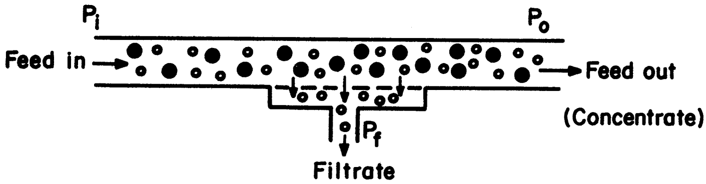

Figure 11.22. Schematic of cross-flow filtration.

For laminar flow, using the Hagen–Poiseuille equation, we can derive the following:

$$P_i - P_0 = \Delta P = \frac{C_1 \mu L V}{d^2} = \frac{C_2 \mu L Q}{d^4} \tag{11.68}$$

where L is the length of the tube, μ is fluid viscosity, Q is the volumetric flow rate of the liquid, and d is diameter of the tube.

For turbulent flow, the following equation is used for ΔP:

$$P_i - P_0 = \Delta P = \frac{C_3 \mu L V^2}{d} = \frac{C_4 f L Q^2}{d^5} \tag{11.69}$$

where f is the Fanning friction factor, which is a function of Re. For cross-flow filtration, turbulent flow is desired.

The average transmembrane pressure drop is

$$\Delta P_M = \frac{P_i + P_0}{2} - P_f \tag{11.70}$$

where P_f is the filtrate pressure, which is usually near atmospheric pressure.

Assuming that $P_f = P_{\text{atm}}$ or P_f is zero gauge pressure, we can relate ΔP_M to ΔP.

$$\Delta P_M = P_i - \tfrac{1}{2}\Delta P \tag{11.71}$$

High inlet pressure and low fluid velocities need to be used to obtain high ΔP_M.

The filtration flux (J), as a function of transmembrane pressure drop, is given by

$$J = \frac{\Delta P_M}{R_G + R_M} \tag{11.72}$$

where R_G and R_M are gel and membrane resistances, respectively. R_M is constant and R_G varies with the solute concentration and the tangential velocity across the membrane, which can retard or eliminate gel formation. Also, the filtration flux (J) is a function of fluid velocity, as described by eqs. 11.71 and 11.72. Usually, there is an optimal fluid velocity range maximizing the filtration rate. At low velocities, the mass transfer coefficient (k) is low, resulting in high gel resistance (R_G) and low filtrate flux. At high fluid velocities, ΔP is high, resulting in low ΔP_M and therefore low J values. Also, there is an optimal value of ΔP_M resulting in maximum flux. This is because of a maximum-pressure limitation on P_i. That is, the maximum value of ΔP_M is P_i, and P_i is limited by the physical properties of the membrane. With modern membranes, especially the new ceramic mem-