or

$$\theta_n = \frac{1}{D_n} = \frac{X_n - X_{n-1}}{r_{x,n}(X_n, S_n)} \tag{9.24b}$$

$$r_{s,n} = \frac{1}{Y_{X/S}} r_{x,n}(X_n, S_n) = D_n(S_n - S_{n-1}) \tag{9.25a}$$

or

$$\theta_n = \frac{1}{D_n} = \frac{S_n - S_{n-1}}{r_{s,n}} \tag{9.25b}$$

$$r_{p,n}(X_n, S_n, \ldots) = D_n(P_n - P_{n-1}) \tag{9.26a}$$

or

$$\theta_n = \frac{1}{D_n} = \frac{P_n - P_{n-1}}{r_{p,n}(X_n, S_R, \ldots)} \tag{9.26b}$$

where $D_n = F/V_n$, θ_n is the mean residence time in the nth stage, and $r_{x,n}$, $r_{s,n}$, and $r_{p,n}$ all represent rates of reaction in the nth stage.

The preceding set of equations lends itself to machine calculations. However, graphical approaches to multistage design can also be used and have the advantage that the functional form of the growth or production rate need not be known. All that is required is a batch growth curve. However, the transfer of the information from batch growth curve to predictions of the multistage system still requires the assumption of balanced growth. Hence, the analysis must be used with caution. In at least one case (the production of spores from *Bacillus*), this approach has made experimentally verifiable predictions of the performance of a six-stage system.

The graphical approaches make use of eqs. 9.24 to 9.26. One approach is to use a plot of $1/(dX/dt)$ versus X or $1/(dP/dt)$ versus P derived from batch growth curves. This corresponds to using eqs. 9.24b and 9.26b. The size of the required reactor is determined by the area of the rectangle described with sides $X_n - X_{n-1}$ and $1/(dX/dt)$ or $P_n - P_{n-1}$ and $1/(dP/dt)$. The area of the rectangle is θ, and if F is known, V can be calculated. An alternative approach avoids some trial-and-error solutions that are necessary with the first approach. This second approach requires plots of dX/dt versus X and dP/dt versus P. The intersection of the reaction curve with a line from the mass balance equation (e.g., eq. 9.24a) determines the exit concentration of X or P, while the slope of the line determines D, and if F is known, V can be found. We illustrate the use of these approaches in Example 9.2.

Example 9.2

Data for the production of a secondary metabolite from a small-scale batch reactor are shown in Fig. 9.4. Assume that two reactors, each with 700-l working volume, are available. You will use exactly the same culture conditions (medium, pH, temperature, and so on) as in the batch reactor. If the flow rate is 100 l/h, predict the outlet concentration of the product. Compare that to the value predicted if a single 1400-l reactor were used. Use both graphical approaches.