Since growth rate is limited by at least one substrate in a chemostat, a simple description of chemostat performance can be made by substituting the Monod equation (eq. 6.30) for μ_g in eq. 6.66:

$$\mu_g = D = \frac{\mu_m S}{K_s + S} \tag{6.67}$$

where S is the steady-state limiting substrate concentration (g/l). If D is set at a value greater than μ_m, the culture cannot reproduce quickly enough to maintain itself and is *washed out*. Equation 6.67 is identical to that for Michaelis–Menten kinetics and, as we discussed in Chapter 3, a plot of $1/\mu_g$ versus $1/S$ can be used to estimate values for μ_m and K_s.

Using eq. 6.67, we can relate effluent substrate concentration to dilution rate for $D < \mu_m$.

$$S = \frac{K_s D}{\mu_m - D} \tag{6.68}$$

A material balance on the limiting substrate in the absence of endogenous metabolism yields

$$FS_0 - FS - V_R \mu_g X \frac{1}{Y_{X/S}^M} - V_R q_P X \frac{1}{Y_{P/S}} = V_R \frac{dS}{dt} \tag{6.69}$$

where S_0 and S are feed and effluent substrate concentrations (g/l), q_P is the specific rate of extracellular product formation (g P/μ_g cells h), and $Y_{X/S}^M$ and $Y_{P/S}$ are yield coefficients (g cell/g S and g P/g S). The use of the superscript M on $Y_{X/S}$ denotes a maximum value of the yield coefficient; such a superscript will be important in discussing the effects of maintenance energy.

When extracellular product formation is negligible and the system is at steady state ($dS/dt = 0$),

$$D(S_0 - S) = \frac{\mu_g X}{Y_{Y/S}^M} \tag{6.70}$$

Since $\mu_g = D$ at steady state if $k_d = 0$,

$$X = Y_{X/S}^M (S_0 - S) \tag{6.71}$$

Using eq. 6.68, the steady-state cell concentration can be expressed as

$$X = Y_{Y/S}^M \left(S_0 - \frac{K_s D}{\mu_m - D} \right) \tag{6.72}$$

In eqs. 6.71 and 6.72, the yield coefficient ($Y_{X/S}$) is assumed to be constant, which is an approximation, since endogenous metabolism has been neglected. Usually, $Y_{X/S}$ varies with the limiting nutrient and growth rate. Consider the effect the inclusion of endogenous metabolism will have; eq. 6.66 becomes

$$D = \mu_g - k_d = \mu_{net} \tag{6.73a}$$

or

$$\mu_g = D + k_d \tag{6.73b}$$