THU-DiscreteMathmatics-Homework-5

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1 Symbolize

1 Use P_1 and P_2 to denote the two points on the plane, predicate $\varphi(P, l)$ to denote point P on the line l, and predicate $\psi(l_1, l_2)$ to denote l_1 and l_2 is the same line:

$$(\exists l)((\varphi(P_1, l) \wedge \varphi(P_2, l)) \wedge (\forall l')((\varphi(P_1, l') \wedge \varphi(P_2, l')) \rightarrow \psi(l, l'))$$

2 Use predicate $\varphi(x)$ to denote $x \in \mathbb{R}$, $\alpha(x, y), \beta(x, y), \gamma(x, y)$ to denote $x < y, x = y, x > y$:

$$(\forall x)(\forall y)((\varphi(x) \wedge \varphi(y)) \rightarrow (\alpha(x, y) \vee \beta(x, y) \vee \gamma(x, y)))$$

3 Use predicate people $\varphi(x)$ to denote x is working in Beijing, predicate $\psi(x)$ to denote the household of x is Beijing:

$$(\exists x)(\varphi(x) \wedge \neg\psi(x))$$

4 Use predicate $\varphi(x)$ to denote x is a kind of metal, predicate $\psi(x)$ to denote x is a kind of liquid, and predicate $\epsilon(x, y)$ to denote x can be dissolved in y:

$$(\forall x)(\exists y)((\varphi(x) \wedge \psi(y)) \rightarrow \epsilon(x, y))$$

2 Translate to natural language

1 For all positive integer x, x is a rational number and it is also a real number.

2 For all positive integer x, x is a rational number, bot not all rational number is positive interger.

3 Translate to propositional logic formula

1

$$\begin{aligned}
&((P(a,a) \rightarrow Q(a,a)) \vee (P(a,b) \rightarrow Q(a,b)) \vee (P(a,c) \rightarrow Q(a,c))) \\
\wedge&((P(b,a) \rightarrow Q(b,a)) \vee (P(b,b) \rightarrow Q(b,b)) \vee (P(b,c) \rightarrow Q(b,c))) \\
\wedge&((P(c,a) \rightarrow Q(c,a)) \vee (P(c,b) \rightarrow Q(c,b)) \vee (P(c,c) \rightarrow Q(c,c)))
\end{aligned}$$

2

$$P(a,a) \vee P(a,b) \vee P(a,c) \vee P(b,a) \vee P(b,b) \vee P(b,c) \vee P(c,a) \vee P(c,b) \vee P(c,c)$$

3

$$\begin{aligned}
&((P(a,a) \vee P(b,a) \vee P(c,a)) \rightarrow (Q(a,a) \wedge Q(b,a) \wedge Q(c,a))) \\
\wedge&((P(a,b) \vee P(b,b) \vee P(c,b)) \rightarrow (Q(a,b) \wedge Q(b,b) \wedge Q(c,b))) \\
\wedge&((P(a,c) \vee P(b,c) \vee P(c,c)) \rightarrow (Q(a,c) \wedge Q(b,c) \wedge Q(c,c)))
\end{aligned}$$