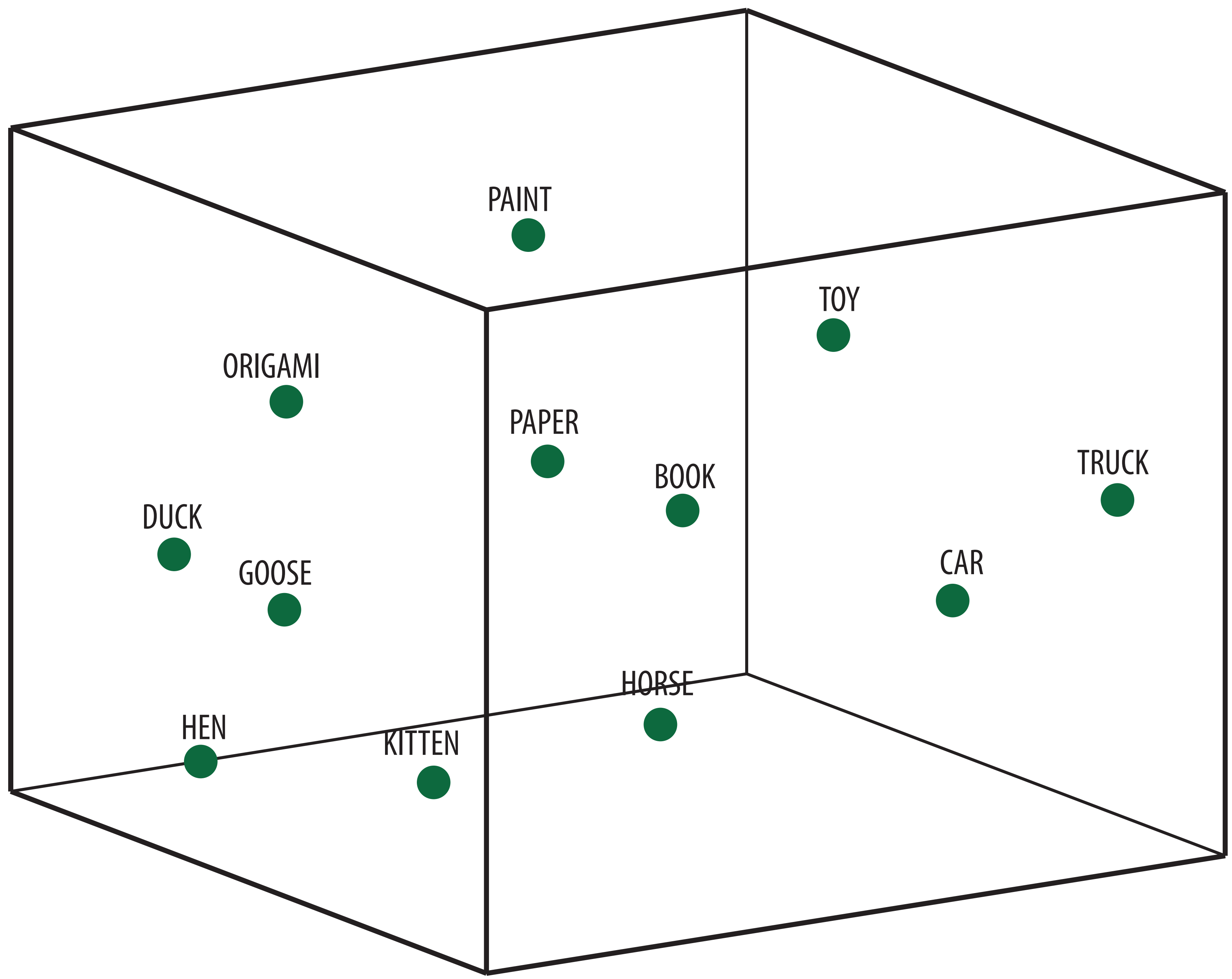
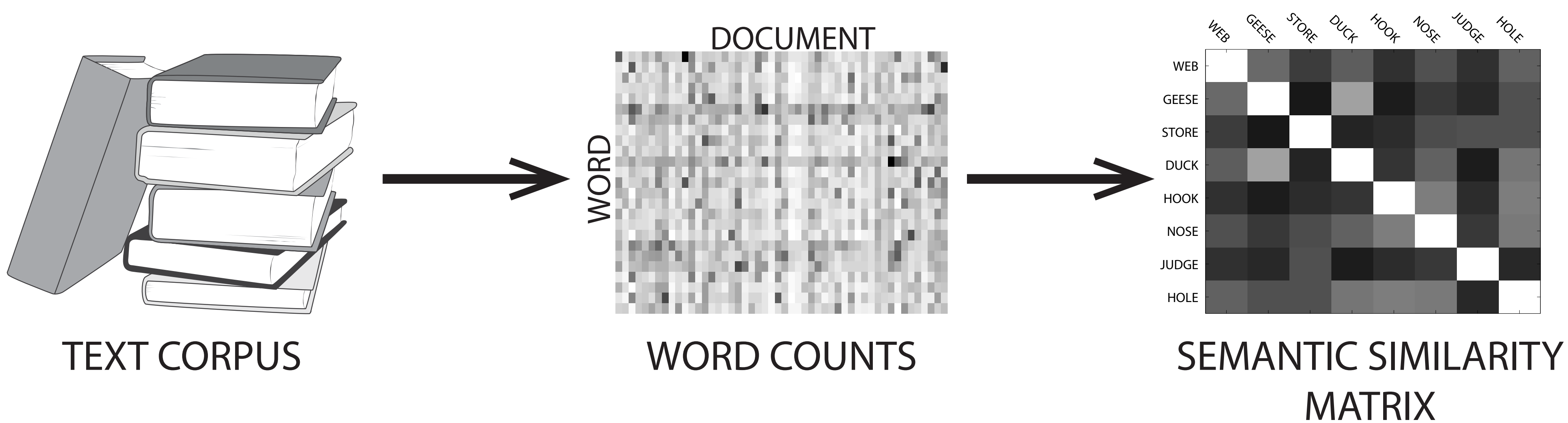
# Recap

- Attribute theory
- Generalized Euclidean distance
- Distance-based similarity
- Cosine-based similarity
- Visualizing and reasoning about high-dimensional spaces

