

Given N , & an array contains N elements from $1 \underline{-} N$

Except 1 element find missing elements. $\underline{ar[N-1]}$

$N=5$: $\underline{ar[4]} : \{ 5, 3, 1, 4 \} : Ans = 2$
 $\underbrace{}_{1-5}$

$N=6$: $\underline{ar[5]} : \{ 3, 2, 4, 6, 1 \} : Ans = 5$
 $\underbrace{}_{1-6}$

$N=8$: $\underline{ar[7]} : \{ 2, 3, 8, 4, 7, 6, 1 \} : Ans = 5$
 $\underbrace{}_{1-8}$

Obs:

1) Then Ans: 1 Mean Ans: N

Check from $1 \underline{-} N$, The element which is not present

$P = 1; P \alpha = N; P + 1 \xrightarrow{O(N)}$
|
|| Check if q is present in $ar[\cdot]$ $\xrightarrow{O(N)}$ }
|
} $T_C: O(N^2)$
 $S_C: O(1)$
Iterate & check right?

2) Obs: $N=9$ $\underline{[1-9]} :$
 $\underline{ar[8]} \Rightarrow$

$\{ 1+2+3+4+5+6+7+8+9 \}$ $\xrightarrow{\frac{(9)(10)}{2}}$
- $\{ 4, 6, 3, 5, 8, 7, 9, 1 \}$ = $\{ 2 \}$
↳ sum of array elements

Generalize

Given N :

Put range $\Rightarrow [-10^9, 10^9]$

long range $\Rightarrow [-10^{18}, 10^{18}]$

Missing Element = $\frac{\text{Sum of all Elements}_{1-N} - \text{Sum of all array Elements}}{(N)(N+1)/2}$

T_C: $O(N)$ S_C: $O(1)$

Constraints:

$1 \leq N \leq 10^6$ $\Rightarrow \frac{(N)(N+1)}{2} \approx \frac{(10^6)(10^6+1)}{2} \approx 10^{12}$ \times overflow?
we cannot store in Integer

Obs2: $a^T a = 0$

$$\cancel{a^T b^T c^T d^T} \cancel{a^T d^T} = \underline{b}$$

Given $N = 9$,

$$[1-9] = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

$$ar[9] = \{6, 4, 1, 3, 2, 9, 7, 5, ?\}$$

$a = \{ \text{sum of all Elements}_{1-N} \}$

$b = \{ \text{sum of all array Elements} \}$

Missing element = $(a^T b)$

b

$a = 0$

$p=1; p \leq N; p++$

$a = a + p$

$b = 0$

$n \text{ for } a$

$T_C: O(N) S_C: O(1)$

202

→ Cryptography
 → Hacking
 → %

% → Remainder

$a \% b \Rightarrow$ Remainder when a is divided by b

$$10 \% 4 \Rightarrow 2 \quad 10 \% 6 \Rightarrow 4$$

Dividend = Divisor × Quotient + Remainder

$$\text{Remainder} = \text{Dividend} - \underbrace{\text{Divisor} \times \text{Quotient}}_{\text{Greater multiple of Divisor} \Leftarrow \text{Dividend}}$$

$$\underline{150 \% 11} = 150 - (11 \times 13) \{ 143 \} \Rightarrow 7$$

$$\underline{100 \% 7} = 100 - (7 \times 14) \Rightarrow 2$$

$$\underline{-40 \% 7} = -40 - (\text{greater multiple of } 7 \text{ } \leftarrow 7) \Rightarrow -42$$

$$\Rightarrow -40 - (-42) = 2$$

$$\underline{-60 \% 9} = -60 - (\text{greater multiple of } 9 \text{ } \leftarrow -63)$$

$$\Rightarrow -60 - (-63) = 3$$

Twist

Don'ts Session

In C/C++/ Java /

vs

Python

?

$$-40 \% 7 = \underline{-5} + \underline{7}$$

$$-60 \% 9 = \underline{-6} + 9$$

$$-30 \% 4 = \underline{-2} + 4$$

$$-40 \% 7 = \underline{2}$$

$$-60 \% 7 = \underline{3}$$

$$-30 \% 4 = \underline{2}$$

C/C++/ Java / TS / . . .

