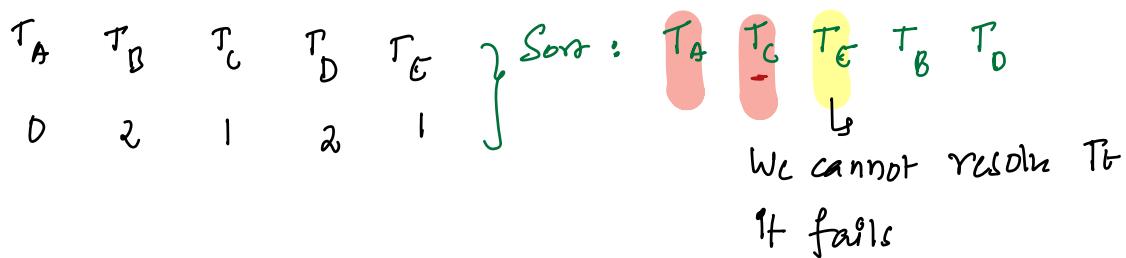
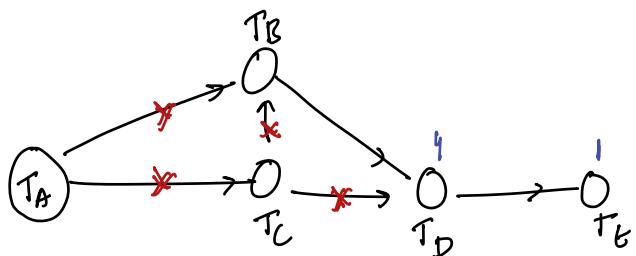


Today's Content:

- Topological Sort ↗ Ironic
- Dijkstra's Algo → { Brief Thursday + Code }

→ Sorting wrt wrk.



Topological Sort:

T_B depends on T_A

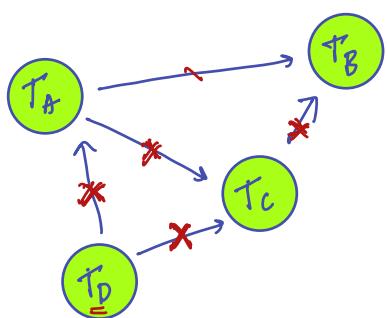
Recursion \rightarrow dp: // $T_A \rightarrow T_B$: first T_A should be done before T_B

a) That means dp depends on Recursion

b) Before doing dp we need to study Recursion

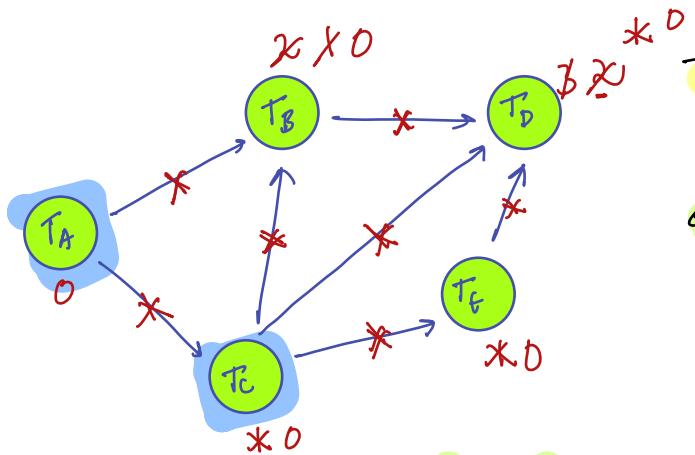
Note: If multiple orders are possible, pick lexicographically smaller

Ex1:



Task Order: $T_D \ T_A \ T_C \ T_B$

Ex2:



