$$\frac{dC_3}{dt} = 0 = H(C_2 - C_3) - k_1 C_3 \tag{c}$$

where C_i is the glucose concentration in compartment i and k_1 is the first-order reaction constant.

The solution to the preceding set of equations is

$$C_1 = \left(2 + \frac{k_1}{H} - \frac{H}{H + k_1} \right) \frac{F}{3k_1 + \left(k_1^2 / H \right)} \tag{d}$$

$$C_2 = \frac{F}{3k_1 + \left(k_1^2 / H \right)} \tag{e}$$

$$C_3 = \frac{H}{(H + k_1)} \frac{F}{\left(3k_1 + \left(k_1^2 / H \right) \right)} \tag{f}$$

If the probe is placed in the middle compartment, then $C_2 = 25$ mg/l if the control system is perfect. Using eq. e, we calculate $F = 3.9$ mg/l-s in the small tank and $F = 4.6$ mg/l-s in the large tank. Values for C_1 and C_3 are found from eqs. d, e, and f. The process is repeated for the second case where the probe is placed in the upper compartment.

The results are summarized as follows:

A. Probe in the middle compartment
   A1. Small tank (10 l); $F = 3.9$ mg/l-s

   $$C_1 = 30.5 \text{ mg/l}; C_2 = 25 \text{ mg/l}; C_3 = 22.4 \text{ mg/l}$$

   A2. Large tank (10,000 l); $F = 4.6$ mg/l-s

   $$C_1 = 51.7 \text{ mg/l}; C_2 = 25 \text{ mg/l}; C_3 = 15 \text{ mg/l}$$

B. Probe in the top compartment
   B1. Small tank (10 l); $F = 3.2$ mg/l-s

   $$C_1 = 25 \text{ mg/l}; C_2 = 20.5 \text{ mg/l}; C_3 = 18.4 \text{ mg/l}$$

   B2. Large tank (10,000 l); $F = 2.2$ mg/l-s

   $$C_1 = 25 \text{ mg/l}; C_2 = 12.1 \text{ mg/l}; C_3 = 7.3 \text{ mg/l}$$

These results illustrate the importance of the alteration of mixing time on nutrient consumption rates (F) and on the concentration gradients in a large vessel. It should also be clear that probe placement in a less than perfectly mixed vessel will have significant effects on reactions within that vessel.