

→ Recap → Target      }  
    → Search Space      }

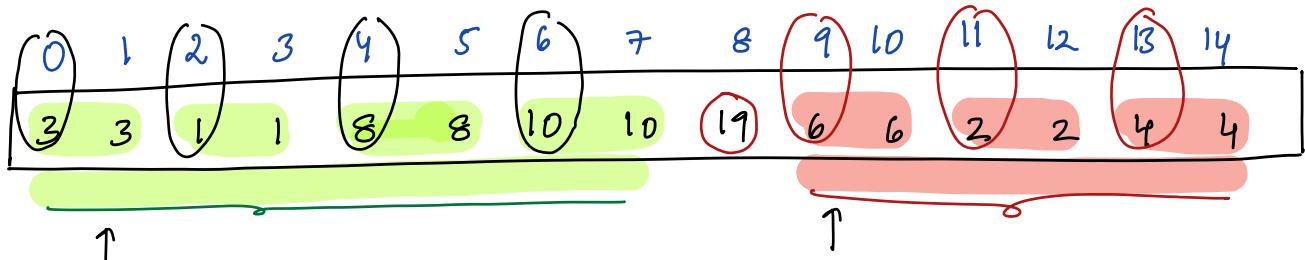
When BS? → Divide search space into 2 parts & discard  
    1 half using condition.

|

50) Every element occurs twice except for 1, find unique element

Note: Duplicates are adjacent to each other.

// BF: XOR of all elements  $\Rightarrow$  TC:  $O(N)$  SC:  $O(1)$



left side:

goto right side

$$l = m + 1$$

All first occurrences are in even indices

right side:

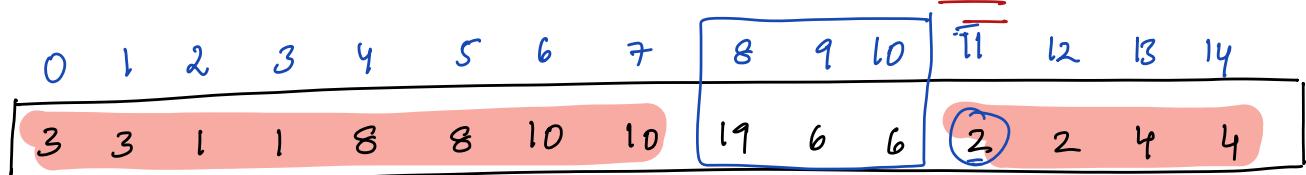
goto left side

$$h = m - 1$$

All first occurrences are in odd indices

$l = 0, h = 14, m = 7 \rightarrow$  2<sup>nd</sup> occurrence }  $\Rightarrow$  goto 1<sup>st</sup> occurrence

$$m = m - 1$$



$l = 0, h = 14 \rightarrow m = 7 \rightarrow$  6: m first occurrence

goto right:  $l = m + 2$

$l = 8, h = 14 \rightarrow m = 11 \rightarrow 11:$

goto left:  $h = m - 1$

### Pseudo Code

is Element

if ( $N == 1$ ) { return  $A[0]$ }

$Tc: O(\log N)$

$Sc: O(1)$

if ( $A[0] \neq A[1]$ ) return  $A[0]$

if ( $A[N-1] \neq A[N-2]$ ) return  $A[N-1]$

$l = 0, h = N - 1$

while ( $l <= h$ ) {

$m = (l + h) / 2$

if ( $ar[m] \neq ar[m-1] \text{ and } ar[m] \neq ar[m+1]$ ) {

} return  $ar[m]$

if ( $ar[m] == ar[m-1]$ ) {

// mid is landing m 2<sup>nd</sup> occurrence

$m = m - 1$

if ( $m \% 2 == 0$ ) {

↳ on left

$\underline{l = m + 2}$

else {

$\underline{h = m - 1}$

Q8 Given an array which is formed by rotating a

Distinct sorted array by  $k$  times, Search for a given a  
 Inc (Back-front) Element of elem in rotated array

0 1 2 3 4 5 6 7 8 9 10 11 12

$\text{ar}[13] = -20 \ -14 \ -8 \ -4 \ 1 \ 2 \ 4 \ 7 \ 11 \ 14 \ 19 \ 23 \ 27$

$k$ : rotation part

$\text{rar}[13] = \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 11 & 14 & 19 & 23 & 27 & -20 & -14 & -8 & -4 & 1 & 2 & 4 & 7 \end{matrix}$

