2-3-24 Ashley Baker

Pt 1

Hands-On

Function $X = f(n)$	Cost	Time
$X = 1$	C_1	X
for $i = 1:n$	C_2	$\sum_{i=1}^{n} \sum_{i=1}^{n} 1$
for $i = 1:n$	$\vdots$	
$X = X + 1$	C_3	n

1. Total Cost $= C_1 + C_2$

$$T(n) = \sum_{i=1}^{n} \sum_{i=1}^{n} 1 + n = \sum_{i=1}^{n} n + n = \frac{n(n+1)}{2} + n = \frac{n^2+n}{2} + n$$

$$T(n) = \frac{1}{2}n^2 + \frac{3}{2}n$$

2. Plot added (Function_X_Original_Graph.png)
Data points file also added. Curve $= \frac{1}{2}n^2 + \frac{3}{2}n$

3. Find $C_1 g(n) \le f(n) \le C_2 g(n)$

$$C_1(n^2) \le \frac{1}{2}n^2 + \frac{3}{2}n \le C_2(n^2)$$

Smallest that $f(n)$ can get is ↓

$$\cancel{\frac{1}{2}n^2 / n^2} + \cancel{\frac{3}{2}n / n^2} = \frac{1}{2} + \frac{3}{2n}$$

$$\lim_{n \to \infty} \frac{1}{2} + \frac{3}{2n} = \frac{1}{2} \qquad C_1 = 1/2$$

largest that $f(n)$ can get once divided by n^2 ↓

$$\lim_{n \to 1} \frac{1}{2} + \frac{3}{2n} = \frac{1}{2} + \frac{3}{2} = 2 \qquad C_2 = 2$$

$$\frac{1}{2}n^2 \le \frac{1}{2}n^2 + \frac{3}{2}n \le 2n^2$$

PT2 2-3-24

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Hands-On

$\frac{1}{2}n^2 \leq \frac{1}{2}n^2 + \frac{3}{2}n \leq 2n^2 \quad g(n) = n^2 \quad f(n) = \frac{1}{2}n^2 + \frac{3}{2}n$

$\lim_{n \to \infty} \frac{f(n)}{g(n)} = \frac{\frac{1}{2}n^2 + \frac{3}{2}n}{n^2} = \frac{1}{2} + \frac{\frac{3\cancel{n}}{2}}{n^{\cancel{2}}} = \frac{1}{2} + \frac{3}{2n}$

$\lim_{n \to \infty} \frac{1}{2} + \frac{3}{2n} = \frac{1}{2}$ Constant

f(n) has same order of growth as g(n)

Big-O:

$O(g(n)) = \{ f(n)$ such that there exists positive constants C and n_0 such that $0 \leq f(n) \leq Cg(n)$ for all $n \geq n_0 \}$

$0 \leq \underbrace{\frac{1}{2}n^2 + \frac{3}{2}n}_{} \leq Cn^2$

↳ will never be less than 0 as long as $n > 0$ $\quad n_0 = 0.1$

From earlier: $C_2 = 2$, then we will say $C = 2$

$C = 2$, $n_0 = 0.1$

$O(n^2)$

Big Omega: $\Omega(g(n)) = \{ f(n)$ such that there exists positive constants C and n_0 such that $0 \leq Cg(n) \leq f(n)$ for all $n \geq n_0$

$g(n) = n^2$

$C\cancel{n^2} \leq \frac{1}{2}\cancel{n^2} + \frac{3}{2}\cancel{n} \cdot \frac{1}{n}$

$C \leq \frac{1}{2} + \frac{3}{2n} \qquad \lim_{n \to \infty} \frac{1}{2} + \frac{3}{2n} = \frac{1}{2} \qquad C = 1/2 \quad n_0 = 0.1$

$\Omega(n^2)$

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Pt 3

Hands-On

Big-Theta: $\Theta(n^2)$ $\frac{1}{2}n^2 \leq \frac{1}{2}n^2 + \frac{3}{2}n \leq 2n^2$

Lower Bounds: $\frac{1}{2}n^2$

Upper Bounds: $2n^2$

4 Plot included: Plot_110.png

$n_0 = 1$ Chosen because that is the last time that $f(n) > C_2 g(n)$ [the upper bounds]

5 No, it just adds n

6 It will increase the results.

7 Merge Sort added + tested (recursive)

[5, 2, 4, 7, 1, 3, 2, 6]

[5] [2] [4] [7] [1] [3] [2] [6]

[2, 5] [4, 7] [1, 3] [2, 6]

[2, 4, 5, 7] [1, 2, 3, 6]

[1, 2, 2, 3, 4, 5, 6, 7]