

Hands-On 4

0) 1) fibonacci.py in link folder

2) Each fib(n) call results in 2 additional calls until n is = 1, 0. This indicates an exponential

$$T(n)^{\text{growth}} = T(n-1) + T(n-2) + O(1)$$

Base: $T(1) = T(0) + 1$

$$T(0) = T(1) = 0$$

$$T(2) = T(1) + T(0) + 1 \\ = 1 + 0 + 1 = 2$$

Assuming $T(n-1) \approx T(n-2)$

$$T(n) = 2T(n-1) + 1 \quad T(n-1) = 2T(n-2) + 1$$

$$= 2(2T(n-2) + 1) + 1$$

$$= 4T(n-2) + 3 \quad T(n-2) = 2T(n-3) + 1$$

$$= 4(2T(n-3) + 1) + 3$$

$$= 8T(n-3) + 7 \quad T(n-3) = 2T(n-4) + 1$$

$$= 8(2T(n-4) + 1) + 7$$

$$\rightarrow = 2(2(2(2T(n-4) + 1) + 1) + 1) + 1$$

$$= 16T(n-4) + 15$$

Pattern for k substitutions

$$2^k T(n-k) + (2^k - 1)$$

K builds until $k = n$

$$T(n) = 2^n T(0) + (2^n - 1) = 2^n + 2^n - 1 = O(2^n)$$

Exponential

3) Improvements:

- Store numbers to avoid re-computation

- Dynamic Programming is more efficient way

Backwards
Substitution