

The actual reaction mechanisms implied in these reactions are likely very complex. Most reactions in a cell are saturable and subject to feedback control. For these reactions, the intracellular level of S is the main control variable. Thus, a reasonable set of equations based on eq. 6.60 for constant V_R is

$$\frac{dS^*}{dt} = X \frac{k_1 S^*}{K_{XS^*} + S^*} \quad (d)$$

where k_1 is rate of uptake per unit cell mass (g S transported/g X -h) and K_{XS^*} is the saturation parameter for uptake. Since S^* is not a cellular component, extrinsic concentrations can be used in the rate expression. For S we write

$$\begin{aligned} \frac{dS}{dt} = X \left\{ \underbrace{\frac{k_1 S^*}{K_{XS^*} + S^*}}_{\text{rate of uptake}} - \alpha_1 \underbrace{\frac{k_2 S/X}{K_{AS} + S/X}}_{\text{rate of } S \text{ used to form } A} (A/X) \right. \\ \left. - \alpha_2 \left[\underbrace{\frac{k_3 S/X}{K_{PSf} + S/X}}_{\text{net rate of } S \text{ consumption to form } P} (A/X) - \frac{k_4 K_{PSd}}{K_{PSd} + S/X} (P/X) \right] \right\} \quad (e) \end{aligned}$$

where k_2 is g A formed/g $\cdot A$ present-h, k_3 is g P formed/g $\cdot A$ present-h, and k_4 is a first-order degradation rate for P (h^{-1}). The parameters α_1 and α_2 are defined in eqs. (b) and (c), while K_{AS} , K_{PSf} , and K_{PSd} are saturation parameters in units of mass fraction. Similarly,

$$\frac{dA}{dt} = X \left[\underbrace{\frac{k_2 S/X}{K_{AS} + S/X}}_{\text{rate of formation of } A} (A/X) - \underbrace{k_5 (A/X)}_{\text{decomposition of } A \text{ to provide maintenance energy}} \right] \quad (f)$$

where k_5 is a first-order coefficient (h^{-1}) and

$$\frac{dP}{dt} = X \left[\frac{k_3 S/X}{K_{PSf} + S/X} (A/X) - \frac{k_4 K_{PSd}}{K_{PSd} + S/X} (P/X) \right] \quad (g)$$

Equations f and g describe the rate of formation of A and P .

The value of k_1 could be determined by the uptake of a nonmetabolizable analog of the substrate, use of vesicles, or material balance information on exponentially growing cells. Values of k_2 and k_3 can be estimated from measurements on total biomass and polymer under conditions where S/X should be large (e.g., exponential growth), while k_4 and k_5 can be estimated from time-dependent measurements of cell composition in the stationary phase, where S/X should be low. If experimental values of S/X are available at various times, then more refined estimates of k_2 , k_3 , k_4 , and k_5 can be made, as well as estimates of the saturation parameters. The rate form chosen in eq. (g) requires that $K_{PSd} \ll K_{PSf}$, so that P is made only when S/X is in excess and degraded only when S/X is low. The values for α_1 and α_2 are essentially