第三部分——

本部分内容要点（Key Points in the Chapter）：

- **0 数字图像处理对象**
- **1 空域中图像处理算子**
- **2 频域中图像处理算子**
- **3 数字图像复原方法**

第三部分——

2 频域中图像处理算子

空域滤波——基于卷积运算法则：

$$g(x,y)=f(x,y)*h(x,y)$$

空域滤波输出图像　原图像　滤波系统的脉冲响应函数

频域滤波——基于卷积定理，运用数字变换作为工具，得到低通、高通、带通或带阻滤波。

$$\Im[g(x,y)]=\Im[f(x,y)*h(x,y)]=F(u,v)H(u,v)$$

其中 $F(u,v)=\Im[f(x,y)],\quad H(u,v)=\Im[h(x,y)]$

记　$G(u,v)=\Im[g(x,y)]$ ，则

$$G(u,v)=F(u,v)H(u,v)$$

1° $f(x,y)\xrightarrow{\Im}F(u,v)$

2° 根据增强的要求设计适当的传递函数 $H(u,v)$

3° 乘法　$G(u,v)\leftarrow F(u,v)H(u,v)$

4° 逆变换 $\Im^{-1}$：　$g(x,y)=\Im^{-1}\left[G(u,v)\right]$

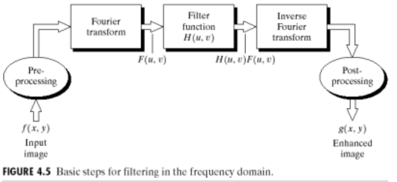


FIGURE 4.5 Basic steps for filtering in the frequency domain.

图像频谱　　$F(u,v)=|F(u,v)|e^{j\varphi(u,v)}$

相位 ψ ——> 0 . $\Im^{-1}\left[\|F(u,v)\|\right]$与 $\Im^{-1}\left[F(u,v)\right]$毫不相同；

幅值 $|F(u,v)|=1$. $\Im^{-1}\left[e^{j\psi(u,v)}\right]$ 保存了原图 $\Im^{-1}\left[F(u,v)\right]$中的许多重要特征。

——>在信号中，相位占据重要地位。

利用 Fourier 变换的平移性

$F[f(x,y)\cdot(-1)^{x+y}]\longrightarrow$ 原点在中心位置的图像频谱

$F(0,0)\propto$图像灰度平均值

频域滤波技术中的关键是要设计一个适当的滤波系统传递函数 $H(u,v)$。

凡是要保留的频率分量——对应的 $H(u,v)=1$或k（k是不等于1的常数）。

凡要抑制或衰减的频率分量——对应的 $H(u,v)=0$ 或 ω（0<ω<1的常数）。

一. 低通滤波——平滑

零相位移、径向对称低通滤波器。

（只改变图像频谱的实部和虚部，不改变频谱的相位——零相位移滤波器。）

➤1. 理想低通滤波器 (Ideal Low Pass Filter —— ILPF)

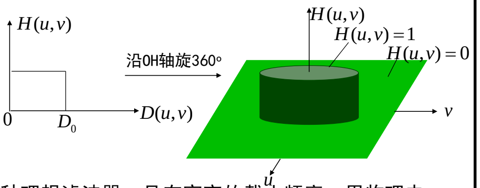
ILPF的传递函数：

$$H(u,v)=\begin{cases}1 & 若D(u,v)\le D_0\\ 0 & 若D(u,v)>D_0\end{cases}$$

其中 D_0—— 截止频率

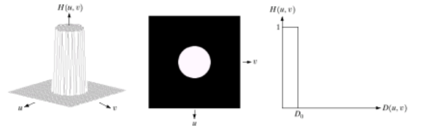
$D(u,v)$—— (u,v) 点到原点的距离，$D(u,v)=\sqrt{u^2+v^2}$

一. 低通滤波——平滑



这种理想滤波器，具有突变的截止频率。用物理电路是得不到的。它是数学滤波器，只能用计算机实现。

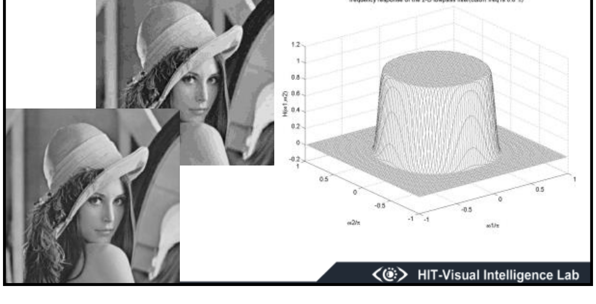
一. 低通滤波——平滑



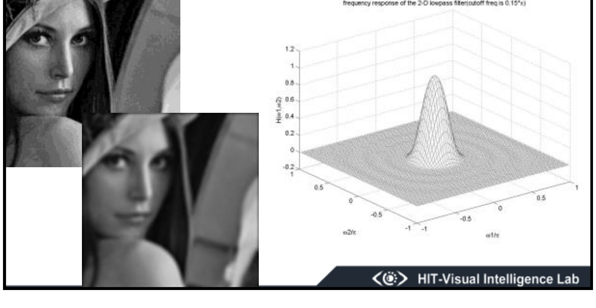
a b c

FIGURE 4.10 (a) Perspective plot of an ideal lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.

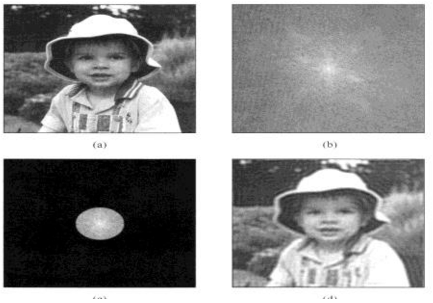
一. 低通滤波——平滑

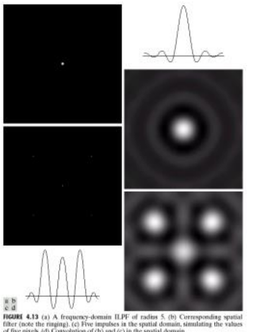


一. 低通滤波——平滑



Another Example





a b
c d

FIGURE 4.13 (a) A frequency-domain ILPF of radius 5. (b) Corresponding spatial filter (note the ringing). (c) Five impulses in the spatial domain, simulating the values of five pixels. (d) Convolution of (b) and (c) in the spatial domain.

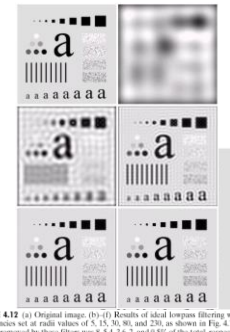
一. 低通滤波——平滑

$$
\begin{aligned}
h(x,y) = \Im^{-1}[H(u,v)] &= \int_{-\infty}^{\infty}\!\!\int H(u,v)e^{j2\pi(ux+vy)}dudv \\
&= \int_0^1 e^{j2\pi ux}du \int_0^1 e^{j2\pi vy}dv \\
&= \left[\frac{e^{j2\pi ux}}{j2\pi x}\right]_0^1 \left[\frac{e^{j2\pi vy}}{j2\pi y}\right]_0^1 \\
&= \frac{1}{j2\pi x}[e^{j2\pi x}-1]\cdot\frac{1}{j2\pi y}[e^{j2\pi y}-1] \\
&= \frac{1}{j2\pi x}e^{j\pi x}(e^{j\pi x}-e^{-j\pi x})\cdot\frac{1}{j2\pi y}e^{j\pi y}(e^{j\pi y}-e^{j\pi y}) \\
&= \frac{1}{j2\pi x}e^{j\pi x}\cdot 2j\sin\pi x\cdot\frac{1}{j2\pi y}e^{j\pi y}\cdot 2j\sin\pi y \\
&= \frac{\sin\pi x e^{j\pi x}}{\pi x}\cdot\frac{\sin\pi y e^{j\pi y}}{\pi y}
\end{aligned}
$$

