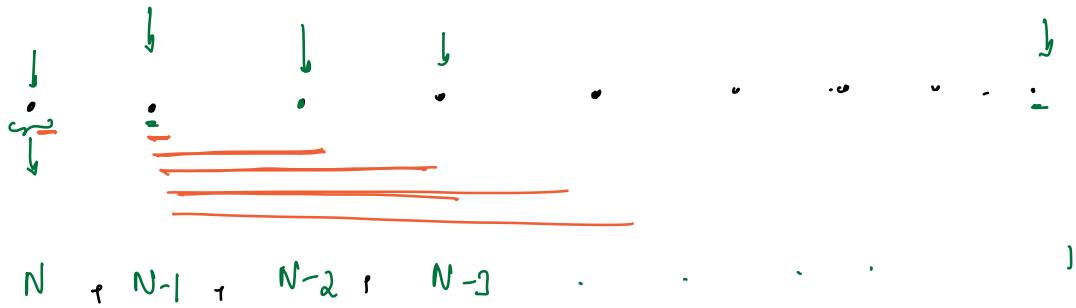


Subarrays : Continuous segment of an array

→ Single Element

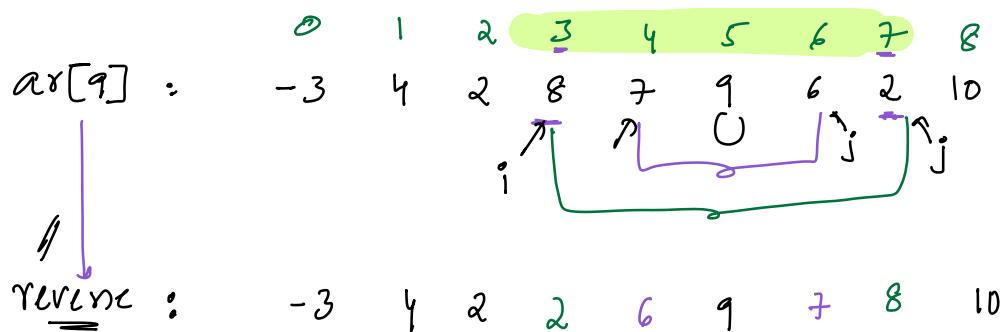
→ Full array

→ $\{s_i, e_i\}$ $s_i \leq e_i$



$$\text{Total Subarrays} = \frac{C(N)(N+1)}{2}$$

Q8) Given an array a , reverse entire subarray $[s^o - e^i]$



```
void reverse( int ar[], int s, int e) {
```

$s=0, e=N-1$

$i=s, j=e;$

$\text{while}(i <= j) \{$

swap $ar[i]$ & $ar[j]$

$i++; j--$

$O(N)$

28) Rotate Array Rotate array from R-L

$\text{ar[7]} = \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & -2 & 1 & 4 & 6 & 9 & 8 \end{matrix}$

Rotate 1: $\begin{matrix} 8 & 3 & -2 & 1 & 4 & 6 & 9 \end{matrix}$

Rotate 2: $\begin{matrix} 1 & 8 & 3 & -2 & 1 & 4 & 6 \end{matrix}$

Rotate 3: $\begin{matrix} 6 & 9 & 8 & 3 & -2 & 1 & 4 \end{matrix}$

Rotate 4: $\begin{matrix} 4 & 6 & 9 & 8 & 3 & -2 & 1 \end{matrix}$

Ex::

$\text{ar[13]} = \begin{matrix} a_0 & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 & a_{10} & a_{11} & a_{12} \end{matrix}$

Rotate 5: $\begin{matrix} a_8 & a_9 & a_{10} & a_{11} & a_{12} & a_0 & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \end{matrix}$

Reverse array:

$\begin{matrix} a_{12} & a_{11} & a_{10} & a_9 & a_8 \end{matrix}$

Reverse subarray [0-4]

$\begin{matrix} a_7 & a_6 & a_5 & a_4 & a_3 & a_2 & a_1 & a_0 \end{matrix}$

Reverse subarray [5, 12]

$a_8 \ a_9 \ a_{10} \ a_{11} \ a_{12} \ a_0 \ a_1 \ a_2 \ a_3 \ a_4 \ a_5 \ a_6 \ a_7$

