



**Figure 9.8.** Schematic of a fed-batch culture.

The rate of change in biomass concentration is

$$\frac{dX}{dt} = \frac{V(dX'/dt) - X'(dV/dt)}{V^2} \quad (9.31)$$

Since  $dX'/dt = \mu_{\text{net}}X'$ ,  $dV/dt = F$ , and  $F/V = D$ , eq. 9.31 becomes

$$\frac{dX}{dt} = (\mu_{\text{net}} - D)X \quad (9.32)$$

When the substrate is totally consumed,  $S \approx 0$  and  $X = X_m = Y_{X/S}^M S_0$ . Furthermore, since nearly all the substrate in a unit volume is consumed, then  $dX/dt = 0$ . This is an example of a quasi-steady state. A fed-batch system operates at quasi-steady state when nutrient consumption rate is nearly equal to nutrient feed rate. Since  $dX/dt = 0$  at quasi-steady state, then

$$\mu_{\text{net}} = D \quad (9.33)$$

If maintenance energy can be neglected,

$$\mu_{\text{net}} = \mu_m \frac{S}{K_s + S}$$

then

$$S \equiv \frac{K_s D}{\mu_m - D} \quad (9.34)$$