

By substituting eq. 6.73b into the steady-state substrate balance, assuming no extracellular product formation (eq. 6.70), we find

$$D(S_0 - S) - 1/Y_{X/S}^M (D + k_d)X = 0 \quad (6.73c)$$

where $Y_{X/S}^M$ denotes the maximum yield coefficient (no endogenous metabolism or maintenance energy). $Y_{X/S}^M$ has a single constant value independent of growth rate. This approach does not allow the direct conversion of S into maintenance energy. Rather, S must be first incorporated into the cell mass, where it is degraded by endogenous metabolism. This viewpoint is the proper one when working with an unstructured model. With a structured model, which explicitly recognizes intracellular S as a subcomponent of the biomass, the direct consumption of S for maintenance functions could be modeled. However, with unstructured models, substrate that is no longer extracellular becomes part of the biomass.

Equation 6.73c can be rearranged to

$$D\left(\frac{S_0 - S}{X}\right) - 1/Y_{X/S}^M (D + k_d) = 0 \quad (6.74)$$

or

$$D\left(\frac{1}{Y_{X/S}^{AP}}\right) - \frac{D}{Y_{X/S}^M} - \frac{k_d}{Y_{X/S}^M} = 0 \quad (6.75a)$$

$$\frac{1}{Y_{X/S}^M} + \frac{k_d}{Y_{X/S}^M \cdot D} = \frac{1}{Y_{X/S}^{AP}} \quad (6.75b)$$

$$\frac{1}{Y_{X/S}^{AP}} = \frac{1}{Y_{X/S}^M} + \frac{m_s}{D} \quad (6.76)$$

where

$$m_s = \frac{k_d}{Y_{X/S}^M} \quad (6.77)$$

where m_s is the *maintenance coefficient* based on substrate S . $Y_{X/S}^{AP}$ is the apparent yield. When $Y_{X/S}$ is written, it should be interpreted as $Y_{X/S}^{AP}$. While $Y_{X/S}^M$ is a constant, $Y_{X/S}^{AP}$ varies with growth conditions if $k_d > 0$.

Values of $Y_{X/S}^M$ and m_s can be obtained from chemostat experiments by plotting $1/Y_{X/S}^{AP}$ against $1/D$. The slope is m_s and the intercept is $1/Y_{X/S}^M$ (see Fig. 6.19).

In the presence of endogenous metabolism ($\mu_{\text{net}} = \mu_m S / (K_s + S) - k_d$), we can also show that

$$S = \frac{K_s(D + k_d)}{\mu_m - D - k_d} \quad (6.78)$$

and

$$X = Y_{X/S}^M [S_0 - S] \cdot \frac{D}{D + k_d} \quad (6.79)$$

In addition to the effects of maintenance, it is often appropriate to consider the conversion of extracellular substrate into extracellular product. With an unstructured