

2024-2025-Autumn-DisMa-8

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1 Homework

1.1 List the element

$$A = \{\{0, -2\}, \{0, -1\}, \{0, 0\}, \{0, 1\}, \\ \{1, -2\}, \{1, -1\}, \{1, 0\}, \{1, 1\}, \\ \{2, -2\}, \{2, -1\}, \{2, 0\}, \{2, 1\}\}$$

1.2 Write the statement of the set

$$A = \{p \mid \forall q \in \mathbb{N}, 2 \leq q \leq p-1, p \bmod q \neq 0\}$$

1.3 Raise examples

Example for 1:

$$A = \emptyset, B = \{\emptyset\}, C = \{\{\emptyset\}\}$$

Example for 2:

$$A = \emptyset, B = \{\emptyset\}, C = \{\emptyset, \{\emptyset\}\}$$

1.4 Prove

Proof to 1: According to the definition of subset, we know for all elements x in the subset B of set C :

$$(x \in B) \wedge (B \subseteq C) \Rightarrow x \in C$$

and it is similar to the situation that element x is also a set.

2 is not correct. Consider

$$A = \{1, \{2\}\}, B = \{\{1, \{2\}\}, 3\}, C = \{\{1, \{2\}\}, 3, \{4\}\}$$

It satisfies $A \in B, B \subseteq C$, but obviously $A \subseteq C$ is false.

1.5 Write the result

The power set of 1:

$$\{\emptyset, \{a\}, \{\{a\}\}, \{a, \{a\}\}\}$$

The power set of 2:

$$\{\emptyset, \{a\}, \{\{b\}\}, \{\emptyset, a\}, \{\emptyset, \{b\}\}, \{a, \{b\}\}, \{\emptyset, a, \{b\}\}\}$$

The power set of 3: we know that $2^\emptyset = \{\emptyset\}$, so we have:

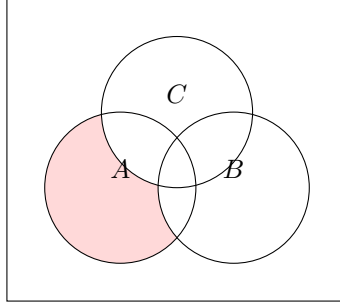
$$\mathcal{P}(\mathcal{P}(\emptyset)) = 2^{\{\emptyset\}} = \{\emptyset, \{\emptyset\}\}$$

so we have

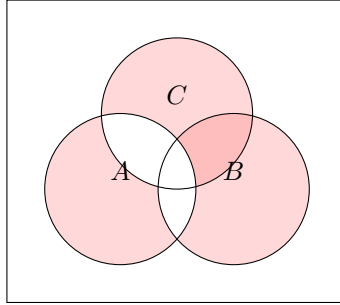
$$\mathcal{P}(\mathcal{P}(\emptyset)) \times \mathcal{P}(\mathcal{P}(\emptyset)) = \{(\emptyset, \emptyset), (\emptyset, \{\emptyset\}), (\{\emptyset\}, \emptyset), (\{\emptyset\}, \{\emptyset\})\}$$

1.6 Draw the Venn diagram

$$A \cap (-B \cup -C)$$



$$A \oplus (B \cup C)$$



1.7 Write the set

The set of Venn diagram 1 is:

$$-A \cap B \cap C$$

The set of Venn diagram 2 is

$$-(A \cup B \cup C) \cup (A \cap B \cap C)$$

1.8 Write the set

$$-(A \cap B) = \{2, 3, 4, 5\}$$

$$\begin{aligned} \mathcal{P}(A) - \mathcal{P}(B) &= \{\emptyset, \{1\}, \{4\}, \{1, 4\}\} - \{\emptyset, \{1\}, \{2\}, \{5\}, \{1, 2\}, \{1, 5\}, \{2, 5\}, \{1, 2, 5\}\} \\ &= \{\{4\}, \{1, 4\}\} \end{aligned}$$