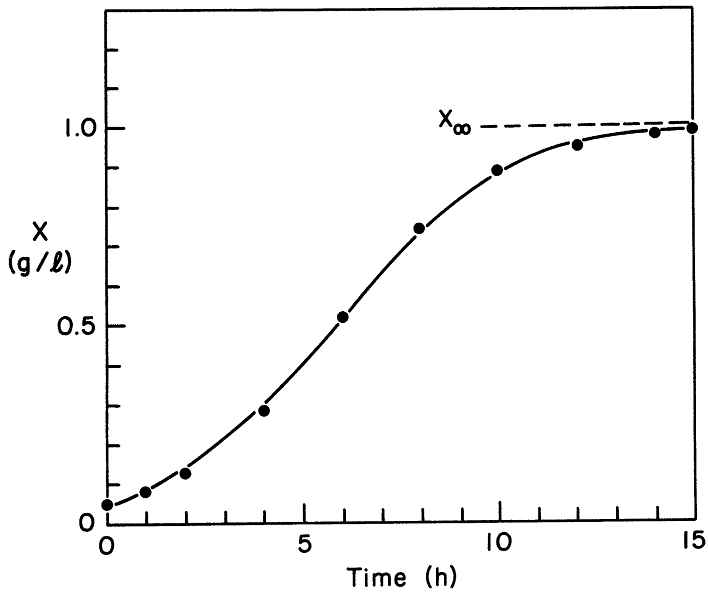

Figure 6.12. Logistic growth curve.

Δt (h)	$\bar{X}$(g/L)	$\frac{1}{\bar{X}}\Delta X / \Delta t$ (h^{-1})	$\left(1-\frac{\bar{X}}{X_\infty}\right)$	k (h^{-1})
2	0.75	0.333	0.931	0.36
3	1.55	0.236	0.856	0.28
5	3.45	0.156	0.681	0.23
5	6.25	0.093	0.416	0.22
5	8.65	0.044	0.200	0.22
5	10.00	0.016	0.074	0.22
5	10.55	0.0057	0.023	0.25

A value of $k = 0.24$ h^{-1} would describe most of the data, although it would slightly underestimate the initial growth rate. Another approach would be to take the log of the above equation to give:

$$\log\frac{1}{X}\frac{dX}{dt} = \log k + \log\left(1-\frac{\bar{X}_0}{X_\infty}\right)$$

and to fit the data to this equation and estimate k from the intercept. In this case k would be about 0.25 h^{-1}.

b) The yields are estimated directly from the data as:

$$Y_{P/S} = \frac{-\Delta P}{\Delta S} = \frac{-(49-0)}{(2-100)} = 0.5\frac{gP}{gS}$$
$$Y_{X/S} = \frac{-\Delta X}{\Delta S} = \frac{-(10.7-0.5)}{(2-100)} = 0.104\frac{gX}{gS}$$

The above estimate of $Y_{X/S}$ is only approximate, as maintenance effects and endogenous metabolism have been neglected.