$$\text{Uncompetitive inhibition:}\quad \mu_g = \frac{\mu_m S}{\left(\dfrac{K_s}{\left(1+\dfrac{I}{K_I}\right)}+S\right)\left(1+\dfrac{I}{K_I}\right)} \tag{6.48}$$

In some cases, the presence of toxic compounds in the medium results in the inactivation of cells or death. The net specific rate expression in the presence of death has the following form:

$$\mu_g = \frac{\mu_m S}{K_s + S} - k'_d \tag{6.49}$$

where k'_d is the death-rate constant (h^{-1}).

6.3.2.3. The logistic equation. When plotted on arithmetic paper, the batch growth curve assumes a sigmoidal shape (see Fig. 6.3). This shape can be predicted by combining the Monod equation (6.30) with the growth equation (6.2) and an equation for the yield of cell mass based on substrate consumption. Combining eqs. 6.30 and 6.2a and assuming no endogenous metabolism yields

$$\frac{dX}{dt} = \frac{\mu_m S}{K_s + S} X \tag{6.50}$$

The relationship between microbial growth yield and substrate consumption is

$$X - X_0 = Y_{X/S}(S_0 - S) \tag{6.51}$$

where X_0 and S_0 are initial values and $Y_{X/S}$ is the cell mass yield based on the limiting nutrient. Substituting for S in eq. 6.50 yields the following rate expression:

$$\frac{dX}{dt} = \frac{\mu_m (Y_{X/S}S_0 + X_0 - X)}{(K_S Y_{X/S} + Y_{X/S}S_0 + X_0 - X)} X \tag{6.52}$$

The integrated form of the rate expression in this phase is

$$\frac{(K_S Y_{X/S} + S_0 Y_{X/S} + X_0)}{(Y_{X/S}S_0 + X_0)} \ln\left(\frac{X}{X_0}\right) - \frac{K_S Y_{X/S}}{(Y_{X/S}S_0 + X_0)} \ln\{(Y_{X/S}S_0 + X_0 - X)/Y_{X/S}S_0\} = \mu_m t \tag{6.53}$$

This equation describes the sigmoidal-shaped batch growth curve, and the value of X asymptotically reaches to the value of $Y_{X/S}S_0 + X_0$.

Equation 6.52 requires a predetermined knowledge of the maximum cell mass in a particular environment. This maximum cell mass we will denote as X_∞; it is identical to the ecological concept of *carrying capacity*. Equation 6.53 is implicit in its dependence on S.

Logistic equations are a set of equations that characterize growth in terms of carrying capacity. The usual approach is based on a formulation in which the specific growth rate is related to the amount of unused carrying capacity: