

### Example 10.2.

After a batch fermentation, the system is dismantled and approximately 75% of the cell mass is suspended in the liquid phase (2 l), while 25% is attached to the reactor walls and internals in a thick film (ca. 0.3 cm). Work with radioactive tracers shows that 50% of the target product (intracellular) is associated with each cell fraction. The productivity of this reactor is 2 g product/l at the 2 l scale. What would be the productivity at 20,000 l scale if both reactors had a height-to-diameter ratio of 2 to 1?

**Solution** Both tanks are geometrically similar, so we can calculate the diameter and the resulting surface area and volume in both tanks.

$$V = \frac{1}{4} \pi D^2 \cdot H = \frac{1}{4} \pi D^2 \cdot 2D = \frac{1}{2} \pi D^3$$
$$S = \pi D \cdot H = \pi D \cdot 2D = 2\pi D^2$$

For the 2 l system

$$D = 10.8 \text{ cm}$$

$$S = 738 \text{ cm}^2$$

$$V = 2000 \text{ cm}^3$$

For the 20,000 l systems,

$$D = 233.5 \text{ cm}$$

$$S = 342,600 \text{ cm}^2$$

$$V = 2 \times 10^7 \text{ cm}^3$$

At the bench scale (2 l) the amount of product made by surface-attached cells is

$$2 \text{ g/l} \cdot 2 \text{ l} \cdot \frac{1}{2} = 2 \text{ g}, \quad \text{for } 738 \text{ cm}^2 \text{ of surface area}$$

The amount of product formed at 20,000 l due to surface-attached growth is

$$342,600 \text{ cm}^2 / 738 \text{ cm}^2 \times 2 \text{ g} = 928 \text{ g}$$

The overall yield in the 2 l system is

$$2 \text{ g/l} \cdot 2 \text{ l} = 4 \text{ g}$$

The overall yield in the 20,000 l system is

$$928 \text{ g} + (2 \text{ g/l} \cdot \frac{1}{2} \cdot 20,000 \text{ l}) = 20,928 \text{ g}$$

If no wall growth had been present, the 20,000 l tank would have yielded 40,000 g. Thus, wall growth, if present, can seriously alter the productivity of a large-scale system upon scale-up.

### Example 10.3.

Consider the scale-up of a fermentation from a 10 l to 10,000 l vessel. The small fermenter has a height-to-diameter ratio of 3. The impeller diameter is 30% of the tank diameter. Agitator speed is 500 rpm and three Rushton impellers are used. Determine the dimensions of the large fermenter and agitator speed for:

- Constant  $P/V$
- Constant impeller tip speed