$$a \% m = \left\{ \begin{array}{l} \text{if } a < 0 \\ a \% m + m \end{array} \right\}$$

Why % Mod? → { % Mod can limit your data to a

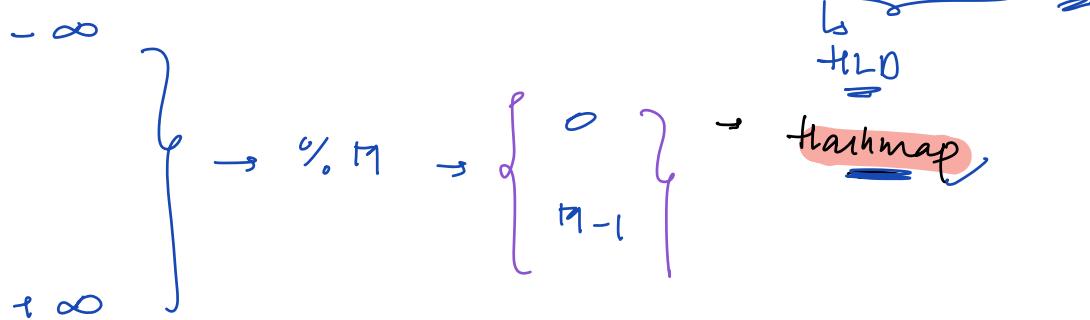
given range }

$$35 \% 10 \Rightarrow \underline{5}$$

← hashing

$$n \% 10 \Rightarrow [0 \quad 9]$$

→ consistent hashing



$\% \mathbb{N} \rightarrow [0, \underline{\underline{m-1}}]$

Modular Arithmetic {+, -, /, %}

$$(a+b) \% m = (\underline{\underline{a \% m + b \% m}}) \% m$$

$$a=6, b=8, m=10$$

$$6 \% 10 \quad 8 \% 10$$

$$6 + 8 = 14$$

$$(6+8) \% 10 \Rightarrow 4$$

1) $(a+b) \% m = (\underline{\underline{a \% m + b \% m}}) \% m \Rightarrow \underline{\underline{}}$

2) $(a+b) \% m = (\underbrace{a \% m + b \% m}_{\text{Exceed } m}) \% m \rightarrow \underline{\underline{}}$

3) $(a-b) \% m = [a(b) \% m] \left. \begin{array}{l} \text{Advanced Scedpm} \\ \text{TODO} \end{array} \right\}$

Q) $\text{power}(a, \underline{\underline{n}}, p) \rightarrow \underbrace{a^n \% p}$

E_n: $a=2, n=5, p=7 \rightarrow 2^5 \% 7 \rightarrow 32 \% 7 = 4$

E₂: $a=3, n=4, p=6 \rightarrow 3^4 \% 6 \rightarrow 81 \% 6 = 3$

```

int power(a, n, P) {
    int ans = 1;
    for (i = 1; i <= N; i++) {
        a = a * a;
    }
    return a % P;
}

```

// Assume no overflow

Constraints : $P \leq 10^7$

| <u>P</u> | <u>a</u> value Befn \Rightarrow | <u>a</u> value Aftn \Rightarrow |
|----------|--------------------------------------|--------------------------------------|
| 1 | <u>a</u> | $a^2 = a^{2^1}$ |
| 2 | a^2 | $a^4 = a^{2^2}$ |
| 3 | a^4 | $a^8 = a^{2^3}$ |
| 4 | a^8 | $a^{16} = a^{2^4}$ |

$a^{2^N} \times a^N$

(Int)

power(a, n, P) {

long ans = 1;

for (i = 1; i <= N; i++) {

$$\text{ans} = (\text{ans} * \text{a}) \% \text{P}$$

$$\text{ans} = (\text{ans} \% \text{P} + \text{a} \% \text{P}) \% \text{P}$$

$$(\text{P}-1 \times \text{P}-1) \approx (\text{P}-1)^2$$

$$\approx 10^4 \Rightarrow$$

$$a = 10, N = 40, p = 100$$

$$\Rightarrow 10^{40} \% 100$$

Can we solve $10^{40} \times$

return (ans \% P);

$$\frac{a^0, P-1}{P}$$

$$\underline{\underline{T_C}}: \underline{\underline{\mathcal{O}(N)}} \quad \underline{\underline{SC}}: \underline{\underline{\mathcal{O}(1)}}$$

