c. Constant Reynolds number
Assume geometric similarity and refer to Table 10.2

Solution Assume the vessel is a cylinder.

Thus,

$$V = (\pi/4)\, D_t^2 H.$$

but $H = 3\, D_t$, so

$$V = (\pi/4)\; 3\, D_t^3 = 10{,}000 \text{ cm}^3$$

Solving for D_t, H, and $D_i = 0.3\, D_t$ gives:

$$D_t = 16.2 \text{ cm}$$

$$H = 48.6 \text{ cm}$$

$$D_i = 4.9 \text{ cm}$$

The scale-up factor is the cube root of the ratio of tank volumes, or

$$(10{,}000 \text{ l} / 10 \text{ l})^{1/3} = 10$$

To maintain geometric simularity the larger vessel will have its dimensions increased by a factor of 10. That is:

$$D_t = 1.62 \text{ m}$$

$$H = 4.86 \text{ m}$$

$$D_i = 0.49 \text{ m}$$

a. For constant P/V to apply $N^3 D_t^2$ must be the same in both vessels. Let subscript 1 refer to the small vessel and 2 to the large vessel.

$$N_2 = N_1 \left(\frac{D_{t1}}{D_{t2}} \right)^{2/3} = 500 \text{ rpm } (1/10)^{2/3} = 107 \text{ rpm}$$

b. For constant tip speed to apply ND_t must be the same in both vessels.

$$N_2 = N_1 \left(\frac{D_{t1}}{D_{t2}} \right) = 500 \text{ rpm } (1/10) = 50 \text{ rpm}$$

c. For constant Re to apply ND_t^2 must be the same in both vessels.

$$N_2 = N_1 \left(\frac{D_{t1}}{D_{t2}} \right)^2 = 500 \text{ rpm } (1/10)^2 = 5 \text{ rpm}$$

Scale-up on the basis of constant P/V would be the most likely choice unless the culture was unusually shear sensitive. Scale-up based on constant Re would not be used.

Example 10.4.†

As fermenters are scaled up, the mixing time usually increases. Mixing time, t_M^{95}, can be defined as the time it takes for the concentration of a compound to return to 95% of the equilib-

†Adapted from J. Jost, Chapter 3 in S. L. Sandler and B. A. Finlayson, eds., *Chemical Engineering Education in a Changing Environment,* Engineering Foundation, New York, 1988.