一. 低通滤波——平滑

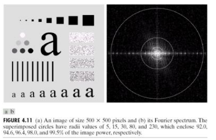
理想低通滤波后的图像，具有“振铃”特性，造成图像有不同程度的模糊。截止距离 D_0 越小，模糊程度越明显。原因在于理想低通滤波器的$H(u,v)$ 在 D_0 处由 1 突变到0。

在空域中表现为同心环的样子。



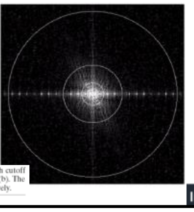
a b
c d
e f

FIGURE 4.12 (a) Original image. (b)–(f) Results of ideal lowpass filtering with cutoff frequencies set at radii values of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). The power removed by these filters was 8, 5.4, 3.6, 2, and 0.5% of the total, respectively.



a b

FIGURE 4.11 (a) An image of size 500 × 500 pixels and (b) its Fourier spectrum. The superimposed circles have radii values of 5, 15, 30, 80, and 230, which enclose 92.0, 94.6, 96.4, 98.0, and 99.5% of the image power, respectively.



a b

一. 低通滤波——平滑

频域滤波的乘积运算 $\xleftrightarrow{\text{相当}}$ 空域滤波的卷积运算

Butterworth 、指数和梯形滤波器的$H(u,v)$渐变的——由1衰减的。

$$f(x,y) * h(x,y) \longrightarrow g(x,y)$$

两个亮点(点脉冲） 同心环 两个同心环

两个被模糊的亮点—“振铃”特性

通过计算证明 $h(x,y)$ 的同心环数与D_0成反比；D_0 越小，同心环数越多。

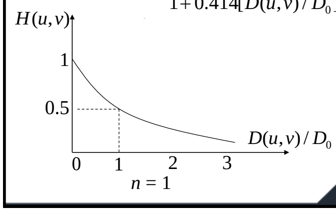
“振铃”特性的存在使理想低通滤波器的实际效用降低。

一. 低通滤波——平滑

➢2. 巴特沃斯（Butterworth）低通滤波器——**BLPF**

BLPF的传递函数如下：

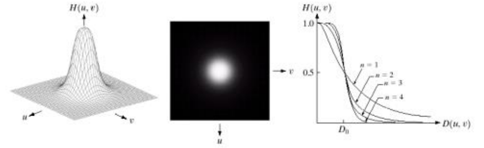
$$H(u,v)=\frac{1}{1+0.414[D(u,v)/D_0]^{2n}} \text{ 或 } \frac{1}{1+[D(u,v)/D_0]^{2n}}$$



BLPF 比用 ILPF 模糊现象轻得多。其原因是传递函数拖长的大尾巴，使相当多的一部分高频分量通过。

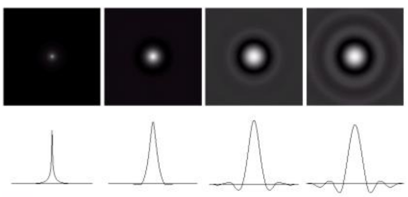
一. 低通滤波——平滑

➢2. 巴特沃斯（Butterworth）低通滤波器——**BLPF**



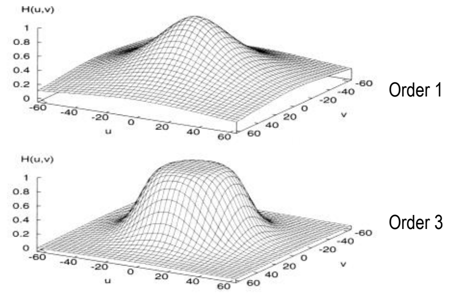
a b c

FIGURE 4.14 (a) Perspective plot of a Butterworth lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.



a b c d

FIGURE 4.16 (a)–(d) Spatial representation of BLPFs of order 1, 2, 5, and 20, and corresponding gray-level profiles through the center of the filters (all filters have a cutoff frequency of 5). Note that ringing increases as a function of filter order.



a b c d e f

FIGURE 4.15 (a) Original image. (b)–(f) Results of filtering with BLPFs of order 2, with cutoff frequencies at radii of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). Compare with Fig. 4.12.

一. 低通滤波——平滑

➢2. 巴特沃斯（Butterworth）低通滤波器——**BLPF**

BLPF处理过的图像中在任何时候都没有明显的振铃效果，这是过滤器在低频和高频之间的平滑过渡的结果。

一. 低通滤波——平滑

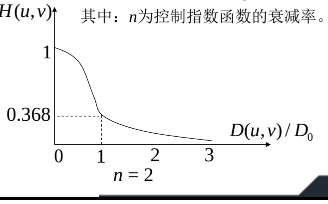
➢3. 指数滤波器(Exponential Low Pass Filter)——**ELPF**

ELPF的传递函数如下：

$$H(u,v)=e^{[\ln(1/\sqrt{2})][D(u,v)/D_0]^n} \quad 或 e^{-[D(u,v)/D_0]^n}$$

$$=e^{-0.347[D(u,v)/D_0]^n}$$

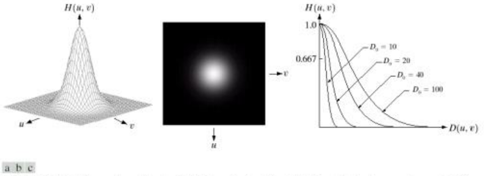
其中：n为控制指数函数的衰减率。



ELPF 比 BLPF 模糊现象增多。因为指数函数下降得快，使高频分量通过不多，但比ILPF轻。

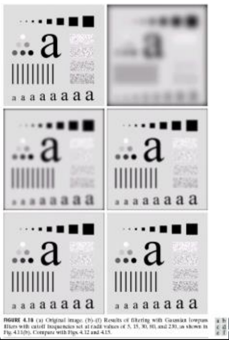
一. 低通滤波——平滑

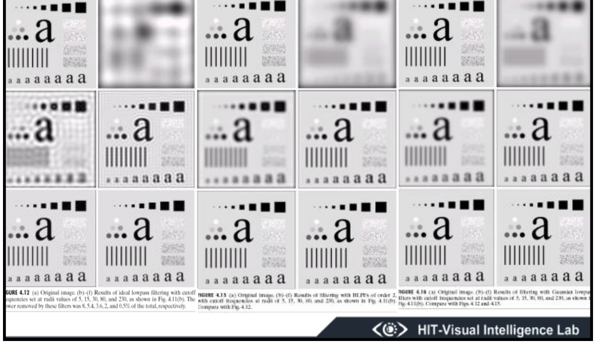
➢3. 指数滤波器(Exponential Low Pass Filter)——**ELPF**



a b c

FIGURE 4.17 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0.



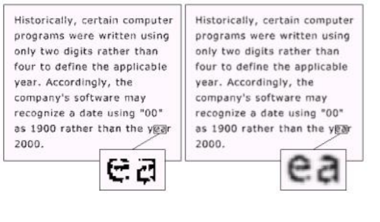


一. 低通滤波——平滑

➢3. 指数滤波器(Exponential Low Pass Filter)——**ELPF**

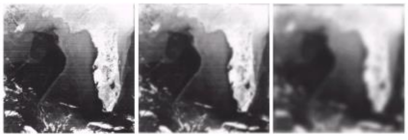
a b

FIGURE 4.19 (a) Sample text of poor resolution (note broken characters in magnified view). (b) Result of filtering with a GLPF (broken character segments were joined).



一. 低通滤波——平滑

➢3. 指数滤波器(Exponential Low Pass Filter)——**ELPF**



a b c

FIGURE 4.21 (a) Image showing prominent scan lines. (b) Result of using a GLPF with $D_0 = 30$. (c) Result of using a GLPF with $D_0 = 10$. (Original image courtesy of NOAA.)

