THU-DiscreteMathmatics-Homework-3

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Octorber 2024

1 Homework

1.1 Write the following formulas

- $P \wedge \neg P$

  1 **CNF**: $P \wedge \neg P$
  2 **DNF**: $P \wedge \neg P$
  3 **PCNF**: $P \wedge \neg P = \bigwedge_{0,1}$
  4 **PDNF**: Empty formula
  5 There is no P that let $P \wedge \neg P$ be True.

- $(P \leftrightarrow Q) \vee ((Q \wedge P) \leftrightarrow (Q \leftrightarrow \neg P))$

  1 **CNF**:

$$\begin{aligned}
&(P \leftrightarrow Q) \vee ((Q \wedge P) \leftrightarrow (Q \leftrightarrow \neg P)) \\
(\textbf{Equivalence Law}) &\Leftrightarrow ((P \to Q) \wedge (Q \to P)) \vee ((Q \wedge P) \leftrightarrow ((Q \to \neg P) \wedge (\neg P \to Q))) \\
(\textbf{Implication Equivalence}) &\Leftrightarrow ((\neg P \vee Q) \wedge (\neg Q \vee P)) \vee ((Q \wedge P) \leftrightarrow ((\neg Q \vee \neg P) \wedge (P \vee Q)))
\end{aligned}$$

We first caculate $(Q \wedge P) \to ((\neg Q \vee \neg P) \wedge (P \vee Q))$:

$$\begin{aligned}
&(Q \wedge P) \to ((\neg Q \vee \neg P) \wedge (P \vee Q)) \\
(\textbf{Implication Equivalence}) &\Leftrightarrow \neg(Q \wedge P) \vee ((\neg Q \vee \neg P) \wedge (P \vee Q)) \\
(\textbf{De Morgan Law}) &\Leftrightarrow (\neg P \vee \neg Q) \vee ((\neg Q \vee \neg P) \wedge (P \vee Q)) \\
(\textbf{Distributive}) &\Leftrightarrow (\neg P \vee \neg Q \vee \neg P \vee \neg Q) \wedge (\neg P \vee \neg Q \vee P \vee Q) \\
(\textbf{Complement, Idempotent}) &\Leftrightarrow (\neg P \vee \neg Q) \wedge \mathsf{T} \\
(\textbf{Identity}) &\Leftrightarrow \neg P \vee \neg Q
\end{aligned}$$

Then we caculate $((\neg Q \vee \neg P) \wedge (P \vee Q)) \to (Q \wedge P)$:

$$\begin{aligned}
&((\neg Q \vee \neg P) \wedge (P \vee Q)) \to (Q \wedge P) \\
(\textbf{Implication Equivalence}) &\Leftrightarrow \neg((\neg Q \vee \neg P) \wedge (P \vee Q)) \vee (Q \wedge P) \\
(\textbf{De Morgan Law}) &\Leftrightarrow ((P \wedge Q) \vee (\neg P \wedge \neg Q)) \vee (P \wedge Q) \\
(\textbf{Idempotent}) &\Leftrightarrow (P \wedge Q) \vee (\neg P \wedge \neg Q)
\end{aligned}$$

So we have

$$\begin{aligned}
&((\neg P \vee Q) \wedge (\neg Q \vee P)) \vee ((Q \wedge P) \leftrightarrow ((\neg Q \vee \neg P) \wedge (P \vee Q)) \\
(\textbf{Equivalence Law}) &\Leftrightarrow ((\neg P \vee Q) \wedge (\neg Q \vee P)) \vee ((\neg P \vee \neg Q) \wedge ((P \wedge Q) \vee (\neg P \wedge \neg Q))) \\
(\textbf{Distributive}) &\Leftrightarrow ((\neg P \vee Q) \wedge (\neg Q \vee P)) \vee (\neg P \wedge \neg Q) \\
(\textbf{Distributive}) &\Leftrightarrow (\neg P \vee Q) \wedge (\neg P \vee \neg Q)
\end{aligned}$$