Can we do

in Int?

looph break

$$\Rightarrow \underline{\underline{(a \% n) \% P = a \% M}}$$

// Ex:

$$\begin{aligned} \underline{a}, \underline{N=5}, \underline{P} \Rightarrow & (a^5) \% P = \\ & (a^4 * a) \% P = (a^4 \% P + a \% P) \% P \\ (a^4) \% P \Rightarrow & \underline{(a^3 * a)} \% P = \underbrace{(a^3 \% P + a \% P) \% P}_{\text{in green}} \end{aligned}$$

//

$$\underline{\text{Ex}}) \underline{(3 \ 8 \ 9)} \% 3 \Rightarrow (3+8+9) \% 3 \neq 0$$

$$\underline{(3 \ 3 \ 6 \ 2 \ 1)} \% 3 = (3+3+6+2+1) \% 3 = 0$$

Divisibility rule of 3:

→ Sum of digits should be divisible by 3

$$(3 \ 5 \ 6 \ 3 \ 2) \% 3 =$$

$$\begin{array}{r} 1 \downarrow 1 \downarrow 1 \downarrow 1 \downarrow \\ 10^4 10^3 10^2 10^1 10^0 \end{array} \left\{ 3 * 10^4 + 5 * 10^3 + 6 * 10^2 + 3 * 10^1 + 2 * 10^0 \right\} \% 3$$

$$\left\{ (3 * 10^4) \% 3 + (5 * 10^3) \% 3 + (6 * 10^2) \% 3 + (3 * 10^1) \% 3 + (2 * 10^0) \% 3 \right\} \% 3$$

$$(3 \% 3 * 10^4 \% 3) \% 3$$

$$1 \% 3 = 1$$

$$10 \% 3 = 1$$

$$10^2 \% 3 = 1$$

$$10^3 \% 3 = 1$$

$$(5 \% 3 * 10^3 \% 3) \% 3$$

$$(6 \% 3 * 10^2 \% 3) \% 3$$

$$\begin{aligned}
 & 1 \% = 1 \\
 & 10 \% = 1 \\
 & 10^2 \% = 1 \\
 & 10^N \% = 1
 \end{aligned}
 \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

$$\begin{aligned}
 & (8\%) \times (10\%) = 80 \\
 & 1 \\
 & (2) \% = 2
 \end{aligned}$$

$$\begin{aligned}
 & ((3\%) + (8\%) + (6\%) + (3\%) + (2\%)) \% = 24 \\
 & \Rightarrow (a\%, b\%, c\%, d\%, e\%) \% = (a+b+c+d+e)\% \\
 & \Rightarrow (3+5+6+3+2) \% = 21
 \end{aligned}$$