# Semantic spaces

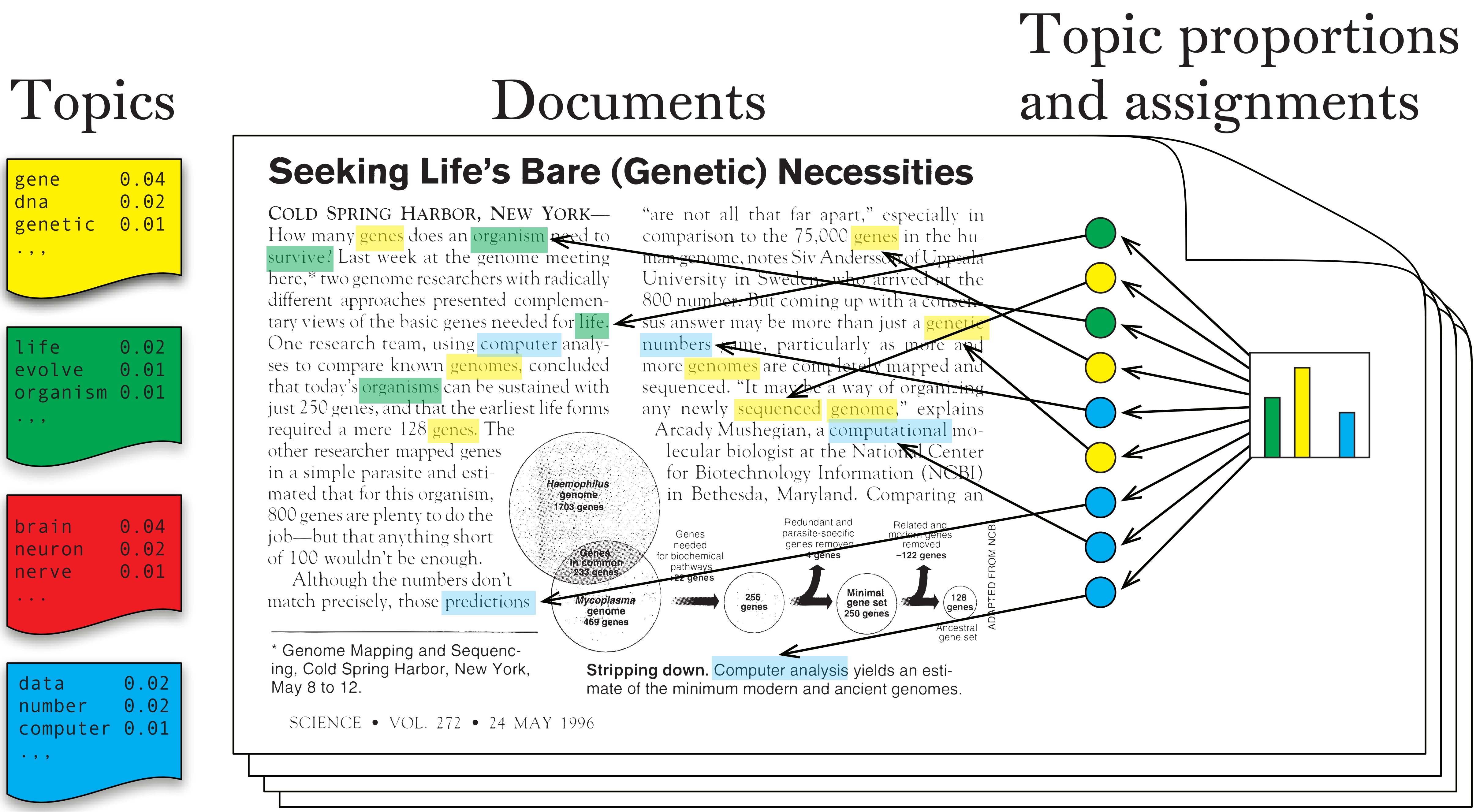


# Word spaces




Manning et al. (2012)

# Topic spaces




Blei et al. (2003)

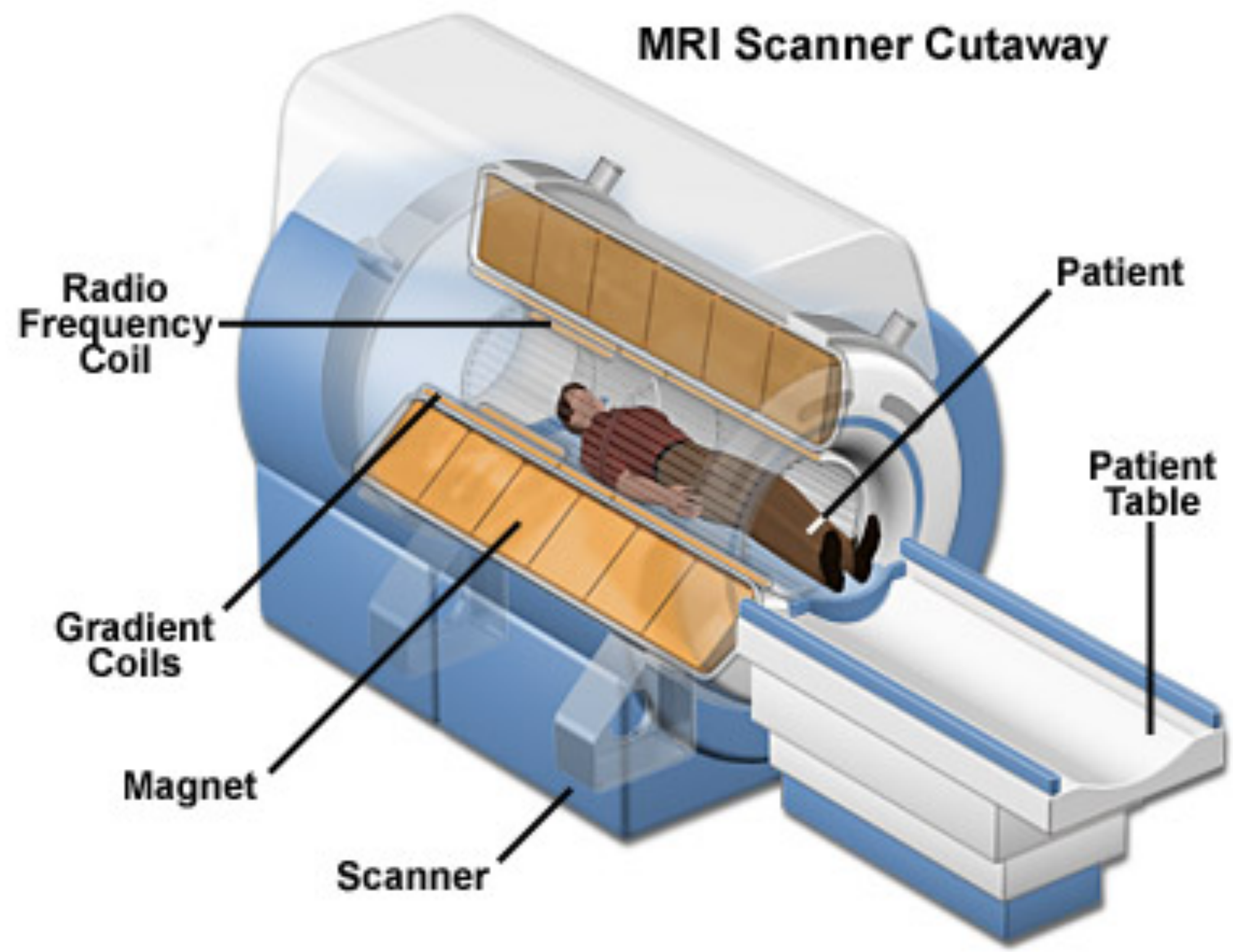
# Other language models

- Psychological experiments; e.g. word association spaces (WAS; Steyvers et al., 2005)
- Detailed hand-labeled models; e.g. Princeton WordNet (Miller, 1998)
- Deep learning models

