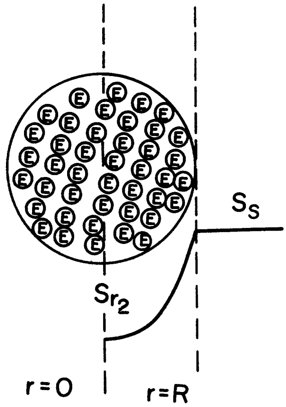

Figure 3.19. Substrate concentration profile in a porous support particle containing immobilized enzymes. Here it is assumed that no external substrate limitation exists so that the bulk and surface concentrations are the same.

$$D_e\left(\frac{d^2[\mathrm{S}]}{dr^2}+\frac{2}{r}\frac{d[\mathrm{S}]}{dr}\right)=\frac{V_m''\,[\mathrm{S}]}{K_m+[\mathrm{S}]} \tag{3.56}$$

with boundary conditions $[\mathrm{S}] = [\mathrm{S}_s]$ at $r = R$ and $d[\mathrm{S}]/dr = 0$ at $r = 0$, where V_m'' is the maximum reaction rate per unit volume of support, and D_e is the effective diffusivity of substrate within the porous matrix.

Equation 3.56 can be written in dimensionless form by defining the following dimensionless variables:

$$\bar{S}=\frac{[\mathrm{S}]}{[S_s]},\qquad \bar{r}=\frac{r}{R},\qquad \beta=\frac{K_m}{[\mathrm{S}_s]}$$
$$\frac{d^2\bar{S}}{d\bar{r}^2}+\frac{2}{\bar{r}}\frac{d\bar{S}}{d\bar{r}}=\frac{R^2V_m''}{S_sD_e}\left(\frac{\bar{S}}{\bar{S}+\beta}\right) \tag{3.57a}$$

or

$$\frac{d^2\bar{S}}{d\bar{r}^2}+\frac{2}{\bar{r}}\frac{d\bar{S}}{d\bar{r}}=\phi^2\,\frac{\bar{S}}{1+\bar{S}/\beta} \tag{3.57b}$$

where

$$\phi=R\sqrt{\frac{V_m''/K_m}{D_e}}=\text{Thiele modulus} \tag{3.57c}$$

With boundary conditions of $\bar{S} = 1$ at $\bar{r} = 1$ and $d\bar{S}/d\bar{r} = 0$ at $\bar{r} = 0$, eq. 3.57 can be numerically solved to determine the substrate profile inside the matrix. The rate of substrate consumption is equal to the rate of substrate transfer through the external surface of the support particle at steady state into the sphere.

$$r_s = N_s = 4\pi R^2 D_e\left.\frac{d[\mathrm{S}]}{dr}\right|_{r=R} \tag{3.58}$$