→ Remainder %

% = remainder

$10 \% 4 = 2$, $99 \% 8 = 3$

$a \% b$ = remainder when a divided by b

Remainder = Divident − Divisor * Quotient

Max multiple of Divisor ≤ Divident

= $99 - 8 * 12$

= $99 - 96 = 3$

Q1: $-40 \% 7$ = -40 − (Max multiple of $7 \leq -40$)

= $-40 - (-42)$

= $-40 + 42 = 2$

$-3/2 \Rightarrow -1.5$

floor

-2

Q2: $-60 \% 9$ = -60 − (Max multiple of $9 \leq -60$)

= $-60 - (-63)$

= $-60 + 63$

= 3

C/C++/Java vs Python

$$\begin{cases} -40 \% 7 = -5 + 7 & 2 \\ -60 \% 9 = -6 + 9 & 3 \end{cases}$$

Why ?

$$\begin{Bmatrix} -\infty \\ 368 \\ +\infty \end{Bmatrix} \% 10 \Rightarrow \begin{Bmatrix} 0 \\ 8 \\ 9 \end{Bmatrix}$$

$$n \longrightarrow \{\% M\} \Rightarrow \begin{Bmatrix} 0 \\ \downarrow \\ M-1 \end{Bmatrix}$$

0 → n is multiple of M

Modular Arthimetic

1) ✓ $\boxed{(a+b)\,\%\,m = \overbrace{(a\,\%\,m + b\,\%\,m)}^{> m}\,\%\,m}$

$a = 4 \quad b = 5 \quad m = 6$

$\underbrace{(9)\,\%\,6}_{3} = (4 + 5) = 9\,\%\,6 = 3$

2) $\boxed{a\,\%\,m = (a + m)\,\%\,m}$ $\Rightarrow (a\,\%\,m + \overbrace{m\,\%\,m}^{\to 0})\,\%\,m$

$\Rightarrow (a\,\%\,m)\,\%\,m$

$\Rightarrow a\,\%\,m$

3) ✓ $\boxed{(a * b)\,\%\,m = (a\,\%\,m * b\,\%\,m)\,\%\,m}$

4) ✓ $\boxed{(a - b)\,\%\,m = (\underbrace{a\,\%\,m}_{[0,\,M-1]} - \underbrace{b\,\%\,m}_{[0,\,M-1]} + M)\,\%\,M}$

$a = 8, \quad b = 4, \quad m = 5$

$(4)\,\%\,5 = 4$

$= (8\,\%\,5 - 4\,\%\,5 + 5)\,\%\,5$

$= (3 - 4 + 5)\,\%\,5$

$= (4)\,\%\,5$

$= 4$

1Q) Given A & B, A > B find no: of M such that

$A \% M = B \% M$ & $M > 1$

Ex1: $A = 16$ $B = 4$, $M = 2, 3, 4, 6, 12$

Ex2: $A = 13$, $B = 7$, $M = 2, 3, 6$

Idea: $A \% M = B \% M$

$A \% M - B \% M = 0$

+ Add M on both sides

$A \% M - B \% M + M = M$

Take % M on both sides

$$\underbrace{(A \% M - B \% M + M)}_{} \% M = 0$$

$(A - B) \% M = 0$ → What all value can M take?

All factor of A-B except 1

sqrt( )

28) Given N array elements, calculate number of pairs (i,j) such that $(arr[i] + arr[j]) \% M = 0$ } M is given → 1 time

Note: (i != j) && { pair (i,j) is same as (j,i) }

0	1	2	3	4	5
4	7	6	5	5	3

Ex1: ar[6] : as above → M = 3

pairs:

i	j	ar[i]	ar[j]	
0	3	4	5	→ 9 % 3
0	4	4	5	→ 9 % 3
1	3	7	5	→ 12 % 3
1	4	7	5	→ 12 % 3
2	5	6	3	→ 9 % 3

count = 5

M = 10

Ex2: ar[] =

0	1	2	3	4	5	6	7	8	9	10	11	12
17	2	5	4	6	23	13	26	14	18	15	30	35

i	j		ar[i]	ar[j]
0	5	:	17	23
0	6	:	17	13
1	9	:	2	18
2	10	:	5	15
2	12	:	5	35

i	j	ar[i]	ar[j]
3	4	4	6
3	7	4	26
4	8	6	14
7	8	26	14
10	12	15	35

Quiz1: ar[] = [13, 14, 22, 3, 32, 19, 16]

	0	1	2	3	4	5	6
ar[]	13	14	22	3	32	19	16

i	j			
0	3	:	13	3
0	5	:	13	19
1	2	:	14	22
4	6	:	32	16

Brute Force: Check all pairs

```
i=0; i<N; i++){
    j=i+1; j<N; j++){
        (ar[i]+ar[j])%17==0
    }       cnt++
}
```

TC: $O(N^2)$ SC: $O(1)$

// observations

$(ar[i] + ar[j]) \% 17 = 0$

$(ar[i]\%17 + ar[j]\%17)\%17 = 0$

	0	1	2	3	4	5	6	7	8	9	10	11	12
ar[] =	17	2	5	4	6	23	13	26	14	18	15	30	35
ar[] =	7	2	5	4	6	3	3	6	4	8	5	0	5

obs: ar[12]: { 24, 16, 8, 7, 17, 23, 35, 20, 26, 28, 34, 21 }

K = 6 :

0	1	2	3	4	5
			6		
24	7	2	21	16	17
		20		28	23
		26		34	35

$1 * 3 = 3$

9 pairs

$$^{N}C_{2} = \frac{(N)(N-1)}{2}$$

obs: ar[ ]