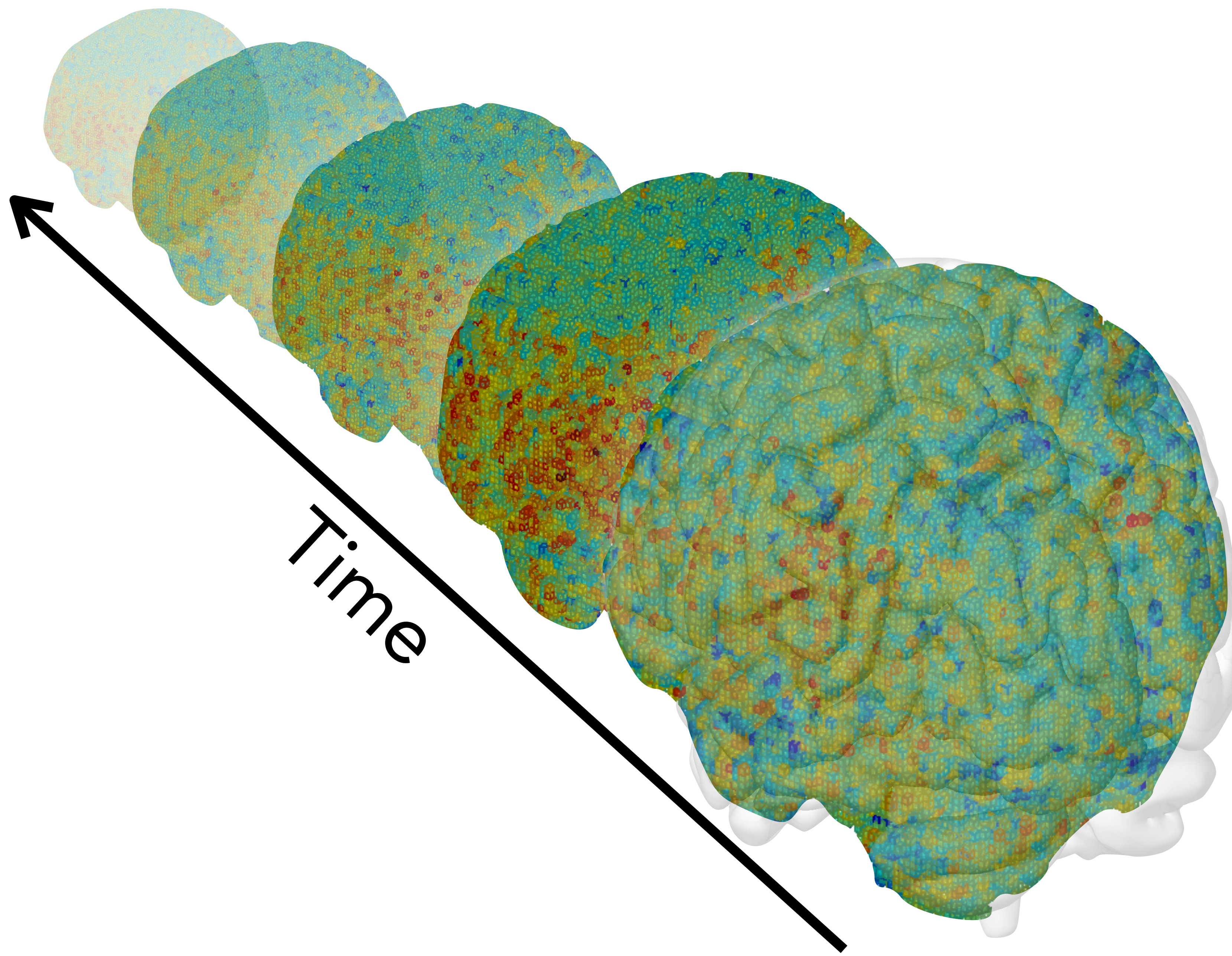
# Brain spaces

- Record from people's brains during an experiment
- Summarize their brain patterns (feature vectors!)
- Idea: if our thoughts come from our brains, then similarities in brain patterns should reflect similarities in thoughts

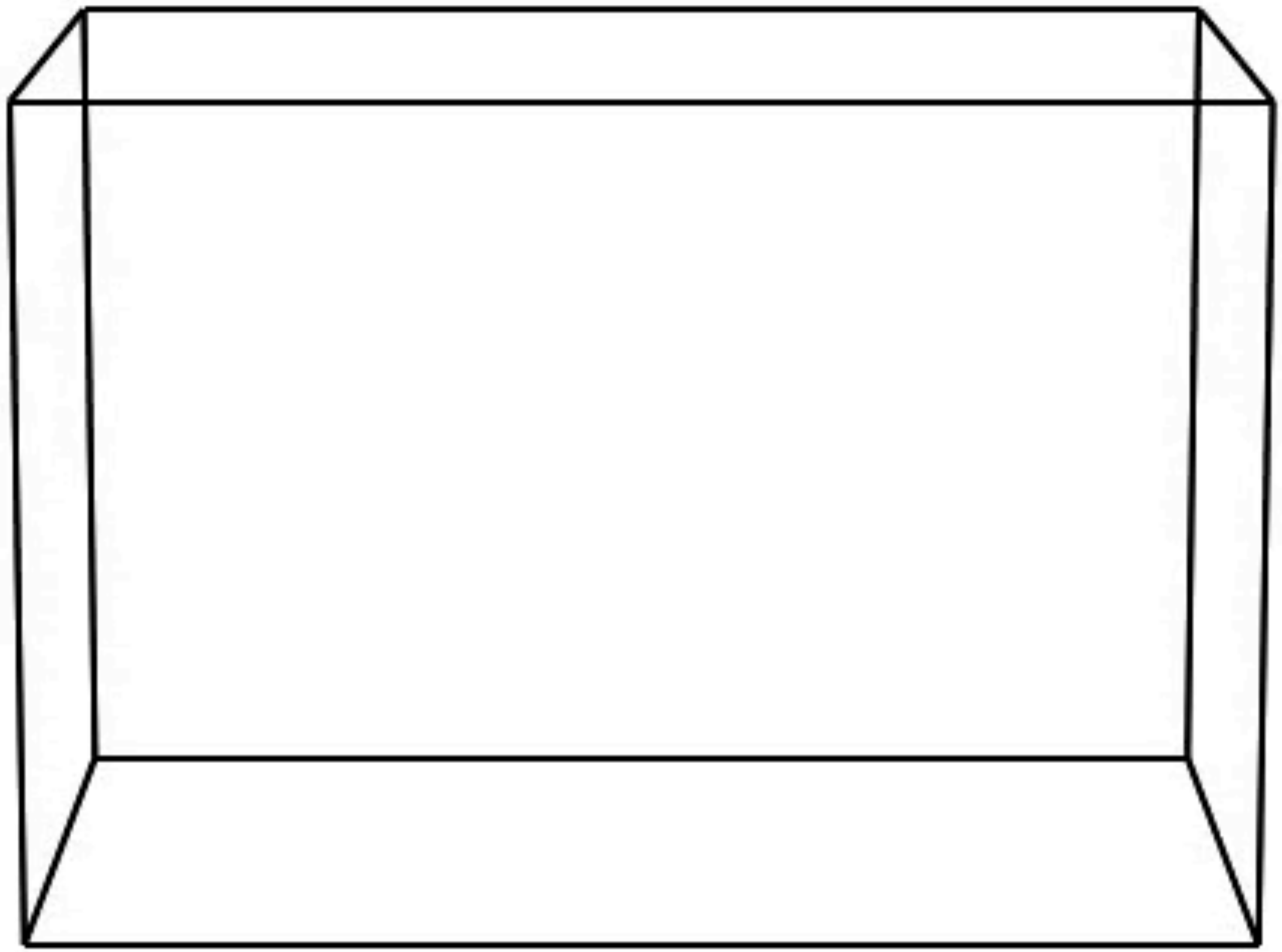
# functional Magnetic Resonance Imaging (fMRI)



