The rate of bioreaction can be approximated to zero order at values of $S >> K_S$. Because K_S is often very small, the zero-order limit usefully describes many systems of practical interest.

$$r_s = \frac{\mu_m X}{Y_{X/S}} = r_m \tag{9.64}$$

The solution to eq. 9.58 in this case is

$$S = S_0 - \frac{r_m}{6D_e}(R^2 - r^2) \tag{9.65}$$

Substrate concentration may be zero at a certain radial distance from the center of the floc according to eq. 9.65. This distance is called the critical radius (r_{cr}) and is determined by setting $S = 0$ at r_{cr} in eq. 9.65.

$$\left(\frac{r_{cr}}{R}\right)^2 = 1 - \frac{6D_e S_0}{r_m R^2} \tag{9.66}$$

When $r_{cr} > 0$—that is, $R > (6D_e S_0/r_m)^{1/2}$—then the concentration of the limiting substrate is zero for $0 < r < r_{cr}$. In this case, the limiting substrate is consumed only in the outer shell of the floc, and the effectiveness factor is given by

$$\eta = \frac{r_m \frac{4}{3}\pi(R^3 - r_{cr}^3)}{\frac{4}{3}\pi R^3 \cdot r_m} = 1 - \left(\frac{r_{cr}}{R}\right)^3 \tag{9.67}$$

or

$$\eta = 1 - \left(1 - \frac{6D_e S_0}{r_m R^2}\right)^{3/2} \tag{9.68}$$

9.4.5. Bioreactor Considerations in Immobilized Cell Systems

Various reactor configurations can be used for immobilized cell systems. Since the support matrices used for cell immobilization are often mechanically fragile, bioreactors with low hydrodynamic shear, such as packed-column, fluidized-bed, or airlift reactors, are preferred. Mechanically agitated fermenters can be used for some immobilized-cell systems if the support matrix is strong and durable. Any of these reactors can usually be operated in a perfusion mode by passing nutrient solution through a column of immobilized cells. Schematic diagrams of immobilized-cell packed-column and fluidized-bed reactors are depicted in Fig. 9.15. These reactors can be operated in batch or continuous mode.

Consider the reactors shown in Fig. 9.15. When the fluid recirculation rate is high, the system approaches CFSTR behavior. One commercial fluidized-bed, immobilized-animal-cell bioreactor system requires high recirculation to maintain uniform conditions in the reactor. The models we have discussed so far can be applied to such systems. The