

Recall the challenge's RSA scheme:

Robin has 5 primes, p_1, p_2, \dots, p_5 , two semi-semi-primes $N_1 = p_1 p_2 p_3$ and $N_2 = p_3 p_4 p_5$, a message $m \in \mathbb{Z}_{N_1} \mathbb{Z}_{N_2}$ (in practice, this just means $m < N_1$ and $m < N_2$), a public exponent $e = 0\mathbf{x}10001$, and two ciphertexts $c_1 \equiv m^e \pmod{N_1}$ and $c_2 \equiv m^e \pmod{N_2}$.

Note that given only N_1, N_2 , we can efficiently compute $\gcd(N_1, N_2) = p_3$.

Robin can easily decrypt the ciphertexts:

By Fermat's little theorem, we can treat the exponents in \mathbb{Z}_{N_1} as if they're under mod $\phi(N_2) = (p_3 - 1) \cdot (p_4 - 1) \cdot (p_5 - 1)$.

(This is one of the corollaries under "Generalizations" in https://en.wikipedia.org/wiki/Fermat's_little_theorem).

Note that there is a natural injection $f : \mathbb{Z}_{N_2} \rightarrow \mathbb{Z}_{p_3}$, which is just taking mod p_3 of the elements of \mathbb{Z}_{N_2} .

But, if we look at the restriction $S = \{x \in \mathbb{Z}_{N_2} : x < p_3\} \subseteq \mathbb{Z}_{N_2}$, then $f : S \rightarrow \mathbb{Z}_{p_3}$ is now a natural bijection.

(What we mean by "natural" above is just that the underlying elements from \mathbb{N} don't change under f .)

In particular, if $m \in S$ (i.e, if m is *small enough*), then we would find $f(m) = m \in \mathbb{Z}_{p_3}$.

This means we might be able to use a different calculation to find m :

$$\begin{aligned} m &\equiv c^{(e^{-1} \pmod{\phi(N_2)})} \pmod{N_2} \\ f(m) &\equiv f\left(c^{(e^{-1} \pmod{\phi(N_2)})}\right) \pmod{p_3} \\ f(m) &\equiv f(c)^{(e^{-1} \pmod{\phi(N_2)} \pmod{\phi(p_3)})} \pmod{p_3} \quad (\text{because FLT must be true of } f(c) \text{ under mod } p_3) \\ f(m) &\equiv f(c)^{(e^{-1} \pmod{\phi(p_3)})} \pmod{p_3} \quad (\text{because } p_3 \text{ divides } N_2) \\ m &\equiv f(c)^{(e^{-1} \pmod{p_3-1})} \pmod{p_3} \end{aligned}$$

See `solve.py` for what this calculation looks like in python.