Introduction to Probability

Preliminaries

We often encounter the use of the term probability in daily life, and it is used in different ways. Some examples:

1. In a cricket match, a fair coin is tossed in the beginning. The probability that this coin will show up heads is $1/2$ and tails is $1/2$. These are however theoretical values of probabilities given a fair coin.

2. In the case above, it is usually believed that the coin is unbiased. However, we really don't know whether that is the case. To verify this, we need to repeat the experiment (in this case coin tossing experiment) say n a number of times and take the limit of the ratio of successes to n, as n becomes large. The probability values are estimated here through repetition of the experiment.

We call the first kind “Classical probability” (or ”Priori probability”)

We call the second kind “Frequentist probability” (or ”Posteriori probability”).

The two kinds of perspectives given above are practically very highly restrictive.

Some instances:
The classical as well as frequentist perspectives have limitations when we have an infinite number of possible outcomes. Examples:
(i) You are picking a real number from the closed interval $[0, \pi]$.
(ii) You are tossing a coin till head appears.
(iii) You wish to model a coin that gives heads with probability say $\frac{1}{e}$.

The problem is solved by unifying the two approaches by using the axiomatic definition of probability. The axiomatic approach not only handles both perspectives but also overcomes the limitations.

First, let us solve one probability problem with infinite sample space

A fair coin is tossed until a head turns up for the first time. Find the probability that
(i) a head turns up for the first time after an even number of tosses.
(ii) a head turns up for the first time after an odd number of tosses.

Note that the number of tosses need not be finite.
All possible outcomes of the experiment are given by the sample space

$$S = \{1, 2, 3, \cdots\}$$

Let E and O denote the events that for the first time a head turns up after an even number of tosses and an odd number of tosses respectively. Then E, O are proper subsets of S and are given by

$$E = \{2, 4, 6, \cdots\},$$

$$O = \{1, 3, 5, \cdots\}.$$

We have $S = \{1, 2, 3, \cdots\}, E = \{2, 4, 6, \cdots\}$ *and* $O = \{1, 3, 5, \cdots\}$.

$$(i) \;\; P[E] = \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \cdots (a \ convergent \ infinite \ series)$$
$$= \frac{\frac{1}{4}}{1 - \frac{1}{4}}$$
$$= \frac{1}{3}$$

$$(ii) \;\; E \cup O = S \ and \ the \ events \ E \ and \ O \ are \ disjoint, \ that \ is, \ E \cap O = \emptyset.$$

$$Hence \ P[O] = 1 - \frac{1}{3} = \frac{2}{3}$$

Definition (Axiomatic Definition of Probability)

The set of all possible outcomes is the sample space, S. A subset of S is called an event. Then $P : S \to [0, 1]$ has to satisfy the following Kolmogorov axioms:

1. $0 \leq P(A) \leq 1$ *for all events* A.
2. $P(S) = 1$.
3. *For any countable collection of events* $A_1, A_2, \cdots$ *which are disjoint, i.e.* $A_i \cap A_j = \emptyset$ *for all* $i \neq j$, *we have*

$$P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i).$$

Result

$$P(\emptyset) = 0.$$

Proof.

By axiom 1, $P(\emptyset)$ lies in the interval $[0, 1]$. As S and $\emptyset$ are disjoint, by axiom 2 and axiom 3 we have

$$\begin{aligned} 1 = P(S) = P(S \cup \emptyset) &= P(S) + P(\emptyset) \\ &= 1 + P(\emptyset) \end{aligned}$$

Hence $P(\emptyset) = 0$.

$\square$

Now, it follows from axiom 3 that if $A_1 \cap A_2 = \emptyset$ then

$$P(A_1 \cup A_2) = P(A_1) + P(A_2).$$

Reason: Choose $A_i = \emptyset$ for $i \geq 3$. Then

$$P(A_1 \cup A_2) = P\left(\bigcup_{i=1}^{\infty} A_i\right)$$

$$= \sum_{i=1}^{\infty} P(A_i) \quad (by \;\; axiom \;\; 3)$$

$$= P(A_1) + P(A_2) + P(\emptyset) + P(\emptyset) + \cdots$$

$$= P(A_1) + P(A_2) + 0 + 0 + \cdots$$

$$= P(A_1) + P(A_2)$$

Result

1. $P(A^c) = 1 - P(A)$.
2. $A \subseteq B \Rightarrow P(A) \leq P(B)$.
3. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Note:
A Venn diagram uses overlapping circles or other shapes to illustrate the logical relationships between two or more sets of items. It is expected to draw an appropriate Venn diagram whenever possible.

