

rium value after a local perturbation in its concentration. Mixing times are experimentally determinable by step addition of an electrolyte. The conductivity can be measured continuously at various locations distant from the injection site.

A production fermenter usually contains multiple impellers. An effective modeling approach is to divide the contents of the large tank into mixing compartments, where each compartment is perfectly mixed. As indicated in Fig. 10.9, a simple model is to consider that a separate compartment is associated with each impeller. In this problem, we let  $H$  be the overall mass transfer coefficient between compartments. The transient mass balances and experimental data can be used to estimate a value of  $H$ .

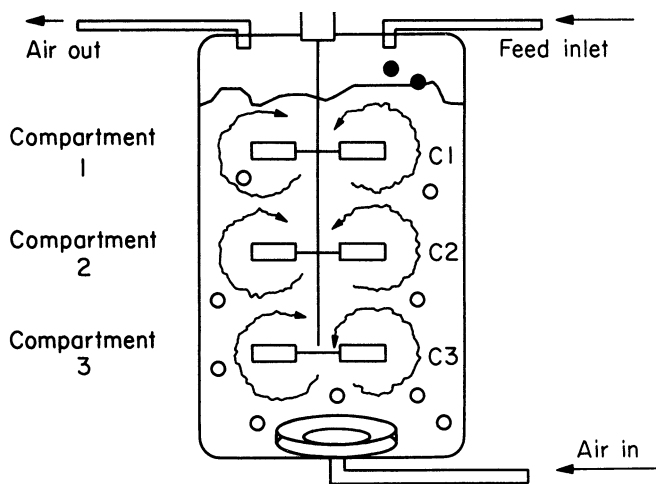
Consider the case where  $H$  has been determined to be  $0.43 \text{ s}^{-1}$  for a 10 l vessel and  $0.075 \text{ s}^{-1}$  for a 10,000 l vessel. With *E. coli* fermentations, glucose feed rates in a fed batch are adjusted to maintain a constant, relatively low concentration of glucose to prevent the formation of toxic metabolites (e.g., acetate) that would limit the ultimate cell concentration. Assume that the desired glucose concentration is 25 mg/l and that the Monod kinetics can be approximated as first order with a rate constant of about  $0.05 \text{ s}^{-1}$ . Assume the cell concentration changes slowly. Assume that the glucose supplemental feed is sufficiently concentrated that the total fluid volume in the reactor is constant. Also, assume that  $F$ , the mass addition rate of glucose per unit reactor volume, changes slowly in comparison to the characteristic mixing and reaction times. Compare the variation in glucose concentrations in the small and large tanks when an ideal probe (no error or lag in measurement) is used to maintain the set-point concentration at 25 mg/l in the middle compartment. Consider the response if the probe is placed in the top compartment instead of the middle compartment.

**Solution** Note that  $F$  and  $k_1$  are closely related. As the cells grow,  $k_1$  increases, which changes the demand for glucose ( $F$ ). Because  $F$  changes more slowly than the characteristic time constants for mixing, we assume a quasi-steady state.

Following Fig. 10.9, we write

$$\frac{dC_1}{dt} = 0 = H(C_2 - C_1) + F - k_1 C_1 \quad (\text{a})$$

$$\frac{dC_2}{dt} = 0 = H(C_1 - C_2) - H(C_2 - C_3) - k_1 C_2 \quad (\text{b})$$



**Figure 10.9.** Schematic of a simple mixing model for Example 10.4.  $C_i$  = concentration of a component in the  $i$ th mixing compartment. (With permission, from J. L. Jost, in S. L. Sandler and B. A. Finlayson, eds., *Chemical Engineering Education in a Changing Environment*, Engineering Foundation, New York, 1980, pp. 21–36.)