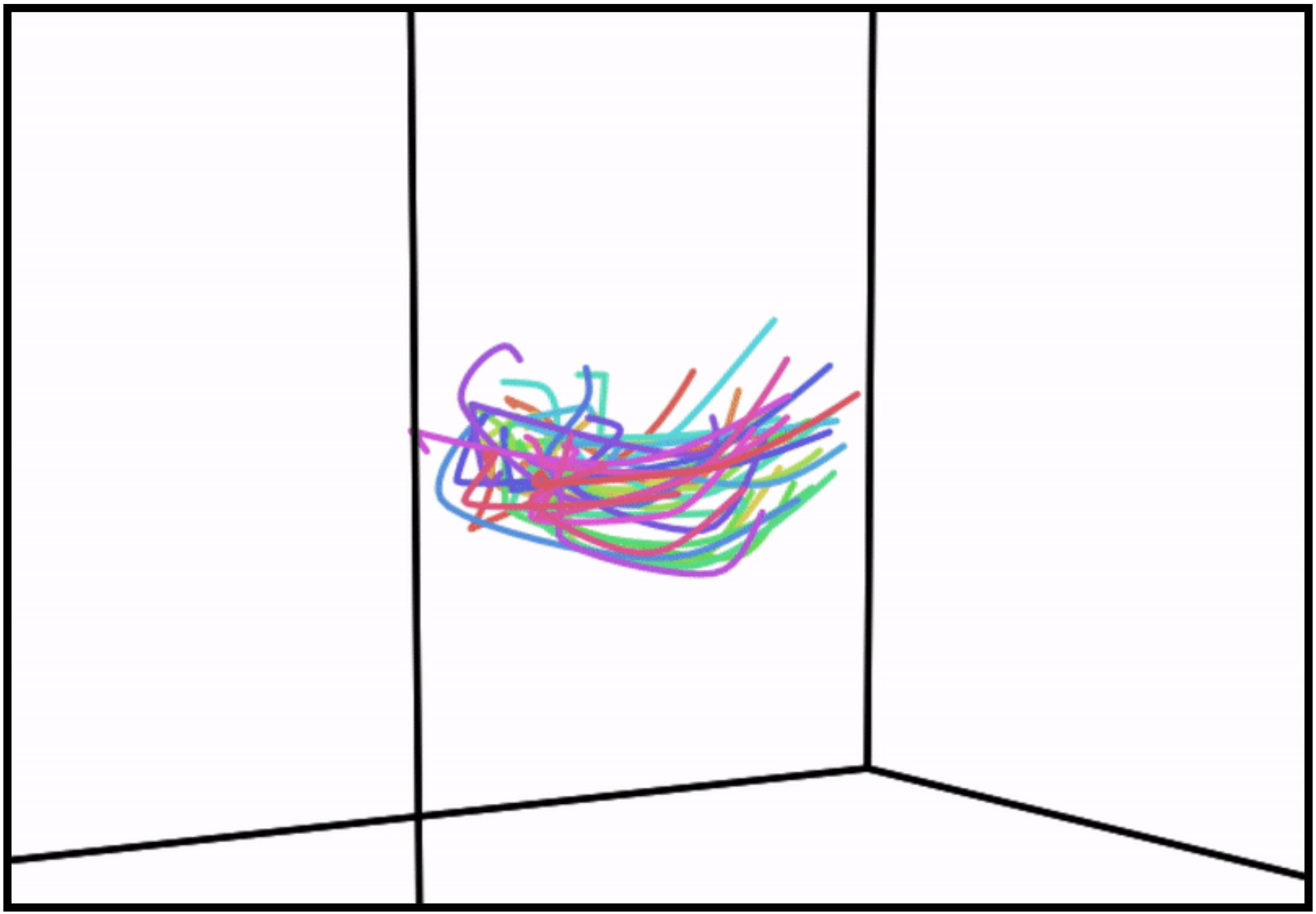
# functional Magnetic Resonance Imaging (fMRI)



# People watching Indiana Jones



# People listening to a story



# Other types of spaces

- Colors and images (pixel RGB values)
- Physical measurements (weather, geometry, velocity, voltage, etc.)
- Qualitative or quantitative properties of anything…

# Laying the groundwork…

- How can we (formally) represent complex thoughts?

- Extensions of strength theory

# What was “wrong” with strength theory?

- Similarity– no notion of which items are similar or how similarity affects memory
- Context– no notion of context or time
- Where does the idea of “strength” even come from?

# The multiple trace hypothesis

$$M = \begin{pmatrix} m_1(1) & m_2(1) & m_3(1) \\ m_1(2) & m_2(2) & m_3(2) \\ m_1(3) & m_2(3) & m_3(3) \\ \vdots & \vdots & \vdots \\ m_1(N) & m_2(N) & m_3(N) \end{pmatrix} = \begin{pmatrix} \mathbf{m}_1 & \mathbf{m}_2 & \mathbf{m}_3 \end{pmatrix}$$

# The multiple trace hypothesis

- Studied items get converted into vectors and stored in memory
- Test items get converted into vectors
- We can compare the test item's vector to each of the studied item's vectors
    - Remember from Chapter 2: serial search, parallel search
- Can people actually do this?

# Experiment: study 2500 items

- “Visual long-term memory has a massive storage capacity for object details” – Brady et al. (2008)
- Examine **capacity** and **fidelity** of visual long-term memory
- Participants study a loooong list of items, looking for repeats (of any previously studied item)
- Then they get a series of recognition tests

- This lasts for 5 grueling hours…
- Then participants take a 10 minute break…
- And then it's time for their memory test!


