the filter medium, and r_c is the resistance of the cake. The value of r_m is characteristic of the filter medium. However, the cake resistance, r_c, increases during filtration, and after a start-up period, r_c exceeds r_m. The value of r_c is given by

$$r_c = \alpha \frac{W}{A} = \alpha \frac{CV}{A} \tag{11.2}$$

where W is the total weight of the cake on filter, C is the weight of the cake deposited per unit volume of filtrate, and α is the average specific resistance of the cake.

The total weight of cake is related to the total volume of filtrate by

$$W = CV \tag{11.3}$$

Substituting eqs. 11.2 and 11.3 into eq. 11.1 with constant A yields

$$\frac{d(V/A)}{dt} = \frac{g_c \Delta p}{\left(r_m + \alpha \dfrac{CV}{A} \right) \mu} \tag{11.4}$$

Integration of eq. 11.4 from $V = 0$ to $V = V$ and $t = 0$ to $t = t$ yields

$$V^2 + 2VV_0 = Kt \tag{11.5}$$

where

$$V_0 = \frac{r_m}{\alpha C} A \quad \text{and} \quad K = \left(\frac{2A^2}{\alpha C \mu} \right) \Delta p \cdot g_c$$

Equation 11.5 is known as the Ruth equation for constant-pressure filtration and can be rearranged to give

$$\frac{t}{V} = \frac{1}{K}(V + 2V_0) \tag{11.6}$$

A plot of t/V versus V yields a straight line with a slope of $1/K$ and intercept of $2V_0/K$, as depicted in Fig. 11.4. The values for r_m and α are calculated from experimentally determined values of K and V_0.

In a rotating drum filter (Fig. 11.3), the drum rotates at a constant speed (n rps) and only a fraction of drum-surface area is immersed in suspension reservoir (φ). The period of time during which filtration is carried out is φ/n per revolution of the drum. Equation 11.5 can be rewritten in this case as

$$\left(\frac{V'}{n} \right)^2 + 2\frac{V'}{n} V_0 = K \frac{\varphi}{n} \tag{11.7}$$

where V' = filtrate volume per unit time (volume/time) and V'/n represents the volume of filtrate filtered for one revolution of the drum.

This analysis of filtration is based on several assumptions; the primary one is an *incompressible cake* which results in constant specific cake resistance. Usually, fermentation cakes are compressible, so α varies with Δp. The concentration of filter aid (1% to 5%) also has a significant effect on specific cake resistance. As depicted in Fig. 11.5, the