一. 低通滤波——平滑

a b c

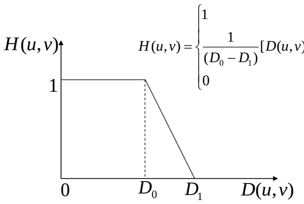
FIGURE 4.20 (a) Original image (1028 × 732 pixels). (b) Result of filtering with a GLPF with $D_0 = 100$. (c) Result of filtering with a GLPF with $D_0 = 80$. Note reduction in skin fine lines in the magnified sections of (b) and (c).

一. 低通滤波——平滑

➢4. 梯形滤波器(Trapezoidal Low Pass Filter)——**TLPF**

TLPF的传递函数如下：

$$H(u,v)=\begin{cases}1 & 若 0\le D(u,v)\le D_0\\ \dfrac{1}{(D_0-D_1)}[D(u,v)-D_1] & 若 D_0\le D(u,v)\le D_1\\ 0 & 若 D(u,v)>D_1\end{cases}$$

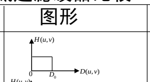
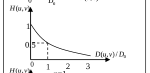
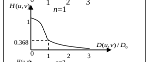
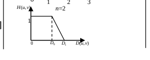


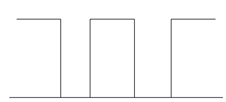
有振荡，但比ILPF小，由折点D_0与D_1引起。

BLPF 与 ELPF 不引起振荡，不发生突变。

一. 低通滤波——平滑

四种低通滤波器比较

滤波器名称	传递函数	图形	性能
ILPF	$\begin{cases}1 & (0,D_0)\\ 0 & (D_0,\infty)\end{cases}$		D_0小，“振铃”大
BLPF	$\dfrac{1}{1+0.414[D(u,v)/D_0]^{2n}}$		好
ELPF	$e^{-[D(u,v)/D_0]^n}$		较好，下降得快
TLPF	$\begin{cases}1 & (0,D_0)\\ \dfrac{1}{(D_0-D_1)}[D(u,v)-D_1] & [D_0,D_1]\\ 0 & (D_1,\infty)\end{cases}$		振荡，比ILPF强



Lowpass

Bandpass

Bandstop

Highpass

二. 高通滤波——锐化

零相位移，径向对称的高通滤波器。

➢1. 理想高通滤波器 (Ideal high pass filter —— **IHPF**)

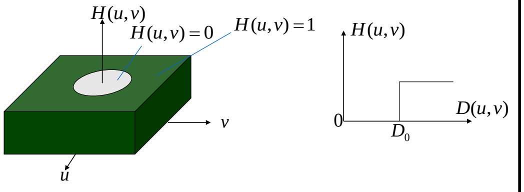
IHPF的传递函数：

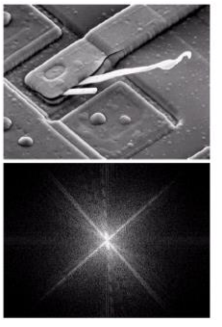
$$H(u,v)=\begin{cases}0 & 若 D(u,v)\le D_0\\ 1 & 若 D(u,v)>D_0\end{cases}$$

其中

$D(u,v)$—— (u,v) 点到原点的距离，$D(u,v)=\sqrt{u^2+v^2}$

二. 高通滤波——锐化





a
b

FIGURE 4.4 (a) SEM image of a damaged integrated circuit. (b) Fourier spectrum of (a). (Original image courtesy of Dr. J. M. Hudak, Brockhouse Institute for Materials Research, McMaster University, Hamilton, Ontario, Canada.)

FIGURE 4.6 Result of filtering the image in Fig. 4.4(a) with a notch filter that set to 0 the $F(0,0)$ term in the Fourier transform.

二. 高通滤波——锐化

➤2. Butterworth高通滤波器——**BHPF**

BHPF的传递函数如下：

$$H(u,v)=\frac{1}{1+[D_0/D(u,v)]^{2n}}$$

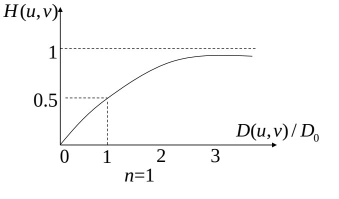
当 $D(u,v)=D_0$时，下降到1/2 最大值。通常使下降到最大值的，则上式为:

$$H(u,v)=\frac{1}{1+[\sqrt{2}-1][D_0/D(u,v)^{2n}]}=\frac{1}{1+0.414[D_0/D(u,v)^{2n}]}$$

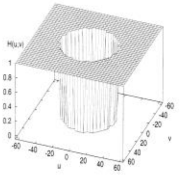
二. 高通滤波——锐化

➤2. Butterworth高通滤波器——**BHPF**

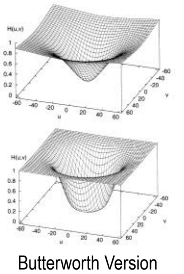
BHPF的切面图:



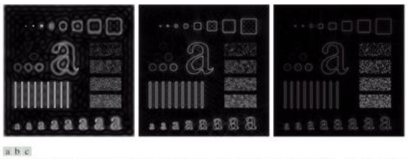
High Pass Filtering



Normal

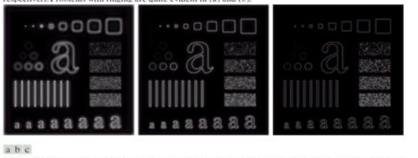


Butterworth Version



a b c

FIGURE 4.24 Results of ideal highpass filtering the image in Fig. 4.11(a) with $D_0=15$, 30, and 80, respectively. Problems with ringing are quite evident in (a) and (b).



a b c

FIGURE 4.25 Results of highpass filtering the image in Fig. 4.11(a) using a BHPF of order 2 with $D_0=15$, 30, and 80, respectively. These results are much smoother than those obtained with an ILPF.

二. 高通滤波——锐化

➢3. 指数高通滤波器——**EHPF**

EHPF的传递函数如下：

$$H(u,v)=e^{-[D_0/D(u,v)]^n}$$

参量 n 控制增长速度。当时，$H(u,v)=e^{-1}$.

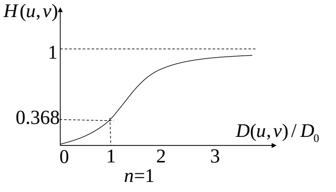
通常使 $D_0=D(u,v)$ 时，使 $H(u,v)=1/\sqrt{2}$ ，则

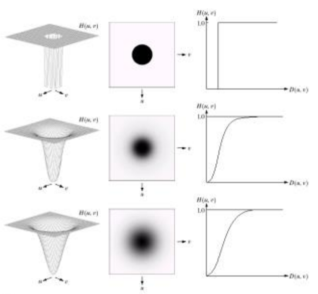
$$H(u,v)=e^{[-\ln 1/\sqrt{2}][D_0/D(u,v)]^n}=e^{-0.347[D_0/D(u,v)]^n}$$

• 二. 高通滤波——锐化

➢3. 指数高通滤波器——**EHPF**

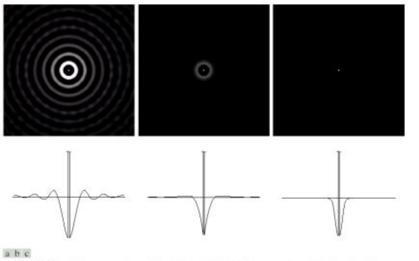
EHPF的切面图:





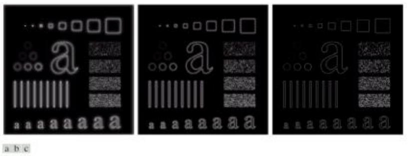
a b c
d e f
g h i

FIGURE 4.22 Top row: Perspective plot, image representation, and cross section of a typical ideal highpass filter. Middle and bottom rows: The same sequence for typical Butterworth and Gaussian highpass filters.



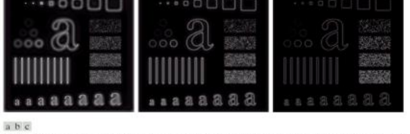
a b c

FIGURE 4.23 Spatial representations of typical (a) ideal, (b) Butterworth, and (c) Gaussian frequency domain highpass filters, and corresponding gray-level profiles.



a b c

FIGURE 4.26 Results of highpass filtering the image of Fig. 4.11(a) using a GHPF of order 2 with D_0 = 15, 30, and 80, respectively. Compare with Figs. 4.24 and 4.25.



a b c

FIGURE 4.25 Results of highpass filtering the image in Fig. 4.11(a) using a BHPF of order 2 with D_0 = 15, 30, and 80, respectively. These results are much smoother than those obtained with an ILPF.

二. 高通滤波——锐化

➢4. 梯形高通滤波器——THPF

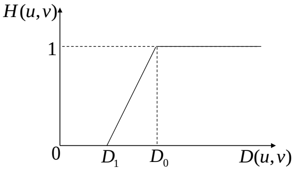
THPF的传递函数为：

$$H(u,v)=\begin{cases}0 & 若0\le D(u,v)\le D_1\\ \dfrac{1}{(D_0-D_1)}[D(u,v)-D_1] & 若D_1\le D(u,v)\le D_0\\ 1 & 若D(u,v)>D_0\end{cases}$$

二. 高通滤波——锐化

➢4. 梯形高通滤波器——THPF

THPF的切面图:



二. 高通滤波——锐化

高通滤波后，由于低频信息衰减，致使图像的平滑区基本消失，丢失了许多必要的信息。对此宜采用高频加强滤波来弥补。

$$G(u,v) = F(u,v)H(u,v)$$

高通滤波 原图频谱 高通滤波传递函数

现 $H'(u,v) = H(u,v) + C, \quad 0 < C < 1$

$$G'(u,v) = F(u,v)H'(u,v) = F(u,v)H(u,v) + CF(u,v)$$

二. 高通滤波——锐化

在高通滤波基础上保留的低频信息。高频信息比一般高通滤波时加强了，这就叫做高频加强滤波。

通常采用高频加强滤波后，还需要再进行直方图均衡化处理，可使图像清晰，质量得到提高。

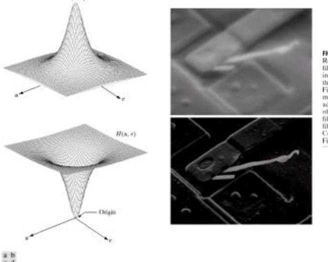


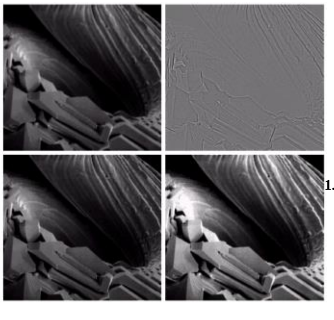
FIGURE 4.8 Result of highpass filtering the image in Fig. 4.4(a) with the filter in Fig. 4.7(c), modified by adding a constant of one-half the filter height to the filter function. Compare with Fig. 4.4(a).

a b
c d

FIGURE 4.7 (a) A two-dimensional lowpass filter function. (b) Result of lowpass filtering the image in Fig. 4.4(a). (c) A two-dimensional highpass filter function. (d) Result of highpass filtering the image in Fig. 4.4(a).

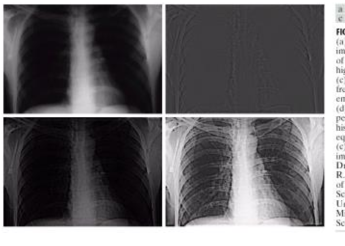
a b
c d

FIGURE 4.29 Same as Fig. 3.43, but using frequency domain filtering. (a) Input image. (b) Laplacian of (a). (c) Image obtained using Eq. (4.4-17) with $A = 2$. (d) Same as (c), but with $A = 2.7$. (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)



1倍原图叠加

1.7倍原图叠加



a b
c d

FIGURE 4.30 (a) A chest X-ray image. (b) Result of Butterworth highpass filtering. (c) Result of high-frequency emphasis filtering. (d) Result of performing histogram equalization on (c). (Original image courtesy Dr. Thomas R. Gest, Division of Anatomical Sciences, University of Michigan Medical School.)

三. 同态滤波 Homomorphic filtering

——特殊的频域滤波技术，起到压缩图像灰度的动态范围，增强对比度的作用。Gray-level range compression and contrast enhancement

所谓同态系统，是服从广义叠加原理的，输入和输出之间可用线性变换表示的系统；若输入量为乘法运算的组合，则称之为乘法同态系统。图像处理中涉及的同态滤波正是这样一类的系统。

图像模型:
亮度 $f(x,y)$
- 入射分量 $i(x,y)$，由光源性质所决定
- 反射分量 $r(x,y)$，由物体的特性决定

三. 同态滤波 Homomorphic filtering

$$f(x,y)=i(x,y)\,r(x,y),\quad 0<i(x,y)<\infty,\ 0<r(x,y)<1$$

f(x,y) → ln → lnf(x,y) → Fourier 变换 → S(u,v) → H(u,v) → Z(u,v) → 逆 Fourier 变换 → Z(x,y) → exp → g(x,y)

同态滤波系统框图

1° 系统对图像 $f(x,y)$ 取对数——> $\ln f(x,y)$

$$\ln f(x,y)=\ln i(x,y)+\ln r(x,y)$$

2° 求 Fourier 变换

$$S(u,v)=\Im[\ln f(x,y)]$$

三. 同态滤波 Homomorphic filtering

$$f(x,y)=i(x,y)\,r(x,y),\quad 0<i(x,y)<\infty,\ 0<r(x,y)<1$$

f(x,y) → ln → lnf(x,y) → Fourier 变换 → S(u,v) → H(u,v) → Z(u,v) → 逆 Fourier 变换 → Z(x,y) → exp → g(x,y)

同态滤波系统框图

3° 选取适当的 $H(u,v)$ 对 $S(u,v)$ 滤波，希望达到压缩灰度动态范围及增强对比度的目的。

$$Z(u,v)=S(u,v)H(u,v)$$

三. 同态滤波 Homomorphic filtering

$$f(x,y)=i(x,y)\,r(x,y),\quad 0<i(x,y)<\infty,\ 0<r(x,y)<1$$

f(x,y) → ln → lnf(x,y) → Fourier 变换 → S(u,v) → H(u,v) → Z(u,v) → 逆 Fourier 变换 → Z(x,y) → exp → g(x,y)

同态滤波系统框图

4° 取逆变换

$$Z(x,y)=\Im^{-1}[Z(u,v)]=\Im^{-1}[S(u,v)H(u,v)]$$

5° 取指数运算——自然对数的逆运算，得到同态滤波系统的输出。$g(x,y)=\exp[Z(x,y)]$

三.同态滤波 Homomorphic filtering

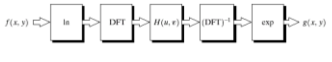


FIGURE 4.31 Homomorphic filtering approach for image enhancement.

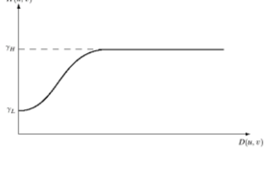


FIGURE 4.32 Cross section of a circularly symmetric filter function. $D(u,v)$ is the distance from the origin of the centered transform.