1) $P(A) + P(A^c) = P(S) = 1$ because A and A^c are disjoint. This implies $P(A^c) = 1 - P(A)$.
(Draw an appropriate Venn Diagram)

2) We have $B = A \cup (B \cap A^c)$.

Then $P(B) = P(A) + P(B \cap A^c) \geq P(A)$.
(Draw an appropriate Venn Diagram)

3) $P(A \cup B) = P(A) + P(B \cap A^c)$.

$P(B) = P(A \cap B) + P(B \cap A^c)$.

Hence $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
(Draw an appropriate Venn Diagram)

Exercise Problem

Show that

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C)$$
$$+P(A \cap B \cap C).$$

Also, deduce the Boole's inequality:

$$P(A \cup B \cup C) \leq P(A) + P(B) + P(C).$$

Result (Inclusion-Exclusion formula in Probability)

$$P\left(\bigcup_{i}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i) - \sum_{1 \leq i < j \leq n} P(A_i \cap A_j)$$

$$+ \sum_{1 \leq i < j < k \leq n} P(A_i \cap A_j \cap A_k) - \cdots + (-1)^{n-1} P(A_1 \cap \cdots \cap A_n).$$

Exercise: Hat Problem

A group of n people enter a restaurant and give their hats to the hat-keeper. On return, the hat-keeper redistributes the hats back at random.

(i) What is the probability P_n that no person gets his/her correct hat?

(ii) What happens to P_n as n tends to infinity? (Answer: $\frac{1}{e}$)

Definition (Independent events)

Two events A and B are independent if

$$P(A \cap B) = P(A)P(B).$$

Otherwise, A and B are said to be dependent.

Result

If A and B are independent then A and B^c are independent.

Proof.

$$\begin{aligned} P(A \cap B^c) &= P(A) - P(A \cap B) \\ &= P(A) - P(A)P(B) \\ &= P(A)(1 - P(B)) \\ &= P(A)P(B^c) \end{aligned}$$

□

Result

If A and B are independent then A^c and B^c are independent.

Proof.

$$\begin{aligned}
P(A^c \cap B^c) &= 1 - P(A \cup B) \\
&= 1 - (P(A) + P(B) - P(A \cap B)) \\
&= 1 - (P(A) + P(B) - P(A)P(B)) \\
&= (1 - P(A))(1 - P(B)) \\
&= P(A^c)P(B^c)
\end{aligned}$$

$\square$

Example (Birthday problem)

How many people are needed in a room for there to be a probability that two people have the same birthday to be at least 0.5? Ignore leap years and assume that all birthdays are equally likely.

Let us assume that there are k people in the room.
We solve this by finding the probability of no birthday match, $1 - P(k)$.
From 365 possible birthdays, the total number of possibilities of birthday combinations is 365^k.
For nobody to have the same birthday, the first person can have any birthday, the second has 364 else to choose, etc. Hence

$$P(\text{a shared birthday}) = 1 - P(\text{no shared birthdays})$$

$$= 1 - \frac{365 \cdot 364 \cdot 363 \cdots (365 - (k-1))}{365^k}$$

$$= 1 - P(k).$$

We have,
$1 - P(22) = 0.4757,$

$1 - P(23) = 0.5073.$

The lowest k for which the probability exceeds 0.5 is, k = 23.
Answer: 23

Example

A person has n keys in his pocket. He selects one at random once and tries to unlock. What is the possibility that he will succeed at the rth trial if
(i) the process is done with replacement.
(ii) the process is done without replacement.

With replacement:
The person has to fail in the first $r-1$ trials and succeed in the rth. So the probability is

$$\frac{(n-1)(n-1)\cdots(n-1)(1)}{n^r} = \frac{(n-1)^{r-1}}{n^r}.$$

Without replacement: The probability is given by

$$\frac{(n-1)(n-2)\cdots(n-r+1)(1)}{n(n-1)\cdots(n-r+1)} = \frac{1}{n}.$$

Conditional Probability

If $P[B] > 0$ then

$$P[A|B] = \frac{P[A \cap B]}{P[B]}.$$

If $P[A] > 0$ then

$$P[B|A] = \frac{P[A \cap B]}{P[A]}.$$

The probability of an event A given that another event B has happened need not be same as the probability of B given that A has happened.

