then cdf of X for $0 \leq x < 1$
Ans = $8/15$

21) If X, Y are indpt with $V(X)=0$, $V(Y)=1$. Then $V(2X-Y) =$ —
Ans = 1

22) $V(2X+3Y) =$ —

23) A, B are mutually exclusive with $P(A)=2/5$ $P(B)=1/4$
Then $P(\overline{A} \cap \overline{B}) = P(\overline{A} \cap B) =$ —
Ans = $5/20$

24) If $P(A)=1/3$, $P(A \cup B)=1/2$. If A, B are indpt, then $P(A'|B') =$ —
Ans = $2/3$

25) If A & B are equally likely events in a sample space s.t $P(A \cup B) = 2P(A \cap B)$. Then $P(B|A) =$ —
Ans = $2/3$

- (26) 4 dice are thrown simultaneously. Find the prob th all of them show same face

$$\text{Ans} = \frac{1}{6^4}$$

- (27) If X, Y are a r.v having pdf $f(x, y) = \frac{6-x-y}{8}$, $0 < x < 2$
 $2 < y < 4$

$$\text{Then } P(Y < 3) = \underline{\hspace{2cm}}$$

$$\text{Ans} = \frac{5}{8}$$

- (28) If c.v. X has normal distⁿ with mean 0 & var 4.

$$P(X > 3) - P(X < -3) = \underline{0}$$

- (29) $X \sim U[-2, 2]$. Then $P(|X-1| \leq 1/2) = \underline{\hspace{2cm}}$

$$\text{Ans} = 1/4$$

① 2 squares of size 1x1 are chosen from 6x6 chess board. What is the prob that they have side in common

Ans = $\frac{1}{18}$

② 2 dice are thrown, $X = \text{sum}$. Use Chebyshev's inequality to find bound for $P(4 < X < 10)$. & verify

Ans

$P(4 < X < 10) \geq 0.35$

$P(4 < X < 10) = 0.65$

③ Let X_1, X_2, X_3 be uncorrelated R.V with $\sigma = 5, 10, 24$ resp. Find ρ_{UV} when $U = X_1 + X_2, V = X_2 + X_3$

Ans

$\rho_{UV} = 0.34$

④ A coin is tossed until H is obtained. What is the prob that the total no of tosses is not divisible by any of 2, 3 & 5?

Ans = 0.508

⑤ A bag has 3R + 4G marbles. 5 marbles are drawn simultaneously at random. Then a fair coin is tossed as many times as no of G marbles are obtained. Given that the total no of H obtained in these tosses is exactly 2, what is the prob that the no of G marbles drawn is first step is 3?

$B_1: 2G + 3R$

$B_2: 3G + 2R$

$B_3: 4G + 1R$

A: 2 heads

$P(B_1) = \frac{{}^4C_2 {}^3C_3}{{}^7C_5}$

$P(B_2) = \frac{{}^4C_3 {}^2C_2}{{}^7C_5}$

$P(B_3) = \frac{{}^4C_4 {}^1C_1}{{}^7C_5}$

$P(A|B_1) = \left(\frac{1}{2}\right)^2$
HH

$P(A|B_2) = \left(\frac{1}{2}\right)^3 \times 3$
HHT, THH

$P(A|B_3) = \left(\frac{1}{2}\right)^4 \times 6$
HHTT, THTT, HTHT, TTHT, HTTH, TTHH

$P(B_2|A) = \frac{12}{19}$

⑥ Let $X \sim N(\mu, \sigma^2)$ with mean $\mu = 5$, $\sigma^2 = 25$. Find $P(X > 1)$

Soln

$$\text{Ans} = 1 - \left(\frac{4}{5}\right)^{25}$$

⑦ The dia of a steel rod follows normal distn. The rods are accepted if the dia is wch 0.01 cm. from 3 cms, otherwise they are rejected. If 5% of the rods are rejected as oversized. and 10% are undersized, then find the mean & var of dia

Soln

$$P(X > 3) = 0.05 \text{ and } P(X < 0.01) = 0.1$$

$$\text{Ans } \mu = 2.999 \quad \sigma = 0.0068$$

⑧ In a large restaurant, an avg 3 out of every 5 customers ask for water with their meal. A random sample of 10 customers ~~are~~ is selected. What is the prob that less than 9 cust ask for water?

Ans

$$0.954$$

⑨ $X \sim N(15, 3.5^2)$, find

- i. $P(X > 16.25)$
- ii. $P(5 < X < 16.25) - P(5 < X < 14)$
- iii. $P(X > 14)$

⑩ If observed that in a communication channel, 90% of the msgs sent are received without any err. Find the prob that among 18 msgs sent, at least 6 are wdt err?

$$\text{Ans} = 0.73$$

⑪ The no of visits to a webpage follows a poisson's distn. The avg no of visits per min is 4. What is the prob that webpage receives atleast 2 visits in one min

$$\text{Ans} = 0.9$$

let (X, Y) be a 2D.r.v with $f(x, y) = \begin{cases} 6xy & 0 \leq x \leq 1 \\ 0 & 0 \leq y \leq \sqrt{x} \\ \text{else} \end{cases}$

Find marginal pdfs & expect^{ns}

of X & Y

Soln

$$g(x) = 3x^2, 0 \leq x \leq 1, \quad h(y) = 3y(1-y^4), 0 \leq y \leq 1$$

$$E(X) = 3/4$$

- 13) There are 2 bags A & B. A has 2w + n balls and B has nw + 2B balls. One of the 2 bags is selected @ random & 2 balls are drawn from it w/o replacement of both are white & the prob that the bag B was used to draw the balls is $6/7$, find n

Ans

$$n = 4$$

- 14) Find 'c' s.t $f(x) = \begin{cases} cx & 0 < x < 2 \\ c + 0.5 & 2 < x < 3 \\ cx^2 + cx & 3 < x < 5 \\ 0 & \text{else} \end{cases}$ is valid pdf
Find $E(X)$

Ans

$$c = 0.0114$$

- 15) 2 cards are drawn at random from a box which has 5 cards numbered 1, 2, 3, 4 & 5. Find joint pdf of (X, Y) where $X \leftarrow \text{sum}$ & $Y \leftarrow \text{max}$ of 2 no^s drawn.
Find f_{xy} .