Given N array elements Rotate by k times

Iterations

Step1: Reverse Entire array. $[N]$

Step2: Reverse $1^{\text{st}} k$ Elements : $[0, k-1]$ —

Step3: Reverse remaining $N-k$ Elements : $[k, N-1]$ —

$T_C = O(N)$ $S_C = O(1)$

if ($k \geq N$) we will earn,
 $\{ k = k \% N \}$ Array out of bounds
 Segmentation

$N/2$

$k/2$

$(N-k)/2$

$\frac{N/2 + k/2 + \frac{N}{2} - k/2}{2}$

$\rightarrow \underline{N}$

Given $arr[6] = a_0 a_1 a_2 a_3 a_4 a_5$

$k = k \% 6$

Rotate by 0 : $a_0 a_1 a_2 a_3 a_4 a_5$

Rotate

$6 = 0$

$12/6 = 0$

Rotate by 1 : $a_5 a_0 a_1 a_2 a_3 a_4$

$74/6 = 1$

$13/6 = 1$

Rotate by 2 : $a_4 a_5 a_0 a_1 a_2 a_3$

$84/6 = 2$

$14/6 = 2$

Rotate by 3 : $a_3 a_4 a_5 a_0 a_1 a_2$

$9 = 3$

$15 = 3$

21

Rotate by 4 : $a_2 a_3 a_4 a_5 a_0 a_1$

$10 = 4$

$16 = 4$

Rotate by 5 : $a_1 a_2 a_3 a_4 a_5 a_0$

$11 = 5$

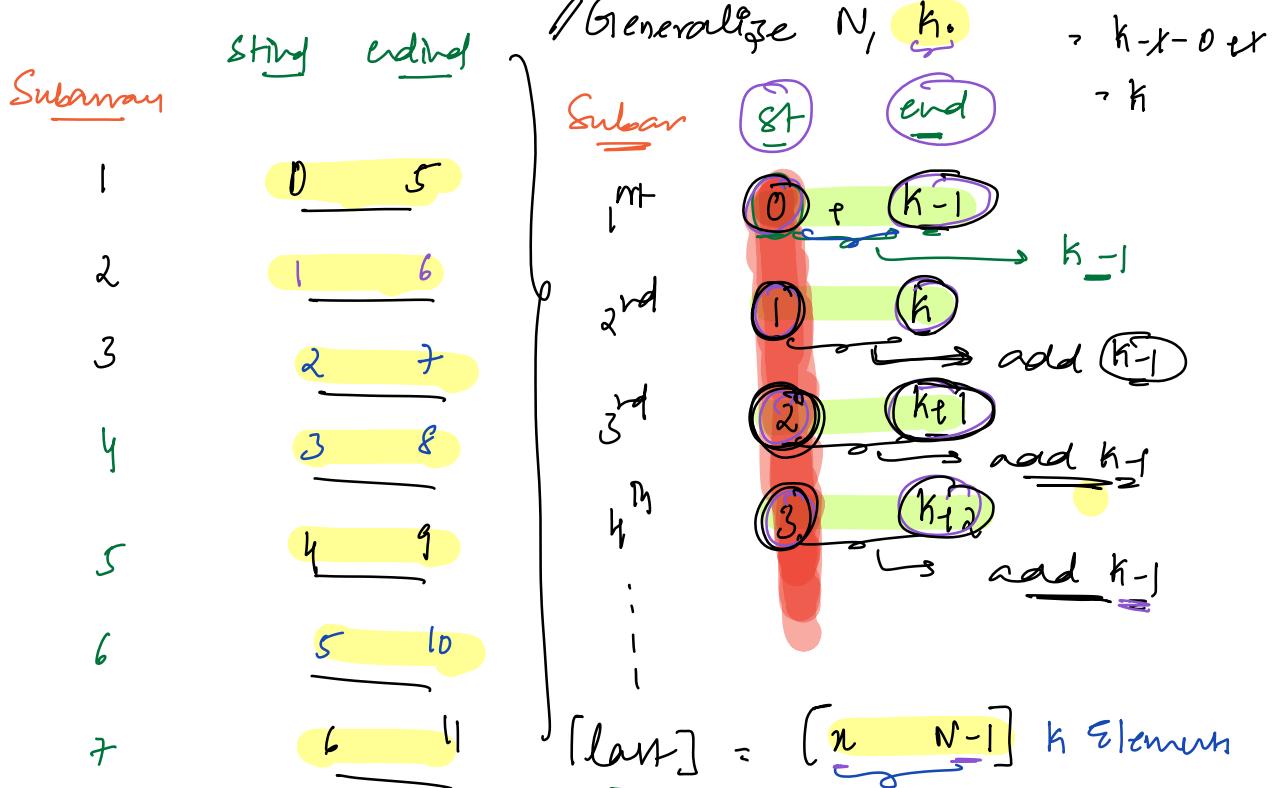
$17 = 5$

Given N , Rotate by k : $k = k \% N$ $\Rightarrow k > N$, $k = k \% N$,

→ TODO: Build from Rotating Left & Right

