$$D = \mu_+ f_+ + \mu_-(1 - f_+) \tag{14.15}$$

Substituting eq. 14.15 into eq. 14.7 after eq. 14.7 is divided by N^t yields

$$df_+ / dt = \mu_+ f_+ - f_+[(\mu_+ f_+) + \mu_-(1 - f_+)] - Rf_+ \tag{14.16}$$

and, after rearrangement,

$$df_+ / dt = f_+^2\, \Delta\mu - f_+(\Delta\mu + R) \tag{14.17}$$

where

$$\Delta\mu = \mu_- - \mu_+ \tag{14.18}$$

Equation 14.17 is a Bernoulli's equation. It can be solved in terms of a dummy variable $v = 1/f^+$, assuming a constant $\Delta\mu$ and R. A constant $\Delta\mu$ and R would be achieved only if the copy number distribution of plasmid-containing cells remained constant during the experiment. Also, strictly speaking, the assumption of constant $\Delta\mu$ is inconsistent with constant N^t. This analysis is best applied to situations where $\Delta\mu$ is not extremely large. The solution is

$$\frac{1}{f_+} = \frac{1}{1 - f_-} = \frac{\Delta\mu}{\Delta\mu + R} + ce^{(\Delta\mu + R)t} \tag{14.19}$$

where c is the constant of integration. The initial condition is $f_- = f_{-0}$ at $t = 0$. Then c must be

$$c = \left(\left[\frac{1}{1 - f_{-0}}\right] - \frac{\Delta\mu}{\Delta\mu + R}\right) \tag{14.20}$$

Once c is evaluated, eq. 14.19 can be rearranged to yield

$$f_- = \frac{(f_{-0}\Delta\mu + R)e^{(\Delta\mu + R)t} - R(1 - f_{-0})}{(f_{-0}\Delta\mu + R)e^{(\Delta\mu + R)t} + \Delta\mu(1 - f_{-0})} \tag{14.21}$$

Since f_{-0} is usually $<< 1$, eq. 14.21 becomes

$$f_- = \frac{(f_{-0}\Delta\mu + R)e^{(\Delta\mu + R)t} - R}{(f_{-0}\Delta\mu + R)e^{(\Delta\mu + R)t} + \Delta\mu} \tag{14.22}$$

Further simplification is possible by considering three limiting cases:

1. $\Delta\mu >> R$ (growth-rate-dependent instability dominant)
2. $\Delta\mu \leq R$ (segregational instability dominant)
3. $\Delta\mu < 0$ and $|\Delta\mu| >> R$ (effective selective pressure)

These limiting cases yield:

Case 1: Where t is sufficiently small so that $\Delta\mu >> (f_{-0}\, \Delta\mu + R)e^{(\Delta\mu t)}$, then

$$f_- \approx (f_{-0} + R/\Delta\mu)e^{\Delta\mu t} \tag{14.23}$$