

## Today's Content

- Find 2 unique elements
- Find 2 missing elements
- Calculate sum of nr of all pairs
- Find max A & B

Q8) Given  $N$  Elements every element repeats twice, except 2 unique elements, find the 2 unique elements.

$$\underline{\text{freq}} = 1$$

Ex:  $\text{ar}[6] \rightarrow \{3, 6, 4, 4, 3, 8\} : 6, 8$

$\text{ar}[4] \rightarrow \{4, 9, 9, 8\} : 4, 8$        $\xrightarrow{\text{hashmap}}$  Avg Case

Ideas1: get freq of all elements, Elements with  $\text{freq} = 1$ , unique element       $\left. \begin{array}{l} \text{TC: } O(N) \\ \text{SC: } O(N) \end{array} \right\}$        $\left. \begin{array}{l} \text{Search} \\ \text{Insert} \\ \text{Delete} \\ \text{Update} \end{array} \right\} O(1)$

Ideas2: Sort & compare adjacent elements, get unique elements       $\left. \begin{array}{l} \text{TC: } O(N \log N + N) \\ \text{SC: } O(1) \end{array} \right\}$

Ideas3: XOR of all Elements =  $\overline{1110}$

$$\{3, 6, 4, 4, 3, 8\} = 6 \oplus 8 = 14$$

1) only  $\text{nr}$  of Elements it won't help.

2)  $\text{nr}$  of all Elements =  $\text{nr}$  of 2 unique Elements

(1010)      (0000)      (1100)      (0110)      (1011)      (1100)  
 $\text{arr[3]} = \begin{matrix} 10 & 8 & 8 & 9 & 12 & 9 & 6 & 11 & 10 & 6 & 12 & 17 \\ (1000) & (1001) & (1001) & (1011) & (1011) & (0110) & (10001) \end{matrix}$

nr of all

Elements = 11^17

4 3 2 1 0  
11 : 0 1 0 1 1

17 : 1 0 0 0 1

11^17 : 1 1 0 1 0

2  
6 =

obs:

At bit pos: 1, 3, 4

Both unique elements

11 & 17 have different bits.

Bit pos = 1 (Reference) ? ✓

Set

Elements, bit pos=1  
set

$\{ 10, 6, 11, 10, 6 \}$   
→ nr of all Elements  
11

Elements, bit pos=1  
unset

$\{ 8, 8, 9, 12, 9 \}$   
 $\{ 12, 17 \}$   
→ nr of all Elements  
17

Bit pos = 3 (Reference) ? ✓

Set

Elements whom  
bit pos=3 set

$\{ 10, 8, 8, 9, 12, 9 \}$   
 $\{ 11, 10, 12 \}$

nr = 11

Elements whom  
bit pos=3 unset

$\{ 6, 6, 17 \}$

17

## Pseudo Code

1) xor of all Elements?

$\Rightarrow N$

val = 0;

$i = 0; i < N; i++ \{$   
    |  
    | val = val  $\wedge$  arr[i]  
    |  
    |  
    |}

2) find set bit pos in val?

$\Rightarrow 31$

$[0 \dots 30]$

pos = -1

$i = 0; i <= 30; i++ \{$

|  
| if (checkBit(val, i)) {  
| | pos = i; break;  
| |}  
| | |  
| | |}

3) Using pos, break array set = 0, unset = 0

into set & unset q

xor of all

$\Rightarrow N$

$i = 0; i < N; i++ \{$

| if (checkBit(arr[i], pos)) {

| | set = set  $\wedge$  arr[i]

| | clr q

| | unset = unset  $\wedge$  arr[i]

TC :  $O(N + 31 + N) \Rightarrow O(N)$

SC :  $O(1)$

4) 2 unique Elements  $\Rightarrow \{set \& unset\}$

23) Given  $N$  array elements, array contains all elements

from  $[1, N+2]$  except 2 elements find 2 missing elements

$\Rightarrow [TODO]$  { Doubts Sess. 7}

$$N=4 \rightarrow [1-6]$$

Missing Elements

$$ar[4] = \{3, 6, 1, 4\} \Rightarrow \{2, 5\}$$

$$N=5 \rightarrow [1-7]$$

$$ar[5] = \{1, 6, 4, 7, 5\} \Rightarrow \{2, 3\}$$

Pieces:

1) Use hashing & find missing elements  $\Rightarrow TC: O(N)$   $SC: O(N)$

2) Sort & missing elements  $\Rightarrow TC: O(N \log N)$ ,  $SC: O(1)$

$\Rightarrow$  3) Use bool ch $[N+2]$ , & get  $\Rightarrow TC: N$ ,  $SC: N$

missing Elements

4<sup>th</sup> App  
 $N=5 \rightarrow [1-7] \Rightarrow \{1, 2, 3, 4, 5, 6, 7\}$

$$ar[5] = \{1, 6, 4, 7, 5\}$$

Every element is repeating twice except 2 unique elements.

