Solution

a. Note that

$$\sigma = \frac{\sigma t_{\max}}{t_{\max}} = \frac{10 \text{ min}}{80 \text{ min}} = 0.125$$

and from eq. 11.88 the yield where $t_1 = 0^{\dagger}$ is

$$Y_{\Delta t} = 0.95 = \tfrac{1}{2}\left[1 + \operatorname{erf}\left(\frac{t - t_{\max}}{\sqrt{2}\,\sigma t_{\max}}\right)\right]$$

$$= 0.95 = \tfrac{1}{2}\left[1 + \operatorname{erf}\left(\frac{t - 80}{\sqrt{2}\ 10 \text{ min}}\right)\right]$$

or $t = 96.4$ min.

b. With twice the flow $t_{\max}$ becomes 40 min. Since for Taylor dispersion $\sigma \propto v^{1/2}$, then

$$\sigma_2 = \sigma_1\left(\frac{v_2}{v_1}\right)^{1/2} = 0.125\left(\frac{40}{20}\right)^{1/2} = 0.177$$

For yield = 0.95 the time required is estimated as

$$0.95 = \tfrac{1}{2} + \operatorname{erf}\left[\frac{t - 40}{\sqrt{2} \cdot 0.177 \cdot 40}\right]$$

or $t = 51.6$ min.

Notice that in this second case (v = 40 cm/h), that the peak is broader and $t_{95\%}/t_{\max}$ = 96.4/80 = 1.205 for v = 20 cm/h and $t_{95\%}/t_{\max}$ = 51.6/40 = 1.29 for v = 40 cm/h.

11.4.10. Electrophoresis

Electrophoresis is used for the separation of charged biomolecules according to their size and charge in an electric field. In an electric field, the drag force on a charged particle is balanced by electrostatic forces when the particle is moving with a constant terminal velocity. A force balance on a charged particle moving with a terminal velocity in an electric field yields the following equation:

$$qE = 3\pi\mu D_P V_t \tag{11.95}$$

or

$$V_t = \frac{qE}{3\pi\mu D_P} \tag{11.96}$$

where q is the charge on the particle, E is electric field intensity, D_P is particle diameter, μ is the viscosity of the fluid, and V_t is the terminal velocity of the particle.

Depending on the pH of the medium, electrostatic charges on protein molecules will be different. When pH > pI, the protein will be negatively charged; and when pH < pI, the

†Note that erf $(-x) = -\text{erf } x$ and erf $(-5.9) = -1$.