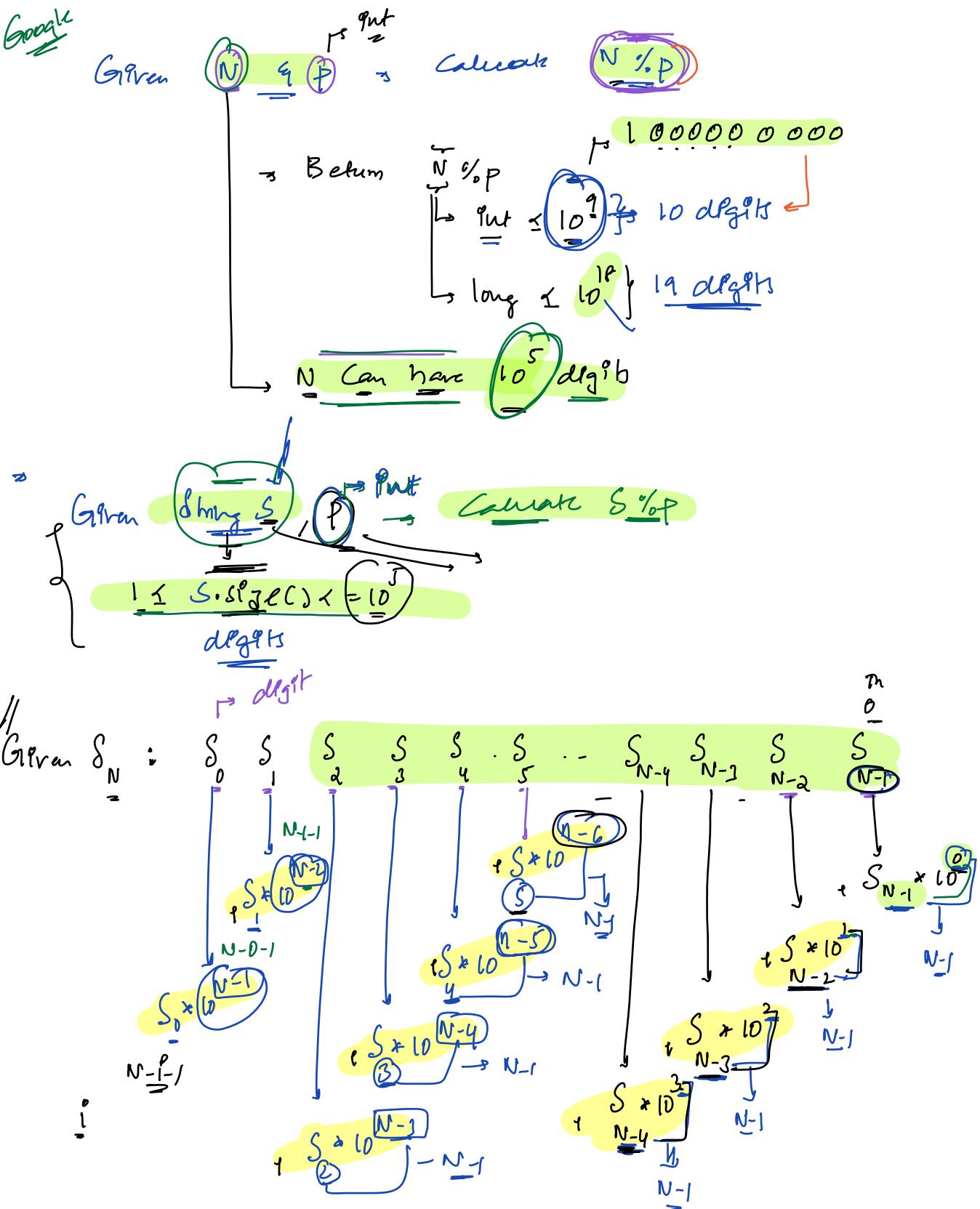
Divisibility by 9 = (Sum of digits divisible by 9)

Divisibility by 4 = (Last 2 digits should be divisible by 4)

$$\Rightarrow (4+8)\%_4 \Rightarrow (4 \times 10^2 + 78)\%_4 \Rightarrow 78\%_4$$

$$\begin{aligned}
 & 100\%_4 = 0 \\
 & 10^3\%_4 = 0 \\
 & 10^4\%_4 = 0 \\
 & 10^5\%_4 = 0
 \end{aligned}
 \quad \Rightarrow (4 \times 10^2 \%_4 + 78)\%_4 = 0$$

Last 3 digits %_4 = 0



$$\begin{aligned}
 S_{N \% p} &= \left(S_0 * 10^{N-1} + S_1 * 10^{N-2} + S_2 * 10^{N-3} + \dots + S_{N-1} * 10^0 \right) \% p \\
 S \% p &= \left\{ \begin{array}{l} \left(S_0 * 10^{N-1} \right) \% p \\ + \left(S_1 * 10^{N-2} \right) \% p \\ + \left(S_2 * 10^{N-3} \right) \% p, \dots \\ + \left(S_{N-1} * 10^0 \right) \% p \end{array} \right\} \% p \\
 &\quad (a * b) \% p \\
 &= (a \% p * b \% p) \% p
 \end{aligned}$$

