For low substrate concentrations in the feed, the rate of substrate consumption is first order and eq. 9.71 has the following form:

$$\ln \frac{S_0}{S_{0i}} = -\frac{\eta r_m LaA}{FK_s} H \tag{9.72}$$

Substrate concentration drops exponentially with the height of the column in this case, and a plot of ln S_0 versus H results in a straight line. Equation 9.71 or 9.72 can be used as the design equation for immobilized–biofilm column reactors to determine the height of the column for a desired level of substrate conversion.

Example 9.4

Glucose is converted to ethanol by immobilized *S. cerevisiae* cells entrapped in Ca-alginate beads in a packed column. The specific rate of ethanol production is $q_P = 0.2$ g ethanol/g cell · h, and the average dry-weight cell concentration in the bed is $\overline{X} = 25$ g/l bed. Assume that growth is negligible (i.e., almost all glucose is converted to ethanol) and the bead size is sufficiently small that $\eta \cong 1$. The feed flow rate is $F = 400$ l/h, and glucose concentration in the feed is $S_{0i} = 100$ g glucose/l. The diameter of the column is 1 m, and the product yield coefficient is $Y_{P/S} \approx 0.49$ g ethanol/g glucose.

a. Write a material balance on the glucose concentration over a differential height of the column and integrate it to determine $S = S(z)$ at steady state.
b. Determine the column height for 98% glucose conversion at the exit of the column.
c. Determine the ethanol concentration in the effluent.

Solution

a. A material balance on the glucose concentration over a differential height of the column yields

$$-F\, dS_0 = \frac{q_P \overline{X}}{Y_{P/S}} dV = \frac{q_P \overline{X}}{Y_{P/S}} A\, dz$$

Integration yields

$$-F \int_{S_{0i}}^{S_n} dS_0 = \frac{q_P \overline{X}}{Y_{P/S}} A \int_0^H dz$$

$$S_{0i} - S_0 = \frac{q_P \overline{X}}{Y_{P/S}} \frac{A}{F} H$$

This equation differs from the form of eq. 9.72 because S_{0i} is high and the reaction rate is effectively zero order.

b. $S_0 = 0.02(100) = 2$ g glucose/l. Substituting the given values into the above equation yields

$$(100 - 2) = \frac{(0.2)(25)}{0.49} \frac{(\pi/4)(10)^2}{400} H$$

$$H = 49 \text{ dm} = 4.9 \text{ m}$$

c. $P = Y_{P/S}\,(S_{0i} - S_0) = 0.49(98) = 48$ g/l.