

- Time Complexity / Space Complexity }
  - $O(\underline{C})$ ,
  - Worst / Best / Avg
  - Why TLE?
- } Thursday's Session

→ Calculating Iterations /

Ques) 21 Subj - } Top 3 Divide & Conquer  
                 Pen / Paper / Try it

Ques)

Given  $N \rightarrow N/2 \rightarrow N/4 \rightarrow N/8 \dots 1 : \log_2^N$  { Recurring  
 Problem Solving }

Ques)  $[3 \underline{10}] \rightarrow 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 : 8$  Numbers  
 $\downarrow$   
 $10 - 3 + 1 = 8$

$[a \ b]$  → All Elements from a to b Included

$$\rightarrow b - a + 1$$

$[a \ b)$  → All Elements from a to b, b Excluded  
 $\rightarrow b - a$

$(a \ b)$  → b - a - 1

Note:  $9/2 \Rightarrow 4$   
 $11/2 \Rightarrow 5$

3Q) Sum of  $N$  Elements in A.P  $\Rightarrow$  Common diff between

$a, a+d, a+2d, a+3d, a+4d, \dots$  2 consecutive elements same.

$a$   $\rightarrow$  first Term,  $d$  - Common Difference

$$S_N = \frac{N}{2} [2a + (n-1)d]$$

$$\underline{\text{Q2}} \quad \log_a^N = N, \quad \log_2^{10} = 10, \quad \log_2^r = r$$

$\xrightarrow{\text{Problems:}}$

1.  $i=1; i \leq N; i++$

$$\left| \begin{array}{l} \\ \\ S = S + i \end{array} \right.$$

$i = \underbrace{[1, \dots, N]}_{\text{Start}} \quad \underbrace{[N]}_{\text{end}}$  Iterations  
 $N-1+1 = \overline{N}$   
 $\Rightarrow \Theta(N)$

2.  $i=1; i \leq N; i \leftarrow 2$

$$\left| \begin{array}{l} \\ \\ S = S + i \end{array} \right.$$

$i = \underbrace{1, 3, 5, 7, 9, \dots}_{\text{Value Iterating}}$   $\rightarrow$   
 All Odd Number  $\underbrace{[1, N]}_{\text{in}} \rightarrow \underbrace{[N+1] / 2}_{\text{out}}$

$N=8$  : Odd  $[1-8] = 4$  Given  $N$ , how many odd  $[1-N]$

$N=9$  : Odd  $[1-9] = 5$

$N=10$  : Odd  $[1-10] = 5$

$N=15$  : Odd  $[1-15] = 8$

ans  $\begin{cases} \frac{N}{2} : N \text{ Even} \\ \frac{N+1}{2} : N \text{ Odd} \end{cases}$  cut:  $\frac{(N+1)}{2}$

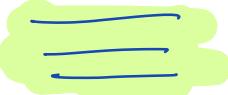
$$\begin{array}{ll} \frac{8}{2} = 4 & \frac{9}{2} = 5 \\ \frac{10}{2} = 5 & \end{array}$$

$$3\otimes \begin{cases} i = 0; \\ \alpha = 100; \\ \beta_{t+1} \end{cases} \quad i = [0, 100] \Rightarrow \text{if } 100 - \alpha_{t+1}$$

  
3  
100  
100 - α<sub>t+1</sub>

$\Rightarrow O(1)$

$$4\otimes \begin{cases} i = 1; \\ i^2 \leq N; \\ \beta_{t+1} \end{cases}$$

  
3  
N  
i^2 ≤ N

$$i^2 i = i^2, \quad i^2 i \leq N \Rightarrow i^2 \leq N \Rightarrow i \leq \sqrt{N}$$

$$\begin{cases} i = 1; \\ i \leq \sqrt{N}; \\ \beta_{t+1} \end{cases} \quad i = [\underline{1}, \overline{\sqrt{N}}] \Rightarrow [\underline{\sqrt{N} - 1}, \overline{1}]$$

1  
√N  
√N - 1  
1  
O(√N)

~~sol~~ - Given  $N > 0$

$$P = N$$

while ( $P \geq 1$ ) {

$$\begin{cases} P = P/2 \\ \downarrow \end{cases}$$

<u>P Before</u>	<u>Total Iterations</u>	<u>P After</u> ( $P = P/2^k$ )
$P \geq 1$	①	$P/2 = P/2^1$
$P/2 \geq 1$	②	$P/4 = P/2^2$
$P/4 \geq 1$	③	$P/8 = P/2^3$
$P/8 \geq 1$	④	$P/16 = P/2^4$
		After <u>k</u> iterations $\rightarrow P/2^k$
		After <u>n</u> iterations $\rightarrow P/2^n = 1$

$$P/2^k = 1, \quad 2^k = N \rightarrow$$

Apply  $\log_2$  on both sides

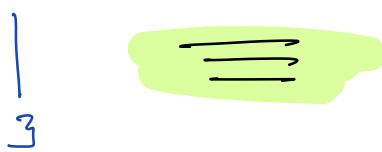
$$\log_2 \frac{k}{2} = \log_2 \frac{N}{2}$$

$$k = \log_2 \frac{N}{2} \quad // \text{Total no. of iterations are } \log_2 \frac{N}{2}$$

$$\Rightarrow O(\log \frac{N}{2})$$

Q2)

$$i = 0; i < N; i = i \times 2 \{$$



3

i Before	Iteration	i After
0	1	0
0	2	0
0	3	0
0	4	0
	...	
	<u>+oo</u>	

Q2)  $\Leftrightarrow$  S2)

$$i = 1; i < N; i = i \times 2 \{$$



3

$\rightarrow \log_2^N$  iterations

When will this loop stop

$$i \geq N$$

$$n = \log_2^N$$

Total Iterations

$$\Theta(\log N)$$

i Before	Iterations	i After
1 $i = N$	1	$2 = 2^1$
2 $i = N$	2	$4 = 2^2$
4 $i = N$	3	$8 = 2^3$
8 $i = N$	4	$16 = 2^4$
		$i = 2^k$

After k iterations

After  $k$  iterations we stop

$$2^k = i \geq N$$

$$2^k \geq N \quad \{ n > \log_2^N \}$$

Q8

$$i = 1; j \alpha = (0; i++ \}) \}$$

$$\boxed{i = 1; j \alpha = N; j++ \} }$$



i	j	Total Iterations
1	[1, N]	$\frac{N}{1}$
2	[1, N]	$\frac{N}{2}$
3	[1, N]	$\frac{N}{3}$
4	[1, N]	$\frac{N}{4}$
.	.	.
10	[1, N]	$\frac{N}{10}$

$= N$

Q9

$$i = 0; \underline{j \alpha N}; i++ \} \}$$

$$\boxed{i = 0; j \alpha N; j++ \} }$$



i	j	Total Iterations
0	[0, N]	$N - 0 = \frac{N}{1}$
1	[0, N]	$N - 0 = \frac{N}{2}$
2	[0, N]	$N - 0 = \frac{N}{3}$
3	[0, N]	$N - 0 = \frac{N}{4}$
.	.	.
N-1	[0, N]	$N - 0 = \frac{N}{N}$

$= N^2$

$\Rightarrow \underline{\underline{O(N^2)}}$

loop

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 $i = 0; i < N; i++ \{$ 
     $j = 0; j < i; j++ \{$ 
         $\dots$ 
     $\}$ 
 $\}$ 

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loop body break

After break, of any

$i$	$j \in [0, i]$	Total Iterations
0	$[0, 0]$	1
1	$[0, 1]$	2
2	$[0, 2]$	3
3	$[0, 3]$	4
$\vdots$		
$N-1$	$[0, N-1]$	$N$

Sum of  
N Natural  
Numbers  
 $\Rightarrow \frac{N(N+1)}{2}$

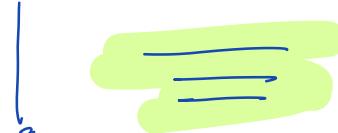
$$\frac{(N)(N+1)}{2} = \frac{N^2 + N}{2} \Rightarrow$$

$$\Rightarrow \frac{N^2}{2} + \frac{N}{2} \Rightarrow O(n^2)$$

118

$$i = 1; j \alpha = N; i = \underline{\underline{i+2}}$$

$$j = 1; j \alpha = i; j++ \}$$

Total Iterations

i	j [i, i]	Total Iterations
1	[1, 1]	$i = 1, j = 1$
3	[1, 3]	$i = 2, j = 2$
5	[1, 5]	$i = 3, j = 3$
7	[1, 7]	$i = 4, j = 4$
.	.	$i = 5, j = 5$
		$i = 6, j = 6$
		$i = 7, j = 7$
		$i = 8, j = 8$
		$i = 9, j = 9$

Sum of all odd Numbers

$[1, N]$

$$\overbrace{1 + 3 + 5 + 7 + 9}^2 \dots - - -$$

$$a = 1 \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{Sum of } N \text{ Terms in AP} \quad N = \frac{(N+1)}{2}$$

$$d = 2$$

$$t \rightarrow \frac{(N+1)}{2}$$

$$S_N = \frac{(N+1)}{2} \left[ 2a + \frac{(N-1)d}{2} \right]$$

$$S_N = \frac{(N+1)}{4} \left[ 2 \cdot 1 + \left[ \frac{N+1-1}{2} \right] 2 \right]$$

$$S_N = \frac{(N+1)}{4} \left[ 2 + \left[ \frac{N+1-2}{2} \right] 2 \right]$$

$$S_N = \frac{(N+1)}{4} \left[ 2 + \frac{N+1-2}{2} \right] \quad O(N^2)$$

$$S_N = \frac{(N+1)(N+1)}{4} \quad \boxed{S_N = \frac{N^2 + 2N + 1}{4}}$$

$$k_{\text{odd}} = k^2$$

11Q)

$i = 1; i < N; i++ \{$

$j = 1; j < N; j = j^2 \}$

$\equiv$

$\underbrace{j}_{\text{?}}$

$i$	$j [1, N]$	Total Iterations
1	$[1, N]$	$\log_2^N$
2	$[1, N]$	$\log_2^N$
3	$[1, N]$	$\log_2^N$
$\vdots$		
$N$	$[1, N]$	$\log_2^N$

$O(\underline{N \log^N})$

12Q)

No overflow

$i = 1; i < \underline{(1 \times N)}; i++ \{$

$\underbrace{i}_{\text{?}}$

$i = 1; i < 2^N; i++ \} \Rightarrow i \rightarrow [1, 2^N]$

$\underbrace{i}_{\text{?}}$

$$a \ll N = a \times 2^N$$

$$1 \ll N = 1 \times 2^N$$

$$1 \times N > 2^N$$

$= 2^N \text{ iterations}$

$= \underline{O(2^N)}$

138)

$$\begin{aligned} \varphi &= 1_j \varphi_{\lambda} = N(\varphi_{\lambda}) \\ \underline{\varphi} &= 1_j \underline{\varphi}_{\lambda} = (1_{\lambda}(\varphi_{\lambda})) \underline{\varphi}_{\lambda} \end{aligned}$$

$i$	$[l_i, u_i]$	$2^i$	Total Iterations
1	$[l_1, u_1]$	$2^1$	
2	$[l_2, u_2]$	$2^2$	
3	$[l_3, u_3]$	$2^3$	
4	$[l_4, u_4]$	$2^4$	
:			
.			
$N$	$[l_N, u_N]$	$2^N$	

Total Iterations =

$$2^1 + 2^2 + 2^3 + 2^4 + \dots + 2^N \quad \underline{\underline{G_{top}}}$$

$$\left\{ \begin{array}{l} a=2 \\ r=2 \\ N \end{array} \right\} \xrightarrow{\text{Sum of } N \text{ Terms in GP} = (r^N - 1)} \frac{(a)(r^N - 1)}{r - 1} = \frac{(2)(2^N - 1)}{2 - 1} = (2)^N + (2^N) - 2$$

## Big(O) Notation

Input size  $N \geq 1$

→ Iterations,  $\rightarrow N$

→ Neglect lower order terms,

lower degree

→ Neglect constant coefficients

$$\underline{\underline{E_{n1}:}} \quad \underline{\underline{100N^2}} + \underline{\underline{5N}} + \underline{\underline{10^4}} = \underline{\underline{O(N^2)}}$$

Higher order:  $N \log N > N$

$$\underline{\underline{E_{n2}:}} \quad \underline{\underline{5N}} + \underline{\underline{6N \log N}} + \underline{\underline{8\sqrt{N}}} \rightarrow \underline{\underline{O(N \log N)}}$$

$\log_2 N, N \geq 1$   
 $\log_2 N > 1$

$$\underline{\underline{E_{n3}:}} \quad \underline{\underline{N^2}} + \underline{\underline{6N\sqrt{N}}} + \underline{\underline{10^4 N \log N}} = \underline{\underline{O(N^2)}}$$

$$\underline{\underline{E_{n4}:}} \quad F(N) = \underline{\underline{4N^2}} + \underline{\underline{3N}} + \underline{\underline{10^6}} = \underline{\underline{O(N^2)}}$$

E<sub>n5</sub>: If iteration are constant:  $O(1)$

$$\log(N) \propto \sqrt{N} \propto N \propto N \log N \propto N\sqrt{N} \propto N^2 \dots \frac{N}{2} \propto N!$$

Doubts

