$$dS/dt = [S_0 - S]D - \frac{\mu_+ n_+}{Ys_+} - \frac{\mu_- n_-}{Ys_-} \tag{14.9}$$

where R is the rate of generation of plasmid-free cells from plasmid-containing cells and Ys_+ and Ys_- are cell-number yield coefficients (i.e., the number of cells formed per unit mass of limiting nutrient consumed). R can be represented by

$$R = P\mu_+ \tag{14.10}$$

where P = probability of forming a plasmid-free cell. P can be estimated by eq. 14.1 if the copy number is known or can be predicted with a more sophisticated structured–segregated model. A value for P could be estimated from an experimentally determined copy-number distribution as in Example 14.1c, which would be more realistic than assuming a monocopy number. As we will soon see, R can be determined experimentally without a knowledge of copy number.

Equations 14.7 through 14.9 assume only simple competition between plasmid-containing and plasmid-free cells. No selective agents are present, and the production of complementing factors from the plasmid is neglected. The simplest assumption for cellular kinetics is

$$\mu_+ = \mu_{+\max} \frac{S}{K_{S_+} + S} \tag{14.11a}$$

$$\mu_- = \mu_{-\max} \frac{S}{K_{S_-} + S} \tag{14.11b}$$

The situation can be simplified even more if we assume that after a few generations in a chemostat with constant operating conditions the total number of cells (N^t) is constant. This approximation will be acceptable in many cases as long as the metabolic burden imposed by plasmid-encoded functions is not too drastic and D is less than 80% of either $\mu_{+\max}$ or $\mu_{-\max}$. For allowable dilution rates, these assumptions allow us to decouple the substrate balance (eq. 14.9) from immediate consideration. If we then add eqs. 14.7 and 14.8, we have

$$dn_+/dt + dn_-/dt = \mu_+ n_+ + \mu_- n_- - D(n_+ + n_-) \tag{14.12}$$

Since N^t is constant and

$$N^t = n_+ + n_- \tag{14.13}$$

at quasi-steady-state, eq. 14.12 becomes

$$0 = \mu_+ n_+ + \mu_- n_- - D(N^t) \tag{14.14a}$$

or

$$0 = \mu_+ f_+ + \mu_- f_- - D \tag{14.14b}$$

where f_+ is the fraction of the total population that is plasmid-containing cells and f_- is the fraction of plasmid-free cells. Since $f_+ + f_- = 1$, then