

→ Remainder %

$$\% = \text{remainder}$$

$$10 \% 4 = 2, \quad 99 \% 8 = 3$$

$a \% b$ = remainder when a divided by b

$$\begin{aligned} \text{Remainder} &= \underset{\text{Dividend}}{\cancel{79}} - \frac{\text{Divisor + Quotient}}{\text{Max multiple of Divisor}} \\ &= 79 - 8^* 12 \\ &= 79 - 96 = 3 \end{aligned}$$

$$\underline{Q_1}: -40 \% 7 = -40 - (\text{Max multiple of } 7 \text{ } \alpha = -40)$$

$$\begin{aligned} &= -40 - (-42) \\ &= -40 + 42 = 2 \end{aligned}$$

-3/2 \rightarrow $\frac{-1+5}{2}$
↓ floor
-2

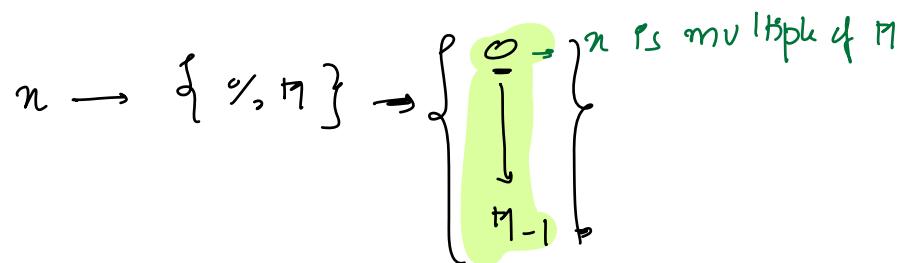
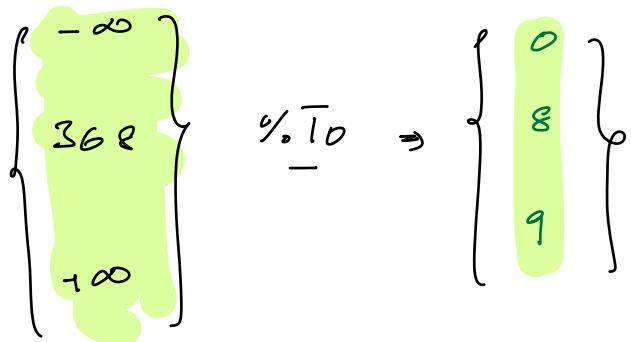
$$\underline{Q_2}: -60 \% 9 = -60 - (\text{Max multiple of } 9 \text{ } \alpha = -60)$$

$$\begin{aligned} &= -60 - (-63) \\ &= -60 + 63 \\ &= 3 \end{aligned}$$

C/C++/ Java vs Python

$$\left\{ \begin{array}{l} -40 \% 7 \\ -60 \% 9 \end{array} \right. = \begin{array}{l} -5 + \underbrace{7}_{\rightarrow} \\ -6 + \underbrace{9}_{\rightarrow} \end{array} \quad \begin{array}{l} 2 \\ 3 \end{array}$$

Why?



Modular Arithmetic

$$1) \quad \boxed{(a+b) \% m = (\underbrace{a \% m + b \% m}_{\text{Sum}}) \% m}$$

$a = 4 \quad b = 5 \quad m = 6$

$$\underbrace{(a \% b)}_{3} = (4 + 5) = 9 \% 6 = 3$$

$$2) \quad \boxed{a \% m = (\underbrace{a + m}_{\text{Sum}}) \% m} \Rightarrow (a \% m + \underbrace{m \% m}_{\text{Sum}}) \% m \xrightarrow{\text{Sum} \rightarrow 0} \\ \Rightarrow (\underbrace{a \% m}_{\text{Sum}}) \% m \Rightarrow a \% m$$

