Example 16.2.

Consider a case of mutualistic growth in a chemostat. How would you write the appropriate equations for this system? Determine the effects of the addition of competition to mutualism.

Solution The physical model is for A and B as two separate species. A produces P_A as a by-product of growth and B produces P_B. Organism B requires P_A to grow, while A requires P_B. The feed to a chemostat contains all essential nutrients except for P_A and P_B, and A and B may compete for substrate, S, in the feed.

For this case the most general description is

$$\frac{dX_A}{dt} = -DX_A + \mu_A X_A \tag{16.6}$$

$$\frac{dX_B}{dt} = -DX_B + \mu_B X_B \tag{16.7}$$

$$\frac{dP_A}{dt} = -DP_A + Y_{P_A}\mu_A X_A - \frac{1}{Y_{X_B/P_A}}\mu_B X_B \tag{16.8}$$

$$\frac{dP_B}{dt} = -DP_B + Y_{P_B}\mu_B X_B - \frac{1}{Y_{X_A/P_B}}\mu_A X_A \tag{16.9}$$

And if the growth of either A or B is limited by S, then we need to consider

$$\frac{dS}{dt} = D(S_0 - S) - \frac{1}{Y_{X_{A/S}}}\mu_A X_A - \frac{1}{Y_{X_{B/S}}}\mu_B X_B \tag{16.10}$$

Note that Y_{X_B/P_A} is the biomass yield of B using P_A as substrate, and Y_{P_A} is the amount of P_A made per unit mass of A. Similar definitions apply to Y_{X_A/P_B} and Y_{P_B}. If we consider the pure mutualistic state, then we ignore eq. 16.10. For a coexistent state to exist, $D = \mu_A = \mu_B$. It is also clear that the rate of production of P_A and P_B must exceed their consumption. Thus

$$Y_{P_A} X_A > \frac{X_B}{Y_{X_B/P_A}} \tag{16.11}$$

and

$$Y_{P_B} X_B > \frac{X_A}{Y_{X_A/P_B}} \tag{16.12}$$

It then follows that

$$Y_{P_A} Y_{P_B} X_A X_B > \frac{1}{Y_{X_B/P_A}} \cdot \frac{1}{Y_{X_A/P_B}} X_A X_B \tag{16.13a}$$

or

$$Y_{P_A} Y_{P_B} > \frac{1}{Y_{X_B/P_A}} \cdot \frac{1}{Y_{X_A/P_B}} \tag{16.13b}$$

It is also clear that

$$D < \min(\mu_{mA}, \mu_{mB}) \tag{16.14}$$