

An energy balance for aerobic growth is

$$Q_0 c \gamma_b + Q_0 d \gamma_p = Q_0 \gamma_s - Q_0 4a \quad (7.13)$$

If  $Q_0$ , the heat evolved per equivalent of available electrons transferred to oxygen, is constant, eq. 7.13 is *not* independent of eq. 7.12. Recall that an observed regularity is 26.95 kcal/g equivalent of available electrons transferred to oxygen, which allows the prediction of heat evolution based on estimates of oxygen consumption.

Equations 7.12 and 7.13 also allow estimates of the fractional allocation of available electrons or energy for an organic substrate. Equation 7.12 can be rewritten as

$$1 = \frac{c \gamma_b}{\gamma_s} + \frac{d \gamma_p}{\gamma_s} + \frac{4a}{\gamma_s} \quad (7.14a)$$

$$1 = \xi_b + \xi_p + \varepsilon \quad (7.14b)$$

where  $\varepsilon$  is the fraction of available electrons in the organic substrate that is transferred to oxygen,  $\xi_b$  is the fraction of available electrons that is incorporated into biomass, and  $\xi_p$  is the fraction of available electrons that is incorporated into extracellular products.

### Example 7.1

Assume that experimental measurements for a certain organism have shown that cells can convert two-thirds (wt/wt) of the substrate carbon (alkane or glucose) to biomass.

- a. Calculate the stoichiometric coefficients for the following biological reactions:



- b. Calculate the yield coefficients  $Y_{X/S}$  (g dw cell/g substrate),  $Y_{X/O_2}$  (g dw cell/g  $\text{O}_2$ ) for both reactions. Comment on the differences.

#### Solution

- a. For hexadecane,

$$\text{amount of carbon in 1 mole of substrate} = 16(12) = 192 \text{ g}$$

$$\text{amount of carbon converted to biomass} = 192(2/3) = 128 \text{ g}$$

$$\text{Then, } 128 = c(4.4)(12); c = 2.42.$$

$$\text{amount of carbon converted to CO}_2 = 192 - 128 = 64 \text{ g}$$

$$64 = e(12), \quad e = 5.33$$

The nitrogen balance yields

$$14b = c(0.86)(14)$$

$$b = (2.42)(0.86) = 2.085$$

The hydrogen balance is

$$34(1) + 3b = 7.3c + 2d$$

$$d = 12.43$$