3Q) Print start & end index of all subarrays ($a \rightarrow b$)

$arr[12] : 0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ [a \ b]$
 $\quad \quad \quad 3 \ 4 \ 2 \ -1 \ 6 \ 7 \ 8 \ 9 \ 3 \ 2 \ -1 \ 4 \ = b - a + 1$
 $k = 6$
 $\boxed{[0, k-1]}$



Pseudo code:

$$i = 0; i \leq N-k; i++) \{$$

$$\boxed{i = i + k-1}$$

↓
Print (i, g)

$$N - i - n + k = k$$

$$N - n = k, \quad k = N - n$$

$$\text{last} = \boxed{[N-k \ N-1]}$$

$$[i - n] = k \quad n - i + 1 = k$$

$$n \rightarrow i+k-1$$

48) Find Max subarray sum with len = k.

	0	1	2	3	4	5	6	7	8	9
$arr[10]$	-3	4	-2	5	3	-2	8	2	-1	4

$$k=5 \\ \Rightarrow \text{sums}$$

$$0-4 : 7$$

$$1-5 : 8$$

$$2-6 : 12$$

$$3-7 : 16$$

$$4-8 : 10$$

$$5-9 : 11$$

$$\text{Max} : 16$$

output: 16

$$TC: (N-k+1) \times (k)$$

for what k we can get max prod.

$$k=N \Rightarrow (N-N+1)(N) = N$$

$$k=1 \Rightarrow (N-1+1)(1) > N$$

$$k \approx N/2 \Rightarrow$$

$$(N-N/2+1) \times (N/2)$$

$$\approx (N_{\frac{N}{2}+1})(N/2) \Rightarrow O(N^2)$$

Idea:
Sum of all subarray of len = k

get max of them

Pseudocode:

$$\text{max} = \frac{0/arr[0]}{\text{INT_MIN}} // \text{Subarrays of } k$$

$$i=0; j=N-k; i++ \{ \quad \boxed{[N-k+1]}$$

$$j = \boxed{i+k-1}$$

$$[i, j] \text{ len } = k$$

$S=0$; we are getting sum

$$l=i; l < j; l++ \{ // \text{key } = k$$

$$S = S + arr[l]$$

// S is subarray sum from $[i, j]$

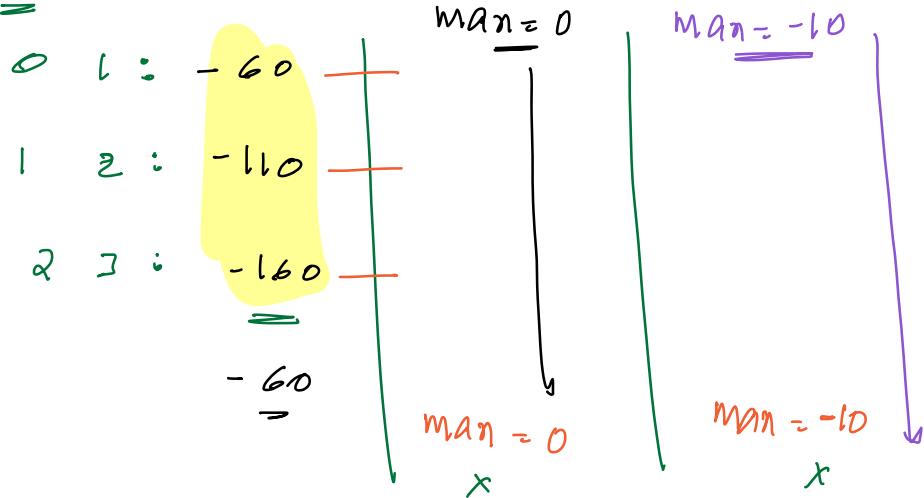
if (S > max) { max = S }

return max

10:45 break

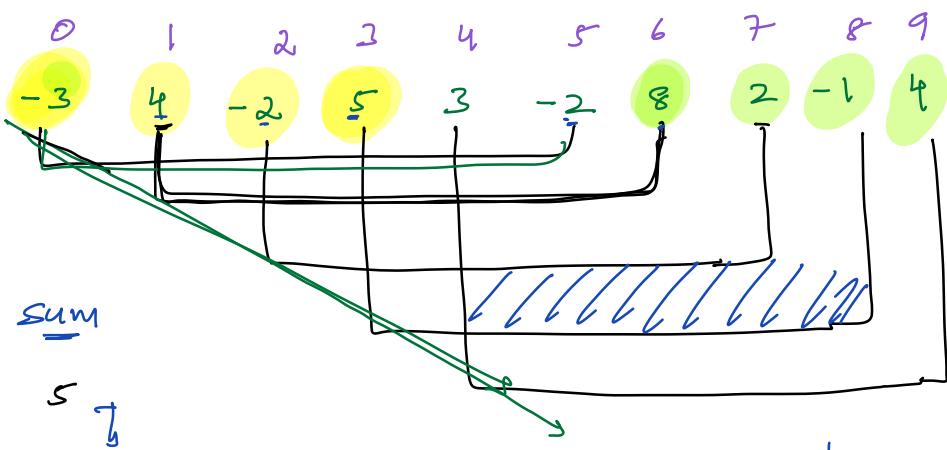
$$\text{Eq: } \begin{matrix} 0 & 1 & 2 & 3 \\ -10 & -50 & -60 & -100 \end{matrix}$$

$k=2$:



$ar[10]$:

$k=6$



$[0-5]$:

5

$$5 - ar[0] + ar[5] = 5 - (-3) + 8 = 16$$

$[1-6]$:

16

$$16 - ar[1] + ar[6] = 16 - (4) + 8 = 14$$

$[2-7]$:

16

$$16 - ar[2] + ar[7] = 16 - (-2) + 2 = 14$$

$[3-8]$:

14

$$14 - ar[3] + ar[8] = 14 - (-4) + -1 = 15$$

$[4-9]$:

15

$$15 - ar[4] + ar[9] = 15 - 5 + 4 = 14$$

// Generalize N array $S \in \mathbb{R}^N \rightarrow k$.

1st subarray : $[0, k-1] = \sum_{i=0}^{k-1} a_i$ get target & get sum

2nd subarray : $[1, k] = S - a_0 + a_k$

3rd subarray : $[2, k+1] = S - a_1 + a_{k+1}$

4th subarray : $[3, k+2] = S - a_2 + a_{k+2}$

5th subarray : $[4, k+3] = S - a_3 + a_{k+3}$

last subarr : $[N-k, N-1] = S - a_{N-k} + a_{N-1}$

$$S = O / ?$$

$$\begin{aligned} i = 0; i < k; i++ \{ & \longrightarrow \} [0, k-1] : k \\ | \quad S = S + a_i & \end{aligned}$$

max = S // S is sum from subarray

$$\begin{aligned} i = 1; i < N-k; i++ \{ & \longrightarrow [1, N-k] : \frac{N-k}{N} \\ | \quad j = [i+k-1] \quad \{ & \end{aligned}$$

$$S = S - a_{i-1} + a_j$$

$\{ \text{if } (S > \text{max}) \{ \text{max} = S \}$

$$TC: \Theta(N)$$

$$SC: \Theta(1)$$

return max

//

Given N array elements, print all subarray sums

$$\text{arr}[4] = \begin{matrix} 0 & 1 & 2 & 3 \\ 3 & -1 & 0 & 2 \end{matrix}$$

$[0, 0] : 3$	$[1, 1] : -1$	$[2, 2] : 0$	$[3, 3] : 2$
$[0, 1] : 2$	$[1, 2] : -1$	$[2, 3] : 2$	
$[0, 2] : 2$	$[1, 3] : 1$		
$[0, 3] : 4$			

