



**Figure 9.4.** Data for Example 9.2. Data are for the production of a secondary product in batch culture.

**Solution** The first step in using either graphical approach is to differentiate the data in the batch growth curve to yield  $dX/dt$  and  $dP/dt$ . The differentiation of experimental data can magnify errors present in the original data, so the values of  $dX/dt$  and  $dP/dt$  must be interpreted cautiously.

For the graphical approach illustrated in Fig. 9.5, we have plotted  $1/(dX/dt)$  versus  $X$  and  $1/(dP/dt)$  versus  $P$ , which corresponds to eqs. 9.24b and 9.26b. For  $\theta_1 = 7$  h (that is,  $700 \text{ 1/100 1/h}$ ), we must determine what value of  $X_1$  will satisfy

$$\theta_1 = (X_1 - X_0) \left( \frac{1}{dX/dt} \right) \Big|_{X_1}$$

or

$$7 \text{ h} = X_1 \left( \frac{1}{dX/dt} \right) \Big|_{X_1}$$

Since a sterile feed is to be used,  $X_0 = 0$ .

By trial and error, we find on the graph that  $X_1 = 7.2 \text{ g/l}$  corresponds to  $1/(dX_1/dt) = 0.95 \text{ h/g/l}$  or  $7.2 \text{ g/l} \cdot 0.95 \text{ h/g/l} = 6.84 \text{ h}$ .

Given the accuracy with which Fig. 9.5 can be read, this is an acceptable solution. The product concentration that corresponds to  $X_1 = 7.2 \text{ g/l}$  is determined from the batch growth curve. As illustrated,  $X_1 = 7.2 \text{ g/l}$  is achieved at 9.4 h after inoculation; at the same time, the value for  $P_1$  is 0.08 g/l.

The effect of the second stage on the process is determined by using eq. 9.26b and noting that again  $\theta_2 = 7 \text{ h}$ . Thus,