There is a nice relation between the two conditional probabilities, and Bayes' theorem presents it.

Bayes' Theorem

Suppose that $B_1, ..., B_n$ are events from sample space S. Let
$B_i \neq \emptyset$ for all i ,
$B_i \cap B_j = \emptyset$ for all $i \neq j$, and
$\bigcup_{i=1}^{n} B_i = S$. Then for any event A with $P(A) > 0$,

$$P[B_j|A] = \frac{P[A \cap B_j]}{P[A]} = \frac{P[A|B_j] \cdot P[B_j]}{\sum_{i=1}^{n} P[A|B_i] \cdot P[B_i]}$$

Proof.

$$Clearly,\ P[B_j|A] = \frac{P[A \cap B_j]}{P[A]} = \frac{P[A|B_j] \cdot P[B_j]}{P[A]}$$

Now, it is enough to show that

$$P[A] = \sum_{i=1}^{n} P[A|B_i] \cdot P[B_i]$$

Note: The statement above is known as the Theorem on Total Probability. Draw an appropriate Venn Diagram.

Proof.

continued..

$$We\ have,\ A = A \cap S = A \cap (\bigcup_{i=1}^{n} B_i) = \bigcup_{i=1}^{n} (A \cap B_i)$$

$$As\ (A \cap B_i) \cap (A \cap B_j) = \emptyset\ for\ all\ i \neq j,\ we\ get$$

$$P[A] = \sum_{i=1}^{n} P[A \cap B_i]$$

$$= \sum_{i=1}^{n} P[A|B_i] \cdot P[B_i]$$

$$Thus,\ P[B_j|A] = \frac{P[A|B_j] \cdot P[B_j]}{\sum_{i=1}^{n} P[A|B_i] \cdot P[B_i]}$$

□

Example

Suppose we have a screening test that tests whether a patient has a particular disease. We denote positive and negative results as positive and negative respectively, and D denotes the person having disease in the population. Suppose that the test is not absolutely accurate, and

$$
\begin{aligned}
P(positive \mid D) &= 95\% \\
P(positive \mid D^c) &= 8\% \\
P(D) &= 0.9\%.
\end{aligned}
$$

What is the probability that a person has the disease given that he received a positive result?

$$P(D \mid positive) = \frac{P(positive \mid D)P(D)}{P(positive \mid D)P(D) + P(positive \mid D^c)P(D^c)}$$

$$= \frac{0.95 \cdot (0.009)}{0.95 \cdot (0.009) + 0.08 \cdot (1 - 0.009)}$$

$$= 0.097347$$

Note that the probability that a person has the disease given that he received a positive result is just 9.7347%.

Problem

A MNC has its branches at Ahmadabad, Bengaluru and Chennai wherein they have 1729, 4104 and 7999 employees respectively. For the purpose of dealing with transfer requests from employees, MNC has divided the employees broadly in to two categories Type I and Type II. At Ahmadabad, Bengaluru and Chennai 17%, 2% , 9% of the employees are of Type II (respectively). The company transfers an employee of Type II. What is the probability that the employee worked at Ahmadabad?

Let II be the event that a Type II employee is transferred (from any location).

Let A, B and C denote the event that a randomly selected employee works at Ahmadabad, Bengaluru and Chennai respectively. Then,

$P[A] = 1729/13382$, $P[B] = 4104/13382$ and $P[C] = 7999/13382$.

We have
$P[II|A] = 17/100$, $P[II|B] = 2/100$, and $P[II|C] = 9/100$.

$$
\begin{aligned}
P[A|II] &= \frac{P(A \cap II)}{P[II]} \\
&= \frac{P[II|A]P[A]}{P[II|A]P[A] + P[II|B]P[B] + P[II|C]P[C]}
\end{aligned}
$$

$$
= \frac{(0.17)(1729/13382)}{(0.17)(1729/13382) + (0.02)(4104/13382) + (0.09)(7999/13382)}
$$

$$
= 0.2682
$$

Problem

A bag contains 90 fair coins $(P(H) = \frac{1}{2} = P(T))$ and 10 unfair coins which flip with $P(H) = \frac{3}{4}, P(T) = \frac{1}{4}$. A coin is picked at random and tossed n times and each one of the n tosses were heads.
(i) What is the probability that the picked coin is the unfair coin?
(ii) Find the least value of n that gives probability that the picked coin is the unfair is at least 90%?