三. 同态滤波 Homomorphic filtering

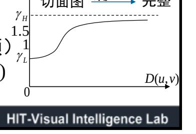
同态滤波系统的传递函数 $H(u,v)$ 的选取：

入射光分量——图像亮度大小，反映灰度恒定分量——低频信息

降低 图像灰度的动态范围降低

反射光分量——物体性质决定反射大小(不同物体吸收光的程度不同，光反射在不同物体的交接两侧并不一样)——高频信息

增加 对比度增加



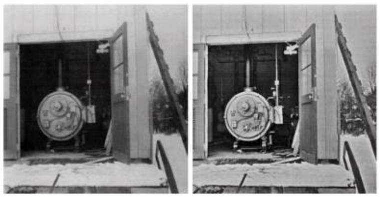
低频部分的 $H(u,v)<1$（意味着减少低频）
高频部分的 $H(u,v)>1$（意味着增强高频）

通常，$r_H=2$，$r_L=0.5$

三. 同态滤波 **Homomorphic filtering**

a b

FIGURE 4.33 (a) Original image. (b) Image processed by homomorphic filtering (note details inside shelter). (Stockham.)



第三部分——

本部分内容要点（Key Points in the Chapter）：

- **0 数字图像处理对象**
- **1 空域中图像处理算子**
- **2 频域中图像处理算子**
- **3 数字图像复原方法**

第三部分——

3 数字图像复原方法

在景物成像过程中，由于目标的高速运动、散射、成像系统畸变和噪声干扰，致使最后形成的图像存在种种恶化，或称之为“退化”(Degradation)。

Case I: -- motion blurring

Orginal image

Blurred image

使退化了的图像尽可能恢复到能真实反映景物图像的处理，我们称为图像的恢复，或称“图像复原”。

图像增强 **Enhancement** } 改善图像的质量 ——主观标准

图像复原 **Restoration** } (以保真原则为前提) ——客观标准

退化图像 ⟶ 真实图像

估计问题

预先了解退化的先验知识

建立退化模型 求逆过程

Enhancement vs. Restoration

- ❖"Better" visual representation ⟷ ❖Remove effects of sensing environment
- ❖Subjective ⟷ ❖Objective
- ❖No quantitative measures ⟷ ❖Mathematical, model dependent quantitative measures

退化过程的逆向估计问题，是一个去卷积的过程。

复原使用一定的估计准则作为判别估计误差的依据，因此它有客观的评价复原图像质量的标准。

进行图像复原
- 可以用连续数学
- 也可以用离散数学
  - 空间域的卷积
  - 频域的相乘

一. 退化模型(Degradation model)

图像复原的关键在于建立退化模型。

设 $f(x,y)$ 为未退化的真实图像(original image)

$g(x,y)$ 为退化图像(degradation) given the observed image

$\eta(x,y)$ 为具有统计性质的附加噪声。

A Model of the Image Degradation / Restoration Process :

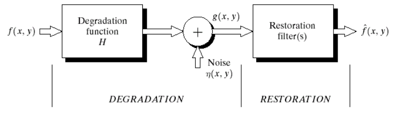


FIGURE 5.1 A model of the image degradation/ restoration process.

Spatial Domain :

$$g(x,y)=h(x,y)*f(x,y)+\eta(x,y)$$

Frequency Domain :

$$G(u,v)=H(u,v)F(u,v)+N(u,v)$$

一. 退化模型(Degradation model)

$$g(x,y)=\mathrm{H}\,f(x,y)+\eta(x,y)$$

复原的关键在于对系统H的基本了解。

系统的特性决定了输入信号与输出信号的联系：

系统的分类
- 线性系统与非线性系统
- 时变系统和非时变系统
- 集中参数系统与分布参数系统
- 连续系统与离散系统

一. 退化模型(Degradation model)

线性系统：

$$H[f_1(x,y)+f_2(x,y)]=g_1(x,y)+g_2(x,y)$$

两个输入之和的响应等于两个响应之和。

非时变系统：

如果系统**H**的参数不随时间 t 变化，称“非时变系统”；否则为时变系统。

空间位移不变系统：

$$H[f(x-\alpha,y-\beta)]=g(x-\alpha,y-\beta)$$

图像中任一点通过系统**H**的响应，只取决于该点的输入值，与该点的位置无关。

一. 退化模型(Degradation model)

- Restoration problem

$$T\{f\}=g$$

$\Rightarrow$ restoration is to find T^{-1}, such that $T^{-1}\{g\}=f$

but, 1. T^{-1} does not exist: singular

2. T^{-1} may exist, but not be unique: ill-conditioned

3. T^{-1} may exist and unique, but there exists ε , which can be made arbitrarily small, such that

$T^{-1}\{g+\varepsilon\}=f+\delta, \quad \delta>>\varepsilon,$ which is not negligible

$\Rightarrow$ Chapter 5 Image Restoration is ill-conditioned at best and singular at worst

Estimation by modeling

☞ Based on either physical characteristics or basic principles

- Eg.1. Physical characteristics: atmospheric turbulence

$$H(u,v)=e^{-k(u^2+v^2)^{5/6}}$$

- Eg.2. Math derivation: motion blur $g(x,y)=\int_0^T f[x-x_0(t), y-y_0(t)]dt$
  - Starting from $H(u,v)=\int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]}dt$
  - After some manipulation(p.259)
  - Setting the motion model, we obtain the degradation func.

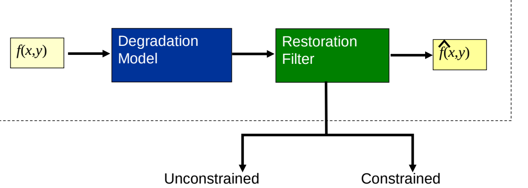
$$H(u,v)=\frac{T}{\pi(ua+vb)}\sin[\pi(ua+vb)]e^{-j\pi(ua+vb)}$$

Motion blur ——applying Math modeling

a b

FIGURE 5.26 (a) Original image. (b) Result of blurring using the function in Eq. (5.6-11) with $a=b=0.1$ and $T=1$.

$$H(u,v)=\frac{T}{\pi(ua+vb)}\sin[\pi(ua+vb)]e^{-j\pi(ua+vb)}$$



Unconstrained
- Inverse Filter
- Pseudo-inverse Filter

Constrained
- Wiener Filter
- Kalman Filter

——消除匀速直线运动模糊

例：设平面匀速运动景物图像 $f(x,y)$，采集时间是 T，并设 $x_0(t)$ 和 $y_0(t)$ 分别是景物在 x 方向和 y 方向的运动分量，由于运动造成的模糊图像为 $g(x,y)$，其它因素忽略，包括噪声，则这一过程可用下面的方程表示：

$$g(x,y)=\int_0^T f[x-x_0(t), y-y_0(t)]dt$$

它的Fourier变换可表示为：

$$G(u,v)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)\exp[-j2\pi(ux+vy)]dxdy$$

——消除匀速直线运动模糊

$$=\int_0^T\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f[x-x_0(t), y-y_0(t)]\exp[-j2\pi(ux+vy)]dxdy\right]dt$$

$$=F(u,v)\int_0^T\exp\{-j2\pi[ux_0(t)+vy_0(t)]\}dt$$

如果定义：

$$H(u,v)=\int_0^T\exp\{-j2\pi[ux_0(t)+vy_0(t)]\}dt$$

可见如果知道了运动分量 $x_0(t)$ 和 $y_0(t)$ ，就可从上式直接得到传递函数 $H(u,v)$ 。因此， $g(x,y)$ 可以恢复出来。

——消除匀速直线运动模糊

下面来计算 $H(u,v)$ 。

设只有沿 x 方向的运动，即$x_0(t)=Nt/T$（当 $t=T$时，$f(x,y)$ 所移动的距离为 N），$y_0(t)=0$ ，则

$$H(u,v)=\int_0^T \exp[-j2\pi(u\cdot N\cdot\frac{t}{T})]dt$$

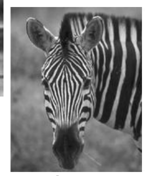
$$=\frac{T}{\pi uN}\sin(\pi uN)\exp[-j\pi uN]$$

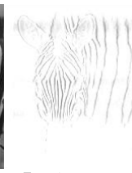
令$N\rightarrow a$，扩展到 $y_0=bt/T$，得到

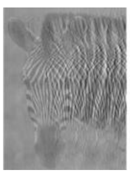
$$H(u,v)=\frac{T}{\pi(ua+vb)}\sin[\pi(ua+vb)]e^{-j\pi(ua+vb)}$$