$\%\boxed{6}$ =

0	1	2	3	4	5
4	2	3	5	2	3

0	3
6	3
18	9
24	15
30	21
	27

6 18 24 30

3 2 1 → 6

4 number 2 →

$$4_{C_2} = \frac{(4)(3)}{2}$$

5

$= 6 + 2*3 + 3*2 + 10 =$

$6 + 6 + 6 + 10 \rightarrow 28$ pairs

ar[ ] : 0 | 1 | 2 | 3 | 4 | 5 | 6 → 7

%7 = 3 2 4 5 2 3 4

cnt

3 + 8 + 12 + 10 = 33

hm[1] : 2

hm[2] : 4

hm[3] : 5

5

hm[6] : 4

Step 1: Modular count

hashmap<int, int> hm; / cnt

i = 0; i < N; i++) {

    hm[ar[i] % M]++;

}

N

ans = $hm[0]C_2$

i = 1, j = M-1; }

while(i < j)

    ans = ans + hm[i] * hm[j]

    i++, j--;

}

if (M % 2 == 0) {

    ans += $hm[M/2]C_2$

}

TC : O(N+M)

SC : O(M)   M >> N

M

If you use array

O(M)

hash map

O(N)

Break : 10:50PM

→ TODO: Given N,

where { $0 <= arr[i] <= N-1$ }, N is size of array

for all Number from [0, to N-1] get it's frequency

TODO

-

Q) Given an array of all distinct integers

where $\{0 <= ar[i] <= N-1\}$, N is size of array.

replace $ar[i]$ with $ar[ar[i]]$

Ex1: $ar[5]$:

	0	1	2	3	4
ar[5] :	3	2	4	1	0

$ar[0] = ar[ar[0]]$ $\quad ar[ar[1]]$ $\quad ar[ar[2]]$ $\quad ar[ar[3]]$ $\quad ar[ar[4]]$

$= ar[3]$ $\quad ar[2]$ $\quad ar[4]$ $\quad ar[1]$ $\quad ar[0]$

: 1 4 0 2 3

Ex2: $ar[7]$ =

	0	1	2	3	4	5	6
ar[7] =	3	1	4	6	5	0	2
	6	1	5	2	0	3	4

Quiz1:

	0	1	2	3	4	5	6
ar[7] =	{ 1,	6,	3,	5,	4,	2,	0 }
	6	0	5	2	4	3	1

// Approach

1) int arr2[N]

```
i = 0; i < N; i++) {
    arr2[i] = arr[arr[i]]
}
```

TC: O(N)

SC: O(N)

// → No Extra Space

		Time
0 : 0 : 0 : 23 hrs →	Day : 0	23 hrs
46 hrs →	Day : 1	22 hrs
100 hrs →	Day : 4	4 hrs
125 hrs →	Day : 5	5 hrs
x hrs →	Day : $x/24$	$x\%24$

Quotient — $x/24$

Remainder — $x\%24$

arr[i]
→ old value : /N Quotient
→ new value : % N Remainder

	0	1	2	3	4
ar[5] :	3	2	4	1	0
N = 5					
Multiply * 5 :	3*5+1	$\boxed{2*5+4}$	4*5+0	1*5+2	0*5+3
ind :	15/5 = 3	= 10/5 = 2	20/5 = 4	5/5 = 1	0/5 = 0
val =	ar[ind]/5 = 1	= ar[2]/5	ar[4]/5 = 0	14/5 = 2	16/5 = 3
	ar[0] + = val	ar[1] + = val			ar[4] + = val

ar[1] = ar[ar[1]] ; ar[2] = ar[ar[2]]

3*5+1 $\boxed{2*5+4}$ 4*5+0 1*5+2 0*5+3

%N 1 4 0 2 3

// Step1: iterate & multiply N ✓

```
i = 0; i < N; i++){
    ind = ar[i]/N
    val = ar[ind]/N
    ar[i] + = val
}
```

$\boxed{ar[i] + = ar[ar[i]/N]/N}$

ind

ar[ind]/N → val

// Step2: iterate & %N →

Inverse modules

$$(a/b) \% M = (a \% M) / (b \% M)$$

$$\Rightarrow (a * b^{-1}) \% M = ((a \% M) * (b^{-1} \% M)) \% M$$

Inverse modulee ?

Given a, m, b is inverse modules of a if

$(a * b) \% m = 1$

// b is exists if gcd(a, m) = 1 — is greatest common divisor

// value of b : [1, M-1]

Ex1: $a = 10 \quad m = 7 \} \quad b : [1, 6]$

$b = 1 \rightarrow (10 * 1) \% 7 \ != 1$

$b = 2$

$b = 3$

$b = 4$

$b = 5 \quad (10 * 5) \% 7 = 1 \rightarrow b = 5$

$$\Rightarrow \boxed{a^{-1} \% m = 5}$$

Ex1: $a = 12$, $m = 5$, $b = 3$

$\Rightarrow (12 * 3) \% 5 = 1$

// Given a, m, gcd (a, m) = 1 find b ?

```
i = 1; i < m; i++) {
    if ((a * i) % m == 1)
        return i
}
```

TC = O(m)

SC = O(1)

// Special Case : If p is prime & $a \% p \, != 0$ (a is not multiple of p)

Fermats Little Theorem

1) $a^{-1} \% p = (a^{p-2}) \% p$

2) $(a^{p-1}) \% p = 1$

$a^{-1} \% p = (a^{p-2}) \% p$ → use inbuilt power function

{Recursion class}

```
powerMod (a, b, p) {
    calculate a^b % p
}
```

Ex1: $a = 10$ $m = 7$

Ex2: $a = 12$ $m = 5$

Special Case :

1) If p is prime number & a % p != 0

$$(a^{p-2}) \% p = a^{-1}$$

$$(a^{p-1}) \% p = 1$$