Let A denote the event that n tosses of the coin gave n heads.
Let B_1 denote the event that the coin is unfair and B_2 denote the event that the coin is fair.

$$(i)\ P[B_1|A] = \frac{P[A|B_1] \cdot P[B_1]}{P[A|B_1] \cdot P[B_1] + P[A|B_2] \cdot P[B_2]}$$

$$= \frac{(\frac{3}{4})^n \cdot (0.1)}{(\frac{3}{4})^n \cdot (0.1) + (\frac{1}{2})^n \cdot (0.9)}$$

$$(ii)\ When\ n = 10,\ we\ get\ P[B_1|A] = 0.864997$$

$$When\ n = 11,\ we\ get\ P[B_1|A] = 0.905757$$

$$Hence\ the\ least\ value\ of\ n\ is\ 11.$$

Some miscellaneous problems with Hints/ Solutions

Problems:

1. A pair of dice is rolled. What is the probability of getting a sum greater than 6? A pair of dice is rolled. What is the probability of rolling a sum neither 5 nor 10?

2. There are 8 positive numbers and 6 negative numbers. 4 numbers are chosen at random and multiplied. What is the probability that the product is a positive number?

3. A number is chosen between 1 and 50. What is the probability that it is divisible by 8?

4. An urn contains 5 red and 10 black balls. 8 of them are placed in another urn. What is the chance that the later then contains 2 red and 6 black balls?

5. A bag contains 8 white and 6 red balls. What is the probability of drawing two balls of the same color?

6. The coefficient A, b, c of the quadratic equation $ax^2 + \mathrm{bx} + \mathrm{c} = 0$ are determined by throwing a die 3 times find the probability that 1. Roots are real 2. Roots are complex.

7. Three group of children contain respectively 3 girls 1 boy, 2 girls 2 boys, 1 girl 3 boys. One child is selected at random from each group. Show that the chance that the 3 selected consist of 1 girl and 2 boys is 13/32.

8. A committee of 4 person is to be appointed from 3 offices of production department, 4 officers from purchase department, 2 officer from the sales department and 1 charted accountant. Find the probability of

1. There must be one from each category.
2. It should have at least one from the purchase department.
3. The CA must be in the committee.

9. What is the probability that a randomly selected year contains 53 Sundays?

10. Each of 2 person A and B tosses 3 fair coins. Find the probability that they get the same number of heads.

11. A and B throw a die alternatively till one of them gets a ‘6’ and wins the game. Find their respective probabilities of winning if A starts first.

Solution: Let S denote the success (getting a ‘6’) and F denote the failure (not getting a ‘6’).
Thus, P(S) =1/6, P(F) =5/6
P(A wins in the first throw) = P(S) = 1/6
A gets the third throw, when the first throw by A and second throw by B result into failures.
Therefore, P(A wins in the 3rd throw) = P(FFS) = P(F)P(F)P(S)= (5/6)(5/6)(1/6)
P(A wins in the 5th throw) = P (FFFFS)= (5/6)(5/6) (5/6)(5/6)(1/6)
Hence, P(A wins) =1/6 + (5/6)(5/6)(1/6) + (5/6)(5/6) (5/6)(5/6)(1/6) + … = 6/11 (G.P infinite sum)
P(B wins) = 1 – P (A wins) = 1- (6/11)= 5/11.

12. A and B throw alternatively a pair of die. A wins if he throws sum 6 before B throws sum 7 and B wins the other way. If A begins, find his chances of winning the game.

13. Six people toss a fair coin one by one. The game is win by the player who throws head. Find the probability of success of the 4^{th} player.