$$3) \quad \boxed{(a * b) \% m = (a \% m * b \% m) \% m}$$

$$4) \quad \boxed{(a - b) \% m = (\underbrace{a \% m - b \% m}_{\text{Difference}} + \overbrace{m}^{\text{Modulus}}) \% m} \xrightarrow{\text{Difference} \in [0, m-1], \text{Modulus} \in [0, m-1]}$$

$$a = 8, \quad b = 4, \quad m = 5$$

$$\begin{aligned} & \Rightarrow (8 \% 5 - 4 \% 5 + 5) \% 5 \\ & \Rightarrow (8 - 4 + 5) \% 5 \\ & \Rightarrow 9 \% 5 \end{aligned}$$

$$\Rightarrow 4 \% 5$$

$$= \underline{4}$$

Q Given $A \in B$, $A > B$ find no:of M such that

$$\underline{A \% M} = \underline{B \% M} \text{ & } \underline{M > 1}$$

Ex1: $A = 16$ $B = 4$ $M = 2, 3, 4, 6, 12$

Ex2: $A = 13$ $B = 7$ $M = 2, 3, 6$

Idea: $\underline{A \% M} = \underline{B \% M}$

$$A \% M - B \% M = 0$$

+ Add M in both sides

$$A \% M - B \% M + M = M$$

Take $\% M$ in both sides

$$(A \% M - B \% M + M) \% M = 0$$

$$(A - B) \% M = 0$$

What all Value can M take?

All factors of $\frac{A-B}{M}$ except 1

Sqr ()

Q8) Given N array elements, calculate number of pairs (i, j)

such that $(arr[i] + arr[j]) \% M = 0 \}$ M is given \rightarrow ① time

Note: $(i \neq j)$ & q q of pair (i, j) is same as (j, i) }

Ex1: $arr[6] : \boxed{4 | 7 | 6 | 5 | 5 | 3} \rightarrow M = 3$

i	j	arr[i]	arr[j]	count =
0	3	4	5	$9 \% 3$
0	4	4	5	$9 \% 3$
1	3	7	5	$12 \% 3$
1	4	7	5	$12 \% 3$
2	5	6	3	$9 \% 3$

$M = 10$

0 1 2 3 4 5 6 7 8 9 10 11 12

Ex2: $arr[] = \boxed{17 | 2 | 5 | 4 | 6 | 23 | 13 | 26 | 14 | 18 | 15 | 30 | 35} =$

i	j	arr[i]	arr[j]	i	j	arr[i]	arr[j]
0	5	17	23	3	4	4	6
0	6	17	13	3	7	4	26
1	9	2	18	4	8	6	14
2	10	5	15	7	8	26	14
2	12	5	35	10	12	15	35

Ques 1: $\text{arr}[] = [13, 14, 22, 3, 32, 19, 16]$

i	j	$\text{arr}[i] + \text{arr}[j] \bmod 17 = 0$
0	3	13 + 3 = 16
0	5	13 + 19 = 32
1	2	14 + 22 = 36
4	6	32 + 16 = 48

Brute force: check all pairs

```

 $i = 0; i < N; i++ \{$ 
     $j = i+1; j < N; j++ \{$ 
         $(\text{arr}[i] + \text{arr}[j]) \% 17 = 0$ 
    }
}

```

cut + p

$Tc: O(N^2) \quad Sc: O(1)$

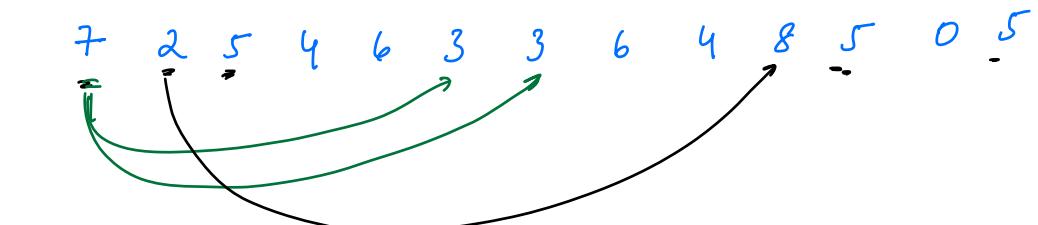
// observations

$$\begin{aligned}
 & (\text{arr}[i] + \text{arr}[j]) \% 17 = 0 \\
 \hookrightarrow & (\underbrace{\text{arr}[i] \% 17}_{\text{arr}[i]} + \underbrace{\text{arr}[j] \% 17}_{\text{arr}[j]}) \% 17 = 0
 \end{aligned}$$