25
20
15
10
5


# People can do this well!

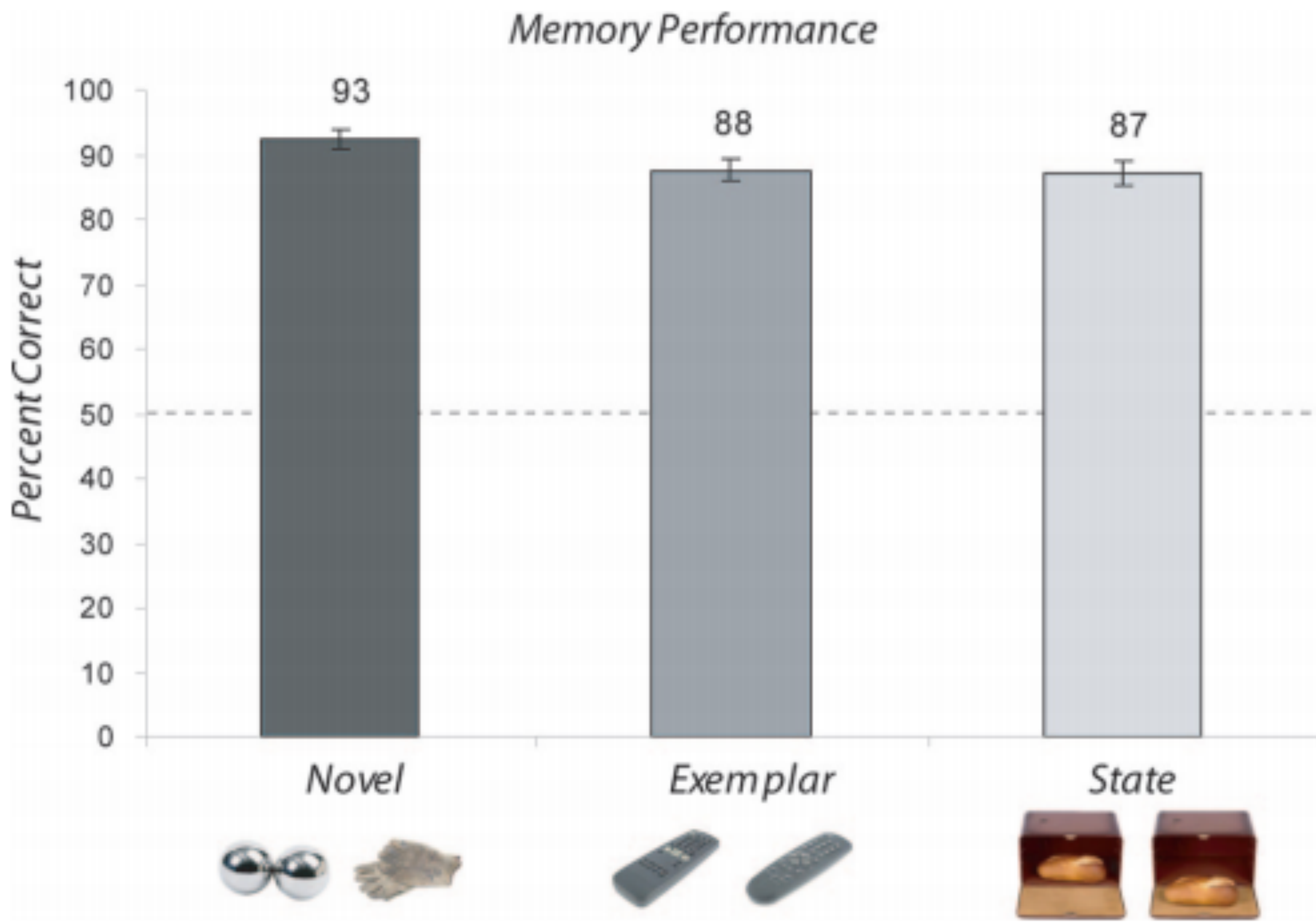


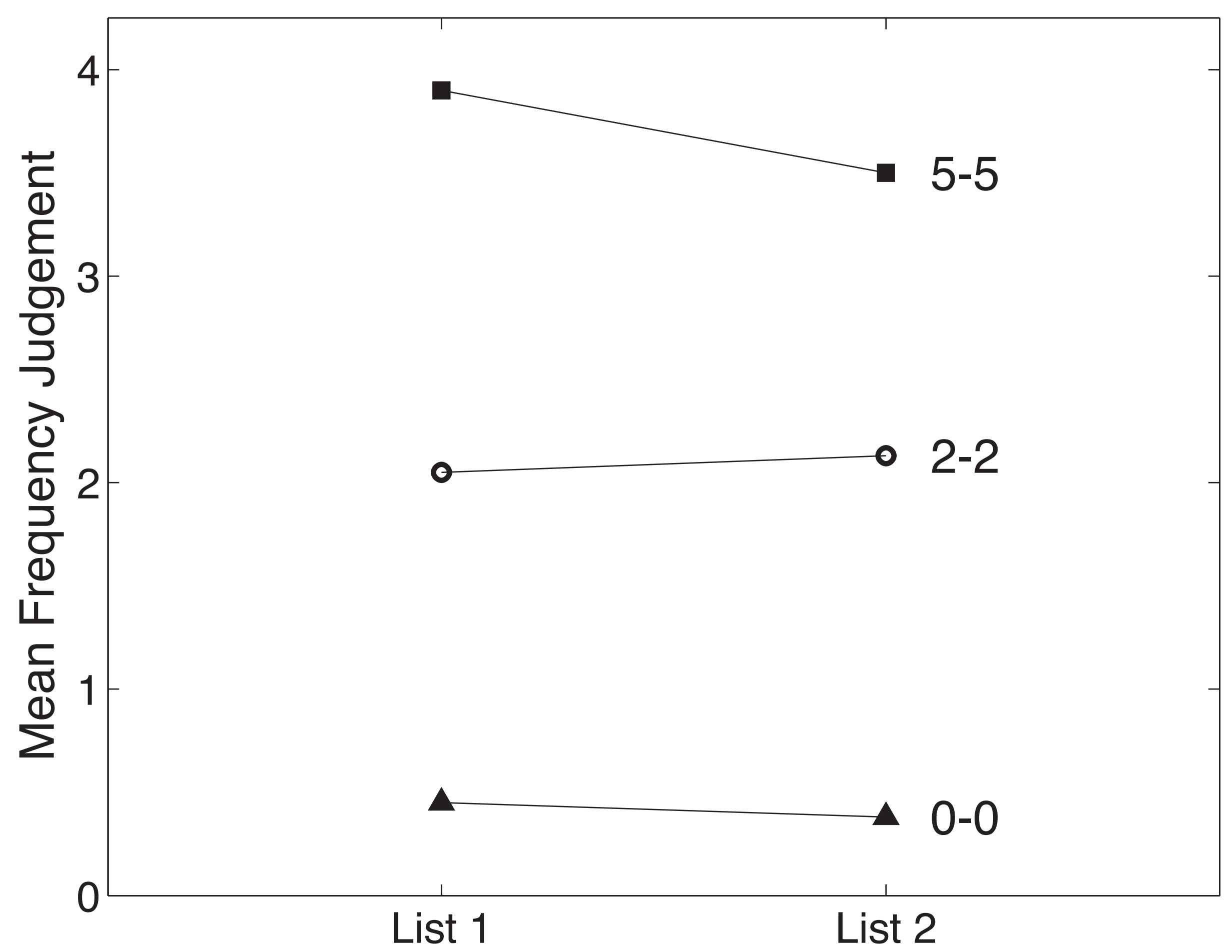

**Fig. 2.** Memory performance for each of the three test conditions (novel, exemplar, and state) is shown above. Error bars represent SEM. The dashed line indicates chance performance.