Answers:

1. 7/12, 29/36
2. 505/1001
3. 6/25
4. 140/429
5. 43/91
6. 43/216, 173/216
7. 13/32
8. 4/35, 195/210, 0.4
9. 1/7 and 2/7(leap year)
10. 5/16
11. Solution
12. For A: 30/61 For B:31/61
13. 4/63

Example: A die is tossed if the number is odd on the face, what is the probability that it is a prime?
$S = \{1,2,3,4,5,6\}$
$A = \{1,3,5\}$ *reduction in the sample space because of the additional information that it is odd.*
$B = \{3,5\}$

$$P(B|A) = \frac{P(B\cap A)}{P(A)} = \frac{2}{3}$$

NOTE:

If A and B are independent events, then $P(B|A) = \frac{P(B\cap A)}{P(A)} = \frac{P(B).P(A)}{P(A)} = P(B).$

If A and B are 2 independent events of S then prove that A& $\bar{B}$, *B &* $\bar{A}$ *and* $\bar{A}$ *&* $\bar{B}$ *are also independent.*

Problems:

1. *If A and B are 2 independent events of S such that* $P(\bar{A} \cap B)=2/15$, $P(A \cap \bar{B})=1/6$ *then find P(B). Answer= 1\6 or 4\5.*
2. *If A and B are 2 independent events of S such that* $P(A)= 1/3$, $P(B)=1/4$, $P(A\cup B) =1/2$ *then find i)* $P(A|B)$ *ii)* $P(B|A)$ *iii)* $P(A\cap \bar{B})$*iv)* $P(A|\bar{B})$.
   Answer: i) 1/3 ii) 1/4 iii)1/4 iv) 1/3
3. *In a certain town 40% have brown hair, 25% have brown eyes, 15% have both brown hair and brown eyes. A person is selected at random.*
   i. *If he has brown hair, then what is the probability that he has brown eyes also.*
   ii. *If he has brown eyes, then what is the probability that he has not have brown hair.*
   iii. *Determine the probability that he neither have brown hair nor brown eyes.*
   Answer: 3/8, 0.4 and 0.5

4. *A bag contains 10 gold coins and 8 silver coins. Two successive drawings of 4 coins are made such that.*
    i. *The coins are replaced before the second trial.*
    ii. *The coins are not replaced before the second trial.*

Find the probability that the first drawing will give 4 gold coins and second drawing will give 4 silver coins.

i) coins are replaced before the second trial

Gold = 10

Silver = 8

first drawing will give 4 gold = $^{10}C_4/^{18}C_4$

second drawing will give 4 silver = $^{8}C_4/^{18}C_4$

probability that the first drawing will give 4 gold and the second 4 silver coins. = $(^{10}C_4/^{18}C_4) * (^{8}C_4/^{18}C_4)$

$= {^{10}C_4} * {^{8}C_4} / (^{18}C_4)^2$

ii) the coins are not replaced before the second trial.

first drawing will give 4 gold = $^{10}C_4/^{18}C_4$

second drawing will give = $^{8}C_4/^{14}C_4$

probability that the first drawing will give 4 gold and the second 4 silver coins. = $(^{10}C_4/^{18}C_4) * (^{8}C_4/^{14}C_4)$

$=(^{10}C_4 * {^{8}C_4}) / (^{18}C_4 . {^{14}C_4})$

5. *Two defective tubes get mixed up with 4 good ones. The tubes are tested one by one, until both defective are found. What is the probability that the last defective tube is obtained on a) 2^{nd} test, b)3^{rd} test, c)6^{th} test.*

Problems:

1. A person has undertaken a construction job. The probabilities are 0.65 that there will be strike, 0.80 that the construction job will be completed on time if there is no strike, and 0.32 that the construction job will be completed on time if there is a strike. Determine the probability that the construction job will be completed on time.

Solution: Let A be the event that the construction job will be completed on time, and B be the event that there will be a strike.
We must find $P(A)$.
We have $P(B) = 0.65$, $P(\text{no strike}) = P(B') = 1 - 0.65 = 0.35$
$P(A|B) = 0.32$, $P(A|B') = 0.80$
Since events B and B′ form a partition of the sample space S, therefore, by theorem on total probability, we have.
$P(A) = P(B)\,P(A|B) + P(B')\,P(A|B')$
$= 0.65 \times 0.32 + 0.35 \times 0.8 = 0.208 + 0.28 = 0.488$

2. suppose 3 companies x y z produce TVs x produces twice as many as y while y and z produce same number. It is known that 2% of x, 2% of y, 4% of z are defected. All the TVs are produced are put into 1 shop and then 1 tv is chosen at random what is the probability that the tv is defecated. Suppose a tv chosen is defective what is the probability that this tv is produced by company x. ?