// Sum of all subarray sums = 14

Sol 1: For every subarray call

q print sum.

$$i = 0; i < N; i++ \{$$

$$j = i; j < N; j++ \{$$

$$\Rightarrow [i, j]$$

$$S = arr[i]$$

$$k = i; k <= j; k++ \{$$

$$S = S + arr[k]$$

print(S)

TC: $O(N^3)$

arr[6]

0	1	2	3	4	5
3	2	-1	6	4	8
5	+ -1 = 4	6	10	4	14
					8
					22

$$[0, 0] : 3$$

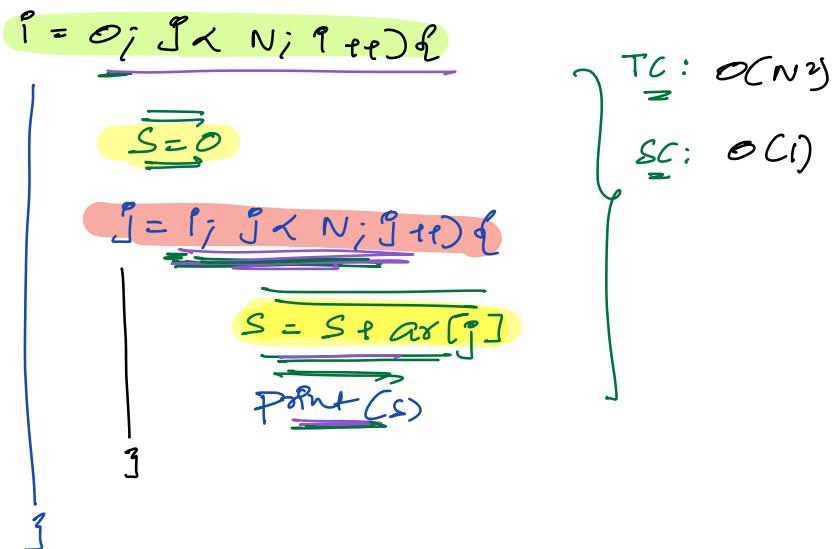
$$[0, 1] : 5$$

$$[0, 2] : 4$$

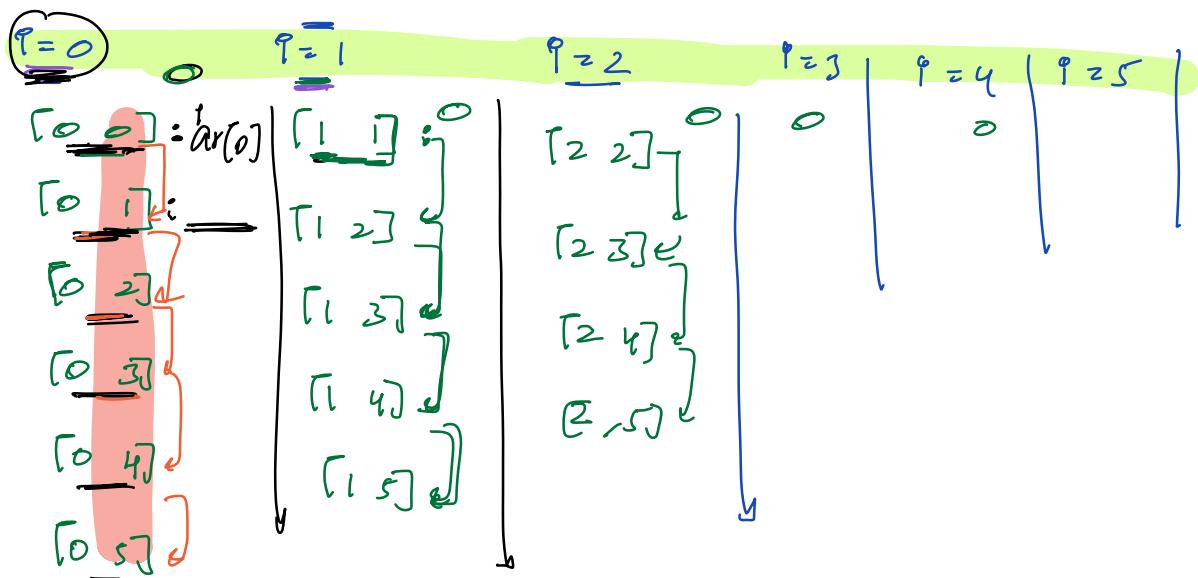
$$[0, 3] : 10$$

$$[0, 4] = [0, 3] + arr[4] = 14$$

$$[0, 5] = [0, 4] + arr[5] = 22$$



$arr[6]$					
0	1	2	3	4	5
3	2	-1	6	4	8



$[i - j] = S$
 $[i, j+1] = S + arr[j+1]$

