$$\mu_g = k\left(1 - \frac{X}{X_\infty}\right) \tag{6.54}$$

Thus,

$$\frac{dX}{dt} = kX\left(1 - \frac{X}{X_\infty}\right) \tag{6.55}$$

The integration of eq. 6.55 with the boundary condition $X(0) = X_0$ yields the logistic curve.

$$X = \frac{X_0 e^{kt}}{1 - \frac{X_0}{X_\infty}(1 - e^{kt})} \tag{6.56}$$

Equation 6.56 is represented by the growth curve in Fig. 6.12.

Equations of the form of 6.56 can also be generated by assuming that a toxin generated as a by-product of growth limits X_∞ (the carrying capacity). Example 6.2 illustrates the use of the logistic approach.

Example 6.2. Logistic Equation

Ethanol formation from glucose is accomplished in a batch culture of *Saccharomyces cerevisiae,* and the following data were obtained.

Time (h)	Glucose (S), g/L	Biomass (X), g/L	Ethanol (P), g/L
0	100	0.5	0.0
2	95	1.0	2.5
5	85	2.1	7.5
10	58	4.8	20.0
15	30	7.7	34.0
20	12	9.6	43.0
25	5	10.4	47.5
30	2	10.7	49.0

a. By fitting the biomass data to the logistic equation, determine the carrying-capacity coefficient k.

b. Determine yield coefficients $Y_{P/S}$ and $Y_{X/S}$.

Solution

a) Equation 6.55 can be rewritten as:

$$\frac{1}{X}\frac{dX}{dt} = k\left(1 - \frac{\overline{X}}{X_\infty}\right)$$

or

$$k = \frac{1}{\overline{X}}\frac{\Delta X}{\Delta t} \div \left(1 - \frac{\overline{X}}{X_\infty}\right)$$

where $\overline{X}$ is average biomass concentration during Δt, and X_∞ is about 10.8 g/L, since growth is almost complete at 30 h. Thus: