Case 2: Where $f_{-0}\ \Delta\mu$ will be $<< R$ and using a binomial expansion,

$$f_{-} \approx 1-(1+\Delta\mu/R)e^{-(\Delta\mu+R)t} + \Delta\mu/R(1+\Delta\mu/R)e^{-2(\Delta\mu+R)t} \tag{14.24}$$

Case 3: The denominator is $\approx \Delta\mu$, since $|\Delta\mu| >> R$ and $f_{-0} << 1$, and we denote $\Delta\mu = -|\Delta\mu|$:

$$f_{-} \approx \frac{R}{|\Delta\mu|} + \left(f_{-0} - \frac{R}{|\Delta\mu|}\right)e^{(R-|\Delta\mu|)t} \tag{14.25}$$

For each of these cases, a straight-line portion of the data will allow estimates of $\Delta\mu$ and R if the basic assumptions for each case are met.

Figure 14.6 shows the behavior that would be expected for each case in a chemostat. The appropriate plots to estimate the parameters $\Delta\mu$ and R are also given (see Fig. 14.6).

In case 1 for $\Delta\mu t > 1$,

$$\ln f_{-} = \ln(f_{-0} + R/\Delta\mu) + \Delta\mu t \tag{14.26}$$

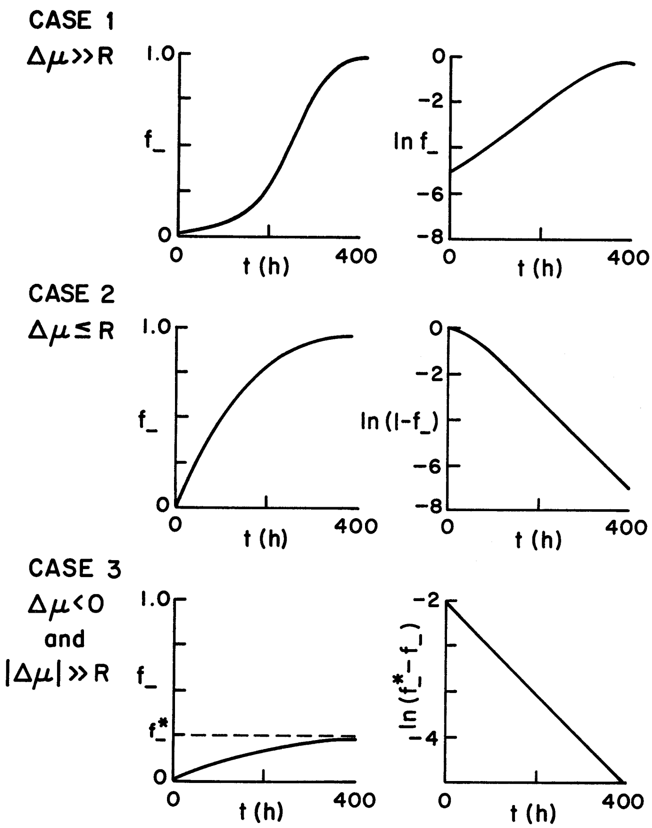

Figure 14.6. The shape of a plot of f_{-} versus t is diagnostic for limiting cases of eq. 14.21. For each case, a plot with a substantial linear region can be constructed and used to evaluate $\Delta\mu$ and R.