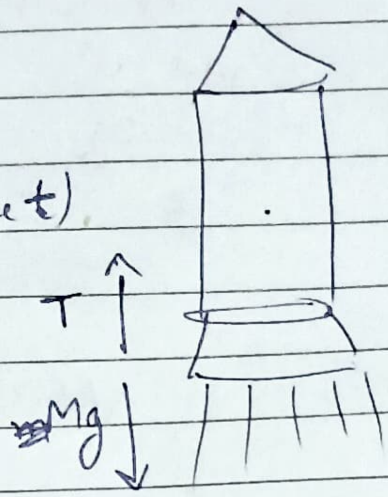
Sol^n (2) (i) let us assume initial mass of the rocket is M_0

$$I_{sp} = \frac{T}{\dot{m} g_0} \qquad (T = \text{Thrust})$$

$$\Rightarrow \boxed{T = I_{sp}\, \dot{m}\, g_0} \quad \text{———} \ (1)$$



$(T - Mg) = Ma$ (M is the mass at time t) (which is $M_0 - \dot{m}t$)

$$\Rightarrow \frac{T}{M} - g = \frac{dv}{dt}$$

$$\int_0^t \left(\frac{T}{M} - g\right) dt = \int_0^V dv$$

$$\int_0^t \frac{T}{M_0 - \dot{m}t}\, dt - gt = V$$

$$\int_0^t \frac{I_{sp}\, \dot{m}\, g_0\, dt}{M_0 - \dot{m}t} - gt = V \quad \text{———} \ (\text{using } (1))$$

$$-I_{sp}\, g_0 \int_0^t \frac{(-\dot{m})\, dt}{M_0 - \dot{m}t} - gt = V$$

$$-I_{sp}\, g_0 \left[\ln(M_0 - \dot{m}t)\right]_0^t - gt = V$$

$$\boxed{V = I_{sp}\, g_0 \ln\left(\frac{M_0}{M_0 - \dot{m}t}\right) - gt}$$

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Sol: 2 (ii) Now, we have the eq^n

$$v = I_{sp} g_o \ln\left(\frac{M_o}{M_o - \dot{m}t}\right) - gt$$

Given that ~~at t =~~ t_b ~~fuel~~ is the burnout time

Velocity at ~~that~~ time $u \text{ (say)} = I_{sp} g_o \ln\left(\frac{M_o}{M_o - \dot{m}t_b}\right) - gt_b$ — ①

Height at that point of time, H (say) can be calculated

$$\frac{dH}{dt} = I_{sp} g_o \ln\left(\frac{M_o}{M_o - \dot{m}t}\right) - gt$$

$$\int_0^H dH = \int_0^{t_b} I_{sp} g_o \ln(M_o)dt - \int_0^{t_b} I_{sp} g_o \ln(M_o - \dot{m}t)dt - \int_0^{t_b} gt\, dt$$

$$H = I_{sp} g_o \ln(M_o) t_b + \frac{I_{sp} g_o}{\dot{m}} (M_o - \dot{m}t_b)\left[\ln(M_o - \dot{m}t_b) - 1\right] - \frac{g t_b^2}{2} \quad \text{②}$$

This is the height at burnout time

$$\text{Maximum height} = H + \frac{u^2}{2g}$$

(H and u are calculated above)

(By putting these in this eq^n, we get max height)