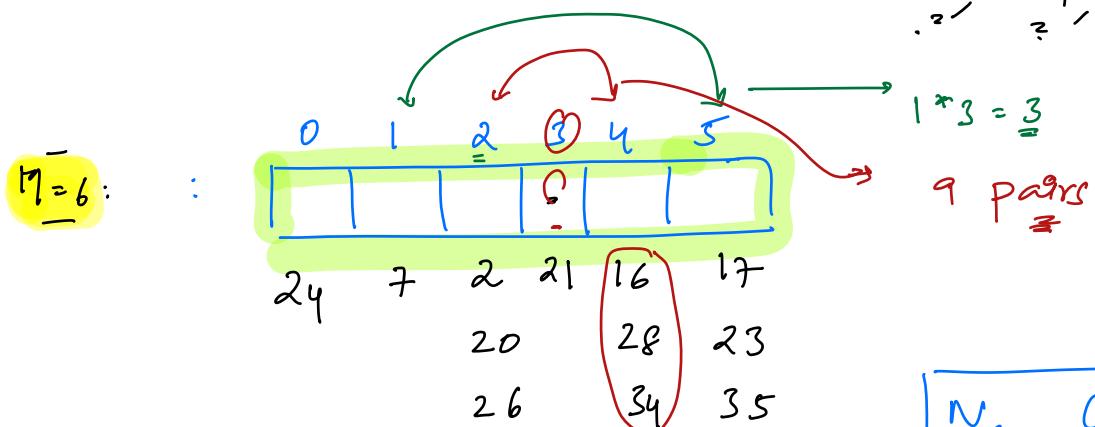
$\text{arr}[] = [0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]$

