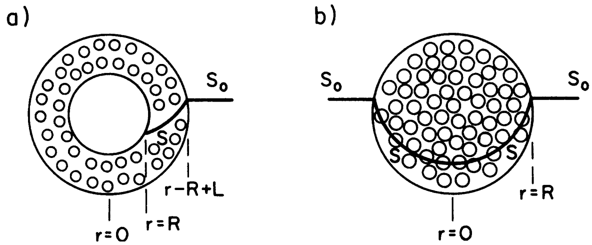

Figure 9.14. (a) Microbial film on inert spherical support particle. (b) Spherical microbial floc.

and

$$\phi = R\sqrt{\frac{\mu_m X}{Y_{X/S} D_e K_s}} = R\sqrt{\frac{r_m}{D_e K_s}} \tag{9.59}$$

The boundary conditions are

$$\bar{S} = 1, \qquad \text{at } \bar{r} = 1$$
$$\frac{d\bar{S}}{d\bar{r}} = 0, \qquad \text{at } \bar{r} = 0$$

For nonspherical particles, a characteristic length is defined as

$$L = \frac{V_P}{A_P} \tag{9.60}$$

where V_P and A_P are the volume and surface area of microbial pellet.

The rate of substrate consumption by a single microbial floc is

$$N_S A_P = -A_P D_e \left.\frac{dS}{dr}\right|_{r=R} = \eta \frac{r_m S_0}{K_s + S_0} V_P \tag{9.61}$$

The effectiveness factor (η) is a function of ϕ and β. Variation of η with ϕ and β is similar to that of Fig. 9.13. However, η values for spherical geometry are slightly lower than those of rectangular geometry for intermediate values of ϕ ($1 < \phi < 10$). An analytical solution to eq. 9.58 is possible for first- and zero-order reaction kinetics.

The reaction rate can be approximated to first order at low substrate concentrations.

$$r_s = \frac{\mu_m S}{Y_{X/S} K_s} X = \frac{r_m}{K_s} S \tag{9.62}$$

where $r_m = (\mu_m / Y_{X/S})X$. The effectiveness factor in this case is given by

$$\eta = \frac{1}{\phi}\left[\frac{1}{\tanh 3\phi} - \frac{1}{3\phi}\right] \tag{9.63}$$

where

$$\phi = \frac{V_P}{A_P}\sqrt{\frac{r_m / K_s}{D_e}}$$