Soln

$$f = 0.8660$$

- 16) On normal distⁿ, 84.13% of items are under 24, 10% of items are over 25. Find mean & var

Soln

$$\mu = 20.4286$$

$$\sigma = 3.5714$$

17) X is a c.v. with $f(x) = xe^{-x}$, $x > 0$. Find $V(X)$

Ans $V(X) = 2$

18) If A, B are indpt, p.t. i) \bar{A}, B are indpt
ii) $\bar{A} \& \bar{B}$ "

19) Each bag in a large box contains 25 tulip bulbs. It is known that 60% of the bag contain bulbs of 5R + 20Y tulips while ~~the~~ the remaining 40% of the bags contain bulbs of 15R + 10Y tulips. A bag selected @ random & a bulb selected ~~at random~~ from this bag is planted. i) What is the prob that it is Yellow tulip
ii) Given that it is Yellow tulip, what is the prob that it comes from a bag that had 5R + 20Y bulbs

Ans
i) $\frac{16}{25}$ ii) 0.75

20) The length of the electric bulbs follow normal distn with mean 1000 hrs & SD 200 hrs. After what period of burning, one would expect that 10% of the bulbs will fail?

Ans = 742

21) Suppose that 0.01% of the population of the city had tot pop 10000 suffer from a disease. Find the prob that there is atleast 2 ppl, who suffer from the disease. If there are 10 such cities ~~each~~, in a state, what is the prob that atleast one city will have atleast one person who suffer from the disease?

i) 0.02462

ii) ~~0.02462~~ 0.9999

(X, Y) has joint pdf $f(x, y) = 9e^{-3y}$, $y > x > 0$.

Find marginal pdfs of x and y

Ans find f_{xy}

soln



$$g(x) = 3e^{-x}, \quad 0 < x < \infty$$

$$h(y) = 9ye^{-3y}, \quad 0 < y < \infty$$

$$E(X) = 1/3 \quad E(Y) = 2/3 \quad E(XY) = 1/9 \quad E(X^2) = 2/9$$

$$V(X) = 1/9 \quad V(Y) = 2/9 \quad \rho = -1/2$$

Q3) A box has 2 balls, one white + one red. I select one ball, I select one ball, put it back to the box & select another ball. Let T be the event of getting the white ball twice, F be the event of picking white ball first, S be the event of picking white ball in the second drawing. Find $P(T)$
 $P(T|F)$

Ans = $1/4$,
= $1/2$

Q4) Let (X, Y) be c.v.r w.d pdf $f(x, y) = \begin{cases} Kx(x-y) & 0 < x < 2 \\ 0 & \text{else} \end{cases}$

Ans

Q4) $K = 1/8$

$$g(x) = \frac{x^3}{4}, \quad 0 < x < 2$$

$$h(y) = \begin{cases} \frac{16+y^3-12y}{4} & 0 < y < 2 \\ \frac{16+5y^3-12y}{4} & -2 < y < 0 \end{cases}$$

Q5) Suppose 2D r.v has joint pdf $f(x, y) = \begin{cases} 2 & 0 < x < y < 1 \\ 0 & \text{else} \end{cases}$
Find P_{xy}

Ans = $1/2$

- 26) In a basketball arena, 10% fans are rooting for home team ii. 25% ~~of~~ of the fans are wearing blue iii. 20% of the fans are wearing blue & are rooting for the away team.
- Are the events of rooting for the away team and wearing blue indep? Are they mutually exclusive?

soln

~~P(A)P(B)~~ Not indep, Not mut excl

- 27) If a machine is correctly set up, it produces 90% acceptable items. If it incorrectly set up, it produces only 40% acceptable items. Past experience shows that 80% of the set ups are correctly done. If you a certain set up, the machine produces acceptable items, find the prob that machine is correctly set up

Ans = 0.45

- 28) The mean score in final exam was 72, SD was 9. The top 10% students are to receive grade A. What is the min score a student must get in order to receive A?

K = 83.61

- 29) X has pdf $f(x) = \begin{cases} 4x - 4x^3 & 0 < x < 1 \\ 0 & \text{else} \end{cases}$. Find $V(X)$, mode

Ans = $11/225$, mode = $1/\sqrt{3}$

- 30) 4 roads A, B, C and D lead away from a jail. A prisoner escaping from the jail selects a road at random. If A is selected, prob of escaping is $1/8$.

for blue to succeed B is $1/6$, C is $1/4$ and A is $1/6$.
 for the prob that the person ~~will~~ will succeed in escaping
 the solm

$$Ans = \frac{161}{480}$$

(31)

a fair die are thrown indptly. 3 events A, B, C are defined as follows
 i. odd face on first die
 ii. odd " second "
 iii. sum of the nos in the 2 dice is odd

Are A, B, C mutually indpt

$$P(A) = 3/6 \quad P(B) = 3/6 \quad P(C) = 0.5$$

$$P(A \cap B) = P(A) P(B) = 1/4 \quad P(A \cap C) = P(A) P(C) = 1/4$$

$$P(B \cap C) = 1/4$$

pairwise indpt

~~∴ P(A ∩ B ∩ C) = 0~~