$$S = \frac{K_s(D + k_d)}{\mu_m - D - k_d} \tag{6.78}$$

Equation 6.81 can be solved for X:

$$X = Y_{X/S}^{M}(S_0 - S)\left(\frac{D}{D + k_d + q_P \dfrac{Y_{X/S}^{M}}{Y_{P/S}}}\right) \tag{6.82}$$

The productivity of a chemostat for product and biomass (Pr_x) can be found from DP and DX, respectively. The dilution rate that maximizes productivity is found by differentiating DP or DX with respect to D and setting the derivative equal to zero. The optimal value of D (D_{opt}) will depend on whether endogenous metabolism and/or product formation are considered. When $k_d = 0$ and $q_P = 0$, eqs. 6.71 and 6.68 apply. Then D_{opt} for biomass production (DX) becomes

$$D_{\text{opt}} = \mu_m\left(1 - \sqrt{\frac{K_s}{K_s + S_0}}\right) \tag{6.83}$$

Since S_0 is usually much greater than K_s, D_{opt} will approach $D = \mu_m$ or the washout point. Stable chemostat operation with $D \approx \mu_m$ is very difficult, unless the flow rate and liquid volume can be maintained exactly constant. Consequently, a value of D slightly less than D_{opt} may be a good compromise between stability and biomass productivity. It should also be apparent that D_{opt} for biomass formation will not necessarily be optimal for product formation.

Examples 6.4 and 6.5 illustrate the use of these equations to characterize the performance of chemostats.

Example 6.4.

A new strain of yeast is being considered for biomass production. The following data were obtained using a chemostat. An influent substrate concentration of 800 mg/l and an excess of oxygen were used at a pH of 5.5 and $T = 35°C$. Using the following data, calculate μ_m, K_s, $Y_{X/S}^{M}$, k_d, and m_s, assuming $\mu_{\text{net}} = \mu_m S/(K_s + S) - k_d$.

Dilution rate (h^{-1})	Carbon substrate concentration (mg/l)	Cell concentration (mg/l)
0.1	16.7	366
0.2	33.5	407
0.3	59.4	408
0.4	101	404
0.5	169	371
0.6	298	299
0.7	702	59

Solution The first step is to plot $1/Y_{X/S}^{AP}$ versus $1/D$. $Y_{X/S}^{AP}$ is calculated from $X/(S_0 - S)$. The intercept is $1/Y_{X/S}^{M} = 1.58$ or $Y_{X/S}^{M} = 0.633$ g X/g S. The slope is 0.06 g S/g X-h, which is the value for m_s. Recall $m_s = k_d/Y_{X/S}^{M}$; then $k_d = m_s Y_{X/S}^{M} = 0.06$ g S/g X-h $\cdot$ 0.633 g X/g $S = 0.038\ h^{-1}$.