$\text{arr}[] = [17, 2, 5, 4, 6, 23, 13, 26, 14, 18, 15, 30, 35]$

$\text{arr}[] = [7, 2, 5, 4, 6, 3, 3, 6, 4, 8, 5, 0, 5]$

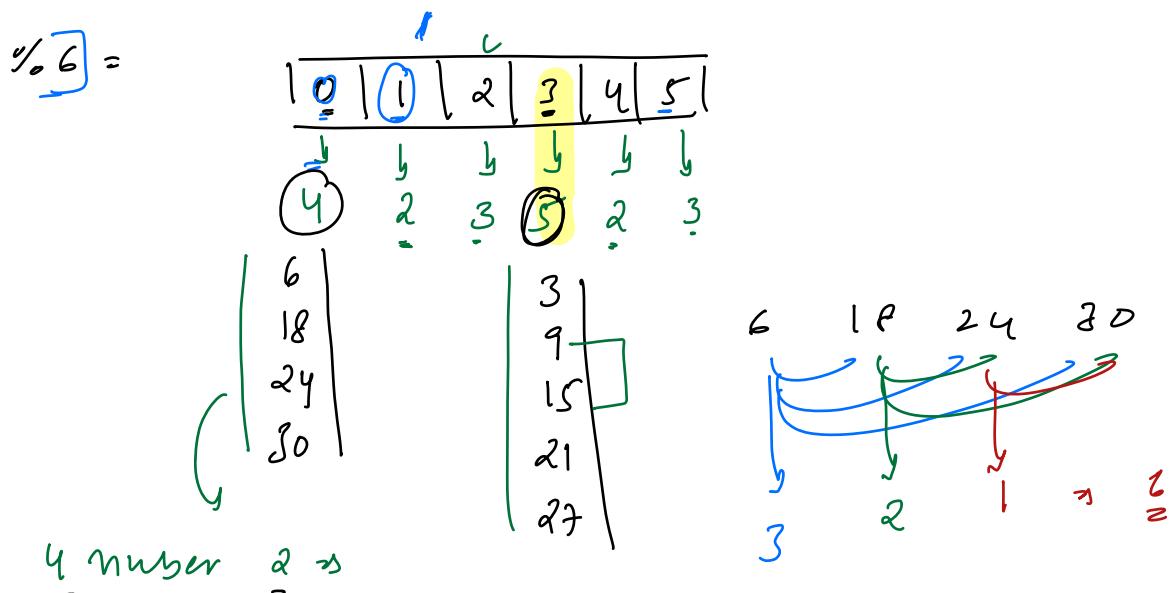


obs: $\text{arr}_{12} : \{ 24, 16, 8, 7, 17, 23, 35, 20, 26, 28, 34, 21 \}$



$$N_{C_2} = \frac{(N)(N-1)}{2}$$

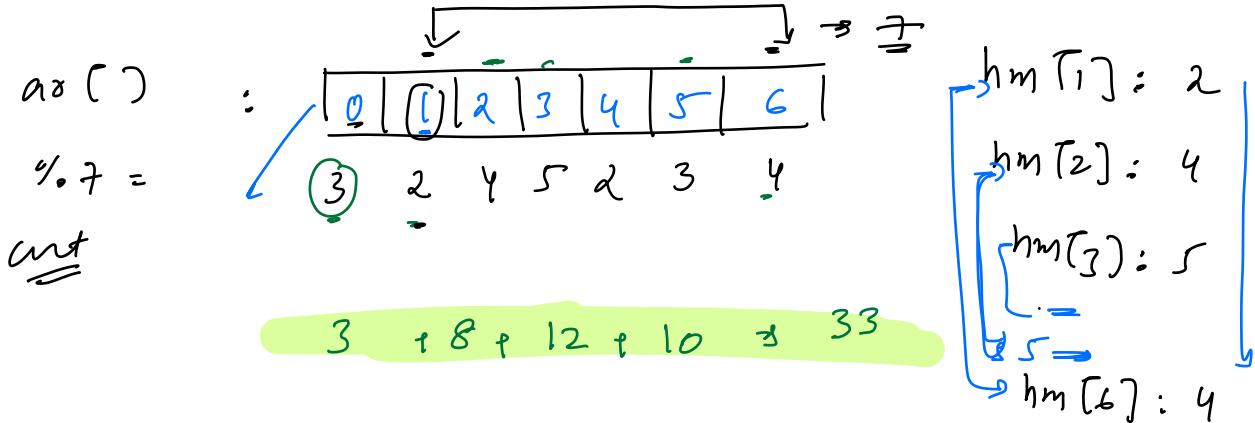
obs: $\text{arr}[]$



$$N_{C_2} = \frac{(4)(3)}{2}$$

$$= \frac{6}{2} + 2 * 3 + 3 * 2 + 10 =$$

6 + 6 + 6 + 10 \Rightarrow 28 pairs



Step 1: Modular count

hashmap <int, int> hm; / ans

```

 $i = 0; i < N; i++) \{$ 
     $hm[\overbrace{\text{arr}[i] \% m}^{\text{hm}[i]}]++;$ 
}

```

$Tc: O(N+m)$

ans = $\sum_{i=0}^{m-1} hm[i]$

Sc: $O(m)$ $m \leq N$

$i = 1, j = m-1; \}$

m if you \downarrow unarray \downarrow hash map

while ($i < j$)

