

The first term in eq. 11.43 is the convective transfer of solute in the bed, the second term represents the time course of change (transient) of the solute concentration in liquid, and the last term represents the rate of solute transfer from the liquid to the solid phase.

The rate of solute transfer from the liquid into the solid phase is

$$\frac{dC_S}{dt} = K_a(C_L - C_L^*) \quad (11.44)$$

where  $K_a$  is the overall mass transfer coefficient describing the internal and external mass transfer resistance, and  $C_L^*$  is the concentration of solute in the liquid phase, which is in equilibrium with  $C_S$ .

Substituting eq. 11.44 into eq. 11.43 yields

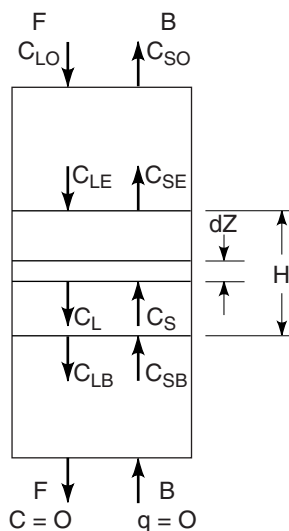
$$U \frac{\partial C_L}{\partial Z} + \varepsilon \frac{\partial C_L}{\partial t} = -(1 - \varepsilon) K_a (C_L - C_L^*) \quad (11.45)$$

Equation 11.45, along with the equilibrium relationship (eq. 11.42), can be solved numerically to determine the solute profile,  $C_L = C_L(Z, t)$ , in the column.

The preceding approach is based on plug flow conditions in the bed. However, in practice, due to back-mixing and dispersion, irregular flow profiles and channeling occur. A dispersion term  $[-D_E(\partial^2 C_L / \partial Z^2)]$  is added to the right side of eq. 11.45 to account for dispersion effects. A fixed-bed adsorption unit can be approximated to a large number of well-mixed tanks in series.

For very long *moving-bed columns* to which adsorbent is added continuously, the operation may be considered to be at steady state (see Fig. 11.15). For the analysis, equilibrium is assumed to occur at the top of the column. If the dispersion effects are negligible, then at steady state eq. 11.45 can be written as

$$U \frac{dC_L}{dZ} = -K_a(1 - \varepsilon)(C_L - C_L^*) \quad (11.46)$$



**Figure 11.15.** Schematic diagram of steady-state adsorption column. (With permission, from D. W. Sundstrom and H. E. Klei, *Wastewater Treatment*, Pearson Education, Upper Saddle River, NJ, 1979, p. 260.)