At steady state, $dS/dt = 0$ and

$$X_1 = \frac{D}{\mu_g} Y^M_{X/S}(S_0 - S \tag{9.11}$$

Substitution of eq. 9.9 when $k_d = 0$ into eq. 9.11 yields

$$X_1 = \frac{Y^M_{X/S}(S_0 - S)}{(1 + \alpha - \alpha C)} \tag{9.12}$$

Therefore, the steady-state cell concentration in a chemostat is increased by a factor of $1/(1 + \alpha - \alpha C)$ by cell recycle. The substrate concentration in the effluent is determined from eq. 9.9 and the Monod eq. 6.30, where endogenous metabolism is neglected, and is

$$S = \frac{K_s D(1 + \alpha - \alpha C)}{\mu_m - D(1 + \alpha - \alpha C)} \tag{9.13}$$

Then eq. 9.12 becomes

$$X_1 = \frac{Y^M_{X/S}}{(1 + \alpha - \alpha C)}\left[S_0 - \frac{K_s D(1 + \alpha - \alpha C)}{\mu_m - D(1 + \alpha - \alpha C)}\right] \tag{9.14}$$

Effluent cell concentrations and productivities in a chemostat with and without cell recycle are compared in Fig. 9.2. Cell concentrations and productivities are higher with cell recycle, resulting in higher rates of substrate consumption. Systems with cell recycle are used extensively in waste treatment and are finding increasing use in ethanol production. The application of cell recycle reactors in waste treatment is detailed in Chapter 16. The equations differ from the case above due to the inclusion of a term for endogenous metabolism (i.e., k_d). The basic concept of operation at flows above the "washout" rate applies when $k_d \neq 0$.

Example 9.1

In a chemostat with cell recycle, as shown in Fig. 9.1, the feed flow rate and culture volumes are $F = 100$ ml/h and $V = 1000$ ml, respectively. The system is operated under glucose limitation, and the yield coefficient, $Y^M_{X/S}$, is 0.5 gdw cells/g substrate. Glucose concentration in the

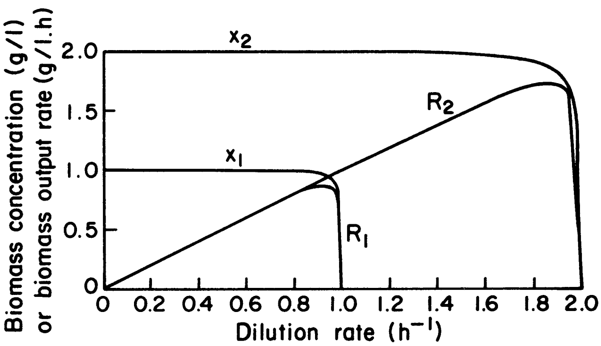

Figure 9.2. Comparison of biomass concentrations and output rates in steady states of chemostat cultures with and without recycle. Symbols: X_1 = biomass concentration in chemostat without recycle; X_2 = biomass concentration in chemostat culture with recycle; R_1 = biomass output rate per unit volume without recycle; R_2 = biomass output rate of chemostat with recycle; $\mu_m = 1.00$ h^{-1}; $S_r = 2.0$ g/l; $K_S = 0.010$ g/l; $Y_{X/S} =$ 0.5 g/g; concentration factor, $C = 2.0$; and recycle rate, $\alpha = 0.5$.