$O(n)$

$ans = ans + hm[i] * hm[j]$

$O(N)$

$i++, j--;$

Break: $10:50PM$

if ($m \% 2 == 0$) {

$ans += hm[m/2]$

TODO: Given N ,

where $\{0 \leq i \leq N-1\}$, N is size of array

for all numbers from $(0, \dots, N-1)$ get its frequency

TODO

Q) Given an array of all distinct integers

where $\{0 \leq i \leq N-1\}$, N is size of array.

replace $\alpha[i]$ with $\alpha[\alpha[i]]$

Diagram illustrating the execution of a reverse array assignment:

Initial array state:

0	1	2	3	4
3	2	4	1	0

Assignment statement: $\text{ar}[0] = \underline{\underline{5}}$

Execution flow:

- $\text{ar}[0] = \text{ar}[\underline{\underline{\text{ar}[0]}}]$ (highlighted in orange)
- $\text{ar}[\underline{\underline{\text{ar}[0]}}] = \underline{\underline{5}}$ (highlighted in orange)
- $\text{ar}[0] = \underline{\underline{5}}$ (highlighted in green)

Updated array state:

0	1	2	3	4
5	4	0	2	3

$$\text{En2: } \alpha[7] = \begin{array}{ccccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ \underline{3} & 1 & 4 & \underline{-6} & 5 & 0 & 2 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ - & & & & & & \\ \underline{6} & 1 & 5 & 2 & 0 & 3 & 4 \end{array}$$

Ques 1:

$$ar[7] = \{1, 6, 3, 5, 4, 2, 0\}$$

↓ ↓ ↓ ↓ ↓ ↓ ↓
 6 0 5 2 4 3 1

// Appraus

1) Int arr[N]

$$T = O(1) \propto N; T \in O(N)$$

$$\left| \begin{array}{l} arr[i] = arr[arr[i]] \\ \end{array} \right.$$

3

$Tc: O(N)$
 $Sc: O(\underline{N})$

// → No Extra Space

$$0 : 0 : 0 : \underline{23 \text{ hrs}} \rightarrow \text{Day: } 0 \quad 23 \text{ hrs}$$

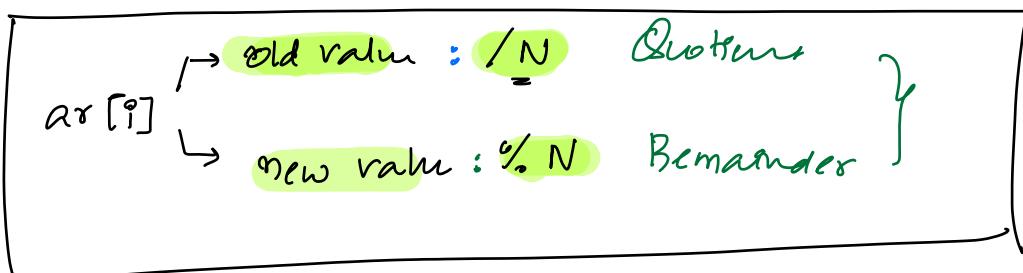
$$46 \text{ hrs} \rightarrow \text{Day: } 1 \quad 22 \text{ hrs}$$

