



**Figure 9.15.** Schematics of a packed-bed and a fluidized-bed biofilm or immobilized-cell bioreactors are shown. In batch operation, only the streams with solid lines exist. In continuous operation, the streams shown by dashed lines are added. For the fluidized bed, fluidization can be accomplished by liquid recirculation only or a mixture of liquid and gas flows.

other extreme involves some waste-treatment systems where the rate of fluid recirculation is low or even zero. In the latter case, the system cannot be modeled as a CFSTR but must be treated as a PFR. To analyze such a system, consider a material balance on the rate-limiting substrate over a differential element:

$$-F dS_0 = N_s a A dz \quad (9.69)$$

where  $S_0$  is the bulk liquid-phase substrate concentration ( $\text{mg S/cm}^3$ ) and is a function of height,  $F$  is the liquid nutrient flow rate ( $\text{cm}^3/\text{h}$ ),  $N_s$  is flux of substrate into the biofilm ( $\text{mg S/cm}^2 \text{ h}$ ),  $a$  is the biofilm or support particle surface area per unit reactor volume ( $\text{cm}^2/\text{cm}^3$ ),  $A$  is the cross-sectional area of the bed ( $\text{cm}^2$ ), and  $dz$  is the differential height of an element of the column ( $\text{cm}$ ). Substituting eq. 9.54 into eq. 9.69 yields the following equation:

$$-F \frac{dS_0}{dz} = \eta \frac{r_m S_0}{K_s + S_0} L a A \quad (9.70)$$

Integration of eq. 9.70 yields

$$K_s \ln \frac{S_{0i}}{S_0} + (S_{0i} - S_0) = \frac{\eta r_m L a A}{F} H \quad (9.71)$$

where  $S_{0i}$  is the inlet bulk substrate concentration,  $L$  is the biofilm thickness or the characteristic length of the support particle ( $L = V_p/A_p$ ), and  $H$  is the total height of the packed bed.