



Figure 9.5. Solution of Example 9.2 for two-stage system, each with  $\theta = 7$  h.

$$\theta_2 = 7 \text{ h} = (P_2 - P_1) \frac{1}{dP_1/dt}$$

By trial and error, we find that at  $P_2 = 0.49$  g/l

$$\begin{aligned} \theta_2 &= (0.49 \text{ g/l} - 0.08 \text{ g/l})(17 \text{ h/g/l}) \\ &= 6.97 \text{ h} \end{aligned}$$

which corresponds reasonably closely to 7 h.

In this solution the reader should note that for the first stage, only solutions that exist for  $X_1$  greater than the value of  $X$  for which  $1/(dX/dt)$  is a minimum are practically obtainable. Washout occurs if  $\theta_1$  is too small.

We can compare the result to a single-stage system with the same total volume as the two-stage system (Fig. 9.6). Here the trial-and-error approach indicates for  $X_1 = 7.35$  g/l that

$$7.35 \text{ g/l} \cdot 1.9 \text{ h/g/l} = 13.97 \text{ h} \approx 14 \text{ h}$$

The value of  $P_1$  that corresponds to  $X_1 = 7.35$  g/l is 0.10 g/l. Thus, the use of the two-stage system in this case increased product concentration from 0.10 to 0.49 g/l.

An alternative graphical approach that eliminates the trial-and-error aspect of the first approach is shown in Fig. 9.7. Here eqs. 9.24a and 9.26a have been used.  $D_1 = 1/\theta_1 =$