$$100 \text{ hrs} \rightarrow \text{Day: } 4 \quad 4 \text{ hrs}$$

$$125 \text{ hrs} \rightarrow \text{Day: } 5 - 5 \text{ hrs}$$

$$\underline{n \text{ hrs}} \rightarrow \text{Day: } \underline{\frac{n}{24}}$$

Quotient Remainder



$\frac{0}{N}$	$\frac{1}{N}$	$\frac{2}{N}$	$\frac{3}{N}$	$\frac{4}{N}$
$ar[5] : \underline{\underline{3}}$	$\underline{\underline{2}}$	$\underline{\underline{4}}$	$\underline{\underline{1}}$	$\underline{\underline{0}}$
$N = 5$				
Multiply $* 5$: $(3 * 5 + 1)$	$\boxed{2 * 5 + 4}$	$4 * 5 + 0$	$1 * 5 + 2$	$0 * 5 + 3$
Ind : $15 / 5 = 3$	$= 10 / 5 = 2$	$20 / 5 = 4$	$5 / 5 = 1$	$0 / 5 = 0$
$val = ar[\underline{\underline{ind}}] / 5 = \underline{\underline{1}}$	$= ar[\underline{\underline{2}}] / 5$	$ar[\underline{\underline{4}}] / 5 = 0$	$14 / 5 = 2 \frac{4}{5}$	$16 / 5 = \underline{\underline{3}}$
$ar[0]_+ = val$	$ar[1]_+ = val$			$ar[4]_+ > val$
$ar[1]_+ = ar[\underline{\underline{ar[1]}}]$	$\}$	$ar[2]_+ = ar[\underline{\underline{ar[2]}}]$		

$$\begin{array}{cccccc}
3 * 5 + 1 & \boxed{2 * 5 + 4} & 4 * 5 + 0 & 1 * 5 + 2 & 0 * 5 + 3 \\
\% N = & \text{---} & \text{---} & \text{---} & \text{---} \\
\underline{\underline{1}} & \underline{\underline{4}} & \underline{\underline{0}} & \underline{\underline{2}} & \underline{\underline{3}}
\end{array}$$

// Step 1: Iterate & multiply N

$i = 0; i < N; i++ \{$

$\underline{\underline{ind}} = ar[i] / N$
 $\underline{\underline{val}} = ar[\underline{\underline{ind}}] / N$
 $ar[i]_+ = val$

\uparrow
 ind
 \uparrow
 $ar[\underline{\underline{ind}}] / N$
 \uparrow
 val

// Step 3: Iterate & $\% N \rightarrow$

Inverse modulus

$$(a/b) \% m = (a \% m) / (b \% m)$$

$$\Rightarrow (a \times b^{-1}) \% m = (\underline{a \% m} \times \underline{(b^{-1} \% m)}) \% m$$

Inverse modulus?

Given a, m , b is inverse modulus of a if

$$(a \times b) \% m = 1$$

↑ greatest common divisor

// b is exists if $\gcd(a, m) = 1$

// value of b : $[1, m-1]$

$$\underline{\underline{\text{Ex:}}} \quad a = 10 \quad m = 7 \quad b: [1, 6]$$

$$b = 1 \rightarrow (10 \times 1) \% 7 = 1 \rightarrow 1$$

$$b = 2$$

$$b = 3$$

$$b = 4$$

$$\underline{b = 5}$$

$$(10 \times 5) \% 7 = 1$$

$$\rightarrow \boxed{a^{-1} \% m = 5}$$

Ex: $a = 12$ $(m = 5)$, $b = 3 \Rightarrow$

$$\Rightarrow (12 \times 3) \% 5 = 1$$

// Given a, m , $\gcd(a, m) = 1$ find b ?

```

p = 1; q < m; p++ {
    if ((a * p) \% m == 1)
        return p
}

```

$Tc \geq O(m)$
 $Sc \geq O(1)$

a is not multiple of p

// Special Case : If p is prime $\bar{a}^1 \% p = 0$

$$1) \bar{a}^1 \% p = \underbrace{(a^{p-2}) \% p}_{\text{Fermat's Little Theorem}}$$

$$2) (a^{p-1}) \% p = 1$$

{Recursion}

$$\bar{a}^1 \% p = \underbrace{(a^{p-2}) \% p}_{\text{uninhibit power branch}}$$

powerMod (a, b, p) {

| calculate $a^b \% p$

$$\underline{\underline{Eqn1}}: \quad a = 10 \quad m = 7$$

$$\underline{\underline{Eqn2}}: \quad a = 12 \quad m = 5$$

Special Case :

i) If p is prime number & $a \% p \stackrel{!}{=} 0$

$$(a^{p-2}) \% p = a^{-1}$$

$$(a^{p-1}) \% p = 1$$