oxidation, the total required tank volume for aeration is reduced by 50% in comparison to the conventional system.

Using rate expressions for microbial growth and substrate utilization and material balances for biomass and substrate, we can determine the required volume of an activated-sludge tank for a certain degree of BOD removal. Since an activated-sludge tank contains a mixed culture of organisms, the actual kinetics of BOD removal are complicated. Usually, interactions among various species are not known. The following analysis is based on pure-culture kinetics, which is only an approximation. It is also assumed that the specific growth-rate expression is given by the Monod equation, with a death rate (or endogenous respiration rate) term

$$\mu_{\text{net}} = \frac{\mu_m S}{K_s + S} - k_d \tag{16.29}$$

Steady-state material balances for biomass and rate-limiting substrates in an activated-sludge tank are (Fig. 16.7)

$$\text{Biomass:} \qquad \left(\frac{\mu_m S}{K_s + S} - k_d\right) XV + \alpha F X_r = (1+\alpha) FX \tag{16.30}$$

$$\text{Substrate:} \qquad FS_0 + \alpha F S_r = \frac{1}{Y_{X/S}^{M}} \left(\frac{\mu_m S}{K_s + S}\right) XV + (1+\alpha) FS \tag{16.31}$$

These equations are very similar to the case of the chemostat with recycle that we discussed in Chapter 9. They differ in that we now include the endogenous respiration term.

Assuming no substrate utilization and cell growth in the settling tank (short residence times), material balances around the settling tank yield

$$\text{Biomass:} \qquad (1+\alpha) FX = (1-\gamma) FX_e + (\alpha + \gamma) FX_r \tag{16.32}$$

$$\text{Substrate:} \qquad (1+\alpha) FS = (1-\gamma) FS_e + (\alpha + \gamma) FS_r \tag{16.33}$$

where α is the ratio of sludge recycle flow rate to feed flow rate and γ is the ratio of excess sludge flow to feed flow rate.

Assuming that the substrate is not separated in the settling tank (that is, $S = S_e = S_r$), eq. 16.33 can be eliminated. By rearranging eq. 16.32, we can obtain

$$(1+\alpha) FX - \alpha F X_r = (1-\gamma) FX_e + \gamma F X_r \tag{16.34}$$

Substituting eqs. 16.34 and 16.29 into eq. 16.30 yields

$$\mu_{\text{net}} VX = (1-\gamma) FX_e + \gamma F X_r \tag{16.35}$$

Defining $\mu_{\text{net}} = 1/\theta_c$, where θ_c is the cells' (solids') residence time, we obtain

$$\theta_c = \frac{1}{\mu_{\text{net}}} = \frac{VX}{F(1-\gamma) X_e + \gamma F X_r} \tag{16.36}$$

which is used to calculate the cellular (solids') residence time in the sludge tank. The value of θ_c is controlled by operator choice of recycle flow rates. Hydraulic (liquid) residence time is