4<sup>th</sup> Sol:  $TC: O(N)$   $SC: O(1)?$

Try it out }

D) Step 1:  $\text{val} = \text{sum of all Elements} \cap \{\text{sum of all } [1, N+2]\}$

Todo 1:  $\text{val} = 0;$

Step 1:  $i = 0; i < N; i++ \{$        $i = 1; i <= (N+2); i++ \{$   
           $\text{val} = \text{val} \wedge \text{arr}[i];$        $\text{val} = \text{val} \wedge \text{arr}[i]$   
           $\}$        $\}$

Step 2:  $\text{pos} = -1$   
 $i = 0; i <= 30; i++ \{$   
     $\text{if}(\text{checkBit}(\text{var}, i)) \{$   
         $\text{pos} = i; \text{break};$   
     $\}$

Step 3:  $\text{set} = 0, \text{unset} = 0 \}$  split initialed data

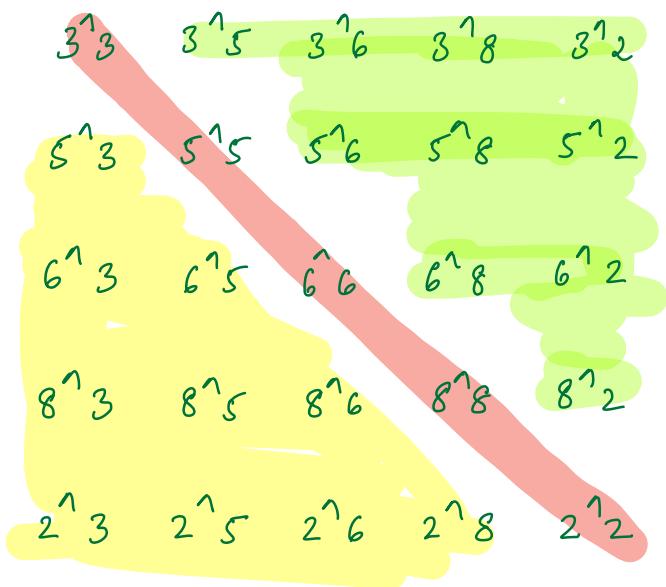
$i = 0; i < N; i++ \{$        $i = 1; i <= (N+2); i++ \{$   
     $\text{if}(\text{checkbit}(\text{arr}[i], \text{pos})) \{$   
         $\text{set} = \text{set} \wedge \text{arr}[i]$   
         $\text{clr} \&$   
         $\text{unset} = \text{unset} \wedge \text{arr}[i]$   
     $\}$   
- }       $\text{if}(\text{checkbit}(i, \text{pos})) \{$   
           $\text{set} = \text{set} \wedge i$   
       $\text{else} \{ \text{unset} = \text{unset} \wedge i \}$   
- }  
     $\text{finally 2 unique elements are set \& unset}$

Q8) Given  $N$  elements. calculate sum of nr of all pairs?

$$ar[5] = \{3, 5, 6, 8, 2\}$$

Idea1: Sum of all pairs  $\rightarrow$

Idea2:



// Work it out

Idea: 3 mins

Idea: All pairs with binary representation.

$sum = 0$  } TC:  $O(N^2)$  SC:  $O(1)$

$i = 0; i < N; i++\}$

$j = i + 1; j < N; j++\}$

$sum = sum + ar[i]^1 ar[j]$

Idea: Iterate on upper part

& get sum  $\rightarrow$

ans =  $2 * sum$

TC:  $O(N^2)$  SC:  $O(1)$

2: 0010, 3: 0011, 5: 0101, 6: 0110, 8: 1000

1d cas:

	2 <sup>3</sup>	2 <sup>2</sup>	2 <sup>1</sup>	2 <sup>0</sup>
3 <sup>1</sup> 5 :	0	1	1	0
3 <sup>1</sup> 6 :	0	1	0	1
3 <sup>1</sup> 8 :	1	0	1	1
3 <sup>1</sup> 2 :	0	0	0	1
5 <sup>1</sup> 6 :	0	0	1	1
5 <sup>1</sup> 8 :	1	1	0	1
5 <sup>1</sup> 2 :	0	1	1	1
6 <sup>1</sup> 8 :	1	1	1	0
6 <sup>1</sup> 2 :	0	1	0	0
8 <sup>1</sup> 2 :	1	0	1	0

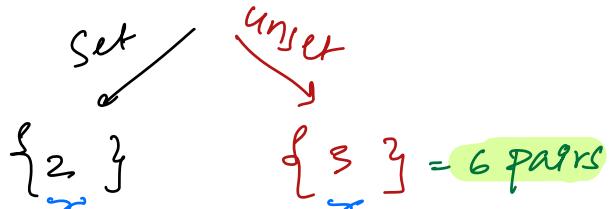
$$4 \quad 6 \quad 6 \quad 6 \\ 2^3 \quad 2^2 \quad 2^1 \quad 2^0$$

$$82 + 24 + 12 + 6$$

$$74 \quad TC \geq 31 + (N^2) \\ 3 \quad 2 * 74 \geq 48 \quad TC \geq O(N^2) \\ SC \geq O(1)$$

1d cas: 4: arr:  $\begin{cases} 0 : \text{same} \\ 1 : \text{different} \end{cases}$

In given array, how many elements 0<sup>m</sup> bit set = ?



