C.

1. Vypočítejte integrál $\displaystyle\int_{\frac{\pi}{2}}^{\frac{\pi}{3}} \frac{dx}{\sin^5 x}$ pomocí substituce $\cos x = t$

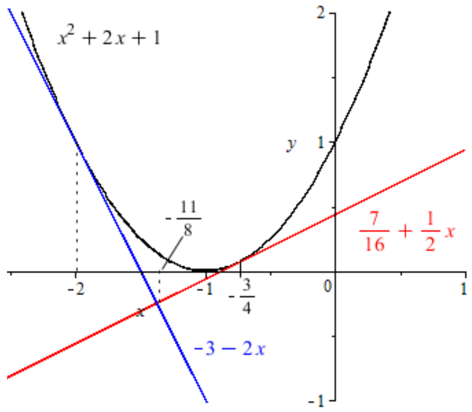
$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{dx}{\sin^5 x} = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{dx}{\sin^6 x}\cdot \sin x\,dx = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{1}{\left(1-\cos^2 x\right)^3}\cdot \sin x\,dx = \left|\begin{array}{cc} \cos x = t & x=\frac{\pi}{3} \Rightarrow t = \frac{1}{2} \\ -\sin x dx = dt & x = \frac{\pi}{2} \Rightarrow t = 0 \end{array}\right| =$$

$$= -\int_{\frac{1}{2}}^{0} \frac{1}{(1+t)^3(1-t)^3}\,dt = \frac{1}{16}\int_{0}^{\frac{1}{2}}\left(\frac{3}{t+1}+\frac{3}{(t+1)^2}+\frac{2}{(t+1)^3}-\frac{3}{t-1}+\frac{3}{(t-1)^2}-\frac{2}{(t-1)^3}\right)dt =$$

$$= \frac{1}{16}\left[3\ln|t+1| - \frac{3}{t+1} - \frac{1}{(t+1)^2} - 3\ln|t-1| - \frac{3}{t-1} + \frac{1}{(t-1)^2}\right]_0^{\frac{1}{2}} =$$

$$= \frac{1}{16}\left(3\ln\frac{3}{2} - 2 - \frac{4}{9} - 3\ln\frac{1}{2} + 6 + 4 - (-3-1+3+1)\right) = \frac{1}{16}\left(3\ln 3 + \frac{68}{9}\right) = \frac{3}{16}\ln 3 + \frac{17}{36}$$

2. Najděte rovnice dvou tečen k parabole $y = x^2 + 2x + 1$; jedna je rovnoběžná s přímkou $x - 2y + 4 = 0$, druhá je na ni kolmá. Vypočítejte obsah části roviny omezené danou parabolou a nalezenými tečnami.
(Pro numerické výpočty po dosazení mezí můžete použít kalkulačku.)



$x - 2y + 4 = 0 \quad \Leftrightarrow \quad y = \frac{1}{2}x + 2$ - hledáme tečny k parabole se směrnicemi $k_1 = \frac{1}{2}, k_2 = -2$.

$y' = 2x + 2 \qquad 2x + 2 = \frac{1}{2} \Leftrightarrow x = -\frac{3}{4},\ f\left(-\frac{3}{4}\right) = \frac{1}{16}, \quad T_1 = \left[-\frac{3}{4}, \frac{1}{16}\right]$

$\qquad\qquad\quad 2x + 2 = -2 \Leftrightarrow x = -2,\ f(-2) = 1, \qquad T_2 = [-2, 1]$

$t_1: \quad y = \frac{1}{16} + \frac{1}{2}\left(x + \frac{3}{4}\right) = \frac{1}{2}x + \frac{7}{16},$

$t_2: \quad y = 1 - 2(x+2) = -2x - 3$

průsečík tečen: $\frac{1}{2}x + \frac{7}{16} = -2x - 3 \Rightarrow x = -\frac{11}{8}$.

$$S = \int_{-2}^{-\frac{3}{4}} \left(x^2 + 2x + 1\right)dx - \int_{-2}^{-\frac{11}{8}} (-3-2x)\,dx - \int_{-\frac{11}{8}}^{-\frac{3}{4}} \left(\frac{7}{16} + \frac{1}{2}x\right)dx = \left[\frac{1}{3}x^3 + x^2 + x\right]_{-2}^{-\frac{3}{4}} + \left[3x + x^2\right]_{-2}^{-\frac{11}{8}} - \left[\frac{7}{16}x + \frac{1}{4}x^2\right]_{-\frac{11}{8}}^{-\frac{3}{4}} =$$

$$= \qquad = \cdots = \frac{125}{768} \doteq 0{,}16275$$