Blur image


Reconstructed image


Original image


Error image


按位移25个像素复原


按位移26个像素复原


按位移27个像素复原

——消除匀速直线运动模糊

在曝光时间 T 内，像素点 0 接受的不只是 $f(0)$ 的信息，而是 N 个采样点的信息在此位置上的叠加。

$$g(0)=f(0)+f(1)+\cdots+f(N-2)+f(N-1)$$

考虑到每个采样点在CCD像素上的曝光量的贡献，上式可写为

$$g(0)=\frac{1}{N}[f(0)+f(1)+\cdots+f(N-2)+f(N-1)]$$

而在CCD像素点1上接受的信息

$$g(1)=\frac{1}{N}[f(1)+f(2)+\cdots+f(N-2)+f(N-1)+f(N)]$$

对在CCD上任意一个像素点 n 来说，

$$g(n)=\frac{1}{N}[f(n)+f(n+1)+\cdots+f(n+N-2)+f(n+N-1)]$$

——消除匀速直线运动模糊

对上式进行Z变换，则有

$$G(z)=\frac{1}{N}[F(z)+F(z)z+F(z)z^2+\cdots+F(z)z^{N-1}]$$

$G(z),F(z)$ 分别是$g(n),f(n)$的 Z变换。

$$G(z)=\frac{1}{N}F(z)[1+z+z^2+\cdots+z^{N-1}]$$

由卷积定理得

$$H(z)=\frac{G(z)}{F(z)}=\frac{1}{N}[1+z+z^2+\cdots+z^{N-1}]=\frac{1}{N}\sum_{i=0}^{n-1}z^i$$

$$=\frac{1}{N}\cdot\frac{1-z^N}{1-z}$$

——消除匀速直线运动模糊

$$\frac{G(z)}{F(z)}=\frac{1}{N}\cdot\frac{1-z^N}{1-z}$$

$$(1-z)\cdot G(z)=\frac{1}{N}\cdot F(z)\cdot(1-z^N)$$

$$G(z)-zG(z)=\frac{1}{N}\cdot[F(z)-z^N F(z)]$$

对方程两边分别进行反 Z变换，则有

$$g(n)-g(n+1)=\frac{1}{N}\cdot[f(n)-f(n+N)]$$

$$f(n)=Ng(n)-Ng(n+1)+f(n+N)$$

——维纳滤波(Wiener Filtering, 1942)

Objective function: find an estimate $\hat{f}$ of f such that the mean square error between them is minimized

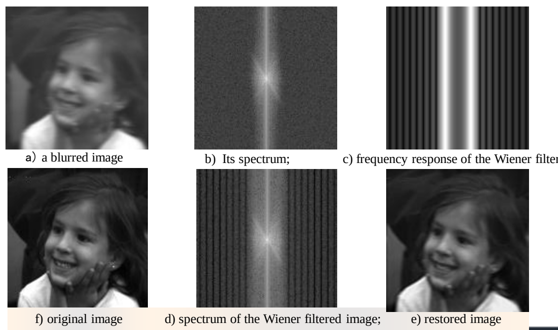
$$e^2 = E\{(f-\hat{f})^2\}$$

——最小均方误差滤波Minimum mean-square error filtering(MMSF)

（又称：最小乘方滤波）

$$\hat{f} = (\frac{1}{\lambda}Q^TQ + H^TH)^{-1}H^Tg$$

$$\hat{F}(u,v) = \frac{1}{H(u,v)}\frac{|H(u,v)|^2}{|H(u,v)|^2 + S_n(u,v)/S_f(u,v)}G(u,v)$$



Wiener filtering

——Using the approximation

K is chosen interactively

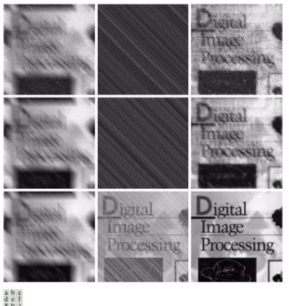
a b c

FIGURE 5.28 Comparison of inverse- and Wiener filtering. (a) Result of full inverse filtering of Fig. 5.25(b). (b) Radially limited inverse filter result. (c) Wiener filter result.

Severe noise

Moderate noise

Negligible noise



Restoration by Wiener filter motion blur

二. 除燥复原

- 1. **Spatial**

  $g(x,y) = f(x,y) + h(x,y)$

- 2. **Frequency**

  G(u,v) = F(u,v) + N(u,v)

Assume noise is uncorrelated to the image.

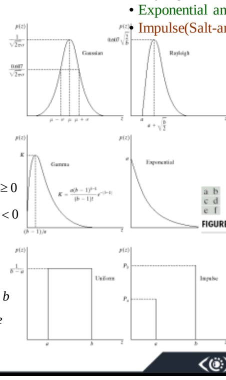
- Gaussian: electronic circuit noise, sensor noise
- Rayleigh: range images
- Exponential and Gamma: laser images
- Impulse(Salt-and-Pepper): faulty switching

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma}e^{-(z-\mu)^2/2\sigma^2}$$

$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-(z-a)^2/b} & for\ z \ge a \\ 0 & for\ z < a \end{cases}$$

$$p(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!}e^{-az} & for\ z \ge 0 \\ 0 & for\ z < 0 \end{cases}$$

$$p(z) = \begin{cases} ae^{-az} & \text{for } z \ge 0 \\ 0 & \text{for } z < 0 \end{cases}$$



a b
c d
e f

FIGURE 5.2 Some important probability density functions.

$$p(z) = \begin{cases} \frac{1}{b-a} & for\ a \le z \le b \\ 0 & otherwise \end{cases}$$

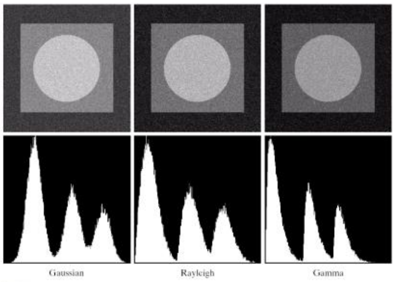
$$p(z) = \begin{cases} P_a & z = a \\ P_b & z = b \\ 0 & otherwise \end{cases}$$

- Gausssian noise(高斯噪声)
- Rayleigh noise(瑞利噪声)
- Gamma noise(Erlang,爱尔兰噪声)
- Exponential noise(指数分布噪声)
- Uniform noise(均匀分布噪声)
- Impulse noise(Salt-and Pepper)(椒盐噪声)
- Modeling a broad range of noise corruption situations found in practice

- GN: 电子电路噪声,低照明，高温等带来的噪声
- RN：特征化噪声现象
- GN and EN:激光成像中的噪声
- UN：模拟随机数产生器
- IN：错误开关

The Test Pattern

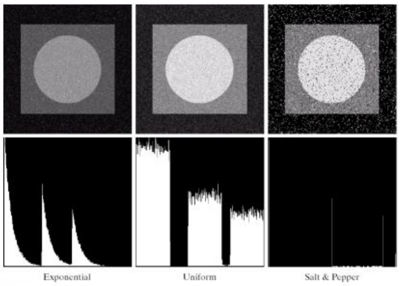
FIGURE 5.3 Test pattern used to illustrate the characteristics of the noise PDFs shown in Fig. 5.2.



Gaussian Rayleigh Gamma

a b c
d e f

FIGURE 5.4 Images and histograms resulting from adding Gaussian, Rayleigh, and gamma noise to the image in Fig. 5.3.

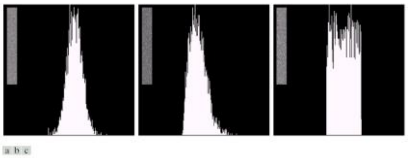


Exponential Uniform Salt & Pepper

g h i
j k l

FIGURE 5.4 (Continued) Images and histograms resulting from adding exponential, uniform, and impulse noise to the image in Fig. 5.3.

