The major forces acting on a solid particle settling in a liquid by gravitational forces are gravitational force (F_G), drag force (F_D), and buoyant force (F_B). When the particles reach a terminal settling velocity, forces acting on a particle balance each other, resulting in a zero net force. That is,

$$F_G = F_D + F_B \tag{11.8}$$

where

$$F_G = \frac{\pi}{6} D_p^3 \rho_p \frac{g}{g_c} \tag{11.9}$$

$$F_B = \frac{\pi}{6} D_p^3 \rho_f \frac{g}{g_c} \tag{11.10}$$

and

$$F_D = \frac{C_D}{2g_c} \rho_f U_0^2 A \tag{11.11}$$

F_D is the drag force exerted by the fluid on solid particles, C_D is the drag coefficient, ρ_f is fluid density, U_0 is the relative velocity between the fluid and particle or the terminal velocity of a particle, and A is the cross-sectional area of the particles perpendicular to the direction of fluid flow; for a sphere, $A = (\pi/4)D_p^2$. For spherical particles, when $\mathrm{Re}_p < 0.3$, the drag force, F_D, is given by the Stokes equation:

$$F_D = 3\pi\mu D_p U_0 \frac{1}{g_c} \tag{11.12}$$

Substitution of eq. 11.12 into eq. 11.11 results in the following relationship between C_D and Re_p:

$$C_D = \frac{24}{\mathrm{Re}_p} \tag{11.13a}$$

When Re_p is between 1 and 10,000, C_D for a sphere is approximated by

$$C_D = \frac{24}{\mathrm{Re}_p} + \frac{3}{\sqrt{\mathrm{Re}_p}} + 0.34 \tag{11.13b}$$

where $\mathrm{Re}_p = D_p U_0 \rho_f / \mu$.

Substitution of eqs. 11.9, 11.10, and 11.12 into eq. 11.8 results in

$$3\pi\mu D_p U_0 = \frac{\pi}{6} D_p^3 (\rho_p - \rho_f) g \tag{11.14}$$

or

$$U_0 = \frac{g D_p^2 (\rho_p - \rho_f)}{18\mu} \tag{11.15}$$

where D_p and ρ_p are particle diameter and density, respectively.