Task Order: $T_A \ T_C \ T_B \ T_E \ T_D$

Obs1: When there is no incoming edge pt's dependence are resolved

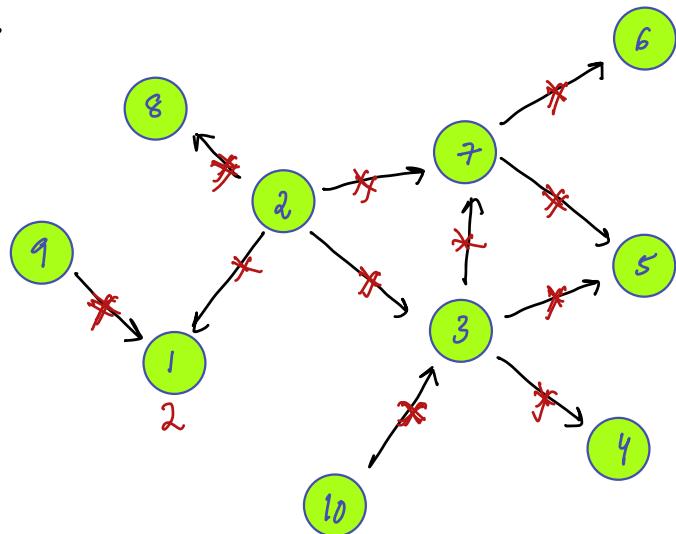
Count Incoming Edges:

T_A	T_B	T_C	T_D	T_E
0	0	2	3	0

ideas: sorting work

$T_A | T_C | T_B | T_E | T_D$

Ex:

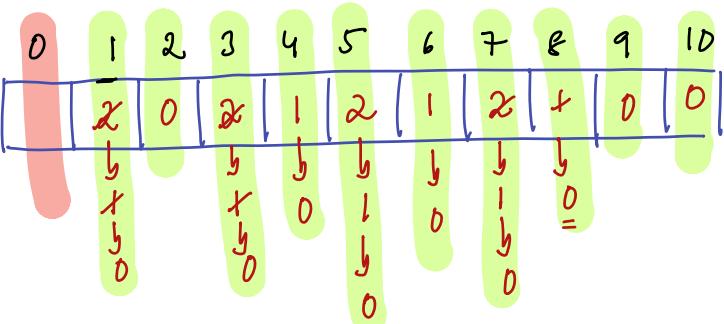


Adj:

0	
1	→ 2
2	→ 1 3 7 8
3	→ 4 5 7
4	→ 5
5	→ 6
6	→ 7
7	→ 5 6
8	→ 6
9	→ 1
10	→ 3

Count Incoming Edges

Pnt `indeg[11]` =



one of the

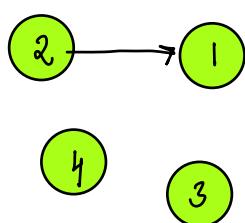
topological →

x | 9 | 10 | 8 | 1 | 7 | x | * | 6 | 5 | 4 | 3 | 2 |

order

↳ above order might not be lexicographically smallest

Ex:



lexicographically smallest order: 2 1 3 4
As per above approach:

x | 3 | 4 | 1 → order → 2 3 4 1

// Given graph as Input N & M Edges $\rightarrow T_C: O(N+E)$

list $g[N+1]$

int $\text{Indeg}[N+1]$

$i = 1; i \leq N; i++\{$
 // for edge will contains u, v . $u \rightarrow v$
 $\text{Indeg}[v]++;$
 $g[u].add(v);$
 $\}$

Queue \leftarrow $\text{Indeg} > 0;$

$i = 1; i \leq N; i++\{$ // i^{th} node is already resolved
 if ($\text{Indeg}[i] == 0\}$ { $q.push(i);$ }
 $\}$

while ($q.size() > 0\}$

int $u = q.front();$

$q.remove();$

$\text{print}(u);$ // indicating Task u is solved, clear its dependencies

$i = 0; i < g[u].size(); i++\{$
 int $v = g[u][i];$ // $u \rightarrow v$
 $\text{Indeg}[v]--;$
 if ($\text{Indeg}[v] == 0\}$ { $q.push(v);$ }
 $\}$

3

$SC: O(N+E)$

To store graph

+
 $\text{Indeg}[i]$

$Q[univ]$

// Lexicographically Smaller One :

→ out of all nodes which have in-degree we should always pick min (min heap) // we insert a node in minheap when it's indeg becomes = 0

