6.3.2.4. Growth models for filamentous organisms. Filamentous organisms such as molds often form microbial pellets at high cell densities in suspension culture. Cells growing inside pellets may be subject to diffusion limitations. The growth models of molds should include the simultaneous diffusion and consumption of nutrients within the pellet at large pellet sizes. This problem is the same one we face in modeling the behavior of bacteria or yeasts entrapped in spherical gel particles (see Chapter 9).

Alternatively, filamentous cells can grow on the surface of a moist solid. Such growth is usually a complicated process, involving not only growth kinetics but the diffusion of nutrients and toxic metabolic by-products. However, for an isolated colony growing on a rich medium, we can ignore some of these complications.

In the absence of mass-transfer limitations, it has been observed that the radius of a microbial pellet in a submerged culture or of a mold colony growing on an agar surface increases linearly with time.

$$\frac{dR}{dt} = k_P = \text{constant} \tag{6.57}$$

In terms of growth rate of a mold colony, eq. 6.57 can be expressed as

$$\frac{dM}{dt} = \rho 4\pi R^2 \frac{dR}{dt} = k_P 4\pi R^2 \rho \tag{6.58a}$$

or

$$\frac{dM}{dt} = \gamma M^{2/3} \tag{6.58b}$$

where $\gamma = k_P(36\pi\rho)^{1/3}$. Integration of eq. 6.58b yields

$$M = \left(M_0^{1/3} + \frac{\gamma t}{3} \right)^3 \approx \left(\frac{\gamma t}{3} \right)^3 \tag{6.59}$$

The initial biomass, M_0, is usually very small compared to M, and therefore M varies with t^3. This behavior has been supported by experimental data.

6.3.3. Models for Transient Behavior

In most practical applications of microbial cultures, the environmental or culture conditions can shift, dramatically leading to changes in cellular composition and biosynthetic capabilities. These cellular changes are not instantaneous but occur over an observable period of time. In this section we examine models that can describe or predict such time-dependent (or transient) changes.

6.3.3.1. Models with time delays. The unstructured growth models we have described so far are limited to balanced or pseudobalanced growth conditions. These unstructured models can be improved for use in dynamic situations through the addition of time delays. The use of time delays incorporates structure implicitly. It is built on the premise that the dynamic response of a cell is dominated by an internal process with a time delay on the order of the response time under observation. Other internal processes