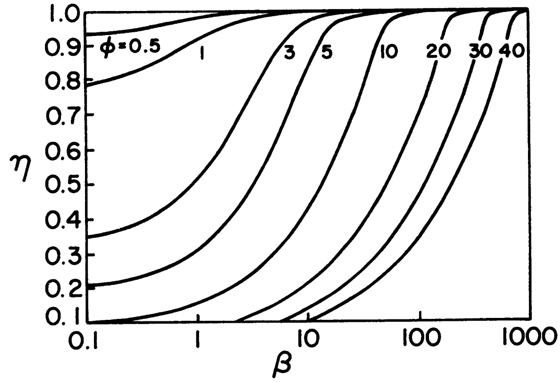

Figure 9.13. Effectiveness factor for a flat biofilm as a function of β, the dimensionless initial substrate concentration, and φ, the Thiele modulus. (With permission, redrawn from B. Atkinson, *Biochemical Reactors*, Pion Ltd., London, 1974, p. 81.)

($\phi < 1$) to eliminate diffusion limitations. As the biofilm grows (slowly), the value of ϕ will gradually increase. If shear forces cause a portion of the film to detach, then ϕ will decrease abruptly.

The effectiveness factor (η) can be calculated as

$$\eta = 1 - \frac{\tanh\phi}{\phi}\left(\frac{\omega}{\tanh\omega} - 1\right), \quad \text{for } \omega \le 1 \tag{9.55}$$

$$\eta = \frac{1}{\omega} - \frac{\tanh\phi}{\phi}\left(\frac{1}{\tanh\phi} - 1\right), \quad \text{for } \omega \ge 1 \tag{9.56}$$

where ω is the modified Thiele modulus and is given by

$$\omega = \frac{\phi(S_0/K_s)}{\sqrt{2}\left(1+\dfrac{S_0}{K_s}\right)}\left[\frac{S_0}{K_s} - \ln\left(1+\frac{S_0}{K_s}\right)\right]^{-1/2} \tag{9.57}$$

Some cells such as molds (*A. niger*) form pellets in a fermentation broth, and substrates need to diffuse inside pellets to be available for microbial consumption. Cells may form biofilms on spherical support particles, as depicted in Fig. 9.14. Similar equations need to be solved in spherical geometry in this case to determine the substrate profile within the floc and the substrate consumption rate. The dimensionless substrate transport equation within the microbial floc is

$$\frac{d^2\bar{S}}{d\bar{r}^2} + \frac{2}{\bar{r}}\frac{d\bar{S}}{d\bar{r}} = \frac{\phi^2\bar{S}}{1+\bar{S}/\beta'} \tag{9.58}$$

where

$$\bar{S} = \frac{S}{S_0}, \quad \bar{r} = \frac{r}{R}, \quad \beta' = \frac{S_0}{K_s}$$