→ instead of Queue use minheap

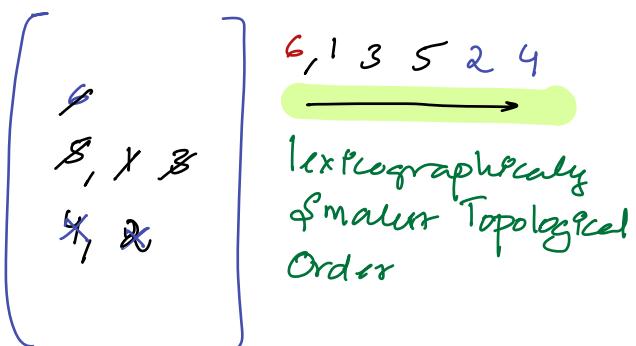
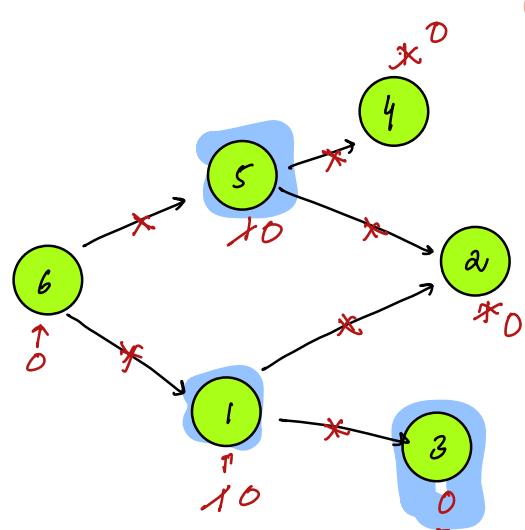
→ TC : () $\Rightarrow \{TC \rightarrow __\}$

min $\rightarrow O(1)$

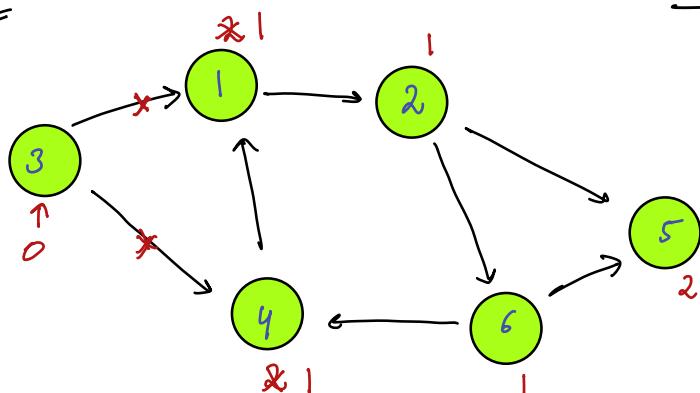
delete min $\rightarrow O(\log N)$

insert $\rightarrow O(\log N)$

MinHeap := Is Present in your langg class



Ex:



$\xrightarrow{\quad}$

\Rightarrow If there is a cycle in

your graph, we cannot

resolve all dependencies

we cannot find topological sort

' Given graph as Input N & M Edges $\text{TC} \rightarrow O(N+G)$

→ To check if Cycle in Directed graph $\text{SC} \rightarrow O(N+G)$

list $g[N+1]$

int $\text{Indeg}[N+1]$

```
i = 1; i <= M; i++ {  
    // for edge will contains u,v. u → v  
     $\text{Indeg}[v]++$   
    g[u].add(v)  
}
```

Queue $\text{Pnt} \rightarrow q;$

```
p = 1; p <= N; p++ {  
    // ith node is already resolved  
    if ( $\text{Indeg}[i] == 0$ ) {  
        q.push(i)  
    }  
}
```

int $c = 0$

while ($q.size() > 0$) {

int $u = q.front();$

$q.pop();$

$\text{print}(u)$ // indicating Task u is solved, clear its dependencies

$c++$

```
p = 0; p < g[u].size(); p++ {  
    int v = g[u][p]; // u → v
```

$\text{Indeg}[v]--;$

```
if ( $\text{Indeg}[v] == 0$ ) {  
    q.push(v)  
}
```

}

// cycle is formed

if ($c != N$) {
 return True
}

Indicate cycle

Unweighted graph:

$S \rightarrow D$: shortest path from $S \rightarrow D$: graph 1

Weighted graph

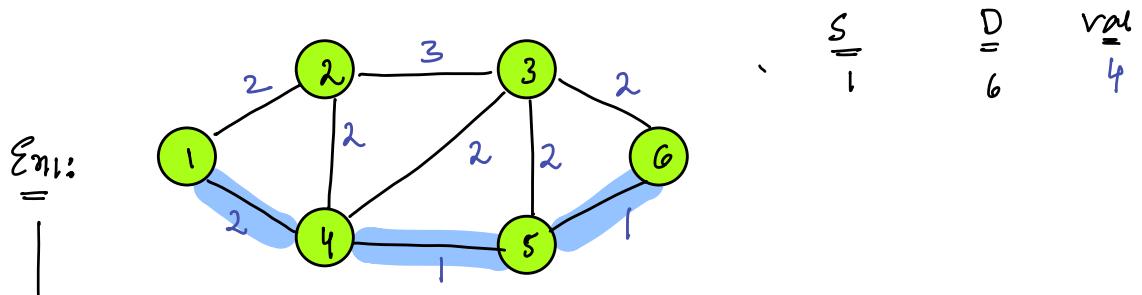
$S \rightarrow D$: shortest path from $S \rightarrow D$

Undirected graph : Cycle detection : graph 2

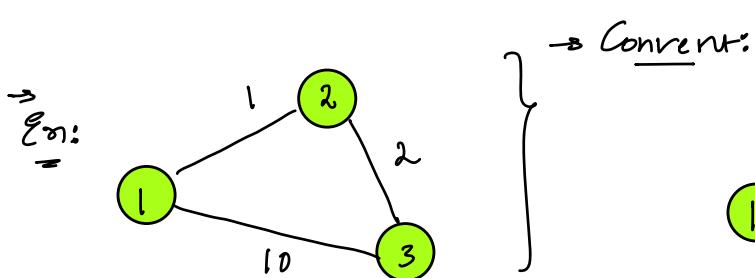
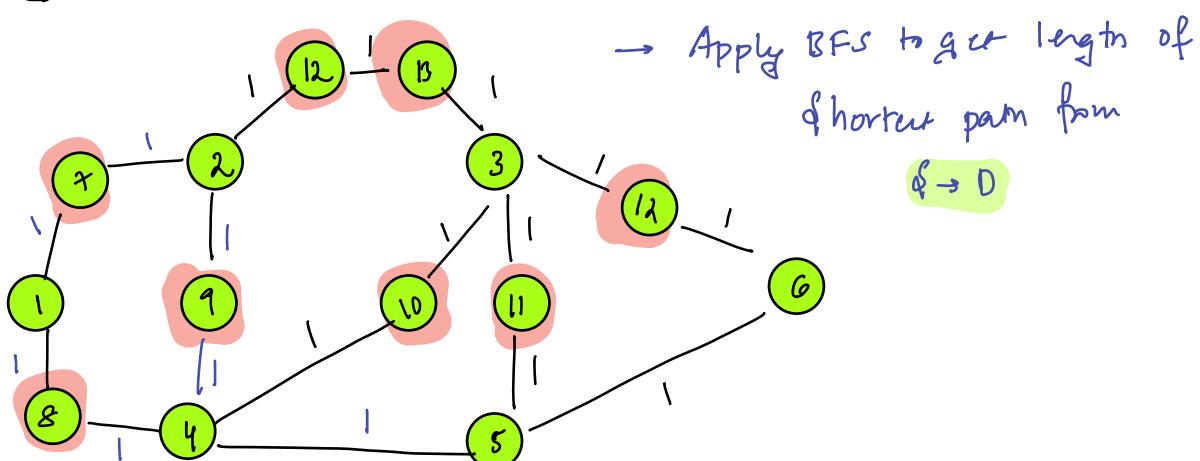
Directed graph : Cycle detection / Cycle detection can be done Using DFS

Break: 10:48 pm

Q8) Given a weighted graph with the weights find the min cost to reach from $S \rightarrow D$



