Solution Equation 11.47 can be written as

$$-\frac{U}{K_a(1-\varepsilon)}\int_{C_L}^{C_{L0}}\frac{dC_L}{C_L-C_L^*}=H$$

Using the equilibrium relationship, we obtain

$$C_L^*=\frac{1}{625}C_S^2$$

and by using the operating-line relationship, which is *F/B* = 10, we note that $C_S = 10C_L$.

$$C_L^*=\frac{1}{625}100C_L^2=\frac{1}{6.25}C_L^2=0.16C_L^2$$

Substituting this equation into eq. 11.47 yields

$$-\frac{U}{K_a(1-\varepsilon)}\int_{C_L}^{C_{L0}}\frac{dC_L}{C_L-0.16C_L^2}=H$$

Integration with the numerical values yields

$$-\frac{1.5\text{ m/h}}{(15\text{ h}^{-1})(0.8)}\int_{0.2}^{5}\frac{dC_L}{C_L-0.16C_L^2}=H$$

$H = 0.6$ m.

11.4.5. Dialysis

Dialysis is a membrane separation operation used for the removal of low-MW solutes such as organic acids (100 < MW < 500) and inorganic ions (10 < MW < 100) from a solution. A well-known example is the use of dialysis membranes to remove urea (MW = 60) from urine in artificial kidney (dialysis) devices. In biotechnology dialysis can be used to remove salts from a protein solution, which is often a step in resolubilizing proteins that were initially in inclusion bodies. A schematic of a dialysis membrane is depicted in Fig. 11.18. Note that the membrane is selective. The dialysis membrane separates two phases containing low-MW and high-MW solutions. Since the MW cutoff of a dialysis membrane is very small, low-MW solutes move from a high- to a low-concentration region. At equilibrium, the chemical potentials of diffusing compounds on both sides of a membrane are equal.

$$\mu_1^\alpha=\mu_1^\beta \tag{11.51}$$

where μ is the chemical potential of the diffusing compound. In terms of concentrations, eq. 11.51 can be written as

$$RT\ln C_1^\alpha\ \gamma_1^\alpha=RT\ln C_1^\beta\ \gamma_1^\beta \tag{11.52}$$

or

$$C_1^\alpha\ \gamma_1^\alpha=C_1^\beta\ \gamma_1^\beta \tag{11.53}$$