long ans = 0, Given S

```

n = S.size();
for (int i = 0; i < n; i++) {
    ans = (ans + S[i] * power(10, N - i - 1, p)) \% p;
}
return ans;

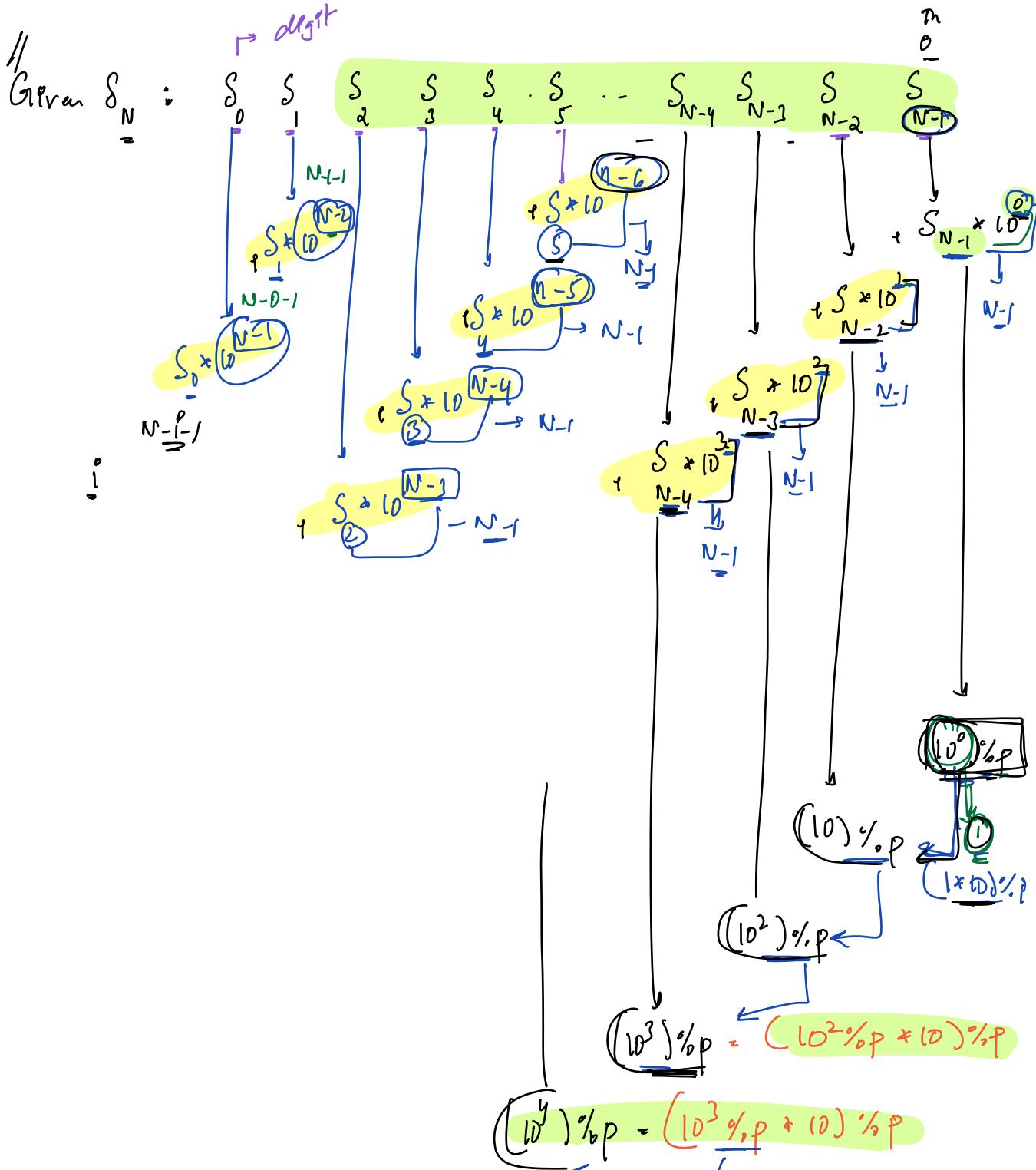
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power(10, N - i - 1, p) $\approx O(N)$

char to int, $O(1)$

Implemented by us

T_c: $O(N^2)$ T_s: $O(1)$



long ans = 0, emp = 1 }

N = S.size();

p = N-1; i = 0; p-- }
 ans = (ans + s[i] * emp) % p
 emp = (emp + 10) % p

return ans

TC: O(N)

SC: O(1)

$$10^3 \% p = \frac{10^4 \% p}{10}$$

$$\frac{1000 \% 7}{100 \% 7} \rightarrow \left(\frac{10 \% p}{10 \% p}\right) \mod 6$$

Details: C/C++/C#/Java vs Python

$$\text{Remainder} = \underbrace{\text{Dividend}}_a - (\underbrace{\text{Quotient}}_{a/b} \times \underbrace{\text{Divisor}}_p)$$

$$\left\{ \begin{array}{l} \text{is always} \\ \text{integer} \end{array} \right.$$

| | |
|----------------------------|---|
| $5/2 \rightarrow 2.5$ | } |
| $7/3 \rightarrow 2.333$ | |
| $10/6 \rightarrow 1$ | |
| $-10/6 \rightarrow -1.666$ | |

$$= -40 - (-40/7) * 7 = -20/3 \rightarrow -6.666$$

$$\rightarrow -40 - (-5) * 7$$

$$\rightarrow -40 + 35$$

$$\rightarrow -5$$

