Equation 16.43 can be integrated to yield a substrate concentration profile throughout the column. Usually, the substrate concentration in bulk liquid is very low, and the biological rate expression can be approximated to first order. That is,

$$-F\frac{dS_0}{dz} \cong \eta \frac{r_m}{K_s} LaAS_0 \tag{16.44}$$

Integration of eq. 16.44 from $z = 0$ to $z = H$ yields

$$\frac{S_0}{S_{0i}} = \exp\left[-\frac{\eta r_m LaAH}{FK_s}\right] \tag{16.45}$$

where S_{0i} is the inlet substrate concentration, and S_0 is concentration of substrate in the liquid phase at a distance z from the inlet. Experimental data of ln S_0/S_{0i} plotted versus z yield a straight line (in at least some cases), supporting the validity of this analysis.

The effectiveness factor can be calculated by using the following expressions:

$$\eta = 1 - \frac{\tanh\phi}{\phi}\left(\frac{\omega}{\tanh\omega} - 1\right), \qquad \text{for } \omega \le 1 \tag{16.46}$$

$$\eta = \frac{1}{\omega} - \frac{\tanh\phi}{\phi}\left(\frac{\omega}{\tanh\omega} - 1\right), \qquad \text{for } \omega \ge 1 \tag{16.47}$$

where

$$\omega = \frac{\phi\left(S_0/K_s\right)}{\sqrt{2}\left(1 + S_0/K_s\right)}\left[\frac{S_0}{K_s} - \ln\left(\frac{S_0}{K_s}\right)\right]^{-1/2} \tag{16.48}$$

and $\phi = L\sqrt{r_m/D_e K_s}$ and $r_m = \mu_m X/Y_{X/S}$. D_e is the effective diffusivity of the substrate within the biofilm (cm^2/s), and $Y_{X/S}$ is the yield coefficient (g cells/g substrate).

A similar, but more empirical design equation that has been used to determine waste-treatment parameters for biological filter is

$$\frac{S_0}{S_{0i}} = \exp\left[-Ka^{1+m}\left(\frac{A}{F}\right)^n\right] \tag{16.49}$$

where a is the specific surface area of the inert support material (cm^2/cm^3), A is the cross-sectional area of the bed, F is liquid flow rate, and K is the apparent rate coefficient. In practice K is used as an adjustable parameter in a curve-fitting procedure. Exponents m and n vary depending on system characteristics such as geometry, hydrodynamics, biological systems, and waste-water characteristics.

Rotating biological contactors (RBC) contain rotating disks that come into contact with waste water periodically as they rotate. The disks are made of polystyrene or polyethylene, and their diameters range from 2 to 4 m. Figure 16.9 shows rotating disks used in biological waste treatment. A biological film forms on the surface of the disks. As the disks come into contact with waste water, nutrients (organics and dissolved oxygen) diffuse through the biofilm and are utilized by a mixed culture of organisms within the