- PDF from small patches



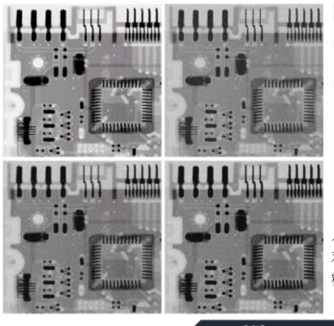
a b c

FIGURE 5.6 Histograms computed using small strips (shown as inserts) from (a) the Gaussian, (b) the Rayleigh, and (c) the uniform noisy images in Fig. 5.4.

①Mean filters - In the Spatial Domain

- Arithmetic mean filter
  - For Gaussian or uniform noise
- Geometric mean filter
  - For Gaussian or uniform noise (Keeps more detail than AMF)
- Harmonic mean filter
  - Works well for salt noise, but fails for pepper noise
  - Does well on Gaussian noise
- Contraharmonic mean filter
  - For the salt(negative Q)-and-pepper(positive Q) noise
  - For Gaussian or uniform noise

Arithmetic & Geometric mean filters for Gaussian noise



a b
c d

FIGURE 5.7 (a) X-ray image. (b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3 × 3. (d) Result of filtering with a geometric mean filter of the same size. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

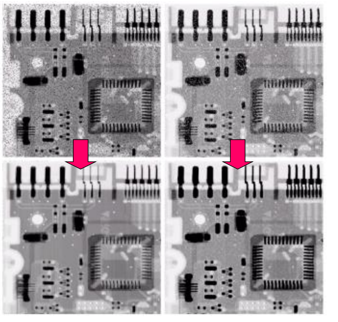
几何均值对图像模糊更少

Contraharmonic filters for salt-and-pepper noise

$$\hat{f}(x,y)=\frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

a b
c d

FIGURE 5.8 (a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a 3 × 3 contraharmonic filter of order 1.5. (d) Result of filtering (b) with $Q = -1.5$.



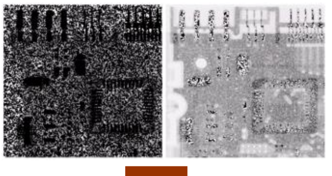
Q<0 : eliminates salt noise

Q=-1 → harmonic mean filter

Q=0 : arithmetic mean filter

Q>0 : eliminates pepper noise

Wrong sign in contraharmonic filters



a b

FIGURE 5.9 Results of selecting the wrong sign in contraharmonic filtering. (a) Result of filtering Fig. 5.8(a) with a contraharmonic filter of size 3 × 3 and $Q = -1.5$. (b) Result of filtering 5.8(b) with $Q = 1.5$.

Disaster!

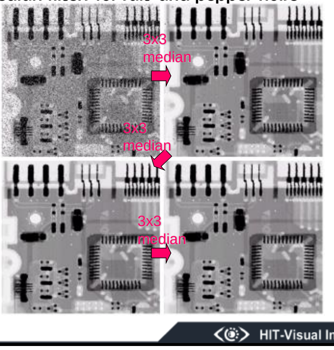
②Order-statistics filters - In the Spatial Domain

- Median filter
  - Particularly well on salt-and-pepper noises
- Max and min filters
  - Max: reduces pepper noises
  - Min: reduces salt noises
- Midpoint filter
  - Combines order statistics and averaging
  - Works best for Gaussian or uniform noises
- Alpha-trimmed mean filter

Median filters for salt-and pepper noise

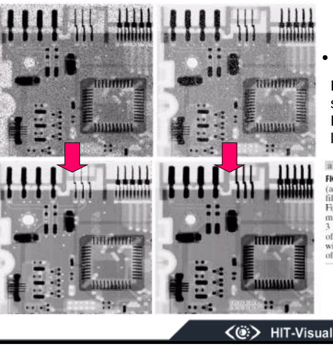
a b
c d

FIGURE 5.10 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.1$. (b) Result of one pass with a median filter of size 3 × 3. (c) Result of processing (b) with this filter. (d) Result of processing (c) with the same filter.



a b
c d

FIGURE 5.8 (a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability.



- Min filter

Removes salt noise
Removes light pixels

a b

FIGURE 5.11 (a) Result of filtering Fig. 5.8(a) with a max filter of size 3 × 3. (b) Result of filtering 5.8(b) with a min filter of the same size.

- Max filter

Removes pepper noise
Removes dark pixels

uniform noise

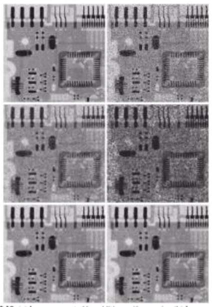
uniform noise + S&P noise

5x5 arithmetic mean

5x5 geometric mean

5x5 median

5x5 alpha-trimmed Mean(d=5)



a b c d e f

FIGURE 5.12 (a) Image corrupted by additive uniform noise. (b) Image additionally corrupted by additive salt-and-pepper noise. Image in (b) filtered with a 5 × 5: (c) arithmetic mean filter; (d) geometric mean filter; (e) median filter; and (f) alpha-trimmed mean filter with $d = 5$.

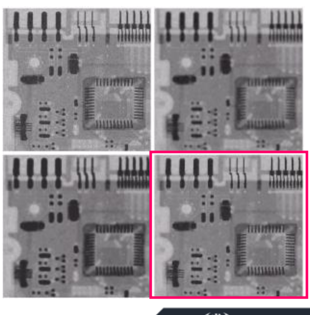
③Adaptive filters - In the Spatial Domain

- Behavior changes locally based on statistical characteristics of local support
- Simple adaptive filter based on mean and variance
  1. If global_var is zero, then f(x,y)=g(x,y)
  2. If local_var>global_var, then f(x,y)=g(x,y) (high local var → edge → should be preserved)
  - If local_var==global_var, then arithmetic mean filtering

Adaptive filters for Gaussian noise

a b c d

FIGURE 5.13 (a) Image corrupted by additive Gaussian noise of zero mean and variance 1000. (b) Result of arithmetic mean filtering. (c) Result of geometric mean filtering. (d) Result of adaptive noise reduction filtering. All filters were of size 7 × 7.



7x7 arithmetic mean

7x7 geometric mean

7x7 Adaptive filters

③Adaptive median filters - In the Spatial Domain

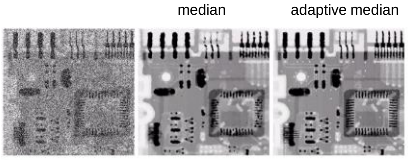
- Cope with impulse noise with large probalility
- Preserve detail while smoothing non-impulse noise

Algorithm

Level A: $A1=z_{med}-z_{min}$
$A2=z_{med}-z_{max}$
If A1>0 AND A2<0, go to level B
Else increase the window size
If window size<=S_{max} repeat level A
Else output z_{xy}
Level B: $B1=z_{xy}-z_{min}$
$B2=z_{xy}-z_{max}$
If B1>0 AND B2<0, output z_{xy}
Else output z_{med}

Adaptive median filters for salt-and-pepper noise

median adaptive median

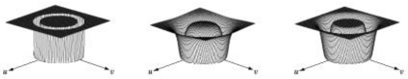


a b c

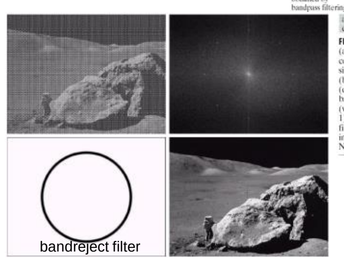
FIGURE 5.14 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.25$. (b) Result of filtering with a 7 × 7 median filter. (c) Result of adaptive median filtering with $S_{max} = 7$.