$k=5$

$\xleftarrow{k \text{ Elements}}$

$\xleftarrow{N-k \text{ Elements}}$

19: ✓

$[0, k-1]$

$[k, N-1]$

7: ✓

$Sol_3): \underset{=}{\text{obs1}}: \begin{cases} 1^{\text{st}} \text{ sorted array: } [0, k-1] \\ l=0, h=k-1, \end{cases}$

24: ✗

$2^{\text{nd}} \text{ sorted array: } [k, N-1]$

$l=k, h=N-1$

Sol:

1) Rotate & Search

T  
 $O(N)$

J  
 $O(\log N)$

Sol\_4):  $\underset{=}{\text{obs1}}:$

All Elements in  $2^{\text{nd}}$  sorted  $<$   $\text{ar}[0]$

2) Iterative Search

$\downarrow$   
 $O(N)$

$T \leq O(\log N)$

$\left\{ \begin{array}{l} \text{if } (\text{ele} < \text{ar}[0]) \{ \\ \quad // 2^{\text{nd}} \text{ sorted } [\text{Apply BS } [k, N-1]] \\ \text{else} \\ \quad // 1^{\text{st}} \text{ sorted } [\text{Apply BS } [0, k-1]] \end{array} \right.$

// Every same :

Rotation factor  $k$  is not given?

Search & get  $k$

$\text{arr}[13] = [11 \ 14 \ 19 \ 23 \ 27 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ -20 \ -14 \ -8 \ -4 \ 1 \ 2 \ 4 \ 7]$

$\uparrow$   
1<sup>st</sup> sorted array

$$l = M+1$$

$\uparrow$   
2<sup>nd</sup> sorted array

$$\text{ans} = \text{mid}$$

$$h = \underline{m-1}$$

//  $k = 0$

$$l = 0, h = N-1$$

while ( $l <= h$ ) {

$$m = (l + h)/2$$

if ( $\text{arr}[m] < \text{arr}[0]$ ) {

// 2<sup>nd</sup> sorted array

$$k = m$$

$$h = m-1;$$

else {  $l = m+1$  }

return  $k$ ;  $\rightarrow$  rotation factor

TC  $\Rightarrow O(\log N)$

SC  $\Rightarrow O(1)$

Ex:

$\text{arr}[]: [0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 3 \ 4 \ 5 \ 8 \ 10 \ 11]$

Apply BS

$l$	$h$	$m$	$\text{arr}[m] < \text{arr}[0]$
0	5	2	$\text{arr}[2] < \text{arr}[0]$
3	5	4	$\text{arr}[4] < \text{arr}[0]$
5	5	5	$\text{arr}[5] < \text{arr}[0]$
6	5	3	= Break

Inital  $\leftarrow k=0$

10: 30 marks

Given  $\underset{N}{\approx}$  find  $SQRT(N)$

$\underset{N}{=}$   $SQRT()$

$16 \underset{4}{=}$

$25 \underset{5}{=}$

$30 \underset{5}{=}$

$39 \underset{6}{=}$

$\underset{N=39}{=}$

ans

1

2

3

4

5

6

$P=1 \rightarrow 1^2 <= 39$

$P=2 \rightarrow 2^2 <= 39$

$P=3 \rightarrow 9 <= 39$

$P=4 \rightarrow 16 <= 39$

$P=5 \rightarrow 25 <= 39$

$P=6 \rightarrow 36 <= 39$

$P=7 \rightarrow 49 <= 39 \times = \text{(incorrect)}$

(8, 9, ...) cannot be ans

TC:  $O(SQRT(N))$

$P=1; ans = \underline{\quad}$

where ( $P^2 <= N$ )

$ans = P;$

++P;