Brady et al., 2008

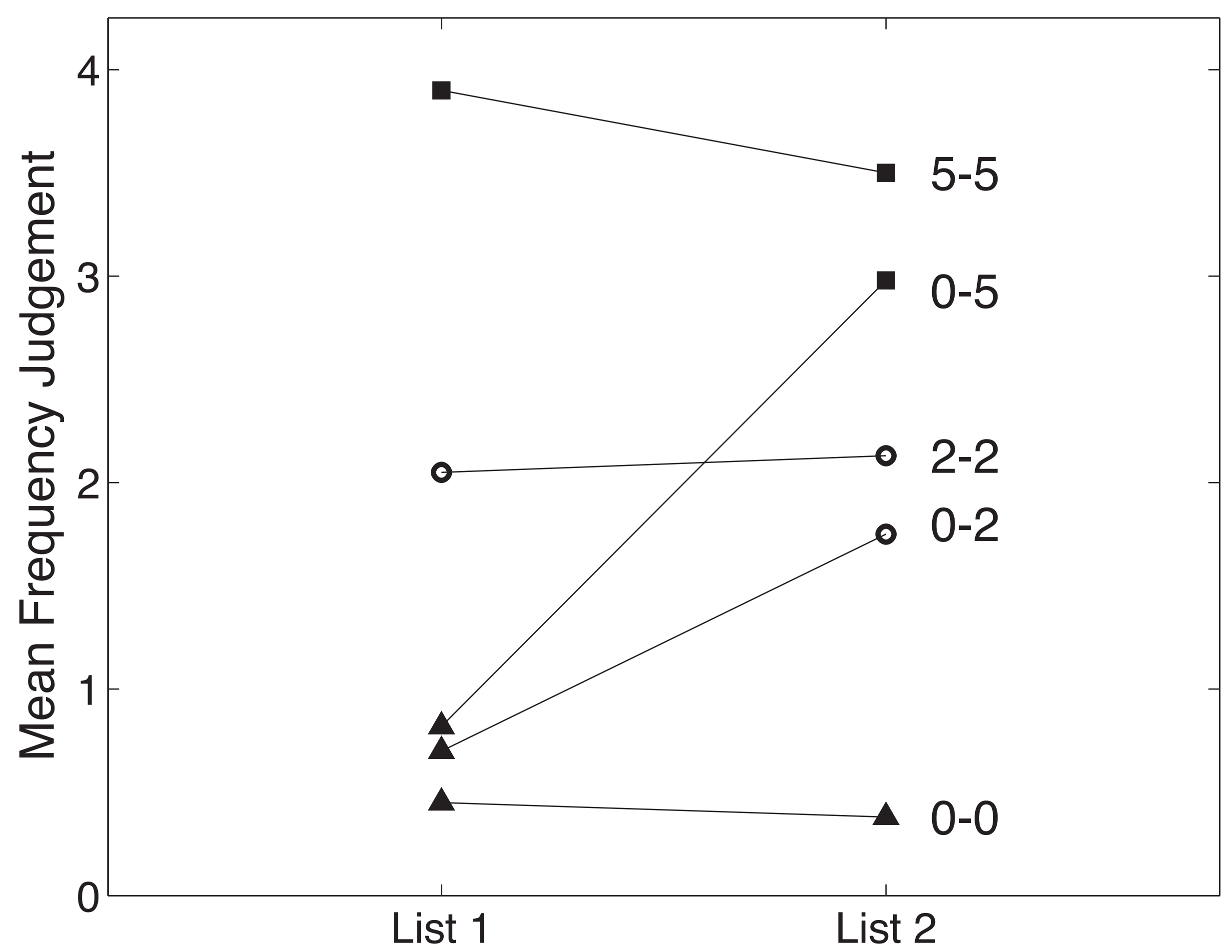
# Judgment of Frequency (JOF)

- Huntsman and Block (1971)
- Two lists separated by a 5 minute break
- Each list has 104 words
  - Half are presented twice; half are presented 5 times (spaced repetitions)
- Unexpected JOF test: rate how many times you saw each item (on each list)

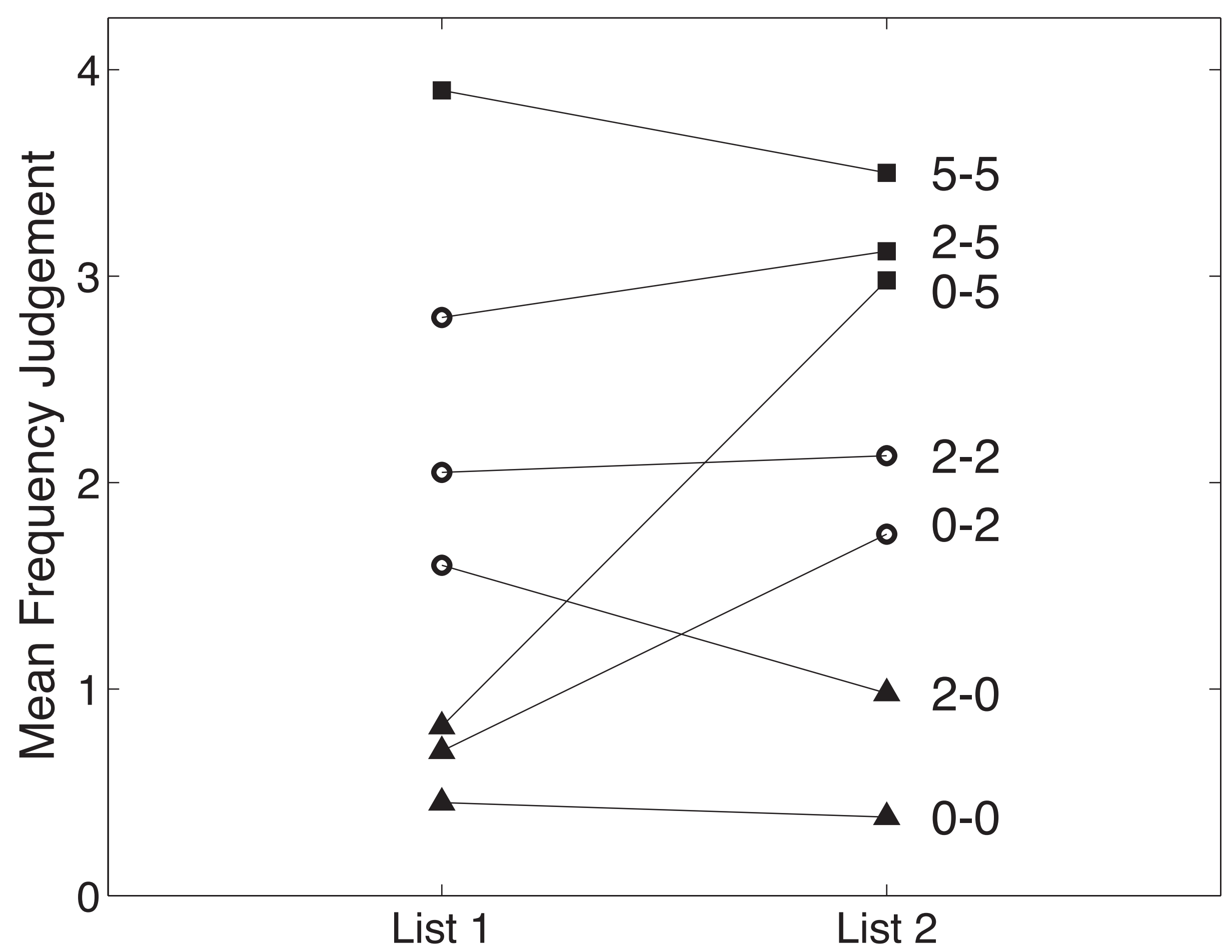
# Judgment of Frequency (JOF)



