$$\mathrm{Sh} = \frac{dk}{D_e} = a\,\mathrm{Re}^b\,\mathrm{Sc}^b \tag{11.65}$$

where $\mathrm{Re} = dv\rho/\mu$, $\mathrm{Sc} = \mu/\rho D_e$, and $\mathrm{Sh} = kd/D_e$, where Sh is the Sherwood number. The value of a is $\frac{1}{3}$ according to the boundary-layer theory; b is approximately 0.5 for laminar flow and 1.0 for turbulent flow. That is, $k \propto v^{0.5}$ for laminar and $k \propto v$ for turbulent flow.

A gel layer is formed on the surface of the UF membrane when slowly diffusing large macromolecules accumulate at the surface. A gel layer forms if the protein concentration in the solution exceeds 0.1%. For biological fluids, protein accumulation is usually dominant (Fig. 11.20). The protein concentration in the gel, C_G, is the maximum value of C_W. In this case, the liquid flux through the filter is given by

$$J = k \ln \frac{C_G}{C_B} \tag{11.66}$$

Gel formation depends on the nature and concentration of the solute, pH, and pressure. Once gel is formed, C_G becomes constant, and liquid flux decreases logarithmically with increasing solute concentration in the bulk liquid. The gel layer causes a hydraulic resistance against flow and acts somewhat like a second membrane. Figure 11.21 depicts the variation of liquid flux with the log of solute concentration in the absence and presence of gel formation.

11.4.8 Cross-flow Ultrafiltration and Microfiltration

Gel formation can be partially eliminated by *cross-flow filtration*, where pressure is not applied directly perpendicular to the membrane, but parallel to the membrane surface. This process is also called *tangential flow filtration.* That is, fluid flows parallel to the membrane surface and passes through the membrane, leaving solutes in a liquid phase above the membrane. Mechanical agitation or vibration of the membrane surface can also be used to alleviate gel formation.

A schematic of cross-flow filtration is shown in Fig. 11.22. The pressure drop driving fluid flow is

$$\Delta P = P_i - P_0 \tag{11.67}$$

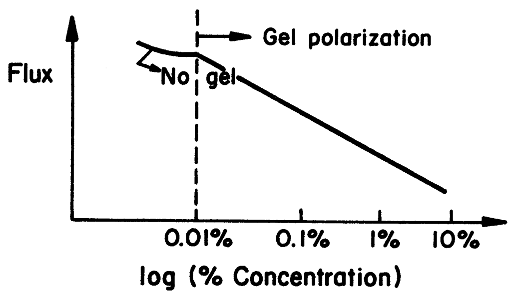

Figure 11.21. Variation of flux with solute concentration.