$\quad // \underline{\underline{Carry forward}}$
 $\quad [0, 5] =$
 $\quad [0, 6] =$

int ans = 0

i = 0; i < N; i++) {

j = i; j < N; j++) {

 [i j]

(s = 0)

k = i; k <= j; k++) {

 s = start[k];

 ans = ans + s // sum of all subarrays

} return ans; TC: O(N^3)

ans = 0

i = 0; i < N; i++) {

s = 0

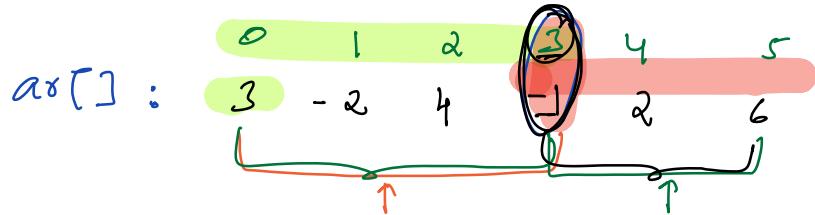
j = i; j < N; j++) {

s = s + arr[j]

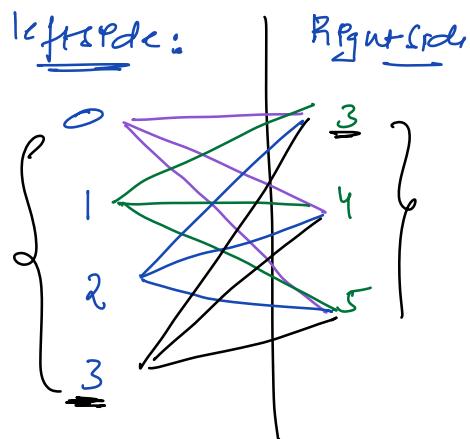
 ans = ans + s // sum of all subarray sums

} return ans; TC: O(N^2)

// Sum of all subarray sums \rightarrow $O(N^3) \rightarrow O(N^2) \rightarrow =$

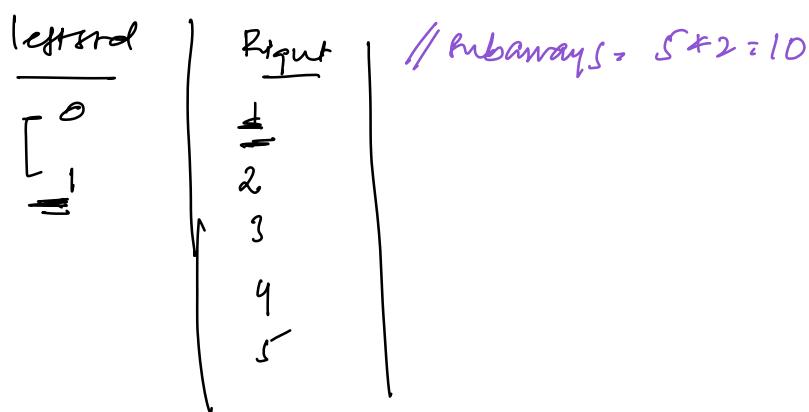
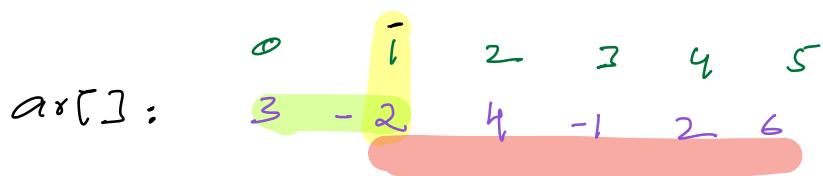