3. There are 3 boxes, the first one containing 1 white, 2 red and 3 black balls: the second one containing 2 white, 3 red and 1 black ball and the third one containing 3 white, 1 red and 2 black balls. A box is chosen at random and from it two balls are drawn at random. One ball is red and the other, white. What is the probability that they come from the second box? Ans: 6/11

4. A randomly selected year has 53 Sundays. Find the probability that it is a leap year. Ans: 0.4

5. Two factories produce identical clocks. The production of the first factory consists of 10,000 clocks of which 100 are defective. The second factory produces 20,000 clocks of which 300 are defective. What is the probability that a particular defective clock was produced in the first factory? Ans:0.25

6. One percent of the population suffers from a certain disease. There is blood test for this disease, and it is **99%** accurate, in other words, the probability that is gives the correct answer is **0.99**, regardless of whether the person is sick or healthy. A person takes the blood test, and the result says that he has the disease. The probability that he actually has the disease, is?

Solution:

A be the event of having the disease.

B be the event of testing positive.

$P(A) = 0.01$

$P(B) = P(B/A)P(A) + P(B/notA)P(notA)$

$P(B) = 0.01 * 0.99 + 0.01 * 0.99 = 0.0198$

$P(A \cap B) = 0.99 * 0.01$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow \frac{0.99 * 0.01}{0.0198}$$

=0.5 or 50%

7. If a machine is correctly set up, it produces 90% acceptable items. If it is incorrectly set up, it produces only 40% acceptable items. Past experience shows that 80% of the set ups are correctly done. If after a certain set up, the machine produces 2 acceptable items, find the probability that the machine is correctly setup.

Solution:
Let A be the event that the machine produces 2 acceptable items.
Also let B_1 represent the event of correct set up and B_2 represent the event of incorrect setup.
Now $P(B_1) = 0.8$, $P(B_2) = 0.2$
$P(A|B_1) = 0.9 \times 0.9$ and $P(A|B_2) = 0.4 \times 0.4$
Therefore $P(B_1|A) = 0.95$

8. It is suspected that a patient has one of the diseases A_1, A_2, A_3. Suppose that the population suffering from this illness are in the ratio 2:1:1. The patient is given a test which turns out to be positive in 25%of the cases of A_1, 50% of the cases of A_2 and 90% of the cases of A_3. Given that out of 3 tests taken by the patient two are positive, then find the probability for each of the diseases. Ans: 0.3128, 0.4170, 0.2703
Solution:
$A_i \rightarrow$ the patient has the illness A_i
$B \rightarrow$ two test results are positive.
$P(A_1) = 2/4$ $\quad$ $P(A_2) = 1/4$ $\quad$ $P(A_3) = 1/4$

$P(B|A_1) = [PPN+PNP+NPP]$
$= {}^3C_2(1/4)^2(3/4)$

$P(B|A_2) = {}^3C_2(1/2)^2(1/2)$

$P(B|A_3) = {}^3C_2(9/10)^2(1/10)$

$P(A_1|B) = ?$ $\quad$ $P(A_2|B) = ?$, $\quad$ $P(A_3|B) = ?$

9. An Archer with an accuracy of 75% fires 3 arrows at one target. The probability of the target falling is 0.6 if he hit once, 0.7 if he his twice. 0.8 if he hits thrice. Given that, the target has fallen find the probability that it was hit twice. Ans:0.411
Solution:
$B_i \rightarrow$ target is hit the i^{th} time.
$B \rightarrow$ target falls
P (anchor hits) =0.75
$P(B|B_1) = 0.6$, $P(B|B_2) = 0.7$, $P(B|B_3) = 0.8$

$P(B_1) = {}^3C_1(1/4)^2(3/4)$ $\quad$ $P(B_2) = {}^3C_2(1/4)(3/4)^2$ $\quad$ $P(B_3) = {}^3C_3(1/4)^0(3/4)^3$

$P(B_2|B) = ?$