$$\frac{d\overline{X}_b}{d\overline{X}_p} = \frac{\mu_b'\overline{X}_b(1-\overline{X}_p)}{-k_d'\overline{X}_p(1-\overline{X}_b)} \tag{16.23}$$

Integration of eq. 16.23 yields

$$\mu_b' \ln \overline{X}_p - \mu_b'\overline{X}_p + k_d' \ln \overline{X}_b - k_d'\overline{X}_b = K \tag{16.24a}$$

or

$$\left(\frac{\overline{X}_p}{e^{\overline{X}_p}}\right)^{\mu_b'}\left(\frac{\overline{X}_b}{e^{\overline{X}_p}}\right)^{k_d'} = e^K \tag{16.24b}$$

where K is an integration constant that is a function of the initial population sizes.

The phase-plane analysis of the system can be made using eq. 16.24. Figure 16.3 describes the limit cycles (oscillatory trajectories) of prey–predator populations for different initial population levels.

The Lotka–Volterra model considers the exponential growth of prey species in the absence of predator and neglects the utilization of substrate by prey species according to Monod form. The Lotka–Volterra oscillations depend on initial conditions and change their amplitude and frequency in the presence of an external disturbance. These types of oscillation are called soft oscillations. The other model based on Monod rate expressions

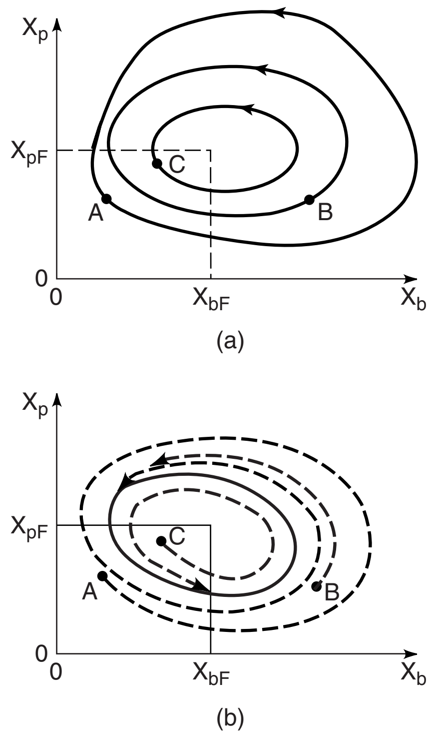

Figure 16.3. Both diagrams are schematics of phase-plane portraits for prey–predator interactions. (a) Limit cycles predicted by Lotka–Volterra (soft oscillations where initial conditions determine the dynamic behavior). (b) Limit-cycle prediction using the model developed in Example 16.3 (hard oscillations where the limit cycle is independent of initial conditions). The predicted steady-state point is defined by X_{pF} and X_{bF}. A, B, and C represent different initial conditions.