2 **DNF**: $(\neg P \wedge Q) \vee (\neg P \wedge \neg Q)$

3 **PCNF**: $\bigwedge_{0,1}$

4 **PDNF**: $\bigvee_{0,1}$

5 When $(P, Q) = (\mathsf{F}, \mathsf{F})$ or $(P, Q) = (\mathsf{F}, \mathsf{T})$.

1.2 Prove

$P \to (Q \to R) \Rightarrow (P \to Q) \to (P \to R)$

1 $A \to B$ is a tautology:

$$\begin{aligned}
& (P \to (Q \to R)) \to ((P \to Q) \to (P \to R)) \\
(\textbf{Implication Equivalence}) \Leftrightarrow & (P \wedge Q \wedge \neg R) \vee (P \wedge \neg Q) \vee \neg P \vee R \\
(\textbf{Identity}) \Leftrightarrow & m_6 \vee (m_4 \vee m_5) \vee (m_0 \vee m_1 \vee m_2 \vee m_3) \vee (m_1 \vee m_3 \vee m_5 \vee m_7) \\
(\textbf{Idempotent}) \Leftrightarrow & \bigvee_{0,\cdots,7}
\end{aligned}$$

So it is a tautology.

2 $A \wedge \neg B$ is a contradicton:

$$\begin{aligned}
& (P \to (Q \to R)) \wedge \neg((P \to Q) \to (P \to R)) \\
(\textbf{Implication Equivalence}) \Leftrightarrow & (\neg P \vee (\neg Q \vee R)) \wedge \neg(\neg(\neg P \vee Q) \vee (\neg P \vee R)) \\
(\textbf{De Morgan Law}) \Leftrightarrow & (\neg P \vee \neg Q \vee R) \wedge ((\neg P \vee Q) \wedge (P \wedge \neg R)) \\
(\textbf{Associative}) \Leftrightarrow & (\neg P \vee \neg Q \vee R) \wedge (P \wedge Q \wedge \neg R) \\
= & \bigwedge_{0,\cdots,7}
\end{aligned}$$

So it is a contradicton.

3 Explain

When event P happens, then when event Q happens, the event R will also happen. So we know, when the event *event P happening leads to the happen of event* Q happens, the event R will also happen. And it's equivalent to $P \to R$.

1.3 Prove

$$\begin{aligned}
& \neg Q \vee S, (E \to \neg U) \to \neg S \\
(\textbf{Conjunction}) \Rightarrow & (\neg Q \vee S) \wedge ((E \to \neg U) \to \neg S) \\
(\textbf{Implication Equivalence}) \Leftrightarrow & (\neg Q \vee S) \wedge ((E \wedge U) \vee \neg S) \\
(\textbf{Distributive}) \Leftrightarrow & (\neg Q \vee S) \wedge ((E \vee \neg S) \wedge (U \vee \neg S)) \\
(\textbf{Conjunction}) \Rightarrow & \neg Q \vee S, E \vee \neg S, U \vee \neg S \\
(\textbf{Simplification}) \Rightarrow & \neg Q \vee S, E \vee \neg S \\
(\textbf{Resolution}) \Rightarrow & \neg Q \vee E \\
(\textbf{Implication Equivalence}) \Leftrightarrow & Q \to E
\end{aligned}$$

1.4 Prove

We use

- P to denote *the state subsidizes agricultural products.*
- Q to denote *the state exercises control over agricultural products.*

- R to denote *the shortage of agricultural products*.

We know $\neg P \rightarrow Q$, $Q \rightarrow \neg R$, and we need to prove $R \rightarrow P$:

$$
\begin{aligned}
& \neg P \rightarrow Q, Q \rightarrow \neg R \\
(\textbf{Hypothetical Syllogism}) \Rightarrow & \neg P \rightarrow \neg Q \\
(\textbf{Contrapositive}) \Leftrightarrow & R \rightarrow P
\end{aligned}
$$