- 2. In the Frequency Domain
  - ①Bandreject filters
  - ②Bandpass filters
  - ③Notch filters

Bandreject filters for periodic noises

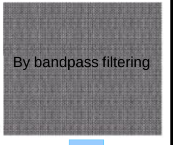


a b c
FIGURE 5.15 From left to right, perspective plots of ideal, Butterworth (of order 1), and Gaussian bandreject filters.

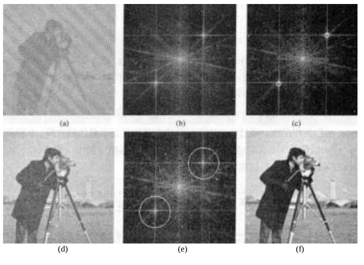


a b
c d
FIGURE 5.16 (a) Image corrupted by sinusoidal noise. (b) Spectrum of (a). (c) Butterworth bandreject filter (white represents 1). (d) Result of filtering. (Original image courtesy of NASA.)

FIGURE 5.17 Noise pattern of the image in Fig. 5.16(a) obtained by bandpass filtering.



Help understanding noise pattern

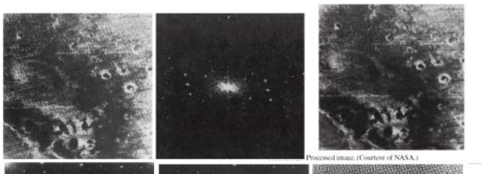


傅立叶变换

函数	$f(t)$	$F(s)$
余弦	$\cos(2\pi ft)$	$\frac{1}{2}[\delta(s+f)+\delta(s-f)]$
正弦	$\sin(2\pi ft)$	$j\frac{1}{2}[\delta(s+f)-\delta(s-f)]$

图(a)为一幅受正弦干扰模式覆盖的图像。
图(b)是(a)的傅立叶频谱幅度图，其上有一对较明显的脉冲白点。
利用带阻滤波器除掉亮点。然后取傅立叶反变换，就可得到图(d)和(f)所示的恢复效果。

a b
FIGURE 5.20 (a) Image of the Martian terrain taken by *Mariner 6*. (b) Fourier spectrum showing periodic interference. (Courtesy of NASA.)



Processed image. (Courtesy of NASA.)

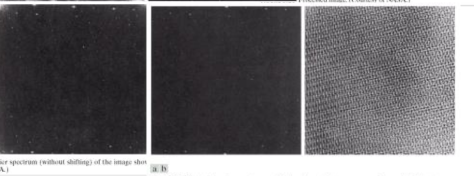


FIGURE 5.21 Fourier spectrum (without shifting) of the image sho... (Courtesy of NASA.)

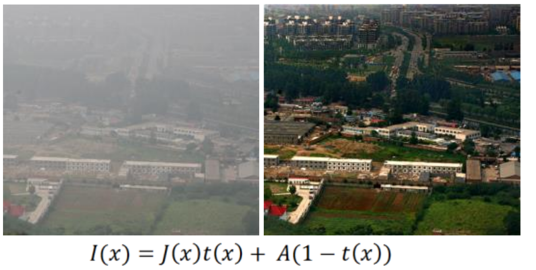
a b
FIGURE 5.22 (a) Fourier spectrum of $N(u, v)$, and (b) corresponding noise interference pattern $\eta(x, y)$. (Courtesy of NASA.)

- 三. 一个例子——“去雾”

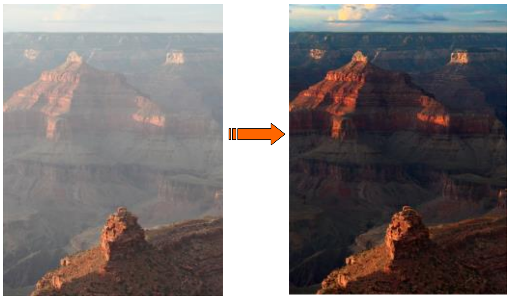
References:

K. He, J. Sun, and X. Tang. “Single image haze removal using dark channel prior”, *IEEE Conference on Computer Vision and Pattern Recognition, pp. 1957–1963, 2009*

– Haze Removal (CVPR2009 **Best Paper**)



$$I(x) = J(x)t(x) + A(1 - t(x))$$



Haze Model

Haze Formation Equation

$$I(x) = J(x)t(x) + A(1 - t(x))$$

$I(x)$ – Observed Intensity

$J(x)$ – Underlying Scene

$t(x)$ – Transmission through Medium (Haze)

$$t(x) = e^{-\beta d(x)}$$

Lambert-Beer Law (assume homogeneous medium)

β– Scattering Coefficient, $d(x)$ – Scene Depth

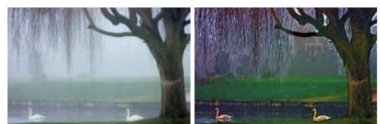
A – Global Airlight

Difficulties with Haze Removal

Under-constrained problem if input is only a single hazy image.

- Unknown depth information
- Some methods have been explored using multiple images or prior knowledge
  - Polarization-based (Schechner et al.)
  - Multiple images of same scene with different weather conditions (Narasimhan et al.)
  - User-input Depth Based (Kopf et al., Narsimhan et al.)

Maximize local contrast (Tan)



Estimate Albedo (Light Reflection Ratio) of Scene and infer Transmission (Fattal)

Dark Channel Prior

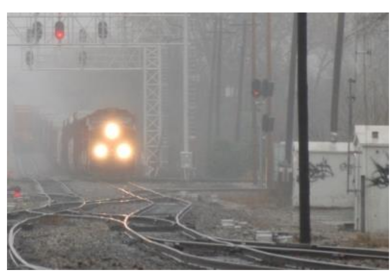
Observation: for haze-free outdoor images, non-sky region has pixels that are very dark

- These pixels can provide estimate of transmitted atmospheric light
- We can find these pixels in regions throughout the image

Dark Channel Defined:

$$J^{dark}(x) = \min_{c \in \{r,g,b\}} \left(\min_{y \in \Omega(x)} (J^c(y)) \right)$$

Dark Channel Example



Input Image

Dark Channel Example

Dark Channel

15x15 patch

Estimating Airlight

- Use information from Dark Channel
  - Find brightest 0.1% pixels from J^{dark}
    - From these locations, the highest intensity pixels in original image, *I*, are selected as the atmospheric light
  - These are most haze opaque regions
    - Depth should be very large (approach infinity)
    - Image content is mostly atmospheric light

$$I(x) = J(x)t(x) + A(1 - t(x))$$

Locations of an estimation

Patch Size: 15x15 $I(x) = J(x)t(x) + A(1 - t(x))$

Locations of an estimation

Patch Size: 31x31 $I(x) = J(x)t(x) + A(1 - t(x))$

Airlight Example: City

Patch Size: 15x15 $I(x) = J(x)t(x) + A(1 - t(x))$

Airlight Example: City

Patch Size: 31x31 $I(x) = J(x)t(x) + A(1 - t(x))$

Transmission Map Estimate

Estimated from normalized input image

$$\tilde{t}(x) = 1 - w \min_{c} \left(\min_{y\in\Omega(x)} \left(\frac{I_c(y)}{A^c} \right) \right)$$

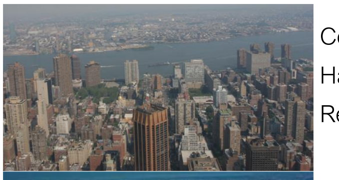
w – Preserve some haze (typ: 0.95)

A^c = Airlight of Color Channel, c

$$I(x) = J(x)t(x) + A(1 - t(x))$$

Coarse Haze Removal

Original Image



Coarsely De-hazed Image

$I(x) = J(x)t(x) + A(1 - t(x))$

Our Result

Authors Result

-Do some histogram processing or gamma correction after running algorithm.

-Unknown what method they use

Original Image

w = 0.95

Synthetic Haze Added

De-hazed Result