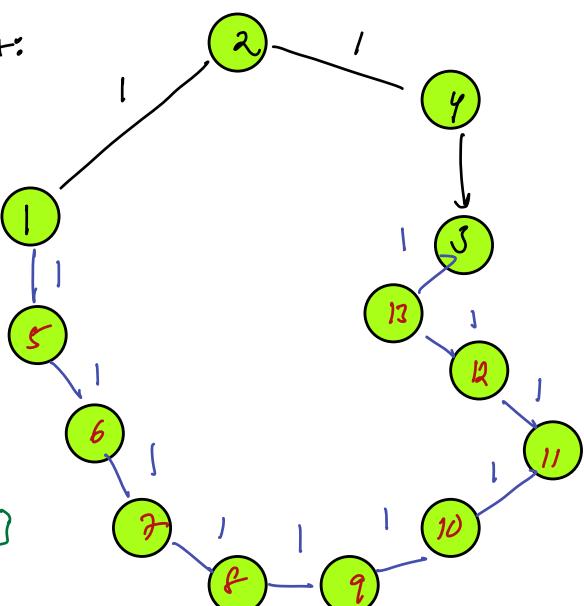
P_{des} : \rightarrow (All weights in graph we are making them $\rightarrow 1$)



Issue of space wasteage:

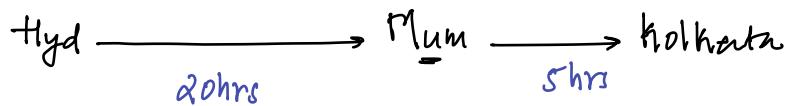
Extra Nodes: \rightarrow Huge Space wasteage

Total sum of weight - {No. of Edges}



→

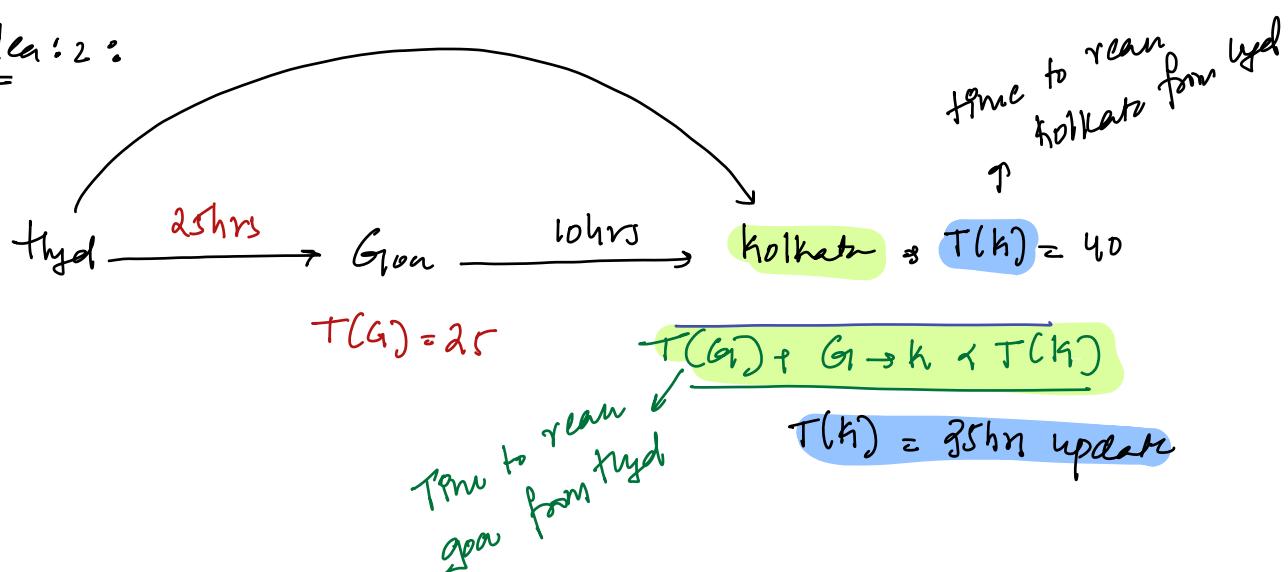
Basic Prerequisites:



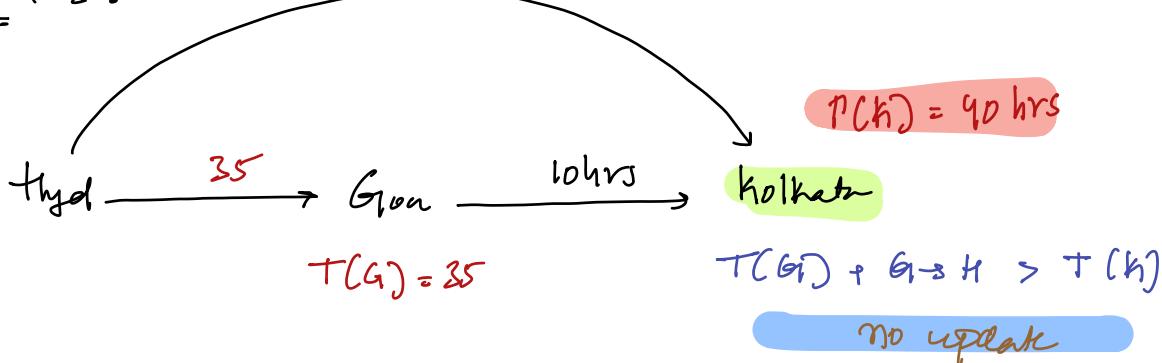
→ Time taken to reach from Hyd → Kolkatta via Mum = **25 hrs**

$$\rightarrow \text{Dist (Mum)} + (\text{Mum} \rightarrow \text{Kolkatta}) = 25$$

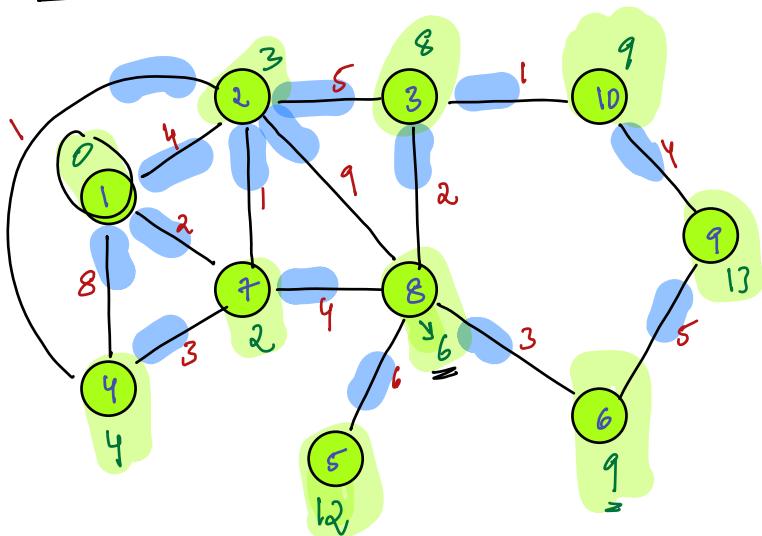
Pdea: 2 :



Pdea: 2 :