return ans

$T.C. = O(\sqrt{N})$

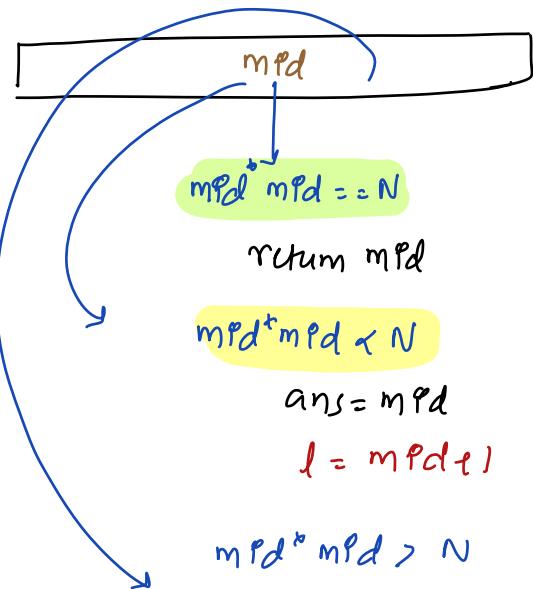
$SC. = O(1)$

No inbuilt functions

Target:  $(\sqrt{N})$

BS: → Search Space:  $[1 \dots N]$

↓ Discard left & right



$N = 39 \underset{=}{}$   $ans = \underline{\quad}$

$\underline{l} \quad \underline{h} \quad \underline{m}$

$20^2 > 39 \quad h = M - 1$

$10^2 > 39 \quad h = M - 1$

$5^2 < 39 \quad a = m$

$l = M + 1$

$7^2 > 39 \quad h = M - 1$

$6^2 < 39 \quad a = m$

$l = M + 1$

+ 6 of Break {

## // Pseudo code

↙ Constraints =

$$1 \leq N \leq 10^9$$

$$1 \leq N \leq 10^{18}$$

$$\Rightarrow N = 10^{18} \quad l = 0 \quad h = 10^{18}$$

— sqrt(N) {

$$\boxed{l = 1, h = N}$$

ans = —

while ( $l <= h$ ) {

$$\text{long } m = (l+h)/2$$

if ( $m^2 = N$ ) {  
return m  
}

if ( $m^2 > N$ ) {  
 $h = m-1$   
}

else {  
 $l = m+1$   
ans = m

$$m = \frac{(l+h)}{2} \Rightarrow \frac{10}{2}$$

$$m^2 = \approx \boxed{\frac{10}{4}}^{36}$$

$$\begin{array}{c} l \leq N \leq 10 \\ \downarrow \quad \downarrow \\ l = \sqrt{N} \quad l = 10^9 \\ \downarrow \quad \downarrow \\ h \end{array}$$

sqrt([1 10]) : [1, 10<sup>18</sup>] cont

$$l \leq N \leq 10^{18}$$

$$\begin{array}{c} l \leq N \leq 10 \\ \downarrow \quad \downarrow \\ l = \text{crt}(N) \quad l = 10^6 \\ \downarrow \quad \downarrow \\ h \end{array}$$

crt([1 10<sup>6</sup>] : [1 10<sup>18</sup>] corr)

Q8) Given an array of +ve Integers, find maximum  $k$  such that, {max subarray sum of len  $k$ }  $\leftarrow \text{Given in Input}$

$$\text{ar}[8] = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{bmatrix} \quad \text{Given } B = 20$$

$$k=5 \quad [0 \ 4] = 20 \quad [2 \ 6] = 25 \quad \left. \begin{array}{l} \text{max} = 25 \\ [1 \ 5] = 20 \quad [3 \ 7] = 22 \end{array} \right\} \text{max} = 25 < 20 \times$$

(Max subarray sum of len)

$$k=1 : \begin{matrix} 7 \\ \underline{=} \end{matrix} \quad k = 20 \quad \begin{matrix} \text{ans} \\ \underline{1} \end{matrix} \quad \begin{matrix} \text{way} \\ \checkmark \checkmark \checkmark \checkmark \checkmark \\ \times \quad \cancel{\times} \cancel{\times} \cancel{\times} \end{matrix}$$

$$k=2 : \begin{matrix} 10 \\ \underline{=} \end{matrix} \quad k = 20 \quad 2 \quad \text{Target : } k \rightarrow \{ \text{Subarray Sums} \}$$

$$k=3 : \begin{matrix} 16 \\ \cancel{\underline{3}} \end{matrix} \quad k = 20 \quad 3 \quad \text{Search : } \begin{bmatrix} 1 & \dots & N \end{bmatrix}$$

$$k=4 : \begin{matrix} 20 \\ \underline{=} \end{matrix} \quad k = 20 \quad 4$$

$$k=5 : \begin{matrix} 25 \\ \underline{=} \end{matrix} \quad k = 20 \quad \cancel{x},$$