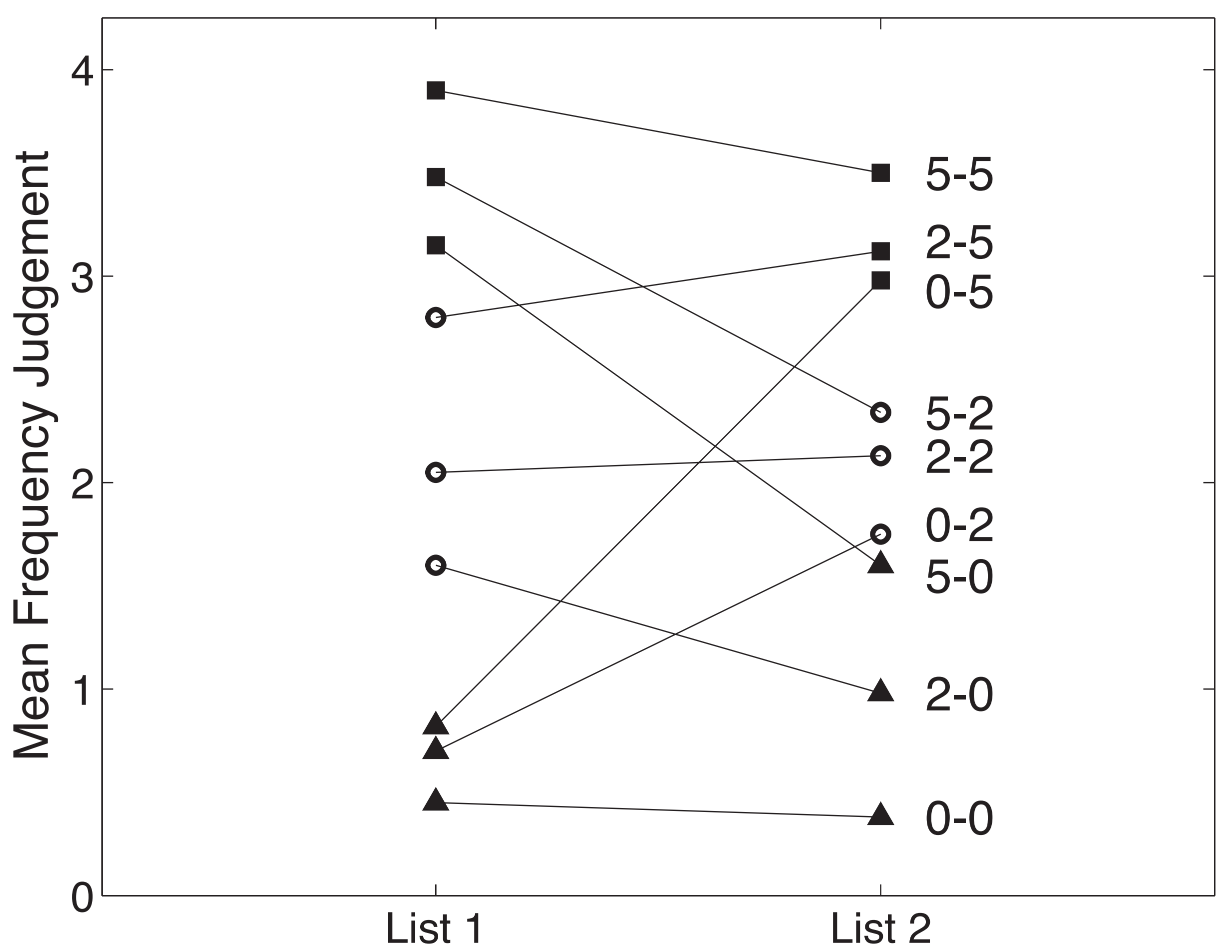
# Judgment of Frequency (JOF)



# Judgment of Frequency (JOF)



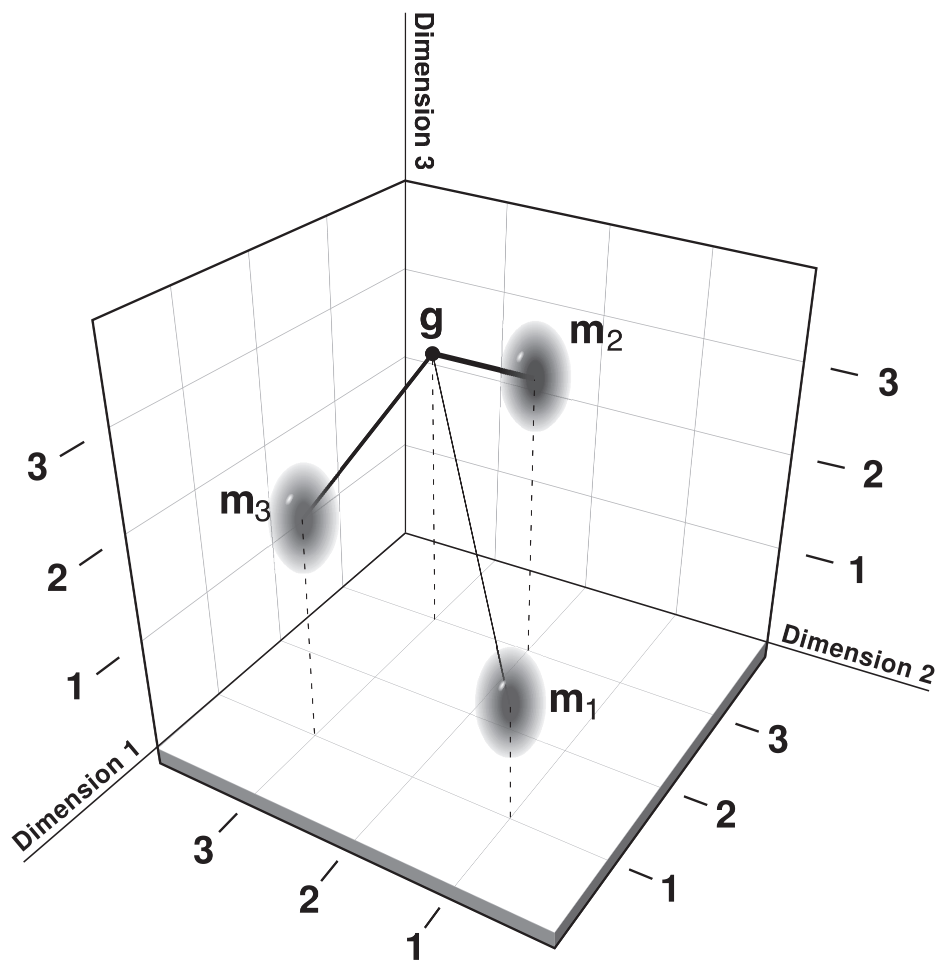
# Judgment of Frequency (JOF)



# Judgment of Frequency (JOF)

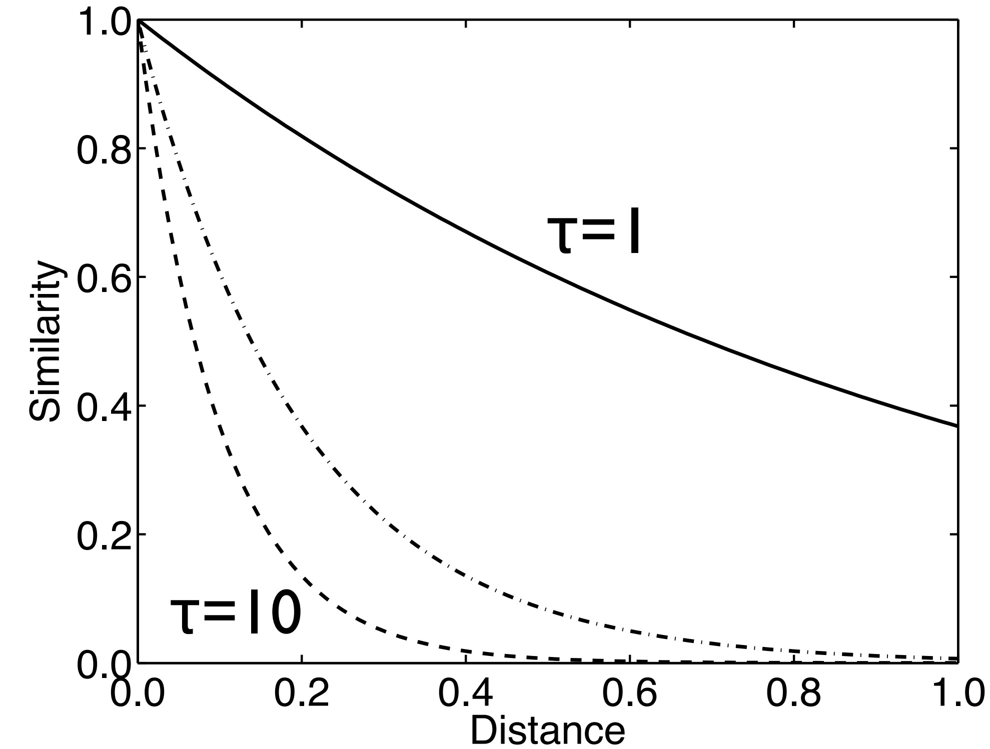
- People store multiple "copies" of each item (one per presentation?)
- Some attributes are related to the *context* (list?) in which the item was encoded
- We can gain access to this information

