This simple premise is rarely, if ever, true; however, the Monod equation empirically fits a wide range of data satisfactorily and is the most commonly applied unstructured, nonsegregated model of microbial growth.

The Monod equation describes substrate-limited growth only when growth is slow and population density is low. Under these circumstances, environmental conditions can be related simply to S. If the consumption of a carbon–energy substrate is rapid, then the release of toxic waste products is more likely (due to energy-spilling reactions). At high population levels, the buildup of toxic metabolic by-products becomes more important. The following rate expressions have been proposed for rapidly growing dense cultures:

$$\mu_g = \frac{\mu_m S}{K_{s0} S_0 + S} \tag{6.31}$$

or

$$\mu_g = \frac{\mu_m S}{K_{s1} + K_{s0} S_0 + S} \tag{6.32}$$

where S_0 is the initial concentration of the substrate and K_{s0} is dimensionless.

Other equations have been proposed to describe the substrate-limited growth phase. Depending on the shape of μ–S curve, one of these equations may be more plausible than the others. The following equations are alternatives to the Monod equation:

$$\text{Blackman equation:} \quad \mu_g = \mu_m, \qquad \text{iff } S \geq 2K_s$$
$$\mu_g = \frac{\mu_m}{2K_s} S, \quad \text{iff } S < 2K_s \tag{6.33}$$

$$\text{Tessier equation:} \quad \mu_g = \mu_m (1 - e^{-KS}) \tag{6.34}$$

$$\text{Moser equation:} \quad \mu_g = \frac{\mu_m S^n}{K_s + S^n} = \mu_m (1 + K_s S^{-n})^{-1} \tag{6.35}$$

$$\text{Contois equation:} \quad \mu_g = \frac{\mu_m S}{K_{sx} X + S} \tag{6.36}$$

Although the Blackman equation often fits the data better than the Monod equation, the discontinuity in the Blackman equation is troublesome in many applications. The Tessier equation has two constants (μ_m, K), and the Moser equation has three constants (μ_m, K_s, n). The Moser equation is the most general form of these equations, and it is equivalent to the Monod equation when $n = 1$. The Contois equation has a saturation constant proportional to cell concentration that describes substrate-limited growth at high cell densities. According to this equation, the specific growth rate decreases with decreasing substrate concentrations and eventually becomes inversely proportional to the cell concentration in the medium (i.e., $\mu_g \propto X^{-1}$).

These equations can be described by a single differential equation as

$$\frac{d\upsilon}{dS} = K \upsilon^a (1 - \upsilon)^b \tag{6.37}$$