THU-DiscreteMathmatics-Homework-2

He Yuhui 2022012050

September 2024

1 Homework

1.1 Transform to Polish Notation

1 $\rightarrow \vee PQ \vee RS$

2 $\leftrightarrow \wedge \neg PR \wedge PQ$

3 $\vee \vee \neg\neg P \wedge WR \neg Q$

4 $\wedge P \rightarrow Q \neg R$

1.2 Prove

1 We have

$$\begin{aligned}
& P \rightarrow Q \\
(\textbf{Implication Equivalence}) &\Leftrightarrow \neg P \vee Q \\
(\textbf{Commutative}) &\Leftrightarrow Q \vee \neg P \\
(\textbf{Double Negation}) &\Leftrightarrow \neg(\neg Q) \vee (\neg P) \\
(\textbf{Implication Equivalence}) &\Leftrightarrow \neg Q \rightarrow \neg P
\end{aligned}$$

2 We have

$$\begin{aligned}
& ((P \rightarrow \neg Q) \rightarrow (Q \rightarrow \neg P)) \wedge R \\
(\textbf{Implication Equivalence}) &\Leftrightarrow ((\neg P \vee \neg Q) \rightarrow (\neg Q \vee \neg P)) \wedge R \\
(\textbf{Implication Equivalence}) &\Leftrightarrow (\neg(\neg P \vee \neg Q) \vee (\neg Q \vee \neg P)) \wedge R \\
(\textbf{De Morgan Law}) &\Leftrightarrow ((P \wedge Q) \vee \neg(Q \wedge P)) \wedge R \\
(\textbf{Commutative}) &\Leftrightarrow ((P \wedge Q) \vee \neg(P \wedge Q)) \wedge R \\
(\textbf{Use } S \textbf{ to denote } P \wedge Q) &\Leftrightarrow (S \vee \neg S) \wedge R \\
(\textbf{Negation}) &\Leftrightarrow \mathsf{T} \wedge R \\
(\textbf{Absorption}) &\Leftrightarrow R
\end{aligned}$$

3 We have

$$\begin{aligned}
& (P \leftrightarrow Q) \leftrightarrow ((P \wedge \neg Q) \vee (Q \wedge \neg P)) \\
(\textbf{Equivalence Law}) &\Leftrightarrow ((P \rightarrow Q) \wedge (Q \rightarrow P)) \leftrightarrow ((P \wedge \neg Q) \vee (Q \wedge \neg P)) \\
(\textbf{Implication Equivalence}) &\Leftrightarrow ((\neg P \vee Q) \wedge (\neg Q \vee P)) \leftrightarrow ((P \wedge \neg Q) \vee (Q \wedge \neg P)) \\
(\textbf{De Morgan Law}) &\Leftrightarrow \neg((P \wedge \neg Q) \vee (Q \wedge \neg P)) \leftrightarrow ((P \wedge \neg Q) \vee (Q \wedge \neg P)) \\
(\textbf{Use } R \textbf{ to denote } (P \wedge \neg Q) \vee (Q \wedge \neg P)) &\Leftrightarrow \neg R \leftrightarrow R \\
(\textbf{Complement Law}) &\Leftrightarrow \mathsf{F} \\
(\textbf{Negation}) &\Leftrightarrow P \wedge \neg P
\end{aligned}$$

4 We have

$$
\begin{aligned}
& P \to (Q \to R) \\
(\textbf{Implication Equivalence}) \Leftrightarrow\ & P \to (\neg Q \vee R) \\
(\textbf{Implication Equivalence}) \Leftrightarrow\ & \neg P \vee (\neg Q \vee R) \\
(\textbf{Associative}) \Leftrightarrow\ & (\neg P \vee \neg Q) \vee R \\
(\textbf{De Morgan Law}) \Leftrightarrow\ & \neg(P \wedge Q) \vee R \\
(\textbf{Implication Equivalence}) \Leftrightarrow\ & (P \wedge Q) \to R
\end{aligned}
$$

1.3 Write the Statement of A and B

A We have

$$
\begin{aligned}
A &= (\neg P \wedge \neg Q) \\
&= \neg(P \vee Q)
\end{aligned}
$$

B We have

$$
\begin{aligned}
B &= \neg(P \wedge \neg Q) \\
&= \neg P \vee Q
\end{aligned}
$$

1.4 Using $\uparrow$ and $\downarrow$ to denote $\neg$, $\wedge$, $\vee$ and $\to$

We know that

$$A \uparrow B \Leftrightarrow \neg(A \wedge B), \quad A \downarrow B \Leftrightarrow \neg(A \vee B)$$

$\neg$ We have

$$
\begin{aligned}
\neg A &= \neg(A \wedge A) = A \uparrow A \\
\neg A &= \neg(A \vee A) = A \downarrow A
\end{aligned}
$$

$\wedge$ We have

$$
\begin{aligned}
A \wedge B &= \neg\neg(A \wedge B) \\
&= \neg(A \uparrow B) \\
&= (A \uparrow B) \uparrow (A \uparrow B) \\
A \wedge B &= \neg(\neg A \vee \neg B) \\
&= (\neg A) \downarrow (\neg B) \\
&= (A \downarrow A) \downarrow (B \downarrow B)
\end{aligned}
$$

$\vee$ We have

$$
\begin{aligned}
A \vee B &= \neg(\neg A \wedge \neg B) \\
&= (\neg A) \uparrow (\neg B) \\
&= (A \uparrow A) \uparrow (B \uparrow B) \\
A \vee B &= \neg\neg(A \vee B) \\
&= \neg(A \downarrow B) \\
&= (A \downarrow B) \downarrow (A \downarrow B)
\end{aligned}
$$

$\rightarrow$ We have

$$
\begin{aligned}
A \rightarrow B &= \neg A \vee B \\
&= \neg(A \wedge \neg B) \\
&= A \uparrow (\neg B) \\
&= A \uparrow (B \uparrow B) \\
A \rightarrow B &= \neg A \vee B \\
&= \neg\neg(\neg A \vee B) \\
&= \neg(\neg A \downarrow B) \\
&= \neg((A \downarrow A) \downarrow B) \\
&= ((A \downarrow A) \downarrow B) \downarrow ((A \downarrow A) \downarrow B)
\end{aligned}
$$