$$\text{In Python } \Rightarrow 5/2 \Rightarrow \underline{\underline{2.5}} \quad \left. \begin{array}{l} \text{Quotient cannot be float} \\ \text{floor} \end{array} \right\}$$

$$8/3 \Rightarrow \underline{\underline{2.666...}}$$

$$\text{Reminder} = \text{Dividend} - \text{Quotient} \times \text{Divisor}$$



$$a = 10, b = 4 \quad a/b = 2.5 \quad \begin{matrix} 2 \\ = \end{matrix}$$

$$a = 10, b = 6 \quad a/b = 2.12.. \quad 2$$

$$a = -14, b = 4 \quad a/b = -3.5.. \quad \begin{matrix} -4 \\ \text{floor} \end{matrix}$$

$$\underline{-60 \% 7} =$$

$$= -60 - \text{floor}(-60/7) \times 7 \quad \begin{matrix} \uparrow \\ \text{floor} \end{matrix}$$

$$-60/7 = \underline{\underline{-8.57}} \quad \begin{matrix} -8 \\ -9 \end{matrix}$$

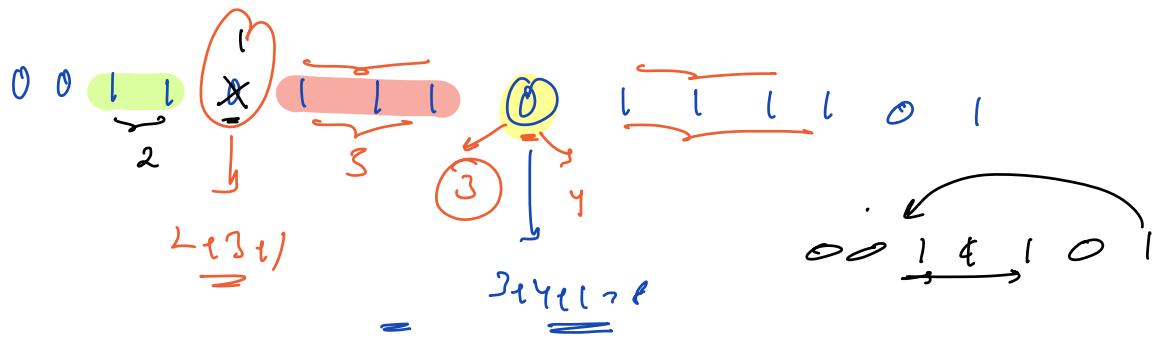
$$= -60 - (-9) \times 7$$

$$= -60 + 63$$

$$\underline{\underline{3}}$$

Divisor : n, q, p

$$\Rightarrow \left\{ \begin{array}{l} \cancel{a \% p} \\ \cancel{a \% p} \\ \cancel{a \% p} \end{array} \right\} \rightarrow \text{remainder} \quad \left. \begin{array}{l} \rightarrow \% p \\ \rightarrow \text{long} \end{array} \right\} \Rightarrow \begin{matrix} n \times 0 : \\ [n \% p + p] \% p \end{matrix}$$



$0 \rightarrow$ centre's left
 \hookrightarrow centre's right

$\% . p \Rightarrow$