// In how many subarrays Index 3 is present

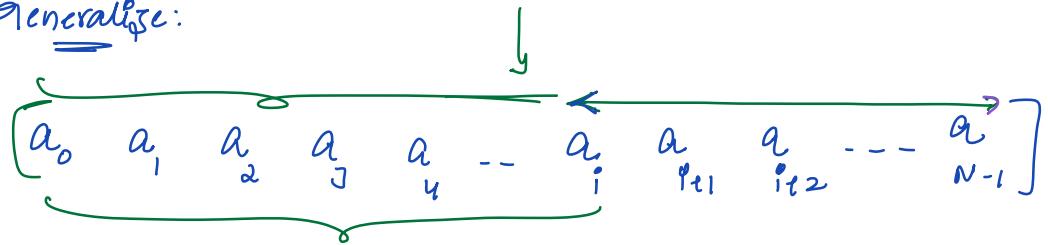


$$\Rightarrow \text{Total pairs} = 12 = \text{Left} * \text{Right} =$$

$$4 * 3 = 12$$



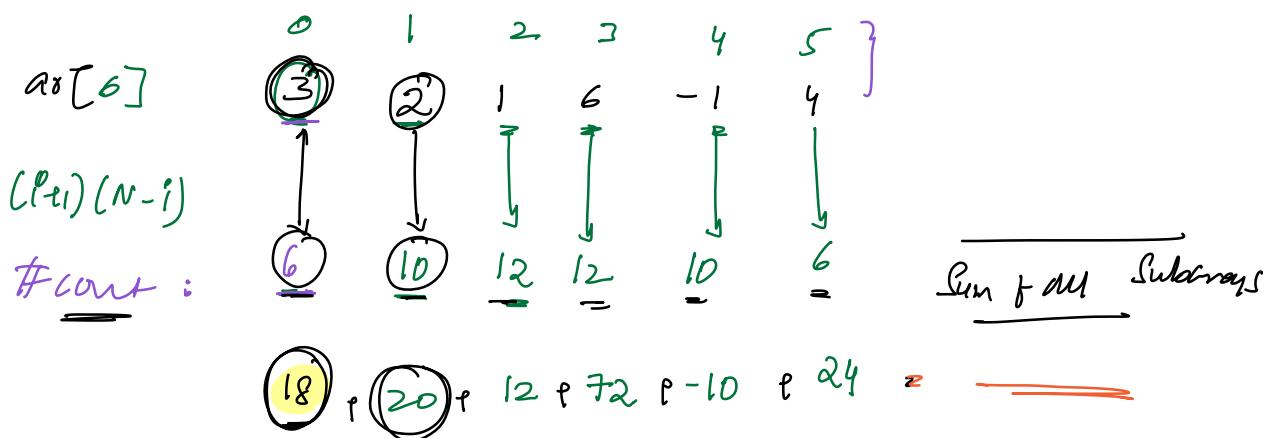
// Generalise:



$$\text{Left side : } \underbrace{[0, i]}_{\text{Left side}} \Rightarrow i+1 \Rightarrow N-i$$

$$\text{Right side : } \underbrace{[i, N-1]}_{\text{Right side}} \Rightarrow N-1-i+1 \Rightarrow N-i$$

// Number of subarrays in which index i is present = $\frac{(i+1)(N-i)}{2}$



// Adding Contribution of Individual Elements

Pseudocode:

$S=0;$

$P=0; i < N; P+=S$

$\left| \begin{array}{l} \text{left} = (i+1), \text{right} = (N-i) \\ S \rightarrow S + ar[i] \times \overbrace{\text{left} \times \text{right}}^{\text{Number of subarrays}} \end{array} \right.$

returns

$T.C: O(N)$
 $S.C: O(1)$

Number of subarrays in which $ar[i]$ present

Doublé