Idea: \rightarrow $Wt \rightarrow \{dest\}$

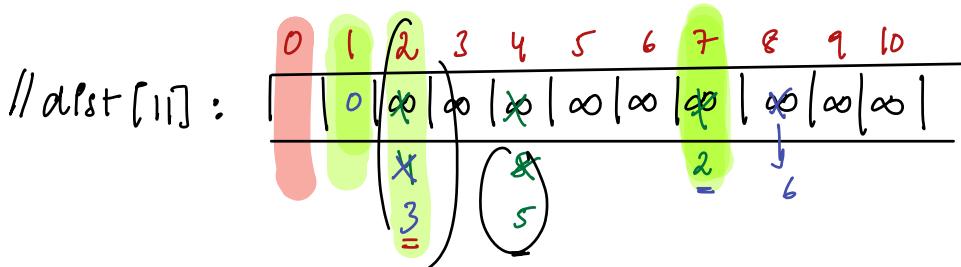


S: 1

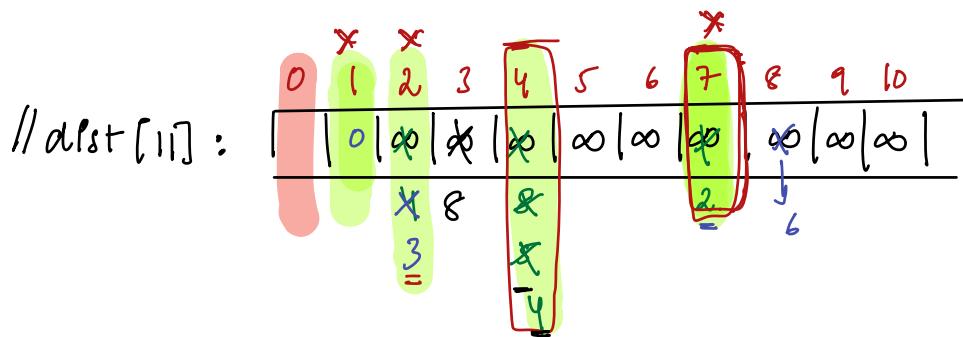
1	{2, 4}, {4, 8}, {7, 2}
2	{1, 4}, {3, 5}, {7, 1}, {8, 9}
3	{2, 5}, {8, 2}, {10, 1}
4	{1, 8}, {2, 1}, {7, 3}
5	{8, 6}
6	{8, 3}, {9, 5}
7	{1, 2}, {2, 1}, {4, 3}, {8, 4}
8	{2, 9}, {3, 2}, {5, 6}, {6, 8}, {7, 4}
9	{6, 5}, {10, 4}
10	{3, 1}, {9, 4}

// $dist[5] = dist$ to reach from $S \rightarrow 5$

// $dist[i] = dist$ to reach from $S \rightarrow i$



$$\textcircled{7} \rightarrow \begin{cases} 2 : dist[2] = 4, & \underbrace{dist[7] + 7 \rightarrow 2}_2 \xrightarrow{1} \\ 8 : dist[8] = \infty, & \underbrace{dist[7] + 7 \rightarrow 8}_2 \xrightarrow{4} \xrightarrow{6} \\ 4 : dist[4] = 8, & \underbrace{dist[7] + 7 \rightarrow 4}_2 \xrightarrow{3} \xrightarrow{5} \end{cases}$$

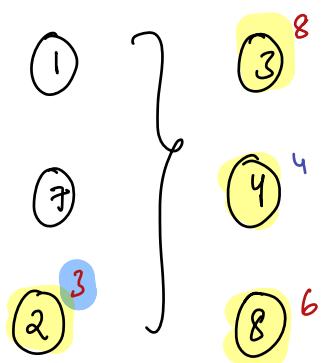
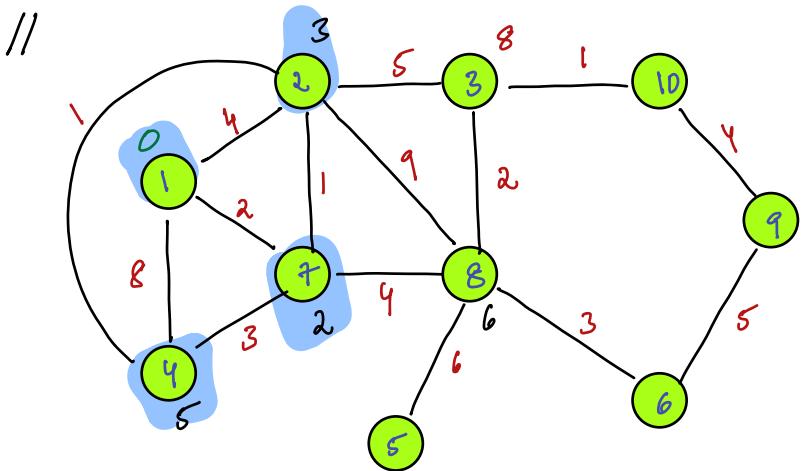


$$\textcircled{2} \rightarrow \begin{cases} 3 : \text{dist}[3] = \infty, \quad \text{dist}[2] + 2 \xrightarrow{?} 3 \Rightarrow \text{dist}[3] = 8 \\ 4 : \text{dist}[4] = 5, \quad \text{dist}[2] + 2 \xrightarrow{?} 4 \Rightarrow \text{dist}[4] = 4 \\ 7 : \text{dist}[7] = 2 \quad \text{dist}[2] + 2 \xrightarrow{?} 7 \quad \text{dist}[7] = 2 \\ 8 : \text{dist}[8] = 6 \quad \text{dist}[2] + 2 + 8 \xrightarrow{?} \quad \text{dist}[8] = 6 \end{cases}$$

⇒ Using the above from given source we can get shortest distance to all other node (edges)

→ At every step node with min dist
 → WPM that node we are updating all of its adjacent nodes

→ Thursday : {Brief + Code}



//