$$\theta_H = \frac{V}{F} = \frac{\theta_c[(1-\gamma)X_e + \gamma X_r]}{X} = \frac{(1-\gamma)X_e + \gamma X_r}{\mu_{\text{net}} X} \tag{16.37}$$

Substituting $S_r = S$ in eq. 16.31 results in

$$F(S_0 - S) = \frac{1}{Y_{X/S}^M} \mu_g XV \tag{16.38}$$

or

$$V = \frac{Y_{X/S}^M F(S_0 - S)}{\mu_g X} = \frac{Y_{X/S}^M \theta_c F(S_0 - S)}{X(1 + k_d \theta_c)} \tag{16.39}$$

Equation 16.39 is used to calculate the required volume of the sludge tank for a certain degree of BOD removal ($S_0 - S$).

The resulting reactor volume can be expressed in terms of the recycle ratio by substituting eq. 16.34 into eq. 16.37 to yield

$$V = F\theta_c \left(1 + \alpha - \alpha \frac{X_r}{X}\right) \tag{16.40}$$

The kinetic parameters of the active organisms in the sludge tank need to be known (i.e., μ_m, K_s, $Y_{X/S}^M$ and k_d), and other physical parameters (V, F, α, X_r, X, and S_0) need to be determined for the design (sizing, aeration requirement) of activated-sludge units. Typical hydraulic residence time for the aeration tank is 4 to 12 hours and typical sludge age is 3–10 days.

A summary of typical values for the kinetic parameters in biological waste treatment is presented in Table 16.2. Despite all simplifying assumptions, the pure-culture model seems to fit steady-state experimental data reasonably well, although it does not predict the dynamic performance very well.

Example 16.4.

An industrial waste with an inlet BOD_5 of 800 mg/l must be treated to reduce the exit BOD_5 level to ≤ 20 mg/l. The inlet flow rate is 400 m^3/h. Kinetic parameters have been estimated for waste as $\mu_m = 0.20\ h^{-1}$, $K_s = 50$ mg/l of BOD_5, $Y_{X/S}^M = 0.5$ mg MLVSS/mg BOD_5, and $k_d = 0.005\ h^{-1}$. A waste treatment unit of 3200 m^3 is available. Assume a recycle ratio of 0.40 and $X_e = 0$. If you operate at a value of $\theta_c = 120$ h, find S and determine if sufficient BOD_5 removal is attained in a well-mixed activated-sludge process to meet specifications. What will be X and the sludge production rate from this process?

Solution Equation 16.29 can be used to estimate S.

$$\mu_{\text{net}} = \frac{1}{\theta_c} = \frac{\mu_m S}{K_s + S} - k_d \tag{16.29}$$

$$S = \frac{K_s(1 + k_d \theta_c)}{\theta_c(\mu_m - k_d) - 1}$$

$$S = \frac{(50\ \text{mg/l})(1 + 0.005\ \text{h}^{-1} \cdot 120\ \text{h})}{120\ \text{h}\ (0.20\ \text{h}^{-1} - 0.005\ \text{h}^{-1}) - 1} = 3.57\ \text{mg/l} \tag{16.29a}$$