$$k=6 : \begin{matrix} x \\ \cancel{\underline{6}} \end{matrix}$$

$$k=7 : \begin{matrix} x \\ \cancel{\underline{7}} \end{matrix}$$

$$k=8 : \begin{matrix} x \\ \cancel{\underline{8}} \end{matrix}$$

$$\underline{\Sigma n} : \text{ar}[4] : 13 \ 14 \ 20 \ 8 \xrightarrow{\text{B} = 5}$$

$$\underline{k=1} : \{ \text{max subsum} : 20 > B \} \cancel{x}$$

$l = 1, h = N, ans = 0$

while ( $l <= h$ ) {

$$k = \lfloor (l + h)/2 \rfloor$$

TC:  $O(N)$

if (Msubsum(arr[], N, k)  $\leftarrow B$ ) {

$$ans = k;$$

Given

$$l = k + 1$$

Since  $k$

else {  $h = k - 1$  }

TC:  $(\log N) * f(N)$

TC:  $O(N \log N)$

SC:  $O(1)$

return ans;

3 mins

## Binary Search

- Sorted Array
- Array is present
- Your own Search Space

→ 1) Review notes

2) Issues in teaching

- Question / English
- Intuition
- Examples
- Fast
- Slow
- Using Break

{ Stark, Way }



// sqrt()

$$l \times N \times = 10^{18}$$

$$l=1, h=10^9$$

$$\frac{m+m}{=}$$

cube root()

$$l \times N \times = 10^{18}$$

$$l=1, h=10^6$$

$$\frac{m^3 m^3 m}{=}$$

4<sup>th</sup> root

$$\frac{m^8 m^8 m^2 m}{=}$$

// 1 Problem

↓  
mid  
→

If mid is ans = return arr[mid]

If mid is landing in 1<sup>m</sup>owna

if (mid % 2 == 0) {

    ↓  
    → left side →

else {

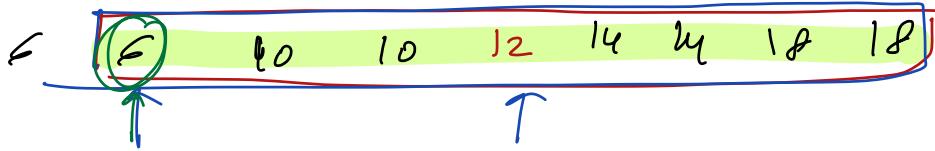
    ↓  
    right side

else {

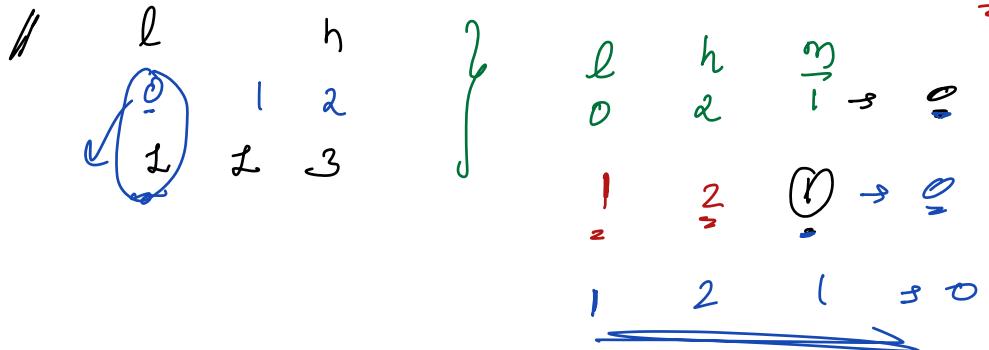
    if (mid % 2 == 0) {

        ↓  
        → right side

    else { left side } }



l<sub>2</sub>m<sub>fj</sub>



1st Element

if ( $N == 1$ ) { return  $A[0]$  }

if ( $A[0] != A[1]$ ) return  $A[0]$

if ( $A[N-1] != A[N-2]$ ) return  $A[N-1]$

$l = 0, h = N-1$

while ( $l <= h$ ) {

$$m = (l + h)/2$$

if ( $ar[m] != ar[m-1] \text{ && } ar[m] != ar[m+1]$ ) {

} return  $ar[m]$

$$\pi = m$$

if ( $ar[m] == ar[m-1]$ ) {

// mid pos landing on 2nd occurrence

$$\pi = \pi - 1$$

if ( $\pi \% 2 == 0$ ) {

    // on left

$$l = \underline{\underline{m+1}}$$

else {

$$h = M - l$$