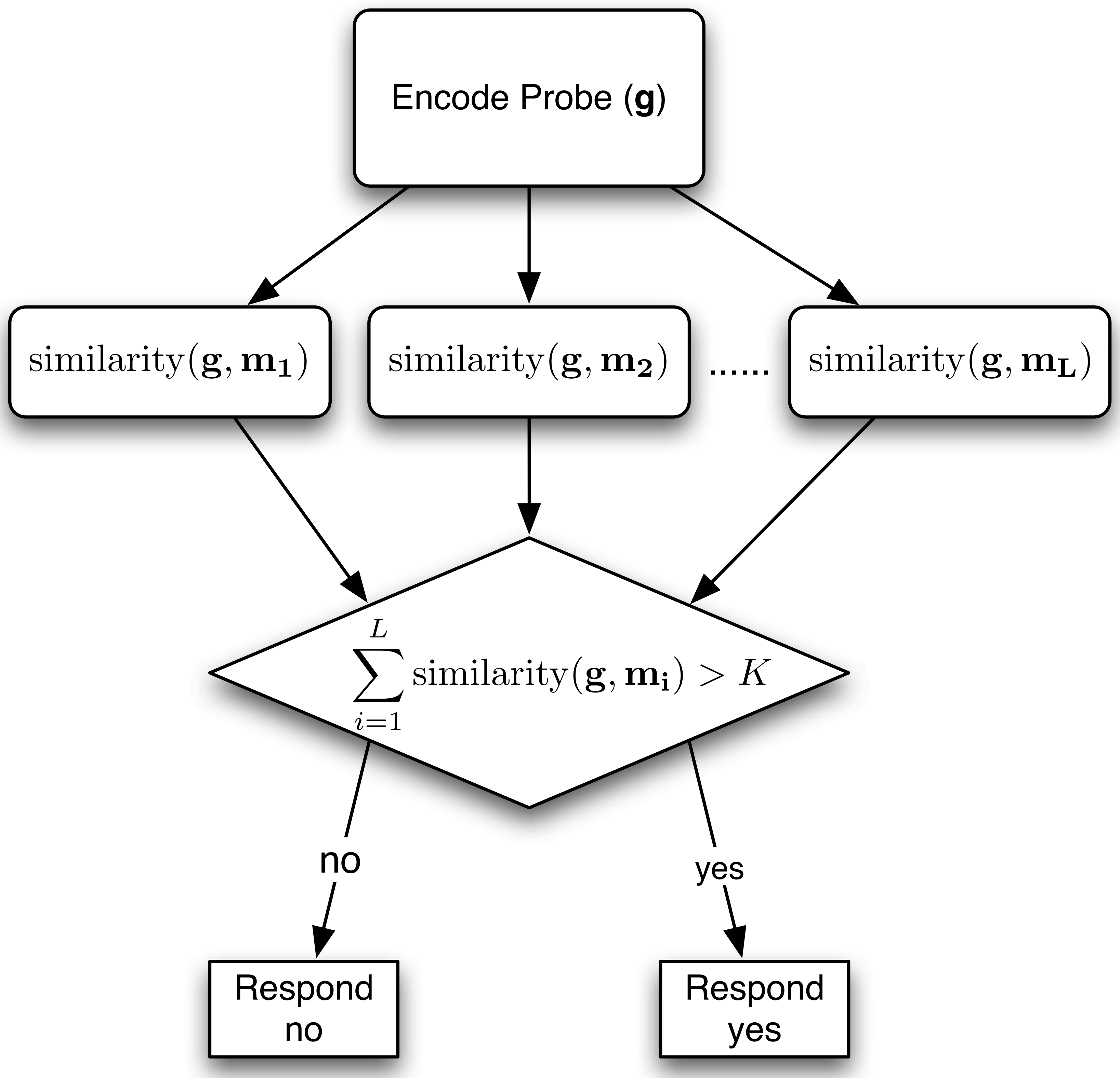
# Summed similarity




Compute the distance between the probe and the studied items (Pythagorean theorem)

$$\text{similarity} = e^{-\tau \text{ distance}}$$





Encode Probe ($\mathbf{g}$)
$\text{similarity}(\mathbf{g}, \mathbf{m_1})$
$\text{similarity}(\mathbf{g}, \mathbf{m_2})$
......
$\text{similarity}(\mathbf{g}, \mathbf{m_L})$
$\sum_{i=1}^{L} \text{similarity}(\mathbf{g}, \mathbf{m_i}) > K$
no
yes
Respond no
Respond yes

Following study of the list, the matrix *M* represents the list in memory

$$M = \begin{pmatrix} \mathbf{m}_1 & \mathbf{m}_2 & \mathbf{m}_3 & \ldots & \mathbf{m}_L \end{pmatrix}$$

**g** denotes a test item, either a target or a lure.

Similarity between **g** and **m**$_i$ =

$$e^{-\tau||\mathbf{g}-\mathbf{m}_i||} = e^{-\tau\sqrt{\sum_{j=1}^{N}(g(j)-m_i(j))^2}}$$

$\tau$ determines the steepness of the exponential function.

$$P(yes) = P\left(\sum_{i=1}^{L} e^{-\tau\sqrt{\sum_{j=1}^{N}(g(j)-m_i(j))^2}} > C\right)$$

The probability of saying "yes" to a probe is equal to the probability that its summed similarity (to the items) exceeds a criterion threshold, C

# Example Problem (tau=1)

| stimulus | Dimension | | |
|---|---|---|---|
| | x | y | z |
| 1 | 3 | 2 | 3 |
| 2 | 1 | 2 | 3 |
| 3 | 3 | 2 | 3 |
| 4 | 2 | 2 | 2 |
| 5 | 1 | 1 | 2 |
| Example Lure | 3 | 3 | 3 |
| Example Target | 1 | 2 | 3 |

| Distance to: | |
|---|---|
| Lure | Target |
| 1 | 2 |
| 2.236 | 0 |
| 1 | 2 |
| 1.732 | 1.414 |
| 3 | 1.414 |

| Similarity to: | |
|---|---|
| Lure | Target |
| 0.37 | 0.14 |
| 0.11 | 1 |
| 0.37 | . 14 |
| 0.18 | 0.24 |
| 0.05 | 0.24 |

| Summed Similarity | Lure | Target |
|---|---|---|
| | 1.07 | 1.76 |

| stimulus | Dimension | | |
|---|---|---|---|
| | x | y | z |
| 1 | 3 | 2 | 3 |
| 2 | 1 | 2 | 3 |
| 3 | 3 | 2 | 3 |
| 4 | 2 | 2 | 2 |
| 5 | 1 | 1 | 2 |
| Example Lure | 3 | 3 | 3 |
| Example Target | 1 | 2 | 3 |

| Distance to: | |
|---|---|
| Lure | Target |
| 1 | 2 |
| 2.236 | 0 |
| 1 | 2 |
| 1.732 | 1.414 |
| 3 | 1.414 |

| Similarity to: | |
|---|---|
| Lure | Target |
| 0.37 | 0.14 |
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| 0.18 | 0.24 |
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| Summed Similarity | Lure | Target |
|---|---|---|
| | 1.07 | 1.76 |