In given array set unset items 1<sup>m</sup> set = 3 2 = 6

In given array set unset items 2<sup>m</sup> set = 2 3 = 6

In given array set unset items 3<sup>m</sup> set = 1 4 = 4

1d ca: At i<sup>M</sup> pos, C<sub>i</sub> number of pairs have i<sup>M</sup> pos

$$\text{a set } \Rightarrow \sum_{i=0}^{30} C_i \neq 2^i$$

$$\left\{ \text{Ans} = 2 * \sum_{i=0}^{30} C_i * 2^i \right.$$

## Pseudo Code

Sum = 0

i = 0; j < N; i++ {

c = 0

j = 0; j < N; j++ {

if (checkbit(ar[j], i) != c++)

Sum = Sum + c \* (N - c) \* (1 << i)

return 2 \* Sum

TC:  $31 \times N + O(N) \quad SC: O(1)$

HPM: break

sol

int: 32 bits: [0 31]

31  $\Rightarrow$  MSB bit, -ve value

Q1) Given N Array elements, choose 2 indices  $i, j$  such that  
 $i \neq j$  &  $\{A[i] \& A[j]\}$  is max }  $\xrightarrow{\text{between and}}$   $\xrightarrow{\text{index}}$

Eg1:  $A[8] = \{27, 18, 20\} \xrightarrow{\{27 \& 18\}} 18 \Rightarrow \underline{\text{ans}}$

$(27, 18)$

$(27, 20)$

$(18, 20)$

$18: 10010$

$27: 11011$

$18: 10010$

$27: \underline{11011}$

$20: \underline{10100}$

$20: \underline{10100}$

18 = 10010

16 = 10000

16 = 10000

Eg2:  $A[4] = \{21, 18, 24, 17\} \Rightarrow \{24, 21, 18, 17\}$

$21: 10101$

$21: 10101$

$\xrightarrow{\text{ans = 17}}$

$18: 10010$

$18: 10001$

$\}$

$24: 11000$

$\underline{10001} \Rightarrow 17$

$17: 10001$

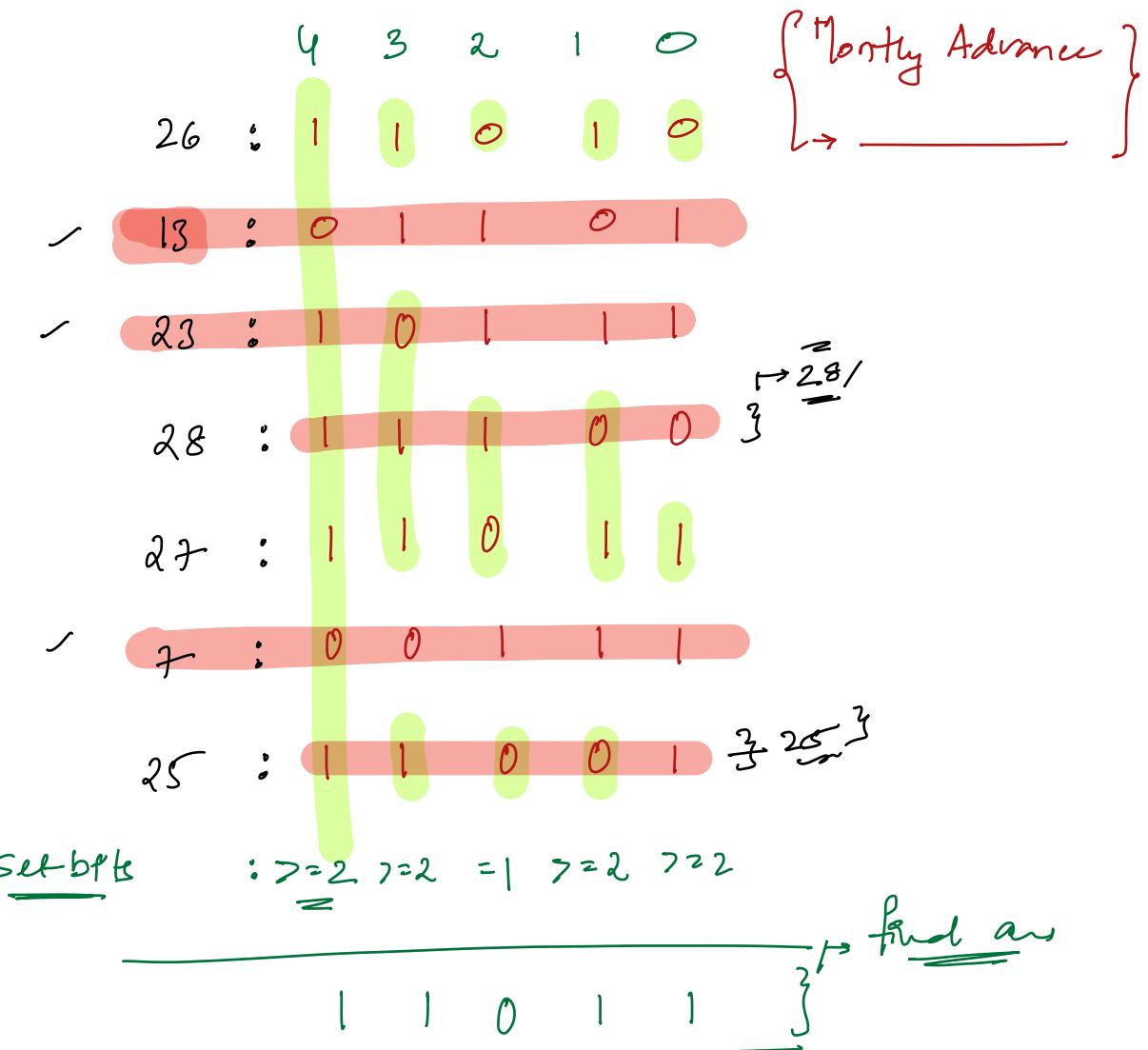
Idea1: Sort, & of adjacent pairs?  $\times$

