The substrate concentration resulting in the maximum reaction rate can be determined by setting $dv/d[S] = 0$. The $[S]_{max}$ is given by

$$[S]_{max} = \sqrt{K'_m K_{S_1}} \tag{3.39}$$

Example 3.2

The following data have been obtained for two different initial enzyme concentrations for an enzyme-catalyzed reaction.

$v([E_0] = 0.015$ g/l) (g/l-min)	[S] (g/l)	$v([E_0] = 0.00875$ g/l) (g/l-min)
1.14	20.0	0.67
0.87	10.0	0.51
0.70	6.7	0.41
0.59	5.0	0.34
0.50	4.0	0.29
0.44	3.3	
0.39	2.9	
0.35	2.5	

a. Find K_m.
b. Find V_m for $[E_0] = 0.015$ g/l.
c. Find V_m for $[E_0] = 0.00875$ g/l.
d. Find k_2.

Solution A Hanes–Woolf plot (Fig. 3.12) can be used to determine V_m and K_m.

$$\frac{[S]}{v} = \frac{K_m}{V_m} + \frac{1}{V_m}[S]$$

$[S]/v$ ($E_0 = 0.015$) (min)	$[S]/v$ ($E_0 = 0.00875$) (min)	[S] (g/l)
17.5	30	20.0
11.5	20	10.0
9.6	16	6.7
8.5	15	5.0
8.0	14	4.0
7.6		3.3
7.3		2.9
7.1		2.5

From a plot of $[S]/v$ versus [S] for $E_0 = 0.015$ g/l, the slope is found to be 0.6 min/g/l and $V_m = 1/0.6 = 1.7$ g/l min. The y-axis intercept is $K_m/V_m = 5.5$ min and $K_m = 9.2$ g [S]/l.

Also, $V_m = k_2 E_0$ and $k_2 = 1.7/0.015 = 110$ g/g enzyme-min. The Hanes–Woolf plot for $E_0 = 0.00875$ g/l gives a slope of 1.0 min/g/l and $V_m = 1.0$ g/l-min; $k_2 = V_m/E_0 = 1.0/0.00875 = 114$ g/g enzyme-min.