Equation 6.60 can also be rewritten in terms of intrinsic concentrations. For simplicity we will use mass fractions (e.g., C_i/X). Note that

$$\frac{d(C_i/X)}{dt} = \left(\frac{1}{X}\frac{dC_i}{dt}\right) - \left(C_i/X\,\frac{dX/dt}{X}\right) \tag{6.61}$$

Recalling

$$\mu = \frac{1}{X}\frac{dX}{dt} \tag{6.2a}$$

we have

$$\frac{d(C_i/X)}{dt} = \left(\frac{1}{X}\frac{dC_i}{dt}\right) - \mu C_i/X \tag{6.62}$$

and, substituting eq. 6.60 for $(1/X)(dC_i/dt)$ after dividing eq. 6.60 by $V_R X$ and assuming V_R is a constant,

$$\frac{d(C_i/X)}{dt} = r_{fi} - \frac{\mu_{\text{net}} C_i}{X} \tag{6.63}$$

In eq. 6.63, the r_{fi} term must be in terms of intrinsic concentrations and the term $\mu_{\text{net}} C_i/X$ represents dilution by growth. These concepts are illustrated in Example 6.3.

The model indicated in Fig. 6.14 is that for a single cell. The single-cell model response can be directly related to culture response if all cells are assumed to behave identically. In this case each cell has the same division cycle. A population will have, at steady state, twice as many cells at "birth" as at division. The average concentrations in the culture will be at the geometric mean (a time equal to $\sqrt{2}$ multiplied by the division time) for each cell component. Used in this way, the model is a structured, nonsegregated model. If, however, a cellular population is divided into subpopulations, with each subpopulation represented by a separate single-cell model, then a population model containing a high level of structure, as well as aspects of segregation, can be built. Such a finite-representation technique has been used and is capable of making good a priori predictions of dynamic response in cultures (see Fig. 6.15). When used in this context, at least one random element in the cell cycle must be included to lead to realistic prediction of distributions. Some examples of such randomness include placement of the cell cross wall, timing of the initiation of chromosome synthesis or cross-wall synthesis, and distribution of plasmids at division.

With this section, the reader should have a good overview of the basic concepts in modeling and some simple tools to describe microbial growth. We need these tools to adequately discuss the cultivation of cells in continuous culture. Example 6.3 illustrates how a structured model might be developed.

Example 6.3.

Write the equations describing the following system. An organism consists of active biomass and a storage component. The storage component is made when the internal concentration of the carbon–energy sources is high and is degraded when it is low. Use the symbols A = active biomass, P = polymeric storage compound, S^* = external concentration of S, and S = internal concentration of S. Assume that S is the growth-rate-limiting nutrient.