Idea2: Max value can be in 1 mc of element?  $\times$

Idea3: For every pair get (Bitwise L) overall max  $\Theta(N^2)$

TC:  
 $\Theta(N^2)$

Ex3: arr[7] : { 26, 13, 23, 28, 27, 7, 25 }



Remove: 13, 7 23      ↴      ↴  
 $\downarrow$        $\downarrow$   
 $28, 25$

no delete  
 in this  
 can

Pseudo Code

int ans = 0

i = 30; i >= 0; i--) {

c = 0;

j = 0; j < N; j++) {

if (checkBit(ar[j], i) { c++ }

}

if (c >= 2) { *i<sup>th</sup> bit have set* }

ans = ans + (1 << i) or ans = ans | (1 << i)

j = 0; j < N; j++) {

if (!checkBit(ar[j], i)) {

ar[j] = 0 }

{  
obsc. Elements which are  
non zero, then are your  
pairs }

return ans; } of main bit with and we can get y

TC  $\Rightarrow$   $31 [N, N] \Rightarrow 62N \Rightarrow O(N)$

SC  $\Rightarrow O(1)$

Checking how  
many elements  
have *i<sup>th</sup> bit set*  
if greater than 2,  
→ Remove those  
elements, whose  
*i<sup>th</sup> bit is set*,  
Instead of  
deleting empty  
make the element  
as = 0

Doubts

	4	3	2	1	0
26 :	1	1	0	1	0
13 :	0	1	1	0	1
23 :	1	0	1	1	1
28 :	1	1	1	0	0
27 :	1	1	0	1	1
7 :	0	0	1	1	1
25 :	1	1	0	0	1
	1	1	0		

$\varphi = 0 ; g_2 N ; \varphi_{eff} \varphi$

$\begin{cases} 1 \\ 2 \\ 3 \end{cases}$

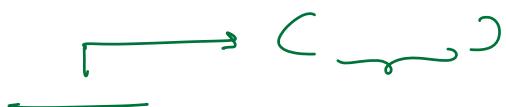
$\varphi = 0 ; i_2 N ; \varphi_{eff} \varphi$

$\begin{cases} 1 \\ 2 \\ 3 \end{cases}$

n or

1	1	0	.	1
1	0	1		1
0	1	1		1

→ doubts: Sunday / Interv → Subseq b/w



{2, 3, 5}

→ {2, 3}  
 $\underbrace{0 \ 1 \ 1}$

(Doubts)

→ Subset / Subsequence

2 3 5  
 $2^n$  Subsets / Subseq /

order doesn't matter

1 2 3

1 1

$$N_{P_0} \in N_{P_1} \in N_{P_2} \dots \in N_{P_N}$$

$$\overbrace{N_{C_0} \in N_{C_1} \in \dots}^{N_{C_N}} \rightarrow$$

⇒ prime subsequence:

