In terms of product concentration, eq. 9.43 can be written as

$$P = P_0 \frac{V_0}{V} + q_P X_m \left(\frac{V_0}{V} + \frac{Dt}{2} \right) t \tag{9.44}$$

Figure 9.9 depicts the variation of V, μ (= D), X, S, and P with time at quasi-steady state in a single cycle of a fed-batch culture.

In some fed-batch operations, part of the culture volume is removed at certain intervals, since the reactor volume is limited. This operation is called the *repeated fed-batch culture*. The culture volume and dilution rate (= μ_{net}) undergo cyclical variations in this operation.

If the cycle time t_w is constant and the system is always at quasi-steady state, then the product concentration at the end of each cycle is given by

$$P_w = \gamma P_0 + q_P X_m \left(\gamma + \frac{D_w t_w}{2} \right) t_w \tag{9.45}$$

where $D_w = F/V_w$, V_w is the culture volume at the end of each cycle, V_0 is the residual culture volume after removal, γ is the fraction of culture volume remaining at each cycle (= V_0/V_w), and t_w is the cycle time.

The cycle time is defined as

$$t_w = \frac{V_w - V_0}{F} = \frac{V_w - \gamma V_w}{F} = \frac{1-\gamma}{D_w} \tag{9.46}$$

Substitution of eq. 9.46 into eq. 9.45 yields

$$P_w = \gamma P_0 + \frac{q_P X_m}{2D_w}(1-\gamma^2) \tag{9.47}$$

An example of fed-batch culture is its use in some antibiotic fermentations, where a glucose solution is intermittently added to the fermentation broth due to the repression of pathways for the production of secondary metabolites caused by high initial glucose concentrations. The fed-batch method can be applied to other secondary metabolite fermenta-

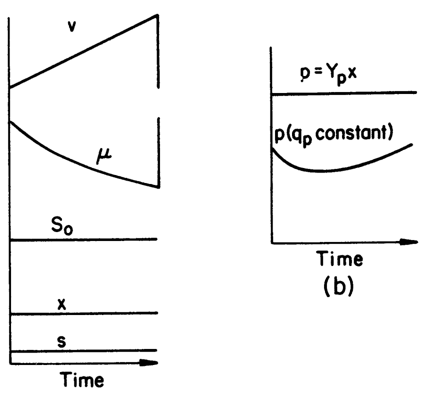

Figure 9.9. (a) Variation of culture volume (V), specific growth rate (μ), cell (X), and substrate (S) concentration with time at quasi-steady state. (b) Variation of product (P) concentration with time at quasi-steady state in a single cycle of a fed-batch culture.